Stepwise pricing in evaluating revenue efficiency in Data Envelopment Analysis: A case study in power plants

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Abstract

Data envelopment analysis (DEA) technique is widely applied for performance assessment of decision making units (DMUs). The revenue efficiency (RE) evaluation is one of the controversial subject matters that can be performed through DEA context. The amount of productions and its prices are crucial factors in the RE. The classical DEA models consider the prices to be fixed and known which are not the case in real world. Also, the classical DEA models considers linear pricing in revenue assessment. However, most of real-world problems deal with nonlinear prices. This paper evaluates the RE given the piecewise linear theory in non-competitive situations. In doing so, a stepwise pricing function is introduced which lets the prices to be changed in relation to the amount of the production. As an innovative idea, a more accurate mathematical modeling for the RE is proposed. We define a dynamic weights' function in maximum revenue optimization model which no longer considers fixed prices. A case study validates our proposed model.

Keywords: Data envelopment analysis (DEA); Revenue efficiency; Stepwise pricing; Mixed integer programming; Big M; Malmquist productivity index (MPI); Piecewise linear functions.

1. Introduction

Data envelopment analysis (DEA) technique is a linear programming (LP) problem that assesses the performance of decision making units (DMUs) involving multiple inputs/outputs. DEA is used in

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theoretical and practical problems. The efficiency measure was first introduced by Farell [1] and then developed by Charnes et al. [2] in the DEA framework.

Since the presentation of the first DEA models, different modifications from variety of aspects have been provided to strengthen the power of DEA. Kuosmanen and Post [3] computed relative efficiency score bounds considering upper and lower scores. They noted that these bounds reveal more accurate approximations for the relative efficiency score. They compute these bounds using LP problems. Kuosmanen and Post [4] corrected an existing technical error in their previous work. Mostafaee and Saljooghi [5] considered the cost efficiency measure in the presence of inadequate price details. Chakraborty et al. [6] emphasize that the products’ requirements alter over time. Fang and Li [7] assessed the cost efficiency in the presence of uncertain prices. They utilized cone-ratio models in DEA with the price information and added weight bounds to the model. Also, Mozaffari et al. [8] introduced cost and revenue models in DEA. Ghiyasi [9] introduced inverse DEA models for assessing both cost and revenue values. Moreover, Fang and Li [10] considered a duality study to build the theoretic attributes of envelopment and multiplier models of cost efficiency assessment assuming uncertain price. Aparicio et al. [11] indicated the way that an output-oriented version of the weighted additive model is treated to satisfactorily indicate revenue efficiency (RE). They introduced an inequality to define the market output price vector. Sahoo et al. [12] claimed that the firms' efficiency along with inputs and outputs varying prices are questionable. They developed new directional measures for cost and revenue assessment of efficiency. Aparicio et al. [13] discussed that in the case that firms face with price fluctuations, overall inefficiency measurement and decomposition are of great importance. Cook and Zhu [14] suggested a piecewise linear pricing DEA model to evaluate the relative efficiency of DMUs. Hosseinzadeh Lotfi et al. [15] developed a modified version of the DEA model which derives suitable benchmarks for inefficient DMUs.

The RE is one of the influential indices for managers and analysts who seek new strategies for gaining more benefits. Konara et al. [16] discussed the RE in banking and emerging markets. Deng et al. [17] utilized DEA to assess the RE of Spanish hotels. Cao et al. [18] studied the decreasing marginal revenue analysis in the agriculture sector.

In the previous RE analysis by DEA models, the prices are assumed to be known and fixed for the inputs/outputs of DMUs. However, the prices are variable in real world problems. Aroche-Reyes [19] reviewed some essential specifications of the price designation methods while considering an input-output model with focus on internal price designation procedure. Johnson and Ruggiero [20] considered a nonparametric measurement of allocative efficiency. They assumed that the output prices are endogenous. Moura [21] presented a two-sector model with two key ingredients for assessing investment shocks with endogenous relative prices. Din and Sun [22] assessed the endogenous choice of prices while taking into account quantities. Cellini et al. [23] proposed a dynamic model of price and quality competition for assessing the cause of competition on quality.

The objective of this paper is to develop a revenue efficiency DEA model to evaluate the relative efficiency of DMUs when the output prices are not fixed. In the conventional DEA models, to assess the RE, linear pricing is assumed. However, in real world problems, the variables have nonlinear behavior. In this study, we assume non-competitive context for the RE assessment. This is the main contribution of this paper that is considered for the first time in DEA. In doing so, here, a stepwise pricing of weight function is introduced to deal with the nonlinear behavior of variables. We develop a mixed integer LP model. Then, the developed model is used for assessing RE using Malmquist productivity index (MPI). The MPI analyzes the progress and regress of DMUs. This paper has the following contributions:

- In this study, we assume non-competitive context for the RE assessment.
- For the first time, a DEA model is introduced to consider nonlinear behavior of variables.
- It is shown that linear pricing does not adequately define the inherent concept of variables.
- In our new model, the prices are not assumed to be fixed.
- A stepwise pricing method is developed considering the theory of piecewise linear functions.
- A case study is given.

This paper proceeds as follows: In Section 2, the DEA preliminaries are reviewed. Section 3 presents our new model. A case study is given in Section 4. Managerial implications are discussed in Section 5. Section 6 concludes the paper.

2. DEA preliminaries

2.1 Revenue efficiency

DEA is a mathematical method for performance assessment of a set of DMUs. One of the hot applications of DEA is to calculate the RE. Capability of producing maximum outputs given current inputs is called RE. Here, we give a concise review of RE in DEA context. DMU_l is the DMU under evaluation. The used notations in this paper are as follows:

x_{ij} :	Indicates the ith input of DMU_j.	x^f_{ij}	Indicates the ith input of DMU_j in period f, $f \in \{t, t+1\}$.
y_{rj} :	Indicates the rth output of DMU_j.	y^f_{rj}	Indicates the rth output of DMU_j in period f, $f \in \{t, t+1\}$.
x_{il} :	Indicates the ith input of DMU_l.	x^k_{il}	Indicates the ith input of DMU_l in period k, $k \in \{t, t+1\}$.
y_{rl} :	Indicates the rth output of DMU_l.	y^k_{rl}	Indicates the rth output of DMU_l in period k, $k \in \{t, t+1\}$.
λ_j :	Intensifier variables for DMU_j.	$y^{k_r}_r$	Indicates the k_rth element of rth output variable.
φ:	The maximum increase in all outputs.	M	Indicates a big scalar.
θ	The maximum decrease in all inputs.	$t^{k_r}_r$	Indicates the k_rth element of rth output for the variable t.
y_r :	Indicates the rth output variable.	v_{k_r}	Indicates the k_rth binary variable.
$D^t(x^t_l, y^t_l)$	θ^* when DMU_l and technology are in period t.	$y^{qk_r}_r$	Indicates the k_rth element of the rth output variable for DMU_j in period q, $q \in \{t, t+1\}$.
$D^{t+1}(x^t_l, y^t_l)$	θ^* when DMU_l is in period t and technology is in period t+1.	$y^{fk_r}_{rj}$	Indicates the k_rth element of the rth output of DMU_j in period f, $f \in \{t, t+1\}$.

$D^{t}(x_l^{t+1}, y_l^{t+1})$	θ^* when DMU_l is in period t+1 and technology is in period t.	$y_{rj}^{k_r}$	Indicates the k_rth element of the rth output of DMU_j.
$D^{t+1}(x_l^{t+1}, y_l^{t+1})$	θ^* when DMU_l and technology are in period t+1.		

$PL.RE.MPI\,(x_l^{t+1}, y_l^{t+1}, x_l^{t}, y_l^{t})$**:** Piecewise linear revenue Malmquist productivity index.

$M\,(x_l^{t+1}, y_l^{t+1}, x_l^{t}, y_l^{t})$: Malmquist productivity index.

Assume that there are *n* DMUs with *m* inputs and *s* outputs which are semi-positive vectors. For each *DMUj*, the input and output vectors are denoted as $X_j = (x_{1j}, x_{2j}, ..., x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, ..., y_{sj})$ respectively for all *j*.

To evaluate the DMU under evaluation (DMU_l) in constant returns to scale environment, Charnes et al. [2] proposed the following LP problem.

$$
\begin{aligned}
\max \quad & \varphi \\
s.t. \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{il} & i = 1,...,m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \ge \varphi y_{rl} & r = 1,...,s, \\
& \lambda_j \ge 0 & j = 1,...,n.
\end{aligned} \tag{2.1}
$$

As addressed by Cooper et al. [24], RE can be obtained from the following procedure which leads to solve the following LP problem. To estimate the RE, output prices are assumed to be fixed and known although it is possible to be changed from one DMU to another DMU. Model (2.2) implies maximal revenue model, Wang et al., [25].

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} w_r y_r \\
s.t. \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{il} && i=1,\dots,m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j = y_r && r=1,\dots,s, \\
& y_r \ge 0 && \mathrm{r}=1,\dots,s, \\
& \lambda_j \ge 0 && j=1,\dots,n.
\end{aligned}
\tag{2.2}
$$

where w_r is the price for each output y_r. Each DMU produces outputs y_r, r=1,...,s at maximal revenue using inputs x_i, i=1,...,m. Therefore, for each DMU_l, the RE is defined as the ratio of its current revenue to the maximum revenue which is the optimal solution of Model (2.2) which is defined as follows:

$$
Revenue \quad efficiency = \frac{\sum_{r=1}^{s} w_r y_{rl}}{\sum_{r=1}^{s} w_r y_r^*} \tag{2.3}
$$

Note that Expression (2.3) should be less than or equal to 1 and also it should be greater than 0. The RE, considering the same level of inputs, shows the extent to which the DMU's revenue is close to the best DMU's revenue.

2.2 Malmquist productivity index (MPI)

DEA models can be used for estimating efficiency and productivity changes over period using Malmquist productivity index (MPI), Caves et al., [26]. The MPI considers two periods (t and t+1) and calculates efficiency variations over time. MPI can be computed by solving the following model by Caves et al., [26] for evaluation of DMU_l.

$$
D(x_l, y_l) = min\{\theta : (\theta x_l, y_l) \in T\} \tag{2.4}
$$

Given the resultant distance function (2.4), consider the following input-oriented CCR (Charnes-Cooper-Rhodes) model:

$$
\begin{aligned}
&D^{f}(x_l^k, y_l^k) = \min \quad \theta \\
&s.t. \quad \sum_{j=1}^{n} x^{f}_{ij}\lambda_j \le \theta x^{k}_{il} \quad i = 1,\dots,m, \\
&\qquad \sum_{j=1}^{n} y^{f}_{rj}\lambda_j \ge y^{k}_{rl} \quad r = 1,\dots,s, \\
&\qquad \lambda_j \ge 0 \quad j = 1,\dots,n.
\end{aligned} \tag{2.5}
$$

Accordingly, four LP problems can be defined. Consider l to be the notion of the unit under evaluation and each of k and f show periods t and t+1. For instance, to assess DMU_l, let $k=t$ and $f=t+1$, $D^{t+1}(x_l^t, y_l^t)$ which shows the coordinates of DMU_l in period t and technology in period t+1. To show progress and regress of DMUs, Caves et al. (1982) proposed MPI which is as follows:

$$
M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \left(\frac{D^t(x_l^{t+1}, y_l^{t+1})D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^{t+1}, y_l^{t+1})D^{t+1}(x_l^t, y_l^t)}\right)^{1/2} \tag{2.6}
$$

The decomposition of this index shows technical efficiency alteration and technology frontier shift while two periods are taken into account (t and t+1).

$$
M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \left(\frac{D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)}\right)\left(\frac{D^t(x_l^{t+1}, y_l^{t+1})D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)D^{t+1}(x_l^t, y_l^t)}\right)^{1/2} \tag{2.7}
$$

If $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) > 1$, then there is a progress in the total productivity of DMU. If $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) < 1$, then there is a regress in the total productivity of DMU. If $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = 1$, then the total productivity is unchanged.

3. Proposed method

3.1 Stepwise pricing for evaluating revenue efficiency

At this juncture, a new method for evaluating the RE is presented. To the best of our knowledge, RE given nonlinear behavior of sales prices has not yet been discussed so far. Consider a non-competitive situation. For instance, in a power plant it is expected that revenue should be increased by producing more electric power. If a power plant produces 9 Mega Watt per Hour (MWH) of electric power, this power will be sold for \$100 per unit for the first 5 units and \$140 per unit for the 4 units beyond the first 5 units. This is a unique case which is discussed in this paper. The existing DEA models just deal with linear pricing. Linear pricing is not applicable in all situations because it may lead to inaccurate results for enterprises like power plants that should use a stepwise pricing system. The average price is not a suitable replacement as the nonlinear behavior of pricing is not considered. This paper considers the conditions in which increments in value are automatically taken into account and occur frequently in real-life situations. So far, in DEA, for calculating RE, a linear function has been considered. Linear function gives an approximate solution. Here, to assess RE, we present a model in which incremental revenue is considered.

Note that the values depend on the amounts of products which is called stepwise rating. Although the values are stable in standard RE measurement, in real world problems this is not the case. Consider y_{rj} as the *rth* output of *j*th DMU. In addition, let w_r is the value of this product. In such a situation, the producer will obtain higher revenue, if a greater amount of the product is produced and sold. In this case, when a stepwise rating system is contemplated, a greater amount of the product will be sold at the higher prices. To get accurate results, it is essential to establish a general framework for RE which considers the real-life market situations. Cook and Zhu [14] discuss that in multiplier DEA model, linear weighting is not adequately capable of indicating the innate behaviors of variables. They argued that some variables have non-linear behavior and linear weighting may lead to a bias. Hosseinzadeh Lotfi et al. [15] presented the modified version of the model in envelop form of CCR model. In Fig. 1, the convex function indicates that the more output, the more revenue.

<< Figure 1 goes here >>

According to the piecewise linear function theory, the estimation of this function can be enhanced by breaking down the scale of the r*th* output into k_r segments in which they are assumed to behave linearly in their segments (see Fig. 2).

R_1 and R_2 represent sets of regular and piecewise linear outputs with increasing magnitude, respectively. Thus, the scale of variable which reveals piecewise linear behavior should be considered as k_r ranges such as $[0,L_1]$, $(L_1,L_2]$, ..., $(L_{kr-1},L_{kr}]$. Consider,

$$t^{k_r}{}_i = \begin{cases} L_{k_r}, & if \quad k_r = 1 \\ L_{k_r} - L_{k_r - 1}, & if \quad k_r = 2,\dots,l_{k_r} \end{cases} \qquad (3.1)$$

<< Figure 2 goes here >>

An expert should determine the number and width of ranges. The vector of profits corresponding to y_r $(r\in R_2)$ which shows nonlinear behavior consists of k_r ranges as $w_r^{k_r}, k_r = 1,\ldots,l_{k_r}, r \in R_2$, where $w_r^{k_r} < w_r^{k_{r+1}}, k_r = 1,\ldots,l_{k_r}, r \in R_2$. We can represent the contribution of the rth output to the weighted aggregate of all outputs in the objective function of maximal revenue model as $\sum_{k_r=1}^{l_{k_r}} w_r^{k_r} y_r^{k_r}$ instead of a single $w_r y_r$.

Assume that y_r is the rth output $(r\in R_2)$ which has stepwise pricing, and w_r is the corresponding vector of prices. For instance, let $k_r = 3$ and consider three ranges for y_r including (y_r^1, y_r^2, y_r^3). We define ranges as [0, 300), [300, 700), and [700, 1000] for y_r^1, y_r^2, and y_r^3, respectively. For $w_r = (w_r^1, w_r^2, w_r^3)$, consider $(650, 700, 750)$, respectively. In traditional approach, $y_r = 800$ and $w_r = 800$ are assumed. In our approach, given the ranges and Equation (3.1), y_r should be replaced with $(y_r^1, y_r^2, y_r^3) = (300, 400, 100)$. Also, w_r should be replaced with $(w_r^1, w_r^2, w_r^3) = (650, 700, 750)$. This conveys the meaning of stepwise pricing in which the output is sold $650 per unit for the first range, $700 per unit for the second range, and $750 per unit for the third range. However, in traditional approach, whole 800 units of outputs are sold $800 per unit.

Our new Model (3.2) can find the optimal $y_r^{k_r}, k_r = 1,\ldots,l_{k_r}, r \in R_2$, which complies with the theory of piecewise function. It means that $y_r^{k_r}, k_r = 1,\ldots,l_{k_r}, r \in R_2$ should be a sequence of sequential values in their specified ranges. Therefore, lower ranges should be filled before higher ones are filled. The stated concepts comply with the reasoning that the scale should be divided to show the nonlinear behavior of the outputs. In Model (3.2), let $v_0 = 0$ and M to be a big positive constant. Note that $k_r, r\in R_2$ illustrates the number of intervals, $(L_{kr-1}, L_{kr}]$. The binary variable v_k forces $y_r^{k_r}$ to become zero. It is obvious that in the case that the lower ranges have not been completely met, it results in $v_k = 1$ in which $y_r^{k_r}$ is forced to become zero. To confine $y_r^{k_r}$ in a way that each portion of this kind of output gets a value according to Equation (3.1), constraints (a), (b), and (c) are added to the model. Furthermore, these variables control the lower ranges to be completed before the upper ranges. In the objective function of Model (3.2), it is clear that each portion of the defined output includes distinct values which are in an increasing order and is based on its magnitude.

$$
\begin{aligned}
Max \quad & \sum_{r\in R_1} w_r y_r + \sum_{r\in R_2} (w_r^1 y_r^1 + w_r^2 y_r^2 + \ldots + w_r^{k_r} y_r^{k_r}) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{il} && i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_r && r \in R_1, \\
& \sum_{j=1}^{n} \lambda_{k_r j} y_{rj}^{k_r} \ge y_r^{k_r} && r \in R_2, k_r = 1,\ldots,l_{k_r}, \\
& y_r^{k_r} \le t_r^{k_r}(1 - v_{k_r-1}) && r \in R_2, k_r = 1,\ldots,l_{k_r}, \quad (a) \\
& v_{k_r} \le (t_r^{k_r} - y_r^{k_r}).M && r \in R_2, k_r = 1,\ldots,l_{k_r}, \quad (b) \\
& (t_r^{k_r} - y_r^{k_r}) \le v_{k_r}.M && r \in R_2, k_r = 1,\ldots,l_{k_r}, \quad (c) \\
& \lambda_j \ge 0, y_r \ge 0 && j = 1,\ldots,n, r \in R_1, \\
& \lambda_{k_r j} \ge 0, y_r^{k_r} \ge 0 && j = 1,\ldots,n, r \in R_2, k_r = 1,\ldots,l_{k_r}, \\
& v_{k_r} \in \{0,1\} && r \in R_2, k_r = 1,\ldots,l_{k_r}.
\end{aligned} \tag{3.2}
$$

The linearization of revenue function may differ among various amounts of y_r. This is shown in Fig. 3. As is seen, the linearization function differs between $y_r = 400$ and $y_r = 750$. Note that when $y_r^{(k+1)_r}, k_r = 1,\ldots,l_{k_r}, r \in R_2$ has a positive value, $y_r^{k_r}, k_r = 1,\ldots,l_{k_r}, r \in R_2$ has also a positive value. Hence, we have $w_r^1 y_r^1 + w_r^2 y_r^2 + w_r^3 y_r^3$.

<< Figure 3 goes here >>

In Model (3.2), the first output has nonlinear behavior in pricing. Thus, in this model instead of single Expression $w_1 y_{1o}$, a linear combination like $w_r^1 y_{1l}^1 + w_r^2 y_{1l}^2 + \ldots + w_r^{l_{k_r}} y_{1l}^{l_{k_r}}$ is replaced. Note that l_{kr} shows the number of defined intervals for variations of the first output. Consider $r = 2,\ldots,s$ and $r=1$ to be the sets of regular and piecewise linear outputs, respectively. Model (3.2) is a mixed integer nonlinear programming as there are integer variables v_{kr}, $r=1$. Finally, a piecewise linear RE for DMU_l is ratio of piecewise linear maximum revenue of the current revenue of DMU_l divided by the optimal solution of Model (3.2) which is as follows:

$$
\text{Revenue efficiency} = \frac{\sum_{r\in R_1} w_r y_{rl} + \sum_{r\in R_2} (w_r^1 y_{rl}^1 + w_r^2 y_{rl}^2 + \ldots + w_r^{k_r} y_{rl}^{k_r})}{\sum_{r\in R_1} w_r y_r^* + \sum_{r\in R_2} (w_r^1 y_r^{*1} + w_r^2 y_r^{*2} + \ldots + w_r^{k_r} y_r^{*k_r})} \tag{3.3}
$$

Theorem 3.1. Model (3.2) is always feasible and the objective function is bounded**.**

Proof: Let $\lambda_l=1$, $\lambda_j=0$, $\forall j=l$, $\lambda_l^{k_r}=1$, $\lambda_j^{k_r}=0$, $\forall j\neq l$, $\forall r=2,\dots,s$, $y_r^{lkr}=t_i^{k_r}$, $\forall r=1$, $\forall k_r=1,\dots,l_{kr}$, $v_0=0$, $v_{kr}=0$, $v_{kr-1}=0$, $\forall r=1$, $k_r=1,\dots,l_{kr}$. Therefore, it can be concluded that Model (3.2) is feasible and the objective function is bounded. After solving Model (3.2), the optimal solution is as $y_2^*, r=2,\dots,s$, $y_1^{*k_r}, r=1, k_r=1,\dots,l_{k_r}$. □

Theorem 3.2. The obtained target point $(x_l, y_r^*, y_r^{*k_r})$ after solving Model (3.2) is Pareto efficient.

Proof: An important point in DEA is that each DMU is compared with to the rest of DMUs. Suppose that the $(y_r^*, y_r^{*k_r})$ is not Pareto efficient. Thus, there is a feasible solution $(\bar{y}_r, \bar{y}_r^{k_r})$ which dominates $(y_r^*, y_r^{*k_r})$. Therefore, it can be concluded that this feasible solution has a greater objective function compared with the obtained optimal $(y_r^*, y_r^{*k_r})$ which conflicts with optimality of $(y_r^*, y_r^{*k_r})$. As a result, it can be concluded that obtained target point $(x_l, y_r^*, y_r^{*k_r})$, after solving Model (3.2), is Pareto efficient. □

Theorem 3.3. Given Model (3.2) and Equation (3.3), at least one DMU is revenue efficient.

Proof: In Model (3.2), at least one inequality constraint which is related to the outputs should be binding. Otherwise it is concluded that all the outputs' inequality constraints are $\sum_{j=1}^{n}\lambda_{k_r j} y_{rj}^{k_r} > y_r^{*k_r}, (r\in R_2, k_r=1,\dots,l_{k_r})$. It is assumed that there is a feasible solution in which $\sum_{j=1}^{n}\lambda_{k_r j} y_{rj}^{k_r} \geq \bar{y}_r^{k_r}, (r\in R_2, k_r=1,\dots,l_{k_r})$. In this case, $\bar{y}_r^{k_r} > y_r^{*k_r}, (r\in R_2, k_r=1,\dots,l_{k_r})$ which is a contradiction. Therefore, at least for one DMU, the output inequality constraints are binding which means the current revenue is equal to the maximum revenue and it is revenue efficient. □

3.2 Stepwise pricing in revenue MPI

Here, piecewise linear revenue MPI (PLREMPI), is introduced given Model (3.2). Consider periods t and t+1. Note that l is as the DMU under evaluation and q and f denote t and t+1, respectively. Assume

k=t and f=t+1, $D^f(x_l^q, y_l^q) = D^{t+1}(x_l^t, y_l^t)$, which shows DMU_l in period t while technology is considered in period t+1. The new model is as follows:

$$
\begin{aligned}
D^f(x_l^q, y_l^q) = Max \quad & \sum_{r \in R_1} w_r y_r^q + \sum_{r \in R_2} (w_r^1 y_r^{q1} + w_r^2 y_r^{q2} + \ldots + w_r^{k_r} y_r^{qk_r}) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^f \le x_{il}^q \qquad i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^f \ge y_r^q \qquad r \in R_1, \\
& \sum_{j=1}^{n} \lambda_{k_r j} y_{rj}^{fk_r} \ge y_r^{qk_r} \qquad r \in R_2, k_r = 1,\ldots,l_{k_r}, \\
& y_r^{qk_r} \le t_r^{k_r}(1 - v_{k_r - 1}) \qquad r \in R_2, k_r = 1,\ldots,l_{k_r}, \quad (a) \\
& v_{k_r} \le (t_r^{k_r} - y_r^{qk_r}).M \qquad r \in R_2, k_r = 1,\ldots,l_{k_r}, \quad (b) \\
& (t_r^{k_r} - y_r^{qk_r}) \le v_{k_r}.M \qquad r \in R_2, k_r = 1,\ldots,l_{k_r}, \quad (c) \\
& \lambda_j \ge 0, y_r \ge 0 \qquad j = 1,\ldots,n, r \in R_1, \\
& \lambda_{k_r j} \ge 0, y_r^{qk_r} \ge 0 \qquad j = 1,\ldots,n, r \in R_2, k_r = 1,\ldots,l_{k_r}, \\
& v_{k_r} \in \{0,1\} \qquad r \in R_2, k_r = 1,\ldots,l_{k_r}.
\end{aligned}
\tag{3.4}
$$

To calculate piecewise linear revenue MPI (PLREMPI), Expression (3.5) is used. Note that DMU is in period t and technology is in period t+1.

$$PL.RE.MPI(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \left(\frac{D^t(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t) D^{t+1}(x_l^t, y_l^t)}\right)^{1/2} \tag{3.5}$$

If the score is more than one, then there is a progress in the productivity from period t to period t+1. If the score is less than one, then there is a regress in the productivity from period t to period t+1. If the score is equal to one, then there is no progress or regress in the productivity from period t to period t+1.

3.3 Illustrative example

Consider five DMUs with one input (I_1) and one output (O_1). For the output, three ranges are considered as o_1^1= [0, 11), o_1^2= [11, 17), and o_1^3= [17, 80). Numbers are in MWH. Table 1 depicts the dataset. Price vector of the output is (w_1^1, w_1^2, w_1^3) = (1000,1200,1500). In classical evaluation, the price is considered as 1000$. Note that the sum of the three ranges is equal to the initial output (O_1). In

Table 2, the piecewise linear evaluation of maximum revenue is represented. Also, the results of classical maximum revenue and the optimal values of output (O_1*) are reported in Table 3.

<< Table 1, 2, and 3 go here >>

For DMU_1 the output is 11 and the current revenue is 11000. After evaluation by Model (3.2), the optimal output is 17.71 and the corresponding revenue is 19265. Given the classical revenue analysis (Model 2.2), the optimal maximum revenue is 12705.88 and the optimal output is 12.71. By comparing Tables 2 and 3, it is seen that Model (2.2) overestimates the outputs. Note that the output of DMU_4 in both Models (3.2) and (2.2) are the same. However, for DMU_5, Model (2.2) overestimates the output.

Here, we just deal with outputs for presenting a stepwise pricing to reach revenue evaluation in DEA context. Thus, some modifications are done in “maximum revenue” optimization model to get a new RE score. A good property of our model is that it does not present a fixed value for the outputs. Instead, given the outputs, it presents different values to get possible maximum revenue score.

4. Case study

Here, we wish to assess Iranian power plants. Most of the Iranian power plants are owned by government and government sells electricity. Due to high electricity consumption in Iran, the generated electricity is insufficient. Here, 20 public power plants in Iran are assessed. Since consumption is higher than electricity generation, Iranian government faces with lack of electricity. Thus, government has to buy electricity from private power plants and sell it to consumers in a subsidized rate. Thus, the price is considered as a penalty paid by government for over-consumption of consumers. In some seasons, there might be periods that demand is less than supply. Thus, the government can sell the extra generated electricity to neighbor countries.

Here, the stepwise pricing for the electricity is considered in non-competitive Iranian power generation market. Therefore, the more price, the more income. In this case, the intervals for changing amount of electricity are considered. For each interval, a specific price is considered. Note that the number and length of the intervals and corresponding prices are determined by the experts. Furthermore, assuming fixed prices is not applicable in real world applications and it may lead to the biased efficiency scores.

The dataset is obtained from Tavanir Management Organization [27]. Dataset dates back to 2004 to 2006. The inputs and outputs are as follows:

I_1: Capacity (MW)

I_2: Internal usage (MWH)

I_3: Fuel (Tera Joule (TJ))

O_1: Electrical power production (MWH)

The dataset related to 2004 is depicted in Table 4.

<< Table 4 goes here >>

In Table 5, given the opinions of experts, the O_1 is divided into four ranges. There is a nonlinear relation between power generation and revenues of the power plants. Therefore, linear pricing cannot give a favorable result. However, using a stepwise rating system, a greater amount of electricity power can be sold for higher prices in noncompetitive situations. Current DEA models cannot consider stepwise pricing in revenue evaluation. Thus, previous DEA models may result in erroneous results.

Given Equation (3.1) and to apply our model, four output ranges are taken into consideration which are as follows:

$$O_1^1= [0,3],\ O_1^2=(3,5],\ O_1^3=(5,7],\ \text{and}\ O_1^4=(7,\infty) \qquad (4.1)$$

The numbers are in MWH. For example, as is depicted in Table 4, the output of DMU_1 is 3297100. The intervals of this output, given Expression (4.1), are defined as (3000000, 297100, 0, 0). The corresponding prices for each of the defined intervals are 1000, 1200, 1500, and 1800, respectively (100 Rials). Hence, it can be said that, here, we deal with a stepwise pricing system.

<< Table 5 goes here >>

Table 6 shows classical RE and piecewise linear revenue efficiency (PL-RE). The results are obtained by solving Model (3.2) and using Equation (3.3). We use GAMS software to solve the problem. As is seen in Table 6, compared with the results of classical RE evaluations, the results of piecewise linear revenue efficiency (PL-RE) might be increasing, decreasing, or unchanged. This could be similar to the classical RE findings in which there is a nonlinear behavior in the outputs. It is clear that our proposed model yields substantial improvement in the RE measurement.

There is a difference between the results of Model (2.2) and Model (3.2). Generally, the efficiency scores obtained from the piecewise linear DEA analysis can be either lower or higher than the standard DEA peers. In the results, compared with the standard RE model, some DMUs have higher efficiency scores and some DMUs have lower efficiency scores. The results are obtained using GAMS software.

<< Table 6 goes here >>

Figure 4 compares the results of RE and PL-RE.

<< Figure 4 goes here >>

Now, consider the output of DMU_5 (7438002) which is indicated as (3000000, 2000000, 2000000, 438002). According to the experts' opinions, the corresponding prices are in an increasing order (1000, 1200, 1500, and 1800). The PL-RE for DMU_5 is 0.98. This measure is derived from dividing the revenue obtained from the current level of the output by the result of Model (3.1). It shows the best possible revenue for DMU_5. The result of Expression (3.1) for DMU_5 is (3000000, 2000000, 2000000, and 218496.09). Given the defined values for each of these ranges, the obtained revenue is 9188403600. However, by multiplying the amount of each interval by the vector value, we obtain 9057003000. In addition, by dividing 9057003000 to 9188403600, the PL-RE is obtained. This measure indicates a crucial debate that DMU_5 can increase its output given the same inputs. This is the most important result that can be obtained from peer revenue evaluation. Another important finding is that the managers can better understand the capability of the system for producing products.

At this juncture, let us contemplate the classical method for deriving RE. The RE score for DMU_5 in the classical method is equal to 1. This score is obtained from Expression (3.3) through dividing the revenue obtained from the current output (7438002000) by the best possible revenue for DMU_5 which is obtained from Model (3.2). This measure demonstrates that the obtained output is the best for DMU_5 and it cannot produce more output given the similar level of inputs. However, using

Model (2.2) and Expression (2.3), the RE score of DMU_{17} is 0.48. This score shows that DMU_{17} can increase its output. Given the classical model (2.2) and Expression (2.3), this measure is obtained via dividing the revenue obtained from the current output by the best possible revenue for DMU_{17}. In a piecewise linear pricing, the revenue obtained from the current output is 3997278000 and the best possible revenue for DMU_{17} is 3997278000. Therefore, by dividing these two numbers, we get 1. In Table 6, the optimal solution of Model (3.1) is reported. There is a significant difference between the two sets of results. The convex function, as illustrated in Figure 1, denotes an increasing function. To estimate the convex function by a linear function, the characteristics of the convex function cannot be displayed precisely. Thus, the results are inaccurate. However, estimating the convex function by a piecewise linear convex function leads to more accurate results. Similar analysis can be repeated for 2005 and 2006. The dataset is reported in Table 7.

<< Table 7 goes here >>

The optimal maximum piece-wise linear revenue and efficiency scores are depicted in Table 8. In Table 8, the optimal maximum piece-wise linear revenue for 2004, 2005, and 2006 are listed.

<< Table 8 goes here >>

Table 9 shows the PL -RE scores in 2005 and 2006. For example, in 2004, the optimal values of DMU_1 for each divided range is (3000000, 2000000, 2000000, and 0). The sum of ranges is 7000000. By comparing the obtained target for the output (7000000) with its initial amount (3297100), we find out that DMU_1 should increase its output to become efficient. In 2005, the initial output is 2222273 and the obtained output target is 7500465.270. The divided ranges are (3000000.000, 2000000.000, 2000000.000, and 500465.27). For the output, we can increase the output by 3.37%. In 2006, the target output is 7528574 and compared with the initial amount (7528574), it is fixed.

As another example, consider DMU_8. In 2004, the obtained target for each of the divided output is (3000000, 2000000, 2000000, and 500465.27) which its sum is 7500465.27. Note that the initial output is 4341330 (3000000, 1341330, 0, and 0). Thus, DMU_8 can increase its output by 1.73%. In 2005, DMU_8 can increase its output by 5.17%. Its initial output and target are 2229153 and 11545325.31, respectively. In 2006, it cannot increase its output as its initial output and target are 9642639 and 9642639, respectively.

<< Table 9 goes here >>

Therefore, to assess performance of the power plants during three years, the MPI can be utilized. Note that the MPI can determine situations where there is no change.

In Table 10, the optimal maximum piecewise linear output resulted from Model (3.2) is listed. The optimal maximum piecewise linear output for DMU_1, DMU_8, DMU_{11}, DMU_{13}, and DMU_{15} are depicted in Table 10 as instances. The ranges are obtained by Expression (3.1). Similarly, we can obtain the results for the other DMUs. The optimal values of the piecewise linear output o_1 are reported in Table 10. As well, the summation of these values for years 2004, 2005, and 2006 is calculated. These results are then used in Expression (3.3) as y^* for calculating the PL-RE for years 2004, 2005, and 2006.

<< Table 10 goes here >>

Knowing progress and regress of DMUs in different periods helps decision makers to better recognize the shortcomings of DMUs. After obtaining the optimal objective function of Model (3.4), using Expression (3.5), the MPI is calculated. The MPIs are listed in Tables 11 and 12. By comparing 2004 with 2005, and also 2005 with 2006, as is depicted in Tables 11 and 12, some DMUs have regress and some DMUs have progress. Note that our method deals with the nonlinear behavior of variables and it affects progress and regress of DMUs during periods.

<< Table 11 and 12 go here >>

5. Managerial implications

Reducing energy consumption is an important issue. This issue should be accepted as a principle in society in both energy production and consumption. An important measure that can help producers to evaluate their performances is the RE score. In this paper, we addressed the main shortcoming of the classical RE which is the fixed price assumption. Also, we modified the previous DEA models to face with the stepwise pricing. The proposed DEA model is based on mixed integer programming and the big M technique. In the case study, we showed that the results of our model are different from the classical model. Furthermore, the progress and regress of DMUs were discussed which give important implications to managers for making crucial decisions.

In Tables 13 and 14, the results of Models (3.2) and (2.2) for DMUs 1, 8, 11, 13, and 15 are reported, respectively. Given Tables 13 and 14, it is clear that our model (Model (3.2)), in most of the

DMUs, increases the output compared with Model (2.2). Since, for the upper intervals higher prices are considered, thus our model tries to reach higher amounts. The maximum revenue obtained from Model (3.2) is higher than Model (2.2). This is an important finding for managers as they are responsible for performance evaluation. The other important finding is the RE score which may decrease, increase, or remain unchanged. This shows the different capability of DMUs for producing products according to the stepwise pricing.

<< Table 13 and 14 go here >>

6. Conclusions

This study modified the classical RE in DEA. The classical RE in DEA has two critical issues. Firstly, its linear pricing cannot show reality of variables which have nonlinear behavior. Secondly, the classical RE assumes the fixed prices which is not applicable in real world problems. In this paper, we used DEA for evaluating the RE and addressed the shortcomings. Our model deals with stepwise pricing systems to get more accurate results in RE assessments. We showed that the results may increase, decrease, or remain unchanged compared with the previous RE models. DMUs can produce more products with higher prices to get more revenues.

For further researches, we suggest developing new DEA models that can deal with competitive settings. Finding optimal value of the big M will be another interesting research topic.

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Conflict of interest: None.

List of figures:

List of Tables:

Table 14. The results of the classical approach

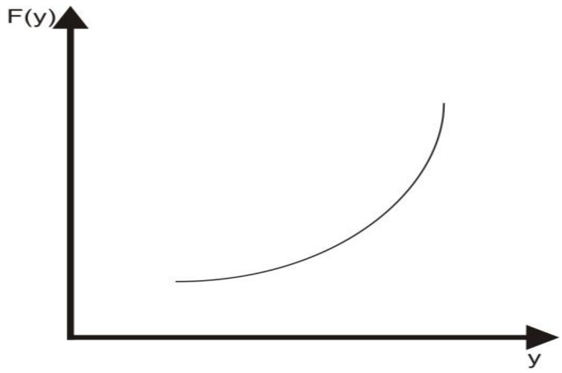

Fig. 1: A convex function

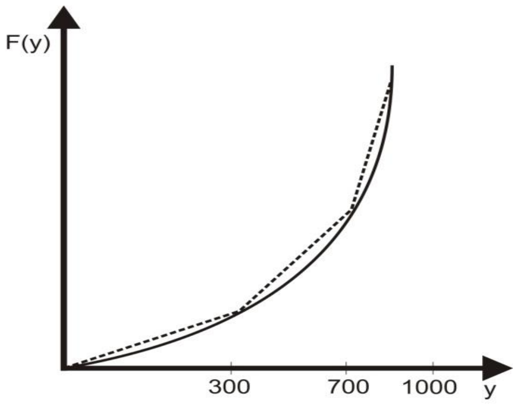

Fig. 2: A convex piecewise linear function

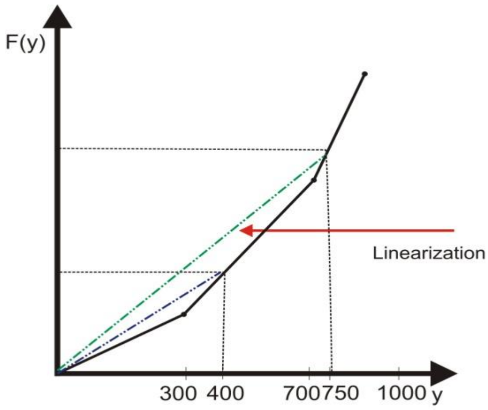

Fig. 3: Linearization

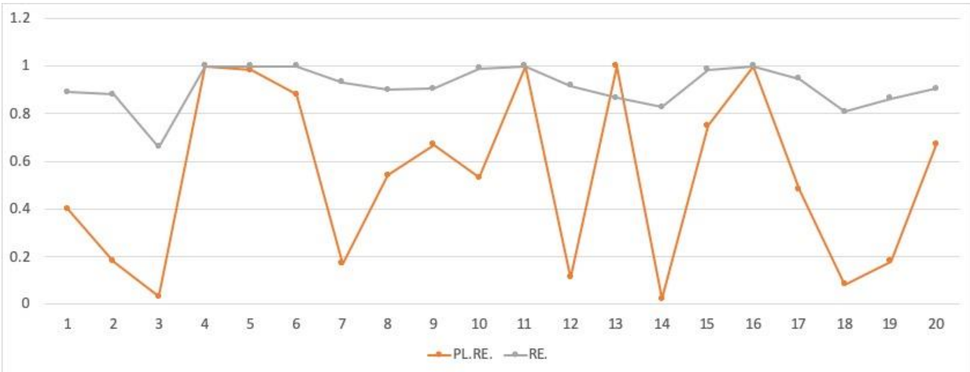

Fig. 4: The results of revenue efficiency (RE) and piece-wise linear revenue efficiency (PL-RE)

Table 1. Dataset of the example

DMUs	Current revenue	I_1	o_1^1	o_1^2	o_1^3	O_1
1	11000	12	11	0	0	11
2	14600	24	11	3	0	14
3	9000	25	9	0	0	9
4	19700	17	11	6	1	18
5	24200	26	11	6	4	21

Table 2. The results of piece-wise linear evaluation

DMUs	Maximum revenue	I_1	o_1^1	o_1^2	o_1^3	Sum of output ranges
1	19265	12	11	6	0.71	17.71
2	23195	24	11	6	3.33	20.33
3	23705	25	11	6	3.67	20.67
4	19700	17	11	6	1.00	18.00
5	24200	26	11	6	4.00	21.00

Table 3. Results of classical maximum revenue

DMUs	Maximum Revenue	I_1	O_1^*
1	12705.88	12	12.71
2	25411.76	24	25.41
3	26470.59	25	26.47
4	18000.00	17	18.00
5	27529.41	26	27.53

Table 4. Dataset of 2004

Power plants (DMUs)	I_1	I_2	I_3	O_1
1	625.88	241139	30.95308	3297100
2	247.5	139505	17.00441	1500253
3	50.0	13039	3.412765	212403
4	1760	301276	107.1448	11000000
5	1300	642909	57.30782	7438002
6	1000	421015	54.61188	6342203
7	240	85307	14.06063	1435991
8	736	361080	42.82585	4341330
9	1000	390708	46.37235	5134547
10	640	350154	38.2554	4210280
11	1890	636643	93.29557	11000000
12	290	81674	7.886569	922587
13	1280	588855	73.2442	7196540
14	60	29698	4.322743	341402
15	835	422673	52.77859	5621431
16	1600	796262	102.4223	11000000
17	600	271901	37.45617	3831065
18	120	63050	7.955432	665887
19	256	140940	15.76105	1492847
20	1000	390708	46.37228	5134547

Table 5. The output ranges

DMUs	o_1^1	o_1^2	o_1^3	o_1^4
1	3000000	297000	0	0
2	1500253	0	0	0
3	212403	0	0	0
4	3000000	2000000	2000000	4000000
5	3000000	2000000	2000000	438002
6	3000000	2000000	1342203	0
7	1435991	0	0	0
8	3000000	1341330	0	0
9	3000000	2000000	134547	0
10	3000000	1210280	0	0
11	3000000	2000000	2000000	4000000
12	922587	0	0	0
13	3000000	2000000	2000000	196540
14	341402	0	0	0
15	3000000	2000000	621431	0
16	3000000	2000000	2000000	4000000
17	3000000	831065	0	0
18	665887	0	0	0
19	1492847	0	0	0
20	3000000	2000000	134547	0

Table 6. The efficiency scores

DMUs	*PL−RE*	*RE*	*DMUs*	*PL−RE*	*RE*
1	0.400	0.891	11	1.000	1.000
2	0.180	0.881	12	0.110	0.917
3	0.031	0.661	13	1.000	0.866
4	1.000	1.000	14	0.020	0.828
5	0.982	1.000	15	0.750	0.985
6	0.881	1.000	16	1.000	1.000
7	0.170	0.931	17	0.480	0.947
8	0.541	0.901	18	0.083	0.807
9	0.670	0.905	19	0.180	0.865
10	0.531	0.991	20	0.670	0.904

Table 7. The dataset (2005 and 2006)

DMUs	2005				2006			
	I_1	I_2	I_3	O_1	I_1	I_2	I_3	O_1
1	260	615989	3.20	2222273	1465	606125	5.780	7528574
2	241	725654	99.340	2692987	1584	54636	45.764	9722169
3	365	285942	57.230	7904814	1271	889503	84.562	9163596
4	627	652939	46.300	1571045	959	440922	45.873	4177877
5	676	323994	38.987	6629453	240	759745	43.673	4448964
6	442	352884	92.345	6789916	279	865304	12.673	6077669
7	1538	41414	53.456	2165922	884	401264	98.563	6948107
8	1088	230039	100.453	2229153	1415	312014	34.674	9642639
9	1931	381196	36.765	6651760	1614	972120	45.632	7206031
10	1978	320838	38.987	11480340	596	693166	89.234	4577178
11	1541	584695	95.980	5699805	961	349921	13.452	7320412
12	835	288844	13.456	1216457	690	468501	103.452	2317777
13	851	203257	7.849	9776371	1241	429415	76.34	6034480
14	1080	23631	62.56	11468004	345	26308	57.836	1285188
15	384	350528	93.456	2056659	805	250166	37.893	6158195
16	1123	740735	85.912	6751574	118	438411	16.75	3046098
17	1042	311695	76.843	11575914	1189	456237	97.543	4964059
18	1540	53701	18.453	8448269	1345	329387	48.341	2964572
19	981	104550	19.457	846810	263	386149	15.894	2866023
20	1931	381196	36.757	6651760	1614	972120	45.602	7206031

Table 8. Optimal maximum piece-wise linear revenues for each year

DMUs	2004	2005	2006
1	15600000000.00	2222273000.0000	9351433200.0000
2	1500253000.000	2692987000.0000	13299904200.000
3	219488279.0000	10028665200.000	12294472800.000
4	15600000000.00	12184718052.617	7386842138.6930
5	9188403600.000	12473798836.527	4738756800.0000
6	7413304500.000	10780237519.055	7016503500.0000
7	1435991000.000	15621206779.588	8322160500.0000
8	4609596000.000	16581585566.974	13156750200.000
9	5601820500.000	16245741346.381	13298200293.057
10	4452336000.000	16464612000.000	7700624605.7850
11	15600000000.00	16636645200.000	8976741600.0000
12	922587000.0000	13372816127.548	7512657957.4250
13	8753772000.000	13397467800.000	11639050395.669
14	362147158.0.00	16442407200.000	1285188000.0000
15	6332146500.000	10214118109.897	7137292500.0000
16	15600000000.00	16636645200.000	3055317600.0000
17	3997278000.000	16636645200.000	11178711052.722
18	725157139.0000	11006884200.000	12521705809.707
19	1537630716.000	1093548938.9070	2866023000.0000
20	5601820500.000	16245741346.378	13298200293.042

Table 9. Piece-wise linear revenue efficiency scores in 2005 and 2006

DMUs	2005	2006	DMUs	2005	2006
1	0.24	1.00	11	0.39	1.00
2	0.27	1.00	12	0.09	0.26
3	1.00	1.00	13	1.00	0.60
4	0.13	0.49	14	1.00	0.10
5	0.63	0.54	15	0.20	0.85
6	0.75	0.78	16	0.48	0.34
7	0.14	0.93	17	1.00	0.48
8	0.13	1.00	18	1.00	0.24
9	0.52	0.66	19	0.06	0.32
10	1.00	0.55	20	0.52	0.67

Table 10. Optimal maximum piecewise linear output

	Outputs	DMU_1	DMU_8	DMU_{11}	DMU_{13}	DMU_{15}
2004	o_1^1	3000000	3000000	3000000	3000000	3000000
	o_1^2	2000000	2000000	2000000	2000000	2000000
	o_1^3	2000000	2000000	2000000	2000000	2000000
	o_1^4	0	67354.51	4000000	196540	3272.77
	Sum	7000000	7067354.51	11000000	7196540	7003272.77
2005	o_1^1	3000000	3000000	3000000	3000000	3000000
	o_1^2	2000000	2000000	2000000	2000000	2000000
	o_1^3	2000000	2000000	2000000	2000000	2000000
	o_1^4	500465.27	4545325.31	4575914	2776371	1007843.39
	Sum	7500465.27	11545325.31	11575914	9776371	8007843.39
2006	o_1^1	3000000	3000000	3000000	3000000	3000000
	o_1^2	2000000	2000000	2000000	2000000	2000000
	o_1^3	2000000	2000000	2000000	2000000	2000000
	o_1^4	528574	2642639	320412	1799472.44	0
	Sum	7528574	9642639	7320412	8799472.44	7000000

Table 11. Malmquist Productivity Index (MPI) comparing 2004 with 2005

DMU	$D^{2004}(x_l^{2004}, y_l^{2004})$	$D^{2005}(x_l^{2005}, y_l^{2005})$	$D^{2004}(x_l^{2005}, y_l^{2005})$	$D^{2005}(x_l^{2004}, y_l^{2004})$	MPI (2004-2005)
1	0.3996	0.2389	0.202	0.142	0.65
2	0.1786	0.2685	0.09	0.173	1.696
3	0.0253	1	0.013	0.643	44.612
4	1	0.1289	0.938	0.101	0.118
5	0.9857	0.6289	0.552	0.503	0.762
6	0.8825	0.75	0.446	0.518	0.994
7	0.171	0.1387	0.086	0.139	1.142
8	0.541	0.1344	0.277	0.143	0.358
9	0.6669	0.5154	0.337	0.537	1.11
10	0.53	1	0.268	1.055	2.728
11	1	0.3877	0.938	0.413	0.413
12	0.1098	0.091	0.055	0.078	1.08
13	1	1	0.526	0.859	1.278
14	0.0219	1	0.021	1.054	48.428
15	0.7533	0.2014	0.381	0.132	0.304
16	1	0.4825	0.938	0.515	0.515
17	0.4759	1	0.242	1.066	3.046
18	0.0793	1	0.04	0.706	14.91
19	0.1777	0.0563	0.09	0.054	0.438
20	0.6649	0.5144	0.338	0.5372	1.112

Table 12. Malmquist Productivity Index (MPI) comparing 2005 with 2006

DMU	$D^{2005}(x_l^{2005}, y_l^{2005})$	$D^{2006}(x_l^{2006}, y_l^{2006})$	$D^{2005}(x_l^{2006}, y_l^{2006})$	$D^{2006}(x_l^{2005}, y_l^{2005})$	MPI (2005-2006)
1	0.2389	1.0000	0.1670	0.5620	3.7530
2	0.2685	1.0000	0.2040	0.7990	3.8170
3	1.0000	1.0000	0.7540	0.7390	0.9900
4	0.1289	0.4917	0.1180	0.2650	2.9270
5	0.6289	0.5402	0.5900	0.2850	0.6440
6	0.7500	0.7816	0.6080	0.4220	0.8500
7	0.1387	0.9271	0.1630	0.5000	4.5310
8	0.1344	1.0000	0.1680	0.7910	5.9250
9	0.5154	0.6596	0.6300	0.5270	1.0350
10	1.0000	0.545	1.2380	0.2940	0.3600
11	0.3877	1.0000	0.4850	0.5400	1.6940
12	0.0910	0.2582	0.0910	0.1390	2.0790
13	1.0000	0.5973	1.0070	0.4180	0.4980
14	1.0000	0.0967	1.2360	0.0770	0.0780
15	0.2014	0.8497	0.1550	0.4290	3.4210
16	0.4825	0.3404	0.6040	0.1840	0.4630
17	1.0000	0.4792	1.2510	0.3220	0.3510
18	1.0000	0.2368	0.8280	0.1780	0.2260
19	0.0563	0.3193	0.0640	0.1720	3.9170
20	0.5156	0.6598	0.6332	0.5272	1.0354

Table 13. The results of our new approach

DMUs	*Y** in Model (3.2)	Output (O_1)	Maximum revenue Model (3.2)	Current revenue Eq. (3.1)	PL- RE Eq. (3.3)
1	7000000	3297100	15600000000	3356400000	0.4
8	7067354	4341330	4609596000	4609596000	0.541
11	1100000	11000000	15600000000	15600000000	1
13	7196540	7196540	8753772000	8753772000	1
15	700327277	5621431	6332146500	6332146500	0.75

Table 14. The results of the classical approach

DMUs	*Y**in Model (2.2)	Output (O_1)	Maximum revenue Model (2.2)	Current revenue (WY)	RE Eq. (2.3)
1	3700169	3297100	3700169133	3297100000	0.891
8	4816983	4341330	4816983946	4341330000	0.901
11	1100000	11000000	11000000000	11000000000	1
13	8307347	7196540	8307347694	7196540000	0.866
15	5702484	5621431	5702484640	5621431000	0.985

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