Reliability and Cost Optimization of a System with k-out-of-n Configuration and Choice of Decreasing the Components Failure Rates

Mani Sharifi *
Faculty of Industrial & Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran,
Tel: +98 (912) 282-5776, Mail: m.sharifi@qiau.ac.ir
* Corresponding author,
Ghasem Cheragh
Faculty of Industrial & Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran,
Tel: +98 (930) 414-6003, Mail: ghasemcheragh@yahoo.com
Kamran Dashti Maljaii
Faculty of Industrial & Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran,
Tel: +98 (912) 661-0656, Mail: kamidashti@yahoo.com
Arash Zaretalab
Department of Industrial Engineering, Amirkabir University of Technology, 424 Hafez Ave., Tehran, Iran,
Tel: +98 (912) 450-1767, Mail: arash_zaretalab@yahoo.com
Mohammadreza Shahriari
Associate Professor, Faculty of Management and accounting, South Tehran Branch, Islamic Azad University, Tehran, Iran,
Tel: +98 (935) 319-1000, Mail: shahriari.mr@gmail.com

Abstract: This paper presents a new redundancy allocation problem for a system with the k-out-of-n configuration at the subsystems' level with two active and cold standby redundancy strategies. The failure rate of components in each subsystem depends on the number of working components. The components are non-reparable, and the failure rate of the component can be decreased with some preventive maintenance actions. The model has two objective functions: maximizing the system's reliability and minimizing the system's costs. The system aims to find the type and number of components in each subsystem, redundancy strategy of subsystems, as well as the decreased values of components failure rates in subsystems. Since the redundancy allocation problem belongs to NP-Hard problems, two Non-Dominated Sorting Genetic Algorithm II (NSGA-II) and Non-Dominated Ranked genetic algorithm (NRGA) metaheuristic algorithms were used to solve the presented model and to tune algorithms parameters we used response surface methodology. Besides, these algorithms were compared using five different performance metrics. Finally, the hypothesis test was used to analyze the results of the algorithms.

Keywords: Reliability, Redundancy allocation problem, NSGA-II, NRGA, Response surface methodology

1. Introduction

Currently, many studies have been conducted in the reliability field to achieve more reliable systems. The redundancy allocation problem (RAP) is the most important in this area. RAP aims to maximize the system's reliability with increasing redundant components of subsystems under some constraints. This problem was first presented by Fyffe, Hines, and Lee [1] and was solved by dynamic programming. Chern [2] proved that RAP belongs to the NP-Hard problem when the number of subsystems increases. Therefore, many heuristic and meta-heuristic methods have been used to solve this problem.

There are many real-world manufacturing and operational systems that increase their reliability through the concepts of RAP, which can be counted, including aircraft engines, the number of pumps at a water pumping station, and so on. Considering the nature of these manufacturing and operating systems, many hypotheses and limitations have been added to the RAP to draw the problem closer to the real-world conditions.

Therefore, researchers categorized this problem in different aspects, including categorization based on the functional status of the components (binary or multi-state), the type of component failure rate (constant or time-dependent), components configuration in the system (active or standby).

Considering the importance of the system's reliability and system's cost in this problem, in many studies, both objectives considered as objective functions, and this problem is transformed into a two-objective problem (and even more than two).

In this paper, we investigate a multi-objective RAP whose failure rates depend on the number of working elements. The subsystems are k-out-of-n, and the failure rate of components can be reduced with spending money. The objective functions of the model are maximizing system reliability and minimizing system weight. The type of each subsystem component and subsystem redundancy strategies are the system variables. Since the model is Np-Hard, we used Non-Dominated Sorting Genetic Algorithm II (NSGA-II) and Non-Dominated Ranked genetic algorithm (NRGA) algorithms for solving the presented model.

This paper organized as follows: In Section 2, we present a literature review to confirm that there exist no studies that exactly meet this research conditions. In Section 3, we discussed the mathematical model and system assumptions. In Section 4, the NSGA-II and NRGA algorithms are presented. In Section 5, a numerical example is presented to compare the algorithms results. Section 6 is the managerial insights, and the final Section deals with the conclusion and further studies.

2. Literature review

In real-world problems, many parameters affect the system's reliability. One of the most important ones is the failure rate of the components. This parameter in RAP studies has two categories: constant failure rate (CFR) and time-dependent failure rate.

Regarding CFR models, Misra and Sharma [3] presented a RAP model with the choice of allocating identical components to each subsystem and active redundancy strategy, then solved the presented model with zero-one programming. Ida [4] used a genetic algorithm to solve RAP with multi-failure components. Coit and Smith [5] presented a RAP model with the choice of allocating non-identical elements to each subsystem and solved the presented model using dynamic programming, integer programming, mixed non-linear integer programming, and compared the results with those of genetic algorithm. Coit and Liu [6] were the first who worked on a system with k-out-of-n sub-systems. They predefined active or cold standby redundancy strategies for each subsystem and solved the problem using integer programming. Hsieh and You [7] presented a new two-stage method based on the immune algorithm to solve the RAP under non-linear weight, volume, and cost constraints. Hsieh and Yeh [8] used penalty guided bees search to solve RAP.

All presented studies have been conducted on single-objective models. Busacca Marseguerra, Zio [9] presented a two-objective model. The model objectives were maximizing the system reliability and net profit and used multi-objective genetic algorithm to solve the problem. Coit and Jin [10] worked on a multi-objective RAP (MORAP) with maximizing system reliability and minimizing the variance of results. Baharanwala, Coit, and Kulturel-Konak [11] used NSGA to solve the MORAP. The objectives of the model are maximizing system reliability, minimizing the system cost, weight, and variance of reliability. In the presented model, each subsystem has a lower and upper limit for components allocation. Salazar, Rocco, and Galván [12] solved three different RAP models using NSGA-II. The objective functions of the models are maximizing system reliability and minimizing the system cost. Kulturel-Konak, Coit, and Baheranwala [13] worked on a MORAP model using the Tabu search algorithm. Taboada, Baheranwala, Coit et al. [14] presented two methods for decreasing the size of Pareto solutions and used the presented method for solving MORAP. Taboada and Coit [15] presented an MOEA evolutionary algorithm to solve the MORAP with a new crossover operator that increased the variations of the solutions. Liang and Lo [16] presented a variable neighborhood search for solving MORAP and solved three different models and compared the results with NSGA-II results. Soylu and Ulusoy [17] worked on MORAP and contributed Pareto solutions to small- and large-scale problems. Then they classified the solutions using the UTADIS method.

Concerning time-dependent failure rate models, Coit [18] presented a new RAP with a switching system. Also, he used the K-Erlang distribution function for components and solved the problem using dynamic programming. Later on, Coit [19] considered two different active and cold standby redundancy strategies for each subsystem. Tavakkoli-Moghaddam, Safari, and Sassani [20], and Safari, Tavakkoli-Moghaddam [21] solved that problem using a genetic algorithm in 2008 and a memetic algorithm in 2010. Amari [22] presented an enumeration method for evaluating the reliability of k-out-of-n systems with a cold standby redundancy strategy. Dhingra [23] used mixed goal programming and goal attainment to produce Pareto optimal solutions in a fourth level system to maximizing reliability and minimizing system costs and weight. Ghafarian Salehi Nezhad, Eshraghniaye Jahromi, Salmani et al. [24] presented a four-phase algorithm to improve reliability in series-parallel systems with redundancy allocation. They combined an Ant Colony Optimization (ACO) algorithm as a meta-heuristic phase, and three other heuristics to develop a solving methodology for RAP. Azaron, Katagiri, Kato et al. [25] used the shortest path method in stochastic graphs to evaluate the reliability of a cold standby system with non-identical elements. Azaron, Perkgoz, Katagiri et al. [26] salved a multi-state cold standby RAP and non-repairable components with the GADSCRRSU method. Ebrahim Nezhad, Maleki, Pasandideh et al. [27] presented a new method for MORAP with the choice of allocating identical elements and a predefined active and cold standby redundancy strategies with maximizing reliability and net profit objective functions. Chambari, Najafi, Rahmati et al. [28] used multi-objective particle swarm optimization and NSGA-II for a multi-objective RAP with the choice of selecting the redundancy strategy of each subsystem. Azimi, Hemmati, and Chambari [29] Solved a RAP with k-out-of-n configuration and non-exponential repairable components. They used optimization via simulation technique to solve the presented RAP. Pourkarim Guilani, Zaretalab, and Niaki [30] worked on a bi-objective reliability model with three-state components. They used multi-objective Strength Pareto Evolutionary Algorithm (SPEA-II) and Non-Dominated Sorting Genetic Algorithm (NSGA-II) to solve the presented model.

In all mentioned research studies, the failure rate of working elements is fixed. For example, for a system with one working component, the failure rate of this component is equal to the failure rates of components in a system with 10 working components. Sharifi, Memariani, and Noorossana. [31] presented a formula for evaluating the reliability of a k-out-of-n system when the components failure rates depend on the number of working elements. In the presented k-out-of-n system, when a component failed, the failure rates of remaining components increases.

Table 1 contains several recent studies on the reliability field, along with a summary of the model behaviors.

Insert Table 1 here

3. Mathematical model

In this paper, the system has s k-out-of-n sub-systems, and its objectives are maximizing system reliability and minimizing system weight under two linear constraints.

3.1. Model assumptions

- The system has s subsystems,
- Each subsystem is k-out-of-n,
- subsystem components are identical,
- Components are binary states,
- The probability of working switching system for cold standby subsystems is equal to p,
- The failure rate of working components depends on the number of working components,
- The components are non-repairable, and
- The system parameters are constant.

3.2. Nomenclature

s: Number of subsystems,

i: Subsystem index,

k_i: Minimum necessary components in i^{th} subsystem,

n_i: Number of components in i^{th} subsystem,

n: The set of n_i, $(n_1, \dots, n_s)$,

$n_{Max,i}$: The upper limit of n_i, $(n_i \leq n_{Max,i}; i = 1, \dots, s)$,

m_i: Number of available component types in i^{th} subsystem, $(i = 1, \dots, s)$,

z_i: Component type index in i^{th} subsystem, $z_i \in (1, \dots, m_i)$,

z: Set of $z_i \in (1, \dots, m_i)$

t: Mission time,

$\lambda_{iz_ik_i}$: The failure rate of type z_i component in i^{th} subsystem when the subsystem working with k components,

$\acute{\lambda}_{iz_ik_i}$: The reduced failure rate of type z_i component in i^{th}subsystem when the subsystem working with k components,

w_{iz_i}: Weight of component, type z_i in i^{th} subsystem,

c_{iz_i}: Cost of component, type z_i in i^{th} subsystem,

$\acute{c}_{iz_i}$: Cost of reducing the failure rate of each component, type z_i in i^{th} subsystem,

θ_{iz_i}: Cost parameter of internal relation for component, type z_i in i^{th} subsystem,

β_{iz_i}: Reducing factor of failure rate for type z_i component in i^{th} subsystem,

W: Total acceptable weight of the system,

A: Index of subsystems with active redundancy strategy,

S: Index of subsystems with cold standby redundancy strategy,

R_{l_1}: Reliability of the subsystem with active redundancy strategy,

R_{l_2}: Reliability of the subsystem with cold standby redundancy strategy,

R_l: System reliability

P: Probability of working the switch in switching time,

3.3. Mathematical model

In the presented mathematical model in this paper, the reliability and cost of the system is designed to be optimized simultaneously. One of the most important system's constraints in the redundancy allocation problems is the system's weight constraint. This constraint is considered in the proposed mathematical model. Another considered constraint is the upper bound for reducing of components failure rates. Given these explanations, the decision variables of this problem are the number of components in each subsystem and reducing the factor of failure rate for all of the components in each subsystem.

$$MaxR_l(t) = \left\{\prod_{i\in A} R_{l_1}(t)\right\} \times \left\{\prod_{i\in S} R_{l_2}(t)\right\} \tag{1}$$

$$MinC = \sum_{i=1}^{s} \left\{c_{iz_i}\left\{n_i + f\left(\theta_{iz_i}\right)\right\} + c'_{iz_i} f(\beta_i)\right\} \tag{2}$$

$s.t:$

$$\sum_{i=1}^{s} w_{iz_i} n_i \le W \tag{3}$$

$$0 \le \beta_i \le a_i \tag{4}$$

$$n_i \in (k_i, 2, \ldots, n_{\max}); \quad i = (1,2,\ldots,s) \tag{5}$$

$$z_i \in (1,2,\ldots,m_i); \quad i = (1,2,\ldots,s) \tag{6}$$

In this model, Equation (1) is the first objective function, i.e., maximizing the system reliability. The Equation (2) is the second objective function, i.e., minimizing the system cost. We consider that the system cost contains the cost of redundant components, cost of internal relation (Wang, Chen, Tang et al. [62]), and the cost of reducing the components failure rates. In this paper, $f(\beta_i)$ and $f(\theta_{iz_i})$ are defined as follows:

$$f(\beta_i) = e^{\beta_i \lambda_{iz_i}}, \quad f(\theta_{iz_i}) = e^{\theta_{iz_i} n_i} \tag{7}$$

The Equation (3) is the constraint of system weight. The Equation (4) is the upper and lower limits of reducing components failure rate and Equations (5) and (6) define the maximum number and type of components in each subsystem, respectively. The first objective function is divided into two parts. The first part is the reliability of the subsystems with an active redundancy strategy, and the second part is the reliability of the subsystems with a cold standby redundancy strategy.

The reliability of a system with n identical component and active redundancy strategy when the failure rate of the component is related to the number of the working component can be calculated as follows (Sharifi, Memariani, and Noorossana [31]):

$$R(t) = \left(\prod_{j=k}^{n} \lambda_j \right) \times \sum_{i=k}^{n} \left\{ \frac{n!}{i(k-1)!} \left(\prod_{\substack{\theta=k \\ \theta \ne i}}^{n} \frac{1}{\theta \times \lambda_\theta - i \times \lambda_i} \right) \times \frac{e^{-i \times \lambda_i \times t}}{\lambda_i} \right\} \tag{8}$$

In Equation (7), λ_i is the failure rate of components when the system is working with i components. In a real situation, when a component fails, the load on other working components increases. The equation (9) makes a relation between the failure rates of working components (Sharifi, Memariani, and Noorossana [31])

$$\lambda_k = \frac{k - \gamma(k-1)}{k} \lambda_1 \tag{9}$$

In Equation (9), $0 \le \gamma \le 1$ can tune the relations between failure rates of the component. When $\gamma = 0$ the failure rate of working components is independent of the number of working components and when $\gamma = 1$ the failure rate of working components is $\lambda_k = \lambda_1/k$. For the presented model, the Equation (9) is transformed into Equation (10).

$$\lambda_{iz_ik_i} = \frac{k_i - \gamma(k_i - 1)}{k_i}\lambda_{iz_i1} \tag{10}$$

We combined the equations (8) and (10), so, the reliability of the systems with active redundancy strategy, and the failure rate depends on the number of working elements can be calculated as follows:

$$R_{l_1}(t) = \sum_{i=k_i}^{n_i} P_i(t) = \left(\prod_{j=k_i}^{n_i} \frac{j - \gamma(j-1)}{j}\lambda_{iz_i1}\right) \times \sum_{i=k_i}^{n_i}\left[\frac{n_i!}{i(k_i-1)!}\left(\prod_{\substack{\omega=k_i\\ \omega\neq i}}^{n_i} \frac{1}{\{\{\omega-\gamma(\omega-1)\}-\{i-\gamma(i-1)\}\}\lambda_{iz_i1}}\right) \times \frac{e^{-\{i-\gamma(i-1)\}\lambda_{iz_i1}t}}{\{i-\gamma(i-1)\}\lambda_{iz_i1}}\right] \tag{11}$$

The reliability formula for a cold standby subsystem is presented in Equation (12); in these subsystems, the switch detects the failures of a working component and changes the failed component with a new one. The switching system is a discrete detection switching and may work in each detection by the probability p and maybe failed in each detection by the probability $(1-p)$.

$$R_{l_2}(t) = \sum_{j=0}^{n_i-k_i-1}\left\{(1-p)p^j \sum_{m=0}^{j} \frac{e^{-k_i\lambda_{iz_ik_i}t}.(k_i\lambda_{iz_ik_i}t)^m}{m!}\right\} + p^{n_i-k_i}\sum_{m=0}^{n_i-k_i}\frac{e^{-k_i\lambda_{iz_ik_i}t}.(k_i\lambda_{iz_ik_i}t)^m}{m!} \tag{12}$$

Since RAP belongs to Np-Hard problems and the first objective function of this problem is non-linear, the exact solutions have less efficiency in solving this problem. So, we used the NSGA-II and NRGA metaheuristic algorithms to solve the presented model. These algorithms are presented in the next section.

4. Solving methods

We used NSGAII and NRGA algorithms to solve the presented problem. These two algorithms are based on population, so the solution structures of both algorithms are the same.

4.1.Solution encoding

Each solution is a $4 \times s$ matrix. The i^{th} column of the matrix belongs to the i^{th} subsystem. The first row of the matrix represents the redundancy strategy of the subsystem components; the second row shows the component type of the subsystem. The third part contains the number of components in each subsystem, and the last row is the failure rate reduction coefficient of the component in the subsystem. The structure of an encoded solution is in Figure 1. In this figure, in the first subsystem, 4 components of type three exist, and the components have an active redundancy strategy. Also, the failure rate of the components in this subsystem reduced by 18.66%.

Insert Figure 1 here

4.2.NSGAII algorithm

In 1994, Srinivas and Deb [63] used Goldberg ideas and presented the concepts of NSGA. This algorithm is an efficient but too complicated algorithm to solve the multi-objective problem. Deb, Agrawal, Pratap et al. [64] presented the NSGAII algorithm to overcome the weakness of the NSGA algorithm regarding particle election and the complexity. In this algorithm, the Pareto solutions are obtained using dominant and non-dominant solutions and the mutation operator of the genetic algorithm used to find the new solutions.

4.3.NRGA algorithm

Improvement of operators is a way to improve the efficiency of multi-objective algorithms. Improvement of selection operators has more effects on the improvement of algorithms efficiency and makes the evolutionary algorithms more converging. So, Al Jadaan, Rajamani, and Rao [65] improved an evolutionary multi-objective algorithm based on Ranked based roulette wheel selection and Pareto-based population ranking and called it NRGA. In this combination, a two-layer ranking is presented based on roulette wheel selection that randomly selects the new generation from old ones based on selecting the best solutions (based on fitness and span). The NRGA can better achieve a wide range of solutions and converge to optimal Pareto versus other evolutionary algorithms.

In both algorithms, the solutions in each population rank are based on its non-dominant rate. The solutions in the first category are the best non-dominant solutions, and the solutions in the last category are the worst non-dominant solutions. So, the solutions in the first category have the maximum fitting, and the solutions in the last category have the minimum fitting. After ranking the categories, the solutions in each category rank are based on the swarm. The solution with maximum swarm has the maximum rank, and the solution with minimum swarm has the minimum rank in the category. NSGAII and NRGA differ in selecting a strategy, ordering the population, and selecting for the next generation. NRGA used RRWS instead of the tournament operator. In this operator, the solutions with better fitting have a higher chance to be selected for reproduction and creation of the next generation. Al Jadaan, Rajamani, and Rao [65] used a selected modified algorithm based on the roulette wheel in which each solution has the fitting value equal to the rank of the solution in the population. The solutions in the population rank are based on two specifications. First, the rank of the containing category of the solution and second the rank of the solution in the category. For selecting a solution, at first, a non-dominant category must be selected. The probability of selecting the i^{th} non-dominant category is calculated using Equation (13) (Al Jadaan, Rajamani, and Rao [65]).

$$p_i = \frac{2 \times rank_i}{N_f \times (N_f + 1)} = \frac{rank_i}{\sum_{i=1}^{p} rank_i} \tag{13}$$

In this Equation, $rank_i$ is the rank of i^{th} category and N_f is the number of the categories. The probability of selecting j^{th} solution in i^{th} non-dominant category is calculated using equation number (14) (Al Jadaan, Rajamani, and Rao [65]).

$$p_{ji} = \frac{2 \times rank_{ji}}{N_j \times (N_j + 1)} = \frac{rank_{ji}}{\sum_{j=1}^{p} rank_{ji}} \tag{14}$$

In this Equation, N_j is the number of solutions in i^{th} category and $rank_{ji}$ is the rank of j^{th} solution in i^{th} category.

In the roulette wheel, the first two real intervals $[0, S_1]$ and $[0, S_2]$ values $S_1 = \sum_{i=1}^{n} p_i$ and $S_2 = \sum_{j=1}^{m} p_j$ are defined. Then the solutions in each category occupy a certain amount of $[0, S_1]$ and $[0, S_2]$ based on the probability of their selection. Then two random numbers are selected between zero and one, and the first random number is used to select in, $[0, S_1]$ and the second random number is used to select an answer in $[0, S_2]$. The flow chart of both algorithms is presented in Figure 2.

Insert Figure 2 here

4.4. Comparison Metrics

Convergence with Pareto optimal solutions and providing density and diversity among the set of solutions are two distinct and somewhat conflicting objectives in multi-objective evolutionary algorithms, a criterion that can be used alone and does not exist in absolute terms for calculating the performance of the algorithm (Deb [66]).

For this reason, we used five performance metrics to evaluate the performance of the two presented algorithms better.

4.4.1. Maximum Spread or Diversity

This index measures the length of the space cubic diameter used by the final values of the targets for the non-dominant solutions. Equation (15) shows the computational procedure of this index. Therefore, the larger this criterion, the more spreading of the archived Pareto fronts.

$$D = \sqrt{\sum_{j=1}^{m} \left(\underset{i}{Max}\ f_i^{\,j} - \underset{i}{Min}\ f_i^{\,j} \right)^2} \qquad (15)$$

4.4.2. Spacing

This scale calculates the relative distance of consecutive solutions using Equation (16).

$$S = \sqrt{\frac{1}{|n|} \sum_{i=1}^{n} (d_i - \bar{d})^2} \qquad (16)$$

In Equation (16), $\bar{d} = \sum_{i=1}^{n} d_i / |n|$ and $d_i = \min_{k \in n \wedge k \neq 1} \sum_{m=1}^{2} \left| f_m^i - f_m^k \right|$. This scale measures the standard deviations of different values d_i. When the solutions are close to the gathering, S has smaller value, and the algorithms with small spacing scales perform better than other algorithms [66].

4.4.3. Number of Pareto Solution (NPS)

This scale shows the number of Pareto solutions in each algorithm.

4.4.4. Mean Ideal Distance (MID)

This scale indicates the distance to the ideal Pareto level and calculated using Equation (17). The lower values of this scale indicate that the algorithm is working properly.

$$MID = \frac{\sum_{i=1}^{n} \sqrt{f_{i1}^2 + f_{i2}^2}}{n} \qquad (17)$$

In Equation (15), f_{i1} and f_{i2} are the first and second objective functions in i^{th} solution.

4.4.5. Time of algorithm

This scale defines the time of algorithm running to satisfy stop criteria.

5. Numerical example

In this section, we present a numerical example to illustrate the effectiveness of the presented algorithms. The example data are obtained from the data of Coit and Smith [5]. The example is a series-parallel system with the k-out-of-n subsystem. Three or four different component types are available for each subsystem, and the redundancy strategy of each subsystem is a variable. The cost, weight, and failure rate of components and a minimum number of components in each subsystem are presented in Table 2. The maximum number of each subsystem component is six. The objectives are maximizing system reliability and minimizing system cost under, and weight constraint (Coit and Liu [6]). Also, the switch reliability is considered as $p = 0.99$ (Coit [19]). The internal connection cost for all subsystems is $\theta_{iz_i} = 0.25$ (Wang, Chen, Tang et al. [62]) and $\gamma = 0.2$, also the cost of reducing components failure rate is $0.75C_{iz_i}$.

Insert Table 2 here

5.1. Parameter tuning

The parameters of NRGA and NSGA-II algorithms are tuned in this section. The RSM is used for parameter tuning. These parameters are population size ($nPop$), crossover rate (P_c), mutation rate (P_{m_1}), and max-min operator(P_{m_2}). Table 3 presents the range of these parameters and Table 4 the results of parameter tuning.

Insert Table 3 here

Insert Table 4 here

5.2. Results

This section deals with comparing the results of NSGA-II and NRGA algorithms. For this comparison, we used a Laptop with $6G$ RAM and 1.73 GH CPU speed, and the algorithm codded using MATLAB 2018. Each algorithm ran five times with the optimal values of parameters. The iteration of each algorithm was considered 100. The results of the algorithm performance were presented in Figure 3. In NPS, concerning diversity and time scales, the NSGA-II has a better performance than those in the NRGA algorithm, and in other scales, NRGA is better than NSGA-II.

Insert Figure 3 here

5.2.1. Results analysis

To find the difference between the results of the indexes, we used a one-way ANOVA test using $1 - \alpha = 0.95$. Table 5 presents the results for the ANOVA test.

Insert Table 5 here

ANOVA test results indicate a meaningful difference between the indexes for two algorithms and reject the hypothesis that the results of two algorithms work the same.

5.2.2. Sensitivity analysis

To further evaluate the model in this section, we intend to solve 33 different numerical examples for the presented multi-objective model using NSGAII and NRGA algorithms. The information for these 33 numerical examples is similar to Table 2. However, the available weight of the system varies from 159 to 191. The result of the performance indices of each algorithm on these 33 numerical examples are graphically illustrated in Figure 4.

Insert Figure 4 here

6. Managerial insights

Increasing the number of parallel components in a system can promote the reliability of this system, but it is not sufficient. Related researches show that improving the performance of each used component in a system can be utilized as another way to improve the reliability of the entire system. Therefore, in this paper, both of these criteria are employed to improve the performance of the system. It should be considered that the required parameters for designing the problem are formed based on the effects of each approach mentioned above and their related costs. On the other hand, if there is a budget capacity constraint, effort in promoting components' availability leads to a less redundancy assignment, and vice versa.

Many realistic examples of failure rate improvement are given in the literature. For instance, consider the action of the installation of a vibration monitoring system for the FD fan and the ID fan. Vibration monitoring can monitor the health condition of the fans, and preventive replacements can be performed to prevent unexpected failures. Therefore, the failure rates of the fans, and thus the failure rate of the generating unit from the 300MW state to 150MW state, can be reduced. Before adopting the vibration monitoring system, the reduction in the failure rates of the fans, i.e., the benefit of installing the vibration monitoring system, can be estimated based on the failure histories, or by the vibration monitoring system provider based on their experiences. Other examples are: installing monitor systems and maintenance planning.

7. Conclusion and further studies

Many parameters affect the reliability of the systems, and the failure rate of components is one of the most important parameters. In this paper, we work on a two-objective reliability model. In this model, the failure rate of the components is constant and depends on the number of working components in the system. The components failure rates may decrease spending some money. The system contains s subsystems, and the subsystems have the k-out-of-n configuration. All subsystems may have two active and cold standby redundancy strategies that are system variables. Besides, the number and the type of each subsystem components and the reduction of the failure of the component rate are other variables of the model. Because RAP belongs to the Np-Hard problem, we used

NSGA-II and NRGA multi-objective algorithms for solving the presented problem. Also, we used 5 different indexes for comparing the algorithm performance. The results showed that NSGA-II has better performance in NPS, diversity, and time indexes, and for other indexes, the NRGA algorithm has better performance.

For further studies, one can consider the components as repairable components. Also, the combination of different components in each subsystem can be considered a new idea. Also, another multi-objective metaheuristic algorithm can be considered for solving the presented problem.

8. References

[1] Fyffe, D. E., Hines, W. W., & Lee, N. K. "System reliability allocation and a computational algorithm", IEEE Transactions on Reliability, 17(2), pp. 64-69 (1968).

[2] Chern, M. S. "On the computational complexity of reliability redundancy allocation in a series system", Operations research letters, 11(5), 309-315 (1992).

[3] Misra, K. B., & Sharma, U. "Multi-criteria optimization for combined reliability and redundancy allocation in systems employing mixed redundancies", Microelectronics Reliability, 31(2-3), pp. 323-335 (1991).

[4] Ida, K. "System reliability optimization with several failure modes by genetic algorithm", In Proceeding of 16th International Conference on Computers. & Industrial Engineering, pp. 349-352 (1994).

[5] Coit, D. W., & Smith, A. E. "Optimization approaches to the redundancy allocation problem for series-parallel systems", In Fourth Industrial Engineering Research Conference Proceedings, pp. 342-349 (1995).

[6] Coit, D. W., & Liu, J. C. "System reliability optimization with k-out-of-n subsystems", International Journal of Reliability, Quality and Safety Engineering, 7(02), pp. 129-142 (2000).

[7] Hsieh, Y. C., & You, P. S. "An effective immune based two-phase approach for the optimal reliability–redundancy allocation problem", Applied Mathematics and Computation, 218(4), pp. 1297-1307(2011).

[8] Hsieh, T. J., & Yeh, W. C. "Penalty guided bees search for redundancy allocation problems with a mix of components in series–parallel systems", Computers & Operations Research, 39(11), pp. 2688-2704 (2012).

[9] Busacca, P. G., Marseguerra, M., & Zio, E. "Multi-objective optimization by genetic algorithms: application to *safety systems",* Reliability Engineering & System Safety, **72**(1), pp. 59-74 (2001).

[10] Coit, D. W., & Jin, T. "Multi-criteria optimization: Maximization of a system reliability estimate and minimization of the estimate variance", In *Proceedings of the 2001 European Safety & Reliability International Conf.(ESREL), Turin, Italy,* (2001).

[11] Baheranwala, F., Coit, D. W., & Kulturel-Konak, S. "Solution of stochastic multi-objective system design problems" In *IIE Annual Conference. Proceedings,* (2004).

[12] Salazar, D., Rocco, C. M., & Galván, B. J. "Optimization of constrained multiple-objective reliability problems using evolutionary algorithms", *Reliability Engineering & System Safety*, ***91***(9), pp. 1057-1070 (2006).

[13] Kulturel-Konak, S., Coit, D. W., & Baheranwala, F. "Pruned Pareto-optimal sets for the system redundancy allocation problem based on multiple prioritized objectives", *Journal of Heuristics*, ***14***(4), pp. 335 (2008).

[14] Taboada, H. A., Baheranwala, F., Coit, D. W., & Wattanapongsakorn, N. "Practical solutions for multi-objective optimization: An application to system reliability design problems", *Reliability Engineering & System Safety*, *92*(3), pp. 314-322 (2007).

[15] Taboada, H. A., & Coit, D. W. "Development of a new multiple objective prioritized genetic algorithm" In *IIE Annual Conference. Proceedings,* pp. 176 (2008

[16] Liang, Y. C., & Lo, M. H. "Multi-objective redundancy allocation optimization using a variable neighborhood search algorithm", *Journal of Heuristics*, ***16***(3), pp. 511-535 (2010).

[17] Soylu, B., & Ulusoy, S. K. "A preference ordered classification for a multi-objective max–min redundancy allocation problem", *Computers & Operations Research*, ***38***(12), pp. 1855-1866 (2011).

[18] Coit, D. W. "Cold-standby redundancy optimization for non-repairable systems", *Iie Transactions*, ***33***(6), pp. 471-478 (2001).

[19] Coit, D. W. "Maximization of system reliability with a choice of redundancy strategies", *IIE transactions*, ***35***(6), pp. 535-543 (2003).

[20] Tavakkoli-Moghaddam, R., Safari, J., & Sassani, F. "Reliability optimization of series-parallel systems with a choice of redundancy strategies using a genetic algorithm", *Reliability Engineering & System Safety*, ***93***(4), pp. 550-556 (2008).

[21] Safari, J., & Tavakkoli-Moghaddam, R. "A redundancy allocation problem with the choice of redundancy strategies by a memetic algorithm", *Journal of Industrial Engineering, International*, ***6***(11), pp. 6-16 (2010).

[22] Amari, S. V. "Reliability of k-out-of-n standby systems with gamma distributions", In *2012 Proceedings Annual Reliability and Maintainability Symposium,* pp. 1-6 (2012).

[23] Dhingra, A. K. "Optimal apportionment of reliability and redundancy in series systems under multiple objectives", *IEEE Transactions on reliability*, ***41***(4), pp. 576-582 (1992).

[24] Ghafarian Salehi Nezhad, A., Eshraghniaye Jahromi, A., Salmani, M. H., & Ghasemi, F. "A four-phase algorithm to improve reliability in series-parallel systems with redundancy allocation", *Scientia Iranica*, ***21***(3), pp. 1072-1082 (2014).

[25] Azaron, A., Katagiri, H., Kato, K., & Sakawa, M. "Reliability evaluation and optimization of dissimilar-component cold-standby redundant systems", *Journal of the Operations Research Society of Japan*, ***48***(1), pp. 71-88 (2005).

[26] Azaron, A., Perkgoz, C., Katagiri, H., Kato, K., & Sakawa, M. "Multi-objective reliability optimization for dissimilar-unit cold-standby systems using a genetic algorithm", *Computers & Operations Research*, ***36***(5), pp. 1562-1571. (2009).

[27] EbrahimNezhad, M., Maleki, V. B., Pasandideh, H. R., & Safari, J. "Increasing the reliability and the profit in a redundancy allocation problem", International Journal of Applied Operational Research, **1**(2), pp. 57-64 (2011).

[28] Chambari, A., Najafi, A. A., Rahmati, S. H. A., & Karimi, A. "An efficient simulated annealing algorithm for the redundancy allocation problem with a choice of redundancy strategies", *Reliability Engineering & System Safety*, *119*, pp. 158-164 (2013).

[29] Azimi, P., Hemmati, M., & Chambari, A. "Solving the redundancy allocation problem of k-out-of-n with non-exponential repairable components using optimization via simulation approach", *Scientia Iranica*, ***24***(3), pp. 1547-1560 (2017).

[30] Guilani, P. P., Zaretalab, A., & Niaki, S. T. A. "A bi-objective model to optimize reliability and cost of k-out-of-n series-parallel systems with tri-state components", *Scientia Iranica. Transaction E, Industrial Engineering*, ***24***(3), pp. 1585-1602 (2017).

[31] Sharifi, M., Memariani, A., & Noorossana, R. "Real time study of a k-out-of-n system: n Identical Elements with Constant Fuzzy Failure", *World applied sciences journal*, ***8***(9), pp. 1136-1143 (2010).

[32] Sharifi, M., Ganjian, M., & Ghajar, H. R. "Expansion of Reliability Models based on Markov Chain with Consideration of Fuzzy Failure Rates: System with two Parallel and Identical Elements with Constant Failure Rates", In *IEEE International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'06),* 2, pp. 1022-1026 (2005).

[33] Lins, I. D., & Droguett, E. L. "Redundancy allocation problems considering systems with imperfect repairs using multi-objective genetic algorithms and discrete event simulation", *Simulation Modelling Practice and Theory*, ***19***(1), pp. 362-381 (2011).

[34] Ouzineb, M., Nourelfath, M., & Gendreau, M. "Tabu search for the redundancy allocation problem of homogenous series–parallel multi-state systems", *Reliability Engineering & System Safety*, ***93***(8), pp. 1257-1272 (2008).

[35] Sharma, V. K., & Agarwal, M. "Ant colony optimization approach to heterogeneous redundancy in multi-state systems with multi-state components", In *8th International IEEE Conference on Reliability, Maintainability and Safety*, pp. 116-121 (2009).

[36] Lins, I. D., & Droguett, E. L. "Multi-objective optimization of availability and cost in repairable systems design via genetic algorithms and discrete event simulation", *Pesquisa Operacional*, ***29***(1), 43-66 (2009).

[37] Ouzineb, M., Nourelfath, M., & Gendreau, M. "A heuristic method for non-homogeneous redundancy optimization of series-parallel multi-state systems", *Journal of Heuristics*, ***17***(1), 1-22 (2011).

[38] Ebrahimipour, V., & Sheikhalishahi, M. "Application of multi-objective particle swarm optimization to solve a fuzzy multi-objective reliability redundancy allocation problem", In *IEEE International Systems Conference*, pp. 326-333 (2011).

[39] Lins, I. D., & Droguett, E. L. "Redundancy allocation problems considering systems with imperfect repairs using multi-objective genetic algorithms and discrete event simulation", *Simulation Modelling Practice and Theory*, ***19***(1), pp. 362-381 (2011).

[40] Garg, H., & Sharma, S. P. "Multi-objective reliability-redundancy allocation problem using particle swarm optimization", *Computers & Industrial Engineering*, ***64***(1), pp. 247-255 (2013).

[41] Garg, H., Rani, M., & Sharma, S. P. "An efficient two phase approach for solving reliability–redundancy allocation problem using artificial bee colony technique", Computers & operations research, 40(12), pp. 2961-2969 (2013).

[42] Levitin, G., Xing, L., Ben-Haim, H., & Dai, Y. "Reliability of series-parallel systems with random failure propagation time", IEEE Transactions on Reliability, 62(3), pp. 637-647 (2013).

[43] Maatouk, I., Châtelet, E., & Chebbo, N. "Availability maximization and cost study in multi-state systems" In IEEE Proceedings Annual Reliability and Maintainability Symposium (RAMS), pp. 1-6 (2013).

[44] Gago, J., Hartillo, I., Puerto, J., & Ucha, J. M. "Exact cost minimization of a series-parallel reliable system with multiple component choices using an algebraic method", Computers & operations research, 40(11), pp. 2752-2759 (2013).

[45] Ebrahimipour, V., Asadzadeh, S. M., & Azadeh, A. "An emotional learning-based fuzzy inference system for improvement of system reliability evaluation in redundancy allocation problem", The International Journal of Advanced Manufacturing Technology, 66(9-12), pp. 1657-1672 (2013).

[46] Liu, Y., Huang, H. Z., Wang, Z., Li, Y., & Yang, Y. "A joint redundancy and imperfect maintenance strategy optimization for multi-state systems", IEEE Transactions on Reliability, 62(2), pp. 368-378 (2013).

[47] Khalili-Damghani, K., Abtahi, A. R., & Tavana, M. "A decision support system for solving multi-objective redundancy allocation problems", Quality and Reliability Engineering International, 30(8), pp. 1249-1262 (2014).

[48] Guilani, P. P., Sharifi, M., Niaki, S. T. A., & Zaretalab, A. "Reliability evaluation of non-reparable three-state systems using Markov model and its comparison with the UGF and the recursive methods", Reliability Engineering & System Safety, pp. 129, 29-35 (2014).

[49] Sharifi, M., Cheragh, G., Maljaii, K. D., Zaretalab, A., Daei, F., & Vahid, A. "Reliability Optimization of a Series-Parallel k-out-of-n System with Failure Rates Depends on Working Component of System, International Journal of Industrial Engineering, 22(4), pp. 438-453(2015).

[50] Mousavi, S. M., Alikar, N., Niaki, S. T. A., & Bahreininejad, A. "Two tuned multi-objective meta-heuristic algorithms for solving a fuzzy multi-state redundancy allocation problem under discount strategies", Applied Mathematical Modelling, 39(22), pp. 6968-6989 (2015).

[51] Zaretalab, A., Hajipour, V., Sharifi, M., & Shahriari, M. R. "A knowledge-based archive multi-objective simulated annealing algorithm to optimize series–parallel system with choice of redundancy strategies", Computers & Industrial Engineering, 80, pp. 33-44 (2015).

[52] Miriha, M., Niaki, S. T. A., Karimi, B., & Zaretalab, A. "Bi-objective reliability optimization of switch-mode k-out-of-n series–parallel systems with active and cold standby components having failure rates dependent on the number of components", Arabian Journal for Science and Engineering, 42(12), pp. 5305-5320 (2017).

[53] Pourkarim Guilani, P., Azimi, P., Sharifi, M., & Amiri, M. "Redundancy allocation problem with a mixed strategy for a system with k-out-of-n subsystems and time-dependent failure rates based on Weibull distribution: An optimization via simulation approach", Scientia Iranica, 26(2), pp. 1023-1038 (2019).

[54] Sharifi, M., Saadvandi, M., & Shahriari, M. R. "Presenting a series-parallel redundancy allocation problem with multi-state components using recursive algorithm and meta-heuristic", Scientia Iranica, 27(2), pp. 970-982 (2020).

[55] Zaretalab, A., & Hajipour, V. "An Extended Simulated Annealing Based on the Memory Structure to Solve Redundancy Allocation Problem", Journal of Advanced Manufacturing Systems, 18(04), pp. 527-548 (2019).

[56] Sharifi, M., Shahriari, M. R., & Zaretalab, A. "The Effects of Technical and Organizational Activities on Redundancy Allocation Problem with Choice of Selecting Redundancy Strategies Using the memetic algorithm", International Journal of Industrial Mathematics, 11(3), pp. 165-176 (2019).

[57] Sharifi, M., & Khoshniat, S. "Optimization the Availability of a System with Short Circuit and Common Cause Failures," International Journal of Industrial Mathematics, 11(4), pp. 239-248 (2019).

[58] Sharifi, M., Moghaddam, T. A., & Shahriari, M. "Multi-objective Redundancy Allocation Problem with weighted-k-out-of-n subsystems", Heliyon, 5(12), e02346 (2019).

[59] Alamdari, A. B., & Sharifi, M. "Solving a Joint Availability-Redundancy Optimization Model with Multistate Components and Metaheuristic Approach", International Journal of Industrial Mathematics, 12(1), pp. 59-70 (2020).

[60] Zaretalab, A., Hajipour, V., & Tavana, M. "Redundancy allocation problem with multi-state component systems and reliable supplier selection", Reliability Engineering & System Safety, 193, 106629 (2020).

[61] Sharifi, M., & Taghipour, S. "Optimizing a Redundancy Allocation Problem with Open-Circuit and Short-Circuit Failure Modes in Components and Subsystems Level", Journal of Engineering Optimization, In press.

[62] Wang, Z., Chen, T., Tang, K., & Yao, X. "A multi-objective approach to redundancy allocation problem in parallel-series systems", In 2009 IEEE Congress on Evolutionary Computation, pp. 582-589 (2009).

[63] Srinivas, N., & Deb, K. "Multi-objective optimization using non-dominated sorting in genetic algorithms", Evolutionary computation, 2(3), pp. 221-248 (1994).

[64] Deb, K., Agrawal, S., Pratap, A., & Meyarivan, T. "A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In International conference on parallel problem solving from nature", Springer, Berlin, Heidelberg, pp. 849-858 (2000).

[65] Al Jadaan, O., Rajamani, L., & Rao, C. R. "Non-dominated ranked genetic algorithm for Solving multi-objective optimization Problems. In NRGA", Journal of Theoretical and Applied Information Technology, pp. 60-67 (2008).

[66] Deb, K. "Multi-objective optimization using evolutionary algorithms", John Wiley & Sons, Vol. 16, (2001).

Table 1.
Some of the recent studies on the reliability area.

Authors	Year	State	Elements type	Algorithm	Fuzzy	Repairable	Penalty function	Objective	Parameter setting	Failure rate
Sharifi, Ganjian, and Ghajar [32]	2005	Binary	Homogeneous	Markov model	✓	-	-	Single	-	Constant
Lins and Droguett [33]	2008	Binary	Heterogeneous	ACO	-	✓	-	Multiple	No	Constant
Ouzineb et al. [34]	2008	Multi-state	Homogeneous	TS	-	-	-	Single	No	Constant
Sharma and Agarwal [35]	2009	Multi-state	Heterogeneous	ACO	-	-	-	Single	No	Constant
Lins and Droguett [36]	2009	Binary	Heterogeneous	GA	-	✓	-	Multiple	No	Constant
Ouzineb, Nourelfath, and Gendreau. [37]	2011	Multi-state	Heterogeneous	GA	-	-	-	Single	No	Constant
Ebrahimipour and Sheikhalishahi [38]	2011	Binary	Heterogeneous	PSO	✓	-	-	Multiple	No	Constant
Lins and Droguett [39]	2011	Multi-state	Heterogeneous	GA	-	✓	✓	Multiple	No	Constant
Garg and Sharma [40]	2013	Binary	Heterogeneous	PSO	-	-	-	Multiple	No	Constant
Garg, Rani, and Sharma [41]	2013	Binary	Heterogeneous	Bee colony	-	-	✓	Single	No	Constant
Levitin, Xing, Ben-Haim et al. [42]	2013	Multi-state	Heterogeneous	GA	-	-	-	Single	No	Constant
Maatouk, Châtelet, and Chebbo [43]	2013	Multi-state	Heterogeneous	GA	-	✓	-	Single	No	Constant
Chambari, Najafi, Rahmati et al. [28]	2013	Binary	Heterogeneous	SA	-	-	✓	Single	No	Constant
Gago, Hartillo, Puerto et al. [44]	2013	Binary	Heterogeneous	Greedy, Walk back	-	-	-	Single	No	Constant
Ebrahimipour, Asadzadeh, and	2013	Binary	Heterogeneous	Fuzzy inference	-	-	-	Single	No	Constant

Azadeh [45]				system (FIS)						
Liu, Huang, Wang et al. [46]	2013	Multi-state	Heterogeneous	Imperfect repair model	✓	✓	-	Single	No	Constant
Khalili-Damghani, Abtahi, and Tavana [47]	2014	Binary	Heterogeneous	e-constraint	-	-	-	Multiple	No	Constant
Guilani, Sharifi, Niaki et al. [48]	2014	Multi-state	Homogeneous	Markov model	-	-	-	Single	-	Constant
Sharifi, Cheragh, Dashti-Maljaii et al. [49]	2015	Binary	Heterogeneous	GA, MA	-	-	✓	Single	RSM	Time dependent - Number dependent
Mousavi, Alikar, Niaki et al. [50]	2015	Multi-state	Homogeneous	CE-NRGA	✓	-	✓	Multiple	Taguchi	Constant
Zaretalab, Hajipour, Sharifi et al. [51]	2015	Multi-state	Homogeneous	MOSA	-	-	✓	Multiple	-	Constant
Miriha, Niaki, Karimi et al. [52]	2017	Binary	Heterogeneous	NSGAII-MOEA/D	-	-	✓	Multiple	Taguchi	Time-dependent
Poorkaim Guilani, Azimi, Sharifi et al. [53]	2019	Binary	Heterogeneous	Simulation	-	-	✓	Single	-	Constant
Sharifi, Saadvandi, and Shahriari [54]	2019	Multi-state	Homogeneous	GA	-	-	✓	Single	RSM	Constant
Zaretalab and Hajipour [55]	2019	Binary	Heterogeneous	SA	-	-	✓	Single	-	Constant
Sharifi, Shahriari, and Zaretalab [56]	2019	Binary	Heterogeneous	Memetic	-	-	✓	Single	-	Constant
Sharifi and Khoshniat [57]	2019	Binary	Heterogeneous	BBO	-	-	✓	Single	RSM	Constant
Sharifi, Moghaddam, and Shahriari [58]	2019	Binary	Heterogeneous	NSGAII-NRGA	-	-	✓	Multiple	RSM	Constant
Borhani Alamdari and Sharifi [59]	2020	Binary	Heterogeneous	GA	-	-	✓	Single	RSM	Constant

Zaretalab, Hajipour, and Tavana [60]	2020	Multi-state	Heterogeneous	GA and MA	-	✓	✓	Single	RSM	Constant
Sharifi andTaghipour [61]	2020	Multi-state	Heterogeneous	GA	-	-	✓	Single	RSM	Constant
Current work	2020	Binary	Heterogeneous	NSGAII-NRGA	-	-	✓	Multiple	RSM	Time dependent - Number dependent

	Subsystem													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy Strategy	A	S	S	A	A	S	A	S	A	A	S	S	A	S
Components Type	2	1	3	2	3	4	3	1	4	2	3	2	1	2
Components Number	4	2	3	4	1	1	3	5	3	2	6	4	2	1
Coefficients of Decreasing Failure Rates	0.1866	0.0134	0.1642	0.0527	0.073	0.1957	0.1921	0.0381	0.1931	0.1312	0.1923	0.1968	0.1205	0.079

Fig. 1: Encoded solution.

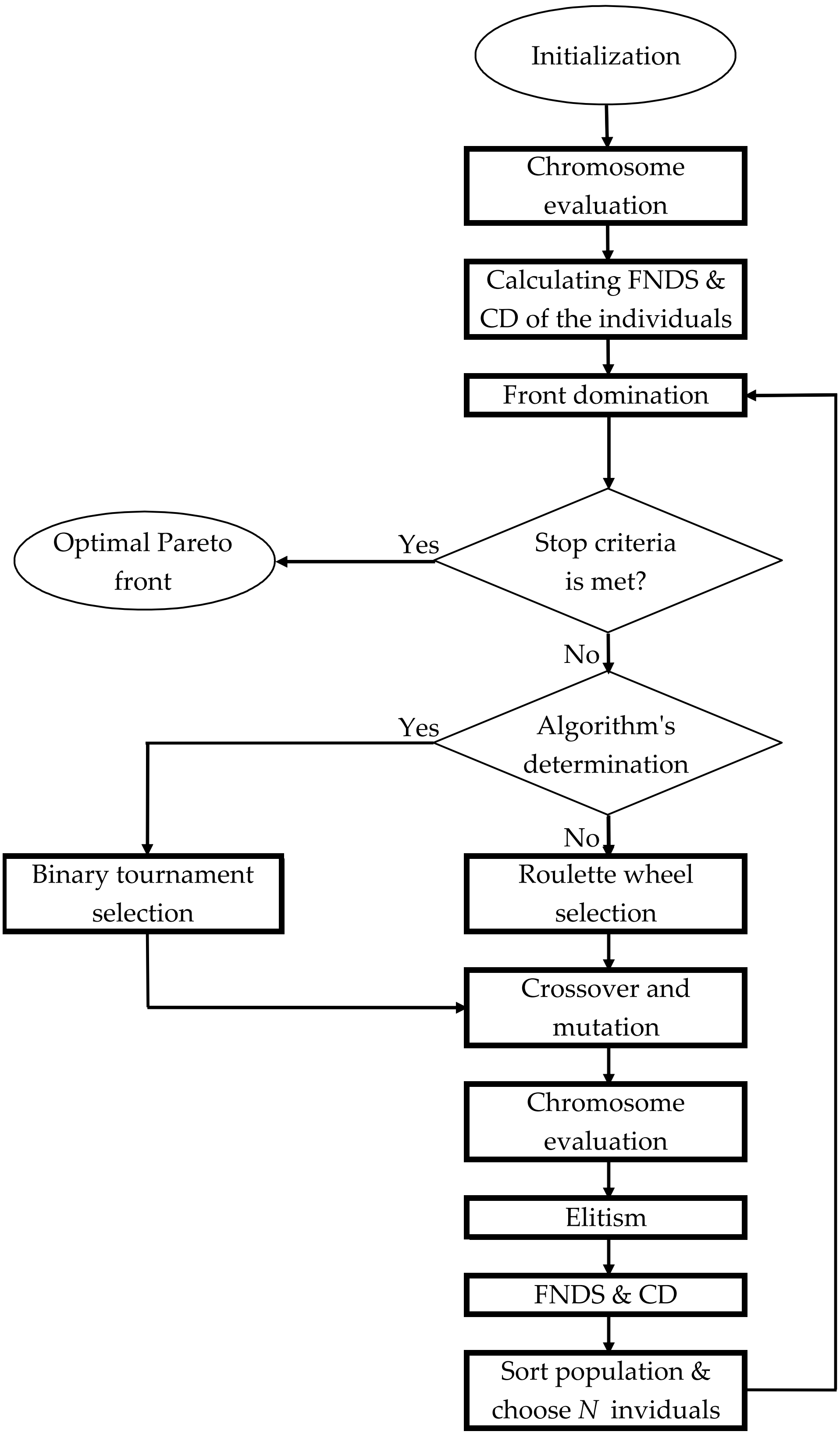

Fig. 2: Flow chart of NSGAII and NRGA algorithms.

Table 2.
Data for Numerical example.

Subsystem		Component type 1			Component type 2			Component type 3			Component type 4		
i	k_i	λ_{i1}	λc_{i1}	w_{i1}	λ_{i2}	λc_{i2}	w_{i2}	λ_{i3}	λc_{i3}	w_{i3}	λ_{i4}	λc_{i4}	w_{i4}
1	1	0.001054	1	3	0.000726	1	4	0.000943	2	2	0.000513	2	5
2	2	0.000513	2	8	0.000619	1	10	0.000726	1	9	-		
3	1	0.001625	2	7	0.001054	3	5	0.001393	1	6	0.000834	4	4
4	2	0.001863	3	5	0.001393	4	6	0.001625	5	4	-		
5	1	0.000619	2	4	0.000726	2	3	0.000513	3	5	-		
6	2	0.000101	3	5	0.000202	3	4	0.000305	2	5	0.000408	2	4
7	1	0.000943	4	7	0.000834	4	8	0.000619	5	9	-		
8	2	0.002107	3	4	0.001054	5	7	0.000943	6	6	-		
9	3	0.000305	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	3	0.001863	4	6	0.001625	4	5	0.001054	5	6	-		
11	3	0.000619	3	5	0.000513	4	6	0.000408	5	6	-		
12	1	0.002357	2	4	0.001985	3	5	0.001625	4	6	0.001054	5	7
13	2	0.000202	2	5	0.000101	3	5	0.000305	2	6	-		
14	3	0.001054	4	6	0.000834	4	7	0.000513	5	6	0.000101	6	9

Table 3.
Range of algorithms tuned parameters.

Solving methodologies	Parameter	Range
NSGAII	$nPop$	50-100
	P_c	0.3-0.6
	P_{m_1}	0.1-0.3
	P_{m_2}	0.1-0.3
NRGA	$nPop$	50-100
	P_c	0.3-0.6
	P_{m_1}	0.1-0.3
	P_{m_2}	0.1-0.3

Table 4.

Optimal values of parameters.

Solving methodologies	Parameter	Optimum value
NSGAII	$nPop$	79
	P_c	0.30
	P_{m_1}	0.30
	P_{m_2}	0.10
NRGA	$nPop$	75
	P_c	0.45
	P_{m_1}	0.20
	P_{m_2}	0.20

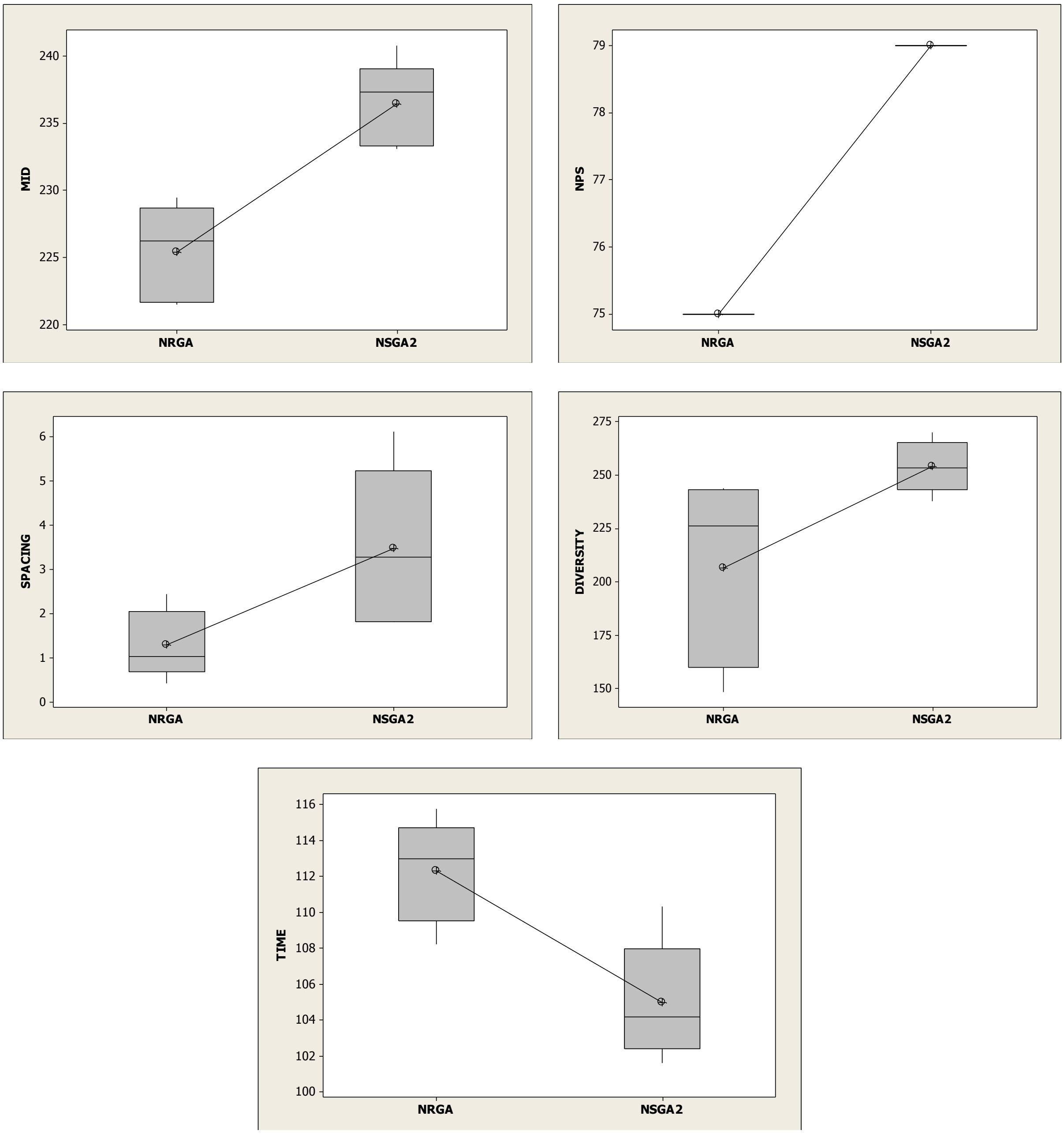

Fig. 3: The results for NSGA-II and NRGA algorithms.

Table 5.
Results of algorithms.

Performance metrics	P-value	Result	Final Result
MID	0.001	H0 is rejected	NRGA
NPS	0.000	H0 is rejected	NSGAII
SPACING	0.039	H0 is rejected	NSGAII
DIVERSITY	0.047	H0 is rejected	NRGA
TIME	0.006	H0 is rejected	NSGAII

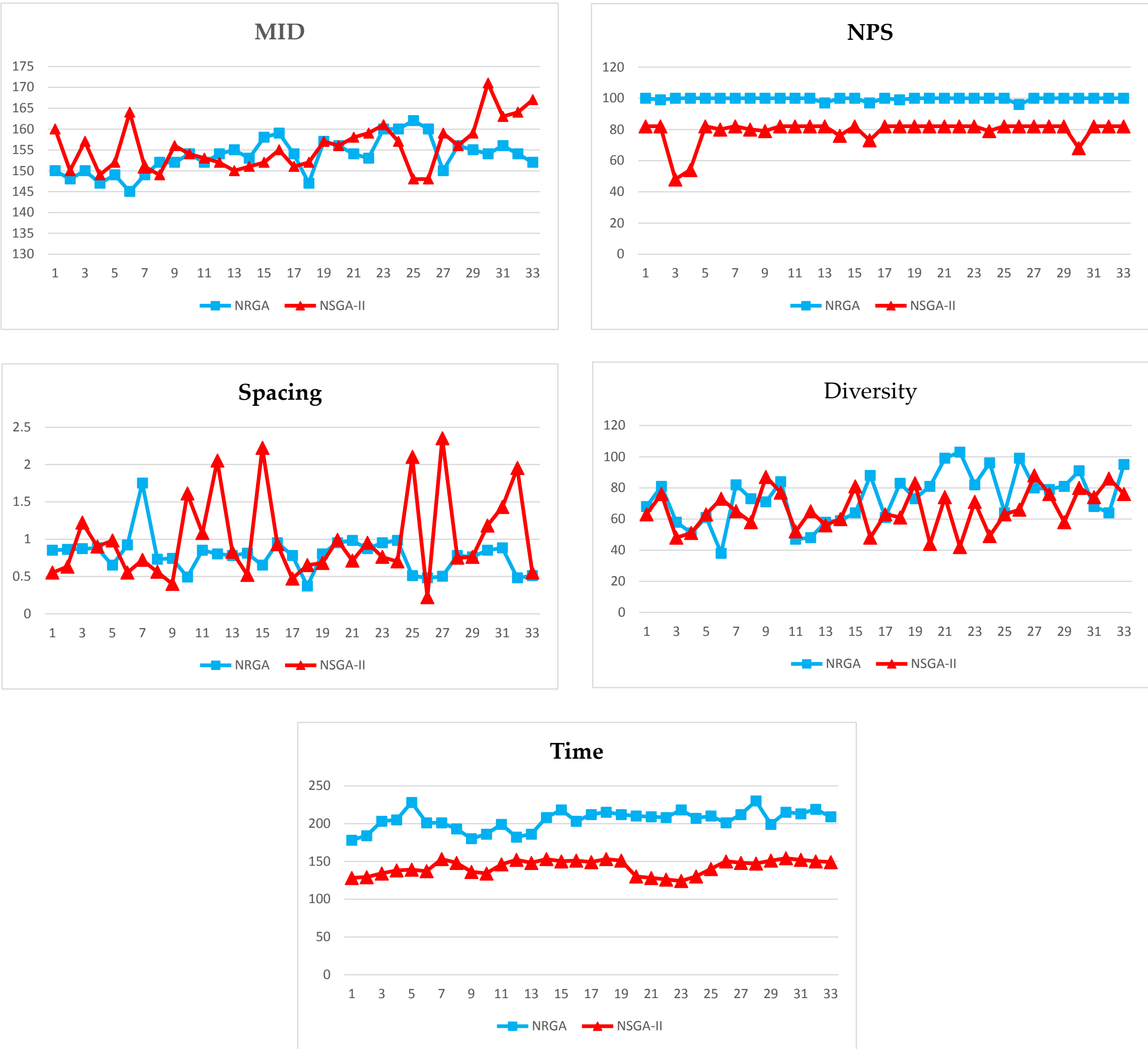

Fig. 4: Sensitivity analysis of the proposed model by NSGA-II and NRGA algorithms.

Mani Sharifi:

Mani Sharifi is an Associate Professor at the Faculty of Industrial and Mechanical Engineering in Qazvin Islamic Azad University, Qazvin, Iran. Nowadays, he is a post-doctoral research fellow at Ryerson University, the Reliability, Risk, and Maintenance Research Laboratory (RRMR Lab). He holds a B.Sc. degree from Qazvin Islamic Azad University, M.Sc. degree from south Tehran branch Islamic Azad University, and Ph.D. degree from Tehran Research and Science Islamic Azad University in Industrial Engineering. He was the Managerial editor of the Journal of Optimization in Industrial Engineering. His area of interest includes Reliability Engineering, Combinatorial Optimization, Statistical Optimization as well as Production Scheduling.

Kamran Dashti:

Kamran Dashti obtained his M.Sc. Degree in Industrial Engineering from Islamic Azad University, Qazvin Branch (QIAU) in Iran. He received his B.Sc. degree in Industrial Engineering from Islamic Azad University, Karaj Branch (KIAU). His research interests are Reliability Engineering, Quality management systems and Facility optimization. He has published a number of papers in journals such as ". International journal of industrial engineering: theory applications and practice, Sharif scientific research journal (Industrial engineering) and Industrial management scientific research journal (Allame Tabatabaei University).

Ghasem Cheragh:

Ghasem Cheragh obtained his M.Sc. Degree in Industrial Engineering from Islamic Azad University, Qazvin Branch (QIAU) in Iran. He received his B.Sc. degree in Industrial Engineering from Islamic Azad University, Karaj Branch (KIAU). His research interests are Reliability Engineering, Quality management systems and Facility optimization. He has published a number of papers in journals such as ". International journal of industrial engineering theory applications and practice, Sharif scientific research journal (Industrial engineering) and Industrial management scientific research journal (Allame Tabatabaei university).

Arash Zaretalab:

Arash Zaretalab obtained his Ph.D. Degree in Industrial Engineering from Amirkabir University of Technology (Tehran Polytechnic) in Iran. He received his B.Sc. and M.Sc. degrees in Industrial Engineering from Islamic Azad University, Qazvin Branch (QIAU). His research interests are Reliability Engineering, Combinatorial Optimization, computational intelligence, machining process optimization and Data Mining. He has published a number of papers in journals such as Reliability engineering & safety system, Computer Operation research, Computers & Industrial Engineering, Applied soft computing, International journal of advanced manufacturing, and among others. He has also reviewed papers for these journals.

Mohammad Reza Shahriari:

Mohammad Reza Shahriari is an assistant professor at Islamic Azad University, South tehran Branch. He holds a BSc degree from South Tehran Islamic Azad University in industrial engineering, and MSc degree from south Tehran branch Islamic Azad University and Ph.D. degree from Tehran Research and Science Islamic Azad University in Operations Research. His research area of interest includes DEA, combinatorial optimization, statistical optimization and fuzzy set theory.