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# Stepwise pricing in evaluating revenue efficiency in data envelopment analysis: A case study of power plants

Z. Moghaddas<sup>a</sup>, M. Vaez-Ghasemi<sup>b</sup>, F. Hosseinzadeh Lotfi<sup>c</sup>, and  
 R. Farzipoor Saen<sup>d,\*</sup>

a. *Department of Mathematics, Qazvin Branch, Islamic Azad University, Qazvin, Iran.*

b. *Department of Mathematics, Rasht Branch, Islamic Azad University, Gilan, Iran.*

c. *Department of Mathematics, Science and Research, Islamic Azad University, Tehran, Iran.*

d. *Faculty of Business, Sohar University, Sohar, Oman.*

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 (MPI);  
 Piece-wise linear  
 functions.

**Abstract.** Data Envelopment Analysis (DEA) technique is widely applied for performance assessment of Decision-Making Units (DMUs). Revenue Efficiency (RE) evaluation is one of the controversial subject matters that can be performed through DEA context. The amount of production and its prices are crucial factors in the RE. The classical DEA models consider the prices fixed and known, which are not the case in the real world. Also, the classical DEA models consider linear pricing in revenue assessment. However, most of real-world problems deal with nonlinear prices. This paper evaluates the RE given the piecewise linear theory in non-competitive situations. In doing so, a stepwise pricing function is introduced that allows the prices to vary with respect to the amount of production. As an innovative idea, a more accurate mathematical modeling for the RE is proposed. A dynamic weight function is defined in the maximum revenue optimization model that no longer considers prices fixed. A case study validates our proposed model.

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## 1. Introduction

Data Envelopment Analysis (DEA) technique is a Linear Programming (LP) problem that assesses the performance of Decision-Making Units (DMUs) involving multiple inputs/outputs. DEA is used in dealing with theoretical and practical problems. The efficiency measure was first introduced by Farrell [1] and then, developed by Charnes et al. [2] in the DEA framework.

Following the presentation of the first DEA models, different modifications in terms of many factors/aspects have been provided to strengthen the power of DEA. Kuosmanen and Post [3] computed relative efficiency score bounds considering upper and lower scores. They noted that these bounds reveal more accurate approximations for the relative efficiency score. They computed these bounds using LP problems. Kuosmanen and Post [4] corrected existing technical error in their previous work. Mostafaei and Saljooghi [5] considered the cost efficiency measure in the presence of inadequate price details. Chakraborty et al. [6] emphasized that the products' requirements would alter over time. Fang and Li [7] assessed the cost efficiency in the presence of uncertain prices. They utilized cone-ratio models in DEA with the price information and added weight bounds to the model. Also, Mozaffari

\*. *Corresponding author. Tel.: +968 26850100;  
 Fax: +96826720102  
 E-mail addresses: Zmoghaddas@qiau.ac.ir (Z. Moghaddas);  
 mohsen.vaez@iaurasht.ac.ir (M. Vaez-Ghasemi);  
 farhad@hosseinzadeh.ir (F. Hosseinzadeh Lotfi);  
 farzipour@yahoo.com (R. Farzipoor Saen)*

et al. [8] introduced cost and revenue models in DEA. Ghiyasi [9] introduced inverse DEA models for assessing both cost and revenue values. Moreover, Fang and Li [10] considered a duality study to build theoretical attributes of envelopment and multiplier models of cost efficiency assessment assuming uncertain price. Aparicio et al. [11] demonstrated that an output-oriented version of the weighted additive model could satisfactorily ensure Revenue Efficiency (RE). They introduced an inequality to define the market output price vector. Sahoo et al. [12] claimed that firms' efficiency as well as inputs and outputs varying prices were questionable. They developed new directional measures for assessment of cost and RE. According to Aparicio et al. [13], in the case that firms face price fluctuations, the overall inefficiency measurement and decomposition would be of great importance. Cook and Zhu [14] suggested a piecewise linear pricing DEA model to evaluate the relative efficiency of DMUs. Hosseinzadeh Lotfi et al. [15] developed a modified version of the DEA model that derived suitable benchmarks for inefficient DMUs.

The RE is one of the influential indices for managers and analysts who sought new strategies for gaining more benefits. Konara et al. [16] discussed the RE in banking and emerging markets. Deng et al. [17] utilized DEA to assess the RE of Spanish hotels. Cao et al. [18] studied the decreasing marginal revenue analysis in the agriculture sector.

In the previous RE analysis by DEA models, the prices are assumed to be known and fixed for the inputs/outputs of DMUs. However, the prices are variable in real-world problems. Aroche-Reyes [19] reviewed some essential specifications of the price designation methods while considering an input-output model with focus on internal price designation procedure. Johnson and Ruggiero [20] considered a nonparametric measurement of allocative efficiency. They assumed that the output prices were endogenous. Moura [21] presented a two-sector model with two key ingredients for assessing investment shocks with endogenous relative prices. Din and Sun [22] assessed the endogenous choice of prices while taking quantities into account. Cellini et al. [23] proposed a dynamic model of price and quality competition for assessing the cause of competition on quality.

The objective of this paper is to develop a revenue efficiency DEA model to evaluate the relative efficiency of DMUs when the output prices are not fixed. In the conventional DEA models, to assess the RE, linear pricing is assumed. However, in real-world problems, the variables have nonlinear behavior. This study assumes a non-competitive context for the RE assessment. This is the main contribution of this paper that has been considered for the first time in DEA. In doing so, here, stepwise pricing of weight function

is introduced to deal with the nonlinear behavior of variables. A mixed integer LP model is developed. Then, the developed model is used for assessing RE using Malmquist Productivity Index (MPI). The MPI analyzes the progress and regress of DMUs. This paper makes the following contributions:

- This paper assumes non-competitive context for the RE assessment;
- For the first time, a DEA model is introduced to consider the nonlinear behavior of variables;
- It is shown that linear pricing does not adequately define the inherent concept of variables;
- In our new model, the prices are not assumed fixed;
- A stepwise pricing method is developed considering the theory of piece-wise linear functions;
- A case study is given.

This paper proceeds as follows: In Section 2, the DEA preliminaries are reviewed. Section 3 presents our new model. A case study is given in Section 4. Managerial implications are discussed in Section 5. Section 6 concludes the paper.

## 2. DEA preliminaries

### 2.1. Revenue Efficiency (RE)

DEA is a mathematical method for assessing the performance of a set of DMUs. One of the hot applications of DEA is to calculate the RE. Capability of producing maximum outputs given current inputs is called RE. Here, a concise review of RE is given in the DEA context. DMU<sub>*l*</sub> is the DMU under evaluation. The used notations in this paper are as follows:

- $x_{ij}$ : The *i*th input of DMU<sub>*j*</sub>
- $y_{rj}$ : The *r*th output of DMU<sub>*j*</sub>
- $x_{il}$ : The *i*th input of DMU<sub>*l*</sub>
- $y_{rl}$ : The *r*th output of DMU<sub>*l*</sub>
- $\lambda_j$ : Intensifier variables for DMU<sub>*j*</sub>
- $\varphi$ : The maximum increase in all outputs
- $\theta$ : The maximum decrease in all inputs
- $y_r$ : The *r*th output variable
- $D^t(x_l^t, y_l^t)$ :  $\theta^*$  when DMU<sub>*l*</sub> and technology are in period *t*
- $D^{t+1}(x_l^t, y_l^t)$ :  $\theta^*$  when DMU<sub>*l*</sub> is in period *t* and technology is in period *t* + 1
- $D^t(x_l^{t+1}, y_l^{t+1})$ :  $\theta^*$  when DMU<sub>*l*</sub> is in period *t* + 1 and technology is in period *t*
- $D^{t+1}(x_l^{t+1}, y_l^{t+1})$ :  $\theta^*$  when DMU<sub>*l*</sub> and technology are in period *t* + 1

- $PL.RE.MPI(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t)$ : Piece-wise linear revenue Malmquist productivity index
- $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t)$ : Malmquist productivity index
- $x_{ij}^f$ : The  $i$ th input of DMU $_j$  in period  $f$ ,  $f \in \{t, t+1\}$
- $y_{rj}^f$ : The  $r$ th output of DMU $_j$  in period  $f$ ,  $f \in \{t, t+1\}$
- $x_{il}^k$ : The  $i$ th input of DMU $_l$  in period  $k$ ,  $k \in \{t, t+1\}$
- $y_{rl}^k$ : The  $r$ th output of DMU $_l$  in period  $k$ ,  $k \in \{t, t+1\}$
- $y_r^{k_r}$ : The  $k_r$ th element of  $r$ th output variable
- $M$ : A big scalar
- $t_r^{k_r}$ : The  $k_r$ th element of  $r$ th output for the variable  $t$
- $v_{k_r}$ : The  $k_r$ th binary variable
- $y_r^{q_{k_r}}$ : The  $k_r$ th element of the  $r$ th output variable for DMU $_j$  in period  $q$ ,  $q \in \{t, t+1\}$
- $y_{rj}^{f_{k_r}}$ : The  $k_r$ th element of the  $r$ th output of DMU $_j$  in period  $f$ ,  $f \in \{t, t+1\}$
- $y_{rj}^{k_r}$ : The  $k_r$ th element of the  $r$ th output of DMU $_j$

Assume that there are  $n$  DMUs with  $m$  inputs and  $s$  outputs that are semi-positive vectors. For each DMU $_j$ , the input and output vectors are denoted by  $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$  and  $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$ , respectively, for all  $j$ 's.

To evaluate the DMU under evaluation (DMU $_l$ ) in constant returns to scale environment, Charnes et al. [2] proposed the following LP problem:

$$\begin{aligned}
 & \max \quad \varphi \\
 & \text{s.t.} : \quad \sum_{j=1}^n x_{ij} \lambda_j \leq x_{il} \quad i = 1, \dots, m, \\
 & \quad \sum_{j=1}^n y_{rj} \lambda_j \geq \varphi y_{rl} \quad r = 1, \dots, s, \\
 & \quad \lambda_j \geq 0 \quad j = 1, \dots, n.
 \end{aligned} \quad (1)$$

As addressed by Cooper et al. [24], RE can be obtained using the following procedure which leads to solving the following LP problem. To estimate the RE, output prices are assumed fixed and known although it is possible for them to change from one DMU to another DMU. Model (2) implies maximal revenue model, as stated by Wang et al. [25]:

$$\begin{aligned}
 & \max \quad \sum_{r=1}^s w_r y_r \\
 & \text{s.t.} : \quad \sum_{j=1}^n x_{ij} \lambda_j \leq x_{il} \quad i = 1, \dots, m,
 \end{aligned}$$

$$\begin{aligned}
 & \sum_{j=1}^n y_{rj} \lambda_j = y_r \quad r = 1, \dots, s, \\
 & y_r \geq 0 \quad r = 1, \dots, s, \\
 & \lambda_j \geq 0 \quad j = 1, \dots, n,
 \end{aligned} \quad (2)$$

where  $w_r$  is the price of each output  $y_r$ . Each DMU produces outputs  $y_r$ ,  $r = 1, \dots, s$  at maximal revenue using inputs  $x_i$ ,  $i = 1, \dots, m$ . Therefore, for each DMU $_l$ , the RE is defined as the ratio of its current revenue to the maximum revenue, which is the optimal solution of Model (2) as defined below:

$$\text{Revenue efficiency} = \frac{\sum_{r=1}^s w_r y_{rl}}{\sum_{r=1}^s w_r y_r^*}. \quad (3)$$

Note that Eq. (3) should be less than, or equal to, 1 and it should also be greater than 0. The RE, considering the same level of inputs, shows the extent to which the DMU's revenue is close to the best DMU's revenue.

## 2.2. Malmquist Productivity Index (MPI)

DEA models can be used for estimating the efficiency and productivity changes over period using MPI [26]. The MPI considers two periods ( $t$  and  $t+1$ ) and calculates efficiency variations over time. MPI can be computed by solving the following model proposed by Caves et al. [26] for evaluation of DMU $_l$ :

$$D(x_l, y_l) = \min \{ \theta : (\theta x_l, y_l) \in T \}. \quad (4)$$

Given the resultant distance function Eq. (4), consider the following input-oriented CCR (Charnes-Cooper-Rhodes) model:

$$\begin{aligned}
 & D^f(x_l^k, y_l^k) = \min \quad \theta \\
 & \text{s.t.} : \quad \sum_{j=1}^n x_{ij}^f \lambda_j \leq \theta x_{il}^k \quad i = 1, \dots, m, \\
 & \quad \sum_{j=1}^n y_{rj}^f \lambda_j \geq y_{rl}^k \quad r = 1, \dots, s, \\
 & \quad \lambda_j \geq 0 \quad j = 1, \dots, n.
 \end{aligned} \quad (5)$$

Accordingly, four LP problems can be defined. Consider  $l$  to be the notion of the unit under evaluation and each of  $k$  and  $f$  shows periods  $t$  and  $t+1$ . For instance, to assess DMU $_l$ , let  $k = t$  and  $f = t+1$ ,  $D^{t+1}(x_l^t, y_l^t)$  which shows the coordinates of DMU $_l$  in period  $t$  and technology in period  $t+1$ . To demonstrate the progress and regress of DMUs, Caves et al. [26] (1982) proposed MPI as given below:

$$M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \left( \frac{D^t(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^t, y_l^t)} \right)^{1/2}. \quad (6)$$

The decomposition of this index shows technical efficiency alteration and technology frontier shift while two periods are taken into account ( $t$  and  $t + 1$ ).

$$M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \left( \frac{D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)} \right) \left( \frac{D^t(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t) D^{t+1}(x_l^t, y_l^t)} \right)^{1/2}. \quad (7)$$

If  $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) > 1$ , then the total productivity of DMU experiences progress. If  $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) < 1$ , then the total productivity of DMU experiences regress. If  $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = 1$ , then the total productivity remains unchanged.

### 3. Proposed method

#### 3.1. Stepwise pricing for evaluating RE

At this juncture, a new method for evaluating the RE is presented. To the best of our knowledge, RE has not yet been discussed so far, given nonlinear behavior of sales prices. Consider a non-competitive situation. For instance, at a power plant, it is expected that revenue be increased by producing greater electric power. If a power plant produces 9 Mega Watt per Hour (MWH) of electric power, this power will be sold for \$100 per unit for the first 5 units and \$140 per unit for the 4 units beyond the first 5 units. This is a unique case that is discussed in this paper. The existing DEA models deal with linear pricing. Linear pricing is not applicable to all situations because it may lead to inaccurate results for enterprises like power plants that should use a stepwise pricing system. The average price is not a suitable replacement as the nonlinear behavior of pricing is not considered. This paper considers the conditions in which increments in value are automatically taken into account and occur frequently in real-life situations. So far, in DEA, for calculating RE, a linear function has been considered. Linear function gives an approximate solution. Here, to assess RE, we present a model in which incremental revenue is considered.

Note that the values depend on the number of products called stepwise rating. Although the values are stable in standard RE measurement, this is not the case in real-world problems. Consider  $y_{rj}$  as the  $r$ th output of the  $j$ th DMU. In addition, let  $w_r$  be the value of this product. In such a situation, the producer will obtain higher revenue, if a larger number

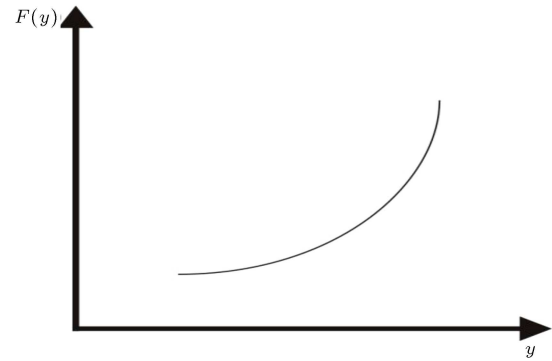


Figure 1. A convex function.

of products are produced and sold. In this case, when a stepwise rating system is contemplated, a larger number of products are sold at higher prices. To get accurate results, it is essential that a general framework be established for RE which considers the real-life market situations. Cook and Zhu [14] argued that in the multiplier DEA model, linear weighting is not adequately capable of indicating the innate behaviors of variables. They maintained that some variables had non-linear behavior and linear weighting might lead to a bias. Hosseinzadeh Lotfi et al. [15] presented a modified version of the model in the envelop form of CCR model. In Figure 1, the convex function indicates that the greater the output, the higher the revenue.

According to the piece-wise linear function theory, the estimation of this function can be enhanced by breaking down the scale of the  $r$ th output into  $k_r$  segments in which they are assumed to behave linearly in their segments (see Figure 2).

$R_1$  and  $R_2$  represent sets of regular and piece-wise linear outputs with increasing magnitude, respectively. Thus, the scale of variable, which reveals piece-wise linear behavior, should be considered as  $k_r$  ranges such as  $[0, L_1], (L_1, L_2], \dots, (L_{k_r-1}, L_{k_r}]$ . Consider:

$$t^{k_r}_i = \begin{cases} L_{k_r}, & \text{if } k_r = 1 \\ L_{k_r} - L_{k_r-1}, & \text{if } k_r = 2, \dots, l_{k_r} \end{cases} \quad (8)$$

An expert should determine the number and width of ranges. The vector of profits corresponding to  $y_r$  ( $r \in R_2$ ), which shows nonlinear behavior, consists of  $k_r$  ranges as  $w_r^{k_r}, k_r = 1, \dots, l_{k_r}, r \in R_2$ , where  $w_r^{k_r} < w_r^{k_r+1}, k_r = 1, \dots, l_{k_r}, r \in R_2$ . We can represent the contribution of the  $r$ th output to the weighted aggregate of all outputs in the objective function of maximal revenue model as  $\sum_{k_r=1}^{l_{k_r}} w_r^{k_r} y_r^{k_r}$  instead of a single  $w_r y_r$ .

Assume that  $y_r$  is the  $r$ th output ( $r \in R_2$ ) which has stepwise pricing and  $w_r$  is the corresponding vector of prices. For instance, let  $k_r = 3$  and consider three ranges for  $y_r$  including  $(y_r^1, y_r^2, y_r^3)$ . We define ranges as  $[0, 300), [300, 700)$ , and  $[700, 1000]$  for

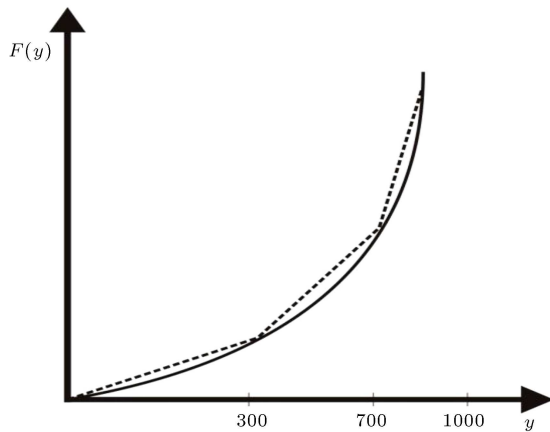


Figure 2. A convex piece-wise linear function.

$y_r^1$ ,  $y_r^2$ , and  $y_r^3$ , respectively. For  $w_r = (w_r^1, w_r^2, w_r^3)$ , consider (650, 700, 750), respectively. In the traditional approach,  $y_r = 800$  and  $w_r = 800$  are assumed. In our approach, given the ranges and Eq. (8),  $y_r$  should be replaced with  $(y_r^1, y_r^2, y_r^3) = (300, 400, 100)$ . Also,  $w_r$  should be replaced with  $(w_r^1, w_r^2, w_r^3) = (650, 700, 750)$ . This conveys the meaning of stepwise pricing in which the output is sold \$650 per unit for the first range, \$700 per unit for the second range, and \$750 per unit for the third range. However, in the traditional approach, the whole 800 units of outputs are sold \$800 per unit.

Our new model (Model (9)) can find the optimal  $y_r^{k_r}, k_r = 1, \dots, l_{k_r}, r \in R_2$ , which complies with the theory of piece-wise function. It means that  $y_r^{k_r}, k_r = 1, \dots, l_{k_r}, r \in R_2$  should be a sequence of sequential values in their specified ranges. Therefore, lower ranges should be filled before higher ones are filled. The stated concepts comply with the reasoning that the scale should be divided to show the nonlinear behavior of the outputs. In Model (9), let  $v_0 = 0$  and  $M$  be a big positive constant. Note that  $k_r, r \in R_2$  illustrates the number of intervals  $[L_{k_r-1}, L_{k_r}]$ . The binary variable  $v_k$  forces  $y_r^{k_r}$  to become zero. It is obvious that in the case that the lower ranges have not been completely met, the result would be  $v_k = 1$  in which  $y_r^{k_r}$  is forced to become zero. To confine  $y_r^{k_r}$  in a way that each portion of this kind of output gets a value according to Eq. (8), Constraints (a), (b), and (c) in Model (9) are added to the mathematical model. Furthermore, these variables control the lower ranges to be completed before the upper ranges. In the objective function of Model (9), it is clear that each portion of the defined output includes distinct values, which are in an increasing order and is based on its magnitude.

$$\text{Max} \quad \sum_{r \in R_1} w_r y_r + \sum_{r \in R_2} (w_r^1 y_r^1 + w_r^2 y_r^2 + \dots + w_r^{k_r} y_r^{k_r})$$

S.t.:

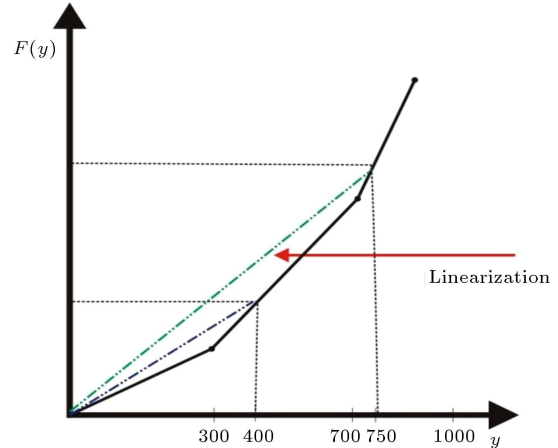


Figure 3. Linearizations of a piece-wise function.

$$\sum_{j=1}^n \lambda_j x_{ij} \leq x_{il} \quad i = 1, \dots, m,$$

$$\sum_{j=1}^n \lambda_j y_{rj} \geq y_r \quad r \in R_1,$$

$$\sum_{j=1}^n \lambda_{k_r,j} y_{rj}^{k_r} \geq y_r^{k_r} \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r},$$

$$y_r^{k_r} \leq t_r^{k_r} (1 - v_{k_r-1}) \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}, \quad (a)$$

$$v_{k_r} \leq (t_r^{k_r} - y_r^{k_r}) \cdot M \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}, \quad (b)$$

$$(t_r^{k_r} - y_r^{k_r}) \leq v_{k_r} \cdot M \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}, \quad (c)$$

$$\lambda_j \geq 0, y_r \geq 0 \quad j = 1, \dots, n, \quad r \in R_1,$$

$$\lambda_{k_r,j} \geq 0, y_r^{k_r} \geq 0 \quad j = 1, \dots, n, \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r},$$

$$v_{k_r} \in \{0, 1\} \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}. \quad (9)$$

The linearization of revenue function may differ among various amounts of  $y_r$ . This is shown in Figure 3. As is seen, the linearization function differs between  $y_r = 400$  and  $y_r = 750$ . Note that when  $y_r^{(k+1)r}, k_r = 1, \dots, l_{k_r}, r \in R_2$  has a positive value,  $y_r^{k_r}, k_r = 1, \dots, l_{k_r}, r \in R_2$  has also a positive value. Hence, we have  $w_r^1 y_r^1 + w_r^2 y_r^2 + w_r^3 y_r^3$ .

In Model (9), the first output has nonlinear behavior in pricing. Thus, in this model, instead of single expression  $w_1 y_{1o}$ , a linear combination like  $w_r^1 y_{1l}^1 + w_r^2 y_{1l}^2 + \dots + w_r^{l_{k_r}} y_{1l}^{l_{k_r}}$  is replaced. Note that  $l_{k_r}$  shows the number of defined intervals for variations of the first output. Consider  $r = 2, \dots, s$  and  $r = 1$  to be the sets of regular and piece-wise linear outputs, respectively. Model (9) is a mixed integer nonlinear programming as there are integer variables  $v_{k_r}, r = 1$ . Finally, a piece-wise linear RE for  $DMU_l$  is the ratio

of piece-wise linear maximum revenue of the current revenue of DMU<sub>*l*</sub> divided by the optimal solution of Model (9) which is as follows:

Revenue efficiency =

$$\frac{\sum_{r \in R_1} w_r y_{rl} + \sum_{r \in R_2} (w_r^1 y_{rl}^1 + w_r^2 y_{rl}^2 + \dots + w_r^{k_r} y_{rl}^{k_r})}{\sum_{r \in R_1} w_r y_r^* + \sum_{r \in R_2} (w_r^1 y_r^{*1} + w_r^2 y_r^{*2} + \dots + w_r^{k_r} y_r^{*k_r})} \quad (10)$$

**Theorem 3.1.** Model (9) is always feasible and the objective function is bounded.

**Proof:** Let  $\lambda_l = 1$ ,  $\lambda_j = 0$ ,  $\forall j = l$ ,  $\lambda_l^{k_r} = 1$ ,  $\lambda_j^{k_r} = 0$ ,  $\forall j \neq l$ ,  $\forall r = 2, \dots, s$ ,  $y_r^{k_r} = t_r^{k_r}$ ,  $\forall r = 1$ ,  $\forall k_r = 1, \dots, l_{k_r}$ ,  $v_0 = 0$ ,  $v_{k_r} = 0$ ,  $v_{k_r-1} = 0$ ,  $\forall r = 1$ ,  $k_r = 1, \dots, l_{k_r}$ . Therefore, it can be concluded that Model (9) is feasible and the objective function is bounded. After solving Model (9), the optimal solution is as  $y_2^*$ ,  $r = 2, \dots, s$ ,  $y_1^{*k_r}$ ,  $r = 1$ ,  $k_r = 1, \dots, l_{k_r}$ .  $\square$

**Theorem 3.2.** The obtained target point  $(x_l, y_r^*, y_r^{*k_r})$  after solving Model (9) is Pareto efficient.

**Proof:** An important point in DEA is that each DMU is compared with the rest of DMUs. Suppose that  $(y_r^*, y_r^{*k_r})$  is not Pareto efficient. Thus, there is a feasible solution  $(\bar{y}_r, \bar{y}_r^{k_r})$  that dominates  $(y_r^*, y_r^{*k_r})$ . Therefore, it can be concluded that this feasible solution has a greater objective function than the obtained optimal  $(y_r^*, y_r^{*k_r})$ , which is conflict with optimality of  $(y_r^*, y_r^{*k_r})$ . As a result, it can be concluded that the obtained target point  $(x_l, y_r^*, y_r^{*k_r})$ , after solving Model (9), is Pareto efficient.  $\square$

**Theorem 3.3.** Given Model (9) and Eq. (10), at least one DMU is revenue efficient.

**Proof:** In Model (9), at least one inequality constraint related to the outputs should be binding. Otherwise, it is concluded that all the outputs' inequality constraints are  $\sum_{j=1}^n \lambda_{k_r j} y_{rj}^{k_r} > y_r^{*k_r}$  ( $r \in R_2, k_r = 1, \dots, l_{k_r}$ ). It is assumed that there is a feasible solution in which  $\sum_{j=1}^n \lambda_{k_r j} y_{rj}^{k_r} \geq \bar{y}_r^{k_r}$  ( $r \in R_2, k_r = 1, \dots, l_{k_r}$ ). In this case,  $\bar{y}_r^{k_r} > y_r^{*k_r}$  ( $r \in R_2, k_r = 1, \dots, l_{k_r}$ ), which is a contradiction. Therefore, at least for one DMU, the output inequality constraints are binding, meaning that the current revenue is equal to the maximum revenue and it is revenue efficient.  $\square$

### 3.2. Stepwise pricing in revenue MPI

Here, Piece-wise Linear Revenue MPI (PLREMPI) is introduced based on Model (9). Consider periods  $t$  and  $t+1$ . Note that  $l$  is similar to the DMU under evaluation and  $q$  and  $f$  denote  $t$  and  $t+1$ , respectively.

Assume  $k = t$  and  $f = t+1$ ,  $D^f(x_l^q, y_l^q) = D^{t+1}(x_l^t, y_l^t)$ , which shows DMU<sub>*l*</sub> in period  $t$  while technology is considered in period  $t+1$ . The new model is as follows:

$$D^f(x_l^q, y_l^q) = \text{Max}$$

$$\sum_{r \in R_1} w_r y_r^q + \sum_{r \in R_2} (w_r^1 y_r^{q1} + w_r^2 y_r^{q2} + \dots + w_r^{k_r} y_r^{qk_r})$$

S.t.:

$$\sum_{j=1}^n \lambda_j x_{ij}^f \leq x_{il}^q \quad i = 1, \dots, m,$$

$$\sum_{j=1}^n \lambda_j y_{rj}^f \geq y_r^q \quad r \in R_1,$$

$$\sum_{j=1}^n \lambda_{k_r j} y_{rj}^{fk_r} \geq y_r^{qk_r} \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r},$$

$$y_r^{qk_r} \leq t_r^{k_r} (1 - v_{k_r-1}) \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}, \quad (a)$$

$$v_{k_r} \leq (t_r^{k_r} - y_r^{qk_r}) \cdot M \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}, \quad (b)$$

$$(t_r^{k_r} - y_r^{qk_r}) \leq v_{k_r} \cdot M \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r}, \quad (c)$$

$$\lambda_j \geq 0, y_r \geq 0 \quad j = 1, \dots, n, \quad r \in R_1,$$

$$\lambda_{k_r j} \geq 0, y_r^{qk_r} \geq 0 \quad j = 1, \dots, n, \quad r \in R_2, \quad k_r = 1, \dots, l_{k_r},$$

$$v_{k_r} \in \{0, 1\} \quad r \in R_2, k_r = 1, \dots, l_{k_r}. \quad (11)$$

To calculate PLREMPI, Eq. (12) is used. Note that DMU is in period  $t$  while technology is in period  $t+1$ .

$$PLREMPI(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) =$$

$$\left( \frac{D^t(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t) D^{t+1}(x_l^t, y_l^t)} \right)^{1/2}. \quad (12)$$

If the score is higher than one, then there is a progress in the productivity from period  $t$  to period  $t+1$ . If the score is less than one, then there is a regress in the productivity from period  $t$  to period  $t+1$ . If the score is equal to one, then there is no progress or regress in the productivity from period  $t$  to period  $t+1$ .

### 3.3. Illustrative example

Consider five DMUs with one input ( $I_1$ ) and one output ( $O_1$ ). For the output, three ranges are considered as  $o_1^1 = [0, 11)$ ,  $o_1^2 = [11, 17)$ , and  $o_1^3 = [17, 80)$ . Numbers

**Table 1.** Dataset of the example.

DMUs	Current revenue	$I_1$	$o_1^1$	$o_1^2$	$o_1^3$	$O_1$
1	11000	12	11	0	0	11
2	14600	24	11	3	0	14
3	9000	25	9	0	0	9
4	19700	17	11	6	1	18
5	24200	26	11	6	4	21

are in MWH. Table 1 depicts the dataset. Price vector of the output is  $(w_1^1, w_1^2, w_1^3) = (1000, 1200, 1500)$ . In the classical evaluation, the price is considered as \$1000. Note that the sum of the three ranges is equal to the initial output ( $O_1$ ). In Table 2, the piece-wise linear evaluation of the maximum revenue is represented. Also, the results of classical maximum revenue and the optimal values of output ( $O_1^*$ ) are reported in Table 3.

For DMU<sub>1</sub>, the output is 11 and the current revenue is 11000. After evaluation by Model (9), the optimal output is 17.71 and the corresponding revenue is 19265. Given the classical revenue analysis (Model (2)), the optimal maximum revenue is 12705.88 and the optimal output is 12.71. By comparing Tables 2 and 3, it is seen that Model (2) overestimates the outputs. Note that the outputs of DMU<sub>4</sub> in both Models (9) and (2) are the same. However, for DMU<sub>5</sub>, Model (2) overestimates the output.

Here, we deal with outputs for presenting stepwise pricing to reach revenue evaluation in DEA context. Thus, some modifications are made to “maximum revenue” optimization model to get a new RE score. A good property of our model is that it does not present a fixed value for the outputs. Instead, given the outputs, it presents different values to get possible maximum revenue score.

#### 4. Case study

Here, we wish to assess Iranian power plants. Most of the Iranian power plants are owned by government and the government sells electricity. Due to high electricity consumption in Iran, the generated electricity is insufficient. Here, 20 public power plants in Iran are assessed. Since consumption is higher than electricity generation, Iranian government faces lack of electricity. Thus, it

**Table 3.** Results of classical maximum revenue.

DMUs	Maximum revenue	$I_1$	$O_1^*$
1	12705.88	12	12.71
2	25411.76	24	25.41
3	26470.59	25	26.47
4	18000.00	17	18.00
5	27529.41	26	27.53

should buy electricity from private power plants and sell it to consumers at a subsidized rate. Thus, the price is considered as a penalty paid by the government for over-consumption of consumers. In some seasons, there might be periods that demand is less than supply. Thus, the government can sell the extra generated electricity to neighboring countries.

Here, the stepwise pricing for the electricity is considered in the non-competitive Iranian power generation market. Therefore, the higher the price, the greater the income. In this case, the intervals for changing amount of electricity are considered. At each interval, a specific price is considered. Note that the number and length of the intervals and corresponding prices are determined by the experts. Furthermore, assuming that fixed prices are not applicable in the real world, biased efficiency scores may be generated.

The dataset is obtained from Tavanir Management Organization [27]. Dataset dates back to the year 2004 to 2006. The inputs and outputs are given below:

$I_1$	Capacity (MW)
$I_2$	Internal usage (MWH)
$I_3$	Fuel (Tera Joule (TJ))
$O_1$	Electrical power production (MWH)

The dataset related to 2004 is depicted in Table 4. In Table 5, given the opinions of experts, the  $O_1$  is divided into four ranges. There is a nonlinear relation between power generation and revenues of the power plants. Therefore, linear pricing cannot give a favorable result. However, using a stepwise rating system, a greater amount of electricity power can be sold for higher prices in noncompetitive situations.

**Table 2.** The results of piece-wise linear evaluation.

DMUs	Maximum revenue	$I_1$	$o_1^1$	$o_1^2$	$o_1^3$	Sum of output ranges
1	19265	12	11	6	0.71	17.71
2	23195	24	11	6	3.33	20.33
3	23705	25	11	6	3.67	20.67
4	19700	17	11	6	1.00	18.00
5	24200	26	11	6	4.00	21.00

**Table 4.** Input and output data related to 2004.

Power plants (DMUs)	$I_1$	$I_2$	$I_3$	$O_1$
1	625.88	241139	30.95308	3297100
2	247.5	139505	17.00441	1500253
3	50.0	13039	3.412765	212403
4	1760	301276	107.1448	11000000
5	1300	642909	57.30782	7438002
6	1000	421015	54.61188	6342203
7	240	85307	14.06063	1435991
8	736	361080	42.82585	4341330
9	1000	390708	46.37235	5134547
10	640	350154	38.2554	4210280
11	1890	636643	93.29557	11000000
12	290	81674	7.886569	922587
13	1280	588855	73.2442	7196540
14	60	29698	4.322743	341402
15	835	422673	52.77859	5621431
16	1600	796262	102.4223	11000000
17	600	271901	37.45617	3831065
18	120	63050	7.955432	665887
19	256	140940	15.76105	1492847
20	1000	390708	46.37228	5134547

**Table 5.** The output ranges defined for the first output.

DMUs	$o_1^1$	$o_1^2$	$o_1^3$	$o_1^4$
1	3000000	297000	0	0
2	1500253	0	0	0
3	212403	0	0	0
4	3000000	2000000	2000000	4000000
5	3000000	2000000	2000000	438002
6	3000000	2000000	1342203	0
7	1435991	0	0	0
8	3000000	1341330	0	0
9	3000000	2000000	134547	0
10	3000000	1210280	0	0
11	3000000	2000000	2000000	4000000
12	922587	0	0	0
13	3000000	2000000	2000000	196540
14	341402	0	0	0
15	3000000	2000000	621431	0
16	3000000	2000000	2000000	4000000
17	3000000	831065	0	0
18	665887	0	0	0
19	1492847	0	0	0
20	3000000	2000000	134547	0

**Table 6.** The efficiency scores obtained from Model (9).

DMUs	PL-RE	RE	DMUs	PL-RE	RE
1	0.400	0.891	11	1.000	1.000
2	0.180	0.881	12	0.110	0.917
3	0.031	0.661	13	1.000	0.866
4	1.000	1.000	14	0.020	0.828
5	0.982	1.000	15	0.750	0.985
6	0.881	1.000	16	1.000	1.000
7	0.170	0.931	17	0.480	0.947
8	0.541	0.901	18	0.083	0.807
9	0.670	0.905	19	0.180	0.865
10	0.531	0.991	20	0.670	0.904

Current DEA models cannot consider stepwise pricing in revenue evaluation. Thus, previous DEA models may produce erroneous results.

Given Eq. (8) and to apply our model, four output ranges are taken into consideration:

$$O_1^1 = [0, 3], \quad O_1^2 = (3, 5],$$

$$O_1^3 = (5, 7], \quad \text{and} \quad O_1^4 = (7, \infty). \quad (13)$$

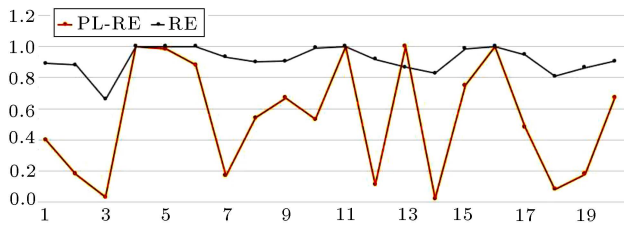
The numbers are in MWH. For example, as is depicted in Table 4, the output of DMU<sub>1</sub> is 3297100. The intervals of this output, given in Eq. (13), are defined as (3000000, 297100, 0, 0). The corresponding prices for each of the defined intervals are 1000, 1200, 1500, and 1800, respectively (100 Rials). Hence, it can be said that we deal with a stepwise pricing system.

Table 6 shows classical RE and Piece-wise Linear-Revenue Efficiency (PL-RE). The results are obtained by solving Model (9) and using Eq. (10). GAMS software is employed to solve the problem. As is seen in Table 6, compared with the results of classical RE evaluations, the results of PL-RE might be increasing, decreasing, or unchanged. This could be similar to the classical RE findings in which outputs follow a nonlinear behavior. It is clear that our proposed model yields substantial improvement in the RE measurement.

There is a difference between the results of Models (2) and (9). Generally, the efficiency scores obtained from the piece-wise linear DEA analysis can be either lower or higher than the standard DEA peers. In the results, compared with the standard RE model, some DMUs have higher efficiency scores, while some have lower efficiency scores. The results are obtained using GAMS software.

Figure 4 compares the results of RE and PL-RE. Now, consider the output of DMU<sub>5</sub> (7438002) as indicated by (3000000, 2000000, 2000000, 438002). According to the experts' opinions, the corresponding prices are in an increasing order (1000, 1200, 1500, and 1800). The PL-RE for DMU<sub>5</sub> is 0.98. This measure





**Figure 4.** The results of Revenue Efficiency (RE) and Piece-wise Linear Revenue Efficiency (PL-RE).

is derived from dividing the revenue obtained from the current level of the output by the result of Eq. (8). It shows the best possible revenue for DMU<sub>5</sub>. The result of Eq. (8) for DMU<sub>5</sub> is (3000000, 2000000, 2000000, and 218496.09). Given the defined values for each of these ranges, the obtained revenue is 9188403600. However, by multiplying the amount of each interval by the vector value, we obtain 9057003000. In addition, by dividing 9057003000 by 9188403600, the PL-RE is obtained. This measure indicates a crucial debate that DMU<sub>5</sub> can increase its output given the same inputs. This is the most important result that can be obtained from peer revenue evaluation. Another important finding is that the managers can understand the capability of the system better for producing products.

At this juncture, let us contemplate the classical method for deriving RE. The RE score for DMU<sub>5</sub> in the classical method is equal to 1. This score is obtained from Eq. (10) by dividing the revenue obtained from the current output (7438002000) by the best possible revenue for DMU<sub>5</sub> which is obtained from Model (9).

This measure demonstrates that the obtained output is the best for DMU<sub>5</sub> and it cannot produce more output given the similar level of inputs. However, by using Model (2) and Expression (3), the RE score of DMU<sub>17</sub> is 0.48. This score shows that DMU<sub>17</sub> can increase its output. Given the classical Model (2) and Eq. (3), this measure is obtained by dividing the revenue obtained from the current output by the best possible revenue for DMU<sub>17</sub>. In piece-wise linear pricing, the revenue obtained from the current output is 3997278000 and the best possible revenue for DMU<sub>17</sub> is 3997278000. Therefore, by dividing these two numbers, we get 1. In Table 6, the optimal solution of Eq. (8) is reported. There is a significant difference between the two sets of results. The convex function, as illustrated in Figure 1, denotes an increasing function. To estimate the convex function by a linear function, the characteristics of the convex function cannot be displayed precisely. Thus, the results are inaccurate. However, estimating the convex function by a piece-wise linear convex function leads to more accurate results. A similar analysis can be repeated for 2005 and 2006. The dataset is reported in Table 7.

The optimal maximum piece-wise linear revenue and efficiency scores are depicted in Table 8. In Table 8, the optimal maximum piece-wise linear revenues for 2004, 2005, and 2006 are listed.

Table 9 shows the PL-RE scores in 2005 and 2006. For example, in 2004, the optimal values of DMU<sub>1</sub> for each divided range are (3000000, 2000000, 2000000, and 0). The sum of ranges is 7000000. Upon comparing the obtained target for the output

**Table 7.** The dataset (2005 and 2006).

DMUs	2005				2006			
	$I_1$	$I_2$	$I_3$	$O_1$	$I_1$	$I_2$	$I_3$	$O_1$
1	260	615989	3.20	2222273	1465	606125	5.780	7528574
2	241	725654	99.340	2692987	1584	54636	45.764	9722169
3	365	285942	57.230	7904814	1271	889503	84.562	9163596
4	627	652939	46.300	1571045	959	440922	45.873	4177877
5	676	323994	38.987	6629453	240	759745	43.673	4448964
6	442	352884	92.345	6789916	279	865304	12.673	6077669
7	1538	41414	53.456	2165922	884	401264	98.563	6948107
8	1088	230039	100.453	2229153	1415	312014	34.674	9642639
9	1931	381196	36.765	6651760	1614	972120	45.632	7206031
10	1978	320838	38.987	11480340	596	693166	89.234	4577178
11	1541	584695	95.980	5699805	961	349921	13.452	7320412
12	835	288844	13.456	1216457	690	468501	103.452	2317777
13	851	203257	7.849	9776371	1241	429415	76.34	6034480
14	1080	23631	62.56	11468004	345	26308	57.836	1285188
15	384	350528	93.456	2056659	805	250166	37.893	6158195
16	1123	740735	85.912	6751574	118	438411	16.75	3046098
17	1042	311695	76.843	11575914	1189	456237	97.543	4964059
18	1540	53701	18.453	8448269	1345	329387	48.341	2964572
19	981	104550	19.457	846810	263	386149	15.894	2866023
20	1931	381196	36.757	6651760	1614	972120	45.602	7206031

**Table 8.** Optimal maximum piece-wise linear revenues for each year.

DMUs	2004	2005	2006
1	15600000000.00	2222273000.0000	9351433200.0000
2	1500253000.000	2692987000.0000	13299904200.000
3	219488279.0000	10028665200.000	12294472800.000
4	15600000000.00	12184718052.617	7386842138.6930
5	9188403600.000	12473798836.527	4738756800.0000
6	7413304500.000	10780237519.055	7016503500.0000
7	1435991000.000	15621206779.588	8322160500.0000
8	4609596000.000	16581585566.974	13156750200.000
9	5601820500.000	16245741346.381	13298200293.057
10	4452336000.000	16464612000.000	7700624605.7850
11	15600000000.00	16636645200.000	8976741600.0000
12	922587000.0000	13372816127.548	7512657957.4250
13	8753772000.000	13397467800.000	11639050395.669
14	362147158.0.00	16442407200.000	1285188000.0000
15	6332146500.000	10214118109.897	7137292500.0000
16	15600000000.00	16636645200.000	3055317600.0000
17	3997278000.000	16636645200.000	11178711052.722
18	725157139.0000	11006884200.000	12521705809.707
19	1537630716.000	1093548938.9070	2866023000.0000
20	5601820500.000	16245741346.378	13298200293.042

(7000000) with its initial amount (3297100), we find out that DMU<sub>1</sub> should increase its output to become efficient. In 2005, the initial output is 2222273 and the obtained output target is 7500465.270. The divided ranges are (3000000.000, 2000000.000, 2000000.000, and 500465.27). For the output, we can increase the output by 3.37%. In 2006, the target output is 7528574 and compared with the initial amount (7528574), it is fixed.

As another example, consider DMU<sub>8</sub>. In 2004, the obtained target for each of the divided outputs is (3000000, 2000000, 2000000, and 500465.27) whose sum is 7500465.27. Note that the initial output is 4341330 (3000000, 1341330, 0, and 0). Thus, DMU<sub>8</sub> can increase its output by 1.73%. In 2005, DMU<sub>8</sub> can increase its output by 5.17%. Its initial output and target are 2229153 and 11545325.31, respectively. In 2006, it cannot increase its output as its initial output and target are 9642639 and 9642639, respectively.

Therefore, to assess the performance of the power plants during three years, the MPI can be utilized. Note that the MPI can determine situations where there is no change.

In Table 10, the optimal maximum piece-wise linear output resulting from Model (9) is listed. The optimal maximum piece-wise linear outputs for DMU<sub>1</sub>, DMU<sub>8</sub>, DMU<sub>11</sub>, DMU<sub>13</sub>, and DMU<sub>15</sub> are depicted in Table 10 as instances. The ranges are obtained by

**Table 9.** Piece-wise linear revenue efficiency scores in 2005 and 2006.

DMUs	2005	2006	DMUs	2005	2006
1	0.24	1.00	11	0.39	1.00
2	0.27	1.00	12	0.09	0.26
3	1.00	1.00	13	1.00	0.60
4	0.13	0.49	14	1.00	0.10
5	0.63	0.54	15	0.20	0.85
6	0.75	0.78	16	0.48	0.34
7	0.14	0.93	17	1.00	0.48
8	0.13	1.00	18	1.00	0.24
9	0.52	0.66	19	0.06	0.32
10	1.00	0.55	20	0.52	0.67

Eq. (8). Similarly, we can obtain the results for other DMUs. The optimal values of the piece-wise linear output  $o_1$  are reported in Table 10. Moreover, the summation of these values for years 2004, 2005, and 2006 is calculated. These results are then used in Eq. (10) as  $y^*$  for calculating the PL-RE for years 2004, 2005, and 2006.

Knowing progress and regress of DMUs in different periods helps decision-makers to better recognize the shortcomings of DMUs. After obtaining the optimal objective function of Model (11), using Eq. (12), the MPI is calculated. The MPIs are listed in Tables

**Table 10.** Optimal maximum piece-wise linear output.

	Outputs	DMU <sub>1</sub>	DMU <sub>8</sub>	DMU <sub>11</sub>	DMU <sub>13</sub>	DMU <sub>15</sub>
2004	$o_1^1$	3000000	3000000	3000000	3000000	3000000
	$o_1^2$	2000000	2000000	2000000	2000000	2000000
	$o_1^3$	2000000	2000000	2000000	2000000	2000000
	$o_1^4$	0	67354.51	4000000	196540	3272.77
	Sum	7000000	7067354.51	11000000	7196540	7003272.77
2005	$o_1^1$	3000000	3000000	3000000	3000000	3000000
	$o_1^2$	2000000	2000000	2000000	2000000	2000000
	$o_1^3$	2000000	2000000	2000000	2000000	2000000
	$o_1^4$	500465.27	4545325.31	4575914	2776371	1007843.39
	Sum	7500465.27	11545325.31	11575914	9776371	8007843.39
2006	$o_1^1$	3000000	3000000	3000000	3000000	3000000
	$o_1^2$	2000000	2000000	2000000	2000000	2000000
	$o_1^3$	2000000	2000000	2000000	2000000	2000000
	$o_1^4$	528574	2642639	320412	1799472.44	0
	Sum	7528574	9642639	7320412	8799472.44	7000000

**Table 11.** Malmquist Productivity Index (MPI) comparing the year 2004 with 2005.

DMU	$D^{2004}(x_i^{2004}, y_i^{2004})$	$D^{2005}(x_i^{2005}, y_i^{2005})$	$D^{2004}(x_i^{2005}, y_i^{2005})$	$D^{2005}(x_i^{2004}, y_i^{2004})$	MPI (2004–2005)
1	0.3996	0.2389	0.202	0.142	0.65
2	0.1786	0.2685	0.09	0.173	1.696
3	0.0253	1	0.013	0.643	44.612
4	1	0.1289	0.938	0.101	0.118
5	0.9857	0.6289	0.552	0.503	0.762
6	0.8825	0.75	0.446	0.518	0.994
7	0.171	0.1387	0.086	0.139	1.142
8	0.541	0.1344	0.277	0.143	0.358
9	0.6669	0.5154	0.337	0.537	1.11
10	0.53	1	0.268	1.055	2.728
11	1	0.3877	0.938	0.413	0.413
12	0.1098	0.091	0.055	0.078	1.08
13	1	1	0.526	0.859	1.278
14	0.0219	1	0.021	1.054	48.428
15	0.7533	0.2014	0.381	0.132	0.304
16	1	0.4825	0.938	0.515	0.515
17	0.4759	1	0.242	1.066	3.046
18	0.0793	1	0.04	0.706	14.91
19	0.1777	0.0563	0.09	0.054	0.438
20	0.6649	0.5144	0.338	0.5372	1.112

11 and 12. Upon comparing the years 2004 and 2005 as well as the years 2005 and 2006, as depicted in Tables 11 and 12, some DMUs have regressed while some have progress. Note that our method deals with the nonlinear behavior of variables and it affects progress and regress of DMUs during periods.

## 5. Managerial implications

Energy consumption reduction is an important issue.

This issue should be accepted as a principle in society in both energy production and consumption. An important measure that can help producers to evaluate their performances is the RE score. This study addressed the main shortcoming of the classical RE, which is the fixed price assumption. Also, the previous DEA models are modified to face the stepwise pricing. The proposed DEA model functions based on mixed integer programming and the big M technique. In the case study, it is shown that the results of our model are different

**Table 12.** Malmquist Productivity Index (MPI) comparing the year 2005 with 2006.

DMU	$D^{2005}(x_i^{2005}, y_i^{2005})$	$D^{2006}(x_i^{2006}, y_i^{2006})$	$D^{2005}(x_i^{2006}, y_i^{2006})$	$D^{2006}(x_i^{2005}, y_i^{2005})$	MPI (2005–2006)
1	0.2389	1.0000	0.1670	0.5620	3.7530
2	0.2685	1.0000	0.2040	0.7990	3.8170
3	1.0000	1.0000	0.7540	0.7390	0.9900
4	0.1289	0.4917	0.1180	0.2650	2.9270
5	0.6289	0.5402	0.5900	0.2850	0.6440
6	0.7500	0.7816	0.6080	0.4220	0.8500
7	0.1387	0.9271	0.1630	0.5000	4.5310
8	0.1344	1.0000	0.1680	0.7910	5.9250
9	0.5154	0.6596	0.6300	0.5270	1.0350
10	1.0000	0.545	1.2380	0.2940	0.3600
11	0.3877	1.0000	0.4850	0.5400	1.6940
12	0.0910	0.2582	0.0910	0.1390	2.0790
13	1.0000	0.5973	1.0070	0.4180	0.4980
14	1.0000	0.0967	1.2360	0.0770	0.0780
15	0.2014	0.8497	0.1550	0.4290	3.4210
16	0.4825	0.3404	0.6040	0.1840	0.4630
17	1.0000	0.4792	1.2510	0.3220	0.3510
18	1.0000	0.2368	0.8280	0.1780	0.2260
19	0.0563	0.3193	0.0640	0.1720	3.9170
20	0.5156	0.6598	0.6332	0.5272	1.0354

**Table 13.** The results of our new approach.

DMUs	Y* in Model (9)	Output ( $O_1$ )	Maximum revenue Model (9)	Current revenue Eq. (8)	PL-RE Eq. (10)
1	7000000	3297100	15600000000	3356400000	0.4
8	7067354	4341330	4609596000	4609596000	0.541
11	1100000	11000000	15600000000	15600000000	1
13	7196540	7196540	8753772000	8753772000	1
15	700327277	5621431	6332146500	6332146500	0.75

**Table 14.** The results of the classical approach.

DMUs	Y* in Model (2)	Output ( $O_1$ )	Maximum revenue Model (2)	Current revenue (WY)	RE Eq. (3)
1	3700169	3297100	3700169133	3297100000	0.891
8	4816983	4341330	4816983946	4341330000	0.901
11	1100000	11000000	11000000000	11000000000	1
13	8307347	7196540	8307347694	7196540000	0.866
15	5702484	5621431	5702484640	5621431000	0.985

from those of the classical model. Furthermore, the progress and regress of DMUs were discussed that gave important implications to managers for making crucial decisions.

In Tables 13 and 14, the results of Models (9) and (2) for DMUs 1, 8, 11, 13, and 15 are reported, respectively. Given Tables 13 and 14, it is clear that

our model (Model (9)) in most of the DMUs increases the output compared with Model (2). At the upper intervals, since higher prices are considered, our model tries to reach higher amounts. The maximum revenue obtained from Model (9) is higher than that from Model (2). This is an important finding for managers as they are responsible for performance evaluation.

Another important finding is the RE score that may decrease, increase, or remain unchanged. This shows the different capability of DMUs for producing products according to the stepwise pricing.

## 6. Conclusions

This study modified the classical Revenue Efficiency (RE) in Data Envelopment Analysis (DEA). The classical RE in DEA had two critical issues. Firstly, its linear pricing could not show the reality of variables with a nonlinear behavior. Secondly, the classical RE assumed the fixed prices not being applicable to real-world problems. This research employed DEA for evaluating the RE and addressed the shortcomings. Our model dealt with stepwise pricing systems to get more accurate results in RE assessments. We demonstrated that the results might increase, decrease, or remain unchanged compared with the previous RE models. Decision-Making Units (DMUs) could produce more products with higher prices to get more revenues.

For further researches, we suggest developing new DEA models that can deal with competitive settings. Finding an optimal value of the big M will be another interesting research topic.

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## Biographies

**Zohreh Moghaddas** is an Assistant Professor at the Department of Mathematics, Qazvin Branch, Islamic Azad University, Qazvin, Iran. Her research interests

include mathematical modeling, optimization, operations research, and data envelopment analysis.

**Mohsen Vaez-Ghasemi** is an Assistant Professor at the Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran. His research interests include supply chain management, optimization, metaheuristic algorithms, operations research, and data envelopment analysis.

**Farhad Hosseinzadeh Lotfi** is a Full Professor at the Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran. His research interests include applied mathematics, mathematical modelling, operations research, optimization, data envelopment analysis, MCDM, and fuzziness.

**Reza Farzipoor Saen** has published over 192 refereed papers in ABDC and ABS ranked journals. His h indices in Google Scholar and Scopus are 41 and 37, respectively. He has 24 years of industrial and consultation experience. Dr. Farzipoor is on the list of top 2% of scientists in the world, ranked by Stanford University (2020; 2021).