Calculation of Urban Transportation Network Reliability Considering Correlation Among the Links Comprising a Route

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Abstract

Recently, the researchers in the field of transportation network planning have become increasingly interested in network reliability, publishing research works focused on the calculation of various types of network reliability. Calculation of network reliability has led the transportation network optimizers toward new approaches where the maximization of reliability has been considered as an objective. Travel time reliability is among the most important reliabilities investigated when analyzing urban transportation networks, with various approaches based on different assumptions proposed for calculating it. In the present research, the uncertainty associated with the demand for travel and the flows passing across links and also the correlations among the links comprising a route were considered to calculate the travel time for each of the network links. Moreover, it was shown that this process follows shifted log-normal distribution. These calculations are expected to serve as a basis for the employment of travel time reliability within the modeling of an transportation system, so as to increase the accuracy and reliability of the simulations. Finally, in order to validate, an urban network with 12 nodes, 21 links, and 4 origin-destination pairs was subjected to the travel time reliability assessment by calculating the travel time over all forming links.

Keywords: *travel time reliability, demand uncertainty, link flow uncertainty, shifted log-normal distribution, correlation among the links comprising a route, urban transportation network.*

1. Introduction

Throughout the 20th century, transportation programs were focused on primary infrastructures for transportation networks. However, in the 21st century, the programs were changed toward network administration and performance. The economy of a country or region is largely dependent on an efficient and reliable transportation system that can provide the required access and safe transportation services for individuals and goods. This is especially the case in metropolitans and confirms the importance of ensuring an acceptable level of transportation service.

Factors affecting travel times across an urban transportation network include (a) the flow of traffic, which is not always fluent across the entire scope of a transportation network but is rather frequently disordered by such crises as quakes, floods, storms, accidents, etc., making the flow of traffic stopped, interfered, or interrupted, with the interruptions lasting for some hours to several years, (b) the link capacity, which may be reduced due to different reasons such as crisis and weather conditions, and (c) the demand, which exhibits occasional fluctuations because of the occurrence of crises or special events. These factors may bring about uncertain travel times. Travel time is one of the most important criteria for evaluating the performance and quality of services provided by a transportation system. Other criteria in this respect (*e.g.*, fuel consumption, accidents, etc.) are dependent on this parameter in one way or another.

In order to evaluate the reliability across a transportation network, three criteria are considered: connectivity reliability, travel time reliability, and capacity reliability [1].

Connectivity reliability: This reliability refers to the probability of connectivity of nodes across a network. In fact, this probability checks for the presence of a route between a particular origin-destination pair. For any particular origin-destination pair, the network that has at least one active and usable route is evaluated as successful.

Travel time reliability: Travel time reliability expresses the probability that a trip between a given pair of origin-destination is successfully performed within a certain time interval at a particular quality of service. This reliability is expressed in terms of the difference between anticipated travel time according to the schedule or average travel time and actual travel time due to traffic congestion or fluctuations in demand.

Capacity reliability: The probability that a network can successfully meet a certain level of demand for a particular origin-destination pair at an acceptable quality of service is known as transportation network capacity reliability, which is an important criterion for system performance evaluation.

There are methods for determining the reliability of an urban passenger transportation network, wherein the route selection model is used to obtain travel time reliability and the reserve capacity model is used to determine the network capacity reliability. In these methods, the travel time and capacity reliabilities are computed by investigating the changes in arc capacities and interruptions incurred by such crises as a quake, storm, flood, etc. and determining thresholds for demand and travel time.

Network reliability is a reflection of the uncertainty associated with the main indices for analyzing a network; various research works have expressed these uncertainties with different models and distributions, as discussed in Section 2 hereunder. The models are expected to simulate the actual network as closely as possible. A bulk of studies have been carried out on the estimation of the reliability of urban transportation networks. Among others, some studies have attempted to estimate the distribution of links across the transportation system by arranging the collected data into statistical distributions; then they generalized the distribution to the whole network using statistical relations considering several assumptions. Improving the accuracy of the reliability estimation, these assumptions include correlation between the neighboring links, uncertainty in demand for travel, and uncertainty in the flows over different links.

The above assumptions have been made in previous studies. However, acknowledging the importance of increasing the accuracy of modeling the travel time, route reliability, and hence the transportation network in general, the present research further makes the following assumptions for addressing a similar problem:

- All links comprising a route are correlated;
- Demands for travel are uncertain;
- Flows across the links are uncertain; and
- Passengers may opt for a different route for the same origin-destination pair in an uncertain way.

The above assumptions are expected to increase the urban transportation network modeling accuracy and reliability.

This paper is organized into five sections. Section 2 reviews the existing literature on methods for calculating travel time reliability in urban transportation networks to identify the existing research gap. Later on in Section 3, modeling and calculation of travel time over a route and distribution of travel time between origin-destination pairs are discussed. In Section 4, a numerical example regarding travel time reliability across a sample urban network is presented. Finally, Section 5 presents conclusions and suggestions for future research.

2. Literature review

Calculation of travel time reliability depends on the calculation of travel time into which numerous factors contribute. The choice of these factors depends on the considered field of study, ease of measurement and computations, simplicity of understanding by user/operator, relation to decision-making, and the capability of representing the users' attitudes toward the risk. Researchers suggest that these factors shall reflect the biased and asymmetric nature of the travel time observations [2]. Many of the studies concerning the travel time distribution estimation have used actual observations (see for example [3-10]). As an instance, in his study, Wardrop [9] stipulated that travel time follows a non-uniform distribution. In the past, different studies used different distributions (*e.g.*, Weibull, gamma, lognormal, and Burr distributions) to fit the travel time data. Most existing studies on the travel time distribution (TTD) have put significant efforts into identifying the best fitting model. The fitted distributions can be classified as single-mode distributions [11-18] and other distributions (multi-mode distribution and truncated distribution) [19-21]. Herman and Lam [4] suggested gamma or log-normal distribution for TTD. Richardson and Taylor [6] used the observations and data available to them to propose log-normal distribution for representing the travel time. Polus [5] concluded that the gamma distribution is better than normal or log-normal distributions as far as the representation of travel time is concerned; and Al-Deek and Emam [3] proposed Weibull distribution for travel time estimation. Wu and Geistefeldt [10] used shifted gamma distribution to describe total travel time behavior. Taylor and Chen [7, 8, 22] used Burr's distribution for estimating travel time assuming independence of the links and compared the results to those of log-normal and gamma distributions. In their study on travel time estimation inspired by the study reported in [2], Zhenliang *et al.* [23] used the so-called Markov chain while considering the correlation of neighboring links. Van Lint *et al.* [24] demonstrated travel time distribution in four different forms based on traffic conditions (free flow, traffic initiation, congestion, traffic dissolution). Pu [25] 25 concluded that these four forms of TTD resemble log-normal distribution. Susilawati *et al.* [26] proposed Burr's distribution type XII for variations of travel time across urban driveways. According to travel time distributions, a large number of criteria have been proposed by various researchers (see for example [24, 27-32]). Given the travel time distribution, reliability criteria can be defined mutually. Other researchers studied the relationship between average travel time per unit distance and standard deviation of travel time (see for example [30, 33-35]), and also optimal routes across a potential network (see for example [2, 36]). Kim *et al.* [37] proposed a mixed gamma distribution for modeling the variations of travel time across a road network. They found that the mixed gamma distribution is the best distribution for travel time estimation, with different dimensions of changes in relation to daily variations and vehicle type.

Wu and Geistefeldt [38] presented a mathematical model for dealing with the standard deviation of total travel time over a freeway. In general, travel time distribution, free flow through links, and distribution of delays in narrow segments can be described using a normal, Erlang, or log-gamma distribution (or any other possible distribution). Parameters of these distributions can be estimated by measurement or simulation. The variance of total travel time through a route can be calculated by summing up the variances of the individual links comprising the route provided that the travel time and delays at each link are statistically independent. However, in reality, such independence is impossible, at least between successive links. In this case, provided the correlation coefficients between successive links are known, one can estimate the variance of total travel time over a route. The coefficients can be either measured or simulated. If the correlation coefficients are known, the standard deviation can be evaluated accordingly.

Wu and Geistefeldt [10] proposed Erlang or shifted gamma distribution for describing the travel time. The Erlang distribution is indeed a special case of the gamma distribution. Because of particular properties of the gamma distribution, the variance of the entire route is equal to the sum of variances of individual links provided the links are statistically independent. Total travel time also follows a gamma distribution. In order to calculate a lower threshold for the travel time, a shifted gamma distribution is used.

Many of the performed studies have mainly used normal and log-normal distributions [39, 40]. In the meantime, it is worth noting that, even though the assumption of normality offers many analytic and computational advantages, it is further associated with such unreasonable restrictions as symmetry and non-zero probability for negative travel times. In a log-normal (two-parameter) distribution, the sub-zero range results in unreasonable free flow velocities [2].

As is evident and emphasized by researchers, measurement and analysis of travel time fluctuations are complicated as these are functions of numerous parameters including the congestion, flow, accidents, specifications of facilities, time intervals per day, road properties, and free-flow travel time [5, 7, 12]. In particular, some studies stipulated that average and standard deviation of travel time are related to one another (see for example [41-45]). One of the other approaches followed to systematically estimate travel time over a route as a random variable is to adopt a model where the travel time is estimated as a function of variable parameters such as random capacity, potential demand, and random route selection. This model has been discussed in [46-50] without considering the correlation of links over a route, and more recently in [51] where random capacity, potential demand, and travel time were assumed for one road narrowing.

Considering previous studies done in this field (as reviewed herein), the present research examines several issues to get closer to the real conditions; given the fact that, in real conditions, in addition to the associated uncertainty with the demand for travel, there are uncertainties in the flows passing through different links (which can be described using suitable distributions), and that all of the links comprising a given route are actually dependent on one another in real conditions, among others, the research works reported in Refs. [2] and [10] are closer to the present work. In Ref. [2], the researchers considered log-normal travel time distribution, without considering the correlation of travel time across all links comprising a given route, to investigate the travel time reliability for different routes. In Ref. [10], not only the correlation assumption was not taken into consideration for all links across a given route, but also travel times were assumed to follow a shifted gamma distribution. Moreover, in both of the research works, distribution of demand among different routes connecting the same origin-destination pair was performed in such a way that the demand was allocated to the route for which travel time reliability was to be calculated, while in the reality, the demand between any particular pair of origin-destination may be allocated to some or even all routes connecting the pair. In the present paper, we aim at calculating the reliability by considering the uncertainty associated with the demand for transportation between an origin-destination pair and the flow passing through links, so that a reliability can be calculated for each route connecting the origin-destination pair considering the threshold set by the deicing-maker, with the ultimate goal of calculating the travel time through each route at a particular level of reliability. Other issues elaborated in the present research include associations among the links comprising a route, log-normality of the flow passing through links, and random route selection by passengers, which are further detailed in the next section.

3. Travel time reliability modeling

3.1. Calculation of travel time

Any urban transportation network is made up of a number of nodes connected via arcs (links). Considering the nature of transportation, each set of passengers selects an origin-destination pair to undertake a trip across this network. Depending on the network type, there may be various routes connecting the same pair of origin and destination nodes as their starting and ending points, respectively. Throughout this trip, at least one link is passed. For instance Figure 1, there are four routes connecting the origin-destination pair (1 – 6). Should a passenger opts for the route marked by the bold line, he/she would pass through three links, namely (1 – 2), (2 – 5), and (5 – 6). Therefore, to determine the reliability of a route, travel time distribution across the route shall be known. In addition, in order to determine travel time distribution across a route, one should determine the travel time distribution for each link over the route.

For each link, the travel time is composed of two parts: (a) free-flow travel time and (b) travel time delay. The former refers to the link travel time when the link hosts only one vehicle so that the vehicle can flow from the starting node toward the ending node of the link at maximum allowed speed. This travel time depends on the vehicle speed and the link length and can be calculated via Equation (1):

$$FreeTime = \frac{Link.lenght * 60}{Link.velocity} \tag{1}$$

Therefore, since the maximum allowed speed via a link and the link length are supposed to be constant, the free-flow travel time is also considered as a constant value. The travel time delay develops due to traffic load over the links, route narrowing, speed bumps, accidents, etc. In different references, the travel time delay has been supposed to follow different distributions such as normal, log-normal, exponential, gamma, and Burr’s distributions, depending on the research objectives.

3.1.1. Link travel time modeling

Considering the study reported in Ref. [2], shifted log-normal distribution better suits to various types of transportation facilities and exhibits a performance no worse than other distributions mentioned in the previous section. Therefore, in the present paper, shifted log-normal distribution was used to model the link travel times (per unit length) for different types of facilities. The following modeling structure is presented for travel time per unit length (t_i) for the i^{th} link.

Considering characteristics of the log-normal distribution, it turns into a normal distribution should one takes a natural logarithm of that. Accordingly, we have:

$$t_i = \gamma_i + \exp(\mu_i + \sigma_i z_i) \tag{2}$$

where γ_i refers to the free-flow link travel time and $\exp(\mu_i + \sigma_i z_i)$ denotes the travel time delay. In this latter expression, z_i is a standard normal random variable (*i.e.*, $z_i \sim N(0,1)$). Therefore, the random variable $l_i = \exp(\mu_i + \sigma_i z_i)$ has a log-normal distribution and the random variable t_i follows a shifted log-normal (SLN) distribution with its parameters denoted as μ_i, σ_i, and γ_i. Mean and variance of the random variable t_i are calculated as follows:

$$E\left[t_i\right] = T_i = \gamma_i + \exp(\mu_i + 0.5\sigma_i^2) \tag{3}$$

$$Var\left[t_i\right] = V_i = \exp(2\mu_i + \sigma_i^2)[\exp(\sigma_i^2) - 1] \tag{4}$$

Thus, the mean travel time delay is given by:

$$Mean_{d_i} = E[t_i] - \gamma_i \tag{5}$$

Considering what was mentioned above, as random variables, the travel time and travel time delay follow a log-normal and SLN distributions, respectively (*i.e.* $t_i \sim SLN(\mu_i, \sigma_i^2, \gamma_i)$).

The coefficient of variation of the time travel time delay is evaluated as follows:

$$CV_i = \frac{\sigma_i}{\mu_i} = \frac{\sqrt{\exp(2\mu_i + \sigma_i^2).[\exp(\sigma_i^2) + 1]}}{\exp(\mu_i + 0.5\sigma_i^2)} = \sqrt{\exp(\sigma_i^2) - 1} \tag{6}$$

The link travel time distribution was modeled according to the function presented by the Federal Highway Administration (FHWA) [52], which is recognized as a valid model in this field. The developed model is presented in the following:

$$T_i = \gamma_i.(1 + \alpha.\left[\frac{V_i}{C_i}\right]^\beta) = \gamma_i + \gamma_i.\alpha.\left[\frac{V_i}{C_i}\right]^\beta \tag{7}$$

where the coefficients α and β can be either presumably set to 0.15 and 4, respectively, or determined using real data. V_i is the flow rate through the i^{th} link and C_i is the capacity of the i^{th} link [14]. The flow through the i^{th} link is equal to the sum of flows through all routes (f_r) including the i^{th} link ($p \in P_i$), as expressed by Equation (8):

$$\sum_{p \in P_i} f_p = v_i \tag{8}$$

The flow passing through a link is a random variable that depends on the demand for travel. Considering the non-negative nature and asymmetry of the log-normal function and the research work performed by Mingxin *et al.* [51], log-normal distribution was herein considered for the travel demand. In the present research, the demand was assumed to be uniformly random-distributed among all routes connecting each origin-destination pair. As a result, the flow passing through each route would also exhibit a log-normal distribution. Each link might be involved in several routes (Equation (8)), and given that sum of several log-normal distributions will also follow a log-normal distribution, the flow passing through each link with log-normal distribution would be in the form of $V_i \sim LN(\mu_{v_i}, \sigma_{v_i}^2)$. In the present research, parameters of this distribution are determined using Monte Carlo simulation. On this basis, the travel time delay of the *i* th link is determined as follows:

$$\begin{aligned} & V_i^{\beta} \sim LN(\beta\mu_{v_i}, \beta^2\sigma_{v_i}^2) \Rightarrow \\ & \frac{\gamma_i.\alpha}{C_i^{\beta}} V_i^{\beta} \sim LN(\ln\left(\frac{\gamma_i.\alpha}{C_i^{\beta}}\right) + \beta\mu_{v_i}, \beta^2\sigma_{v_i}^2) \end{aligned} \tag{9}$$

and the parameters of the travel time over *i* th link ($t_i \sim SLN(\mu_i, \sigma_i^2, \gamma_i)$, as a random variable) are as follows:

$$\mu_i = \ln\left(\frac{\gamma_i.\alpha}{C_i^{\beta}}\right) + \beta\mu_{v_i} \tag{10}$$

$$\sigma_i^2 = \sigma_{d_i}^2 = \beta^2\sigma_{v_i}^2 \tag{11}$$

3.2. Calculation of travel time reliability

When it comes to obtaining the probability distribution of travel time over a route connecting a particular origin-destination pair, since the route is made up of more than one link, the obtained distribution is equal to the sum of probability distributions of the links comprising the route. As far as the link travel time, as a random variable, follows the SLN distribution, the cumulative probability function of the route travel time (which corresponds to the sum of those for links) does not have a closed form; therefore, the travel time over the route *p* is estimated by the log-normal distribution (Fenton–Wilkinson's approach) [53] with the parameters μ_p, σ_p, and γ_p, so that its average and standard deviation are given via the following relationships:

$$Mean_p = \gamma_p + \exp(\mu_p + 0.5\sigma_p^2) = \sum_{i \in p} \gamma_i + \sum_{i \in p} \exp(\mu_i + 0.5\sigma_i^2) \tag{12}$$

$$\begin{aligned} Var_p &= \exp(2\mu_p + \sigma_p^2)[\exp(\sigma_p^2) - 1] = \sum_{i \in p} \exp(2\mu_i + \sigma_i^2)[\exp(\sigma_i^2) - 1] \\ &+ \sum_{i \in p, j \in p} \rho_{ij} \left\{\exp(2\mu_i + \sigma_i^2)[\exp(\sigma_i^2) - 1]\right\}^{0.5} \left\{\exp(2\mu_j + \sigma_j^2)[\exp(\sigma_j^2) - 1]\right\}^{0.5} \end{aligned} \tag{13}$$

In the above relationships, ρ_{ij} is the correlation coefficient between travel times of the i th and j th links over the route p. Accordingly, the reliability of the travel time over the route p is calculated by the estimated route travel time distribution ($t_p \sim SLN(\mu_p, \sigma_p^2, \gamma_p)$).

As was mentioned in Section 1, route travel time reliability refers to the probability that the travel time over the route (t_p) is equal to or smaller than a predetermined threshold (T_0). The threshold T_0 indicates the expected travel time plus some extra time considered to ensure the on-time accomplishment of the travel. Therefore:

$$R_p = P[t_p < T_0] \tag{14}$$

For a route X (which is represented by a vector composed of the comprising links), considering the law of large numbers, the above probability is calculated using a standard normal distribution, as follows:

$$R_p(X) = \Phi\left(\frac{T_0 - Mean_p}{\sqrt{Var_p}}\right) \tag{15}$$

Furthermore, once the reliability becomes known, one can calculate the total travel time for each route via the following relationship:

$$TT_p = Z_R * \sqrt{Var_p} + Mean_p \tag{16}$$

where ZR is the inverse of the standard normal function for the reliability R (e.g. 80% or 95%) and TTp is total travel time through the route p at the reliability R.

Figure 2 shows the step-by-step procedure followed to calculate the travel time reliability.

4. Numerical example

In this section, a part of an urban transportation system is investigated, and reliabilities of the routes connecting given pairs of origins and destinations are examined. This numerical example was coded using MATLAB R2017a on a PC powered by an Intel Core™ i5 2410 processor computing at 2.30 GHz.

Made up of 12 nodes and 21 links (G (12, 21)), the considered network is demonstrated in **Error! Reference source not found.Error! Reference source not found.**. The lengths of the links are presented in Table 1.

The considered origin-destination pairs and their details are presented in **Error! Reference source not found.**. It was assumed that the demand for travel through the origin-destination pair follows a random log-normal distribution.

Based on the above-provided information, the free-flow travel time for each link is given in Table 1 (calculated from Equation (1)).

Considering what was mentioned in Section 3.1.1, the flow through the links follows a log-normal distribution. According to Ref. [2] and given that the passengers do not know details of available routes, the passengers were assumed to select their routes on a random basis. As explained in Section 3.1.1, existing demand was uniformly random-distributed over available routes, so that the flow through individual links also followed a log-normal distribution. In the present research, it is desired to determine the routes of higher reliability using the initial network information.

Based on the discussions delivered in Section 3.1.1, the parameters of the log-normal distribution were obtained using Monte Carlo simulation. Accordingly, performing the route selection process by passengers for 200 iterations, average and variance of the flow via each link (which also follows a log-normal distribution) are further given in Table 1. Moreover, upon performing the iterations, the matrix of the correlation coefficient between each and any link was determined.

Available routes connecting the origin-destination pairs are demonstrated in **Error! Reference source not found.**. There are 8, 7, 8, and 4 routes for the origin-destination pairs (1 – 11), (2 – 12), (1 -12), and (2 – 11), respectively. The average and variance of the travel time for each route were calculated using the relationships presented in Section 3.2, and the results are listed in **Error! Reference source not found.Error! Reference source not found.**.

In order to determine the travel time reliability using Equation (15), a threshold time was considered for each origin-destination pair (9, 33, 7.5, and 43, respectively), and the results are reported in **Error! Reference source not found.Error! Reference source not found.**.

Moreover, knowing the reliability, one can determine total travel time over each route using Equation (16). The corresponding results to reliabilities of 80%. 90%, 95%, and 99% are presented in **Error! Reference source not found.**.

4.1. Comparing the results to previous research works

Among the outstanding research works in this scope (as was mentioned in Section 2), one can refer to the studies reported by Srinivasan *et al.* [2] and Wu and Geistefeldt [10]. The former discussed travel time reliability by assuming log-normality of the travel time without considering dependence relationships among all links, and the latter performed the same by assuming that the travel time follows a shifted gamma distribution with independent links. In this section, the results of this research are compared to these of the mentioned studies at 95% confidence level. The obtained values of total travel times are plotted in Figure 4 through Figure 7.

The first thing to infer from the above curves is that, at a given level of reliability, the routes for which total travel time is minimal in the three studies are not significantly different. Therefore, the results of this research are consistent with those of [2] and [10]. In the second place, comparing our study with that reported in Ref. [2], reliability of all routes connecting the origin-destination pairs is smaller or equal, *i.e.* at a given level of reliability, to the total travel time over each and any route connecting the corresponding origin-destination pair, the total travel time in the present research is equal to or greater than those in [2] as we considered the correlation among all of the links comprising a route connecting each origin-destination pair, and the longer the route or the higher the number of the links over a route, the longer the total travel time difference (travel time delay) because of the correlation. Regarding the comparison between the present research and that reported in Ref. [10], since skewness and kurtosis of a log-normal distribution are higher than those of a gamma distribution with the same mean and standard deviation, then, at a given level of confidence, the travel times obtained in the present research and reported in Ref. [2] are commonly greater than or equal to those reported in Ref. [10], especially in the case of longer routes.

5. Conclusion

Several previous works have investigated the estimation of travel time and travel time reliability over links and routes and hence across an urban transportation network. In all cases, assumptions were made to resemble the real world condition as closely as possible. In an urban transportation network, finding a probability distribution that can accurately estimate the travel time and its reliability has always been a crucial task. Our results in the present study showed the capability of the shifted log-normal distribution for modeling the travel time and travel time reliability over a link or route based on several assumptions, including the correlation among all links comprising a route, uncertain demand for travel, uncertain flow over links, and random route selection by passengers.

In order to estimate the parameters of a route composed of several links, one may begin with calculating the parameters for each link followed by adopting the mentioned relationships in Section 3.2 to obtain those for the route. Using the travel time parameters, one can calculate the reliability for a route or network.

Application of the proposed method for evaluating the travel time reliability was demonstrated on a part of a transportation network composed of 21 links and 12 nodes, making up four origin-destination pairs. The results (**Error! Reference source not found.**) showed that, for the origin-destination pair (1–11), the route 1 – 3 – 8 – 11 provided for a travel time of 10.03 min at a reliability of 95%. Following another approach, for the origin-destination pair (2–11), one might arrive at the destination through the route 2 – 4 – 3 – 8 – 11 within 43 min at a reliability of 95.17%.

In the present paper, considering correlations among the links, we obtained closer-to-reality modeling results. This can help passengers select routes of higher reliability, and largely helps urban transportation management enhance the route capacities. The following topics are recommended for future research works:

1. Application and calculation of route capacity reliability and investigation of its relationship to travel time reliability.
2. Increasing and optimizing link capacities to minimize associated costs and maximize route reliability.
3. Determination of a route selection algorithm for passengers to replace the random route selection by them.
4. Using the results of this article in modeling urban transportation networks by means of bi-level programming models and solving the models via meta-heuristic algorithms like genetic algorithm, backtracking search algorithm, and particle swarm optimization, and then comparing the results to those of the present study. Considering solution approaches, interested readers may refer to [54-56].
5. Considering the effects of different uncertainties in the model by handling fuzzy optimization (see for example [57-59]).

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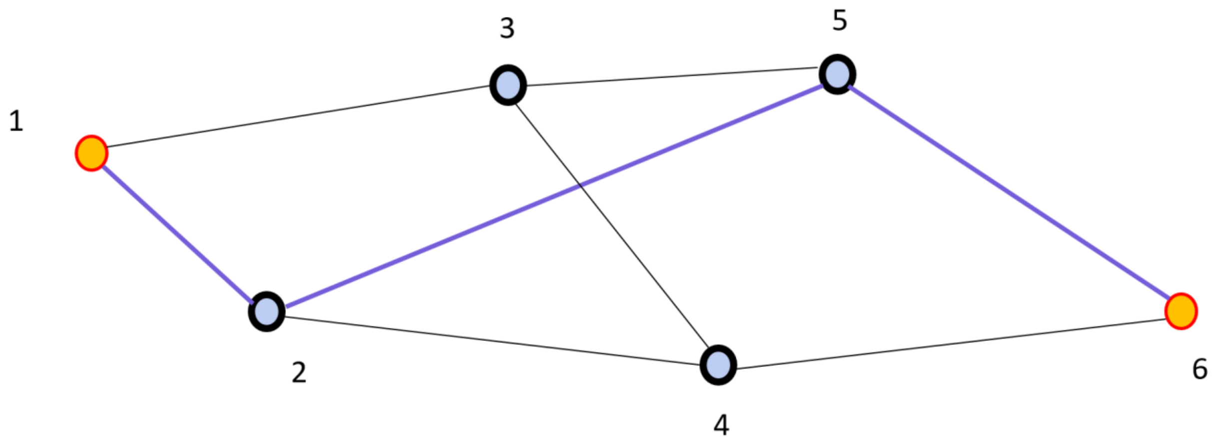


Figure 1- A network composed of 6 nodes and 8 links.

Step 0	Input urban transportation network information
Step 1	Find all routes connecting each origin-destination pair
Step 2	Generate the demand for each origin-destination pair using log-normal distribution
Step 3	Determine the flow through all routes connecting the origin-destination pair (distribute the demands generated in Step 2 among all routes connecting the corresponding origin-destination pair)
Step 4	Determine the flow over each link using Equation (8)
Step 5	Repeat Steps 2 to 4 (rerun the simulation) and store the flow through each link at each iteration. Proceed to Step 6 once finished with the iterations
Step 6	Calculate the average and variance of the flow over each link and matrix of correlation of the links using the entire set of the above-mentioned iterations
Step 7	Compute parameters of the travel time over each link, as a random variable, that is μ, σ, and γ using Equations (10), (11), and (1), respectively
Step 8	Calculate the average and variance of travel time over all routes using Equations (12) and (13), respectively
Step 9	Calculate route travel time reliability using a predetermined threshold via Equation (15)

Figure 2- The step-by-step procedure followed to calculate the travel time reliability

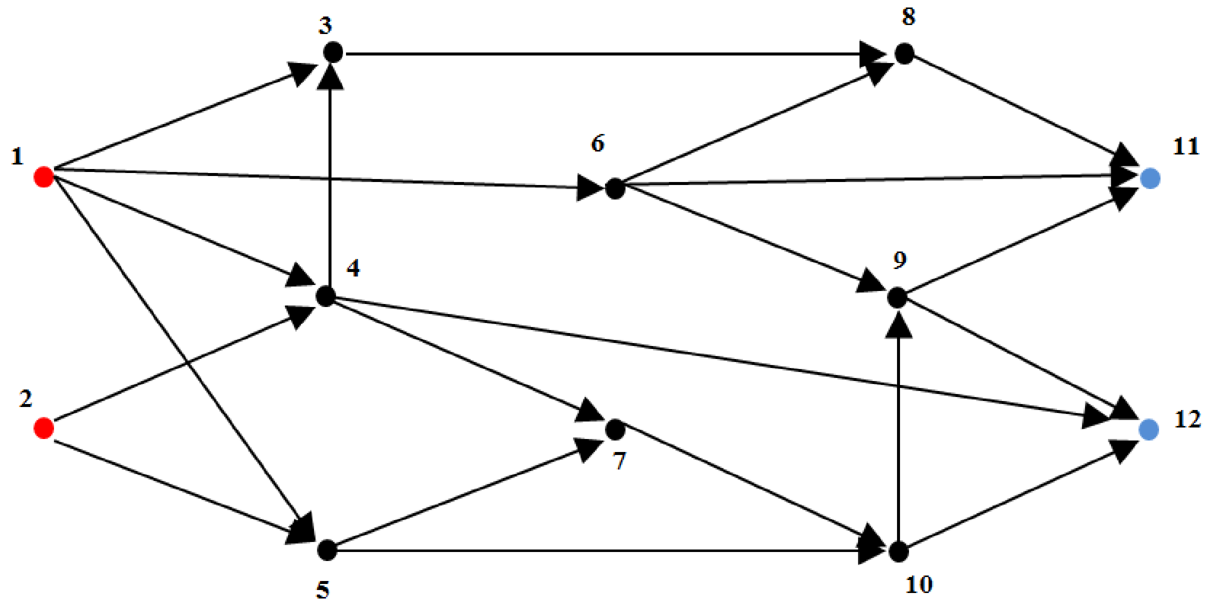


Figure 3- Considered network.

Table 1- Characteristics of the considered links.

Link name	Link length	Link capacity	Link allowed speed	Free-flow travel time	Average flow through the link	Standard deviation of the flow over the link
1-3	1	150	80	0.75	49.31	28.19
4-3	0.5	100	50	0.6	149.50	61.19
1-4	1	200	50	1.2	186.20	45.72
2-4	2	150	50	2.4	294.53	72.60
1-5	1.5	100	50	1.8	247.73	69.38
2-5	2.5	270	80	1.875	355.29	72.70
1-6	3	160	80	2.25	186.67	53.74
4-7	1	120	50	1.2	255.88	64.22
5-7	1	150	50	1.2	316.98	121.17
3-8	3	200	80	2.25	198.81	67.12
6-8	2	80	50	2.4	50.28	28.27
6-9	2	140	50	2.4	84.62	32.52
10-9	1.5	170	50	1.8	628.67	109.66
5-10	2	250	80	1.5	286.05	69.82
7-10	2.5	200	50	3	572.85	120.56
6-11	4	150	80	3	51.78	29.48
8-11	2.5	250	80	1.875	249.08	75.30
9-11	2	130	50	2.4	449.25	79.89
4-12	4.5	150	50	5.4	75.36	34.81
9-12	2	110	50	2.4	264.04	64.47
10-12	3	120	80	2.25	230.23	55.20

Table 2- Characteristics of existing origin-destination pairs.

Origin-destination pair	Average demand	Variance of demand	Expected time for traveling from origin to destination
1-11	400	4	9
2-12	300	3	33
1-12	270	2	7.5
2-11	350	3.5	43

Table 3- Results of calculations for finding characteristics of available routes connecting the origin-destination pairs

Origin-destination pair	Available routes	Average total travel time	Variance of total travel time	Travel time reliability (%)	80% reliable travel time (min)	90% reliable travel time (min)	95% reliable travel time (min)	99% reliable travel time (min)
1-11	1-6-11	6.29	5.70	87.15	8.30	9.35	10.22	11.85
	1-3-8-11	5.98	6.05	89.01	8.05	9.13	10.03	11.70
	1-6-8-11	8.29	23.39	55.81	12.36	14.49	16.25	19.55
	1-6-9-11	70.05	4922.98	19.21	129.11	159.97	185.46	233.28
	1-4-3-8-11	8.35	41.52	53.99	13.78	16.61	18.95	23.34
	1-5-10-9-11	146.38	13800.21	12.11	245.24	296.92	339.60	419.66
	1-4-7-10-9-11	176.77	17265.28	10.08	287.36	345.17	392.90	482.45
	1-5-7-10-9-11	195.95	23910.21	11.33	326.09	394.11	450.29	555.67
2-12	2-4-12	15.59	184.44	90.01	27.02	32.99	37.93	47.18
	2-5-10-12	13.64	122.76	95.97	22.97	27.84	31.87	39.42
	2-4-7-10-12	67.52	3450.27	27.84	116.95	142.79	164.13	204.17
	2-5-7-10-12	63.22	4967.65	33.41	122.53	153.54	179.15	227.18
	2-5-10-9-12	86.56	5945.51	24.36	151.46	185.38	213.39	265.94
	2-4-7-10-9-12	140.44	10706.33	14.96	227.52	273.04	310.63	381.15
	2-5-7-10-9-12	136.13	13260.55	18.52	233.05	283.71	325.54	404.02
1-12	1-4-12	6.96	1.28	68.43	7.91	8.41	8.82	9.59
	1-5-10-12	28.49	1310.43	28.10	58.95	74.88	88.03	112.70

	1-6-9-12	25.08	880.10	27.67	50.05	63.10	73.88	94.10
	1-4-7-10-12	58.89	3313.77	18.60	107.33	132.66	153.57	192.80
	1-5-7-10-12	78.06	7027.88	20.00	148.61	185.50	215.95	273.08
	1-5-10-9-12	101.41	7854.24	14.47	175.99	214.98	247.18	307.58
	1-6-11	6.29	5.70	87.15	8.30	9.35	10.22	11.85
	1-3-8-11	5.98	6.05	89.01	8.05	9.13	10.03	11.70
2-11	1-6-8-11	8.29	23.39	55.81	12.36	14.49	16.25	19.55
	1-6-9-11	70.05	4922.98	19.21	129.11	159.97	185.46	233.28
	1-4-3-8-11	8.35	41.52	53.99	13.78	16.61	18.95	23.34
	1-5-10-9-11	146.38	13800.21	12.11	245.24	296.92	339.60	419.66

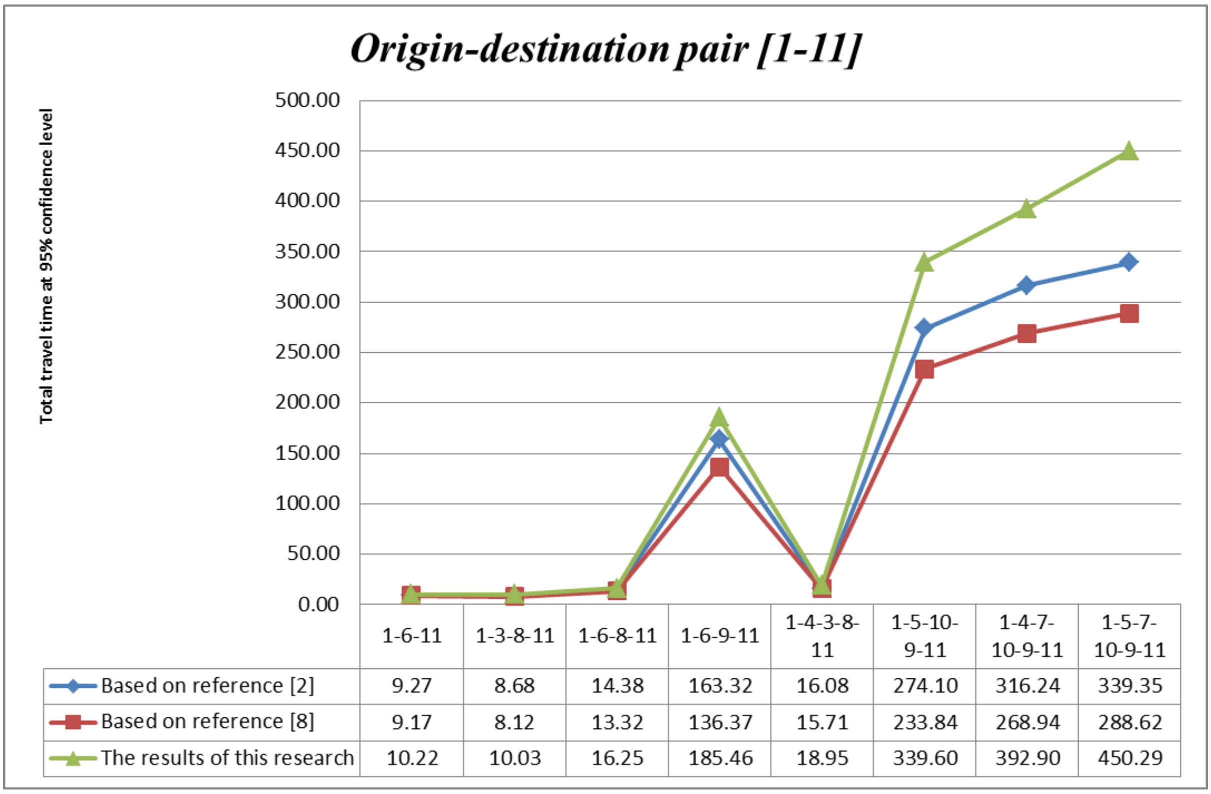
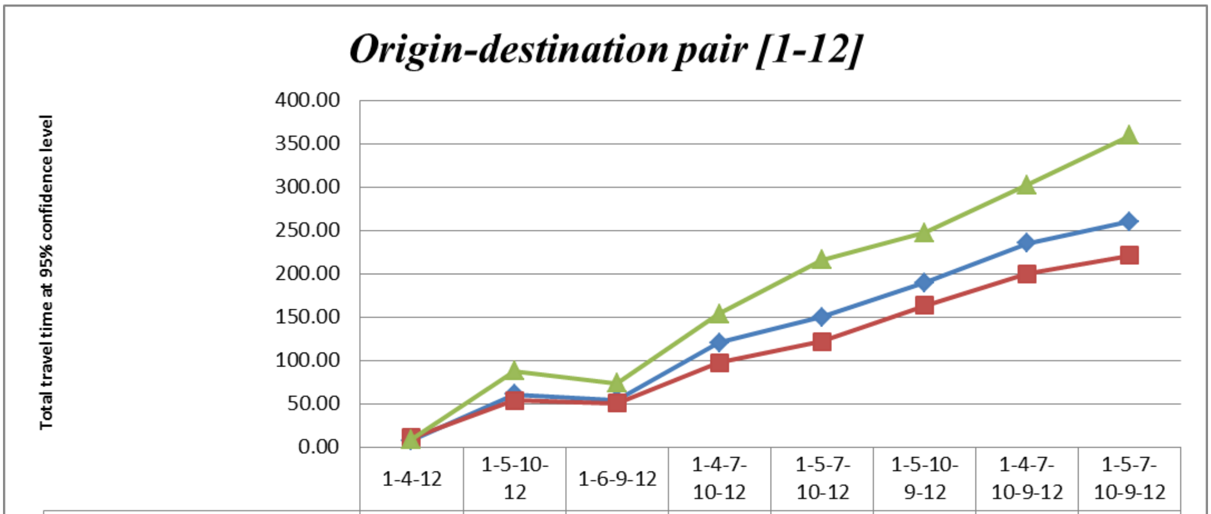
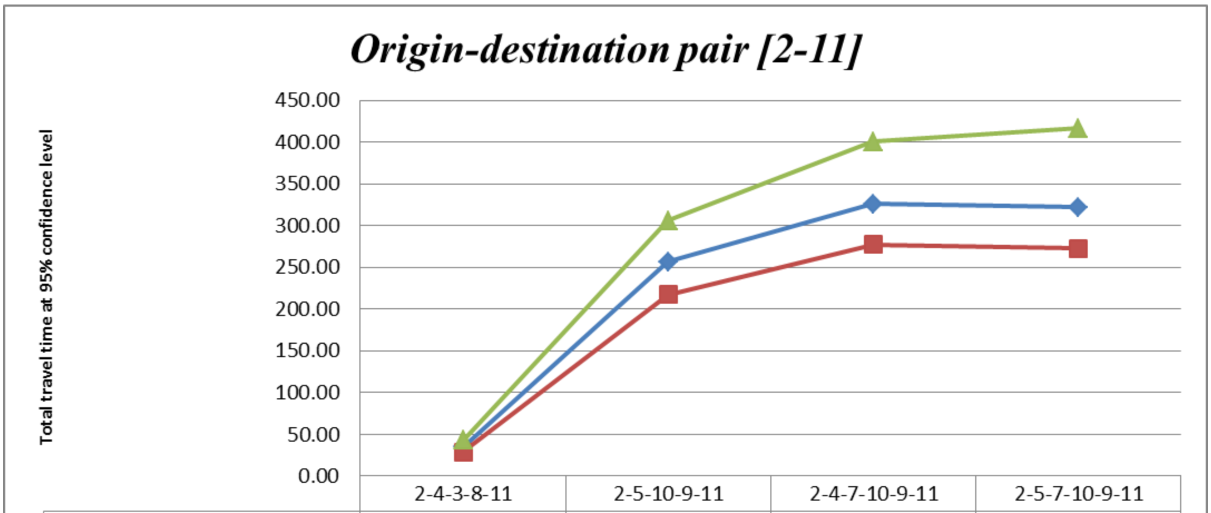


Figure 4- Comparison of total travel time for the routes connecting the [1–11] origin-destination pair at 95% confidence level.



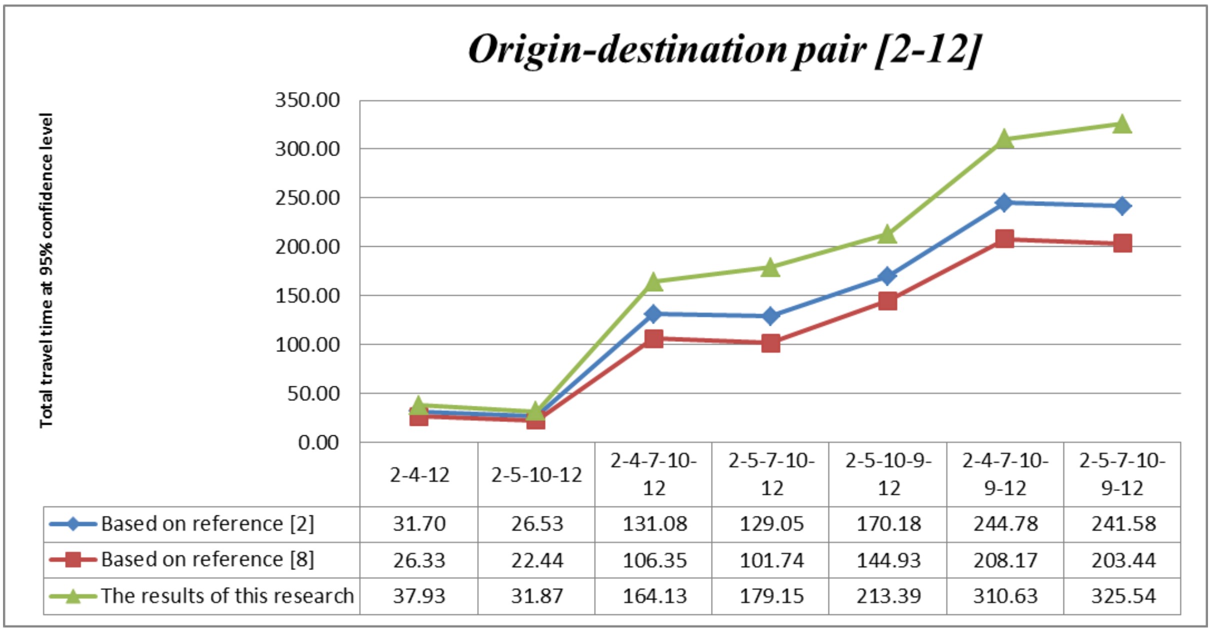
	1-4-12	1-5-10-12	1-6-9-12	1-4-7-10-12	1-5-7-10-12	1-5-10-9-12	1-4-7-10-9-12	1-5-7-10-9-12
Based on reference [2]	7.77	60.90	54.48	120.60	149.63	189.59	235.15	259.85
Based on reference [8]	11.15	53.77	50.31	97.40	121.75	163.09	199.83	220.66
The results of this research	8.82	88.03	73.88	153.57	215.95	247.18	302.35	359.31

Figure 5- Comparison of total travel time for the routes connecting the [1–12] origin-destination pair at 95% confidence level.



	2-4-3-8-11	2-5-10-9-11	2-4-7-10-9-11	2-5-7-10-9-11
Based on reference [2]	34.69	256.23	325.58	321.97
Based on reference [8]	28.85	217.13	277.11	272.34
The results of this research	42.74	305.77	400.45	416.37

Figure 6- Comparison of total travel time for the routes connecting the [2–11] origin-destination pair at 95% confidence level.



	2-4-12	2-5-10-12	2-4-7-10-12	2-5-7-10-12	2-5-10-9-12	2-4-7-10-9-12	2-5-7-10-9-12
Based on reference [2]	31.70	26.53	131.08	129.05	170.18	244.78	241.58
Based on reference [8]	26.33	22.44	106.35	101.74	144.93	208.17	203.44
The results of this research	37.93	31.87	164.13	179.15	213.39	310.63	325.54

Figure 7- Comparison of total travel time for the routes connecting the [2–12] origin-destination pair at 95% confidence level