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Reliability and cost optimization of a system with k-out-of-n configuration and choice of decreasing the components failure rates

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Abstract. This paper presents a new redundancy allocation problem for a system with the k-out-of-n configuration at the subsystems level with two active and cold standby redundancy strategies. The failure rate of components in each subsystem depends on the number of working components. The components are non-reparable, and the failure rate of the component can be decreased through some preventive maintenance actions. The model has two objective functions: maximizing the system reliability and minimizing the system costs. The system aims to find the type and number of components in each subsystem, redundancy strategy of subsystems, as well as the decreased values of components failure rates in subsystems. Since the redundancy allocation problem belongs to NP-hard problems, two metaheuristic algorithms including Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Non-dominated Ranked Genetic Algorithm (NRGA) were used to solve the presented model. To tune algorithms parameters, we used response surface methodology. Besides, these algorithms were compared using five different performance metrics. Finally, the hypothesis test was used to analyze the results of the algorithms.

1. Introduction

Currently, many studies have been conducted in the field of reliability to achieve more reliable systems. The Redundancy Allocation Problem (RAP) is the most important in this area. RAP aims to maximize the system reliability by increasing redundant components of subsystems under some constraints. This problem was first presented by Fyffe et al. [1] and was solved by dynamic programming. Chern [2] proved that RAP belongs to the NP-hard problem when the number of subsystems increases. Therefore, many heuristic and meta-heuristic methods have been used to solve this problem.

There are much real-world manufacturing and operational systems that increase their reliability through the concepts of RAP, which can be counted, including aircraft engines, the number of pumps at a water pumping station, and so on. Considering the nature of these manufacturing and operating systems, many hy-

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potheses and limitations have been added to the RAP to draw the problem closer to real-world conditions.

Therefore, researchers categorized this problem from different aspects, including categorization based on the functional status of the components (binary or multi-state), the type of component failure rate (constant or time-dependent), components configuration in the system (active or standby).

Considering the importance of the system reliability and system cost, in many studies, both objectives are considered as objective functions, and this problem is transformed into a two-objective problem (and even more than two).

In this paper, we investigate a Multi-Objective RAP (MORAP) whose failure rates depend on the number of working elements. The subsystems are k-out-of-n, and the failure rate of components can be reduced with spending money. The objective functions of the model are maximizing system reliability and minimizing system weight. The type of each subsystem component and subsystem redundancy strategies are the system variables. Since the model is NP-hard, we used Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Non-dominated Ranked Genetic Algorithm (NRGA) for solving this model.

This paper is organized as follows: In Section 2, we present a literature review to confirm that there exist no studies that exactly meet these research conditions. In Section 3, we discussed the mathematical model and system assumptions. In Section 4, the NSGA-II and NRGA algorithms are presented. In Section 5, a numerical example is presented to compare the algorithms results. Section 6 is the managerial insights, and the final section deals with the conclusion and further studies.

2. Literature review

In real-world problems, many parameters affect the system reliability. One of the most important factors is the failure rate of the components. This parameter in RAP studies has two categories: Constant Failure Rate (CFR) and time-dependent failure rate.

Regarding CFR models, Misra and Sharma [3] presented a RAP model with the choice of allocating identical components to each subsystem and active redundancy strategy, then solved the presented model with zero-one programming. Ida [4] used a genetic algorithm to solve RAP with multi-failure components. Coit and Smith [5] presented a RAP model with the choice of allocating non-identical elements to each subsystem and solved the presented model using dynamic programming, integer programming, mixed non-linear integer programming, and compared the results with those of genetic algorithm. Coit and Liu [6] were the first who worked on a system with k-out-of-n sub-systems. They predefined active or cold standby redundancy strategies for each subsystem and solved the problem using integer programming. Hsieh and You [7] presented a new two-stage method based on the immune algorithm to solve the RAP under non-linear weight, volume, and cost constraints. Hsieh and Yeh [8] used penalty-guided bees search to solve RAP.

All presented studies have been conducted on single-objective models. Busacca et al. [9] presented a two-objective model. The model objectives were maximizing the system reliability and net profit and used a multi-objective genetic algorithm to solve the problem. Coit and Jin [10] worked on a MORAP with maximizing system reliability and minimizing the variance of results. Baharanwala et al. [11] used NSGA to solve the MORAP. The objectives of the model are maximizing system reliability, minimizing the system cost, weight, and variance of reliability. In the presented model, each subsystem has a lower and upper limit for components allocation. Salazar et al. [12] solved three different RAP models using NSGA-II. The objective functions of the models are maximizing system reliability and minimizing the system cost. Kulturel-Konak [13] worked on a MORAP model using the Tabu search algorithm. Taboada et al. [14] presented two methods for decreasing the size of Pareto solutions and used the presented method for solving MORAP. Taboada and Coit [15] presented an MOEA evolutionary algorithm to solve the MORAP with a new crossover operator that increased the variations of the solutions. Liang and Lo [16] presented a variable neighborhood search for solving MORAP and solved three different models and compared the results with NSGA-II results. Soylu and Ulusoy [17] worked on MORAP and contributed Pareto solutions to small- and large-scale problems. Then they classified the solutions using the UTADIS method.

Concerning time-dependent failure rate models, Coit [18] presented a new RAP with a switching system. Also, he used the K-Erlang distribution function for components and solved the problem using dynamic programming. Later on, Coit [19] considered two different active and cold standby redundancy strategies for each subsystem. Tavakkoli-Moghaddam et al. [20], Safari and Tavakkoli-Moghaddam [21] solved that problem using a genetic algorithm in 2008 and a memetic algorithm in 2010. Amari [22] presented an enumeration method for evaluating the reliability of k-out-of-n systems with a cold standby redundancy strategy. Dhingra [23] used mixed goal programming and goal attainment to produce Pareto optimal solutions in a fourth-level system to maximizing reliability and minimizing system costs and weight. Ghafarian Salehi Nezhad et al. [24] presented a four-phase algorithm to improve reliability in series-parallel systems with redundancy allocation. They combined an Ant Colony

Optimization (ACO) algorithm as a meta-heuristic phase, and three other heuristics to develop a solving methodology for RAP. Azaron et al. [25] used the shortest path method in stochastic graphs to evaluate the reliability of a cold standby system with non-identical elements. Azaron et al. [26] salved a multi-state cold standby RAP and non-repairable components with the GADSCRRSU method. Ebrahim Nezhad et al. [27] presented a new method for MORAP with the choice of allocating identical elements and predefined active and cold standby redundancy strategies with maximizing reliability and net profit objective functions. Chambari et al. [28] used multi-objective particle swarm optimization and NSGA-II for a MORAP with the choice of selecting the redundancy strategy of each subsystem. Azimi et al. [29] Solved a RAP with k-out-of-n configuration and non-exponential repairable components. They used optimization via simulation technique to solve the presented RAP. Pourkarim Guilani et al. [30] worked on a bi-objective reliability model with three-state components. They used multi-objective Strength Pareto Evolutionary Algorithm II (SPEA-II) and NSGA-II to solve the presented model.

In all mentioned research studies, the failure rate of working elements is fixed. For example, for a system with one working component, the failure rate of this component is equal to the failure rates of components in a system with 10 working components. Sharifi et al. [31] presented a formula for evaluating the reliability of a k-out-of-n system when the components failure rates depend on the number of working elements. In the presented k-out-of-n system, when a component failed, the failure rates of the remaining components increases.

Table 1 contains several recent studies on the reliability field, along with a summary of the model behaviors.

3. Mathematical model

In this paper, the system has s k-out-of-n sub-systems, and its objectives are maximizing system reliability and minimizing system weight under two linear constraints.

3.1. Model assumptions

The main assumptions of the current paper are as follows:

- The system has s subsystems
- Each subsystem is k-out-of-n
- Subsystem components are identical
- Components are binary states
- The probability of working switching system for cold standby subsystems is equal to p
- The failure rate of working components depends on the number of working components
- The components are non-repairable, and
- The system parameters are constant

The notations which are used in this paper are as follows:

s	Number of subsystems
i	Subsystem index
k_i	Minimum necessary components in ith subsystem
n_i	Number of components in ith subsystem
n	The set of n_i $(n_1, ..., n_s)$
$n_{\max,i}$	The upper limit of n_i $(n_i \leq n_{\max,i}; i = 1, ..., s)$
m_i	Number of available component types in ith subsystem $(i = 1, ..., s)$
z_i	Component type index in ith subsystem $z_i \in (1, ..., m_i)$
z	Set of $z_i \in (1, ..., m_i)$
t	Mission time
$\lambda_{iz_ik_i}$	The failure rate of type z_i component in ith subsystem when the subsystem working with k components
$\lambda'_{iz_ik_i}$	The reduced failure rate of type z_i component in ith subsystem when the subsystem working with k components
w_{iz_i}	Weight of component, type z_i in ith subsystem
c_{iz_i}	Cost of component, type z_i in ith subsystem
c'_{iz_i}	Cost of reducing the failure rate of each component, type z_i in ith subsystem
θ_{iz_i}	Cost parameter of internal relation for component, type z_i in ith subsystem
β_{iz_i}	Reducing factor of failure rate for type z_i component in ith subsystem
W	Total acceptable weight of the system
A	Index of subsystems with active redundancy strategy
S	Index of subsystems with cold standby redundancy strategy
R_{l_1}	Reliability of the subsystem with active redundancy strategy
R_{l_2}	Reliability of the subsystem with cold standby redundancy strategy
R_l	System reliability
P	Probability of working the switch in switching time

3.2. Mathematical model

In the presented mathematical model, the reliability

Table 1. Some of the recent studies on the reliability area.

Authors	Year	State	Elements type	Algorithm	Fuzzy	Repairable	Penalty function	Objective	Parameter setting	Failure rate
Sharifi et al. [32]	2005	Binary	Homogeneous	Markov model	√	–	–	Single	–	Constant
Lins and Droguett [33]	2011	Binary	Heterogeneous	ACO	–	√	–	Multiple	No	Constant
Ouzineb et al. [34]	2008	Multi-state	Homogeneous	TS	–	–	–	Single	No	Constant
Sharma and Agarwal [35]	2009	Multi-state	Heterogeneous	ACO	–	–	–	Single	No	Constant
Lins and Droguett [36]	2009	Binary	Heterogeneous	GA	–	√	–	Multiple	No	Constant
Ouzineb et al. [37]	2011	Multi-state	Heterogeneous	GA	–	–	–	Single	No	Constant
Ebrahimipour and Sheikhalishahi [38]	2011	Binary	Heterogeneous	PSO	√	–	–	Multiple	No	Constant
Lins and Droguett [39]	2011	Multi-state	Heterogeneous	GA	–	√	√	Multiple	No	Constant
Garg and Sharma [40]	2013	Binary	Heterogeneous	PSO	–	–	–	Multiple	No	Constant
Garg et al. [41]	2013	Binary	Heterogeneous	Bee colony	–	–	√	Single	No	Constant
Levitin et al. [42]	2013	Multi-state	Heterogeneous	GA	–	–	–	Sing le	No	Constant
Maatouk et al. [43]	2013	Multi-state	Heterogeneous	GA	–	√	–	Single	No	Constant
Chambari et al. [28]	2013	Binary	Heterogeneous	SA	–	–	√	Single	No	Constant
Gago et al. [44]	2013	Binary	Heterogeneous	Greedy, Walk back	–	–	–	Single	No	Constant
Ebrahimipour et al. [45]	2013	Binary	Heterogeneous	Fuzzy Inference System (FIS)	–	–	–	Single	No	Constant
Liu et al. [46]	2013	Multi-state	Heterogeneous	Imperfect repair model	√	√	–	Single	No	Constant

Table 1. Some of the recent studies on the reliability area (continued).

Authors	Year	State	Elements type	Algorithm	Fuzzy	Repairable	Penalty function	Objective	Parameter setting	Failure rate
Khalili-Damghani et al. [47]	2014	Binary	Heterogeneous	e-constraint	–	–	–	Multiple	No	Constant
Guilani et al. [48]	2014	Multi-state	Homogeneous	Markov model	–	–	–	Single	–	Constant
Sharifi et al. [49]	2015	Binary	Heterogeneous	GA, MA	–	–	√	Single	RSM	Time dependent -Number dependent
Mousavi et al. [50]	2015	Multi-state	Homogeneous	CE-NRGA	√	–	√	Multiple	Taguchi	Constant
Zaretalab et al. [51]	2015	Multi-state	Homogeneous	MOSA	–	–	√	Multiple	–	Constant
Miriha et al. [52]	2017	Binary	Heterogeneous	NSGA-II-MOEA/D	–	–	√	Multiple	Taguchi	Time-dependent
Pourkarim Guilani et al. [53]	2019	Binary	Heterogeneous	Simulation	–	–	√	Single	–	Constant
Sharifi et al. [54]	2020	Multi-state	Homogeneous	GA	–	–	√	Single	RSM	Constant
Zaretalab and Hajipour [55]	2019	Binary	Heterogeneous	SA	–	–	√	Single	–	Constant
Sharifi et al. [56]	2019	Binary	Heterogeneous	Memetic	–	–	√	Single	–	Constant
Sharifi and Khoshniat [57]	2019	Binary	Heterogeneous	BBO	–	–	√	Single	RSM	Constant
Sharifi et al. [58]	2019	Binary	Heterogeneous	NSGA-II-NRGA	–	–	√	Multiple	RSM	Constant
Borhani Alamdari and Sharifi [59]	2020	Binary	Heterogeneous	GA	–	–	√	Sin gle	RSM	Constant
Zaretalab et al. [60]	2020	Multi-state	Heterogeneous	GA and MA	–	√	√	Single	RSM	Constant
Sharifi and Taghipour [61]	2020	Multi-state	Heterogeneous	GA	–	–	√	Single	RSM	Constant
Current work	2020	Binary	Heterogeneous	NSGA-II-NRGA	–	–	√	Multiple	R SM	Time dependent -Number dependent

and cost of the system are designed to be optimized simultaneously. One of the most important system constraints in the RAPs is the system weight constraint. This constraint is considered in the proposed mathematical model. Another considered constraint is the upper bound for reducing components failure rates. In light of these explanations, the decision variables of this problem are the number of components in each subsystem and reducing the factor of failure rate for all of the components in each subsystem.

$$\max R_l\left(t\right) = \left\{\prod_{i\in A} R_{l_1}\left(t\right)\right\} \times \left\{\prod_{i\in S} R_{l_2}\left(t\right)\right\}, \tag{1}$$

$$\min C = \sum_{i=1}^{s} \left\{c_{iz_i}\left\{n_i + f\left(\theta_{iz_i}\right)\right\} + c'_{iz_i} f\left(\beta_i\right)\right\}, \tag{2}$$

s.t.:

$$\sum_{i=1}^{s} w_{iz_i} n_i \le W, \tag{3}$$

$$0 \le \beta_i \le a_i, \tag{4}$$

$$n_i \in \left(k_i, 2, ..., n_{\max}\right); \quad i = (1, 2, ..., s), \tag{5}$$

$$z_i \in (1, 2, ..., m_i); \quad i = (1, 2, ..., s). \tag{6}$$

In this model, Eq. (1) is the first objective function, i.e., maximizing the system reliability. Eq. (2) is the second objective function, i.e., minimizing the system cost. We consider that the system cost contains the cost of redundant components, the cost of internal relation [62], and the cost of reducing the components failure rates. In this paper, $f(\beta_i)$ and $f(\theta_{iz_i})$ are defined as follows:

$$f\left(\beta_i\right) = e^{\beta_i \lambda_{iz_i}}, \quad f(\theta_{iz_i}) = e^{\theta_{iz_i} n_i}. \tag{7}$$

Eq. (3) is the constraint of system weight. Eq. (4) is the upper and lower limits of reducing components failure rate and Eqs. (5) and (6) define the maximum number and type of components in each subsystem, respectively. The first objective function is divided into two parts. The first part is the reliability of the subsystems with an active redundancy strategy, and the second part is the reliability of the subsystems with a cold standby redundancy strategy.

The reliability of a system with n identical component and active redundancy strategy when the failure rate of the component is related to the number of the working component can be calculated as follows [31]:

$$R(t) = \left(\prod_{j=k}^{n} \lambda_j\right) \times \sum_{i=k}^{n} \left\{ \frac{n\,!}{i(k-1)!} \left(\prod_{\substack{\theta=k \\ \theta\neq i}}^{n} \frac{1}{\theta \times \lambda_\theta - i \times \lambda_i}\right) \times \frac{e^{-i\times\lambda_i\times t}}{\lambda_i} \right\}. \tag{8}$$

In Eq. (7), λ_i is the failure rate of components when the system is working with i components. In a real situation, when a component fails, the load on other working components increases. Eq. (9) makes a relation between the failure rates of working components [31]:

$$\lambda_k = \frac{k - \gamma\left(k-1\right)}{k}\lambda_1. \tag{9}$$

In Eq. (9), $0 \le \gamma \le 1$ can tune the relations between failure rates of the component. When $\gamma = 0$ the failure rate of working components is independent of the number of working components and when $\gamma = 1$ the failure rate of working components is $\lambda_k = \lambda_1/k$. For the presented model, Eq. (9) is transformed into Eq. (10):

$$\lambda_{iz_ik_i} = \frac{k_i - \gamma\left(k_i - 1\right)}{k_i}\lambda_{iz_i1}. \tag{10}$$

We combined Eqs. (8) and (10), so, the reliability of the systems with active redundancy strategy, and the failure rate depends on the number of working elements and can be calculated in Eq. (11) as shown in Box I.

$$R_{l_1}\left(t\right) = \sum_{i=k_i}^{n_i} P_i\left(t\right) = \left(\prod_{j=k_i}^{n_i} \frac{j-\gamma\left(j-1\right)}{j}\lambda_{iz_i1}\right) \times \sum_{i=k_i}^{n_i}\left[\frac{n_i\,!}{i\left(k_i-1\right)!}\left(\prod_{\substack{\omega=k_i \\ \omega\neq i}}^{n_i} \frac{1}{\left\{\left\{\omega-\gamma\left(\omega-1\right)\right\}-\left\{i-\gamma\left(i-1\right)\right\}\right\}\lambda_{iz_i1}}\right) \times \frac{e^{-\left\{i-\gamma(i-1)\right\}\lambda_{iz_i1}t}}{\left\{i-\gamma\left(i-1\right)\right\}\lambda_{iz_i1}}\right]. \tag{11}$$

Box I

The reliability formula for a cold standby subsystem is presented in Eq. (12); in these subsystems, the switch detects the failures of a working component and changes the failed component with a new one. The switching system is a discrete detection switching and may work in each detection by the probability p and maybe failed in each detection by the probability $(1-p)$.

$$R_{l_2}(t) = \sum_{j=0}^{n_i-k_i-1} \left\{ (1-p)\, p^j \sum_{m=0}^{j} \frac{e^{-k_i \lambda_{iz_ik_i} t}.\, (k_i \lambda_{iz_ik_i} t)^m}{m\,!} \right\} + p^{n_i-k_i} \sum_{m=0}^{n_i-k_i} \frac{e^{-k_i \lambda_{iz_ik_i} t}.\, (k_i \lambda_{iz_ik_i} t)^m}{m\,!}. \tag{12}$$

Since RAP belongs to NP-hard problems and the first objective function of this problem is nonlinear, the exact solutions have less efficiency in solving this problem. So, we used the NSGA-II and NRGA metaheuristic algorithms to solve the presented model. These algorithms are presented in the next section.

4. Solving methods

We used NSGA-II and NRGA algorithms to solve the presented problem. These two algorithms are based on population, so the solution structures of both algorithms are the same.

4.1. Solution encoding

Each solution is a $4 \times s$ matrix. The ith column of the matrix belongs to the ith subsystem. The first row of the matrix represents the redundancy strategy of the subsystem components; the second row shows the component type of the subsystem. The third part contains the number of components in each subsystem, and the last row is the failure rate reduction coefficient of the component in the subsystem. The structure of an encoded solution is shown in Figure 1. In this figure, in the first subsystem, 4 components of type three exist, and the components have an active redundancy strategy. Also, the failure rate of the components in this subsystem was reduced by 18.66%.

4.2. NSGA-II algorithm

In 1994, Srinivas and Deb [63] used Goldberg ideas and presented the concepts of NSGA. This algorithm is efficient but too complicated, unable to solve multi-objective problems. Deb et al. [64] presented the NSGA-II algorithm to overcome the weakness of the NSGA algorithm regarding particle election and the complexity. In this algorithm, the Pareto solutions are obtained using dominant and non-dominant solutions, and the mutation operator of the genetic algorithm is used to find the new solutions.

4.3. NRGA algorithm

Improvement of operators is a way to improve the efficiency of multi-objective algorithms. Improvement of selection operators has more effects on the improvement of algorithms efficiency and makes the evolutionary algorithms more converging. So, Al Jadaan et al. [65] improved an evolutionary multi-objective algorithm based on Ranked based roulette wheel selection and Pareto-based population ranking and called it NRGA. In this combination, a two-layer ranking is presented based on roulette wheel selection that randomly selects the new generation from old ones based on selecting the best solutions (based on fitness and span). The NRGA can better achieve a wide range of solutions and converge to optimal Pareto versus other evolutionary algorithms.

In both algorithms, the solutions in each population rank are based on their non-dominant rate. The solutions in the first category are the best non-dominant solutions, and the solutions in the last category are the worst non-dominant solutions. So, the solutions in the first category have the maximum fitting, and the solutions in the last category have the minimum fitting. After ranking the categories, the solutions in each category rank are based on the swarm. The solution with maximum swarm has the maximum rank, and the solution with minimum swarm has the minimum rank in the category. NSGA-II and NRGA differ in selecting a strategy, ordering the population,

	Subsystem													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy Strategy	A	S	S	A	A	S	A	S	A	A	S	S	A	S
Components Type	2	1	3	2	3	4	3	1	4	2	3	2	1	2
Components Number	4	2	3	4	1	1	3	5	3	2	6	4	2	1
Coefficients of Decreasing Failure Rates	0.1866	0.0134	0.1642	0.0527	0.073	0.1957	0.1921	0.0381	0.1931	0.1312	0.1923	0.1968	0.1205	0.079

Figure 1. Encoded solution.

and selecting for the next generation. NRGA used RRWS instead of the tournament operator. In this operator, the solutions with better fitting have a higher chance to be selected for reproduction and creation of the next generation. Al Jadaan et al. [65] used a selected modified algorithm based on the roulette wheel in which each solution has the fitting value equal to the rank of the solution in the population. The solutions in the population rank are based on two specifications. First, the rank of the containing category of the solution, and second the rank of the solution in the category. For selecting a solution, at first, a non-dominant category must be selected. The probability of selecting the ith non-dominant category is calculated using Eq. (13) [65]:

$$p_i = \frac{2 \times rank_i}{N_f \times (N_f + 1)} = \frac{rank_i}{\sum_{i=1}^{p} rank_i}, \tag{13}$$

where $rank_i$ is the rank of ith category and N_f is the number of the categories. The probability of selecting jth solution in ith non-dominant category is calculated using Eq. (14) [65]:

$$p_{ji} = \frac{2 \times rank_{ji}}{N_j \times (N_j + 1)} = \frac{rank_{ji}}{\sum_{j=1}^{p} rank_{ji}}, \tag{14}$$

where N_j is the number of solutions in ith category and $rank_{ji}$ is the rank of jth solution in ith category.

In the roulette wheel, the first two real intervals $[0, S_1]$ and $[0, S_2]$ values $S_1 = \sum_{i=1}^{n} p_i$ and $S_2 = \sum_{j=1}^{m} p_j$ are defined. Then the solutions in each category occupy a certain amount of $[0, S_1]$ and $[0, S_2]$ based on the probability of their selection. Then two random numbers are selected between zero and one, and the first random number is used to select in $[0, S_1]$ and the second random number is used to select an answer in $[0, S_2]$. The flow chart of both algorithms is presented in Figure 2.

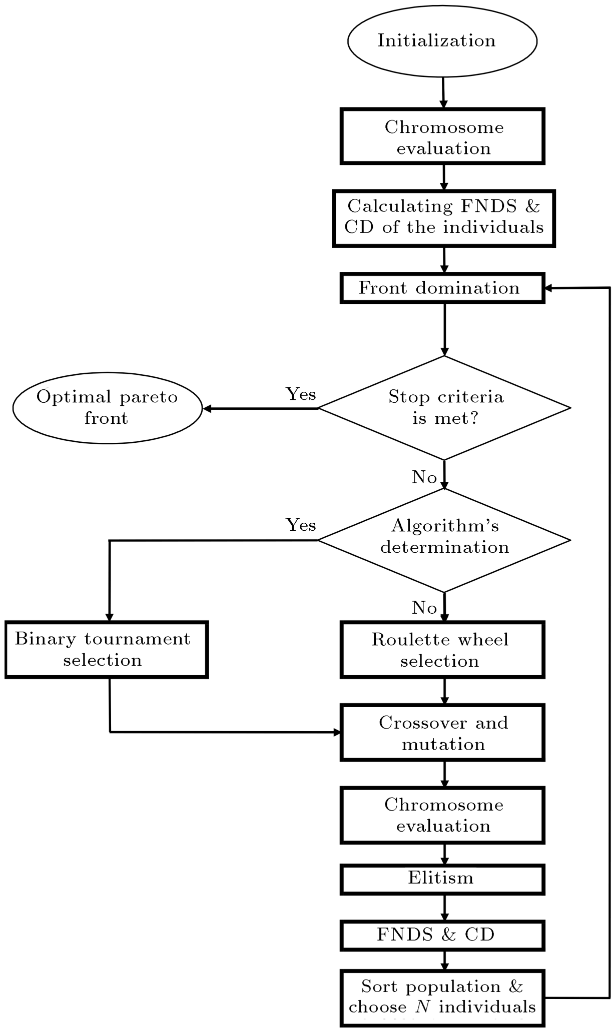

Figure 2. The results for Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Non-dominated Ranked Genetic Algorithm (NRGA) algorithms.

4.4. Comparison metrics

Convergence with Pareto optimal solutions and providing density and diversity among the set of solutions are two distinct and somewhat conflicting objectives in multi-objective evolutionary algorithms, a criterion that can be used alone and does not exist in absolute terms for calculating the performance of the algorithm [66].

For this reason, we used five performance metrics to better evaluate the performance of the two presented algorithms better.

4.4.1. Maximum spread or diversity

This index measures the length of the space cubic diameter used in the final values of the targets for the non-dominant solutions. Eq. (15) shows the computational procedure of this index. Therefore, the larger this criterion, the more the archived Pareto front spreads.

$$D = \sqrt{\sum_{j=1}^{m} \left(\max_i f_i^j - \min_i f_i^j \right)^2}. \tag{15}$$

4.4.2. Spacing

This scale calculates the relative distance of consecutive solutions using Eq. (16):

$$S = \sqrt{\frac{1}{|n|} \sum_{i=1}^{n} (d_i - \bar{d})^2}, \tag{16}$$

where $\bar{d} = \sum_{i=1}^{n} d_i / |n|$ and $d_i = \min_{k \in n k \neq 1} \sum_{m=1}^{2} |f_m^i - f_m^k|$. This scale measures the standard deviations of different values d_i. When the solutions are close to the gathering, S has smaller a value, and the performance of the algorithm with a small spacing scale is better than other algorithms [66].

Table 2. Data for numerical example.

Subsystem		**Component type 1**			**Component type 2**			**Component type 3**			**Component type 4**		
i	k_i	λ_{i1}	$\lambda_{c_{i1}}$	w_{i1}	λ_{i2}	λc_{i2}	w_{i2}	λ_{i3}	λc_{i3}	w_{i3}	λ_{i4}	λc_{i4}	w_{i4}
1	1	0.001054	1	3	0.000726	1	4	0.000943	2	2	0.000513	2	5
2	2	0.000513	2	8	0.000619	1	10	0.000726	1	9		–	
3	1	0.001625	2	7	0.001054	3	5	0.001393	1	6	0.000834	4	4
4	2	0.001863	3	5	0.001393	4	6	0.001625	5	4		–	
5	1	0.000619	2	4	0.000726	2	3	0.000513	3	5		–	
6	2	0.000101	3	5	0.000202	3	4	0.000305	2	5	0.000408	2	4
7	1	0.000943	4	7	0.000834	4	8	0.000619	5	9		–	
8	2	0.002107	3	4	0.001054	5	7	0.000943	6	6		–	
9	3	0.000305	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	3	0.001863	4	6	0.001625	4	5	0.001054	5	6		–	
11	3	0.000619	3	5	0.000513	4	6	0.000408	5	6		–	
12	1	0.002357	2	4	0.001985	3	5	0.001625	4	6	0.001054	5	7
13	2	0.000202	2	5	0.000101	3	5	0.000305	2	6		–	
14	3	0.001054	4	6	0.000834	4	7	0.000513	5	6	0.000101	6	9

4.4.3. Number of Pareto Solution (NPS)
This scale shows the NPSs in each algorithm.

4.4.4. Mean Ideal Distance (MID)
This scale indicates the distance to the ideal Pareto level and is calculated using Eq. (17). The lower values of this scale indicate that the algorithm is working properly.

$$MID = \frac{\sum_{i=1}^{n} \sqrt{f_{i1}^2 + f_{i2}^2}}{n}, \tag{17}$$

where f_{i1} and f_{i2} are the first and second objective functions in ith solution.

4.4.5. Time of algorithm
This scale defines the time of the algorithm running to satisfy stop criteria.

5. Numerical example

In this section, we present a numerical example to illustrate the effectiveness of the presented algorithms. The example data are obtained from the data of Coit and Smith [5]. The example is a series-parallel system with the k-out-of-n subsystem. Three or four different component types are available for each subsystem, and the redundancy strategy of each subsystem is a variable. The cost, weight, and failure rate of components and a minimum number of components in each subsystem are presented in Table 2. The maximum number of each subsystem component is six. The objectives are maximizing system reliability and minimizing system cost underweight constraint [6].

Table 3. Range of algorithms tuned parameters.

Solving methodologies	**Parameter**	**Range**
NSGA-II	$nPop$	50–100
	P_c	0.3–0.6
	P_{m_1}	0.1–0.3
	P_{m_2}	0.1–0.3
NRGA	nPop	50–100
	P_c	0.3–0.6
	P_{m_1}	0.1–0.3
	P_{m_2}	0.1–0.3

Also, the switch reliability is considered as $p = 0.99$ [19]. The internal connection cost for all subsystems is $\theta_{iz_i} = 0.25$ [62] and $\gamma = 0.2$, also the cost of reducing components failure rate is $0.75C_{iz_i}$.

5.1. Parameter tuning
The parameters of NRGA and NSGA-II algorithms are tuned in this section. The Response Surface Methodology (RSM) is used for parameter tuning. These parameters are population size ($nPop$), crossover rate (P_c), mutation rate (P_{m_1}), and max-min operator(P_{m_2}). Table 3 presents the range of these parameters and Table 4 shows the results of parameter tuning.

5.2. Results
This section deals with comparing the results of NSGA-II and NRGA algorithms. For this comparison, we used a Laptop with 6G RAM and 1.73 GH CPU

Table 4. Optimal values of parameters.

Solving methodologies	Parameter	Optimum value
NSGA-II	$nPop$	79
	P_c	0.30
	P_{m_1}	0.30
	P_{m_2}	0.10
NRGA	$nPop$	75
	P_c	0.45
	P_{m_1}	0.20
	P_{m_2}	0.20

speed, and the algorithm codded using MATLAB 2018. Each algorithm ran five times with the optimal values of parameters. The iteration of each algorithm was considered 100. The results of the algorithm performance were presented in Figure 3. In NPS, concerning diversity and time scales, the NSGA-II has a better performance than those in the NRGA algorithm, and in other scales, NRGA is better than NSGA-II.

5.2.1. Results analysis

To find the difference between the results of the indexes, we used a one-way ANOVA test using $1-\alpha=0.95$. Table 5 presents the results for the ANOVA test.

The ANOVA test results show that there are meaningful differences between the indexes of the two algorithms, and reject the assumption that the results of the two algorithms are the same.

5.2.2. Sensitivity analysis

To further evaluate the model in this section, we intend to solve 33 different numerical examples for the presented multi-objective model using NSGA-II and NRGA algorithms. The information for these 33 numerical examples is similar to Table 2. However, the available weight of the system varies from 159 to 191. The result of the performance indices of each algorithm on these 33 numerical examples are graphically illustrated in Figure 4.

6. Managerial insights

Increasing the number of parallel components in a system can promote the reliability of this system, but it is not sufficient. Related researches show that

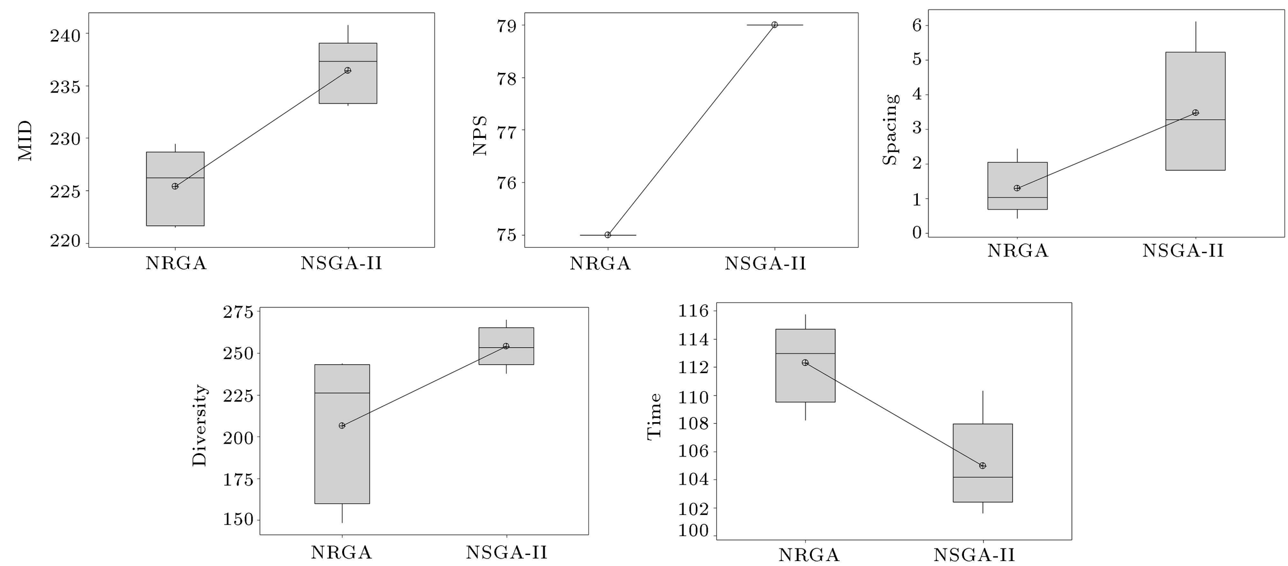

Figure 3. The results for Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Non-dominated Ranked Genetic Algorithm (NRGA) algorithms.

Table 5. Results of algorithms.

Performance metrics	P-value	Result	Final result
MID	0.001	H0 is rejected	NRGA
NPS	0.000	H0 is rejected	NSGA-II
Spacing	0.039	H0 is rejected	NSGA-II
Diversity	0.047	H0 is rejected	NRGA
Time	0.006	H0 is rejected	NSGA-II

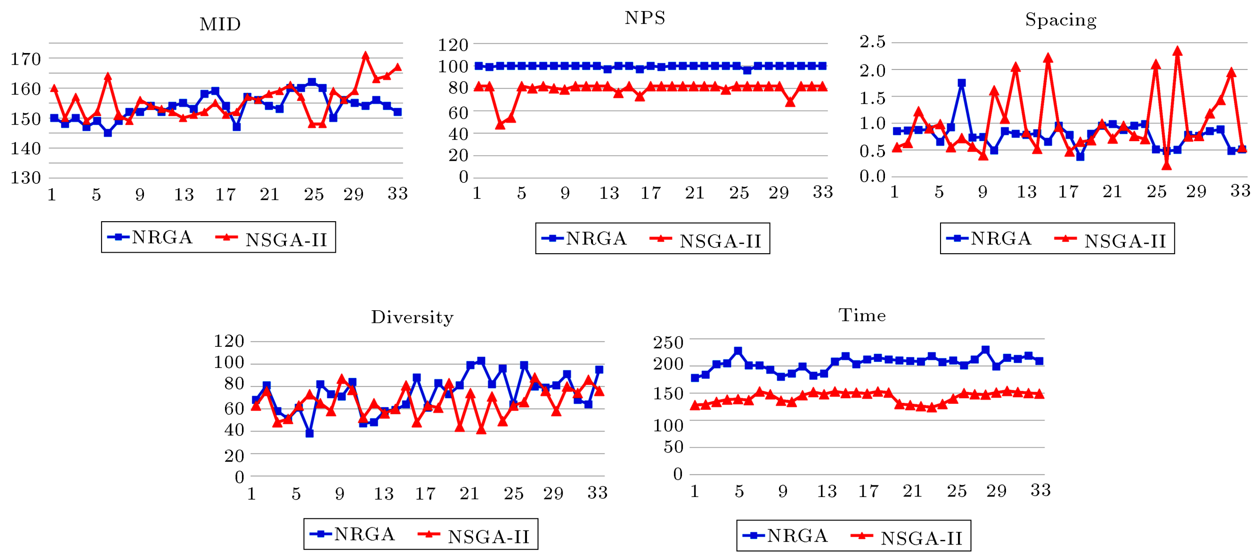

Figure 4. Sensitivity analysis of the proposed model by Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Non-dominated Ranked Genetic Algorithm (NRGA) algorithms.

improving the performance of each used component in a system can be utilized as another way to improve the reliability of the entire system. Therefore, in this paper, both of these criteria are employed to improve the performance of the system. It should be considered that the required parameters for designing the problem are formed based on the effects of each approach mentioned above and their related costs. On the other hand, if there is a budget capacity constraint, effort in promoting components' availability leads to a less redundancy assignment and vice versa.

Many realistic examples of failure rate improvement are given in the literature. For instance, consider the action of the installation of a vibration monitoring system for the FD fan and the ID fan. Vibration monitoring can monitor the health condition of the fans, and preventive replacements can be performed to prevent unexpected failures. Therefore, the failure rates of the can be reduced, thereby reducing the failure rate of the generating unit from the 300 MW state to the 150 MW state. Before adopting the vibration monitoring system, the reduction in the failure rates of the fans, i.e., the benefit of installing the vibration monitoring system, can be estimated based on the failure histories, or by the vibration monitoring system provider based on their experiences. Other examples are: installing monitor systems and maintenance planning.

7. Conclusion and further studies

Many parameters affect the reliability of the systems, and the failure rate of components is one of the most important parameters. In this paper, we worked on a two-objective reliability model. In this model, the failure rate of the components is constant and depends on the number of working components in the system. The components failure rates could be cost-effective. The system contains s subsystems, and the subsystems have the k-out-of-n configuration. All subsystems may have two active and cold standby redundancy strategies that are system variables. Besides, the number and the type of each subsystem component and the reduction of the failure of the component rate are other variables of the model. Because Redundancy Allocation Problem (RAP) belongs to the NP-hard problem, we used Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Non-dominated Ranked Genetic Algorithm (NRGA) multi-objective algorithms for solving the presented problem. Also, we used 5 different indexes for comparing the algorithm performance. The results showed that NSGA-II has better performance in Number of Pareto Solution (NPS), diversity, and time indexes, and for other indexes, the NRGA has better performance.

For further studies, the components can be considered as repairable components. In addition, the combination of different components in each subsystem can also be considered a new idea. Also, another multi-objective metaheuristic algorithm can be considered for solving the proposed problem.

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