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## Complex q-rung orthopair fuzzy variation coefficient similarity measures and their approach to medical diagnosis and pattern recognition

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#### **KEYWORDS**

Complex q-rung orthopair fuzzy sets; Interval-valued complex q-rung orthopair fuzzy sets; Jaccard similarity measures; Hybrid vector similarity measures; Variation coefficient similarity measures.

Abstract. The q-Rung Orthopair Fuzzy Set (q-ROFS) as a generalization of Fuzzy Set (FS) is characterized by membership and non-membership. The sum of their q-powers is restricted to [0, 1]. In this study, a new Complex q-ROFS (Cq-ROFS) is proposed by combining q-ROFS with Complex FS (CFS). Cq-ROFS is a better way to process uncertain and imprecise information in decision-making, which is characterized by complex-valued membership and complex-valued non-membership. First, some fundamental operational laws, score function and accuracy function, and a comparison method are proposed. Further, because the Vector Similarity Measures (VSMs) play a key role in statistics, physics, and engineering, some VSMs including Jaccard Similarity Measures (JSMs), Dice Similarity Measures (DSMs), and Cosine Similarity Measures (CSMs) for Cq-ROFSs and Interval-Valued Cq-ROFS (IVCq-ROFS) are investigated. Moreover, Hybrid VSMs (HVSMs) called Variation Co-efficient Similarity Measures (VCSMs) for Cq-ROFSs and IVCq-ROFSs are also proposed and their properties are discussed. Finally, in order to demonstrate the feasibility of the investigated HVSMs, the existing similarity measures about Complex Pythagorean Fuzzy Sets (CPFSs) and Complex Intuitionistic Fuzzy Sets (CIFSs) are compared with the proposed methods by numerical examples of medical diagnosis and pattern recognition.

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#### 1. Introduction

Intuitionistic Fuzzy Set (IFS), initiated by Atanassove [1], is an extension of Fuzzy Set (FS) [2] and is characterized by membership and non-membership grades, the sum of which is restricted to [0, 1]. IFS is an

important tool to deal with awkward and complicated information in real decision problems and now, it is receiving greater attention in the domain of distance measures [3–5] and entropy measures for IFSs [6–8]. De et al. [9] introduced the application of IFSs in medical diagnosis. Szmidt and Kacprzyk [10,11] utilized IFS to process the medical diagnosis problems. Wei [12] established geometric aggregation operators, and Xu [13] established power aggregation operators based on IFS. Liu [14] developed Hamacher aggregation operators based on interval-valued IFS. Further, Alkouri and Salleh [15] initiated the idea of Complex IFSs (CIFSs) as a generalization of Complex FS (CFS) [16]. The

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CIFS is characterized by complex-valued membership and complex-valued non-membership functions, whose range is not limited to [0, 1] but extended to unit disc in the complex plane. Moreover, Kumar and Bajaj [17] combined the notions of CIFS and soft sets [18] and proposed complex intuitionistic fuzzy soft sets. At the same time, complex intuitionistic fuzzy relation was developed by Alkouri and Salleh [19]. Recently, Rani and Garg [20,21] gave different ideas of distance measures and power aggregation operators based on CIFS. Garg and Rani [22,23] developed different ideas of power aggregation operators and robust correlation coefficient for CIFS in Multi-Attribute Group Decision-Making (MAGDM) problems.

In addition, the idea of Pythagorean Fuzzy Set (PFS), initiated by Yager [24], is characterized by membership and non-membership grades whose constraint is that the sum of the square of membership and square of non-membership grades is limited to [0,1]. Yager and Abbasov [25] provided an example to explain the meaning of PFS. When a decision-maker provides  $\frac{\sqrt{3}}{2}$  for membership grades and  $\frac{1}{2}$  for non-membership grades, the sum of two values is greater than 1 and hence, they cannot be expressed by an IFS, but described by PFS since:

$$(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \le 1.$$

It is clear that the PFS is more general than IFS in coping with uncertain information. Further, Ullah et al. [26] proposed the notion of Complex PFS (CPFS) as a generalization of CIFS and CFS to describe effectively complicated information in the environment of MAGDM problems. Obviously, CPFS is superior to CIFS, CFS, IFS, and FS in dealing with uncertain and complex information in real-life problems.

However, when a decision-maker provides 0.8 for membership and 0.9 for non-membership grades, the sum of the square of membership grade and square of non-membership grade is greater than 1. Hence, they cannot be expressed by PFS and IFS, but are expressible by q-Rung Orthopair Fuzzy Set (q-ROFS), which was proposed by Yager [27]. The constraint of q-ROFS is that the sum of q-power of membership grade and q-power of non-membership grade is limited to [0, 1]. Obviously, when q = 1, q-ROFS is reduced to IFS; when q = 2, it is reduced to PFS, i.e., q-ROFS is more general than PFS, IFS, and FS, and now it has attracted much attention. Liu et al. [28] proposed a Multiple Attribute Decision-Making (MADM) method under q-ROFS. Liu et al. [29] developed the q-rung orthopair fuzzy heronian mean operators. Wang et al. [30] investigated a number of similarity measures for q-ROFS. Jan et al. [31] proposed the idea of qrung orthopair fuzzy graph. Liu and Liu [32] developed q-rung orthopair fuzzy Bonferroni mean operators for aggregating the q-rung orthopair fuzzy numbers. Liu and Wang [33,34] proposed Archimedean Bonferroni mean operators for q-ROFSs. Peng et al. [35] suggested the exponential operators for q-ROFS. Liu et al. [36] extended the power Maclaurin symmetric mean operators for q-ROFS. Yang and Peng [37] investigated the idea of q-rung orthopair fuzzy partitioned Bonferroni mean operators. Li et al. [38] developed the q-rung orthopair linguistic heronain mean operators with their application in MAGDM. Liu and Liu [39] further developed the power Bonferroni mean operators for linguistic q-rung orthopair fuzzy numbers.

Although the q-ROFS can describe complicated uncertain information effectively by reducing restrictions of membership grade and nonmembership grade, when a decision-maker provides  $(0.91)e^{i}2\pi(0.77)$  for the complex-valued membership grade and  $(0.87)e^{i}2\pi(0.72)$  for the complex-valued nonmembership grade, the CIFS, CPFS, and q-ROFS cannot effectively described them. To handle such types of information, the Complex q-ROFS (Cq-ROFS) becomes a powerful tool to deal with uncertain and complicated information in fuzzy set theory. The advantage of Cq-ROFS is that the sum of q-power of real part (also for imaginary part) of complex-valued membership grade and that (also for imaginary part) of complex-valued non-membership grade is limited to [0,1]. The Cq-ROFS is an extension of CIFS and CPFS to cope with complexity of real-life problems.

In addition, the distance and similarity measures are two important tools that deal with the fuzzy information, which are extensively used in pattern recognition [40,41], machine learning [42-44], and MAGDM [45,46]. The most popular distance measures include hamming, Euclidean, and generalized distance measures, and the hamming and Euclidean distance measures are the special cases of the generalized distance measures. Based on the importance of similarity and distance measures for q-ROFSs, many researchers have developed some reasonable similarity and distance measures for q-ROFSs. Du [47] investigated the Makowski-type similarity measures for q-ROFS. Peng and Dai [48] developed multiparametric similarity measures for q-ROFS. The similarity and distance measures of q-ROFSs are complementary to each other. The similarity measures quantify the closeness degree between q-ROFSs, while the distance measures depict the difference between q-ROFSs.

However, the above-mentioned similarity and distance measures are defined based on general distance metrics and they have a weaker discrimination capability. Therefore, they may lead to counter-intuitive results in some special cases. Some derived measures have complex forms, yet without a specific physical meaning. It is necessary to develop an effective distance measure with a relatively concise expression and a clear physical meaning from both the mathematical and practical points of view. In particular, because the Cq-ROFS can better express the complex uncertain information and it is more general than some other fuzzy sets, it is important to propose some novel similarity and distance measures for Cq-ROFS and then, to use them to solve the MADM or MAGDM problems. Thus, the motivation and goals of this paper are shown as follows:

- 1. We propose the notions of Cq-ROFS and intervalvalued Cq-ROFS (IVCq-ROFS) and discuss their fundamental properties. At the same time, some numerical examples are given to explain their meanings;
- 2. The Jaccard Similarity Measures (JSMs), Dice Similarity Measures (DSMs), Cosine Similarity Measures (CSMs), and Vector Similarity Measures (VSMs) for Cq-ROFS and IVCq-ROFS are proposed and then, the Hybrid VSMs (HVSMs) are also presented for them;
- 3. We utilize some numerical examples in pattern recognition and medical diagnosis to examine the superiority and effectiveness of the proposed methods;
- 4. The comparisons between the proposed and existing measures are also discussed in detail to examine the validity and superiority of the established measures.

In order to achieve these goals, the construct of this manuscript is shown as follows. Section 2 reviews some basic concepts of q-ROFSs, IVq-ROFSs, Cq-ROFSs, and IVCq-ROFSs as well as their prop-In addition, the JSMs, DSMs, and CSMs erties. are reviewed. Section 3 proposes the JSMs, DSMs, CSMs, and HVSMs for Cq-ROFSs and IVCq-ROFSs. Based on HVSMs for Cq-ROFSs and IVCq-ROFSs, Section 4 gives some numerical examples about pattern recognition and medical diagnosis to show the superiority and dominance of the proposed methods. The comparisons between the proposed methods and some existing methods are also discussed in detail. The conclusion is given in the last section.

## 2. Preliminaries

The notions of q-ROFSs and Cq-ROFSs are briefly reviewed in this section and then, the JSMs, DSMs, and CSMs are also introduced. Further, the Cq-ROFSs, IVCq-ROFSs, and their fundamental properties are proposed. For convenience, k is used to represent the finite universal set in this article.

## 2.1. The q-ROFSs and IVq-ROFSs

This subsection introduces q-ROFSs and Cq-ROFSs and discusses their basic properties. Further, JSM, DSM, and CSM are discussed.



Figure 1. Geometrical interpretation of q-ROFS.

## **Definition 1 [27]:** A q-ROFS is given by:

$$A = \{(k, T'_A(k), F'_A(k)) : k \in K\},$$
(1)

where  $T'_A, F'_A : K \to [0, 1]$  represents the membership and non-membership degrees, with a condition that  $0 \leq T'_A + F'_A \leq 1, q \geq 1$ . The q-Rung Orthopair Fuzzy Number (q-ROFN) is denoted by  $A = (T'_A, F'_A)$ . The geometrical interpretation of q-ROFSs is shown in Figure 1.

**Definition 2 [27]:** Let  $A = (T'_A, F'_A)$  be a q-ROFN. Then, the score and accuracy functions of A are given by:

$$S(A) = T_A'^q - F_A'^q, S(A) \in [-1, 1],$$
(2)

$$H(A) = T'_{A} + F'_{A}, H(A) \in [0, 1].$$
(3)

**Definition 3 [27]:** Let  $A = (T'_A, F'_A)$  and  $B = (T'_B, F'_B)$  be two q-ROFNs. Then, the comparison method for q-ROFNs A and B is given as follows:

- 1. If  $S(A) \succ S(B) \Rightarrow A \succ B$ ;
- 2. If  $S(A) = S(B) \Rightarrow$ 
  - 1) If  $H(A) \succ H(B) \Rightarrow A \succ B$ ;
  - 2) If  $H(A) = H(B) \Rightarrow A = B$ .

### **Definition 4 [49]:** An IVq-ROFS is given by:

$$A = \{ \left( k, \left[ T_A'^L(k), T_A'^U(k) \right], \left[ F_A'^L(k), F_A'^U(k) \right] \right) : k \in K \},$$
(4)

where  $T_A^{\prime L}, T_A^{\prime U}, F_A^{\prime L}, F_A^{\prime U} : K \rightarrow [0, 1]$  represent the lower and upper membership degrees and the lower and upper non-membership degrees, with a condition that  $0 \leq T_A^{\prime Uq} + F_A^{\prime Uq} \leq 1, q \geq 1$ . The Interval-Valued q-ROFN (IVq-ROFN) is denoted by  $A = ([T_A^{\prime L}, T_A^{\prime U}], [F_A^{\prime L}, F_A^{\prime L}]).$ 

**Definition 5 [49]:** Let  $A = ([T'_A, T'_A], [F'_A, F'_A])$ 

and  $B=([T_B^{\prime L},T_B^{\prime U}],[F_B^{\prime L},F_B^{\prime U}])$  be two IVq-ROFNs. Then, we have:

$$\begin{split} A^{c} &= ([F_{A}^{\prime L}, F_{A}^{\prime U}], [T_{A}^{\prime L}, T_{A}^{\prime U}]); \\ A \lor B &= ([\max(T_{A}^{\prime L}, T_{B}^{\prime L}), \max(T_{A}^{\prime U}, T_{B}^{\prime U})], \\ & [\min(F_{A}^{\prime L}, F_{B}^{\prime L}), \min(F_{A}^{\prime U}, F_{B}^{\prime U})]); \\ A \land B &= ([\min(T_{A}^{\prime L}, T_{B}^{\prime L}), \min(T_{A}^{\prime U}, T_{B}^{\prime U})], \\ & [\max(F_{A}^{\prime L}, F_{B}^{\prime L}), \max(F_{A}^{\prime U}, F_{B}^{\prime U})]). \end{split}$$

**Definition 6 [49]:** Let  $A = ([T'^L_A, T'^U_A], [F'^L_A, F'^U_A])$  be an IVq-ROFN. Then, the score and accuracy functions of A are given by:

$$S(A) = \frac{T_A^{\prime Lq} + T_A^{\prime Uq}}{2} - \frac{F_A^{\prime Lq} + F_A^{\prime Uq}}{2},$$
  

$$S(A) \in [-1, 1],$$
  

$$H(A) = \frac{T_A^{\prime Lq} + T_A^{\prime Uq}}{2} + \frac{F_A^{\prime Lq} + F_A^{\prime Uq}}{2},$$
(5)

$$H(A) \in [0,1].$$
 (6)

**Definition 7 [49]:** Let  $A = ([T'_A, T'_A], [F'_A, F'_A])$ and  $B = ([T'_B, T'_B], [F'_B, F'_B])$  be two IVq-ROFNs. Then, the comparison method for IVq-ROFNs A and B is given as follows:

- 1. If  $S(A) \succ S(B) \Rightarrow A \succ B$ ;
- 2. If  $S(A) = S(B) \Rightarrow$ 
  - 1) If  $H(A) \succ H(B) \Rightarrow A \succ B$ ;
  - 2) If  $H(A) = H(B) \Rightarrow A = B$ .

Next, we give the definitions of the existing JSMs [50], DSMs [51], CSMs [52], and VCSMs [53].

**Definition 8** [50]: A JSM is given by:

$$J(K,Y) = \frac{KY}{\|K\|^2 + \|Y\|^2 - KY}$$
$$= \frac{\sum_{i=1}^n k_i y_i}{\sum_{i=1}^n k_i^2 + \sum_{i=1}^n y_i^2 - \sum_{i=1}^n k_i y_i},$$
(7)

where  $K = (k_1, k_2, \dots, k_n)$  and  $Y = (y_1, y_2, \dots, y_n)$ represent two vectors whose Euclidean norm and inner product are followed as  $||K|| = \sqrt{\sum_{i=1}^{n} k_i^2}, ||Y|| = \sqrt{\sum_{i=1}^{n} y_i^2}$ , and  $KY = \sum_{i=1}^{n} k_i y_i$ . Then, Eq. (7) holds the following conditions:

- 1.  $0 \le J(K, Y) \le 1;$
- 2. J(K,Y) = J(Y,K);
- 3. J(K, Y) = 1 if K = Y, i.e.,  $k_i = y_i (i = 1, 2, \dots, n)$ .

**Definition 9 [51]:** A DSM is given by:

$$D(K,Y) = \frac{2KY}{\|K\|^2 + \|Y\|^2} = \frac{2\sum_{i=1}^n k_i y_i}{\sum_{i=1}^n k_i^2 + \sum_{i=1}^n y_i^2},$$
(8)

where  $K = (k_1, k_2, \dots, k_n)$  and  $Y = (y_1, y_2, \dots, y_n)$ represent two vectors whose Euclidean norm and inner product are followed as  $||K|| = \sqrt{\sum_{i=1}^n k_i^2}, ||Y|| = \sqrt{\sum_{i=1}^n y_i^2}$ . Then, Eq. (8) holds the following conditions:

- 1.  $0 \le D(K, Y) \le 1;$
- 2. D(K,Y) = D(Y,K);
- 3. D(K,Y) = 1 if K = Y i.e.,  $k_i = y_i (i = 1, 2, \dots, n)$ .

**Definition 10 [52]:** A CSM is given by:

$$C(K,Y) = \frac{KY}{\|K\| + \|Y\|} = \frac{\sum_{i=1}^{n} k_i y_i}{\sqrt{\sum_{i=1}^{n} k_i^2} + \sqrt{\sum_{i=1}^{n} y_i^2}}, \quad (9)$$

where  $K = (k_1, k_2, \dots, k_n)$  and  $Y = (y_1, y_2, \dots, y_n)$ represent two vectors whose Euclidean norm and inner product are followed as  $||K|| = \sqrt{\sum_{i=1}^n k_i^2}$ ,  $||Y|| = \sqrt{\sum_{i=1}^n y_i^2}$ . Then, Eq. (9) holds the following conditions:

- 1.  $0 \le C(K, Y) \le 1;$
- 2. C(K, Y) = C(Y, K);
- 3. C(K,Y) = 1 if K = Y i.e.,  $k_i = y_i (i = 1, 2, \dots, n)$ .

**Definition 11** [53]: A VCSM is given by:

$$V(K,Y) = \rho \frac{2KY}{\|K\|^2 + \|Y\|^2} + (1-\rho) \frac{KY}{\|K\| \cdot \|Y\|}$$
$$= \rho \frac{2\sum_{i=1}^n k_i y_i}{\sum_{i=1}^n k_i^2 + \sum_{i=1}^n y_i^2} + (1-\rho) \frac{\sum_{i=1}^n k_i y_i}{\sqrt{\sum_{i=1}^n k_i^2} \cdot \sqrt{\sum_{i=1}^n y_i^2}},$$

where  $K = (k_1, k_2, \dots, k_n)$  and  $Y = (y_1, y_2, \dots, y_n)$ represent two vectors whose Euclidean norm and inner product are followed as  $||K|| = \sqrt{\sum_{i=1}^n k_i^2}$ ,  $||Y|| = \sqrt{\sum_{i=1}^n y_i^2}$ , and  $KY = \sum_{i=1}^n k_i y_i$  with  $\rho \ge 1$ . Then, Eq. (10) holds the following conditions:

- 1.  $0 \le V(K, Y) \le 1;$
- 2. V(K, Y) = V(Y, K);
- 3. V(K,Y) = 1 if K = Y i.e.,  $k_i = y_i (i = 1, 2, \dots, n)$ .

These four similarity measures come with the same meaning in the domain of interval values. If  $k_i = 0$  and  $y_i = 0$ , then the JSMs and DSMs are undefined. If  $k_i = 0$  or  $y_i = 0$ , then the CSMs are undefined.

## 2.2. Cq-ROFSs and IVCq-ROFSs

This subsection proposes the Cq-ROFSs and IVCq-ROFSs and discusses their basic properties.

**Definition 12:** A Cq-ROFS is given by:

$$A = \{ (k, T'_A(k), F'_A(k)) : k \in K \},$$
(11)

where  $T'_A = T_A e^{i2\pi(\varphi_{T_A})}$  and  $F'_A = F_A e^{i2\pi(\varphi_{F_A})}$ represent complex-valued membership and complexvalued non-membership, with a condition that:  $0 \leq T^q_A + F^q_A \leq 1$ , and  $0 \leq \varphi^q_{T_A} + \varphi^q_{F_A} \leq 1$  for  $q \geq 1$ . The complex q-ROFN (Cq-ROFN) is denoted by:

$$A = \left( T_A e^{i2\pi(\varphi_{T_A})}, F_A e^{i2\pi(\varphi_{F_A})} \right).$$

**Definition 13:** Let  $A = (T_A e^{i2\pi(\varphi_{T_A})}, F_A e^{i2\pi(\varphi_{F_A})})$ and  $B = (T_B e^{i2\pi(\varphi_{T_B})}, F_B e^{i2\pi(\varphi_{F_B})})$  be two Cq-ROFNs. Then, we have:

$$\begin{aligned} A^{c} &= (F_{A}e^{i2\pi(\varphi_{F_{A}})}, T_{A}e^{i2\pi(\varphi_{T_{A}})}); \\ A \lor B &= (\max(T_{A}, T_{B})e^{i2\pi(\max(\varphi_{T_{A}}, \varphi_{T_{B}}))}, \\ \min(F_{A}, F_{B})e^{i2\pi(\min(\varphi_{F_{A}}, \varphi_{F_{B}}))}); \\ A \land B &= (\min(T_{A}, T_{B})e^{i2\pi(\min(\varphi_{T_{A}}, \varphi_{T_{B}}))}, \end{aligned}$$

$$\max(F_A, F_B)e^{i2\pi(\max(\varphi_{F_A}, \varphi_{F_B}))}).$$

**Definition 14:** Let  $A = (T_A e^{i2\pi(\varphi_{T_A})}, F_A e^{i2\pi(\varphi_{F_A})})$  be a Cq-ROFN. Then, the score and accuracy functions of A are given by:

$$S(A) = \frac{1}{2} (T_A^q + \varphi_{T_A}^q - \varphi_{F_A}^q - F_A^q),$$
  

$$S(A) \in [-1, 1],$$
(12)

$$H(A) = \frac{1}{2} (T_A^q + \varphi_{T_A}^q + \varphi_{F_A}^q + F_A^q),$$
  

$$H(A) \in [0, 1].$$
(13)

**Definition 15:** Let  $A = (T_A e^{i2\pi(\varphi_{T_A})}, F_A e^{i2\pi(\varphi_{F_A})})$ and  $B = (T_B e^{i2\pi(\varphi_{T_B})}, F_B e^{i2\pi(\varphi_{F_B})})$  be two Cq-ROFNs. Then, the comparison method for Cq-ROFNs A and B is given as follows:

- 1. If  $S(A) \succ S(B) \Rightarrow A \succ B$ ; 2. If  $S(A) = S(B) \Rightarrow$ 
  - 1) If  $H(A) \succ H(B) \Rightarrow A \succ B$ ;
  - 2) If  $H(A) = H(B) \Rightarrow A = B$ .

**Example 1:** Let  $A = (0.8e^{i2\pi(0.7)}, 0.9e^{i2\pi(0.67)})$  and  $B = (0.88e^{i2\pi(0.75)}, 0.67e^{i2\pi(0.66)})$  be two Cq-ROFNs for q = 5. Then, we have:

$$A^{c} = (0.9e^{i2\pi(0.67)}, 0.8e^{i2\pi(0.7)});$$

 $A \lor B = (\max(0.8, 0.88)e^{i2\pi(\max(0.7, 0.75))},$ 

$$\min(0.9, 0.67)e^{i2\pi(\min(0.67, 0.66))})$$

$$=(0.88e^{i2\pi(0.75)}, 0.67e^{i2\pi(0.66)});$$

$$A \wedge B = (\min(0.8, 0.88)e^{i2\pi(\min(0.7, 0.75))},$$
$$\max(0.9, 0.67)e^{i2\pi(\max(0.67, 0.66))})$$
$$= (0.88e^{i2\pi(0.7)}, 0.9e^{i2\pi(0.67)}).$$

Next, we calculate the score values of A and B as follows:

$$S(A) = \frac{1}{2} \left( (0.8)^5 + (0.7)^5 - (0.9)^5 - (0.67)^5 \right)$$
$$= \frac{1}{2} (0.50 - 0.73) = -0.115,$$
$$S(B) = \frac{1}{2} ((0.88)^5 + (0.75)^5 - (0.67)^5 - (0.66)^5)$$
$$= \frac{1}{2} (0.77 - 0.26) = 0.255.$$

So,  $B \succ A$ .

**Definition 16:** An IVCq-ROFS is given by:

$$A = \{(k, T'_A(k), F'_A(k)) : k \in K\},$$
(14)

where  $T'_A = [T^L_A, T^U_A] \cdot e^{i2\pi[\varphi^L_{T_A}, \varphi^U_{T_A}]}$  and  $F'_A = [F^L_A, F^U_A] \cdot e^{i2\pi[\varphi^L_{F_A}, \varphi^U_{F_A}]}$  represent interval-valued complex membership and interval-valued complex non-membership, with a condition that  $0 \leq T^{Uq}_A + F^{Uq}_A \leq 1$  and  $0 \leq \varphi^{Uq}_{T_A} + \varphi^{Uq}_{F_A} \leq 1$ ,  $q \geq 1$ . The interval-valued Cq-ROFN (IVCq-ROFN) is denoted by  $A = ([T^L_A, T^U_A] \cdot e^{i2\pi[\varphi^L_{T_A}, \varphi^U_{T_A}]}, [F^L_A, F^U_A] \cdot e^{i2\pi[\varphi^L_{F_A}, \varphi^U_{F_A}]})$ .

**Definition 17:** Let  $A = ([T_A^L, T_A^U] \cdot e^{i2\pi[\varphi_{T_A}^L, \varphi_{T_A}^U]}, [F_A^L, F_A^U] \cdot e^{i2\pi[\varphi_{F_A}^L, \varphi_{F_A}^U]})$  and  $B = ([T_B^L, T_B^U] \cdot e^{i2\pi[\varphi_{T_B}^L, \varphi_{T_B}^U]})$ ,  $[F_B^L, F_B^U] \cdot e^{i2\pi[\varphi_{F_B}^L, \varphi_{F_B}^U]})$  be two IVCq-ROFNs; then, we have:

$$\begin{aligned} A^{c} &= ([F_{A}^{L}, F_{A}^{U}] \cdot e^{i2\pi[\varphi_{F_{A}}^{L}, \varphi_{F_{A}}^{U}]}, [T_{A}^{L}, T_{A}^{U}] \\ &\cdot e^{i2\pi[\varphi_{T_{A}}^{L}, \varphi_{T_{A}}^{U}]}); \\ A \lor B &= ([\max(T_{A}^{L}, T_{B}^{L}), \max(T_{A}^{U}, T_{B}^{U})] \end{aligned}$$

$$\cdot e^{i2\pi[\max(\varphi_{T_A}^L,\varphi_{T_B}^L),\max(\varphi_{T_A}^U,\varphi_{T_B}^U)]}, \\ [\min(F_A^L,F_B^L),\min(F_A^L,F_B^U)] \\ \cdot e^{i2\pi[\min(\varphi_{F_A}^L,\varphi_{F_B}^L),\min(\varphi_{F_A}^U,\varphi_{F_B}^U)]});$$

$$\begin{split} A \wedge B &= \left( [\min(T_A^L, T_B^L), \min(T_A^U, T_B^U)] \right. \\ &\cdot e^{i2\pi [\min(\varphi_{T_A}^L, \varphi_{T_B}^L), \min(\varphi_{T_A}^U, \varphi_{T_B}^U)]}, \\ &\left. [\max(F_A^L, F_B^L), \max(F_A^U, F_B^U)] \right. \\ &\cdot e^{i2\pi [\max(\varphi_{F_A}^L, \varphi_{F_B}^L), \max(\varphi_{F_A}^U, \varphi_{F_B}^U)]}). \end{split}$$

**Definition 18:** Let  $A = ([T_A^L, T_A^U] \cdot e^{i2\pi [\varphi_{T_A}^L, \varphi_{T_A}^U]}, [F_A^L, F_A^U] \cdot e^{i2\pi [\varphi_{F_A}^L, \varphi_{F_A}^U]})$  be an IVCq-ROFN. Then, the score and accuracy functions of A are given by:

$$S(A) = \frac{1}{2} \left( \frac{T_A^L + T_A^U}{2} + \frac{\varphi_{T_A}^L + \varphi_{T_A}^U}{2} - \frac{F_A^L + F_A^U}{2} - \frac{\varphi_{F_A}^L + \varphi_{F_A}^U}{2} \right), \quad S(A) \in [-1, 1],$$
(15)

$$H(A) = \frac{1}{2} \left( \frac{T_A^L + T_A^U}{2} + \frac{\varphi_{T_A}^L + \varphi_{T_A}^U}{2} + \frac{F_A^L + F_A^U}{2} + \frac{\varphi_{F_A}^L + \varphi_{F_A}^U}{2} + \frac{\varphi_{F_A}^L + \varphi_{F_A}^U}{2} \right), \quad H(A) \in [0, 1].$$
(16)

**Definition 19:** Let  $A = ([T_A^L, T_A^U] \cdot e^{i2\pi[\varphi_{T_A}^L, \varphi_{T_A}^U]}, [F_A^L, F_A^U] \cdot e^{i2\pi[\varphi_{F_A}^L, \varphi_{F_A}^U]})$  and  $B = ([T_B^L, T_B^U] \cdot e^{i2\pi[\varphi_{T_B}^L, \varphi_{T_B}^U]}, [F_B^L, F_B^U] \cdot e^{i2\pi[\varphi_{F_B}^L, \varphi_{F_B}^U]})$  be two IVCq-ROFNs. Then, the comparison method for IVCq-ROFNs A and B is given as follows:

- 1. If  $S(A) \succ S(B) \Rightarrow A \succ B$ ;
- 2. If  $S(A) = S(B) \Rightarrow$ 
  - 1) If  $H(A) \succ H(B) \Rightarrow A \succ B$ ;
  - 2) If  $H(A) = H(B) \Rightarrow A = B$ .

**Example 2:** Let  $A = ([0.8, 0.82]e^{i2\pi[0.7, 0.72]}, [0.9, 0.92] e^{i2\pi[0.67, 0.69]}$  and  $B = ([0.88, 0.9]e^{i2\pi[0.75, 0.77]}, [0.67, 0.69]e^{i2\pi[0.66, .0.68]})$  be two Cq-ROFNs for q = 5. Then, we have:

$$A^{c} = ([0.9, 0.92]e^{i2\pi[0.67, 0.69]}, [0.8, 0.82]e^{i2\pi[0.7, 0.72]});$$

 $A \lor B = ([\max(0.8, 0.88), \max(0.82, 0.9)])$ 

 $e^{i2\pi[\max(0.7, 0.75), \max(0.72, 0.77)]}, [\min(0.9, 0.67),$ 

 $\min(0.92, 0.69)]e^{i2\pi[\min(0.67, 0.66), \min(0.69, 0.68)})$ 

 $= ([0.88, 0.9]e^{i2\pi} [0.75, 0.77],$ 

 $[0.67, 0.69]e^{i2\pi [0.66, .0.68]});$ 

$$\begin{split} A \wedge B = &([\min(0.8, 0.88), \min(0.82, 0.9)] \\ &e^{i2\pi[\min(0.7, 0.75), \min(0.72, 0.77)]}, [\max(0.9, 0.67), \\ &\max(0.92, 0.69)]e^{i2\pi[\max(0.67, 0.66), \max(0.69, 0, 68))} \end{split}$$

$$= ([0.8, 0.82]e^{i2\pi[0.7, 0.72]},$$
$$[0.9, 0.92]e^{i2\pi[0.67, 0.69]}).$$

Next, we calculate the score values of A and B and get

$$\begin{split} S(A) &= \frac{1}{2} (((0.8)^5 + (0.82)^5)/2 + ((0.7)^5 + (0.72)^5)/2 \\ &- ((0.9)^5 + (0.92)^5)/2 - ((0.67)^5 + (0.69)^5)/2) \\ &= \frac{1}{2} (0.35 + 0.18 - 0.62 + 0.15) = -0.12, \\ S(B) &= \frac{1}{2} (((0.88)^5 + (0.9)^5)/2 + ((0.75)^5 + (0.77)^5)/2 \\ &- ((0.67)^5 + (0.69)^5)/2 - ((0.66)^5 + (0.68)^5)/2) \\ &= \frac{1}{2} (0.56 + 0.25 - 0.15 + 0.14) = 0.2. \\ \text{So, } B \succ A. \end{split}$$

# 3. Hybrid vector similarity measures for Cq-ROFSs

This section investigates some VSMs called Jaccard, Dice, and cosine similarity measures and HVSM called variation coefficient similarity measures for Cq-ROFSs. For convenience, in this paper, the weighted vector is defined by:  $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}^T, \omega_i \in [0, 1], i = 1, 2, \cdots, n$  with a condition that  $\sum_{i=1}^n \omega_i = 1$ . In addition,  $A = (T_A e^{i2\pi(\varphi_{T_A})}, F_A e^{i2\pi(\varphi_{F_A})})$  and  $B = (T_B e^{i2\pi(\varphi_{T_B})}, F_B e^{i2\pi(\varphi_{F_B})})$  are used to denote the Cq-ROFNs.

## 3.1. The VSMs for Cq-ROFSs

The Jaccard, Dice, and cosine similarity measures are important tools to find the degree of similarity between objects. In this subsection, we extend them to Cq-ROFSs and propose JSM, Weighted JSM (WJSM), DSM, Weighted DSM (WDSM), CSM, and Weighted CSM (WCSM) for Cq-ROFSs.

**Definition 20:** The JSM for Cq-ROFS is given by Eq. (17) as shown in Box I. Eq. (17) holds the following conditions:

- 1.  $0 \le J_{jc}(A, B) \le 1$ ,
- 2.  $J_{jc}(A,B) = J_{jc}(B,A),$
- 3.  $J_{jc}(A, B) = 1$ , for A = B i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

$$J_{jc}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{T_A^{q}(k_i)T_B^{q}(k_i) + F_A^{q}(k_i)F_B^{q}(k_i)}{(T_A^{2q}(k_i) + F_A^{2q}(k_i)) + (T_B^{2q}(k_i) + F_B^{2q}(k_i)) - (T_A^{q}(k_i)T_B^{q}(k_i) + F_A^{q}(k_i)F_B^{q}(k_i))} + \frac{\varphi_{T_A}^{q}(k_i)\varphi_{T_B}^{q}(k_i) + \varphi_{F_A}^{q}(k_i)\varphi_{F_B}^{q}(k_i)}{(\varphi_{T_A}^{2q}(k_i) + \varphi_{F_A}^{2q}(k_i)) + (\varphi_{T_B}^{2q}(k_i) + \varphi_{F_B}^{2q}(k_i)) - (\varphi_{T_A}^{q}(k_i)\varphi_{T_B}^{q}(k_i) + \varphi_{F_A}^{q}(k_i)\varphi_{F_B}^{q}(k_i))} \right).$$
(17)

Box I

$$J_{wjc}(A,B) = \frac{1}{2} \sum_{i=1}^{n} \omega_i \left( \frac{T_A^{q}(k_i) T_B^{q}(k_i) + F_A^{q}(k_i) F_B^{q}(k_i)}{(T_A^{2q}(k_i) + F_A^{2q}(k_i)) + (T_B^{2q}(k_i) + F_B^{2q}(k_i)) - (T_A^{q}(k_i) T_B^{q}(k_i) + F_A^{q}(k_i) F_B^{q}(k_i))} + \frac{\varphi_{T_A}^{q}(k_i) \varphi_{T_B}^{q}(k_i) + \varphi_{F_A}^{q}(k_i) \varphi_{F_B}^{q}(k_i)}{(\varphi_{T_A}^{2q}(k_i) + \varphi_{F_A}^{2q}(k_i)) + (\varphi_{T_B}^{2q}(k_i) + \varphi_{F_B}^{2q}(k_i)) - (\varphi_{T_A}^{q}(k_i) \varphi_{T_B}^{q}(k_i) + \varphi_{F_A}^{q}(k_i) \varphi_{F_B}^{q}(k_i))} \right).$$
(18)



**Definition 21:** The WJSM for Cq-ROFS is given by Eq. (18) as shown in Box II. Eq. (18) holds the following conditions:

- 1.  $0 \le J_{wjc}(A, B) \le 1$ ,
- 2.  $J_{wic}(A,B) = J_{wic}(B,A),$
- 3.  $J_{wjc}(A,B) = 1$ , for A = B i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

**Definition 22:** The DSM for Cq-ROFS is given by Eq. (19) as shown in Box III. Eq. (19) holds the following conditions:

$$1. \quad 0 \le J_d(A, B) \le 1,$$

- 2.  $J_d(A,B) = J_d(B,A),$
- 3.  $J_d(A, B) = 1$ , for A = B, i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

**Definition 23:** The WDSM for Cq-ROFS is given by Eq. (20) as shown in Box IV. Eq. (20) holds the following conditions

- 1.  $0 \le J_{Wd}(A, B) \le 1$ ,
- 2.  $J_{Wd}(A, B) = J_{Wd}(B, A),$
- 3.  $J_{Wd}(A, B) = 1$ , for A = B i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

$$J_{d}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{2(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i}))}{(T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i})) + (T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i}))} + \frac{2(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i}))}{(\varphi_{T_{A}}^{2q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i})) + (\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i}))} \right).$$
(19)

$$J_{Wd}(A,B) = \frac{1}{2} \sum_{i=1}^{n} \omega_i \left( \frac{2(T_A^q(k_i)T_B^q(k_i) + F_A^q(k_i)F_B^q(k_i))}{(T_A^{2q}(k_i) + F_A^{2q}(k_i)) + (T_B^{2q}(k_i) + F_B^{2q}(k_i))} + \frac{2(\varphi_{T_A}^q(k_i)\varphi_{T_B}^q(k_i) + \varphi_{F_A}^q(k_i)\varphi_{F_B}^q(k_i))}{(\varphi_{T_A}^{2q}(k_i) + \varphi_{F_A}^{2q}(k_i)) + (\varphi_{T_B}^{2q}(k_i) + \varphi_{F_B}^{2q}(k_i))} \right).$$
(20)

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$$J_{c}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{2(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i}))}{\sqrt{(T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i}))} \cdot \sqrt{(T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i}))}} + \frac{2(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{(\varphi_{T_{A}}^{2q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i}))} \cdot \sqrt{(\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i}))}}} \right).$$
(21)



$$J_{Wc}(A,B) = \frac{1}{2} \sum_{i=1}^{n} \omega_{i} \left( \frac{2(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i}))}{\sqrt{(T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i}))} \cdot \sqrt{(T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i}))}} + \frac{2(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{(\varphi_{T_{A}}^{2q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i}))} \cdot \sqrt{(\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i}))}} \right).$$
(22)



**Definition 24:** The CSM for Cq-ROFS is given by Eq. (21) as shown in Box V. Eq. (21) holds the following conditions:

- 1.  $0 \leq J_c(A, B) \leq 1$ ,
- 2.  $J_c(A,B) = J_c(B,A),$
- 3.  $J_c(A,B) = 1$ , for A = B i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

**Definition 25:** The WCSM for Cq-ROFS is given by Eq. (22) as shown in Box VI. Eq. (22) holds the following conditions:

- 1.  $0 \le J_{Wc}(A, B) \le 1$ ,
- 2.  $J_{Wc}(A, B) = J_{Wc}(B, A),$
- 3.  $J_{Wc}(A, B) = 1$ , for A = B i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

#### 3.2. The VSMs for IVCq-ROFSs

This subsection combines Jaccard, Dice, and cosine similarity measures with IVCq-ROFS and proposes JSM, WJSM, DSM, WDSM, CSM, and WCSM for IVCq-ROFSs. For convenience, the membership and non-membership values in the domain of IVCq-ROFSs are given as follows:

$$2\Omega T_{A}^{q}(k_{i}) = (T_{A}^{L}(k_{i}) + T_{A}^{U}(k_{i}))/2,$$
  

$$2\Omega F_{A}^{q}(k_{i}) = (F_{A}^{L}(k_{i}) + F_{A}^{U}(k_{i}))/2,$$
  

$$2\Omega \varphi_{T_{A}}^{q}(k_{i}) = (\varphi_{T_{A}}^{L}(k_{i}) + \varphi_{T_{A}}^{U}(k_{i}))/2,$$
  

$$2\Omega \varphi_{F_{A}}^{q}(k_{i}) = (\varphi_{F_{A}}^{L}(k_{i}) + \varphi_{F_{A}}^{U}(k_{i}))/2,$$

and:

$$2\Omega T_{B}^{q}(k_{i}) = (T_{B}^{L}(k_{i}) + T_{B}^{U}(k_{i}))/2,$$
  

$$2\Omega F_{B}^{q}(k_{i}) = (F_{B}^{L}(k_{i}) + F_{B}^{U}(k_{i}))/2,$$
  

$$2\Omega \varphi_{T_{B}}^{q}(k_{i}) = (\varphi_{T_{B}}^{L}(k_{i}) + \varphi_{T_{B}}^{U}(k_{i}))/2,$$
  

$$2\Omega \varphi_{F_{B}}^{q}(k_{i}) = (\varphi_{F_{B}}^{L}(k_{i}) + \varphi_{F_{B}}^{U}(k_{i}))/2.$$

Then, we propose the following similarity measures.

**Definition 26:** The JSM for IVCq-ROFS is given by Eq. (23) as shown in Box VII. Eq. (23) holds the following conditions:

- $$\begin{split} 1. & 0 \leq J_{jc}^{IV}(A,B) \leq 1, \\ 2. & J_{jc}^{IV}(A,B) = J_{jc}^{IV}(B,A), \\ 3. & J_{jc}^{IV}(A,B) = 1, \text{ for } A = B \text{ i.e., } T \end{split}$$
- 3.  $J_{jc}^{IV}(A, B) = 1$ , for A = B i.e.,  $T_A^L = T_B^L$ ,  $T_A^U = T_B^U$ ,  $F_A^L = F_B^L$ ,  $F_A^U = F_B^U$ ,  $\varphi_{T_A}^L = \varphi_{T_B}^L$ ,  $\varphi_{T_A}^U = \varphi_{T_B}^U$ ,  $\varphi_{F_A}^L = \varphi_{F_B}^L$ , and  $\varphi_{F_A}^U = \varphi_{F_B}^U$ .

**Definition 27:** The WJSM for IVCq-ROFS is given by Eq. (24) as shown in Box VIII. Eq. (24) holds the following conditions:

- 1.  $0 \le J_{Wic}^{IV}(A, B) \le 1$ ,
- 2.  $J_{Wic}^{IV}(A,B) = J_{Wic}^{IV}(B,A),$
- 3.  $J_{Wjc}^{IV}(A,B) = 1$ , for A = B i.e.,  $T_A^L = T_B^L$ ,  $T_A^U = T_B^U$ ,  $F_A^L = F_B^L$ ,  $F_A^U = F_B^U$ ,  $\varphi_{T_A}^L = \varphi_{T_B}^L$ ,  $\varphi_{T_A}^U = \varphi_{T_B}^U$ ,  $\varphi_{F_A}^L = \varphi_{F_B}^L$ , and  $\varphi_{F_A}^U = \varphi_{F_B}^U$ .

$$J_{jc}^{IV}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i})}{((\Omega T_{A}^{q}(k_{i}))^{2} + (\Omega F_{A}^{q}(k_{i}))^{2} + ((\Omega T_{B}^{q}(k_{i}))^{2} + (\Omega F_{B}^{q}(k_{i}))^{2}) - \begin{pmatrix}\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i})}{+\Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i})} \\ + \frac{\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i})}{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2} + ((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2}) - \begin{pmatrix}\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) \\ +\Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}) \end{pmatrix} \right)^{-} (23)$$

Box VII





Definition 28: The DSM for IVCq-ROFS is given by Eq. (25) as shown in Box IX. Eq. (25) holds the following conditions:

1.  $0 \leq J_d^{IV}(A, B) \leq 1$ ,

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**Definition 29:** The WDSM for IVCq-ROFS is given by Eq. (26) as shown in Box X. Eq. (26) holds the following conditions:

- 1.  $0 \leq J_{Wd}^{IV}(A, B) \leq 1$ ,

$$J_{d}^{IV}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{2(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i}))}{((\Omega T_{A}^{q}(k_{i}))^{2} + (\Omega F_{A}^{q}(k_{i}))^{2}) + ((\Omega T_{B}^{q}(k_{i}))^{2} + (\Omega F_{B}^{q}(k_{i}))^{2})} + \frac{2(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}))}{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2}) + ((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2})} \right).$$
(25)

$$J_{Wd}^{IV}(A,B) = \frac{1}{2} \sum_{i=1}^{n} \omega_{i} \begin{pmatrix} \frac{2(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i}))}{((\Omega T_{A}^{q}(k_{i}))^{2} + (\Omega F_{A}^{q}(k_{i}))^{2}) + ((\Omega T_{B}^{q}(k_{i}))^{2} + (\Omega F_{B}^{q}(k_{i}))^{2})} \\ + \frac{2(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}))}{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2}) + ((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2})} \end{pmatrix}.$$
(26)

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$$J_{c}^{IV}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i}))}{\sqrt{((\Omega T_{A}^{q}(k_{i}))^{2} + (\Omega F_{A}^{q}(k_{i}))^{2})} \cdot \sqrt{((\Omega T_{B}^{q}(k_{i}))^{2} + (\Omega F_{B}^{q}(k_{i}))^{2})}} + \frac{(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2})} \cdot \sqrt{((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2})}} \right).$$
(27)



**Definition 30:** The CSM for IVCq-ROFS is given by Eq. (27) as shown in Box XI. Eq. (27) holds the following conditions:

- 1.  $0 \leq J_c^{IV}(A, B) \leq 1$ ,
- 2.  $J_c^{IV}(A,B) = J_c^{IV}(B,A),$
- 3.  $J_c^{IV}(A,B) = 1, \text{ for } A = B \text{ i.e., } T_A^L = T_B^L, T_A^U = T_B^U,$  $F_A^L = F_B^L, F_A^U = F_B^U, \varphi_{T_A}^L = \varphi_{T_B}^L, \varphi_{T_A}^U = \varphi_{T_B}^U,$  $\varphi_{F_A}^L = \varphi_{F_B}^L, \text{ and } \varphi_{F_A}^U = \varphi_{F_B}^U.$

**Definition 31:** The WCSM for IVCq-ROFS is given by Eq. (28) as shown in Box XII. Eq. (28) holds the following conditions:

- 1.  $0 \leq J_{Wc}^{IV}(A, B) \leq 1$ ,
- 2.  $J_{Wc}^{IV}(A,B) = J_{Wc}^{IV}(B,A),$
- 3.  $J_{Wc}^{IV}(A,B) = 1$ , for A = B, i.e.,  $T_A^L = T_B^L$ ,  $T_A^U = T_B^U$ ,  $F_A^L = F_B^L$ ,  $F_A^U = F_B^U$ ,  $\varphi_{T_A}^L = \varphi_{T_B}^L$ ,  $\varphi_{T_A}^U = \varphi_{T_B}^U$ ,  $\varphi_{F_A}^L = \varphi_{F_B}^L$ , and  $\varphi_{F_A}^U = \varphi_{F_B}^U$ .

## 3.3. The HVSMs for Cq-ROFSs

This subsection investigates the notion of HVSM called VCSM and Hybrid Weighted VSM (HWVSM) called

Weighted VCSM (WVCSM) for Cq-ROFSs and IVCq-ROFSs. The properties of the proposed methods are discussed in detail.

**Definition 32:** The HVSM for Cq-ROFS is given by Eq. (29) as shown in Box XIII. Eq. (29) holds the following conditions:

- 1.  $0 \le H_V(A, B) \le 1$ ,
- 2.  $H_V(A,B) = H_V(B,A),$
- 3.  $H_V(A, B) = 1$ , for A = B i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

**Proof:** The proofs of the above three points are shown as follows:

1. The DSM and CSM for Cq-ROFS are proposed in Eqs. (19) and (21); we have  $0 \leq J_d(A, B) \leq 1$  and  $0 \leq J_c(A, B) \leq 1$ . Then, we get  $H_V(A, B) =$  $\theta J_d(A, B) + (1 - \theta)J_c(A, B) \leq \theta + 1 - \theta = 1$ . Further,  $0 \leq J_d(A, B)$  and  $0 \leq J_c(A, B)$ ; therefore,  $H_V(A, B) \geq 0$  for any values of  $\theta \in [0, 1]$ . Hence,  $0 \leq H_V(A, B) \leq 1$  holds clearly;

$$J_{Wc}^{IV}(A,B) = \frac{1}{2} \sum_{i=1}^{n} \omega_{i} \begin{pmatrix} \frac{(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i}))}{\sqrt{((\Omega T_{A}^{q}(k_{i}))^{2} + (\Omega F_{A}^{q}(k_{i}))^{2})} \cdot \sqrt{((\Omega T_{B}^{q}(k_{i}))^{2} + (\Omega F_{B}^{q}(k_{i}))^{2})} \\ + \frac{(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2})} \cdot \sqrt{((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2})}} \end{pmatrix}.$$
(28)

$$H_{V}(A,B) = \frac{1}{n} \begin{pmatrix} \frac{2(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i}))}{((T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i})) + ((T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i})))} \\ + \frac{2(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i}))}{(\varphi_{T_{A}}^{2q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i})) + (\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i}))} \end{pmatrix} \\ + \frac{(1-\theta)}{2} \sum_{i=1}^{n} \begin{pmatrix} \frac{(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i}))}{\sqrt{((T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i}))} \cdot \sqrt{((T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i}))}} \\ + \frac{(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{((\varphi_{T_{A}}^{2q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i})) \cdot \sqrt{(\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i}))}} \end{pmatrix} \end{pmatrix}$$

$$(29)$$

## Box XIII

$$H_{WV}(A,B) = \begin{pmatrix} \frac{\theta}{2} \sum_{i=1}^{n} \omega_{i} \begin{pmatrix} \frac{2(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i})) \\ ((T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i})) + ((T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i}))) \\ + \frac{2(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i})) \\ + \frac{2(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i})) + (\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i})) \\ + \frac{(1-\theta)}{2} \sum_{i=1}^{n} \omega_{i} \begin{pmatrix} \frac{(T_{A}^{q}(k_{i})T_{B}^{q}(k_{i}) + F_{A}^{q}(k_{i})F_{B}^{q}(k_{i})) \\ \sqrt{((T_{A}^{2q}(k_{i}) + F_{A}^{2q}(k_{i})) \cdot \sqrt{((T_{B}^{2q}(k_{i}) + F_{B}^{2q}(k_{i})))} \\ + \frac{(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i})) \\ + \frac{(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i})) \\ + \frac{(\varphi_{T_{A}}^{q}(k_{i})\varphi_{T_{B}}^{q}(k_{i}) + \varphi_{F_{A}}^{q}(k_{i})\varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{(\varphi_{T_{A}}^{2q}(k_{i}) + \varphi_{F_{A}}^{2q}(k_{i})) \cdot \sqrt{(\varphi_{T_{B}}^{2q}(k_{i}) + \varphi_{F_{B}}^{2q}(k_{i}))}} \end{pmatrix} \right).$$
(30)



- 2.  $H_V(A, B) = H_V(B, A)$  holds obviously;
- 3. If we consider  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ , then the values of  $J_d(A, B) = 1$  and  $J_c(A, B) = 1$  are achieved; therefore,  $H_V(A, B) = 1$  is obtained, as well.

**Definition 33:** The HWVSM for Cq-ROFS is given by Eq. (30) as shown in Box XIV. Eq. (30) holds the following conditions:

- 1.  $0 \le H_{WV}(A, B) \le 1$ ,
- 2.  $H_{WV}(A,B) = H_{WV}(B,A),$
- 3.  $H_{WV}(A,B) = 1$ , for A = B, i.e.,  $T_A = T_B$ ,  $\varphi_{T_A} = \varphi_{T_B}$ ,  $F_A = F_B$ , and  $\varphi_{F_A} = \varphi_{F_B}$ .

**Definition 34:** The HVSM for IVCq-ROFS is given by Eq. (31) as shown in Box XV. Eq. (31) holds the following conditions:

1.  $0 \le H_V^{IV}(A, B) \le 1$ ,

- 2.  $H_V^{IV}(A,B) = H_V^{IV}(B,A),$
- $\begin{array}{ll} 3. \quad H_V^{IV}(A,B)=1, \mbox{ for } A=B \mbox{ i.e., } T_A^L=T_B^L, \ T_A^U=\\ T_B^U, \ F_A^L=F_B^L, \ F_A^U=F_B^U, \ \varphi_{T_A}^L=\varphi_{T_B}^L, \ \varphi_{T_A}^U=\varphi_{T_B}^U, \\ \varphi_{F_A}^L=\varphi_{F_B}^L, \mbox{ and } \varphi_{F_A}^U=\varphi_{F_B}^U. \end{array}$

**Proof:** The proofs of the above three points are shown as follows:

- 1. The DSM and CSM for IVCq-ROFS are proposed in Eqs. (25) and (27); we have  $0 \leq J_d^{IV}(A,B) \leq 1$ and  $0 \leq J_c^{IV}(A,B) \leq 1$  and then, get  $H_V^{IV}(A,B) =$  $\theta J_d^{IV}(A,B) + (1-\theta) J_c^{IV}(A,B) \leq \theta + 1 - \theta = 1$ . Further,  $0 \leq J_d^{IV}(A,B)$  and  $0 \leq J_c^{IV}(A,B)$ ; thus,  $H_V^{IV}(A,B) \geq 0$  for any values of  $\theta \in [0,1]$ . Hence,  $0 \leq H_V^{IV}(A,B) \leq 1$  holds clearly;
- 2.  $H_V^{IV}(A,B) = H_V^{IV}(B,A)$  holds clearly;
- 3. If we consider  $T_A^L = T_B^L$ ,  $T_A^U = T_B^U$ ,  $F_A^L = F_B^L$ ,  $F_A^U = F_B^U$ ,  $\varphi_{T_A}^L = \varphi_{T_B}^L$ ,  $\varphi_{T_A}^U = \varphi_{T_B}^U$ ,  $\varphi_{F_A}^L = \varphi_{F_B}^L$ , and  $\varphi_{F_A}^U = \varphi_{F_B}^U$ , then the values of  $J_d^{IV}(A, B) =$

$$H_{V}^{IV}(A,B) = \frac{1}{n} \begin{pmatrix} \frac{2(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i}))}{(((\Omega T_{A}^{2q}(k_{i}))^{2} + (\Omega F_{A}^{2q}(k_{i}))^{2}) + (((\Omega T_{B}^{2q}(k_{i}))^{2} + (\Omega F_{B}^{2q}(k_{i}))^{2}) \\ + \frac{2(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}))}{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2}) + ((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2}) \\ + \frac{(1 - \theta)}{2} \sum_{i=1}^{n} \begin{pmatrix} \frac{(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i}))}{\sqrt{(((\Omega T_{A}^{2q}(k_{i}))^{2} + (\Omega F_{A}^{2q}(k_{i}))^{2}) \cdot \sqrt{(((\Omega T_{B}^{2q}(k_{i}))^{2} + (\Omega F_{B}^{2q}(k_{i}))^{2})}} \\ + \frac{(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i}))}{\sqrt{(((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2}) \cdot \sqrt{(((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2})}} \end{pmatrix} \end{pmatrix}$$
(31)

Box XV

$$H_{WV}^{IV}(A,B) = \begin{pmatrix} \frac{\theta}{2} \sum_{i=1}^{n} \omega_{i} \begin{pmatrix} \frac{2(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i})) \\ \frac{1}{(((\Omega T_{A}^{2q}(k_{i}))^{2} + (\Omega F_{A}^{2q}(k_{i}))^{2}) + (((\Omega T_{B}^{2q}(k_{i}))^{2} + (\Omega F_{B}^{2q}(k_{i}))^{2}) \\ + \frac{2(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i})) \\ + \frac{((\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{A}}^{q}(k_{i}))^{2}) + (((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2}) \\ + \frac{(1-\theta)}{2} \sum_{i=1}^{n} \omega_{i} \begin{pmatrix} \frac{(\Omega T_{A}^{q}(k_{i})\Omega T_{B}^{q}(k_{i}) + \Omega F_{A}^{q}(k_{i})\Omega F_{B}^{q}(k_{i})) \\ \sqrt{(((\Omega T_{A}^{2q}(k_{i}))^{2} + (\Omega F_{A}^{2q}(k_{i}))^{2}) \cdot \sqrt{(((\Omega T_{B}^{2q}(k_{i}))^{2} + (\Omega F_{B}^{2q}(k_{i}))^{2})} \\ + \frac{(\Omega \varphi_{T_{A}}^{q}(k_{i})\Omega \varphi_{T_{B}}^{q}(k_{i}) + \Omega \varphi_{F_{A}}^{q}(k_{i})\Omega \varphi_{F_{B}}^{q}(k_{i})) \\ + \frac{(\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} + (\Omega \varphi_{T_{A}}^{q}(k_{i}))^{2} \cdot \sqrt{((\Omega \varphi_{T_{B}}^{q}(k_{i}))^{2} + (\Omega \varphi_{F_{B}}^{q}(k_{i}))^{2})} \end{pmatrix} \end{pmatrix}$$
(32)



1 and  $J_c^{IV}(A, B) = 1$  are achieved and therefore,  $H_V^{IV}(A,B) = 1$  is obtained, as well.

Therefore, the proofs are completed.

Definition 35: The HWVSM for IVCq-ROFS is given by Eq. (32) as shown in Box XVI. Eq. (32) holds the following conditions:

- 1.  $0 \le H_{WV}^{IV}(A, B) \le 1$ ,
- $\begin{array}{l} 2. \quad H^{IV}_{WV}(A,B) = H^{IV}_{WV}(B,A), \\ 3. \quad H^{IV}_{WV}(A,B) = 1, \mbox{ for } A = B \mbox{ i.e., } T^L_A = T^L_B, \ T^U_A = T^U_B, \ F^L_A = F^L_B, \ F^U_A = F^U_B, \ \varphi^U_{T_A} = \varphi^U_{T_B}, \ \varphi^U_{T_A} = \varphi^U_{T_B}, \\ \varphi^U_{F_A} = \varphi^L_{F_B}, \ \mbox{and } \varphi^U_{F_A} = \varphi^U_{F_B}, \end{array}$

where  $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}^T$ ,  $\omega_i \in [0, 1]$ ,  $i = 1, 2, \cdots, n$  with a condition that  $\sum_{i=1}^n \omega_i = 1$  represent the weighted vectors. If we consider  $\omega_i = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)^T$ , then Eqs. (18), (20), and (22) are converted into Eqs. (17), (19), and (21), respectively.

Similarly, if we consider  $\omega_i = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$ , then Eqs. (24), (26), and (28) are converted into Eqs. (23), (25), Eqs. (23), (25), and (27), respectively. Further, if we consider  $\omega_i = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^{T}$ ; then, Eqs. (30) and (32) are converted into Eqs. (29) and (31), respectively.

## 4. Applications based on VSMs for Cq-ROFNs

This section gives two applications such as medical diagnosis and pattern recognition.

## 4.1. An application for medical diagnosis

We diagnose the symptoms of diseases in a patient to determine a disease, and the corresponding steps are shown as follows.

- 1. Construct the matrix for symptoms  $k_i$  (j = $1, 2, \cdots, n$ ) of diseases  $B_j$  (j = 1, 2, 3, 4);
- 2. Give the symptoms of the patient P;

<b>Table 1.</b> Representation of all diagnoses.						
<b>Symbol representations</b> $B_1$ $B_2$ $B_3$ $B_4$						
Diseases	Viral fever	Malaria	Typhoid	Stomach problem		
Table 2. Representation of all symptoms.						

Table 3	3.	Decision	matrix	of	Ca-ROFNs

 $k_2$ 

Headache

 $k_3$ 

Stomach pain

 $k_4$ 

Cough

 $k_1$ 

Temperature

Values	$B_1$	$B_2$	$B_3$	$B_4$
$k_1$	$\left(0.8e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.2)}\right)$	$\left(0.6e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.5)}\right)$	$\left(0.3e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.1)}\right)$	$\left(0.5e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.6)}\right)$
$k_2$	$\left(0.9e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.2)} ight)$	$\left(0.4e^{i2\pi(0.9)}, 0.5e^{i2\pi(0.1)} ight)$	$\left(0.8e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.6)} ight)$	$\left(0.3e^{i2\pi(0.1)}, 0.6e^{i2\pi(0.3)} ight)$
$k_3$	$\left(0.7e^{i2\pi(0.8)}, 0.2e^{i2\pi(0.1)} ight)$	$\left(0.5e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.3)} ight)$	$\left(0.7e^{i2\pi(0.6)}, 0.2e^{i2\pi(0.2)} ight)$	$\left(0.8e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.5)}\right)$
$k_4$	$\left(0.8e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.1)} ight)$	$\left(0.4e^{i2\pi(0.9)}, 0.5e^{i2\pi(0.1)} ight)$	$\left(0.2e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.2)} ight)$	$\left(0.1e^{i2\pi(0.3)}, 0.6e^{i2\pi(0.5)} ight)$

Table 4. Patients' information.

Symbols	$k_1$	$k_2$	$k_3$	$k_4$
P	$\left(0.8e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.2)} ight)$	$\left(0.9e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.2)}\right)$	$\left(0.7e^{i2\pi(0.8)}, 0.2e^{i2\pi(0.1)} ight)$	$\left(0.6e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.4)}\right)$

 Using Eq. (30) to calculate the similarity measures of patient P with the diseases B<sub>j</sub> (j = 1, 2, 3, 4);

Symbol representations

Symptoms

- 4. Rank the similarity measures with all diseases  $B_j$ (j = 1, 2, 3, 4);
- 5. Determine the disease of the patient;
- 6. End.

Based on the above-mentioned steps, an example is used to express the reliability and effectiveness of the proposed approaches, taken from [23].

Numerical Example 1: We consider a set of all diseases shown in Table 1. Next, we consider a set of all symptoms listed in Table 2.

The steps for solving this problem are as follows:

- 1. Construct a matrix for symptoms  $k_j$   $(j = 1, 2, \dots, n)$  of diseases  $B_j$  (j = 1, 2, 3, 4) as shown in Table 3 (suppose the weighted vector of symptoms  $\omega = \{0.3, 0.2, 0.1, 0.4\}^T$  and q = 1).
- 2. Determine the symptoms of the patient P in terms of Cq-ROFSs in Table 4.
- 3. Employ Eq. (30) to calculate the VSMs of the patient P with the all diseases and get:

$$H_{WV}(B_1, P) = 0.96;$$
  $H_{WV}(B_2, P) = 0.85;$ 

$$H_{WV}(B_3, P) = 0.79;$$
  $H_{WV}(B_4, P) = 0.68.$ 

4. Rank the similarity measures with all diseases and obtain:

$$B_1 \ge B_2 \ge B_3 \ge B_4$$

- 5. The disease of the patient is  $B_1$ , which is a viral fever.
- 6. End.

The comparison between the proposed and existing methods with respect to this example is shown in Table 5. Further, we can give a graphical interpretation of Table 5, which is shown in Figure 2.

From Table 5, we know that all methods get the same disease for this patient, i.e., it is  $B_1$ .

Next, we will give another example for pattern recognition.

## 4.2. Another application for pattern recognition

The steps for solving the application of pattern recognition are shown as follows:

- 1. Construct the decision matrix for known patterns  $B_i$   $(i = 1, 2, \dots, m)$  and criteria  $(j = 1, 2, \dots, n)$ ;
- Get the criteria values associated with the unknown pattern P;
- 3. Use Eq. (32) to examine the similarity measures between known patterns  $B_i$  (j = 1, 2, 3, 4) with respect to unknown pattern P;

Methods	Score functions	Ranking
CIFS proposed by Rani and Garg [20]	$H_{WV}(B_1, P) = 0.96; H_{WV}(B_2, P) = 0.85;$ $H_{WV}(B_3, P) = 0.79; H_{WV}(B_4, P) = 0.68$	$B_1 > B_2 > B_3 > B_4$
CIFS proposed by Garg and Rani [23]	$H_{WV}(B_1, P) = 0.02; \ H_{WV}(B_2, P) = 0.006;$ $H_{WV}(B_3, P) = 0.008; \ H_{WV}(B_4, P) = 0.01$	$B_1 > B_4 > B_3 > B_2$
CPFS proposed by Ullah et al. [26]	$H_{WV}(B_1, P) = 0.29; \ H_{WV}(B_2, P) = 0.24;$ $H_{WV}(B_3, P) = 0.26; \ H_{WV}(B_4, P) = 0.28$	$B_1 > B_4 > B_3 > B_2$
Proposed methods in this paper for $q = 1$	$H_{WV}(B_1, P) = 0.96; H_{WV}(B_2, P) = 0.85;$ $H_{WV}(B_3, P) = 0.79; H_{WV}(B_4, P) = 0.68$	$B_1 > B_2 > B_3 > B_4$
Proposed methods in this paper for $q = 2$	$H_{WV}(B_1, P) = 0.23; \ H_{WV}(B_2, P) = 0.15;$ $H_{WV}(B_3, P) = 0.14; \ H_{WV}(B_4, P) = 0.08$	$B_1 > B_2 > B_3 > B_4$
Proposed methods in this paper for $q = 5$	$H_{WV}(B_1, P) = 0.019; H_{WV}(B_2, P) = 0.006;$ $H_{WV}(B_3, P) = 0.008; H_{WV}(B_4, P) = 0.001$	$B_1 > B_3 > B_2 > B_4$

Table 5. Comparison between the proposed method and existing methods.

<b>Table 6.</b> Representation of all alternatives.					
Symbol representations	$B_1$	$B_2$	$B_3$	$B_4$	
Alternatives	Car company	Food company	Computer company	Arms company	



Symbol representations	$k_1$	$k_2$	$k_3$	$k_4$
Criteria	Risk analysis	Growth analysis	Social impact	Political impact



Figure 2. Graphical interpretation of the proposed work with existing works in Table 5.

- 4. Rank the similarity measures  $B_i$  (j = 1, 2, 3, 4);
- 5. Choose the most desirable pattern  $B_i$  (j = 1, 2, 3, 4) to be classified with pattern P;
- 6. End.

Numerical Example 2: Here we give detailed information. Firstly, a set of all possible alternatives is considered in Table 6. Next, a set of all the criteria listed in Table 7 is considered. The steps of this decision are shown below:

- 1. Construct the decision matrix for the alternatives with respect to all the criteria, which is expressed by IvCq-ROFNs shown in Table 8 (the weighted vector of criteria is  $\omega = \{0.3, 0.2, 0.1, 0.4\}^T$ );
- 2. Determine the information on the criteria associated with alternative P, as given in Table 9;

Values	$B_1$	$B_2$	$B_3$	$B_4$
$k_1$	$\begin{pmatrix} [0.7, 0.9]e^{i2\pi[0.5, 0.9]}, \\ [0.1, 0.5]e^{i2\pi[0.2, 0.6]} \end{pmatrix}$	$\begin{pmatrix} [0.5, 0.7]e^{i2\pi[0.3, 0.5]}, \\ [0.2, 0.4]e^{i2\pi[0.5, 0.9]} \end{pmatrix}$	$\begin{pmatrix} [0.1, 0.5]e^{i2\pi[0.7, 0.9]}, \\ [0.4, 0.6]e^{i2\pi[0.2, 0.4]} \end{pmatrix}$	$\begin{pmatrix} [0.4, 0.6]e^{i2\pi[0.2, 0.4]}, \\ [0.3, 0.9]e^{i2\pi[0.4, 0.8]} \end{pmatrix}$
$k_2$	$\begin{pmatrix} [0.1, 0.5]e^{i2\pi[0.5, 0.7]}, \\ [0.1, 0.5]e^{i2\pi[0.2, 0.6]} \end{pmatrix}$	$\begin{pmatrix} [0.3, 0.5]e^{i2\pi[0.89, 0.91]}, \\ [0.5, 0.9]e^{i2\pi[0.2, 0.4]} \end{pmatrix}$	$\begin{pmatrix} [0.7, 0.9]e^{i2\pi[0.2, 0.4]}, \\ [0.2, 0.4]e^{i2\pi[0.7, 0.9]} \end{pmatrix}$	$\begin{pmatrix} [0.1, 0.5]e^{i2\pi[0.01, 0.19]}, \\ [0.7, 0.9]e^{i2\pi[0.2, 0.4]} \end{pmatrix}$
$k_3$	$\begin{pmatrix} [0.1, 0.6]e^{i2\pi [0.7, 0.9]}, \\ [0.2, 0.6]e^{i2\pi [0.1, 0.5]} \end{pmatrix}$	$\begin{pmatrix} [0.4, 0.6]e^{i2\pi[0.4, 0.6]}, \\ [0.4, 0.6]e^{i2\pi[0.4, 0.6]}, \\ [0.4, 0.6]e^{i2\pi[0.4, 0.6]} \end{pmatrix}$	$\begin{pmatrix} [0.2, 0.1]^{e} \\ [0.6, 0.8] e^{i 2\pi [0.5, 0.7]}, \\ [0.39, 0.41] e^{i 2\pi [0.39, 0.41]} \end{pmatrix}$	$ \begin{pmatrix} [0.7, 0.9]e^{i2\pi[0.2, 0.4]}, \\ [0.2, 0.4]e^{i2\pi[0.4, 0.6])} \end{pmatrix} $
$k_4$	$\begin{pmatrix} [0.7, 0.9] e^{i2\pi [0.5, 0.9]}, \\ [0.2, 0.6] e^{i2\pi [0.1, 0.5]} \end{pmatrix}$	$\begin{pmatrix} [0.3, 0.5]e^{i2\pi[0.89, 0.91]}, \\ [0.5, 0.9]e^{i2\pi[0.1, 0.5]} \end{pmatrix}$	$\begin{pmatrix} [0.39, 0.41]e^{i2\pi[0.5, 0.9]}, \\ [0.7, 0.9]e^{i2\pi[0.39, 0.41]} \end{pmatrix}$	$\begin{pmatrix} ([0.01, 0.19]e^{i2\pi[0.2, 0.4]}, \\ [0.5, 0.7]e^{i2\pi[0.4, 0.6]} \end{pmatrix}$

 Table 8. Decision matrix of IVCq-ROFNs in Example 2.

Table 9. Information for alternative P.

Symbols	$k_1$	$k_2$	$k_3$	$k_4$
Р	$\begin{pmatrix} [0.7, 0.9]e^{i2\pi[0.5, 0.7]}, \\ [0.2, 0.4]e^{i2\pi[0.3, 0.5]} \end{pmatrix}$	$\begin{pmatrix} [0.89, 0.91]e^{i2\pi[0.6, 0.8]}, \\ [0.2, 0.4]e^{i2\pi[0.3, 0.5]} \end{pmatrix}$	$\begin{pmatrix} [0.6, 0.8]e^{i2\pi[0.7, 0.9]}, \\ [0.1, 0.7]e^{i2\pi[0.2, 0.4]} \end{pmatrix}$	$\begin{pmatrix} [0.4, 0.8]e^{i2\pi[0.4, 0.6]}, \\ [0.3, 0.5]e^{i2\pi[0.5, 0.7]} \end{pmatrix}$

Table 10. Decision matrix of Cq-ROFNs in Example 3.

Values	$B_1$	$B_2$	$B_3$	$B_4$
$k_1$	$\left(0.8e^{i2\pi(0.7)}, 0.3e^{i2\pi(0.4)} ight)$	$\left(0.6e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.7)} ight)$	$\left(0.3e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.3)} ight)$	$\left(0.5e^{i2\pi(0.3)}, 0.6e^{i2\pi(0.8)} ight)$
$k_2$	$\left(0.9e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.4)} ight)$	$\left(0.4e^{i2\pi(0.9)}, 0.7e^{i2\pi(0.3)} ight)$	$\left(0.8e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.8)} ight)$	$\left(0.3e^{i2\pi(0.1)}, 0.8e^{i2\pi(0.5)} ight)$
$k_3$	$\left(0.7e^{i2\pi(0.8)}, 0.4e^{i2\pi(0.3)} ight)$	$\left(0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)} ight)$	$\left(0.7e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.4)}\right)$	$\left(0.8e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.7)} ight)$
$k_4$	$\left(0.8e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.3)}\right)$	$\left(0.4e^{i2\pi(0.9)}, 0.7e^{i2\pi(0.3)} ight)$	$\left(0.4e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.4)} ight)$	$\left(0.1e^{i2\pi(0.3)}, 0.8e^{i2\pi(0.7)}\right)$

Table 11. Decision matrix for unknown values.

$\mathbf{Symbols}$	$k_{1}$	$k_2$	$k_3$	$k_4$
Р	$\left(0.8e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.4)}\right)$	$\left(0.9e^{i2\pi(0.7)}, 0.3e^{i2\pi(0.4)} ight)$	$\left(0.7e^{i2\pi(0.8)}, 0.4e^{i2\pi(0.3)} ight)$	$\left(0.6e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.6)}\right)$

3. Use Eq. (32) to calculate the VSMs and get:

$$H_{WV}^{IV}(B_i, P) = 0.075;$$
  $H_{WV}^{IV}(B_2, P) = 0.042;$ 

$$H_{WV}^{IV}(B_3, P) = 0.045;$$
  $H_{WV}^{IV}(B_4, P) = 0.020.$ 

4. Rank all IvCq-ROFNs and obtain:

$$B_1 \ge B_3 \ge B_2 \ge B_4.$$

- 5. Thus,  $B_1$  is the best alternative, which is car company;
- 6. End.

### 4.3. Comparative analysis

This subsection utilizes a numerical example to demonstrate the reliability of the proposed method in comparison to the existing methods. **Numerical Example 3:** This example is revised from Numerical Example 1.

- 1. The symptoms for all diseases are shown in Table 10 (suppose the weighted vectors  $\omega = \{0.3, 0.2, 0.1, 0.4\}^T$  and q = 2);
- 2. All symptoms of the patient P in terms of Cq-ROFSs are given in Table 11;
- Eq. (30) is used to calculate the VSMs of the patient P with the all diseases as follows:

$$H_{WV}(B_1, P) = 0.075;$$
  $H_{WV}(B_2, P) = 0.042;$ 

$$H_{WV}(B_3, P) = 0.045;$$
  $H_{WV}(B_4, P) = 0.028.$ 

4. The similarity measures for all diseases are ranked below:

$$B_1 \ge B_3 \ge B_2 \ge B_4$$

Methods	Score functions	Ranking
CIFS proposed by Rani and Garg [20]	Cannot be calculated	Cannot be calculated
CIFS proposed by Garg and Rani [23]	Cannot be calculated	Cannot be calculated
CPFS proposed by Ullah et al. [26]	$H_{WV}(B_1, P) = 0.070; \ H_{WV}(B_2, P) = 0.036;$ $H_{WV}(B_3, P) = 0.039; \ H_{WV}(B_4, P) = 0.014$	$B_1 > B_3 > B_2 > B_4$
Proposed methods	$H_{WV}(B_1, P) = 0.075; H_{WV}(B_2, P) = 0.041;$ $H_{WV}(B_3, P) = 0.044; H_{WV}(B_4, P) = 0.019$	$B_1 > B_3 > B_2 > B_4$

Table 12. Comparison between the proposed methods and existing methods for Example 3.

Values	$B_1$	$B_2$	$B_3$	$B_4$
$k_1$	$\left(0.8e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.4)}\right)$	$\left(0.6e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.7)}\right)$	$\left(0.89e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.3)} ight)$	$\left(0.86e^{i2\pi(0.3)}, 0.6e^{i2\pi(0.8)}\right)$
$k_2$	$\left(0.9e^{i2\pi(0.6)}, 0.8e^{i2\pi(0.4)} ight)$	$\left(0.4e^{i2\pi(0.9)}, 0.9e^{i2\pi(0.3)} ight)$	$\left(0.8e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.8)}\right)$	$\left(0.87e^{i2\pi(0.1)}, 0.8e^{i2\pi(0.5)}\right)$
$k_3$	$\left(0.7e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.3)} ight)$	$\left(0.5e^{i2\pi(0.5)}, 0.9e^{i2\pi(0.5)}\right)$	$\left(0.7e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.4)}\right)$	$\left(0.8e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.7)} ight)$
$k_4$	$\left(0.8e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.3)} ight)$	$\left(0.4e^{i2\pi(0.9)}, 0.8e^{i2\pi(0.3)} ight)$	$\left(0.8e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.4)} ight)$	$\left(0.9e^{i2\pi(0.3)}, 0.8e^{i2\pi(0.7)} ight)$

Table 13. Decision matrix of Cq-ROFNs in Example 4.

Table 14. Symptoms of the patient P in Example 4.

$\mathbf{Symbols}$	$k_1$	$k_2$	$k_3$	$k_4$
Р	$\left(0.8e^{i2\pi(0.6)}, 0.8e^{i2\pi(0.4)} ight)$	$\left(0.9e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.4)}\right)$	$\left(0.7e^{i2\pi(0.8)}, 0.89e^{i2\pi(0.3)}\right)$	$\left(0.6e^{i2\pi(0.5)}, 0.88e^{i2\pi(0.6)}\right)$



Figure 3. Comparison between the proposed methods and existing methods for Example 3.

The disease of the patient is  $B_1$ , which is a viral fever.

#### 5. End.

The comparison between the proposed method and existing methods with respect to Example 3 is shown in Table 12. Further, a graphical interpretation of Table 12 is shown in Figure 3.

From Table 12, it is clear that our proposed method is more effective and more general to cope

with uncertain and complex information. Obviously, in this example, the CIFS cannot be used to solve this decision problem; thus, the methods proposed in [20,23] are invalid. In addition, it can be seen that the CPFS can solve this example and get the same ranking result as that produced by the proposed method; hence, the validity of the proposed method is proven. In the next example, the advantage of the proposed method versus the method proposed by Ullah et al. [26] is shown further. **Numerical Example 4:** This example is the modified form of Numerical Example 1.

- 1. The symptoms for all diseases are shown in Table 13 (suppose the weighted vectors  $\omega = \{0.3, 0.2, 0.1, 0.4\}^T$  and q = 2);
- 2. All symptoms of the patient *P* in terms of Cq-ROFSs are denoted in Table 14;
- 3. Eq. (30) is employed to calculate the VSMs of the patient P with the all diseases:

 $H_{WV}(B_1, P) = 0.086;$   $H_{WV}(B_2, P) = 0.053;$ 

$$H_{WV}(B_3, P) = 0.056;$$
  $H_{WV}(B_4, P) = 0.039.$ 

4. The similarity measures for all diseases are ranked below:

 $B_1 \ge B_3 \ge B_2 \ge B_4.$ 

Thus, the disease of the patient is  $B_1$ , which is a viral fever;

5. End.

The comparison between the proposed method and existing methods with respect to Example 4 is shown in Table 15.

From Table 15, it is clear that our proposed method is more effective and more general to cope with uncertain and complex information. Further, a graphical interpretation of Table 15 is shown in Figure 4. Obviously, the proposed method can provide a much information expression in decision-making problems.

## 4.4. Advantages of the proposed similarity measures

The similarity measures investigated in this paper are the extension of existing ones in [54], and the proposed method is the generalization of the existing methods discussed in [54].

**Remark 1:** When q = 2, Eqs. (17) and (18) are converted into JSM and WJSM of CPFSs, and when q = 1, Eqs. (17) and (18) are converted to JSM and WJSM for CIFSs.

**Remark 2:** When q = 2, Eqs. (19) and (20) are converted into DSM and WDSM of CPFSs, and when q = 1, Eqs. (19) and (20) are converted into DSM and WDSM for CIFSs.

**Remark 3:** When q = 2, Eqs. (21) and (22) are converted into CSM and WCSM of CPFSs, and when q = 1, Eqs. (21) and (22) are converted into CSM and WCSM for CIFSs.

**Remark 4:** When q = 2, Eqs. (23) and (24) are converted into JSM and WJSM of IVCPFSs, and when q = 1, Eqs. (23) and (24) are converted into JSM and WJSM for IVCIFSs.

Table 15. Comparison between the proposed methods and existing methods for Example 4.

${f Methods}$	Score functions	Ranking
CIFS proposed by Rani and Garg [20]	Cannot be calculated	Cannot be calculated
CIFS proposed by Garg and Rani [23]	Cannot be calculated	Cannot be calculated
CPFS proposed by Ullah et al. [26]	Cannot be calculated	Cannot be calculated
Proposed methods	$H_{WV}(B_1, P) = 0.086; H_{WV}(B_2, P) = 0.053;$ $H_{WV}(B_2, P) = 0.056; H_{WV}(B_4, P) = 0.039;$	$B_1 \ge B_3 \ge B_2 \ge B_4$



Figure 4. Comparison between the proposed methods and existing methods for Example 4.

**Remark 5:** When q = 2, Eqs. (25) and (26) are converted into DSM and WDSM of IVCPFSs, and when q = 1, Eqs. (25) and (26) are converted into DSM and WDSM for IVCIFSs.

**Remark 6:** When q = 2, Eqs. (27) and (28) are converted into CSM and WCSM of IVCPFSs, and when q = 1, Eqs. (27) and (28) are converted into CSM and WCSM for IVCIFSs.

**Remark 7:** When q = 2, Eqs. (29) and (30) are converted into HVSM and HWVSM of CPFSs, and when q = 1, Eqs. (29) and (30) are converted into HVSM and HWVSM for CIFSs.

**Remark 8:** When q = 2, Eqs. (31) and (32) are converted into HVSM and HWVSM of IVCPFSs, and when q = 1, Eqs. (31) and (32) are converted into HVSM and HWVSM for IVCIFSs.

In a word, our proposed method is more reliable and more general than existing methods, to be described below.

Few years ago, the CIFS [15] was successfully applied in different areas; yet, it faced some real-world problems. The CIFS cannot effectively elaborate on complex kinds of information in real decision-making. In order to handle these problems, Ullah et al. [26] proposed the concept of CPFS and its constraint condition was that the sum of the square of the real membership (also for imaginary) and square of the real non-membership (also for imaginary) was limited to [0, 1]. Obviously, the CPFS was more general than CIFS. However, the problem that remains is that when a decision-maker provides a flawed type of information, it does not satisfy the conditions of CIFS and CPFS. To deal with this constraint, the Cq-ROFSs, characterized by complex-valued membership and complex-valued non-membership, are proposed here. The advantage of the Cq-ROFS is that the sum of the q-power of the real membership (also for imaginary) and q-power of the real non-membership (also for imaginary) is bound to [0, 1]. The Cq-ROFSs are more generalizable than CIFSs and CPFSs in dealing with uncertain and complicated information in the fuzzy set theory. For example, when a decision-maker assigns  $(0.9e^{i2\pi(0.78)})$  to complex-valued membership grade and  $(0.88e^{i2\pi(0.80)})$ to complex-valued non-membership grade, respectively, the validity holds only for the case of Cq-ROFS. On the other hand, all the complex intuitionistic fuzzy degrees and complex Pythagorean fuzzy degrees are part of the complex q-rung orthopair fuzzy degrees, thus demonstrating that Cq-ROFS is more powerful and more comprehensive than CIFS and CPFS.

In addition, the similarity measures for IFS, CIFS, PFS, q-ROFS, and CPFS are the special cases

of the proposed similarity measures. Therefore, the proposed method is more general and more reliable in solving the real-life problem more accurately than existing methods.

### 5. Conclusion

This study introduced the background of IFSs, CIFSs, PFSs, CPFSs, and q-ROFSs in detail and explained their limitations in expressing uncertain and complex information. Further, the Cq-ROFS and IVCq-ROFS were proposed and their fundamental properties were discussed. At the same time, some numerical examples were given to explain these concepts. Obviously, the Cq-ROFS was an extension of CFS, CPFS, PFS, IFS, and FS, and it managed to express the uncertain and complicated fuzzy information better. In addition, the Cq-ROFS contains two-dimensional information in a single set whose restriction is that the sum of the q-power of the real part of the membership (also for imaginary part) and q-power of the real part of the non-membership (also for imaginary part) is less than or equal to 1. Based on these advantages, the VSMs including Jaccard, dice, and cosine similarity measures for Cq-ROFSs and IVCq-ROFSs were investigated. Further, the HVSMs called VCSMs for Cq-ROFSs and IVCq-ROFSs were proposed and their fundamental properties were discussed. To demonstrate the feasibility of the proposed HVSMs, the existing similarity measures based on CPFSs and CIFSs were compared with the proposed methods with respect to some numerical examples such as medical diagnosis and pattern recognition.

In the future, we will research some new similarity measures for different fuzzy sets such as similarity measures [55,56], linguistic neutrosophic set [57], probabilistic linguistic information [58], linguistic D number [59], and interval type-2 fuzzy set [60] or apply them to solve the MADM or MAGDM problems [61–64].

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