Designing a risk-adjusted CUSUM control chart based on DEA and NSGA-II approaches (a case study in healthcare: Cardiovascular patients)

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Abstract. Researchers have recently devoted a lot of attention to the development of control charts for monitoring healthcare systems. Accordingly, the purpose of this paper is to design a risk-adjusted cumulative sum (CUSUM) control chart to detect decreasing shifts. The proposed chart is used to monitor the survival times of patients who may be subject to an assignable cause such as human mistakes during a surgery. To this end, risk adjustment is performed to consider the impact of each patient's preoperative risks on survival times using survival analysis regression models. However, using the risk-adjusted CUSUM requires that the chart parameters are determined. Hence, a multi-objective economic-statistical model is proposed and a two-stage solution method including non-dominated sorting genetic algorithm (NSGA-II) and Data Envelopment Analysis (DEA) is implemented to solve the model and obtain the optimal design parameters. The performance of the proposed approach is also studied in a real cardiac surgery center. Finally, to confirm the effectiveness of the proposed multi-objective design, two comparisons with the bi-objective and pure economic designs are made. The results show that the performance of the risk-adjusted CUSUM obtained from the proposed model is better than the two other designs considering statistical and economic properties.

KEYWORDS: Control chart design; Risk adjustment; Survival analysis regression models; Non-dominated sorting genetic algorithm (NSGA-II); Data Envelopment Analysis (DEA).

1. Introduction

Monitoring hospital and clinical processes has become an important part of modern healthcare systems which improves the quality of surgical and other treatment services. The most important tools to monitor the effectiveness in the field of healthcare are control charts which are commonly used to detect shifts in process parameters [1]. Among them, cumulative sum (CUSUM) control charts are widely applied for statistical monitoring and detecting small shifts in healthcare systems [2]. To this end, four parameters should be determined while making the use of this chart to effectively monitor processes. This includes the sample size, sampling interval, control limits and the parameter designed for optimal detection of a specific shift. Thus, designing a control chart is defined as the selection of these parameters.

There are several economic and statistical aspects for designing a control chart. The first economic design model was suggested by Duncan [3] to determine the control chart parameters in the presence of an assignable cause with minimum average cost. Duncan's cost model includes the expenses for sampling, out-of-control alarms, detection and repair of assignable causes and receiving defective product. The other popular model was developed by Lorenzen and Vance [4] which is more flexible compared with Duncan's model. It allows the process to pause or continue throughout the search for detecting and eliminating assignable causes. Since then, a great amount of research was carried out on economic design of control charts. Jiao and Helo [5] suggested an economic design for CUSUM chart based on Taguchi's loss function. Lee [6] studied the design of CUSUM chart for process monitoring with correlated samples. Celano et al. [7] carried out the economic design of CUSUM *t* chart to monitor short production runs. Fallahnezhad and Golbafian [8] introduced a mathematical model based on average number of inspected items for economic design of cumulative count of confronting control charts.

However, the economic design of control charts may result in weak statistical features and the managers hesitate to take decisions for using such control charts. Statistical properties include the probability of type I and type II errors or the in-control or out-of-control average run length (ARL). To improve the statistical characteristics, Saniga [9] added the probability of type I and type II errors as constraints to Duncan's economic model [3] and presented economic-statistical design. Thus, economic-statistical design helps to gain statistical and economic features simultaneously. Asadzadeh and Khoshalhan [10] proposed a multi-objective model for economic-statistical design of $\bar{x}$ control charts which, in addition to minimizing an objective function of average expected costs, seeks to maximize the objective functions of test power and in-control average run length (ARL). Niaki et al. [11] compared the economic and economic-statistical design of MEWMA control chart and suggested a particle swarm optimization method to solve it. Ketabi and Moghadam [12] used a full adaptive method for economic and economic-statistical designs of attribute *np* control charts in which the cost model was developed by Markov Chain approach.

As mentioned earlier, monitoring hospital outputs and clinical processes are considered as an essential part of healthcare systems. Patients, who are treated in these systems, create a heterogeneous population due to their different preoperative features such as age, gender, diabetes, blood pressure, etc. As the result, each patient's mortality after surgery depends not only on surgeon's skills but also on unique risk factors before the surgery. Therefore, monitoring and evaluating surgery performance should be adjusted for the effect of patients' risk factors. It should be noted that it is quite impossible to assess the quality of surgery properly without performing risk-adjustment procedure [2]. Axelrod et al. [13] evaluated the liver and kidney transplantation performance using risk-adjusted CUSUM (RACUSUM) which detects clinically significant changes in center performance more rapidly. Keefe et al. [14] used a new Bernoulli RACUSUM chart to monitor outputs from cardiac surgery that incorporates outcome information as soon as it is available. Begun et al. [15] developed a monitoring method based on the RACUSUM chart for early detection of changes in the revision rates after hip replacement. Kim et al. [16] utilized a RACUSUM chart, which monitors the surgical failure, to evaluate the learning curve and establish an appropriate training program for laparoscopic pancreaticoduodenectomy.

The careful investigation of literature reveals that the economic-statistical design of risk-adjusted control charts in healthcare systems has never been addressed. Hence, the present study intends to develop a multi-objective design model for RACUSUM control chart with economic and statistical considerations. In the proposed model, the design parameters of the RACUSUM chart are determined to minimize the average expected cost as well as the out-of-control average run length (ARL_1), while the in-control average run length (ARL_0) is maximized. There are several methods to solve the suggested model from which non-dominated sorting genetic algorithm II (NSGA-II) and Data Envelopment Analysis (DEA) are the two powerful methods for optimization. NSGA-II is one of the most popular multi-objective evolutionary algorithms due to its capacity to improve the quality of solutions [17]. Therefore, this algorithm is used to determine non-dominated solutions for the suggested model. Then, DEA is applied to find the most efficient solutions. DEA is used to compare and evaluate the relative efficiency of similar units with the same responsibilities [18]. It is widely used to solve multi-criteria problems in economic and management, including the evaluation of hospital services performance and supply chain of a shipping company [19, 20].

The remainder of the paper is as follows. In section 2, the construction and development of the RACUSUM control chart is mainly discussed. Section 3 describes the cost function used in the economic design. Moreover, the multi-objective model for the economic-statistical design of the RACUSUM control chart is developed. In Section 4, NSGA-II and DEA are studied as the solving methods to optimize the model. Section 5 provides a real case study in the cardiac surgery center in Iran to show the application of the proposed procedure in healthcare systems. Section 6 addresses two performance comparisons to validate the effectiveness of the proposed multi-objective economic-statistical design. Finally, concluding remarks are given in the last section.

2. Monitoring procedure based on risk-adjusted CUSUM control chart

The challenge of community heterogeneity is apparent in healthcare systems monitoring due to the unique preoperative features of patients entering the surgical process. Preoperative features of patients, known as risk factors, include age, gender, diabetes, blood pressure, etc. that affect the monitoring of surgery performance [21]. For instance, patients with more severe risks may experience worse outputs even if they receive the best care. As the result, a patient with a low surgical risk is much more likely to survive than a patient with a high surgical risk. Therefore, the surgery performance for each patient depends not only on the surgery quality, but also on the unique health record of the patient prior to the operation. Consequently, in order to have an effective monitoring plan, it is necessary to identify the relationship between the mortality rate of a particular disease and its risk factors or to have the monitoring procedure risk-adjusted. Thus, the purpose of risk-adjusted monitoring scheme is to rapidly detect shifts in patient mortality rates taking into account their risk factors. Considering discrete and continuous outputs, various studies were devoted to developing risk-adjusted control charts. Among the existing methods, a monitoring scheme that is based on patient's survival time is more sensitive to the detection of shifts in the mortality rate compared with other procedures. It should be noted that the survival time is a variable with a reliability nature with some specific features such as being censored. The censoring mechanism is used to reduce time and cost in the surgical process which sometimes causes the survival times not to be recorded accurately. Thus, the reliability nature of survival times in line with the heterogeneities among patients leads us to use survival analysis regression models. Accelerated failure time (AFT) and proportional hazards (PH) are the two major regression models in survival analysis which are widely employed to justify the heterogeneity in patients' population and to establish the relationship between survival times and influential covariates (risk factors). The AFT model presents one or more parameters as a function of covariates [22]. The present study makes use of the AFT model for model building and risk adjustment. Accordingly, the parameters of the baseline distribution could be expressed as $\psi_i = g(\beta, X_i)$, where ψ_i is a vector of distribution parameters, and β and X_i are vectors of regression parameters and covariates respectively which reflect the risk factors for the i^{th} patient. Moreover, survival data are often modeled using a member of location-scale and log-location-scale distributions. Weibull is one of the most practical distributions which is known to be helpful in various conditions [23]. As the result, based on the analysis obtained from the real case study, it is assumed that the output variable (survival times) follows Weibull distribution. The probability density and the survival functions of this distribution, denoted by f and s respectively, are as follows:

$$f(t)=\frac{\kappa}{\eta}(\frac{t}{\eta})^{\kappa-1}\exp(-(\frac{t}{\eta})^{\kappa})$$
$$s(t)=\exp(-(\frac{t}{\eta})^{\kappa}) \tag{1}$$

where, t is the recorded value for patients' survival time. Also, $\kappa > 0$ and $\eta > 0$ are the shape and scale parameters respectively. Based on the AFT model, if one relates the scale parameter to the unique covariate in the real case study, the probability density and survival functions are restated as:

$$f(t\,|\,x)=\kappa\exp(-(\beta_0+\beta_1 x_i))(t\exp(-(\beta_0+\beta_1 x_i)))^{\kappa-1}\exp(-(t\exp(-(\beta_0+\beta_1 x_i)))^{\kappa})$$
$$s(t\,|\,x)=\exp(-(t\exp(-(\beta_0+\beta_1 x_i)))^{\kappa}) \tag{2}$$

where β_0 and β_1 are parameters of regression model. It should be noted that in the previous equation, the fixed part $\exp(\beta_0)$ has a similar interpretation as η. As the result, the Weibull distribution has the shape parameter κ and the scale parameter $\eta\exp(\beta_1 x_i)$. Hence, the vector of in-control parameters is defined as $\psi_{i0}=(\kappa,\eta\exp(\beta_1 x_i))$. It is assumed that $\psi_{i1}=(\kappa,\upsilon\eta\exp(\beta_1 x_i))$ represents the out-of-control condition where υ is a predetermined shift that the CUSUM chart is designed for its optimal detection.

Having defined the relationship between the survival time and preoperative risk factor using the AFT model, it seems essential to design and develop the RACUSUM control chart to monitor survival times. Given the sensitive nature of the healthcare systems with high and irretrievable cost of mistakes with regard to patients' life, there is an attempt to detect decreasing shifts in the mean survival times [24]. As a result, a one-way control chart is proposed and employed for this purpose. The statistic for the risk-adjusted CUSUM chart is calculated as follows:

$$Q_i=\min(0,Q_{i-1}-w_i),\ \ i=1,2,...$$
$$Q_0=0 \tag{3}$$

in which w_i is the CUSUM score and is computed in the following manner:

$$w_i=\log\{\frac{L(t_i\,|\,\psi_{i1})}{L(t_i\,|\,\psi_{i0})}\} \tag{4}$$

where L represents the likelihood function. As mentioned earlier, ψ_{i0} and ψ_{i1} are the in-control and out-of-control nominal values for parameters related to the output variable or survival times.

So, it is apparent that the proposed CUSUM chart is designed for the rapid detection of shift from ψ_{i0} to ψ_{i1}. Since a one-way CUSUM chart is used in the present study to detect decreasing shifts in survival time mean, a lower control limit should be set. In this regard, LCL is the lower control limit and is selected in the way that a desirable in-control ARL is reached. However, to develop the RACUSUM chart, the similar procedure discussed in Sego et al. [21] is implemented to construct the likelihood function in the chart scores. This procedure considers the right-censored mechanism and the patients' survival time is only observed until a pre-determined time or a fixed censored time denoted by c. Doing so, the likelihood function is obtained as follows:

$$L(z_i \mid \psi_i, \delta_i) = [f(z_i, \psi_i)]^{\delta_i}[s(z_i, \psi_i)]^{1-\delta_i} \tag{5}$$

in which

$$\begin{aligned} &z_i = \min(t_i, c) \\ &\delta_i = \begin{cases} 1 & if \quad t_i \le c \\ 0 & if \quad t_i > c \end{cases} \end{aligned} \tag{6}$$

Substituting the AFT-based probability density and survival functions in equation (5), the likelihood function for the RACUSUM chart is obtained. Subsequently, to optimally detect a shift of size υ in the survival time mean, the log-likelihood score of the Weibull RACUSUM control chart is given by the following equation:

$$\begin{aligned} w_i &= \log \frac{[\frac{\kappa \exp(-\beta_1 x_i)}{\upsilon\eta}(\frac{z_i \exp(-\beta_1 x_i)}{\upsilon\eta})^{k-1} \exp(-(\frac{z_i \exp(-\beta_1 x_i)}{\upsilon\eta})^{\kappa})]^{\delta_i}[\exp(-(\frac{z_i \exp(-\beta_1 x_i)}{\upsilon\eta})^{\kappa})]^{1-\delta_i}}{[\frac{\kappa \exp(-\beta_1 x_i)}{\eta}(\frac{z_i \exp(-\beta_1 x_i)}{\eta})^{k-1} \exp(-(\frac{z_i \exp(-\beta_1 x_i)}{\eta})^{\kappa})]^{\delta_i}[\exp(-(\frac{z_i \exp(-\beta_1 x_i)}{\eta})^{\kappa})]^{1-\delta_i}} = \\ &[(1-\upsilon^{-\kappa})(\frac{z_i \exp(-\beta_1 x_i)}{\eta})^{\kappa}] - \delta_i \kappa \log \upsilon \end{aligned} \tag{7}$$

Having obtained the RACUSUM scores, the calculation of the control chart statistics is straightforward using equation (3). As soon as the updated statistic is less than LCL, the RACUSUM control charts triggers a signal.

3. Economic and economic-statistical designs of the risk-adjusted CUSUM control chart

The economic design is defined as determining the parameters of a RACUSUM control chart through minimizing a proper cost function. These parameters are n (sample size), h (sampling interval), LCL (lower control limit) and υ (coefficient used in the CUSUM for optimal detection of a shift) as discussed earlier. In the present study, the Lorenzen and Vance [4] cost function is employed to determine the optimal values of RACUSUM chart parameters. The cost function could be divided into three major parts. The first part includes the sampling costs. The second part shows the costs imposed on hospitals (healthcare systems) during the out-of-control surgery conditions, and finally, the last part introduces the cost of detecting and implementing corrective actions for an assignable cause. Moreover, it is assumed that process begins from an in-control condition till an assignable cause occurs. As the assignable cause happens, the process changes to out-of-control condition and remains in that situation until it is found and repaired; then, a new cycle begins. The expected cost in unit time is calculated by dividing the total expected cost during a cycle by the expected length of a cycle. Also, the expected length in a cycle includes four parts: (1) The time that the process remains in-control. It is usually assumed that this time is an exponential random variable with mean $\frac{1}{\lambda}$; (2) The time when the process is out-of-control; (3) The time to detect and identify the assignable cause, and (4) the time for implementing corrective action to repair the assignable cause. Therefore, taking all the four parts into account, the expected cycle length equals to:

$$E_T = \frac{1}{\lambda} + [(h.ARL_1) - \tau] + TF + TD \tag{8}$$

Subsequently, the total expected cost of a cycle equals:

$$E_C = A.\frac{\frac{1}{\lambda} + [(h.ARL_1) - \tau] + \gamma_1.TF + \gamma_2.TD}{h} + CO.ARL_1 + (CF + CD) \tag{9}$$

where the parameters of the above equations are summarized as follows:

- ARL_0 : In-control average run length
- ARL_1 : Out-of-control average run length
- τ : Expected time of occurrence of the assignable cause between the j^{th} and $j+1^{\text{st}}$ samples calculated by the below equation:

$$\frac{1-(1+\lambda.h).e^{-\lambda.h}}{\lambda.(1-e^{-\lambda.h})} \tag{10}$$

- TF : The average time for search and identification of the assignable cause
- TD : The average time to perform the corrective actions

- A: Sampling costs for each patient
- CO: Cost imposed on hospital due to out-of-control surgery for each patient because of the occurrence of the assignable cause
- CF: Cost of the search and identification of the assignable cause
- CD: Cost of corrective actions to repair the assignable cause
- γ_1: 1 if the process does not stop during the search and investigation for the assignable cause and 0 otherwise
- γ_2: 1 if the process does not stop during the repair and elimination of the assignable cause and 0 otherwise

Finally, the expected cost in unit time imposed on the process is as follows:

$$E_A = \frac{E_C}{E_T} \tag{11}$$

Therefore, the economic design of the RACUSUM chart includes determining the optimal parameters which minimize E_A. Following Lorenzen and Vance [4], it is practical and helpful to model the costs of designing control charts in accordance with the average run length (ARL). It is noteworthy that the present study makes use of simulation method to calculate the ARLs values.

Next, the main concentration is given to the multi-objective design of the RACUSUM control chart which is believed to be more beneficial in real practice. The major disadvantage of designing a control chart from economic point of view is the lack of statistical properties, called the probability of type I and II errors. Therefore, to overcome the statistical weakness of pure economic design of RACUSUM control chart, a multi-objective decision model (MODM) is proposed to consider both the statistical and economic criteria simultaneously. In general, MODM makes use of mathematical programming methods to solve optimization problems with the aim of satisfying several conflicting objective functions at the same time [25]. The multi-objective model of the RACUSUM control chart consists of one economic and two statistical objectives followed by a set of constraints as below:

$$\begin{aligned}
&Min \quad E_A(D) \\
&Max \quad ARL_0(D) \\
&Max \quad 1/ARL_1(D) \\
&s.t. \\
&E_A \le E_A^U \\
&ARL_0 \ge ARL_0^L \\
&ARL_1 \le ARL_1^U
\end{aligned} \tag{12}$$

where E_A^U is the desired upper bound on the expected cost. ARL_0^L and ARL_1^U are also the pre-determined values showing the lower and upper bounds for ARL_0 and ARL_1 respectively. It is remarkable that the MODM model intends to minimize the expected cost for each time unit (E_A), Maximize the in-control ARL (ARL_0) to decrease the false alarm rate, and Maximize the inverse of out-of-control ARL ($1/ARL_1$) to detect out-of-control conditions in a timely manner. Furthermore, $D=(n,h,LCL,\upsilon)$ is a possible combination of design parameters which needs to be optimally determined. Therefore, selecting a combination of design parameters for the RACUSUM control chart with optimal objective values is the main goal of the proposed MODM model. Next section elaborates on the solution algorithm based on the integration of Non-dominated sorting genetic algorithm (NSGA-II) and data envelopment analysis (DEA) for optimizing the multi-objective model introduced in equation (12).

4. Solution approach

To solve the proposed multi-objective economic-statistical model, several algorithms can be proposed. However, optimizing multiple objectives simultaneously creates Pareto solutions. Non-dominated sorting genetic algorithm (NSGA-II), introduced by Deb et al. [26], is one of the most popular multi-objective evolution algorithms for solving a variety of problems which can be used as an efficient method to identify the optimal Pareto set. Several studies dealing with the control chart design, such as the one described in Safaei et al. [17], have implemented NSGA-II to create the optimal Pareto set. Similarly, in this article, NSGA-II is employed to optimally solve the model introduced in equation (12). However, there is a challenge while using this algorithm because it often provides many solutions and it is quite difficult to choose the most efficient solution from the optimal Pareto set. To overcome this problem, data envelopment analysis is introduced to rank Pareto solutions and select the most efficient ones. Therefore, in this paper, a two-step solution method based on NSGA-II and DEA is devised to determine the optimal solutions of the multi-objective economic-statistical model of the RACUSUM control chart. To this end, NSGA-II is shortly summarized, and then the DEA method is introduced.

4.1. Non-dominated sorting genetic algorithm (NSGA-II)

NSGA-II makes use of an explicit diversity-preserving mechanism to find Pareto solutions for multi-objective programming problems. In this algorithm, instead of using the offspring populations solely, offspring and parent populations are combined to sort a set of non-dominant items. The offspring population is then created from parent population applying tournament selection, crossover and mutation operators. The tournament selection operator assumes that each solution i has two attributes in the population, namely a non-dominant rank (r_i) and crowding distance (d_i). Thus, it can be mentioned that solution i wins the competition against solution j if $r_i < r_j$ or $r_i = r_j$, and $d_i > d_j$. This approach ensures better selection from a set of non-dominated solutions [25]. The goal of crossover operator is to exchange information between two parent chromosomes to produce two new offspring for the next population. According to the literature, there are various types of crossover such as one-point, two-point, uniform, and round. In this study, round crossover operator is applied to produce new chromosomes [27]. This operator is expressed by:

$$\begin{aligned} Ch_1 &= round(Par_1 \times Alpha + Par_2 \times (1 - Alpha)) \\ Ch_2 &= round(Par_2 \times Alpha + Par_1 \times (1 - Alpha)) \end{aligned} \tag{13}$$

where Par_1 and Par_2 are the selected parents, respectively. *Alpha* varies between 0 and 1, and it has the same dimension as the chromosome matrix. Besides, Ch_1 and Ch_2 are the resulting children. The mutation operator is applied soon after the crossover operator. This operator generates offspring by randomly changing one or several genes in a chromosome. Offspring may thus possess different characteristics from their parents. Mutation prevents local searches of the search space and increases the probability of finding global optimum [25].
To find the Pareto optimal set of multi-objective economic-statistical design model, NSGA-II is defined as:

1. Randomly, generate initial population of size *n*-pop (the number of chromosomes).
2. Compute ARL_0, $1/ARL_1$ and E_A for each chromosome.
3. Rank the initial population using non-dominated criteria.
4. Compute crowding distance for the initial population.
5. Employ the crossover and mutation operators to generate offspring population of size *n*-pop.
6. Evaluate objectives and constraints for the mentioned offspring population.
7. Combine the two (parent and offspring) populations, rank them and compute crowding distance.
8. Select a new population of size *n*-pop from the best individuals based on the computed rank and crowding distance.
9. Go to step 3 and repeat until the termination criterion (number of generations) has been reached.

In this study, (n, h, LCL, υ) are the parameters of the RACUSUM control chart which are actually the decision variables of model (12). Figure 1 shows an example of a chromosome consisting of four genes as design parameters.

Insert Figure 1 about here

It should be noted that the design parameters (decision variables) will be reduced to (LCL, υ) due to the described situation in the real case study.

4.2. Data Envelopment Analysis (DEA)

DEA is a mathematical programming-based approach which measures the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. In this study, to rank the Pareto solutions obtained from NSGA-II, DEA methods are used. For this purpose, non-dominated solutions or design parameters of RACUSUM control chart are considered as DMUs. Also, the two statistical objectives, namely ARL_0 and $1/ARL_1$, are determined as outputs, and the cost objective (E_A) is the only input of the model. Then, the efficiency of each DMU is calculated using additive model. Finally, the most efficient DMU is detected through cross-efficiency evaluation technique. These methods are defined briefly in the following subsections.

4.2.1. Additive model

The additive model is one of the most important models to determine the efficiency in DEA which is the basis of definition for many other models [25]. One of the main reasons for the importance of this model is that it computes the efficiency completely since it directly tempts to minimize slack variables. However, in other models, such as the CCR and BCC, the detection of slack variables in efficiency is generally performed using another model similar to the additive model at a second stage of efficiency measurement. Therefore, one of the advantages of the additive model is that it does not require a two-step approach and the efficiency evaluation of a unit is done by solving a single model. Suppose that there are *m* DMUs, each with *a* inputs and *b* outputs. The values of inputs and outputs for DMU$_i$ ($i = 1, 2, ..., m$) are denoted by p_{ji} ($j = 1, 2, ..., a$) and q_{ri} ($r = 1, 2, ..., b$) respectively. The efficiency of a DMU is specified by the additive model expressed as a mathematical programming (14). The mathematical formula for the DMU$_i$ is as follows:

$$
\begin{aligned}
& Max \quad E_i(D) = \sum_{r=1}^{b} u_r q_{ri}(D) - \sum_{j=1}^{a} e_j p_{ji}(D) - T \\
& s.t. \\
& \sum_{r=1}^{b} u_r q_{ri}(D) - \sum_{j=1}^{a} e_j p_{ji}(D) - T \le 0, \qquad for \quad other \quad design \ D \\
& u_r, e_j \ge 1, \\
& T \quad is \quad free.
\end{aligned}
\tag{14}
$$

where e_j and u_r are the input and output weights respectively, and T represents the returns to scale. Model (14) should be formulated for each DMU or a combination of design parameters in order to reach a set of weights for maximizing the efficiency of a given DMU. If $E_i^* = 1$, DMU$_i$ is called efficient, while for the case of $E_i^* < 1$, DMU$_i$ is not efficient.

4.2.2. Cross-efficiency evaluation

Cross-efficiency evaluation was developed as an extension of DEA to rank efficient DMUs and determine the most efficient one. In a cross-efficiency evaluation, the performance of each efficient DMU is measured according to its optimal weights and the optimal weights of other efficient DMUs [18]. Assuming that the optimal weights of the model (14) for DMU$_d$, is $(e_{jd}^*, u_{rd}^*, T_d^*)$, the efficiency of DMU$_i$ $(i = 1, 2, .., m)$ considering the DMU$_d$ weights in a peer-evaluated process is calculated as follows:

$$E_{di} = \frac{\sum_{r=1}^{b} u_{rd}^* q_{ri}}{\sum_{j=1}^{a} e_{jd}^* p_{ji} - T_d^*} \tag{15}$$

The mean of all E_{di} is called cross-efficiency and the DMU with the highest cross-efficiency has the best rank. The calculation of mean is straightforward as follows:

$$\bar{E}_i = \frac{\sum_{d=1}^{m} E_{di}}{m}, \ \ i =,1, 2, ..., m \tag{16}$$

Finally, to clarify the application of the RACUSUM control chart in a surgical center using the proposed multi-objective design, Figure 2 is provided to illustrate the summarized steps of the approach.

Insert Figure 2 about here

5. The case study in the cardiac surgery center

Cardiovascular disease is the major cause of death around the world and many people die every year because of cardiac diseases. Cardiac surgery is one of the most common surgeries among adults, and given that a person’s life depends mainly on his/her heart performance, the sensitivity of this surgery is quite high so that monitoring the patient's survival time after the surgery seems essential. Therefore, the application of the proposed approach is investigated in Imam Ali cardiac surgery center located in the west of Iran.

For this purpose, a special type of operation called Coronary Artery Bypass Grafting (CABG) surgery was selected, and data were collected on 100 patients including surgery date, surgeon's name, surgery procedure, survival time. Note that the Parsonnet score is used to determine the preoperative risks for each patient as the only covariate affecting the survival time in the cardiac surgery process [28]. The Parsonnet score is computed based on the sum of various scores given in Table 1.

Insert Table 1 about here

Once the Parsonnet scores are calculated for each patient, its impact on the survival time should be moderated by the AFT model. Due to the hospital regulations, the survival times of patients who survived during the study were censored at 21 days. To begin with, the data collected from 100 patients were used to find appropriate distribution and estimate the value of in-control parameters. The results revealed that the Parsonnet score data follow gamma distribution with a scale parameter of 5.117 and a shape parameter of approximately 4.208. Then, the maximum likelihood estimation (MLE) was used to estimate the values of in-control parameters associated with the AFT Weibull model. Doing so, these values were estimated to be $\eta = 183744.22$, $\kappa = 1.2066$ and $\beta_1 = -0.2144$ respectively. Consequently, using equations (2) and (7), the probability density and survival functions of AFT Weibull model in line with the RACUSUM scores can be calculated.

However, the most important part is the deployment of the RACUSUM control chart in CABG process; thus, it is necessary to determine the four design parameters of the proposed chart. As noted earlier in this study, all patients are monitored individually and sequentially because of the high sensitivity of healthcare systems. Hence, the value of n is constant and is equal to 1. Furthermore, since patients undergo surgery every four hours at Imam Ali Hospital, the h value is also constant which is equal to 4. On the other hand, the two other parameters of the RACUSUM chart, namely the coefficient for optimal detection (υ) and the lower bound of the control chart (LCL), need to be determined in the process of cardiac surgery, so that both statistical and economic properties are satisfied. As a result, our proposed MODM model is used to select a combination of (LCL, υ) parameters to achieve the desired objectives of minimum expected cost and maximum statistical properties.

In the CABG process, an assignable cause due to the human-resource mistake occurs at $\lambda = 0.01875h$ rate, reducing the patient's survival time by 95%. The sampling cost is 840000 Rials (Iran currency) for each patient because of filling out the Parsonnet questionnaire, carrying out check-ups, and taking actions to obtain Parsonnet score records. In addition, when an assignable cause occurs, the CABG procedure goes to out-of-control condition. In this case, the cost of check-ups, echocardiography, angiography, surgery, consultant, operation room, consumable products, anesthesia, consumable drugs, ICU beds, nursing services and public beds, is approximately 21623500 Rials which is imposed on the hospital. The details of the costs imposed on hospital in the out-of-control condition are reported in Table 2.

Insert Table 2 about here

In addition, when the CABG process is out-of-control, a specialized committee called morbidity or mortality is formed to investigate the root cause of the problem. On average, 4 hours are spent on these actions, and after the root cause of the assignable cause is identified, it takes 2 hours on average to implement corrective action. An average cost of 16000000 Rials is estimated to find the human-oriented assignable cause and an average cost of 8000000 Rials is spent for corrective action. Also, the process continues to work while the identification and repair of the assignable cause is being done. Then, according to the estimated parameters with regard to the Parsonnet score and the AFT Weibull model, simulation studies are performed to calculate the statistical indices. It should be noted that in order to minimize the simulation error, the procedure is repeated 10000 times. For each combination of design parameters, the ARL_0 values are calculated considering no shift in the data, while the ARL_1 values are recorded when there exists a 95% reduction in survival time. Moreover, to avoid the high incidence rate of false alarms, to achieve acceptable probability of detection power, and to consider the budget constraints using the RACUSUM control chart in the CABG surgery process, a lower bound of 20, an upper bound of 5, and an upper bound of 1900000 Rials have been considered for ARL_0, ARL_1, and E_A respectively. It is noteworthy that the following limits are applied to the design parameters: $0.01 \leq \upsilon \leq 0.2$ and $-1.5 \leq LCL \leq -0.01$. Therefore, the expected cost per time unit associated with the application of the proposed RACUSUM chart to Imam Ali cardiac surgery center is obtained via Equation (11).

Finally, in order to optimize the multi-objective economic-statistical model of the RACUSUM control chart and to achieve the best possible combination of design parameters, a two-step solution approach is implemented. It should be noted that all calculations related to the solution approach were facilitated under the coded programs in MATLAB (version R2016a) environment. Initially, due to the features of the proposed MODM model and the proper performance of NSGA-II, the set of non-dominated solutions were identified using the described algorithm. In other words, the optimal Pareto solutions were determined by implementing NSGA-II with 1000 replications, *n*-pop of size 100, the crossover operator with probability of 0.2 and the mutation operator with probability of 0.9. The results are reported in Table 3. Also, the Pareto front for the three objective E_A, ARL_0 and $1/ARL_1$ is also shown in Figure 3.

Insert Table 3 about here

Insert Figure 3 about here

After the non-dominated solutions were identified, DEA methods were used to prioritize and select the most effective solution for establishing the RACUSUM control chart at the surgery center. In DEA, any combination of design parameters, namely (LCL, υ), is considered a DMU. Since DEA methods select the most efficient DMU with the minimum input value and maximum output value, the cost function was considered as the only input while the statistical properties were the two outputs. The additive model was then used to identify the efficient DMUs. Based on the results from additive model, 10 DMUs were selected as the combination of efficient design parameters. Finally, these 10 DMUs were considered as input data for the cross-efficiency evaluation technique, and the most efficient DMU was detected. The results are shown in Table 4.

Insert Table 4 about here

From Table 4, it is remarkable that the cross-efficiency evaluation technique offers $\upsilon = 0.08$ and $LCL = -1.38$ as the most efficient combination of design parameters for the RACUSUM control chart with the best economic and statistical properties ($E_A = 1555232.69$ Rials, $ARL_0 = 55.118$, and $1/ARL_1 = 0.306$).

6. Performance comparison

In this section, the performance of the proposed multi-objective economic-statistical design model is compared with the bi-objective statistical design and the pure economic design model to investigate its effectiveness. The bi-objective model is similar to the multi-objective model, presented by equation (12) in Section 3, with this difference that the expected cost for each time unit (E_A) is omitted. Therefore, this model can be introduced with two statistical objectives, ARL_0 and $1/ARL_1$, which is rewritten with a set of constraints as follows:

$$\begin{aligned}
&Max \quad ARL_0(D) \\
&Max \quad 1/ARL_1(D) \\
&s.t. \\
&E_A \le E_A^U \\
&ARL_0 \ge ARL_0^L \\
&ARL_1 \le ARL_1^U
\end{aligned} \tag{17}$$

To compare this model with the multi-objective one, its application to the cardiac surgery center has been studied. According to the characteristics of the bi-objective model, NSGA-II was applied for realizing non-dominated solutions and the Pareto front was determined using this algorithm. The Pareto front for ARL_0 and $1/ARL_1$ of the bi-objective design is shown in Figure 4.

Insert Figure 4 about here

After the non-dominated solutions were recognized, $\upsilon = 0.02$ and $LCL = -0.69$ were selected as the best combination of bi-objective design parameters for the RACUSUM control chart. The optimal values corresponding to multi-objective and bi-objective designs are given in Table 5.

Insert Table 5 about here

Table 5 indicates that the bi-objective design model increases the ARL_0 effectively, while the multi-objective design has better performance with regard to ARL_1 and E_A. The bi-objective design managed to increase the ARL_0 by 69.3%; However, the ARL_1 and the E_A values have been negatively raised 15.2% and 7.1% respectively. Thus, the results confirm that the multi-objective design outperforms the bi-objective design in terms of detection power and the expected cost.
Finally, the performance of the multi-objective design is compared with the pure economic design model. Table 6 depicts the optimal parameters of multi-objective design and pure economic design. It is apparent that as the E_A increases by 1.3% in the multi-objective design compared to the economic design, the ARL_0 of multi-objective design increases 96% as well. However, no significant difference is observed for ARL_1 in both designs. Therefore, the comparisons revealed that the ARL_0 increases dramatically with a slight increase in the cost. Consequently, the multi-objective design could be effectively applied with a significant improvement in statistical properties of the RACUSUM control chart.

Insert Table 6 about here

7. Conclusion

Considering the importance of healthcare systems, this paper proposed a multi-objective economic-statistical model for the design of the risk-adjusted CUSUM (RACUSUM) control chart to effectively monitor patients' lifetime. First, the RACUSUM chart was devised based on a class of survival analysis regression models called the accelerated failure time (AFT) model taking the preoperative risks of each patient into account. It was assumed that the cardiac surgery process in a hospital is influenced by an assignable cause resulting from the human mistakes which causes a decrease in the survival time of patients. Thus, a multi-objective economic-statistical design model was addressed to determine the parameters of RACUSUM chart, so that both the economic and statistical properties could be met simultaneously. Due to the constant sample size and sampling interval, while implementing the RACUSUM chart in healthcare system, the control chart design parameters were considered to be the lower control limit and the coefficient for optimal shift detection denoted by LCL and υ respectively. In order to determine the optimal values of these parameters, a two-stage solution algorithm was employed. The NSGA-II was used in order to obtain the optimal Pareto set taken from design parameters, and the DEA methods were implemented to rank the solutions and choose the most efficient one. It should be noted that each combination of design parameters was considered as a DMU, and as the efficient DMUs were determined using the additive model, a Cross-efficiency evaluation method was used to select the final solution. Finally, the application of the proposed multi-objective model and the proposed solution method was described in the real cardiac surgery center (hospital) located in the west of Iran. Furthermore, two comparisons were performed with the bi-objective and the pure economic design models. The results clearly revealed that the performance of the multi-objective design is relatively superior to the bi-objective design. Likewise, in comparison with pure economic design, the multi-objective design offers better statistical properties although it slightly increases the imposed costs. As a result, in general, the proposed approach in designing the RACUSUM control charts can be effectively applied taking the economic and statistical properties into account while monitoring the survival times of patients in healthcare systems. An interesting area worthy of continued research efforts includes the multi-objective design of the RACUSUM control charts in the presence of multiple assignable causes.

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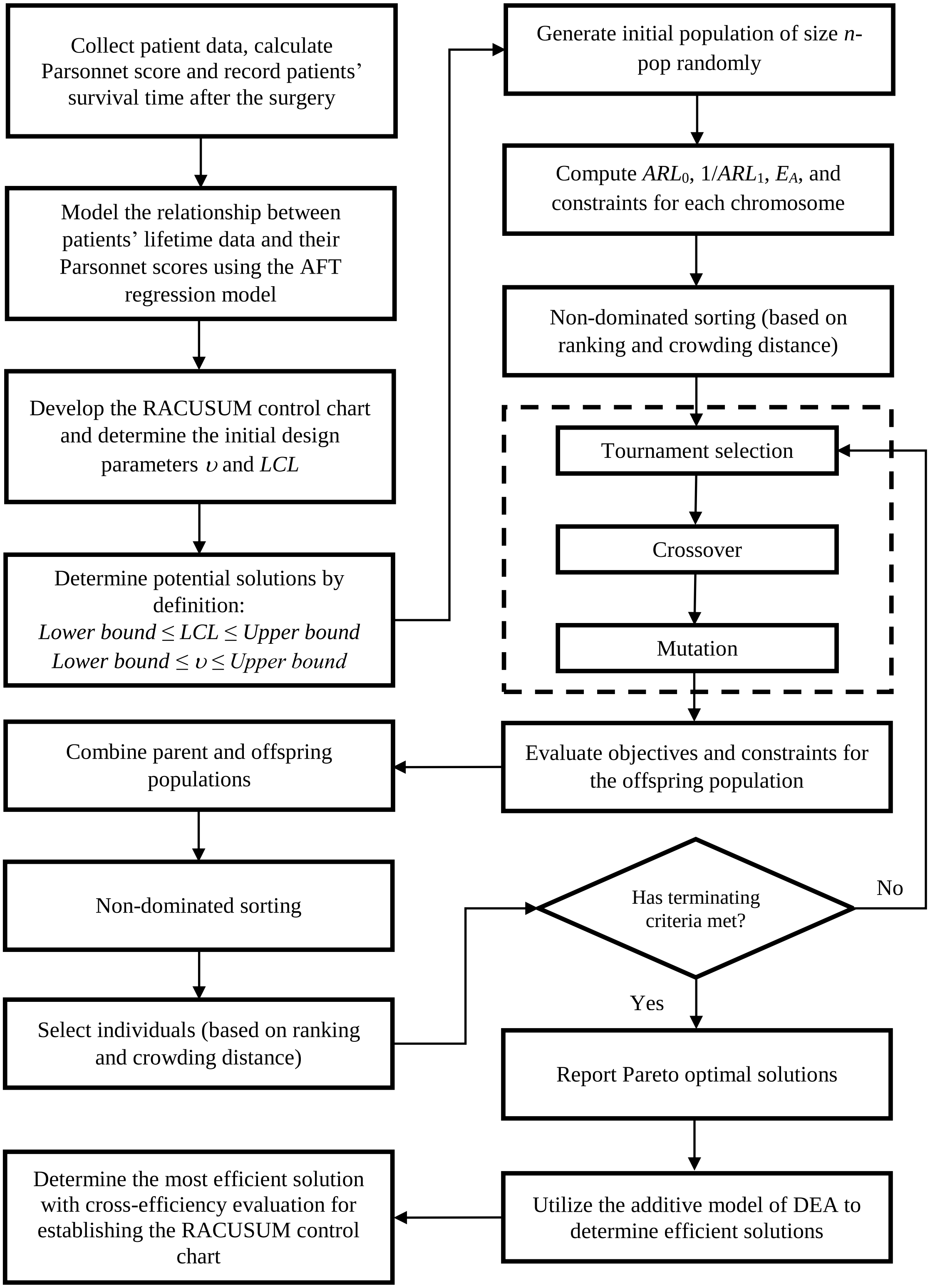
Figures and table captions:

Figure 1. A sample chromosome.
Figure 2. Flowchart of the multi-objective design of RACUSUM control chart in a surgery center.
Figure 3. Pareto front for E_A, ARL_0 and $1/ARL_1$.
Figure 4. Pareto front for ARL_0 and $1/ARL_1$.
Table 1. The preoperative risks of patients used for the calculation of Parsonnet scores.
Table 2. The details of the costs imposed on hospital in the out-of-control condition.
Table 3. Pareto optimal solutions for the multi-objective economic-statistical design of the RACUSUM control chart.
Table 4. Efficient solutions obtained from the DEA additive model and cross-efficiency evaluation technique.
Table 5. Comparison of the multi-objective and bi-objective designs.
Table 6. Comparison of the multi-objective and pure economic design.

n	h	LCL	υ

Figure 1.



Collect patient data, calculate Parsonnet score and record patients' survival time after the surgery
Model the relationship between patients' lifetime data and their Parsonnet scores using the AFT regression model
Develop the RACUSUM control chart and determine the initial design parameters υ and LCL
Determine potential solutions by definition:
$Lower\ bound \le LCL \le Upper\ bound$
$Lower\ bound \le \upsilon \le Upper\ bound$
Combine parent and offspring populations
Non-dominated sorting
Select individuals (based on ranking and crowding distance)
Determine the most efficient solution with cross-efficiency evaluation for establishing the RACUSUM control chart
Generate initial population of size *n*-pop randomly
Compute ARL_0, $1/ARL_1$, E_A, and constraints for each chromosome
Non-dominated sorting (based on ranking and crowding distance)
Tournament selection
Crossover
Mutation
Evaluate objectives and constraints for the offspring population
Has terminating criteria met?
No
Yes
Report Pareto optimal solutions
Utilize the additive model of DEA to determine efficient solutions

Figure 2.

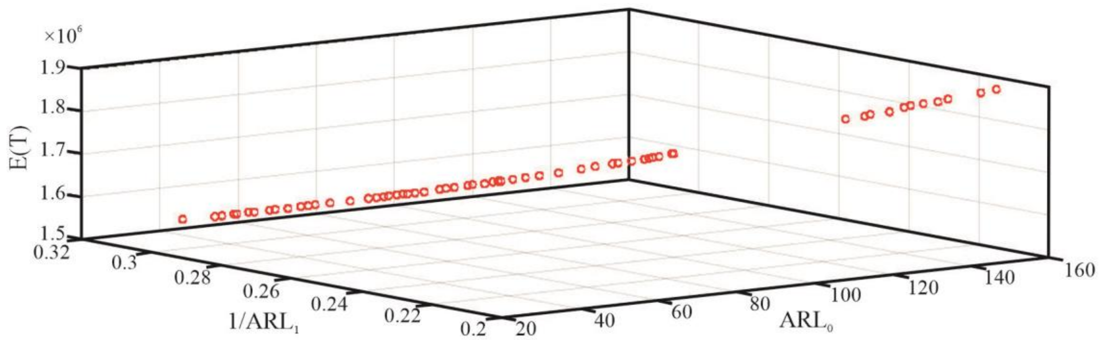


Figure 3.

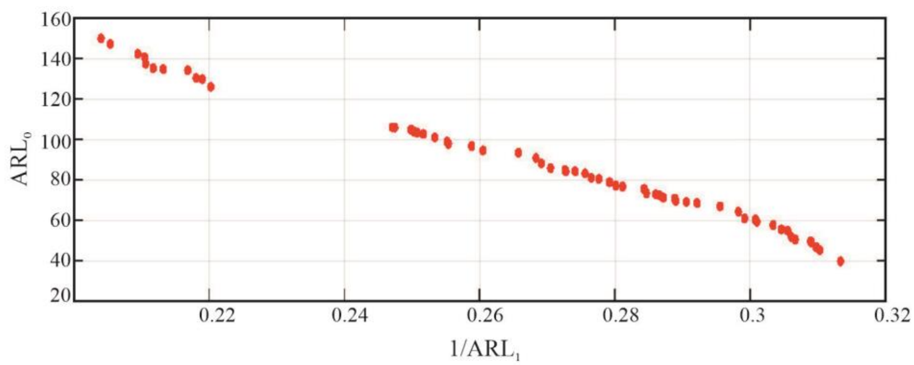


Figure 4.

Table 1.

Preoperative Risk

Risk Factor		*Score*	*Risk Factor*	*Score*
Female gender		6	Left-main disease	2.5
Age	70-75	2.5		
	76-79	7	Morbid obesity	1
	80+	11		
Congestive failure		2.5	preoperative IABP	4
COPD, severe		6	Reoperation	10
				20
Diabetes		3	One valve, aortic	0
Ejection fraction	30-49%	6.5	One valve, mitral	4.5
	<30%	8		
Hypertension		3	Valve + ACB	6

Special Conditions

Cardiac	*Score*
Cardiogenic shock (urinary < 10 cc/hr)	12
Endocarditis, active	6.5
Left-ventricular aneurysm resected	1.5
One valve, tricuspid: procedure proposed	5
Transmural acute myocardial infraction within 48 hr	4
Ventricular septal defect, acute	12
Ventricular tachycardia, ventricular fibrillation, aborted sudden death	1

Pulmonary	*Score*
Asthma	1
Endotracheal tube, preoperative	4
Idiopathic thrombocytopenic purpura	12
Pulmonary hypertension (mean pressure > 30)	11

Hepato-renal	*Score*
Cirrhosis	12.5
Dialysis dependency	13.5
Renal failure, acute or chronic	3.5

Vascular	*Score*
Abdominal aortic aneurysm, asymptomatic	0.5
Carotid disease (bilateral or 100% unilateral occlusion)	2
Peripheral vascular disease, severe	3.5

Miscellaneous	*Score*
Blood products refused	11
Severe neurologic disorder	5
PTCA or catheterization failure	5.5
Substance abuse	4.5

COPD: *Chronic obstructive pulmonary disease*
IABP = *Intra-aortic balloon pump*
PTCA: *Percutaneous transluminal coronary angioplasty*
ACB: *Aortocoronary bypass*

Table 2.

Action	**Cost (in Rial)**	**Action**	**Cost (in Rial)**
Check-ups	215000	Anesthesia	324000
Echocardiography	100000	Consumable products	2950000
Angiography	485000	Consumable drugs	370000
Consultant	52000	ICU bed for two nights	3811000
Surgery	1904000	Nursing services	809000
Operation room	483000	Public bed for eight nights	10120000

Table 3.

Design parameters		**Objective function**		
υ	LCL	E_A	ARL_0	$1/ARL_1$
0.02	-1.50	1725006.17	105.995	0.247
0.05	-1.44	1589433.19	68.479	0.292
0.07	-1.44	1557444.81	55.752	0.305
0.02	-1.43	1725937.34	106.124	0.247
0.12	-1.42	1547031.46	49.667	0.309
0.08	-1.38	1555232.69	55.118	0.306
0.02	-1.37	1710771.79	102.641	0.252
0.02	-1.35	1715010.48	103.799	0.250
0.06	-1.35	1571124.64	61.068	0.299
0.03	-1.34	1635706.18	83.233	0.276
0.05	-1.34	1580663.99	66.783	0.296
0.06	-1.34	1567163.32	60.558	0.301
0.02	-1.33	1716775.65	104.959	0.250
0.03	-1.33	1644061.58	84.273	0.273
0.02	-1.32	1713477.95	103.062	0.251
0.03	-1.28	1629783.14	80.740	0.278
0.02	-1.24	1704920.52	101.020	0.253
0.04	-1.23	1593599.92	69.038	0.291
0.08	-1.22	1553779.67	51.749	0.306
0.06	-1.20	1560712.49	57.823	0.303
0.02	-1.19	1698858.61	99.045	0.255

0.05	-1.19	1573502.28	63.979	0.298
0.03	-1.17	1632935.42	81.016	0.277
0.04	-1.16	1597916.06	69.616	0.289
0.03	-1.11	1624978.55	78.935	0.279
0.01	-1.07	1899125.94	150.118	0.204
0.03	-1.02	1622594.21	77.414	0.280
0.02	-1.01	1698107.60	97.951	0.255
0.03	-1.01	1619648.85	76.877	0.281
0.05	-1.00	1566452.67	59.089	0.301
0.10	-0.94	1543648.94	45.386	0.310
0.02	-0.92	1687088.36	96.549	0.259
0.02	-0.88	1681706.96	94.539	0.260
0.12	-0.84	1536168.95	39.993	0.313
0.03	-0.83	1610764.21	75.610	0.284
0.03	-0.81	1605936.12	73.015	0.286
0.01	-0.80	1892779.57	147.285	0.205
0.01	-0.70	1874373.26	142.585	0.209
0.02	-0.69	1665124.77	93.358	0.266
0.03	-0.69	1604541.67	72.218	0.287
0.02	-0.68	1657098.92	90.893	0.268
0.02	-0.66	1,654906.38	88.064	0.269
0.06	-0.66	1552325.62	50.880	0.307
0.03	-0.63	1609934.16	73.659	0.285
0.01	-0.62	1869120.10	137.427	0.211
0.01	-0.60	1869355.63	140.914	0.211
0.03	-0.54	1598277.20	70.672	0.289
0.06	-0.49	1546684.20	49.154	0.309
0.01	-0.48	1,864226.13	135.155	0.212
0.03	-0.46	1603011.16	71.204	0.287
0.01	-0.42	1857463.66	134.889	0.213
0.01	-0.41	1841894.75	134.194	0.217
0.02	-0.34	1650667.91	86.008	0.271
0.06	-0.25	1544854.32	46.867	0.310
0.01	-0.22	1836949.32	130.496	0.218
0.02	-0.16	1639932.48	84.086	0.274
0.02	-0.14	1644105.69	84.916	0.273
0.01	-0.09	1832831.55	129.904	0.219
0.01	-0.03	1827862.32	126.074	0.220

Table 4.

DMUs	Optimal input weight	Optimal output weight		Optimal weight	Efficiency	Cross-Efficiency
(LCL,υ)	e_1^*	u_1^*	u_2^*	T^*	*Additive model*	
(-1.42, 0.12)	1.29	1685.837	125360.81	1872173.26	1	0.93
(-1.38, 0.08)	1.20	1858.815	37505.95	1752593.07	1	0.94
(-1.34, 0.05)	1.66	5683.617	390072.56	2129647.87	1	0.91
(-1.20, 0.06)	1.00	4090.747	2538116.05	554369.47	1	0.93
(-1.19, 0.05)	1.49	4099.163	319192.26	1980022.60	1	0.92
(-1.07, 0.01)	1.62	7658.709	902715.92	1740017.21	1	0.70
(-0.84, 0.12)	1.15	420.5552	44140.31	1741218.29	1	0.90
(-0.69, 0.02)	1.70	6153.724	343148.33	2165886.81	1	0.82
(-0.68, 0.02)	1.66	5626.94	297469.39	2167488.56	1	0.83
(-0.49, 0.06)	1.00	1122.886	1.00	1491261.38	1	0.93

Table 5.

Design	(LCL,υ)	ARL_0	ARL_1	E_A
Multi-objective design	(-1.38, 0.08)	55.118	3.268	1555232.69
Bi-objective design	(-0.69, 0.02)	92.358	3.763	1665124.77

Table 6.

Design	(LCL,υ)	ARL_0	ARL_1	E_A
Multi-objective design	(-1.38, 0.08)	55.118	3.268	1555232.69
Economic design	(-0.17, 0.18)	28.097	3.185	1534632.37

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