Fuzzy Mathematical Models for Maximizing Contractors’ and Clients’ Satisfaction by Considering Flexible Start Date of the Project

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Abstract

Resource-Constrained Multi-Project Scheduling Problem (RCMPSP) is considered a significant topic in project management studies and many kinds of research have been carried out in this field which have been proposed numerous approaches. However, in most of them, the viewpoints of clients and contractors, two important stakeholders of the project are not directly considered. The current research tries to introduce a new approach of RCMPSP in order to schedule the project portfolio and allocate the budget as a limited resource simultaneously. In this way, first, the client's and contractors’ budget satisfaction is defined. Then some budget allocation models have been proposed to maximize the clients’ and the contractors’ satisfaction. These models consider constraints such as the minimum cost required for each project, the maximum budget for each period, and the flexibility for the start date of each project. To illustrate the proposed models, a real case of the project portfolio is considered.

**Keywords:** Budget allocation; satisfaction; fuzzy linear programming; project management; Resource-Constrained Multi-Project Scheduling Problem (RCMPSP)

1. Introduction

The constrained resources in project management are considered an essential issue. One of the significant resources is the budget. In some projects, clients’ fund can radically change over time due to the uncertainty of the economy [1]. On the other hand, contractors rarely rely on their savings in projects, so the proper financing by the clients is contractors’ first concern because this is a fact that the constraints of the budget allocated to a project affect contractor’s indirect costs [2]. Therefore, it can be concluded that not only money is a vital project resource whose distribution method is one of the essential factors, but also the choice of suitable techniques for project portfolio programming is importance [3].
Resource-constrained Multi-Project Scheduling problem (RCMPSP) is a well-known concept for project portfolio programming to ensure the balanced access of all projects to resources in proper timing and it was derived from the Resource-constrained project scheduling problem (RCPSP) models. There are currently two main ways to solve RCMPSP problems. Firstly, all projects are assumed as a single major project and are solved using RCPSP methods. Despite the first one, in the second approach, all projects are considered independent with their exclusive critical path; the methods attempt to specify the start and end date of each project according to the resource-constrained [4] and projects are not considered as the precedence of each other. Therefore, projects can start in the midterm of another project on the contrary with the first approach. The second approach has been studied in many pieces of researches; it can be due to more realism and less attention in past researches and thus more opportunity for improvement [5].

In order to solve RCMPSP with independent projects approach (second approach), some researchers have proposed a dual-level approach. In this style, the project managers make low-level decisions independently to schedule activities, while senior managers make high-level decisions using a set of resources (total resources) to allocate [6]. In this case, the project portfolio manager focuses on the scheduling of all projects and specifies the amount of resources for each project at different intervals; the portfolio manager monitors only its implementation.

Regardless of what approaches are used, RCMPSPs are more complicated to solve by ordinary mathematical methods. Hence, several heuristic methods have been proposed in different studies. Most of the heuristic methods used in RCMPSP belong to the Priority Rule (PR) methods, and numerous studies have been conducted in this regard such as Gonçalves et al. [7] and Herroelen [8]. PRs represent managers’ tendencies for project scheduling which can be used separately or simultaneous [9]. The project managers need to make fast calculation [10]. Therefore, PRs are the tools which help managers achieve satisfaction.

The RCMPSP solution methods have some significant shortcomings. In some existing approaches, there is no difference between the independent projects and project portfolio, so it causes disagreements
between the project and portfolio manager. There is also a remarkable difference between research and practical stream in constrained resources situation [10]. On the other hand, due to the dynamic environment of the project in the real world, project scheduling may be restricted to notable uncertainty; for example, activities may be done with shorter or longer time, and resources may be temporarily unavailable [4]. Therefore, the practical schedule may not align with the models' output.

Another drawback of the previous RCMPSP methods is that they do not focus on stakeholders’ viewpoints. Allocating resources, especially financial resources, has a considerable impact on the performance of the project stakeholders. It is a fact that a project manager always competes with other project managers to get more resources, so the initial distribution of resources is based on the competition of project managers. It can result in dissatisfaction [11]. On the other hand, the client must respect the contractors’ capacities. Project managers and contractors make their decisions independently and autonomously to minimize their executive costs [12]. However, the benefits of collaboration will ultimately lead to greater satisfaction of them [13]. These facts show that considering the project managers and contractors’ viewpoints in budget allocations is extremely important. This issue must be addressed by considering several compromises to reach an acceptable optimal point [14].

One of the best ways to involve the stakeholder's viewpoints in decision making is to use the concept of satisfaction. But the main question is how satisfaction in budget allocation can be calculated? In this way Dubois and Fortemps [15] introduced the definition of constrain satisfaction in the resource allocation field so that each resource has a capacity with a lower and upper boundary as constrains. The allocation will be satisfied if the upper boundary is completely allocated and will not be satisfied if the lower boundary is not reached. So if the client's and contractors' viewpoints about the budget are converted into the interval form include minimum-maximum budget expectation, by Dubois and Fortemps approach, it is possible to calculate the satisfaction value of budget allocation according to the client's and contractors' viewpoints.

In this research, we propose a new approach to solve RCMPSP in which satisfaction is considered as the main objective. So three distinct goals are emphasized in this approach: 1) maximizing the client’s
satisfaction; 2) maximizing the contractors’ satisfaction; 3) maximizing the client's and contractors’ satisfaction simultaneously. Therefore, we classify the approach into three categories and develop optimization models for each category. These models are, in fact, a developing form of Dubois and Fortemps [15] satisfaction concept and Zimmermann's approach [16] which is maximizing the satisfaction derived from the budget allocation. The models distribute the budget as cumulative payment with flexibility at the start date of project versus starting all project in the first period of portfolio.

This article is organized in six sections. The second section consists of the literature review. The methodology can be found in the third section. Section four presents real case projects portfolio evaluation. Section five includes discussion of the results, and the last section contains the conclusion.

2. Literature review

Scheduling multi-projects with limited resources is a significant challenge for most of project-based companies. The managers should share common resources with the number of projects in order to achieve the goals defined by firm’s strategies. Therefore, solving RCMPSP is a substantial concern of managers. Pritsker et al. [17] made the first attempts to solve such problems. They presented a zero-one model and claimed that their model could solve many real-world problems regarding scheduling in limited resources, solving the job shop problems, etc. Three limitations of “decreased time of all projects, decreased the project portfolio makespan and decreased penalties” were considered in their issue.

Basically, RCMPSPs are more complicated than can be solved by traditional mathematical methods [18]. Accordingly, numerous studies have suggested the use of heuristic and meta-heuristic approaches. For example, Gonçalves et al. [7] and Tseng [19] utilize genetic algorithm to solve RCMPS problems. The results obtained from their study conducted on different projects represented the acceptable performance of the genetic algorithm. Majazi Dalfard and Ranjbar [20] used a genetic algorithm to solve scheduling problems and compared the results with the traditional method. Can and Ulusoy [21]
developed a two-step method to allocate the budget. In their method, all projects were initially changed into macro activities and were solved using GA with maximum Net Present Value (NPV) limit. The shortest execution makespan was determined by owing to the start time and the resources specified for each project.

In this stream, most of researchers have emphasized using PRs as an adequate tool to solve theirs heuristic and meta-heuristic methods. Gonçalves et al. [7] stressed the development of heuristic methods based on PRs. Moreover, Chakrabortty et al. [22] stated that doing activities in proper time (based on resource allocation) have a sustainable effect on project complication time. Therefore, it is essential that a suitable PR be selected. Kurtulus and Narula [23] introduced PRs for project portfolio scheduling and compared their performance with other PRs. Their results revealed that when a problem is small, the rules which are based on resource usage are suitable, and when the problem is big, techniques based on slack reduction are appropriate.

However, there are significant discrepancies between the results of the researches. For example, using 6 heuristic RCPS problem-solving methods, Russell [24] showed that project time reduction methods would not necessarily lead to an increase in NPV. In contrast, Chiu and Tsai [25] examined a heuristic approach and concluded that any PR which minimized project delays could result in increased NPV. Therefore, it could be helpful to use PR that minimized project runtime. Nevertheless, Herroelen [26] pointed out that management should focus on sustainability rather than reduction of project portfolio time in order to create a stable scheduling program with a desirable duration to prevent small disruptions throughout the program.

In spite of all the efforts made to study the PRs, the important question is which PR is appropriate. In order to answer this question, Browning and Yassine [5] reviewed various PRs and compared their performance. They introduced some PRs. They emphasized that one PR could not be considered superior or inferior. Also, Vázquez et al. [27] studied a lot of PRs and concluded that it is difficult to categorize and identify all PRs for all working conditions. Furthermore, Wang et al. [4] reviewed the performance of PRs in a contingency state. According to the literature, 20 PRs were used to solve
RCMPS problems. Their results showed that the performance of the PRs depends on the given constraints and they have significantly different. They concluded that it is hard to say that one PR is suitable for all modes.

On the other hand, regardless of what method or PR has been used, two general approaches are used to solve RCMPS problems: (1) single project approach which by using dummy tasks and precedence arrows converts multi-projects into a mega-project. (2) Multi-projects approach that keeps up the projects with their exclusive critical path and tries to allocate them common resources [28]. Nabipoor Afruzi et al. [29] and Zhang et al. [30] used the first approach to allocate resources of multi-projects. However, Kurtulus and Davis [31] presented a plan to examine the performance of a single project approach versus multi-project. Their research showed that the multi-project approach performed much better than the time when several projects were converted into a virtual single project. Also, Lova et al. [32] concluded that a parallel scheduling generation scheme along with a multi-project approach could lead to more appropriate results in allocating limited resources compared to the conversion of several projects into a virtual single project.

In conclusion, reviewing literature shows that although a wide range of approaches has been proposed, but based on our studies, mostly the objective of methods have not concentrated on the viewpoints, experiences, and satisfaction of project stakeholders such as project managers and contractors. Indeed, the methods try to achieve objectives like shortening the makespan, reducing penalties, reducing resource consumption, or increasing some factors like NPV, while attaining such goals can only be obtained with the participation of project executive agents. Therefore, planning should be accompanied by their satisfaction which is not considered in former researches. Thus, involving the concept of satisfaction into the RCMPSP can be examined as a significant point in the development of past methods.
3. Methodology

The fuzzy mathematical models presented in this study are obtained by developing Zimmermann's approach [16] and are classified into three categories. The first category contains those optimization models whose purpose is to allocate the budget to the projects to maximize the client’s satisfaction. The aim of the models proposed in the second category is to maximize the contractors’ satisfaction with the budget allocated to the projects in successive periods. Finally, the third category includes models which allocate the budget with the aim of maximizing the satisfaction of the client and the contractors, simultaneously. These models were based on the following assumptions.

1) In the models which the projects are allowed to start by delay, the cost required for the same phases of each project may vary in different periods;

2) The projects must be completed in successive and nonstop periods. In other words, after allocating the first part of the cost to a project, jumping over the next period without allocating any part of the rest cost needs to complete the project is not allowed.

In the following, 6 models are presented and are described in three different categories.

3.1. Budget allocation with the aim of maximizing the client’s satisfaction

The budget must be allocated to the projects required the specific budget in each period. However, the optimal allocated budget, from the client's point of view, may be a certain amount for each period in a way that overpayment reduces the client's satisfaction. Fig.1 shows the client's satisfaction of the total budget paid in the period \( j \), where \( x_{ij} \) is the budget allocated to the project \( i \) in the period \( j \) \((i = 1, ..., n; j = 1, ... t)\), \( n \) is the number of projects, \( t \) is the number of periods, \( \sum_{i=1}^{n} x_{ij} \) is total budget paid in the period \( j \), \( B^L_j \) and \( B^U_j \) are the highest amount of satisfying budget and the lowest amount of unsatisfying budget which can pay in period \( j \) according to the client's point of view, \( \tilde{A} \) is the fuzzy set of client’s satisfaction from the budget paid in different periods, \( \mu_{\tilde{A}}(\sum_{i=1}^{n} x_{ij}) \) is the client’s satisfaction degree from the budget paid in the period \( j \). If the budget paid in the period \( j \) \((\sum_{i=1}^{n} x_{ij}) \) exceed \( B^L_j \), the
client's satisfaction decreases gradually in a way that the budget payment equal to $B_j^U$ is not satisfying for the client.

The start dates of projects can be either unchangeable or flexible. Model I has presented for situation that the start dates of projects are fixed and unchangeable. However, if the start dates are flexible we can use model II given in Appendix A. These models, both, were proposed based on the cumulative budget assumption. In another word, we assumed that the clients give their satisfaction from the budget allocated in each period and previous periods cumulatively.

Model I

$$\begin{align*}
\text{max } & \lambda \\
\text{s.t. } & \\
\lambda & \leq \frac{AB_j^U - \sum_{j=1}^{n} \sum_{i=1}^{t} x_{ij}}{AB_j^U - AB_j^L}, \quad \forall j, \quad (2) \\
\sum_{j=1}^{t} x_{ij} & = C_i, \quad \forall i, \quad (3) \\
x_{ij} & \geq C_{ij}, \quad \forall i, j \quad (4)
\end{align*}$$

Where:

$\lambda$ the client’s fixed satisfaction;

$x_{ij}$ the budget paid for the project $i$ in the period $j$ ($i=1-n; j=1-t$);

$AB_j^L$ the satisfying cumulative budget from the client point of view for allocating up to the end of period $j$;

$AB_j^U$ the unsatisfying cumulative budget from the client point of view for allocating up to the end of period $j$;

$C_i$ total cost of the project $i$;

$C_{ij}$ the minimum cost required for the project $i$ at the period $j$;

$n_i$ the number of successive periods required to complete the project $i$.

Models I and II aim to maximize the client’s satisfaction which cannot exceed the client's satisfaction in each period. It is guaranteed by the objective function (1) and the constraints (2) in all models.
Moreover, the total budget paid to the project \( i \) at all periods should be equal to the total cost of the project guaranteed by the constraint (3) in these models. Also, the minimum budget required for projects in each period must be provided. It is guaranteed by constraint (4) in both models. The model I is used when the start date of each project is fixed and unchangeable. However, in Models II, the start date of the projects may vary so that their optimal start date is calculated after solving the given models. Therefore, the binary variable \( y_{ij}^k \) is used which indicates that project \( i \) starts at period \( k \) or doesn't start. However, the project \( i \) has only one start period which is guaranteed by constraint (5).

### 3.2. Budget allocation with the aim of maximizing the contractors’ satisfaction

Contractor \( i \) requires a determined budget at period \( j \) to progress its project. If, the allocated budget is lower than the required budget, its satisfaction decreases. Fig. 2 represents the satisfying budget for the contractor \( i \) at the period \( j \), where \( x_{ij} \) is the allocated budget to the project \( i \) at the period \( j \) \((i = 1 - n, j = 1 - t)\). \( U_{ij} \) is the minimum budget for the project \( i \) at the period \( j \) that completely satisfies the contractor \( i \). \( L_{ij} \) is the maximum budget for the project \( i \) at period \( j \) which is completely unsatisfying for the contractor \( i \). \( \tilde{B} \) is the fuzzy set of the satisfaction of the contractor \( i \) and \( \mu_i(x_{ij}) \) is the satisfaction degree of the contractor \( i \) depended on the allocated budget at the period \( j \).

\( U_{ij} \) is the minimum value for \( x_{ij} \) with the membership degree of 1 in fuzzy set \( \tilde{B} \). If the budget allocated to the project \( i \) in the period \( j \) \((x_{ij}) \) is less than \( U_{ij} \), the satisfaction of the contractor \( i \) decreases. \( L_{ij} \) is the maximum value for \( x_{ij} \) with the membership degree of zero in fuzzy set \( \tilde{B} \) which has no satisfaction for the contractor \( i \) at the period \( j \).

As same as section 3.1, the start dates of projects can be unchangeable or flexible. Model III has presented for situation that the start dates of projects are fixed and unchangeable. However, if the start dates are flexible we can use model IV given in Appendix B. These models, both, were proposed based on the cumulative budget assumption. In other word, we assumed that the contractors give their satisfaction from the budget allocated in each period and previous periods cumulatively.
Model III

\[ \max \lambda \] \hspace{1cm} (8)

s.t.

\[ \lambda \leq \sum_{h=1}^{j} x_{ih} - AC_{ij}^L \quad \forall i, j, \] \hspace{1cm} (9)

\[ \sum_{j=1}^{n} x_{ij} \leq B_j \quad \forall j, \] \hspace{1cm} (10)

\[ \sum_{j=1}^{t'} x_{ij} = C_i \quad \forall i, \] \hspace{1cm} (11)

\[ x_{ij} \geq 0 \quad \forall i, j \] \hspace{1cm} (12)

Where:

\( \lambda \) the contractors’ final satisfaction;

\( B_j \) the total budget assigned to the period \( j \);

\( AC_{ij}^L \) the unsatisfying cumulative cost from the contractor point of view for the project \( i \) up to the end of the period \( j \);

\( AC_{ij}^U \) the satisfying cumulative cost from the contractor point of view for the project \( i \) up to the end of the period \( j \);

The models III & IV aim to maximize the contractors’ total satisfaction which cannot exceed the contractors’ satisfaction in each period. It is guaranteed by the objective function (8) and the constraints (9) in these models. Moreover, the total budgets allocated to the projects at each period cannot exceed the budget assigned to that period; it is guaranteed by the constraint (10) in both models. Finally, the sum of budgets paid for the project \( i \) in all periods should be equal to the total cost needed for the project; it is guaranteed by the constraint (11) in these models.

The model III is used when the contractors determine their satisfying costs up to the end of each period cumulatively; the start dates of the projects are unchangeable. In models IV, the start date of the projects vary. Like models II, this model needs to use the binary variable \( y_{ij}^k \) and the constraint (13). In other words, the constraint (13) in model IV guarantees that the project \( i \) has only one start date. \( M \) in this model is a large positive number.
3.3. Budget allocation with the aim of maximizing both client's and contractors’ satisfaction

In budgets allocation, client and contractors sometimes follow contradictory goals. In other words, the client usually prefers to allocate the majority of the budget in the final periods of the project's lifecycle, while contractors often prefer to receive most of the budget in the initial periods. In this section, optimization models are proposed to allocate the budget; they maximize the satisfaction of both the client and the contractors. In other words, these models are a combination of the models presented in A and B. Two conditions are considered here, and accordingly, different models are presented; the start date of projects can be fixed and unchangeable versus flexible and variable. If the start dates are fixed, the model V presented as follow is used. However, if the start dates are flexible, the model VI given in Appendix C is applied. In this case, all parameters and variables excepted λ are those given in models I - IV. Here, λ represents the final satisfaction of both client and contractors.

Model V

\[
\text{max } \lambda \\
\text{s.t.} \\
\lambda \leq \frac{AB^U_j - \sum_{i=1}^{n} \sum_{j=1}^{L} x_{d_j}}{AB^U_j - AB^I_j} \quad \forall j, \quad (15)
\]

\[
\lambda \leq \sum_{h=1}^{j} x_{ih} - AC^U_{ij} - AC^L_{ij} \quad \forall i,j, \quad (16)
\]

\[
\sum_{j=1}^{n} x_{ij} = C_i \quad \forall i, \quad (17)
\]

\[
x_{ij} \geq 0 \quad \forall i,j. \quad (18)
\]

The models V & VI aim to maximize both client and contractors’ final satisfaction simultaneously which cannot exceed the client and contractors’ satisfaction at each period. It is guaranteed by the objective function (15) and the constraints (16) and (17) in both models. Moreover, Constraints (18) in these models ensure that the total budgets for the projects are allocated. Finally, constraints (20) show that
every project has only one start date. The model $V$ is used when the start dates of the projects are unchangeable. However, in models $VI$, the start dates of the projects are varied.

4. Numerical illustration

In order to illustrate the presented models, a real project portfolio of Khuzestan Steel Company consist of five projects is used. These projects “A, B, C, D and E” require total costs as shown in Table (1), and must be completed in specified successive periods. The company as the client may express its satisfaction of allocable budgets in different ways; fixed budget for each period or fuzzy cumulative budget up to the end of each period. These are illustrated by the data given in Tables (2), and (3). The fuzzy specification for the fuzzy budgets given in Table (3) are corresponding to "should be less than" and the membership function form has been shown as Fig 1.

Contractors may also express their satisfaction of allocated costs for their projects in different ways; fixed cost for each period or fuzzy cumulative cost up to the end of each period. These are illustrated by contractors given in Tables (4) and (5). The fuzzy specification for the fuzzy costs given in Table (5) is corresponding to "should be more than" and the membership function form has been shown as Fig. 2.

Depending on the satisfaction described by the client and the contractors (Tables 3 to 5), six different examples were modeled and solved. It should be noted that in odd examples, the projects start at period 1. In even examples, however, the start dates of the projects should not be necessarily the first period, although they all must finish up to the eight periods. In other words, in these examples, project B which requires six successive periods is allowed to start at period 1, 2 or 3. Project C which requires four successive periods can start at period 1, 2, 3 or 4, etc. However, project A which requires eight consecutive periods should start at period 1 in all examples. In these examples, it is also assumed that if a project starts with a delay, its costs do not change.

The aim of examples 1 and 2 is to maximize the client’s satisfaction in different conditions. The optimal solution to these examples is shown in Table (6). However, examples 3 and 4 aim to maximize the
contractors’ satisfaction; the optimal solution to these examples is given in Table (7). Finally, examples 5 and 6 which try to maximize the satisfaction of both client and contractors (overall satisfaction) under different conditions; the optimal solution is given in Table (8).

4.1. Example 1:

Assume that the client expresses the satisfaction of the allocable budget in the form of fuzzy budget cumulatively up to the end of each period, where the contractors express their satisfaction as the fixed cost in each period (Tables 3 and 4). In this case, model I is used to maximize the client's satisfaction.

4.2. Example 2:

In example 1, if projects are allowed to start from each period, the client’s maximum satisfaction can be obtained by solving the model II.

4.3. Example 3:

The client may express the satisfaction as the fixed budget for each period (Table 2), while contractors may express their satisfaction in a fuzzy manner up to the end of each period (Table 5). In this case, model III is used to maximize contractors’ satisfaction.

4.4. Example 4:

Providing that the projects are flexible to start from each period in example 3, contractors’ maximum satisfaction is obtained from the model IV.

4.5. Example 5:

Assume that both the client and the contractors express their satisfaction in a cumulative fuzzy up to the end of each period, in this case, the maximum total satisfaction is obtained by solving the model V.
4.6. Example 6:
Provided that the projects are allowed to start from each period in example 5, the total satisfaction is obtained by solving the model $VI$.

5. Discussion

Project managers focus on solving resource-constrained project portfolio scheduling problems. Previous studies have emphasized the application of various PRs. This study focused on the satisfaction for allocating the resources to the projects. Satisfaction is a binary relationship between resources and constraints \cite{33}, and it means that the resource is allocated in a way that, the allocated resource is accepted by the distributor (the client) and the recipient (the contractors). The models presented in this paper focused on the budget distribution and considered the tolerances determined by the client and the contractors so as to increase their satisfaction and it conforms to the model proposed by Dubios and Fortemps \cite{15} for solving the constraint satisfaction problems. The features and objectives of these models are summarized in Table (9).

In current research, three different categories allocation model in two different cases (start date of project is constant or flexible) have been presented. In first category the client's satisfaction has been just measured and the contractors' viewpoints has not been considered. On the contrary, only the contractors' satisfaction has been calculated in second category and the client's satisfaction has not been studied. These categories can be used depending on what strategy is followed by the portfolio manager. But if the manager wants to consider the client and contractors' point of view simultaneously, the third category is proposed. These models are applied independently so that the manager's strategies define which one should be used.

According to the results, models with flexibility at the start dates result in more satisfaction than those with the constant start dates. For example, in example 1 in which projects must start from the first period, the client's satisfaction is 0.109, while it increases to 0.556 by making the start dates flexible in
example 2. In example 2, projects B starts at period 2 and other projects start at periods 3, which allow
the client to offset the budget deficit in the first period in subsequent periods. In case the budget
presented cumulatively, budget deficit in each period can be compensated in subsequent cumulative
payments.

The models presented in this research can be compared with the two-stage methods proposed by
Kurtulus and Davis [31] and Lova et al. [32] which the portfolio manager defines the restrictions for
each project according to resource constraint at first stage. Then, the project managers, according to
their priorities, use the results of the proposed model as a constraint and carry out their project
scheduling independently. In other words, the results of the model presented in this paper can be input
data for RCPSP models such as those presented by Kaveh and Vazirinia [34], Tavaana et al. [35],
Paraskevopoulos et al. [36].

The results of the real case show that the models avoid delays caused by resource deficit by providing
sustainable budget during the implementation. It is consistent with that of the study conducted by
Chakrabortty et al. [22] who stated that choosing an appropriate method could affect the sustainability
of the project time. It is also mentioned by Herroelen [26] who emphasized that the primary purpose of
a RCMPS solution should be to stabilize scheduling.

Adhau et al. [11] also pointed to the practical problems and lack of information exchange. These
problems have been solved in the models presented in this paper because the client's and the contractors’
viewpoints have been considered simultaneously. Wang et al. [37], Wanke et al. [38] and Mirzaei et al.
[39] also pointed to the data uncertainty; it was tried in this study to regard ambiguity in data as fuzzy
concepts in the proposed models.

6. Conclusions and future research

The models proposed in this research give an approach to solve RCMPSP based on maximizing the
client's and the contractors’ satisfaction. Three distinct goals were considered in this research:
maximizing the client’s satisfaction, maximizing the contractors’ satisfaction and maximizing the client's and contractors’ satisfaction simultaneously. In each case, cumulative payments were assumed. Moreover, two ways were considered for each payment assumption: flexibility at the start dates of the projects and starting all the projects in the first period. The results of real case projects portfolio using the proposed models showed that the proposed approach had a high ability to allocate budget appropriately. Due to the simultaneous participation of the two main projects partners, i.e., the client and the contractors, the present models are more likely to be applicable. Because input information is provided by the stakeholders, their commitment to complying with the limits on the implementation phase increases. Moreover, employing the models presented in this research leads to a reduction in many of the project managers' objections and lobbies in funding. Also, improving the satisfaction level will increase the focus on lower-level scheduling.

Although, the proposed models in current research reveal appropriate abilities, nevertheless, they have some constraints in the application. Firstly, the data gathering in these models needs considerable collaboration between project managers and contractors. This issue needs coordination and time. Secondly, the models are solved by mathematical programs that the computation of them can be complicated in large scale multi-projects and need high-performance computers. Finally, the satisfaction of the budget is directly related to the experience of the client and contractors. It causes that the performance of the models has been influenced by users.

One method cannot indeed be used to solve all RCMPSP problems, but having a variety of methods in different situations improves the scheduling outcomes. Therefore, it is suggested in future researches that the proposed approach is combined with other PRs. In the present model, the weight of all projects was considered equal regarding importance; thus, it is suggested that projects with different weights and importance are considered in the model. On the other hand, in the model assumptions, the implementation and payment periods were considered sequential and consecutive; in practice, however, delays may occur during the execution of the projects. To improve, the possibility of delays in projects or changes in scheduling can also be examined. Furthermore, in this research, the payment is assumed
cumulatively which can be done in some projects individually. It is suggested that these models are examined by different payments.

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**References**


**Figures:**

Fig (1): Membership function of client's satisfaction of the budget paid in the period $j$

Fig (2): Membership function of the satisfaction of contractor $i$ in the period $j$

**Tables:**

Table 1: The information of projects portfolio of Khuzestan Steel Company

Table 2: The fixed budget for each period

Table 3: The fuzzy allocable cumulative budgets up to the end of each period

Table 4: The fixed costs needed for projects in each period

Table 5: The fuzzy cumulative cost for projects up to the end of each period

Table 6: Optimal budget allocating for examples 1 & 2 in order to maximize the client's satisfaction

Table 7: Optimal budget allocating for examples 3 & 4 in order to maximize the contractors’ satisfaction

Table 8: Optimal budget allocating for examples 5 & 6 in order to maximize both the client's and contractors’ satisfaction

Table 9: Features of examples 1 to 6

---

**Figures**

![Fig 1](image1)

Fig 1
Fig 2

### Tables

#### Table 1

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APPENDIX

APPENDIX A

The model used for maximizing the client’s satisfaction when the start dates of projects are flexible

Model II

\[
\text{max } \lambda \\
\text{s.t.}
\]

\[
\lambda \leq \frac{AB_j^{\prime} - \sum_{i=1}^{n} \sum_{l=1}^{j} \sum_{k=1}^{l-1} x_{i,l,k}^k}{AB_j^{\prime} - AB_j} \quad \forall j, \quad (A1)
\]

\[
\sum_{j=1}^{k-1} x_{i,j}^k - C_i^k \times y_i^k = 0 \quad \forall i, \quad (A2)
\]

\[
x_{i,j}^k - C_{ij} \times y_i^k = 0 \quad \forall i,j,k, \quad (A3)
\]

\[
\sum_k y_i^k = 1 \quad \forall i, \quad (A4)
\]

\[
x_{i,j}^k \geq 0 \quad \forall i,j, \quad (A5)
\]

\[
y_i^k = 0, 1 \quad \forall i,k \quad (A6)
\]

where:

\( x_{i,j}^k \) the budget paid for the project \( i \) in the period \( j \) provided that the project starts at the period \( k \);

\( y_i^k \) \{ 0 if project \( i \) doesn't start at period \( k \\
1 if project \( i \) starts at period \( k \) \};

\( C_{i,j}^k \) the cost required for the project \( i \) at the period \( j \) provided that the project starts at the period \( k \);

\( C_i^k \) total cost of the project \( i \) provided that the project starts at the period \( k \).

APPENDIX B

The model used for maximizing the contractors’ satisfaction when the start dates of projects are flexible

Model IV

\[
\text{max } \lambda \quad (B8)
\]

s.t.

\[
\frac{\sum_{i=1}^{n} \sum_{l=1}^{j} \sum_{k=1}^{l-1} x_{i,l,k}^k}{AB_j^{\prime} - AB_j} \quad \forall j, \quad (B9)
\]

\[
\sum_{j=1}^{k-1} x_{i,j}^k - C_i^k \times y_i^k = 0 \quad \forall i, \quad (B10)
\]

\[
x_{i,j}^k - C_{ij} \times y_i^k = 0 \quad \forall i,j,k, \quad (B11)
\]

\[
\sum_k y_i^k = 1 \quad \forall i, \quad (B12)
\]

\[
x_{i,j}^k \geq 0 \quad \forall i,j, \quad (B13)
\]

\[
y_i^k = 0, 1 \quad \forall i,k \quad (B14)
\]
\[
- \sum_{h=k}^{j} x_{ih} + (AC_{ij}^{kU} - AC_{ij}^{kL}) \times \lambda \leq M (1 - y_{i}^{k}) - AC_{ij}^{kL} \quad \forall i, j, k , \tag{B9}
\]
\[
\sum_{i=1}^{n} \sum_{k=1}^{j} x_{ij}^{k} \leq B_{j} \quad \forall j, \tag{B10}
\]
\[
\sum_{i=k}^{k+n_{i} - 1} x_{ij}^{k} - C_{i}^{k} \times y_{i}^{k} = 0 \quad \forall i, k , \tag{B11}
\]
\[
x_{i}^{k} \geq 0 \quad \forall i, j , \tag{B12}
\]
\[
\sum_{i} y_{i}^{k} = 1 \quad \forall i, \tag{B13}
\]
\[
y_{i}^{k} = 0, 1 \quad \forall i, k \tag{B14}
\]

where:

\( AC_{ij}^{kL} \) the cumulative satisfying cost for the project \( i \) up to the end of the period \( j \) provided that the project starts at the period \( k \);

\( AC_{ij}^{kU} \) The maximum unsatisfying budget for the project \( i \) up to the end of the period \( j \) provided that the project starts at the period \( k \);

**APPENDIX C**

The model used for maximizing the client and contractors’ satisfaction when the start dates of projects are flexible

Model XII

\[
\max \lambda \tag{C15}
\]

s.t.

\[
\lambda \leq \frac{AB_{j}^{U} - \sum_{i=1}^{n} \sum_{k=1}^{j} \sum_{k=1}^{L} x_{i}^{k}}{AB_{j}^{L} - AB_{j}^{U}} \quad \forall j , \tag{C16}
\]
\[
- \sum_{h=k}^{j} x_{ij}^{k} + [AC_{ij}^{kU} - AC_{ij}^{kL}] \times \lambda \leq M (1 - y_{i}^{k}) - AC_{ij}^{kL} \quad \forall i, j, k , \tag{C17}
\]
\[
\sum_{i=k}^{k+n_{i} - 1} x_{ij}^{k} - C_{i}^{k} \times y_{i}^{k} = 0 \quad \forall i , \tag{C18}
\]
\[
x_{i}^{k} \geq 0 \quad \forall i, j , \tag{C19}
\]
\[
\sum_{i} y_{i}^{k} = 1 \quad \forall i , \tag{C20}
\]
\[
y_{i}^{k} = 0, 1 \quad \forall i, k \tag{C21}
\]
Biography

Farhad Khazaeli received his BSc degree from Islamic Azad University Ahvaz branch in civil engineering and MS degree from Islamic Azad University Science and Research branch in engineering and construction management. He currently studies in engineering and construction management in PhD course. His research interests include, project management, budget allocating, meta-heuristic optimization methods and multi-criteria decision-making.

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