A New Nonparametric Composite Exponentially Weighted Moving Average Sign Control Chart

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Abstract

This study proposed a dual EWMA statistics based nonparametric sign control chart to monitor the process deviation from the targeted value. The performance of the proposed chart is compared with the existing nonparametric version of the EWMA sign and CUSUM mean control charts by using out-of-control average run length for various shifts in the process. The simulation study showed the superiority of the suggested chart over the existing counterparts. A real dataset is also considered for the application of the proposed chart.

Keywords: Average Run Length; Binomial Distribution; EWMA Statistic; Nonparametric Control Charts; Statistical Process Control

1. Introduction

Statistical Process Control (SPC) plays an essential role in monitoring and identifying the changes in the process by using well-established statistical tools. These changes are measured in terms of variations which can be classified into common and assignable causes of variations. The common causes of variations being an inherent part of the process leading it to the state of the statistically in-control [1]. The SPC has gained considerable progress after the development of the Shewhart control chart [2]. Subsequently, the Cumulative Sum (CUSUM) control chart [3] and the Exponentially Weighted Moving Average (EWMA) control chart [4] have made a perceptible improvement. The Shewhart control chart is useful in detecting larger shifts in the process as it is based on current sample information only whereas the CUSUM and EWMA control charts utilize both the current and previous observations and therefore are more sensitive to identify small to moderate shifts.

In order to combine the advantages of the Shewhart control charts in detecting larger shifts and the CUSUM control charts for detecting smaller shifts, combined Shewhart-CUSUM chart was developed by adding Shewhart limits in the CUSUM chart [5]. A similar scheme was proposed by Lucas and Saccucci [6] by combining the Shewhart scheme with the EWMA control chart. Moreover, Abbas et al. [7] proposed a mixed EWMA-CUSUM control chart to enhance the sensitivity of the conventional memory control charts for smaller shifts. In the same context, Haq [8, 9] developed a Hybrid Exponentially Weighted Moving Average (HEWMA) control chart by using dual EWMA charting statistics. This methodology of combining charts allows us to deal with a range of shift values effectively. To be more precise, these charts detect shifts more quickly than the classical CUSUM, EWMA, and traditional Shewhart control charts.

In the aforementioned control charts, the critical quality characteristic of the process is assumed to be normally distributed. However, in many practical situations, the distribution of the underlying process is either unknown or non-normal. This as a result seriously affects the efficiency of the typical control charts [10]. One of the possible alternatives in this scenario is the use of the distribution-free or nonparametric control charts which are generally robust, flexible, and more effective. According to Chakraborti and Graham [11], the in-control (IC) average run-length of all continuous distributions remains the same in distribution-free or nonparametric control charts.

In the context of nonparametric control charts, a new idea was proposed by Yang and Cheng [12] by introducing the process proportion and suggested a nonparametric CUSUM mean chart. Subsequently, Yang et al. [13] designed the nonparametric EWMA sign control chart and showed that their suggested chart is capable of detecting small shifts in a process target when the underlying distribution is either non-normal or unknown. The adaptive Phase-II nonparametric EWMA control chart was proposed by Liu et al. [14] to detect a range of shifts in the location parameter. Lu [15] developed a nonparametric extension of the EWMA control chart called Generally Weighted Moving Average (GWMA) sign control chart to enhance the detection ability for small process shifts. Abbasi et al. [16] investigated the performance of the nonparametric EWMA control chart by using asymptotic, time-varying, and fast initial response based control limits. A mixed version of the EWMA-CUSUM sign chart was proposed by Abbasi et al. [17] using arcsine transformation and showed that the performance of their chart is better than other counterparts in terms of shift detection. The synthetic version of the EWMA sign control chart was proposed by Haq [18] by integrating the existing EWMA sign control chart with the conforming run length. Recently, Raza et al. [19] proposed the nonparametric homogeneously weighted moving average control charts based on sign and signed-rank statistics. For more details on nonparametric control charts, we refer the interested readers to, for example, Altukife [20, 21], Amin et al. [22], Aslam et al. [23], Bakir [24-26], Chowdhury et al. [27], Graham et al. [28-31], Koutras and Triantafyllou [32], Mahmood et al. [33], Raza et al. [34] and the references cited therein.

In this article, we propose a nonparametric Composite Exponentially Weighted Moving Average (CEWMA) control chart by combining two EWMA statistics that enhance the sensitivity of the chart. Note that Haq [8, 9] designed a HEWMA control chart by using two EWMA statistics under the assumption that the underlying distribution of the quality characteristic is normal. However, this chart is not effective in case of violation of normality assumption or when the distribution is unknown which happens to be the most practical situation. Furthermore, the variance expression derived by Haq [8, 9] becomes undefined when $\lambda_1 = \lambda_2$. The nonparametric version of a double EWMA control chart was proposed by Riaz and Abbasi [35] using arcsine transformation and assumed the same value of smoothing constant λ for both EWMA statistics. This makes the use of their chart limited for practitioners if they want to use different values for smoothing parameters. On the contrary, our proposed chart is simple as it considers a sign test without any transformation to define a composite EWMA charting statistic. Furthermore, the CEWMA chart provides more flexibility as compared to the one proposed by Riaz and Abbasi [35] by allowing practitioners to consider all possible combinations of smoothing parameters λ_1 and λ_2. Besides, we have evaluated the performance of the proposed control chart under various distributions which had not been considered by Riaz and Abbasi [35].

The rest of this paper is organized as follows: The design structure of the proposed CEWMA sign control chart is given in Section 2. Performance evaluations of the proposed charts are given in Section 3. Section 4 reports comparisons with the existing control charts by using out-of-control average run length for various shifts and simulated as well as real-life data examples. Finally, some conclusions are drawn in Section 5.

2. The Proposed Nonparametric CEWMA sign Chart

Suppose that in a production process, a certain quality characteristic Y has a median η as a target value. Moreover, assume that $Z = Y - \eta$ denotes the process deviation from the target value with probability $p = P(Z > 0)$. The proposed chart is based on the sign test in which we test the hypothesis that the probability of a plus sign of Z equals to the probability of a minus sign, which is equivalent to test the hypothesis that the process has a specific target value, say η. Therefore, the process is said to be IC if $p = p_0 = 0.5$, otherwise, the process is deemed to be out-of-control (OOC), i.e., $p = p_1 \neq p_0$. Let $\{Y_{it}, 1 \leq i \leq n\}$ be independently and identically distributed sample taken at the time t from Y to investigate the variation from the process target value η. Then define

$$Z_{it} = Y_{it} - \eta \quad and \quad I_{it} = \begin{cases} 1, & if\ Z_{it} > 0 \\ 0, & otherwise \end{cases} \qquad for\ i = 1, 2, \ldots \text{ and } t = 1, 2, \ldots$$

Let S_t be the number of observations for which Z_{it}, $i = 1, 2, \ldots, n$ is greater than zero, then the statistic $S_t = \sum_{i=1}^{n} I_{it}$ follows the binomial distribution with parameters $(n, 0.50)$ for an IC process.

Based on this assumption, we propose CEWMA sign chart by combining the two EWMA statistics as follows:

$$HE_{S_t} = \lambda_1 E_{S_t} + (1-\lambda_1) HE_{S_{t-1}} \quad 0 < \lambda_1 \le 1 \tag{1}$$

$$E_{S_t} = \lambda_2 S_t + (1-\lambda_2) E_{S_{t-1}} \quad 0 < \lambda_2 \le 1 \tag{2}$$

Here S_t is the t^{th} successively recorded number of $Z_t(>0)$. Note that the mean value of S_t is taken as the initial value of the EWMA statistics, i.e., $HE_{S_0} = E_{S_0} = \mu = np_0 = n/2$ and HE_{S_t} is the monitoring statistic of the proposed CEWMA control chart. By expanding Equations (1) and (2) we obtain

$$HE_{S_t} = (1-\lambda_1)^t HE_{S_0} + \lambda_1 \sum_{i=0}^{t-1} (1-\lambda_1)^i E_{S_{t-i}} \tag{3}$$

$$E_{S_t} = (1-\lambda_2)^t E_{S_0} + \lambda_2 \sum_{j=0}^{t-1} (1-\lambda_2)^j S_{t-j}, \tag{4}$$

and then by substituting Equation (4) into Equation (3), we get

$$HE_{S_t} = (1-\lambda_1)^t \mu + \lambda_1 \sum_{i=0}^{t-1} (1-\lambda_1)^i (1-\lambda_2)^{t-i} \mu + \lambda_1 \lambda_2 \sum_{i=0}^{t-1} (1-\lambda_1)^i \sum_{j=0}^{t-i-1} (1-\lambda_2)^j S_{t-i-j} \tag{5}$$

$$= (1-\lambda_1)^t (np_0) + \lambda_1 \sum_{i=0}^{t-1} (1-\lambda_1)^i (1-\lambda_2)^{t-i} (np_0) + \lambda_1 \lambda_2 \sum_{i=1}^{t} \left(\left(1-\lambda_2\right)^{t-i} \sum_{j=0}^{t-i} \left(\frac{1-\lambda_1}{1-\lambda_2} \right)^j \right) S_i \tag{6}$$

After some algebraic computation of Equation (6), we can obtain the mean of HE_{S_t} as:

$$E\left(HE_{S_t}\right) = np_0 \tag{7}$$

Now by definition, the variance of HE_{S_t} can be obtained as:

$$V\left(HE_{S_t}\right) = (\lambda_1 \lambda_2)^2 \sum_{i=1}^{t} (1-\lambda_1)^{2(t-i)} \left(\sum_{j=0}^{t-i} \left(\frac{1-\lambda_1}{1-\lambda_2} \right)^j \right)^2 V(S_i),$$

where $V(S_i) = np_0(1-p_0)$. After some algebraic computation, we obtain the variance of HE_{S_t} as:

$$V\left(HE_{S_t}\right) = \left(\frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \right)^2 \left[\sum_{i=1}^{2} \frac{\left(1-\lambda_i\right)^2 \left(1-(1-\lambda_i)^{2t}\right)}{1-(1-\lambda_i)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^t (1-\lambda_2)^t\}}{1-(1-\lambda_1)(1-\lambda_2)} \right] \left\{ np_0(1-p_0) \right\}. \tag{8}$$

For $\lambda_1 = \lambda_2 = \lambda$, Equation (5) reduces to

$$HE_{S_t} = (1+\lambda t)(1-\lambda)^t (np_0) + \lambda^2 \sum_{i=1}^{t} (t-i+1)(1-\lambda)^{t-i} S_i \tag{9}$$

By definition of variance, we obtain

$$V(HE_{S_t}) = \lambda^4 \sum_{i=1}^{t} (t-i+1)^2 (1-\lambda)^{2(t-i)} V(S_i)$$

which yields

$$V(HE_{S_t}) = \lambda^4 \sum_{i=1}^{t} i^2 (1-\lambda)^{2(i-1)} \{np_0(1-p_0)\}$$

and after some algebraic work, we finally obtain the following expression for the variance of Equation (9):

$$V(HE_{S_t}) = \lambda^4 \frac{\{1+(1-\lambda)^2\} - (1-\lambda)^{2t}[(1+t)^2 - (-1+2t+2t^2)(1-\lambda)^2 + t^2(1-\lambda)^4]}{\{1-(1-\lambda)^2\}^3} \{np_0(1-p_0)\} \tag{10}$$

Now, the center line and control limits of the CEWMA sign chart are computed as follows:

$$UCL = np_0 + k\sqrt{V(HE_{S_t})} \tag{11}$$

$$CL = np_0 \tag{12}$$

$$LCL = np_0 - k\sqrt{V(HE_{S_t})} \tag{13}$$

where k denotes the distance of the control limits from the center line in multiples of the standard deviation of the CEWMA statistic (HE_{S_t}). The three parameters λ_1, λ_2, and k are chosen in such a way that a specific IC average run length *(ARL)* is achieved. Note that if any $HE_{S_t} \geq UCL$ or $HE_{S_t} \leq LCL$, the process is considered to be OOC. Furthermore, if we select $\lambda_1 = 1$ or $\lambda_2 = 1$, the CEWMA sign control chart reduces to the EWMA sign chart proposed by Yang et al. [13].

3. Performance Evaluation

The average run length *(ARL)* is commonly used to assess the performance of a control chart. Recall that the *ARL* is the average number of points that are plotted on a control chart before an OOC signal is shown. In this study, the Monte Carlo simulation is used to calculate the coefficient values k for various combinations of λ_1 and λ_2 after fixing ARL_0=370 and $n=10$. To get the average results, we use 100,000 repetitions for each Monte Carlo simulation. Since the distribution of S_t in CEWMA sign statistic follows a binomial distribution, the exact ARL_0=370 is not always attained. We, therefore, find the k values such that the ARL_0 is approximately equal to 370 (i.e., $ARL_0 \approx 370$). Some choices of design parameters of the CEWMA sign chart under a nominal ARL_0 value of 370 are given in Table 1.

[Insert Table 1 here]

The control limits of the proposed chart depend on $n, k, \lambda_1,$ and λ_2. It can be observed from Table 1 that the values of k are symmetrical with respect to λ_1 and λ_2. There is an increasing trend in k by increasing either λ_1, λ_2 or both, except for larger values of λ_1 and λ_2. The same strategy can be applied to find the values of k for various n. In addition to the ARL_0, the OOC run length *(ARL_1)* is also calculated which is expected to be adequately small (for a fixed ARL_0) to detect the process shift quickly. Considering the IC process proportion $p_0 = 0.50$ and $\lambda_1 = \lambda_2 = \lambda = 0.05,$ the ARL_1 values of the CEWMA sign chart are computed in Table 2 by using different values of n and the expected OOC process proportion (p_1). Besides, the performance of the CEWMA sign chat is also investigated for $\lambda_1 \neq \lambda_2$. In this scenario, the *ARL* values for smoothing constants $(\lambda_1 = 0.05, \lambda_2 = 0.10)$ and $(\lambda_1 = 0.05, \lambda_2 = 0.20)$ under various n and p_1 are presented in Tables 3 and 4, respectively. When $p = 0.50$, the attained ARL_0 values are approximately equal to the desired value of 370. From Tables 2-4, it can be noticed that ARL_1 values are inversely related to $|p_1 - 0.50|$ and n. It can also be observed that k changes marginally as the sample size n increases. Moreover, for smaller shifts, the ARL_1 values increase marginally with the increase in one of the smoothing constants λ_1 and λ_2.

[Insert Tables 2-4 here]

In the following, we further investigate the performance of the proposed CEWMA control chart under the assumption that the critical quality characteristic of a process follows some specific distribution. The performance of the proposed CEWMA control chart under various symmetrical and asymmetrical distributions is assessed by using different run-length characteristics such as *ARL*, median run length *(MDRL)*, and standard deviation run length *(SDRL)*. To be more precise, by setting ARL_0=370, the run length distribution for $\lambda_1 = \lambda_2 = 0.05$ and $n = 10$ are computed under the standard normal distribution, $N(0,1)$; the Student's t distribution with 4 and 8 degrees of freedom; the heavy-tailed symmetric distributions which include the Laplace (or double exponential) distribution $Laplace\left(0,1/\sqrt{2}\right)$, the Logistic distribution $LG\left(0,\sqrt{3}/\pi\right)$, the Weibull distribution $\text{Weibull}\,(2,1)$, the Gamma distribution $\text{Gamma}\,(4,1)$, and the contaminated normal (CN) distribution. Recall that the CN distribution is obtained by combining two normal distributions $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$ using the relation $(1-\varphi)N(0,\sigma_1^2)+\varphi N(0,\sigma_2^2)$, where φ denotes the proportion of contamination. This distribution is generally used to evaluate the outlier resistance power of the control charts. In our study, we set $\sigma_1^2 = 4\sigma_2^2$ and the proportion of contamination $\varphi = 0.10$. To determine the OOC run length characteristics, the shift in the process mean is introduced in terms of process standard deviation (σ), i.e., $\mu_1 = \mu_0 + \delta\sigma$ where δ determines the magnitude of the shift. The results are reported in Table 5. It can be noticed from Table 5 that the ARL_0 values remain the same under all continuous symmetrical and asymmetrical distributions which validate the theory of nonparametric control charts. Furthermore, the ARL_1 values decline rapidly with the increase in the size of the shift for all the distributions considered in this study. However, the rate of decline in ARL_1, $MDRL_1$, and $SDRL_1$ values varies with the distribution.

[Insert Table 5 here]

4. Performance Comparison

In this section, we compare the performance of the proposed chart with the existing nonparametric EWMA sign and CUSUM mean control chart by using ARL_1 values for various shifts. Besides, two examples are presented in this section. Considering $n = 10, 15, 20$, $p_0 = 0.50$, and $ARL_0=370$, the ARL_1 curves of the nonparametric EWMA sign, CUSUM mean, and CEWMA sign control charts are shown in Figures 1-3 for various shifts in the process proportion. It can be observed that the nonparametric EWMA sign and CUSUM mean control charts produce almost similar results and the proposed CEWMA sign control chart remains on the lower side for all shifts. This shows that the proposed chart outperforms for all shifts of p_1 and for various sample sizes.

[Insert Figures 1-3 here]

4.1 Example 1: To illustrate the shift detection ability of the CEWMA sign and EWMA sign control charts, the following example with description taken from Montgomery [36] is considered.

"The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height".

In this example, we have sample size =10, total subgroups=15, and the target value=0. By choosing $\lambda = 0.05$ and $k = 2.49$ at $ARL_0 \approx 370$, we obtain 4.36 and 5.63 as the lower and upper control limits for the EWMA sign chart, respectively. The monitoring statistic E_{S_t} of the EWMA sign chart is computed and plotted against their respective control limits in Figure 4. We see that the nonparametric EWMA sign chart detects an OOC signal at sample 13. For rational evaluation of the EWMA sign scheme, the proposed nonparametric CEWMA sign chart is constructed by considering $\lambda_1 = \lambda_2 = \lambda = 0.05$ and $k = 1.954$ as shown in Figure 5. The last two columns of Table 6 show time-varying lower and upper control limits of the CEWMA sign chart. Here, we notice that the proposed chart has identified the OOC signal at sample 12. This shows that the nonparametric CEWMA sign chart performs better than the nonparametric EWMA sign chart regarding the quick detection of the process shift.

[Insert Table 6 here]

[Insert Figures 4-5 here]

4.2 Example 2: To demonstrate the detection ability of the proposed CEWMA sign chart for the dataset from a skewed distribution, 40 samples of size 15 each are generated from the gamma distribution. The first 30 samples are generated from a gamma distribution with parameters shape=4 and rate=1 (IC process). Assuming that the parameters of the gamma distribution shifted to shape=4.25 and rate=1 (OOC process), the next 10 samples are then generated from this shifted gamma distribution. The average values of the two different gamma-distributed samples are 3.91 and 4.3, respectively. By using the aforementioned information, parametric and nonparametric control charts are constructed to assess the performance of the suggested chart.

[Insert Table 7 here]

Choosing ARL_0=370, we construct the Shewhart $\bar{X}$ control chart as shown in Figure 6 and the time-varying EWMA control chart with $\lambda = 0.05$ and $L = 2.492$ as shown in Figure 7 with the assumption that the quality characteristic is normally distributed. Furthermore, we construct three nonparametric control charts, i.e., the EWMA sign chart with parameters $\lambda = 0.05$ and $k = 2.49,$ the CUSUM mean control chart with parameters $K_1 = 0.75$ and $H_1 = \pm 11.30,$ and the proposed CEWMA sign chart with parameters $\lambda_1 = \lambda_2 = \lambda = 0.05$ and $k = 1.958,$ to guarantee that $ARL_0 \approx 370$. The results of the nonparametric control charts are provided in Table 7 and displayed graphically in Figures 8-10. In these schemes, the statistic E_{S_t} is plotted against the control limits LCL=6.7279 and UCL=8.2721 for the nonparametric EWMA sign chart, $C_t^+ 's$ and $C_t^- 's$ are plotted against the decision interval H_1 for the nonparametric CUSUM mean chart, and HE_{S_t} is plotted against time-varying control limits for the proposed CEWMA sign chart.

It is apparent from Figure 10 that the suggested control chart triggers OOC signals at samples 35-40 whereas Figures 6-9 show that the Shewhart $\bar{X}$, classical EWMA, nonparametric EWMA sign, and nonparametric CUSUM mean control charts fail to identify this shift in the given dataset. This example points out the superiority of the proposed chart in identifying small shifts in the process over the four other competing parametric and nonparametric control charts.

[Insert Figures 6-10 here]

5. Conclusion

In this study, a new nonparametric CEWMA sign chart has been proposed which is the combination of two EWMA statistics. The proposed chart can be used to detect process shifts quickly, particularly, smaller shifts by using different combinations of the sensitivity parameters λ_1 and λ_2. The suggested CEWMA sign chart can attract wider applicability as the performance of the CEWMA sign chart is superior to the existing nonparametric EWMA Sign Chart and the nonparametric CUSUM mean chart. Furthermore, the nonparametric EWMA Sign Chart is a special case of the proposed CEWMA chart when either λ_1 or λ_2 equals to one. The practical implementation of our proposed chart is illustrated using a real-life example. Moreover, results based on the simulation study also showed that the suggested chart completely dominates the other competing parametric and nonparametric control charts in detecting smaller shifts. Hence, the use of CEWMA sign control chart is recommended for the practitioners, if the complete knowledge about underlying process distribution is not available. One can further extend this scheme by developing the CEWMA sign chart using other nonparametric tests such as the Mann-Whitney test.

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Tables Captions

Table 1. The *k* values with different combinations of (λ_1, λ_2) for $ARL_0 \approx 370$ and $n = 10\cdot$

Table 2. The *ARL* values of the CEWMA sign control chart for $\lambda_1 = \lambda_2 = \lambda = 0.05.$

Table 3. The *ARL* values of the CEWMA sign control chart for $\lambda_1 = 0.05$ and $\lambda_2 = 0.10.$

Table 4. The *ARL* values of the CEWMA sign control chart for $\lambda_1 = 0.05$ and $\lambda_2 = 0.20.$

Table 5. The run-length characteristic of the proposed CEWMA chart under various distributions for $\lambda_1 = \lambda_2 = 0.05, n = 10,$ and $k = 1.954$ at $ARL_0 \approx 370\cdot$

Table 6. Existing nonparametric EWMA sign and the proposed nonparametric CEWMA sign control charts for the dataset from Montgomery [36], the fill volume of soft-drink beverage bottle.

Table 7. The Shewhart $\overline{X}$, classical EWMA, nonparametric EWMA sign, nonparametric CUSUM mean, and nonparametric CEWMA sign control charts for the simulated dataset.

Figures Captions

Figure 1: *ARL* curves of the nonparametric CEWMA sign, EWMA sign, and CUSUM mean control charts at $n = 10$ and $ARL_0 \approx 370$.

Figure 2: *ARL* curves of the nonparametric CEWMA sign, EWMA sign, and CUSUM mean control charts at $n = 15$ and $ARL_0 \approx 370$.

Figure 3: *ARL* curves of the nonparametric CEWMA sign, EWMA sign, and CUSUM mean control charts at $n = 20$ and $ARL_0 \approx 370$.

Figure 4: Nonparametric EWMA sign control chart for the fill height data using $\lambda = 0.05$ and $k = 2.49$ at $ARL_0 \approx 370$.

Figure 5: Nonparametric CEWMA sign chart for the fill height data using $\lambda_1 = \lambda_2 = 0.05$ and $k = 1.954$ at $ARL_0 \approx 370$.

Figure 6: Shewhart $\overline{X}$ chart for the simulated data set using $A_2 = 0.223$ at $ARL_0 \approx 370$.

Figure 7: Classical EWMA chart for the simulated data set using $\lambda = 0.05$ and $L = 2.492$ at $ARL_0 \approx 370.$

Figure 8: Nonparametric EWMA sign chart for the simulated data using $\lambda = 0.05$ and $k = 2.49$ at $ARL_0 \approx 370.$

Figure 9: Nonparametric CUSUM mean control chart for the simulated data using $K_1 = 0.75$ and $H_1 = 11.30$ at $ARL_0 \approx 370.$

Figure 10: Nonparametric CEWMA sign control chart for the simulated data using $\lambda_1 = \lambda_2 = 0.05$ and $k = 1.958$ at $ARL_0 \approx 370.$

Table 1.

λ_1	λ_2										
	0.05	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.05	**1.954**	2.092	2.227	2.271	2.304	2.353	2.392	2.428	2.455	2.481	2.505
0.1	2.092	**2.245**	2.387	2.429	2.468	2.526	2.562	2.598	2.629	2.66	2.682
0.2	2.227	2.387	**2.528**	2.571	2.61	2.666	2.711	2.744	2.772	2.795	2.822
0.25	2.271	2.429	2.571	**2.619**	2.655	2.71	2.751	2.784	2.809	2.832	2.852
0.3	2.304	2.468	2.61	2.655	**2.692**	2.743	2.782	2.811	2.837	2.855	2.871
0.4	2.353	2.526	2.666	2.71	2.743	**2.792**	2.825	2.853	2.871	2.878	2.883
0.5	2.392	2.562	2.711	2.751	2.782	2.825	**2.856**	2.875	2.884	2.889	2.892
0.6	2.428	2.598	2.744	2.784	2.811	2.853	2.875	**2.884**	2.89	2.893	2.885
0.7	2.455	2.629	2.772	2.809	2.837	2.871	2.884	2.89	**2.894**	2.882	2.868
0.8	2.481	2.66	2.795	2.832	2.855	2.878	2.889	2.893	2.882	**2.868**	2.855
0.9	2.505	2.682	2.822	2.852	2.871	2.883	2.892	2.885	2.868	2.855	**2.82**

Table 2.

	n	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
	k	**1.956**	**1.954**	**1.962**	**1.955**	**1.953**	**1.961**	**1.958**	**1.966**	**1.956**	**1.955**	**1.963**	**1.957**	**1.964**	**1.957**	**1.955**	**1.964**	**1.957**
	0.05	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.10	1.3	1.3	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.15	1.6	1.6	1.3	1.3	1.4	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0
	0.20	2.0	2.0	1.6	1.6	1.7	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.3	1.2	1.2	1.1	1.1
	0.25	2.8	2.6	2.3	2.2	2.2	1.9	1.9	1.7	1.7	1.7	1.5	1.5	1.6	1.4	1.4	1.3	1.3
	0.30	4.1	3.8	3.4	3.2	3.2	2.8	2.7	2.4	2.4	2.4	2.1	2.2	2.1	2.0	2.0	1.8	1.8
	0.35	7.0	6.3	5.7	5.4	5.1	4.7	4.4	4.1	4.0	3.9	3.6	3.5	3.5	3.2	3.1	2.9	2.9
	0.40	13.8	12.8	11.5	10.9	10.3	9.5	8.9	8.3	8.0	7.8	7.2	7.0	6.9	6.5	6.3	6.0	5.9
p	**0.45**	42.1	38.6	35.4	33.6	31.6	29.7	28.0	26.2	25.3	24.7	23.0	22.3	21.7	20.5	20.0	18.9	18.6
	0.50	**371.0**	**370.8**	**370.5**	**369.5**	**371.8**	**371.7**	**371.5**	**370.4**	**370.4**	**371.4**	**371.3**	**371.9**	**379.1**	**372.8**	**369.4**	**370.9**	**371.6**
	0.55	42.0	38.9	35.1	33.8	31.6	29.3	28.1	26.1	25.2	24.3	23.1	22.1	21.7	20.6	20.0	18.8	18.7
	0.60	14.0	12.8	11.5	10.8	10.2	9.5	9.0	8.3	7.9	7.8	7.2	7.1	6.8	6.5	6.3	6.0	5.8
	0.65	7.o	6.3	5.7	5.4	5.1	4.7	4.4	4.1	3.9	3.9	3.5	3.5	3.4	3.2	3.1	2.9	2.9
	0.70	4.1	3.8	3.4	3.3	3.2	2.8	2.7	2.4	2.4	2.4	2.1	2.1	2.1	2.0	2.0	1.8	1.8
	0.75	2.8	2.6	2.3	2.2	2.2	1.9	1.9	1.7	1.7	1.7	1.5	1.5	1.6	1.4	1.4	1.3	1.3
	0.80	2.0	2.0	1.6	1.6	1.7	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.3	1.1	1.2	1.1	1.1
	0.85	1.6	1.6	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0
	0.90	1.3	1.3	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.95	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 3.

	n	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	k	**2.097**	**2.092**	**2.105**	**2.098**	**2.10**	**2.101**	**2.098**	**2.096**	**2.101**	**2.098**	**2.096**	**2.098**	**2.095**	**2.101**	**2.097**	**2.095**	**2.096**
	0.05	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.10	1.3	1.3	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.15	1.6	1.6	1.3	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0
	0.20	2.1	2.1	1.7	1.7	1.7	1.4	1.4	1.5	1.3	1.3	1.4	1.2	1.2	1.1	1.2	1.2	1.1
	0.25	3.0	2.8	2.4	2.3	2.2	2.0	2.0	2.0	1.7	1.7	1.7	1.6	1.6	1.4	1.4	1.5	1.4
	0.30	4.4	4.1	3.6	3.4	3.3	2.9	2.9	2.8	2.5	2.5	2.4	2.3	2.2	2.0	2.0	2.0	1.9
	0.35	7.4	6.9	6.1	5.9	5.4	4.9	4.7	4.6	4.2	4.1	4.0	3.7	3.6	3.3	3.3	3.3	3.1
	0.40	14.7	13.4	12.3	11.6	10.8	10.0	9.6	9.2	8.6	8.2	7.9	7.5	7.2	6.8	6.6	6.5	6.2
p	**0.45**	44.9	41.9	37.6	35.7	33.9	31.0	29.7	28.8	26.8	25.9	24.7	23.4	22.8	21.5	21.1	20.7	19.7
	0.50	**372.5**	**370.9**	**371.0**	**370.9**	**370.9**	**370.1**	**371.5**	**369.7**	**370.7**	**371.2**	**371.3**	**370.3**	**369.9**	**371.5**	**371.9**	**370.7**	**370.4**
	0.55	44.9	41.5	37.9	35.6	33.6	31.1	29.8	28.5	26.6	25.8	24.9	23.8	22.7	21.8	21.1	20.7	19.6
	0.60	14.6	13.5	12.2	11.6	10.9	10.0	9.6	9.2	8.6	8.2	7.9	7.5	7.3	6.8	6.6	6.5	6.2
	0.65	7.3	6.8	6.1	5.7	5.4	4.9	4.8	4.6	4.2	4.1	4.0	3.7	3.6	3.3	3.3	3.3	3.1
	0.70	4.5	4.2	3.6	3.5	3.3	2.9	2.9	2.8	2.5	2.5	2.4	2.2	2.2	2.0	2.0	2.0	1.9
	0.75	3.0	2.8	2.4	2.3	2.2	2.0	2.0	2.0	1.7	1.7	1.7	1.6	1.6	1.4	1.4	1.5	1.4
	0.80	2.1	2.1	1.7	1.7	1.7	1.4	1.4	1.5	1.3	1.3	1.4	1.2	1.2	1.1	1.2	1.2	1.1
	0.85	1.6	1.6	1.3	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0
	0.90	1.3	1.3	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.95	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 4.

	n	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
	k	**2.235**	**2.227**	**2.228**	**2.230**	**2.229**	**2.227**	**2.231**	**2.230**	**2.229**	**2.232**	**2.228**	**2.230**	**2.230**	**2.230**	**2.232**	**2.228**	**2.2**
	0.05	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.10	1.3	1.3	1.3	1.1	1.1	1.2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.15	1.6	1.6	1.6	1.4	1.4	1.4	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0
	0.20	2.2	2.1	2.0	1.7	1.7	1.7	1.5	1.5	1.5	1.4	1.4	1.2	1.2	1.3	1.2	1.2	1.2
	0.25	3.0	2.9	2.8	2.4	2.4	2.3	2.0	2.0	2.0	1.8	1.8	1.6	1.6	1.6	1.5	1.5	1.5
	0.30	4.6	4.3	4.0	3.6	3.5	3.3	3.0	2.9	2.8	2.6	2.5	2.3	2.3	2.3	2.1	2.1	2.1
	0.35	7.6	7.0	6.6	6.0	5.7	5.4	4.9	4.7	4.6	4.2	4.2	3.8	3.7	3.7	3.4	3.4	3.3
	0.40	15.1	13.9	12.9	11.8	11.2	10.6	9.9	9.4	9.0	8.6	8.2	7.7	7.5	7.3	6.9	6.7	6.6
p	**0.45**	47.1	43.4	40.8	37.3	35.5	33.5	31.2	29.5	28.4	26.7	25.9	24.7	23.7	23.2	22.0	21.4	20.7
	0.50	**371.2**	**369.1**	**370.4**	**371.4**	**371.8**	**370.8**	**369.8**	**370.3**	**370.7**	**370.3**	**371.6**	**369.7**	**370.5**	**369.2**	**370.7**	**371.1**	**370.0**
	0.55	47.8	43.8	40.8	37.4	35.6	33.0	31.3	29.7	28.5	27.0	25.9	24.3	23.9	23.0	22.0	21.2	20.8
	0.60	15.1	13.9	12.9	11.9	11.2	10.7	9.9	9.4	9.0	8.6	8.3	7.8	7.5	7.3	6.9	6.7	6.6
	0.65	7.6	7.0	6.5	6.0	5.7	5.4	5.0	4.7	4.6	4.3	4.2	3.8	3.7	3.7	3.4	3.4	3.3
	0.70	4.6	4.3	4.0	3.6	3.5	3.4	3.0	2.9	2.8	2.6	2.5	2.3	2.3	2.3	2.1	2.1	2.1
	0.75	3.0	2.9	2.8	2.4	2.4	2.3	2.0	2.0	2.0	1.8	1.8	1.6	1.6	1.6	1.5	1.5	1.5
	0.80	2.1	2.1	2.0	1.7	1.7	1.7	1.5	1.5	1.5	1.4	1.4	1.2	1.2	1.3	1.2	1.2	1.2
	0.85	1.6	1.6	1.6	1.4	1.4	1.4	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0
	0.90	1.3	1.3	1.3	1.1	1.1	1.2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.95	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 5.

Distribution	Characteristic	δ										
		0	**0.05**	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**1.50**	**2.00**	**2.5**	**3.0**
$N(0,1)$	***ARL***	370.8	141.9	54.2	12.9	4.1	2.2	1.6	1.1	1.0	1.0	1.0
	SDRL	423.8	150.8	50.6	10.4	3.0	1.3	0.7	0.4	0.1	0.0	0.0
	MDRL	235	97	42	10	3	2	1	1	1	1	1
$t(4)$	***ARL***	371.3	99.3	35.4	8.2	2.8	1.7	1.4	1.1	1.0	1.0	1.0
	SDRL	416.3	100.7	31.4	6.4	1.8	0.8	0.5	0.3	0.2	0.1	0.0
	MDRL	239	71	28	7	2	2	1	1	1	1	1
$t(8)$	***ARL***	371.1	126.4	45.7	10.9	3.5	2.0	1.5	1.1	1.0	1.0	1.0
	SDRL	417.9	129.7	41.9	8.7	2.5	1.1	0.7	0.3	0.2	0.1	0.0
	MDRL	237	88	36	9	3	2	1	1	1	1	1
CN	***ARL***	370.3	128.1	47.2	11.1	3.6	2.1	1.5	1.1	1.0	1.0	1.0
	SDRL	417.3	133.4	42.9	8.9	2.6	1.2	0.7	0.3	0.1	0.0	0.0
	MDRL	234	88	37	9	3	2	1	1	1	1	1
$LG\left(0,\sqrt{3}/\pi\right)$	***ARL***	368.4	123.2	45.0	10.6	3.4	2.0	1.5	1.1	1.0	1.0	1.0
	SDRL	417.8	127.7	40.7	8.5	2.4	1.1	0.7	0.3	0.2	0.1	0.0
	MDRL	235	86	36	9	2	2	1	1	1	1	1
$Laplace\left(0,1/\sqrt{2}\right)$	***ARL***	369.2	69.8	25.1	6.4	2.6	1.7	1.4	1.1	1.0	1.0	1.0
	SDRL	412.8	67.2	21.2	4.9	1.6	0.8	0.6	0.3	0.2	0.1	0.1
	MDRL	237	52	20	5	2	2	1	1	1	1	1
$Gamma(4,1)$	***ARL***	371.6	132.7	50.5	12.4	4.1	2.3	1.7	1.2	1.0	1.0	1.0
	SDRL	415.9	140.3	46.4	10.0	3.0	1.4	0.8	0.4	0.2	0.1	0.0
	MDRL	237	91	39	10	3	2	2	1	1	1	1
$Weibull(2,1)$	***ARL***	369.7	165.1	68.5	17.5	6.5	3.9	2.8	1.9	1.6	1.4	1.3
	SDRL	415.6	174.6	64.9	14.3	5.1	2.8	1.8	1.0	0.7	0.6	0.5
	MDRL	237	113	52	14	5	3	2	2	1	1	1

Table 6.

Sample	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	S_t	E_{S_t}	HE_{S_t}	*LCL*	*UCL*
1	2.5	0.5	2	-1	1	-1	0.5	1.5	0.5	-1.5	7	5.1000	5.0050	4.9923	5.0077
2	0	0	0.5	1	1.5	1	-1	1	1.5	-1	6	5.1450	5.0120	4.9834	5.0166
3	1.5	1	1	-1	0	-1.5	-1	-1	1	-1	4	5.0878	5.0158	4.9733	5.0267
4	0	0.5	-2	0	-1	1.5	-1.5	0	-2	-1.5	2	4.9334	5.0117	4.9624	5.0376
5	0	0	0	-0.5	0.5	1	-0.5	-0.5	0	0	2	4.7867	5.0004	4.9510	5.0490
6	1	-0.5	0	0	0	0.5	-1	1	-2	1	4	4.7474	4.9878	4.9393	5.0607
7	1	-1	-1	-1	0	1.5	0	1	0	0	3	4.6600	4.9714	4.9274	5.0726
8	0	-1.5	-0.5	1.5	0	0	0	-1	0.5	-0.5	2	4.5270	4.9492	4.9156	5.0844
9	-2	-1.5	1.5	1.5	0	0	0.5	1	0	1	5	4.5506	4.9292	4.9038	5.0962
10	-0.5	3.5	0	-1	-1.5	-1.5	-1	-1	1	0.5	3	4.4731	4.9064	4.8922	5.1078
11	0	1.5	0	0	2	-1.5	0.5	-0.5	2	-1	4	4.4495	4.8836	4.8808	5.1192
12	0	-2	-0.5	0	-0.5	2	1.5	0	0.5	-1	3	4.3770	**4.8582**	4.8696	5.1304
13	-1	-0.5	-0.5	-1	0	0.5	0.5	-1.5	-1	-1	2	**4.2581**	**4.8282**	4.8588	5.1412
14	0.5	1	-1	-0.5	-2	-1	-1.5	0	1.5	1.5	4	**4.2452**	**4.7991**	4.8483	5.1517
15	1	0	1.5	1.5	1	-1	0	1	-2	-1.5	5	**4.2830**	**4.7733**	4.8381	5.1619

Table 7.

Sample	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}	$\bar{Y}$	EWMA	S_t	C_t^-	C_t^+	E_{S_t}	HE_{S_t}	*LCL*	*UCL*
1	2.20	4.35	3.02	1.69	5.70	1.88	2.90	4.33	1.39	3.35	4.67	3.11	8.31	5.86	7.77	4.04	4.00	7	0	0	7.48	7.50	7.49	7.51
2	4.33	1.58	7.00	1.75	2.18	3.33	1.58	7.94	4.68	4.22	1.69	4.32	2.54	4.12	2.88	3.61	3.98	7	0	0	7.45	7.50	7.48	7.52
3	1.42	6.39	2.40	3.51	6.65	6.71	2.14	10.49	1.46	7.43	7.47	5.54	6.21	7.60	4.32	5.31	4.05	10	0	2.5	7.58	7.50	7.47	7.53
4	3.49	1.43	2.33	4.23	3.14	8.91	6.76	7.17	3.44	2.45	3.62	6.02	1.84	1.66	1.78	3.88	4.04	5	-1.75	0	7.45	7.50	7.45	7.55
5	6.42	4.53	1.47	2.24	6.30	1.94	2.94	1.73	0.87	9.53	3.11	4.39	3.32	4.17	5.58	3.90	4.03	7	-1.5	0	7.43	7.49	7.44	7.56
6	5.98	3.02	1.26	5.19	6.22	2.27	3.15	3.64	3.66	2.27	1.08	4.21	5.63	2.52	3.45	3.57	4.01	5	-3.25	0	7.31	7.49	7.43	7.57
7	2.07	4.76	4.12	10.49	1.33	1.63	4.15	2.76	4.04	5.13	7.55	3.73	3.52	3.16	2.22	4.04	4.01	7	-3	0	7.29	7.48	7.41	7.59
8	5.32	5.17	3.63	5.36	2.45	5.20	3.96	5.95	1.27	5.89	1.53	1.91	4.67	1.82	4.10	3.88	4.01	9	-0.75	0.75	7.38	7.47	7.40	7.60
9	2.60	2.54	3.61	0.90	2.34	6.25	3.09	3.75	5.34	1.87	2.39	5.92	5.73	5.81	7.51	3.98	4.00	6	-1.5	0	7.31	7.46	7.38	7.62
10	5.50	3.53	1.89	0.75	1.31	6.10	7.40	4.33	3.81	5.77	6.19	2.91	7.27	4.77	0.84	4.16	4.01	8	-0.25	0	7.34	7.46	7.37	7.63
11	4.22	0.69	1.94	4.44	1.47	2.91	3.88	4.81	2.25	2.16	4.54	2.56	4.02	4.47	4.70	3.27	3.97	7	0	0	7.32	7.45	7.35	7.65
12	1.17	5.34	4.83	3.48	2.15	4.73	2.74	6.20	5.07	2.33	5.09	4.24	5.88	3.42	3.38	4.00	3.98	8	0	0	7.36	7.45	7.34	7.66
13	5.59	2.47	5.89	3.79	3.91	4.60	5.89	3.08	1.14	0.65	3.66	6.38	6.19	2.59	5.25	4.07	3.98	8	0	0	7.39	7.44	7.33	7.67
14	3.27	5.70	3.45	3.39	4.79	4.37	6.16	5.54	5.94	1.10	9.19	3.62	6.06	6.45	3.53	4.84	4.02	9	0	0.75	7.47	7.44	7.31	7.69
15	3.36	5.46	3.10	3.07	0.77	5.78	2.34	3.67	1.95	4.46	3.48	9.23	5.49	3.17	2.04	3.83	4.01	5	-1.75	0	7.35	7.44	7.30	7.70
16	5.35	2.08	6.67	5.89	6.35	4.14	4.10	5.64	5.56	2.79	3.12	4.10	3.85	3.70	1.97	4.35	4.03	9	0	0.75	7.43	7.44	7.29	7.71
17	4.40	2.53	6.73	2.24	2.36	2.25	6.28	3.87	4.02	2.83	1.93	5.70	3.07	1.85	4.46	3.64	4.01	6	-0.75	0	7.36	7.43	7.28	7.72
18	4.31	1.31	4.02	3.66	5.67	5.65	5.11	3.42	5.01	3.27	2.46	1.60	4.46	2.62	2.68	3.68	3.99	7	-0.5	0	7.34	7.43	7.27	7.73
19	2.04	3.24	3.86	4.15	3.05	3.96	6.33	3.05	3.12	2.99	2.19	2.36	6.73	5.00	3.37	3.70	3.98	5	-2.25	0	7.22	7.42	7.26	7.74
20	3.97	3.33	3.68	4.10	2.50	2.89	1.91	3.33	2.83	3.84	5.70	3.83	2.58	4.56	3.57	3.51	3.96	4	-5	0	7.06	7.40	7.25	7.75
21	3.94	8.24	3.11	1.97	2.72	3.50	4.56	3.25	4.43	1.64	1.99	4.94	1.89	3.00	3.23	3.49	3.93	5	-6.75	0	6.96	7.38	7.24	7.76
22	3.38	3.98	3.03	3.05	10.18	1.80	5.23	2.89	4.49	4.29	4.26	4.23	2.75	5.44	3.82	4.19	3.95	8	-5.5	0	7.01	7.36	7.23	7.77
23	3.13	3.81	1.70	0.71	3.16	4.37	5.24	6.16	5.20	5.07	3.47	3.95	2.43	4.46	5.05	3.86	3.94	8	-4.25	0	7.06	7.35	7.22	7.78
24	3.90	6.32	3.17	5.34	3.52	4.47	2.88	5.59	3.78	2.88	3.99	1.26	2.36	4.97	2.12	3.77	3.93	6	-5	0	7.01	7.33	7.21	7.79

Sample	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}	$\overline{Y}$	**EWMA**	S_t	C_t^-	C_t^+	E_{S_t}	HE_{S_t}	*LCL*	*UCL*
25	8.20	3.84	4.32	6.15	3.78	3.73	2.50	4.94	2.80	4.23	1.06	4.39	3.44	4.51	6.26	4.28	3.95	8	-3.75	0	7.06	7.32	7.20	7.80
26	2.79	3.10	1.28	10.05	4.47	2.54	8.42	2.44	6.80	3.30	1.95	2.02	2.61	3.20	2.62	3.84	3.94	4	-6.5	0	6.90	7.30	7.19	7.81
27	5.79	1.39	4.91	3.74	4.38	2.37	5.52	6.78	3.01	1.63	4.26	3.75	3.16	5.57	3.57	3.99	3.95	7	-6.25	0	6.91	7.28	7.19	7.81
28	4.12	6.81	4.43	3.70	0.40	5.05	2.94	0.68	1.70	4.33	1.66	1.07	0.88	5.25	4.37	3.16	3.91	7	-6	0	6.91	7.26	7.18	7.82
29	3.85	3.95	2.38	1.54	5.59	1.61	8.64	2.10	4.58	2.51	3.98	7.82	3.01	2.86	2.76	3.81	3.90	6	-6.75	0	6.87	7.24	7.17	7.83
30	3.15	1.23	3.24	4.26	5.63	2.49	2.07	6.78	3.87	3.19	2.83	3.35	3.81	4.23	4.52	3.64	3.89	5	-8.5	0	6.77	7.22	7.17	7.83
31	6.33	5.03	3.97	3.91	5.09	3.77	7.13	4.79	1.44	1.93	1.87	6.99	1.14	2.15	8.90	4.30	3.91	9	-6.25	0.75	6.89	7.20	7.16	7.84
32	8.72	3.45	8.00	1.97	6.80	5.93	1.82	3.60	3.61	2.41	5.64	7.57	3.71	6.67	2.11	4.80	3.95	7	-6	0	6.89	7.18	7.15	7.85
33	5.27	5.26	5.79	4.98	6.93	3.11	5.80	6.06	7.86	2.24	3.63	3.33	3.89	3.08	11.49	5.25	4.02	9	-3.75	0.75	7.00	7.17	7.15	7.85
34	3.02	3.29	2.66	1.92	3.00	3.73	2.67	4.87	2.74	3.14	1.63	6.42	3.54	6.29	1.08	3.33	3.98	3	-7.5	0	6.80	7.16	7.14	7.86
35	2.76	3.43	7.00	3.13	9.55	3.04	4.01	3.52	6.99	1.48	2.81	6.42	2.50	6.90	6.70	4.68	4.02	7	-7.25	0	6.81	7.14	7.14	7.86
36	4.89	2.13	7.52	4.24	2.35	1.06	2.86	1.78	4.11	6.63	5.03	4.94	4.43	1.99	3.47	3.83	4.01	8	-6	0	6.87	7.12	7.14	7.86
37	3.10	3.58	10.00	4.90	3.14	3.93	3.18	3.17	1.72	2.90	4.94	5.47	4.22	4.26	5.59	4.27	4.02	8	-4.75	0	6.92	7.11	7.13	7.87
38	3.04	7.89	3.35	3.14	7.39	5.97	3.69	3.09	2.26	4.90	3.71	2.86	5.67	7.14	4.67	4.58	4.05	7	-4.5	0	6.93	7.10	7.13	7.87
39	4.13	6.32	3.93	1.46	8.67	4.55	2.45	7.41	3.60	5.12	6.09	2.38	3.21	7.25	2.54	4.61	4.08	9	-2.25	0.75	7.03	7.10	7.12	7.88
40	2.21	3.96	1.64	4.97	5.39	2.79	4.07	7.09	2.11	2.30	2.32	2.59	3.09	4.41	1.78	3.38	4.04	6	-3	0	6.98	7.10	7.12	7.88

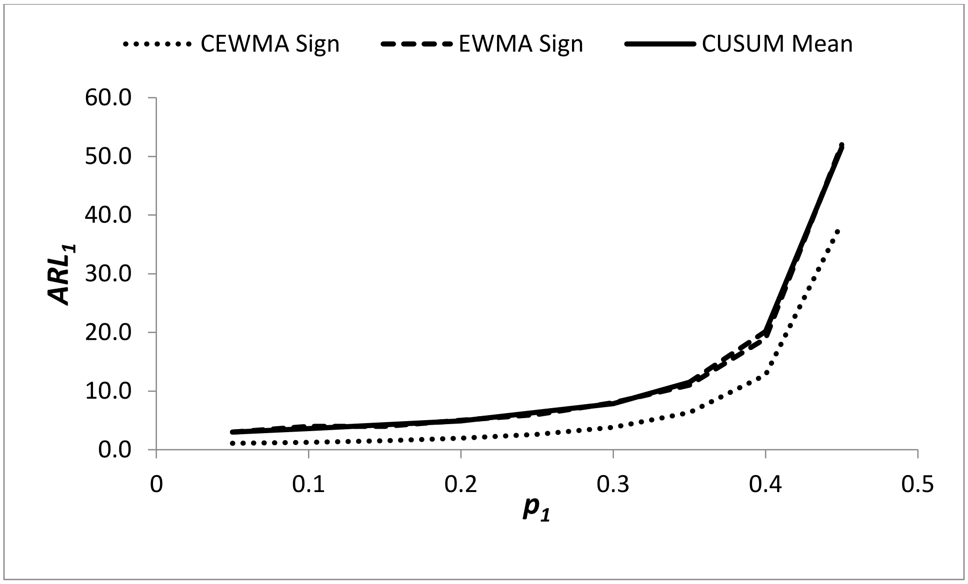
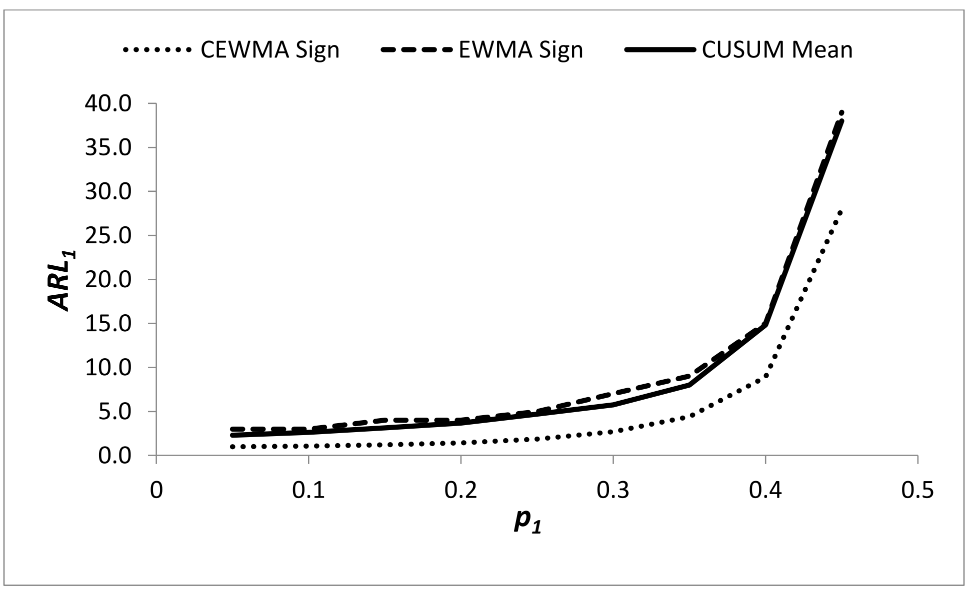


Figure 1.



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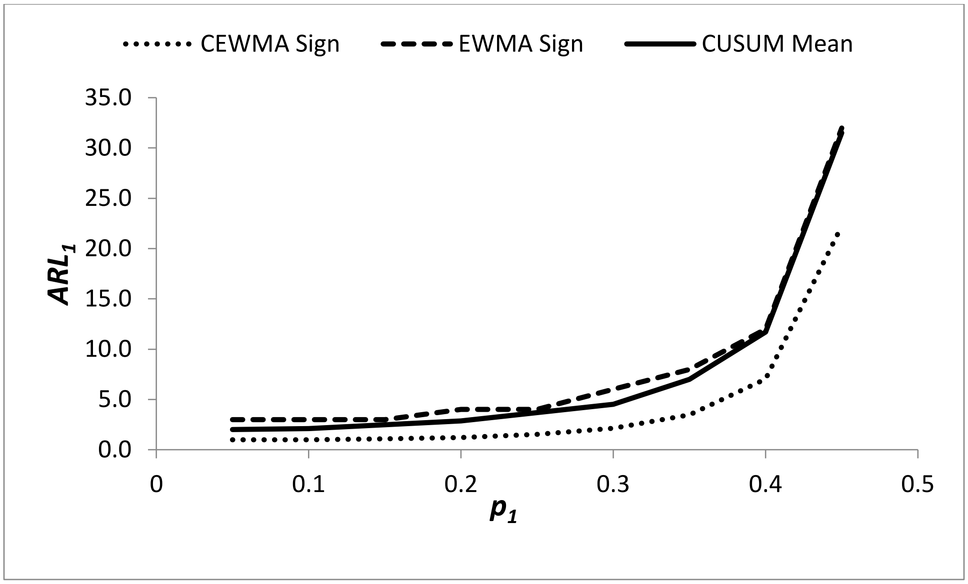

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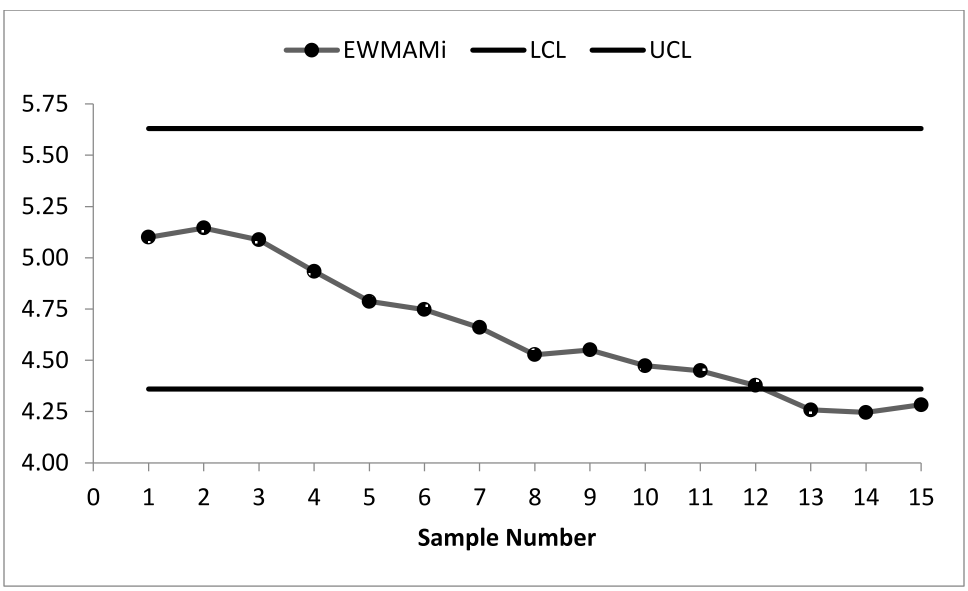

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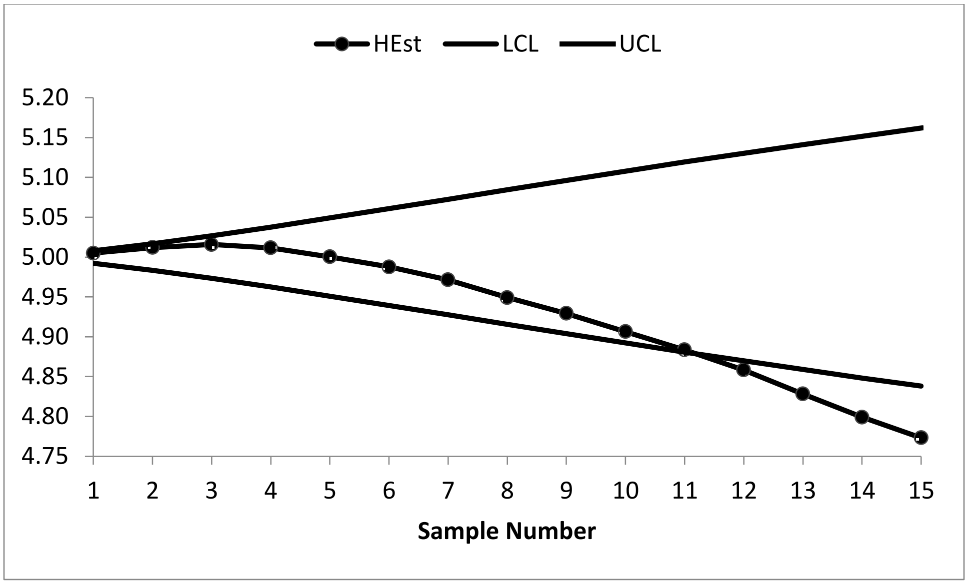

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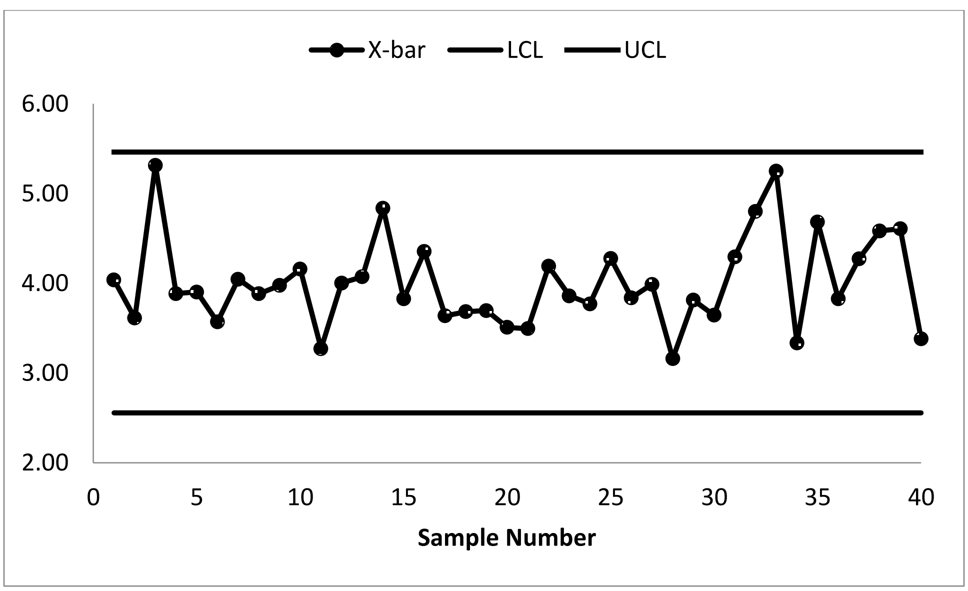

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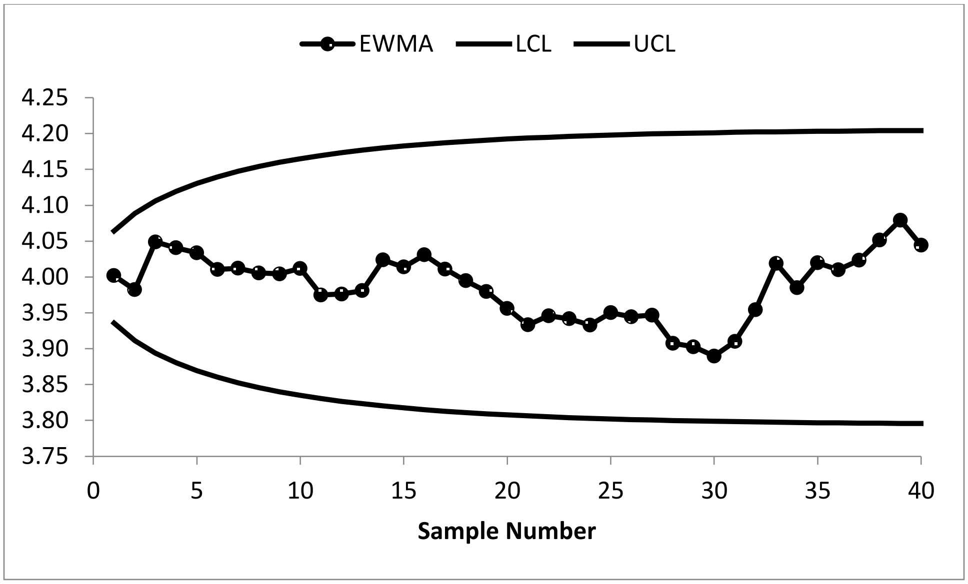

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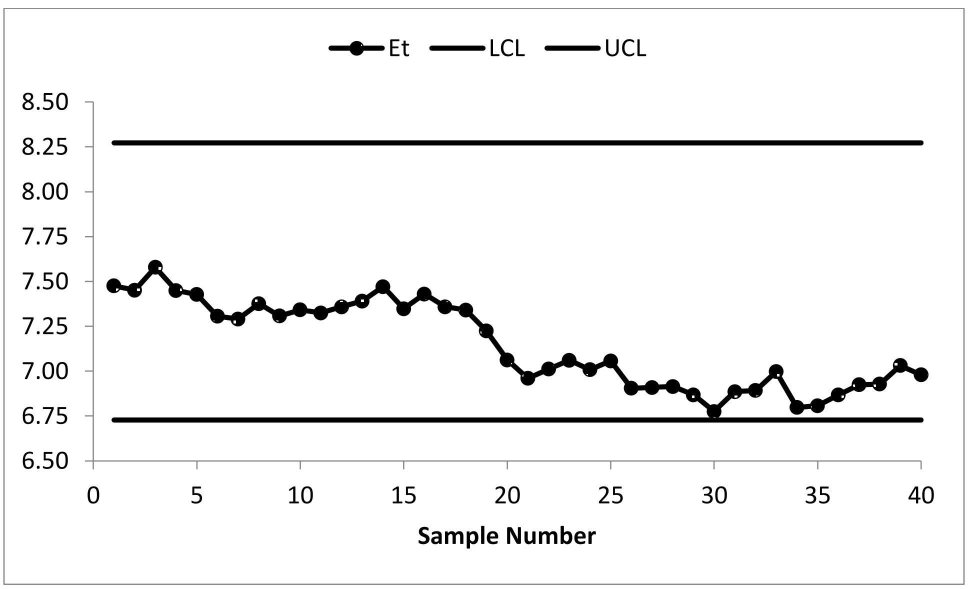

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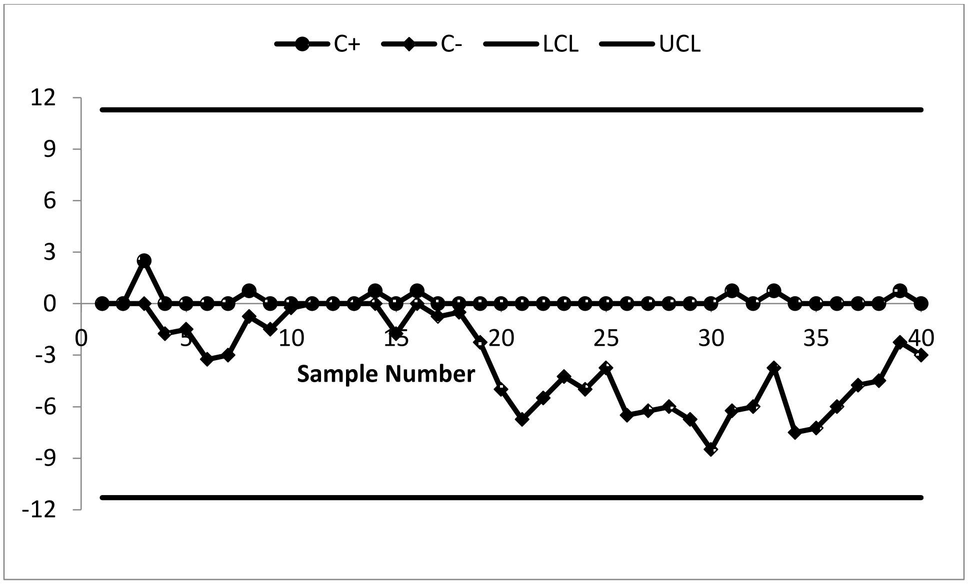

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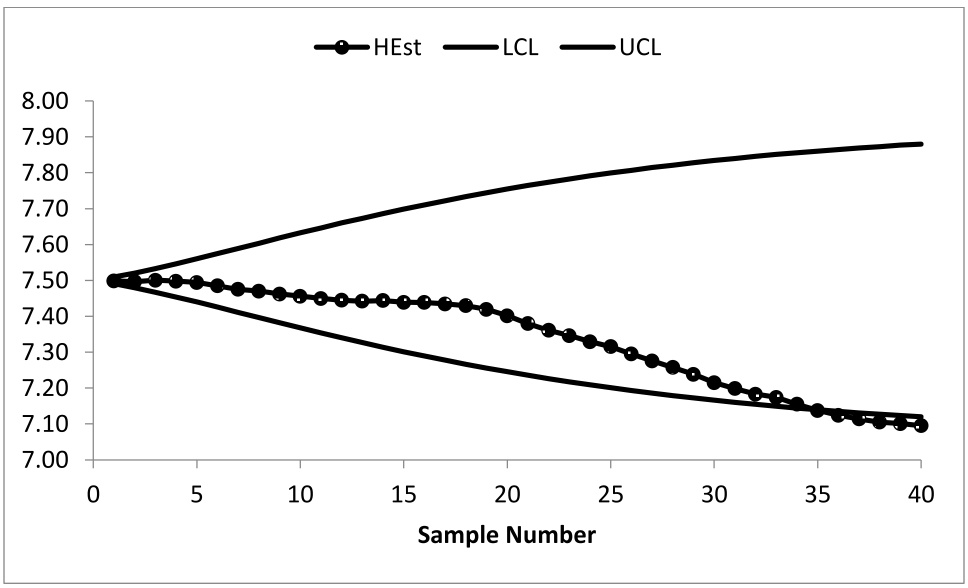

Figure 10.

Author's Biographies

Muhammad Ali Raza obtained his MSc and MPhil in Statistics from the University of the Punjab Lahore, Pakistan. He earned his Ph.D. in Statistics from Shanghai Jiao Tong University, the People's Republic of China in June 2019. He is working as an Assistant Professor in the Department of Statistics, Government College University Faisalabad, Pakistan, since October 2011. He has more than 20 refereed publications in various research journals. His research interests include statistical process control, probability and probability distributions, and applied statistics.

Muhammad Aslam earned his M.Sc. in Statistics (2004) from GC University Lahore with Chief Minister of the Punjab merit scholarship, M. Phil in Statistics (2006) from GC University Lahore with the Governor of the Punjab merit scholarship, and Ph.D. in Statistics (2010) from National College of Business Administration & Economics Lahore under the supervision of Prof. Dr. Munir Ahmad. He worked as a lecturer of Statistics in Edge College System International from 2003-2006. He also worked as a Research Assistant in the Department of Statistics, GC University Lahore from 2006 to 2008. Then he joined the Forman Christian College University as a lecturer in August 2009. He worked as Assistant Professor at the same University from June 2010 to April 2012. He worked in the same department as Associate Professor from June 2012 to October 2014. He worked as Associate Professor of Statistics in the Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia from October 2014 to March 2017. He taught summer courses as Visiting Faculty of Statistics at Beijing Jiao tong University, China in 2016**.** Currently, he is working as a Full Professor of Statistics in the Department of Statistics, King Abdulaziz University Jeddah, and Saudi Arabia. He has published more than **395** research papers in national and international well-reputed journals including, for example, IEEE Access, Journal of Applied Statistics, European Journal of Operation Research, Information Sciences, Journal of Process Control, Journal of the Operational Research Society, Applied Mathematical Modelling, International Journal of Fuzzy Systems, Symmetry, International Journal of Advanced Manufacturer Technology, Communications in Statistics, Journal of Testing and Evaluation, and Pakistan Journal of Statistics, etc. His papers have been cited more than 3700 times with h-index 32 and i-10 index 106 (Google Scholar). His papers have been cited more than 2200 times with h-index 25 (Web of Science Citations). He is the author of two books published by VDM Germany and Springer, respectively. He has published 8 chapters in well-reputed books. He is also HEC approved Ph.D. supervisor since 2011. He supervised 5 Ph.D. theses, more than 25 M.Phil. theses, and 3 M.Sc. theses. He is a reviewer of more than 70 well-reputed international journals. He has reviewed more than 155 research papers for various well-reputed international journals. He received a meritorious services award in research from the National College of Business Administration & Economics Lahore in 2011. He received Research Productivity Award for the year 2012 by Pakistan Council for Science and Technology. His name listed in 2nd position among Statistician in the Directory of Productivity Scientists of Pakistan 2013. His name listed in 1st Position among Statistician in the Directory of Productivity Scientists of Pakistan 2014. He got 371 positions in the list of top 2210 profiles of Scientist of Saudi Institutions 2016. He is selected for the “Innovative Academic Research & Dedicated Faculty Award 2017” by SPE, Malaysia. He Received King Abdulaziz University Excellence Awards in Scientific Research for the paper entitled “**Aslam, M**., Azam, M., Khan, N. and Jun, C.-H. (2015). A New Mixed Control Chart to Monitor the Process, *International Journal of Production Research*, 53 (15), 4684–4693. He Received the King Abdulaziz University citation award for the paper entitled “Azam, M., **Aslam, M**. and Jun, C.-H. (2015). Designing of a hybrid exponentially weighted moving average control chart using repetitive sampling, International *Journal of Advanced Manufacturing Technology*, 77:1927–1933 in 2018.

Prof. Muhammad Aslam pioneer in the area of Neutrosophic Statistical Quality Control (NSQC). He is the founder of Neutrosophic Inferential Statistics (NIS) and NSQC. His contribution is the development of neutrosophic statistics theory for the inspection and process control. He originally developed the theory in these areas under the neutrosophic statistics. He extended the classical statistics theory to neutrosophic statistics originally in 2018.

He is a member of the editorial board of Electronic Journal of Applied Statistical Analysis, Neutrosophic Sets and Systems, Asian Journal of Applied Science and Technology, and Pakistan Journal of Commence and Social sciences. He is also a member of the Islamic Countries Society of Statistical Sciences. He is appointed as an external examiner for the 2016/2017-2018/2019 triennium at The University of Dodoma, Tanzania. His areas of interest include reliability, decision trees, industrial statistics, acceptance sampling, rank set sampling, neutrosophic statistics, and applied statistics.

Muhammad Farooq earned his Ph. D. in Statistics from the University of Stuttgart, Germany in October 2018. He is working as an assistant professor in the Department of Statistics, Government College University Lahore, Pakistan. His research interests include statistical learning, kernel-based learning methods, mathematical statistics, neuro-statistics, and Bayesian analysis.

Rehan Ahmed Khan Sherwani is an associate professor in the College of Statistical and Actuarial Sciences, University of the Punjab, Lahore, Pakistan. His area of research includes mixed models, statistical quality control, neutrosophic statistics, computational, and applied statistics.

Sajjad Haider Bhatti received his Ph.D. in Applied Statistics and Econometrics from the University of Dijon, France. He is currently working as an Assistant Professor in the Department of Statistics, Government College University Faisalabad, Pakistan. His research interests include regression diagnostics, modified estimators, and statistical quality control.

Tanvir Ahmad received his Ph.D. in Statistics from Islamia University Bahawalpur, Pakistan. He partially completed his Ph.D. research in Queen Mary, University of London. He received his M.Sc. from the Department of Statistics, Bahauddin Zakariya University, Multan. He joined the University of Southampton as a post-doctoral fellow in 2015. He is currently working as Associate Professor of Statistics in the Department of Statistics, Government College University Faisalabad, Pakistan. His research interests include response surface methodology, robust optimization, epidemiology, and survival analysis.