Dynamic pricing in a semi-centralized dual-channel supply chain with a reference price dependent demand and production cost disruption: the case of Iran Khodro Company

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Abstract

During the years of imposed sanctions against Iran, Iran Khodro Company (IKCO) got into a hazardous situation due to CKD parts’ purchasing cost increment and emersion of new product variants in the competitive market. To examine such situation, this study examines a multi-period semi-centralized dual-channel supply chain where a common retailer (free market) and two manufacturers’ (IKCO and Saipa as a major competitor) direct channels are confronted with reference price dependent and stochastic demand. The problem is analyzed under Stackelberg and cooperative games scenarios using heuristic algorithm and a League Championship algorithm respectively, as solution methods. Results obtained from solving the problem with IKCO data proves higher profitability of the cooperative game and its remarkable resilience for all products’ memory types i.e. short/long term memory against production cost disruption which is imposed to IKCO in some periods. Besides calculating Saipa’s optimal wholesale price in the disruption periods, our approach with support of experimental analyses is able to assign a supply chain’s degree of resilience against disruptions to its product’s memory type and also power structure.

1 Introduction

In industrial world, considerable increase in number of manufacturers stimulates their concern about continuity and as a result, persuades them to enhance their product to meet customer’s preferences and interests. Therefore, the manufacturers should have a close relationship with the customers. Dual-channel supply chain is a kind of supply chain providing this
relationship. The dual-channel supply chain, as it is evident from its name, has two selling channels, including retail channel (also known as traditional channel) in which product is offered by retailer and E-direct channel wherein selling of the product is conducted by the manufacturer and ordinarily using internet. Academic researches orientation towards the dual-channel supply chain and using this type of supply chain by top manufacturers of the world like Samsung, HP, IBM, Sony, Dell, Lenovo, Panasonic, and Pioneer Electrics demonstrate the profitability and its vital role in survival of a manufacturer [1]. Some Iranian companies like IKCO, Saipa, Pars Shahab Light Company and Rasa Nour Neishabour Company have two selling channels.

Pricing is one of the crucial decisions in the supply chain and is influential in its profitability. There is a strong literature in supply chain pricing field. Due to the fact that most of the researches utilized game theoretical approach, some of them are indicated briefly in this section. Sana et al (2014) studied a three-layer supply chain and considered both collaborative and Stackelberg scenarios [2]. Modak et al (2016) studied a closed loop supply chain and considered cooperative and Stackelberg games scenarios that in the stackelberg game, the retailers’ competition was modeled under Cournot and collusion games [3]. Modak et al (2016) investigated a supply chain wherein they considered manufacturer-led Stackelberg vertical game in which the retailers have three different behaviors namely Cournot, Collusion and Stackelberg. They utilized all units discount contract with franchise fee as a channel conflict respondent [4]. Sana et al (2017) considered knowledge management approach in agro-industrial supply chain of cocoa wherein they took into consideration collection centers-led Estackelberg and collaboration scenarios [5]. Modak et al (2018) considered a two-echelon closed loop supply chain in which in addition to considering three possible collection activities of used product for recycling namely retailer led collection, manufacturer led collection and third party led collection, they introduced the concept of sub-game perfect equilibrium and alternative offer bargaining strategy to resolve the channel conflict and distribute surplus profit [6] Roy et al (2018) studied a two-echelon supply chain and obtained optimal order quantity under Estackelberg, Bertland, Cournot-Bertland and integrated scenarios [7].

It is worth mentioning that, the importance of pricing in the dual-channel supply chain is higher than the one-channel supply chain due to the existence of price competition that generates channel conflict. Multiplicity of the channel conflict-related researches in the literature proves the critical importance of this field. These researches have analyzed the channel conflict and introduced some strategies to decline the effects of the conflict, additionally. The strategies are divided into two groups, including use of contracts and improvement of sales services as well as customer loyalty. In the first group, Tsay and Agraval (2004) utilized game theory to examine the channel conflict and coordinate the chain members as well [8]. Mukhapaday (2006) instituted profit sharing contract while the retailer has the ability to add on some values to the product
Yan (2008) used game theoretic approach in order to analyze strategic role of the profit sharing among the chain members and improve coordination and performance of the chain [10]. Later, Chen (2015) and Panda et al. (2015) utilized the profit sharing contract in their researches [11, 12]. Cay et al. (2009) investigated the influence of quantity discount contract on competition of the dual-channel supply chain [13]. Cay (2010) studied the effects of channel coordination contract on the members in two one-channel and two dual-channel supply chains scenarios [14]. Yan and Pei (2011) considered a multiple-channel supply chain consisting of numerous manufacturers each of which has private information about customers’ demand, and one retailer. They represented information sharing contract [15]. Cao (2013) presented optimum wholesale price contract in a dual-channel supply chain under cost information asymmetry and total information asymmetry scenarios [16]. Modak et al (2014) studied a dual-channel supply chain and took into consideration social responsibility in both decentralized and centralized scenarios. As a channel coordination strategy, they introduced all unit quantity discounts with an agreement of a franchise fee and surplus profit division using bargaining [17].

Matsui (2016) suggested that direct channel’s price should be determined before wholesale price or simultaneously to prevent channel conflict effects [18]. Li et al. (2016) studied a green dual-channel supply chain and represented two-part tariff contract to coordinate the decentralized supply chain [19]. Li et al. (2016) considered a supply chain with a risk-neutral manufacturer and a risk-averse retailer and introduced risk-sharing contract for the coordination of the chain members [20]. Liu et al. (2016) investigated the effects of risk aversion on the optimal strategies of a dual-channel supply chain under complete symmetric and asymmetric information scenarios. They concluded that the information asymmetry increases the wholesale and retail prices yet reduces the direct sale price [21]. Wang et al (2017) considered the pricing and service decisions of complementary products in a dual-channel supply chain with two manufacturers and one retailer that one of the manufacturers uses direct channel besides the retailer [22]. Xu et al (2018) considered the coordination problem in a dual-channel supply chain under mandatory carbon emission capacity regulation. Regarding that, costumers are either brand loyal or retailer loyal; they investigated the problem in centralized and decentralized scenarios. They also introduced online channel price discount and offline channel price discount contracts for the coordination. Two different types of pricing form and different structures were analyzed using backward induction [23]. Modak and kelle (2019) considered the coordination problem in a dual-channel supply chain with one manufacturer and one retailer all of which has a price and delivery time dependent stochastic demand. They proposed a hybrid all-unit quantity discount along with a franchise fee contract for the coordination of both centralized and decentralized scenarios [24].

The second group is dedicated to the sales services improvement along with the customer loyalty. Customers’ preferences for channel choice are influential incredibly in the sales level. As the manufacturer sells its product through the direct
channel, the retailer tries to obtain the customers’ satisfaction and hence increases the sales scale by means of improving the sales services. Dan (2012) studied optimal decisions for the pricing and sales services of the retailer in both centralized and decentralized dual-channel supply chain scenarios [25]. Zhao et al (2017) considered the pricing problem in a dual-channel supply chain with two manufacturers and one retailer along with complementary products. They represented four pricing game models and evaluated them based on the customer loyalty and level of complementarity [26].

Other effective factor in the supply chain profit is unpredicted disruptions. Disruption management is one of the most important aspects of supply chain management, since they affect directly the chain performance. The researches carried out in recent years separated the disruptions into four groups. The intended groups are demand disruption [27, 28, 29, 30, 31] supply disruption [32, 33, 34], production cost disruption [35], simultaneous demand and production cost disruptions [36, 37, 38, 39].

There are only three researches in the dual-channel supply chain literature that are related to risk and disruption. Huang et al. (2012, 2013) respectively studied demand and production cost disruptions in a centralized/ decentralized dual-channel supply chain [40, 41]. Xu et al. (2014) proposed a pricing model in a risk-averse centralized/ decentralized dual-channel supply chain [42].

Reference price that has been used abundantly in economic and marketing literature has a very weak literature in the supply chain area. The reference price is an expected buying price of a product formed by the customers, and referring to the literature, it reflects the customers’ last purchasing experiences [43]. The customers conceive an expected price to buy a commodity. This expected price is known as the reference price. There are two possible situations in the market; if the reference price is lower than observed price (actual price of the commodity), then the demand increases, otherwise the demand decreases. There are several ways to determine the reference price amount like selling price of the best brand and buying price of the last bought brand [44]. The idea of the reference price is originated from Adaptation– level theory and Assimilation [45]. There are only four researches in the supply chain literature that utilize the reference price. Geng et al. (2010) presented a pricing strategy in a supply chain with the reference price and deterministic demand [46]. Zhang et al. (2013) studied the influence of advertisement on the reference price quantity in a supply chain [43]. Zhang et al. (2014) considered a supply chain wherein there are the reference price effects, and members determine their pricing strategy independently [47]. Lin (2016) analyzed price promotion in a supply chain and concluded that the reference price mitigates double-marginalization effects and improves the chain efficiency [48]. Taleizadeh et al (2017) studied a three-level closed loop supply chain and proposed a joint optimization model of pricing strategies, quality levels, effort decisions and return policy regarding the reference price under different game structure scenarios [49].
The reviewed literature in this study reveals some weaknesses as follows:

- The majority of the researches carried on in the dual-channel supply chain pricing area have been taken into consideration from managerial point of view. However, there are minor researches that have taken into account this problem from operational perspective.

- Disruption in the dual-channel supply chain has a weak literature.

- The researches frequently used a supply chain with one manufacturer and one retailer, and some of which have a supply chain with one manufacturer and multiple retailers. There are some researches that consider supply chains comprising multiple manufacturers. However, in real world, there are considerable numbers of supply chains that have more than one manufacturer.

Furthermore, sanctions imposed against Iran industry led to substantial losses and economic stagnations. Automobile industry, one of the biggest industries of Iran, confronted with profound problems in supplying raw material requirements in sanctions periods. Peugeot 206 is among IKCO’S automobiles that faced critical issues as IKCO had been prohibited from Peugeot Company; thus, its CKD parts supply had been disrupted. First strategy of IKCO to challenge this disruption was meeting its requirements via intermediate countries. Thereby, purchasing cost of CKD parts had a considerable increment. On the other hand, Saipa Company proceeded to purchasing production lines of Chinese automobiles, and launched producing these automobiles and entering them into Iran automobile market. The act of Saipa which is regarded as a threat to IKCO has stimulated its concern, since the Chinese automobiles are absorbing demand due to lower prices; hence, they are regarded as competitors of IKCO’S automobiles. For these reasons, this paper is conducted in the dual-channel supply chain disruption and pricing area. This paper considers a dual-channel supply chain that consists of two competing manufacturers (IKCO and Saipa) each owning two selling channels and a common retailer. One manufacturer (IKCO) suffered from the production cost (excluded to purchasing cost of CKD parts) disruption that occurs in some periods.

2 Problem Definition

The research considers a dual-channel supply chain with two competing manufacturers and a common retailer. Each manufacturer produces a different yet substitutable product and sells them through the direct channel and the retailer known as the retail channel. There is price competition among these four channels. The issue has multiple periods and in some periods, one manufacturer faces disruption in the production cost which is due to purchasing cost of the raw materials. The popularity of the corresponding manufacturer is lowered inconsiderably because of its higher brand. Demand \((D_{ijt})\) faced all selling channels is stochastic and it depends on the reference price and other channels’ selling price. In each period, the
retailer orders the quantity $q_{irit}$ to manufacturer $i$ considering its expected demand ($d_{irit}$) and positive inventory of the last previous period ($I^*_{irit-1}$). Manufacturer $i$ produces the product at unit cost $C_{it}$ which has a different value in disruption periods, and provides the retailer’s order completely at unit wholesale price $w_{it} > C_{it}$, as well as allocates the quantity $q_{idt}$ to its direct channel regarding its expected demand ($d_{idt}$). Considering product $i$’s quantity in channel $r$ as $q_{irit}$ (sum of $q_{irit}$ and $I^*_{irit-1}$), if the retailer’s demand (manufacturer $i$’s demand) is less than $q_{irit}$ ($q_{idt}$), the retailer (manufacturer $i$) endures holding cost of $h_{r}$ ($h_{i}$) per inventory unit. Conversely, if $q_{irit} < D_{irit}$ ($q_{idt} < D_{idt}$), then the retailer (manufacturer $i$) bears lost sales cost of $b_{r}$ ($b_{i}$) per unit. Production level ($Q_{it}$) of manufacturer $i$ in a period is calculated by adding $q_{irit}$ to $q_{idt}$ and subtracting $I^*_{irit-1}$ (positive inventory of the last previous period in case of $q_{idt-1} > D_{idt-1}$) from it. Broadly, decision variables for manufacturer $i$ are $p_{idt}$, $w_{it}$, $q_{idt}$ and $Q_{it}$ and for the retailer are $q_{irit}$ and $p_{irit}$.

This paper is conducted in IKCO. IKCO is the manufacturer facing disruption in the production cost and the other manufacturer is Saipa. These are the most carmaker companies in Iran and most probably in the Middle East. The Retailer is assumed free market that sells the mentioned manufacturers’ product. The product for each manufacturer is selected in a way that they have about equal prices. Therefore, Peugeot 206 for IKCO and Brilliance (H230) for Saipa are selected. The aim of this paper is developing an appropriate pricing model such that it can represent optimum level of prices for each period in order to maximize each member’s profit.

As shown in Figure (1), IKCO and Saipa are in control of Competition Committee, a State organization that investigates prices to control inflation and create a stable and sustainable economic growth, hence semi-centralized dual-channel supply chain is innovated. This type of supply chain is a compound of centralized and decentralized supply chains, that is to say, some parts of the chain act centrally but these parts along with the others has a decentralized relationship.

3 Mathematical Modeling

As it was mentioned in section (2), the proposed problem is a multi-period one and the extra inventory of a period is transferred to the next period. For this reason and due to the existence of the reference price effects on the periods’ demand, dynamic mathematical model is used to model the problem. This section represents each member’s mathematical model of the chain after representing parameter definition as follows.

Sets:

- $j, k, j, k = \{r, d\}$ Channel indexes.
- $i, l, i, l = \{1, 2\}$ Manufacturer (Product) indexes.
Parameters:

\( R \)  
Retail channel index.

\( D \)  
Direct channel index.

\( D_{ijt} \)  
Total demand for product \( i \) in channel \( j \) in period \( t \).

\( d_{ijt} \)  
Expected demand for product \( i \) in channel \( j \) in period \( t \).

\( \beta_{ijt} \)  
Elasticity coefficient for product \( i \) in channel \( j \) in period \( t \).

\( y_{ij} \)  
Cross effect coefficient of \( p_{klt} \).

\( \theta_{ij} \)  
Market share of \( j \)th channel of product \( i \).

\( p_{it} \)  
Average observed price of \( i \)th product in period \( t \).

\( p_{ijt} \)  
Price of product \( i \) in channel \( j \) in period \( t \).

\( \mu_{ij} \)  
Potential demand of \( i \)th product in channel \( j \) if \( p_{ijt} = 0 \).

\( M \)  
Market potential demand if \( \forall i, j, p_{ijt} = 0 \).

\( A \)  
Memory effect.

\( g_1 \)  
Customers’ sensitivity coefficient for reference and observed prices difference in no disruption periods.

\( g_2 \)  
Customers’ sensitivity coefficient for reference and observed prices difference in disruption periods.

\( \varepsilon_{ijt} \)  
Stochastic element of demand function for product \( i \) in channel \( j \) in period \( t \).

\( M_{it} \)  
Price of product \( i \) which is determined by marketing department of manufacturer \( i \).

\( Q_{it} \)  
Production quantity of manufacturer \( i \) in period \( t \).

\( q_{it} \)  
Order level of \( i \)th product of the retailer in period \( t \).

\( q'_{irt} \)  
\( i \)th product’s quantity of the retailer in period \( t \).

\( q_{iadt} \)  
Level of products allocated to channel \( d \) of manufacturer \( i \) in period \( t \).

\( I_{it} \)  
Inventory level of manufacturer \( i \) in period \( t \).

\( I^*_{it} \)  
Positive inventory level of manufacturer \( i \) in period \( t \).

\( I_{rt} \)  
Inventory level of \( r \)th channel of product \( i \) in period \( t \).

\( I^*_{rt} \)  
Positive inventory level of \( r \)th channel of product \( i \) in period \( t \).

\( c_{it} \)  
Unit production cost of product \( i \) in period \( t \).

\( \Delta c_{it} \)  
Unit production cost variation of product \( i \) in period \( t \).

\( C_{it} \)  
Total unit production cost of product \( i \) in period \( t \).
\( h_r \)  Unit holding cost of the retailer.

\( h_i \)  Unit holding cost of manufacturer \( i \).

\( b_r \)  Unit lost sales cost of the retailer.

\( b_i \)  Unit lost sales cost of manufacturer \( i \).

\[ \Pi_{ir} \]  Profit function of \( r \)th channel of product \( i \) in period \( t \).

\[ \Pi_r \]  Profit function of the retailer in period \( t \).

\[ \Pi_i \]  Profit function of manufacturer \( i \) in period \( t \).

\[ \Pi_{cp} \]  Profit function of the centralized part in period \( t \).

\[ \Pi_{ec} \]  Total profit function of the dual-channel supply chain.

Binary Parameters:

\[ \varphi = \begin{cases} 
1 & \text{If it is a no disruption period} \\
0 & \text{O.W.} 
\end{cases} \]

3.1 Demand function

IKCO historical data have revealed that variation between the amount of real and estimated demands is somewhat low. Moreover, they have indicated that the demand of a period depends on its previous periods’ demand. Therefore, the demand faced all selling channels is the reference price and other channels’ selling price dependent and stochastic.

Now, demand function for each selling channel in every period is denoted by

\[ D_{ijt} = d_{ijt} + \varepsilon_{ijt} \]

(1)

Eq. (1) indicates that the demand function comprises two parts namely deterministic and stochastic demands. The expected deterministic demand function is defined as

\[ d_{ijt} = \mu_{ij} - \beta_{ij} p_{ijt} + \sum_{v \in \{k,l\} \neq i} \sum_{k \neq i} y_{ijk} p_{ks} + (\varphi g_1 + (1-\varphi) g_2) (r_{it} - p_{it}) \]

(2)

Where \( \mu_{ij} = \mu \times \theta_{ij} \), and \( \sum_{i=1}^{2} \sum_{j=1}^{r} \theta_{ij} = 1 \). \( \beta_{ij} \) is elasticity coefficient of \( i \)th product of \( j \)th channel in period \( t \). This coefficient indicates the reverse effects of price changes of a channel on its demand. \( y_{ijk} \) is named cross effect coefficient of \( i \)th product of channel \( k \) in period \( t \). This coefficient measures the direct impact of a channel’s price changes on the other channels’ demand. \( g_1 \) and \( g_2 \) are customers’ sensitivity coefficients for the reference and observed prices difference respectively, in no
disruption and disruption periods. If the reference and observed prices have a considerable difference, \( r \) has a greater value. \( \phi \) is binary disruption parameter and gets 1 and zero respectively, in no disruption and disruption periods. \( r \) is called the reference price which is calculated as

\[
 r_{it} = \alpha r_{i(t-1)} + (1-\alpha) p_{i(t-1)} \\
0 \leq \alpha \leq 1
\]  

(3)

Where \( \alpha \), which is the crucial parameter in determining the reference price, is called memory effect. The memory effect is customers’ memory about previous prices. If the product has a short-time memory, the amount of \( \alpha \) is near to zero, else if the product has a long-time memory, the quantity of \( \alpha \) is given near to 1.

\[
p_{i(t-1)} = \frac{(p_{i(t-1)} + p_{d(i(t-1))})}{2}
\]

(4)

The other conditions are assumed as follows [50]:

\[
\frac{\partial d_{ijt}}{\partial p_{ijt}} < 0 \quad \begin{cases} i = \{1, 2\}, j = \{r, d\} 
\end{cases}
\]

(5)

\[
\frac{\partial d_{ijt}}{\partial p_{ijt}} > 0 \quad \begin{cases} i, l = \{1, 2\}, j, k = \{r, d\}, i \neq l \end{cases}
\]

(6)

\[
B_{ijt} > y_{lkt} \quad \begin{cases} i, l = \{1, 2\}, j, k = \{r, d\} 
\end{cases}
\]

(7)

Eq. (5) relates that \( d_{ijt} \) decreases with the increase in \( p_{ijt} \). This amount of decrease is transferred to the other manufacturer’s channels or other channel of the corresponding manufacturer. Eq. (6) demonstrates \( d_{ijt} \) increases when \( p_{ijt} \) decreases. Eq. (7) indicates higher value of \( \beta_{ijt} \) compared with \( y_{lkt} \).

3.2 Profit functions

In consideration of total revenue and total cost of the retailer and defining \( q'_{i(t)} = q_{i(t)} + I'_{i(t-1)} \) as the amount of product \( i \), the retail channel \( i \)’s expected profit function can be given as

\[
\Pi_{i(t)} = \begin{cases} p_{i(t)} D_{i(t)} - w_{i(t)} q_{i(t)} - h_i(q_{i(t)} - D_{i(t)}); D_{i(t)} < q_{i(t)} \\
p_{i(t)} q_{i(t)} - w_{i(t)} q_{i(t)} - b_i(D_{i(t)} - q_{i(t)}); D_{i(t)} > q_{i(t)} 
\end{cases}
\]

(8)

Defining \( z_{i(t)} = q'_{i(t)} - d_{i(t)} \) as stocking factor, the relevant expected profit function can be represented as

\[
\Pi_{i(t)} = \begin{cases} p_{i(t)} (d_{i(t)} + e_{i(t)}) - w_{i(t)} (d_{i(t)} + z_{i(t)} - h_i (z_{i(t)} - e_{i(t)})) + w_{i(t)} I'_{i(t-1)}; e_{i(t)} < z_{i(t)} \\
p_{i(t)} (d_{i(t)} + z_{i(t)}) - w_{i(t)} (d_{i(t)} + z_{i(t)} - h_i (z_{i(t)} - e_{i(t)})) + w_{i(t)} I'_{i(t-1)}; e_{i(t)} > z_{i(t)} 
\end{cases}
\]

(9)

The expected profit function can be written as
Given Equations (11), (12), the expected profit function can be reduced to (Proof is represented in Appendix A)

\[
E(\Pi_{it}) = \sum_{i=1}^{2} \left\{ (p_{it} - w_{it})d_{it} - \left\{ (w_{it} + h_{i})\Phi(z_{it}) + (p_{it} + b_{i} - w_{it})\Theta(z_{it}) \right\} + \left( p_{it} + w_{it} \right) E(\varepsilon_{it}) + I_{it-1}^{+} w_{it} \right\}
\]

(13)

The retailer’s constraints are

\[
q_{it} = z_{it} + d_{it} - I_{it-1}^{+}
\]

(14)

\[
I_{it} = I_{it-1}^{+} + q_{it} - (d_{it} + \varepsilon_{it})
\]

(15)

\[I_{it-1}^{+} = \max\left( I_{it-1}, 0 \right)\]

(16)

\[p_{it} \leq M_{i}\]

(17)

Constraints (14) and (15) calculate the order quantity of product \(i\), and the inventory amount respectively, in period \(t\), where \(I_{it-1}^{+}\) is the positive inventory of the last previous period and it can be calculated using Eq. (16). Eq. (17) is associated with constraining \(p_{it}\) such that it should be less than \(M_{i}\) that is applied by marketing department.

Expected profit function of manufacturer \(i\) can be written as

\[
\Pi_{it} = \begin{cases} 
 w_{it} q_{it} + p_{it}D_{it} - C_{it}Q_{it} - h_{i} (q_{it} - D_{it}) ; & D_{it} < q_{it} \\
 w_{it} q_{it} + p_{it}q_{it} - C_{it}Q_{it} - b_{i} (D_{it} - q_{it}) ; & D_{it} > q_{it} 
\end{cases}
\]

(18)

Where \(C_{it}\) has a different value in each period on the basis that if the disruption occurs or not, and is calculated as

\[C_{it} = c_{it} + (1 - \varphi) \Delta c_{i}\]

(19)

Where \(c_{it}\) is the production cost in no disruption period and is regarded as the basic production cost. \(\varphi\) is the binary disruption parameter.

Manufacturer \(i\)’s expected profit function can be represented as Eq. (20) using \(z_{it} = q_{it} - d_{it}\) and Equations (11) and (12).

\[
E(\Pi_{i}) = w_{it} q_{it} + p_{it}D_{it} - C_{it}Q_{it} - \left\{ h_{i} \Phi(z_{it}) + (p_{it} + b_{i})\Theta(z_{it}) \right\} + 2E(\varepsilon_{it})
\]

(20)

The inventory quantity and the production amount in period \(t\) is calculated as
\[ I_u = q_{idt} - D_{idt} \]  
\[ Q_u = q_{idt} + q_{idt} - I_{idt-1} \]

Now the expected profit of the centralized part is given as

\[ \Pi_{ctr} = \sum_{r=1}^{T} \sum_{i=1}^{r} E(\Pi_{r}) \]  
\[ E(\Pi_{r}) = \sum_{r=1}^{T} \{ E(\Pi_{ctr}) + E(\Pi_{r}) \} \]

4 Stackelberg Game

As it was shown in Figure (1), the semi-centralized supply chain consists of two parts. First, Competition Committee determines the optimal wholesale prices, afterwards, the retailer proceeds to set the optimal level of the retail channels’ price; hence, the problem has two stages and the Stackelberg game should be used in this problem wherein the centralized part is the leader and the retailer is the follower. In the second stage of the Stackelberg game, the Nash equilibrium is used due to the existence of price competition among the channels. The Nash equilibrium represents an optimum level for all channels’ price and quantity. Subsequently, responses obtained for the Nash equilibrium variables are replaced in \( \Pi_{ctr} \), and the two-stage mathematical model is converted into a one-stage mathematical model resultantly. In this stage, concavity of \( \Pi_{ctr} \) is analyzed and the optimum response for each \( w_{idt} \) is obtained in the case of concavity.

In the Nash equilibrium, concavity of \( \Pi_{r} \) for \( p_{irt} \) is analyzed to find the best responses for the retailer. Proposition (1) gives the obtained results.

Proposition (1): \( \Pi_{r} \) with respect to \( p_{irt} \) is negative definite. It indicates that \( \Pi_{r} \) is strictly concave. Accordingly, the optimum response for \( p_{irt} \) is

\[ \frac{\partial \Pi_{r}}{\partial p_{irt}} = 0 \Rightarrow p_{irt} = w_{it} + \frac{\Theta(z_{irt}) - d_{irt} - (p_{(3-irt)} - w_{(3-irt)}) \frac{\partial d_{(3-irt)}}{\partial p_{m}}}{\frac{\partial p_{irt}}{\partial p_{m}}} \]

\[ (25) \]

In Eq. (25), denominator has a negative value (\( \frac{\partial d_{(3-irt)}}{\partial p_{m}} = \beta_{p} + g \)). In numerator, all parameters and variables have negative sign other than \( \Theta(z_{irt}) \). Sum of these negative values divided by negative sign of the denominator is converted to a positive value. As a result, only \( \Theta(z_{irt}) \) has a negative effect on \( p_{irt} \). Taking into consideration the stoking factor Eq. and also Eq. (11), it is evident that \( \Theta(z_{irt}) \) has a positive relation with the inventory and thus has a negative impact on the profit. The following notes can be induced.
The inventory has a negative effect on \( p_{ir} \) level.

It is obvious that Eq. (25) has a positive value and hence \( p_{ir} > 0 \) and \( p_{ir} > w_{ir} \).

Apparently, \( p_{ir} \) and \( p_{(3-i)ir} \) are price variables of the retail channels that can be obtained using Eq. (25) simply. Now, looking Eq. (25), there is \( p_{(3-i)ir} \) in it. In order to have an applicable equation to yield the optimal amount of each retail channel’s price particularly in section (4.1), \( p_{(3-i)ir} \) should be omitted from Eq. (25) somehow. Referring to Zhang et al (2012), the existing approach in the literature is to replace \( p_{(3-i)ir} \) with its Equation (Eq. (25) corresponding to \( p_{(3-i)ir} \)). It is feasible to do so, if Hessian Matrix of \( \pi_{ir} \) with respect to \( p_{ir} \) and \( p_{(3-i)ir} \) is strictly concave [51]. Proposition (2) gives the results.

**Proposition (2):** The Hessian Matrix of \( \Pi_{ir} \) with respect to \( p_{ir} \) and \( p_{(3-i)ir} \) is negative definite (Proof is represented in Appendix B). As a result, \( \Pi_{ir} \) is strictly concave with respect to \( p_{ir} \) and \( p_{(3-i)ir} \); hence, it is allowable to place \( p_{(3-i)ir} \) in Eq. (25). The simplified Eq. for \( p_{ir} \) is

\[
p_{ir} = \left( \frac{x_{ir} - f(z_{ir})}{xy} - \frac{d_{ir}}{xy} - \frac{1 - g}{xy} \right) + \frac{w_{ir}(1 - g) - w_{ir}(1 - f)}{y} \]

(26)

Where

\[
x = \frac{\partial d_{(3-i)ir}}{\partial p_{(3-i)ir}}, \quad y = \frac{\partial d_{ir}}{\partial p_{ir}}, \quad g = \frac{\partial d_{ir}}{\partial p_{(3-i)ir}}, \quad f = \frac{\partial d_{(3-i)ir}}{\partial p_{ir}}
\]

In this section, heuristic algorithm that is used as a solution method for this problem is explained. The reason for selecting the heuristic method is that, in the first stage of the Stackelberg game, \( \Pi_{crt} \) is not concave in \( w_{ir} \). So an existing heuristic algorithm in the literature is utilized.

To determine the amount of \( z_{ir} \), Proposition (3), that is represented in the literature [52], is used.

**Proposition (3):** If \( z \) is defined in a range \([-A, A]\), the optimal level for \( z \) is the greatest amount obtained from solving \( \partial(\Pi_{cr})/\partial z = 0 \). Otherwise, the optimal level for \( z \) is \((-A)\).

According to the literature, a proper range for service level is \([0.85, 0.95]\). It is worth mentioning that in this study, the equation \( \partial(\Pi_{cr})/\partial z = 0 \) was solved to set the optimal quantity for \( z_{ir} \). The obtained results have demonstrated that no solution for \( z_{ir} \) has been found. Therefore, Referring to Proposition (3), the optimal level for \( z_{ir} \) is \( F^{-1}(0.85) \). The level for \( z_{ir} \) is equal for all periods referring to Proposition (3).

In this section, the existing heuristic algorithm in the literature [52] is represented in the following.
4.1 Heuristic Algorithm

Step (1): For \( i = 1, 2 \) and \( t = 1 \ldots T \), set a value to \( w_{it} \) randomly regarding \( C_{it} \) and set \( L = 0 \).

Step (2): Determine the amount of other variables at random using \( w_{it} \) for \( i = 1, 2 \) and \( t = 1 \ldots T \), and calculate \( \Pi_r \) and \( \Pi_{opt} \). Subsequently, select a random number for \( i \) from set \( i = \{1, 2\} \).

Step (3): For \( t = 1 \ldots T \), choose a random amount for \( w_{it} \) from a small group that its lower bound is larger than \( c_{it} \). Generate \( p_{db} \) randomly in a small range which its lower bound is greater than \( w_{it} \).

Step (4): For \( i = 1, 2 \) and \( t = 1 \ldots T \), obtain the amount of \( p_{db} \) using Eq. (26), and solve the problem. If the difference between \( \Pi_r \) and its previous calculated amount is less than \( \Delta \), the quantity of \( p_{cr}, p_{db}, w_{it} \) and \( \Pi_r (i = 1, 2 \text{ and } t = 1 \ldots T) \) are the Nash equilibrium quantities and go to Step (6), else go to Step (5).

Step (5): For \( i = 1, 2 \) and \( t = 1 \ldots T \), add up \( \varepsilon \) to each bound of the initial range of \( p_{db} \) and generate \( p_{db} \) regarding the new range and go back to Step (4).

Step (6): Calculate \( \Pi_{opt} \) using the equilibrium quantities. If the amount of \( \Pi_{opt} \) is less than its previous calculated amount, the obtained equilibrium amounts are the Stackelberg point and hence are the best achieved responses for the problem and stop. Else if the amount of \( L \) is smaller than 10, add up \( \varepsilon \) to every bounds of the last previous range of \( w_{it} \) and set \( L = L + 1 \) and go back to Step (3). If the quantity of \( L \) is greater than 10, set \( i = 3 - i \) and go back to Step (3).

Figure (2) yields flowchart of the intended heuristics algorithm.

5 Cooperative Game

In this section, the problem is solved using the cooperative game as a strategy to respond to the production cost disruption. In this game, the supply chain members make their decision cooperatively. The cooperative game is a one-stage game and its fitness function is sum of the fitness function of all members. Solution method that is used to solve the problem and is defined in this section is League Championship Algorithm (LCA) metaheuristic. In every cooperative game problem, a bargaining model divides the obtained profit among the members. The utilized bargaining model and LCA is briefly explained in the following.

5.1 Bargaining Model

In the cooperative game, the obtained profit should be divided among the chain members by an appropriate bargaining model. In this paper, Nash equilibrium bargaining model is utilized (Xu et al. 2014). If the total profit of the centralized part and the retailer in the cooperative and Stackelberg games are given respectively by \( \Pi_{CCP}, \Pi_{CR} \) and \( \Pi^*, \Pi_{cp}^* \), and the gained
profit in the cooperative game is denoted by $\Pi^*$, then the optimal level for each part of the chain in the cooperative game is given by [53]

$$\Pi^*_c = \frac{1}{2} (\Pi^*_c - \Pi^*_p + \Pi^*_r)$$  \hspace{1cm} (27)

$$\Pi^*_cp = \frac{1}{2} (\Pi^*_c + \Pi^*_p - \Pi^*_r)$$  \hspace{1cm} (28)

5.2 League Championship Algorithm

LCA is among stochastic and population based metaheuristic algorithms and is used for solving continuous optimization problems. LCA mimics the champion process in sport league. In addition to the nature, culture, politics, human, etc. as general sources of inspiration of various algorithms, the metaphor of sporting competitions is used for the first time in LCA. In LCA, every solution array containing $n$ members makes role as a sport team and play in pairs in an artificial league and in multiple weeks. Broadly, LCA solves the problem using the following steps:

Step (1): set $t = 1$ (Week index).

Step (2): Initialize League size ($L$), the number of seasons ($s$). Then generate $L$ number of Solutions randomly. Every solution is considered as a team and is displayed as a $(1\times n)$ array ($n$ is the number of variables of the intended problem). Fitness function of $i$th team ($i = 1 \ldots n$) is $(f(x^i))$ and is considered as the best obtained solution for team $i$.

Step (3): As the teams play in pairs in an artificial league and in multiple weeks, obtain a league schedule utilizing single round-robin method that is used in real football leagues.

Step (4): In week $t$, the teams match in pairs, and the winner of a match is determined stochastically and regarding the fitness function (playing strength) of the two following artificial teams. If team $i$ and team $j$ with formation (solution) $x^i_t$ and $x^j_t$, as well as playing strength $f(x^i_t)$ and $f(x^j_t)$ respectively, play against each other at week $t$, and $^*$ is defined as an ideal point; then calculate expected value for team $i$’s chance to beat team $j$ is

$$p'^i = \frac{f(x^i_t) - f^*}{f(x^i_t) + f(x^j_t) - 2f^*}$$  \hspace{1cm} (29)

To determine the winner, generate a number randomly in range $[0, 1]$ if the generated number is less (greater) than or equal to $p'^i$, then team $i$ is the winner (loser) of the game. Choose the best obtained solution for team $i$ ($i = 1 \ldots n$) and display it as $B^i = [b'^{i1}, b'^{i2} \ldots b'^{in}]$. 

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Step (5): If \( \text{mod}(t, L - 1) = 0 \), the season has ended, and player transfer between clubs should be conducted. Therefore, go to Step (5-1) (In a real league, after \((L-1)\) consecutive weeks \((\text{mod}(t, L - 1) = 0)\) player transfer between clubs is carried out). Else, go to Step (6).

Step (5-1): For team \( i, i = 1 \ldots n \), create a set \( M_i = \{ m \mid m \neq i, f(B_m^i) < f(B_i^i) \} \), and for \( d = 1 \ldots n \), choose a random number from \([0, 1]\). If this number is less than a threshold value \( T_r \), Choose an index from \( M_i \) randomly and update \( b_{td} \) by the value of \( b'_{md} \). Afterwards, generate a new league schedule and go to Step (6).

Step (6): If \( t \geq S \times (L - 1) \), LCA ends, else go to Step (7).

Step (7): For team \( i, (i = 1 \ldots L) \), set up a new team formation. To assign a new formation to a team, SWOT decision matrix is utilized. In LCA, strengths and weaknesses of the team as internal criteria and threats and opportunities (strengths and weaknesses of the rival team) as external criteria should be analyzed. Broadly, one of the following Equations obtains new formation of a team.

\[
\text{(S/T equation): } x_{id}^{t+1} = b_{id}^i + y_{id}^i (\Psi_1 r_{id}(x_{id}^i - x_{id}^i) + \Psi_2 r_{2id}(x_{id}^i - x_{id}^i)) \quad \forall d = 1, \ldots, n
\] (30)

\[
\text{(S/O equation): } x_{id}^{t+1} = b_{id}^i + y_{id}^i (\Psi_1 r_{id}(x_{id}^i - x_{id}^i) + \Psi_2 r_{2id}(x_{id}^i - x_{id}^i)) \quad \forall d = 1, \ldots, n
\] (31)

\[
\text{(W/T equation): } x_{id}^{t+1} = b_{id}^i + y_{id}^i (\Psi_1 r_{id}(x_{id}^i - x_{id}^i) + \Psi_2 r_{2id}(x_{id}^i - x_{id}^i)) \quad \forall d = 1, \ldots, n
\] (32)

\[
\text{(W/O equation): } x_{id}^{t+1} = b_{id}^i + y_{id}^i (\Psi_1 r_{id}(x_{id}^i - x_{id}^i) + \Psi_2 r_{2id}(x_{id}^i - x_{id}^i)) \quad \forall d = 1, \ldots, n
\] (33)

Where \( d \) is dimension index, \( r_{1id} \) and \( r_{2id} \) are uniform random numbers in \([0, 1]\). \( B_i^t = \{ b_{i1}, b_{i2} \ldots b_{in} \} \) is the best-obtained solution for team \( i \) until week \( t \). \( \Psi_1 \) and \( \Psi_2 \) are respectively, coefficients of retreat and approach. \( Y_i^t = (y_{i1}^t, y_{i2}^t \ldots y_{in}^t) \) is a binary change array in which \( y_{id}^t \) is binary change variable. In this array, number of ones is equal to \( q_i^t \). Number of changes of \( B_i^t \) is calculated as

\[
q_i^t = \left[ \frac{\ln(1 - (1 - (1 - p_c)^{r_{-q_i^t} - q_0^t}) r)}{\ln(1 - p_c)} \right] + q_0^t - 1 \quad q_i^t \in \{ q_0^t, q_0^t + 1, \ldots, n \}
\] (34)

Where \( r \) is a stochastic variable in \([0, 1]\), \( p_c \) (\( p_c < 1, p_c \neq 0 \)) is a control parameter. The least change that is determined in artificial play analysis process is demonstrated by \( q_0^t \). After simulating the changes, using Eq. (34), \( q_i^t \) number of elements is selected at random from \( B_i^t \) and their amount is changed using one of Equations (30) through (33). After assigning new formations, set \( t = t + 1 \) and go back to Step (4) [54, 55].
6 Numerical Example

As mentioned before, the case of this paper is IKCO wherein IKCO, as the manufacturer faced disruption, and Saipa along with free market, as the other manufacturer and common retailer respectively, have established the intended dual-channel supply chain. In this section, a numerical example is used for solving the problem that the required data has been assigned by consulting IKCO staffs. The example has assumed 2011 as no disruption year and 2012 as disruption year. According to the staff’s statements, IKCO proceeds to hold meeting in order to change the selling price of its products every four months on average. Therefore, each year has been divided into three periods. Peugeot 206 and Brilliance H230 that have nearly equal prices have been selected as the products of IKCO and Saipa respectively. The required data for this section which is given from IKCO is yielded in Table (1).

To solve the Stackelberg game problem, the represented heuristic algorithm is implemented in MatLab (R2012B) Software. The assumed ranges for $w_{it}$ and $p_{idt}$ in Million-Toman (currency of Iran) units are as follows: $p_{1dt-min} = (16,16,16,20, 26, 31)$ , $p_{1dt-max} = (17,17, 17, 22, 28, 34)$ , $p_{2dt-min} = (16, 16, 16, 17, 17, 17)$, $p_{2dt-max} = (17,17, 17, 19,.5 19.5, 19.5)$ .

The heuristic algorithm and LCA have solved the problem using the numerical example which is yielded in Table (1). Table (2) and Table (3) represent the obtained results respectively.

The members’ profit using the Nash equilibrium bargaining model are determined as

$$\pi^*_{CR} = 221,938,852,457$$ $\pi^*_{C} = 587,068,622,106$

Remark (1): The comparisons reveal that the cooperative game problem is more profitable for all members than the Stackelberg game problem. The reason for higher profit of the cooperative game problem is referred to the production quantity and the demand level. Looking at the results clarifies that the level of both $Q_1$ and $Q_2$ in Table (2) are greater than those of in Table (3). It illustrates that in the cooperative game scenario, the demand of four selling channels is higher compared with those of in the Stackelberg game scenario. As a result, the achieved profit of each member in the cooperative game scenario is higher than that of in the Stackelberg game scenario.

7 Sensitivity Analysis

In this section, sensitivity of fitness function on main parameters such $\beta_{irt}$, $y_{irt}$, $\alpha$ and $g$ is analyzed. As it is evident from the demand function, the demand quantity and then the fitness amount become lower with the gradual increase in $\beta_{irt}$. In addition, the demand function demonstrates that the amount of profit decreases as $y_{irt}$ increases. Outcomes of the conducted experiments have proved the mentioned results and because of their clarity, they have not been represented in the paper.
The main parameters that are analyzed in this section are the reference price-related parameters. Firstly, $\alpha$ is analyzed. The acquired results for $r_{it}$ from the conducted experiments are yielded in Table (4) for $0 < \alpha < 1$. For the sake of analysis convenience, the obtained results are depicted in Figure (3). Figure (3) indicates that lower amount of $\alpha$ causes $r_{it, i=1, 2}$ to be influenced in higher degree by $p_{i(t-1)}$. This effect will result in decrease in the gap between $p_{it}$ and $r_{it}$. In Figure (3-a), due to the occurred disruption in periods 5 and 6, curves of $t = 5$ and $t = 6$ illustrate the considerable decrease in $r_{it}$ that is resulted in increase in $\alpha$. The lower amount of $\alpha$ causes that $p_{i(t-1)}$ affects significantly $r_{it}$ and thus leads to raise the quantity of $r_{it}$ and subsequently, reduces the difference between $r_{it}$ and $p_{it}$.

Therefore, decrease in the demand occurs somewhat. Resultantly, the disruption has less destructive effects on the supply chain. Regarding $\alpha$ as the memory parameter, the achieved results are explained in Remark (2).

**Remark (2):** When the disruption occurs, the demand quantity for products with short-time memory—lower amount of $\alpha$—decreases somewhat with the increase in $p$. As a result, the demand can be adapted to the happened situation effectively. Whereas, substantial analyses about increase in the selling price of a product with long-time memory—higher amount of $\alpha$—should be conducted and the selling price should be determined as low as possible due to the remarkable impact of $r_{t-1}$ on $r_{t}$. This result can be verified considering the quantity of $r_{it}$ when $\alpha$ is allocated higher value in periods 5 and 6 as Figure (3-a) illustrates.

In this section, the effects of $\alpha$, $g_1$ and $g_2$ on the fitness function in both the cooperative and Stackelberg games are considered analytically. In this analysis, there are 10 steps and in each step, the amount of $\alpha$ and $(g_2 - g_1)$ is determined increasingly at a rate of 0.1. Obtained results are depicted in Figures (4), (5) and (6).

In Figures (4) and (5) (Stackelberg-related ones), the maximum level of the fitness function is obtained in the second Step (0.2 increase). It may be possible to explain the mentioned result regarding Table (4) and Figures (3-a) and (3-b). It is evident that in most periods, maximum level of $r_{it, i=1,2}$ is obtained when $\alpha = 0.2$. This result points out that in 0.2 increase, the difference between $r_{it}$ and $p_{it}$ is at the lowest level in most periods. Thus, if the quantity of $\alpha$ and $(g_2 - g_1)$ increase 0.2, maximum profit of each chain member is achieved in the Stackelberg game. Broadly, the obtained results demonstrate that lower level of the corresponding parameters result in higher level for the product’s demand and profit as well. Comparing Figures (4) and (5) reveals that in the Stackelberg scenario, the amount of $\Pi_{it}$ decreases in great slop after step2. In Figure (4), $\Pi_{i}$ has a negative great slop in Step 7. Since the free market has two selling channels and only the first channel ($i = 1$) has disrupted, and moreover, the mentioned channel has lower demand compared with the relevant product’s direct channel demand. Therefore, it can be declared that the retail channel of IKCO product is more resistant than its direct channel. The reason why Competition Committee has imposed substantial losses is that the largest percentage of the total market demand
has been allocated to IKCO and the disruption has caused the gap between $r_1$, and $p_1$, to increase, and $q_1$, to decrease significantly as well. Furthermore, looking Table (4) yields that the gap between $r_2$, and $p_2$, tends to increase slowly with the periods passing; hence $\Pi_2$, has decreased somewhat. For these reasons, Competition Committee has been imposed considerable losses as the product has long-time memory and the disruption had occurred.

To compare the Stackelberg and cooperative scenarios, Table (5) that demonstrates total profit of the dual-channel supply chain is presented.

Figure (6) depicts that there is a considerable decrease in Step 7. This obtained result and also numbers of Table (5) confirms the better performance of the cooperative game rather than the Stackelberg game. Remark (3) yields the main obtained results.

**Remark (3):** In disruption occurring conditions, the cooperative game is resistant for products with any kinds of memory and profitable for the chain all members compared with the Stackelberg game. In the intended case study, considering automobile as a product with long-time memory (since the quantity of $\alpha$ is in $[0.3, 0.5]$ according to statements of IKCO staffs), it is suggested to Competition Committee to utilize the cooperative game as an efficient strategy to counteract effects of the disruption occurred to IKCO.

### 7.1 Optimal level for the wholesale price in the case study

In this section, the optimal level for the second manufacture’s wholesale price of the case study-Saipa–is analyzed. For this purpose, the quantity of $w_2$, for each $t = 4, 5,$ and $6$ (disruption periods) is analyzed in different ranges in Million Toman units as follows respectively: $\{(15, 16), (16, 17), (17, 18)\}$, $\{(15, 17), (17, 19), (19, 21), (21, 23)\}$ and $\{(14, 17), (17, 20), (20, 23), (23, 26), (26, 28)\}$. Figure (7) depicts the obtained results for the profit of Competition Committee. The main achieved result from Figures (7) is that in competitive market, as soon as a manufacturer increases selling price of its product, it is not an appropriate strategy for its competitors to raise their product’s selling price approximately equal to the intended product’s selling price. To maximize the competitors’ profit in such a situation, they should conduct some profound analyses and then determine the optimal selling price. Figures (7) yield the optimal level for $w_2$, given in Table (6).

In such a supply chain with a disrupted manufacturer, sometimes, increase in the selling price of the other manufacture approximately as high as the selling price of the disrupted manufacturer is not only detrimental to the intended manufacturer but also profitable to the chain other members. In Figures (7-a) and (7-b), if Competition Committee selects $w_2$, from $[19, 21]$ in the second disruption period, the free market is allocated more profit than the case of being $w_2$ in $[17, 19]$, however, unlike the free market, Competition Committee gains lower profit than the optimal range $([17, 19])$. Moreover, in the third
disruption period, if Competition Committee selects $w_2$ in [26, 28] with the goal of gaining more profit, looking Figures (7-a) and (7-b), the free market earns more profit than the other defined cases, while Competition Committee does not achieve its maximal level of the profit.

8 Management Insights

The most important management notifications which were obtained through this study are given as follows:

1. In disruption occurring conditions, it is more possible for products with long-term memory to be at risk comparing with products with short-term memory.

2. To preserve demand from high variations, profound analyses should be carried out to determine optimal amount of the selling price.

3. The cooperative game is suggested to the products with any kinds of memory as a proper strategy to respond to the occurred disruption due to its remarkable resilience against the disruption as well as its profitability for all members of the supply chain.

4. Upon increasing the selling price of a manufacturer, other competitors should conduct deep analyses to find the best response of their selling price.

9 Conclusion

This research considered the sanctions disruption which had been imposed against IKCO analytically. For this purpose, it regarded a semi-centralized dual-supply chain that comprises two manufacturers (IKCO and Saipa) and a retailer (Free market). Each manufacturer has two selling channels i.e. retail and direct channel. Demand faced all selling channels is reference price and other channels’ selling price dependent and stochastic. One manufacturer (IKCO) suffered from disruption in production cost. It was assumed that the intended manufacturer loses its demand trivially due to its higher popularity in the market. To solve the problem, Stackelberg game was used, and the existing price competition among the selling channels caused to utilize the Nash equilibrium in the second stage of the Stackelberg game. Moreover, cooperative game was introduced as a strategy for disruption management. Results obtained from solving the problem with the data collected from IKCO demonstrated that the cooperative game is more profitable for the chain all members due to all channels’ demand increase and subsequently, all manufacturers’ production level increase. Results obtained from the experiments indicated that the supply chains which produces product with short-time memory struggle with disruptions resiliently, they demonstrated that the cooperative game is beneficial for products of any types of memory e.g. long-time
and short-time memory in disruption conditions. It was concluded from the mentioned results that the cooperative game could be used to manage the disruption. The results represented optimal range for the wholesale price of the second manufacturer (Saipa) in the disruption periods as well.

As it was stated, this study is a research about a real world problem. Regarding the nature of the intended problem, the semi-centralized supply chain was introduced for the first time in the supply chain literature. In addition to introducing semi-centralized supply chain concept, main contribution of the study is analyzing the influence of the reference price on the real-world disrupted supply chain problem. The reference price has a determinant role in the resilience of the supply chain against the occurred disruptions. In a Stackelberg game, the longer the product’s memory time is, the higher the effects of the occurred disruption on the supply chain will be. However, the supply chain is resilient regardless of the memory type in the cooperative game. The proposed method with support of the experimental results demonstrated that the memory type of the product is effective in the degree of resilience in the Stackelberg game. As a result, the memory type can be considered as one of the influential factors in determining power structure and disruption management as well.

The most important limitation of the study was that upon analyzing the second stage of the manufacturer’s problem in the Stackelberg scenario, obtaining the amount of the wholesale price was rather impossible so that heuristics algorithm had to be utilized. Proposing an appropriate approach with the aim of solving this issue could be regarded as a future opportunity.

One suggestion for this matter is considering a uniform distribution for the stochastic demand function in order to make Expression (23) simpler. Additionally, in the study, the problem only considered one product of IKCO and one product of Saipa to have a less-complicated model. Considering the other products of IKCO and Saipa including their own brands as well as foreign brands and analyzing their effects on the profit of IKCO as well as taking into consideration the impacts of the exchange rate variations on the profit function may yield a more proper insight to IKCO. Another possible research opportunity may be taking into account other common types of existing disruption in the literature like supply, demand, simultaneous demand and production cost disruption.

As the last point, the main purpose of conducting the research is referred to Iran sanctions years. Over these years, industry of Iran witnessed economic depression. Events of the intended years demonstrated that large number researches should be carried out in Iran industry. This issue can be regarded as the considerable opportunity for future researches as well.

References


[41] Huang, S., Yang, C. and Liu, H. “Pricing and production decisions in a dual-channel supply chain when production costs are disrupted”, Econ. Model., 30, pp. 521-538 (2013).

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Figure 1. Semi-centralized dual-channel supply chain structure of the case study
For $i = 1, 2$ and $t = 1 \ldots T$, set a value to $w_i$ randomly regarding $C_i$ and set $L = 0$

Determine the amount of other variables at random using $w_i$ for $i = 1, 2$ and $t = 1 \ldots T$, and calculate $\Pi_r$ and $\Pi_{cp}$. Subsequently, select a random number for $i$ from set $i = \{1, 2\}$

For $t = 1 \ldots T$, choose a random amount for $w_i$ from a small group that its lower bound is larger than $c_{ip}$. Generate $p_{it}$ randomly in a small range which its lower bound is greater than $w_i$.

For $i = 1, 2$ and $t = 1 \ldots T$, obtain the amount of $p_{it}$ using Eq. (26), and solve the problem.

If the difference between $\Pi_r$ and its previous calculated amount is less than $\Delta$ the quantity of $p_{it}$ and $w_i$ ($i = 1, 2$ and $t = 1 \ldots T$), are Nash equilibrium quantities.

For $i = 1, 2$ and $t = 1 \ldots T$, add up $\varepsilon$ to each bound of the initial range of $p_{it}$ and generate $p_{it}$ regarding new range.

Calculate $\Pi_{cp}$ using equilibrium quantities.

If the amount of $\Pi_{cp}$ is less than its previous calculated amount, obtained equilibrium amounts are the Stackelberg point and hence are the best achieved responses for the problem.

If the amount of $L$ is smaller than 10, add up $\varepsilon$ to every bounds of the last previous range of $w_i$, and set $L = L + 1$

Stop

**Figure 2.** Flowchart of Heuristic Algorithm
Figure 3. Effects of $\alpha$ on $a = r_{1t}$, $b = r_{2t}$

Figure 4. Retailer (Free market)'s profit, Stackelberg game scenario
Figure 5. Centralized Part (Competition Committee)'s profit, Stackelberg game scenario
Figure 6. Cooperative game scenario’s profit
Figure 7. Effects of $w_2$ on profits of a = Centralized part, b = Retailer, c = Cooperative game
Table 1. Data acquired from IKCO

<table>
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<th>$c_{ij}$</th>
<th>$\mu = 30,000$</th>
<th>$b_j = [100,000 \ 100,000]$</th>
<th>$h_i = 100,000$</th>
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<td>$b_i = 100,000$</td>
<td>$\theta_j = [0.3 \ 0.1]$</td>
<td>$\epsilon_{i,j} \sim N(0,10)$</td>
</tr>
<tr>
<td>$h_j = [100,000 \ 100,000]$</td>
<td>$\phi = 0.45 \ 0.15$</td>
<td>$\Delta = 1%$</td>
<td>$\epsilon_{2,i} \sim N(0,6)$</td>
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<tr>
<td>$r_i = [12,800,000 \ 12,800,000]$</td>
<td>$T = 6$</td>
<td>$\alpha = 0.3$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\beta_d$</th>
<th>$\beta_i$</th>
<th>$\gamma_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 0 &amp; 0 \ 0 &amp; 0 \ 0 &amp; 0 \ 1 &amp; 1 \ 1 &amp; 1 \ 1 &amp; 1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.0012 &amp; 0.0014 \ 0.00089 &amp; 0.00166 \ 0.00083 &amp; 0.00156 \ 0.00087 &amp; 0.001806 \ 0.00065 &amp; 0.0014 \ 0.00075 &amp; 0.00085 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.00168 &amp; 0.00095 \ 0.00172 &amp; 0.00095 \ 0.0017 &amp; 0.00088 \ 0.00176 &amp; 0.001 \ 0.00115 &amp; 0.00082 \ 0.00072 &amp; 0.0009 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.00049 &amp; 0.0004 \ 0.0008 &amp; 0.00008 \ 0.00074 &amp; 0.00008 \ 0.00085 &amp; 0.00008 \ 0.0005 &amp; 0.00008 \ 0.00045 &amp; 0.00009 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma_i$</th>
<th>$\Delta c$</th>
<th>$Q$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 0.0000045 &amp; 0.00085 \ 0.0000027 &amp; 0.00085 \ 0.0000053 &amp; 0.00081 \ 0.0000005 &amp; 0.000849 \ 0.000007 &amp; 0.00077 \ 0.000000005 &amp; 0.00024 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 &amp; 0 \ 0 &amp; 0 \ 0 &amp; 0 \ 5000000 &amp; 0 \ 6000000 &amp; 0 \ 5000000 &amp; 0 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 21000 &amp; 6000 \ 21000 &amp; 6000 \ 21000 &amp; 6000 \ 15500 &amp; 6500 \ 15500 &amp; 8500 \ 15000 &amp; 9000 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 13250000 &amp; 13000000 \ 13500000 &amp; 13500000 \ 13900000 &amp; 13900000 \ 18500000 &amp; 17800000 \ 24000000 &amp; 17900000 \ 29000000 &amp; 17950000 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
Table 2. Obtained results for the Stackelberg problem

<table>
<thead>
<tr>
<th>period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{1t})</td>
<td>13,169,649</td>
<td>13,178,360</td>
<td>13,181,144</td>
<td>18,199,910</td>
<td>24,184,222</td>
<td>29,027,680</td>
</tr>
<tr>
<td>(w_{2t})</td>
<td>13,371,213</td>
<td>13,549,644</td>
<td>13,665,012</td>
<td>14,353,282</td>
<td>14,397,907</td>
<td>14,654,882</td>
</tr>
<tr>
<td>(p_{1t})</td>
<td>16,460,272</td>
<td>16,560,185</td>
<td>16,769,966</td>
<td>21,335,481</td>
<td>27,833,529</td>
<td>32,782,388</td>
</tr>
<tr>
<td>(p_{2t})</td>
<td>16,103,801</td>
<td>16,114,036</td>
<td>16,797,380</td>
<td>18,022,308</td>
<td>18,417,229</td>
<td>19,049,395</td>
</tr>
<tr>
<td>(p_{1r})</td>
<td>17,598,323</td>
<td>17,428,245</td>
<td>17,202,854</td>
<td>21,256,310</td>
<td>28,067,053</td>
<td>33,912,553</td>
</tr>
<tr>
<td>(p_{2r})</td>
<td>16,397,983</td>
<td>16,503,021</td>
<td>16,732,648</td>
<td>18,062,517</td>
<td>18,353,116</td>
<td>18,207,555</td>
</tr>
<tr>
<td>(q_{1t})</td>
<td>7,678</td>
<td>7,536</td>
<td>7,053</td>
<td>5,848</td>
<td>5,047</td>
<td>4,260</td>
</tr>
<tr>
<td>(q_{2t})</td>
<td>1,841</td>
<td>1,856</td>
<td>2,086</td>
<td>4,838</td>
<td>3,503</td>
<td>3,061</td>
</tr>
<tr>
<td>(q_{1r})</td>
<td>14,206</td>
<td>14,061</td>
<td>14,551</td>
<td>11,709</td>
<td>11,011</td>
<td>11,029</td>
</tr>
<tr>
<td>(q_{2r})</td>
<td>5,003</td>
<td>5,021</td>
<td>4,333</td>
<td>5,420</td>
<td>6,938</td>
<td>7,418</td>
</tr>
<tr>
<td>(Q_{1t})</td>
<td>21,884</td>
<td>21,297</td>
<td>21,604</td>
<td>17,557</td>
<td>16,058</td>
<td>15,289</td>
</tr>
<tr>
<td>(Q_{2t})</td>
<td>6,844</td>
<td>6,877</td>
<td>6,419</td>
<td>10,258</td>
<td>10,441</td>
<td>10,479</td>
</tr>
</tbody>
</table>

Total Profit: 783,114,480,012

Retailer’s Profit: 208,992,355,183
Cent. Part’s Profit: 574,122,124,830
Table 3. Obtained results for the cooperative problem

<table>
<thead>
<tr>
<th>period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{1t}$</td>
<td>13,744,757</td>
<td>13,757,778</td>
<td>13,893,086</td>
<td>18,760,960</td>
<td>24,983,642</td>
<td>29,297,395</td>
</tr>
<tr>
<td>$w_{2t}$</td>
<td>13,181,421</td>
<td>13,635,705</td>
<td>13,921,629</td>
<td>14,652,660</td>
<td>14,837,604</td>
<td>14,980,727</td>
</tr>
<tr>
<td>$p_{1lt}$</td>
<td>17,065,314</td>
<td>17,194,105</td>
<td>17,286,475</td>
<td>22,285,889</td>
<td>28,869,059</td>
<td>34,385,837</td>
</tr>
<tr>
<td>$p_{2lt}$</td>
<td>16,132,030</td>
<td>16,293,663</td>
<td>16,826,085</td>
<td>17,687,272</td>
<td>18,427,982</td>
<td>19,470,202</td>
</tr>
<tr>
<td>$p_{1lt}$</td>
<td>16,906,758</td>
<td>16,908,651</td>
<td>17,101,218</td>
<td>21,138,203</td>
<td>26,499,352</td>
<td>34,083,534</td>
</tr>
<tr>
<td>$p_{2lt}$</td>
<td>16,738,134</td>
<td>16,930,685</td>
<td>17,003,686</td>
<td>19,167,620</td>
<td>19,204,443</td>
<td>19,312,219</td>
</tr>
<tr>
<td>$q_{1lt}$</td>
<td>9,648</td>
<td>9,314</td>
<td>7,829</td>
<td>8,413</td>
<td>9,021</td>
<td>5,998</td>
</tr>
<tr>
<td>$q_{2lt}$</td>
<td>1,995</td>
<td>2,026</td>
<td>2,031</td>
<td>4,206</td>
<td>3,340</td>
<td>2,857</td>
</tr>
<tr>
<td>$q_{1lt}$</td>
<td>13,777</td>
<td>13,874</td>
<td>14,342</td>
<td>11,785</td>
<td>10,971</td>
<td>10,959</td>
</tr>
<tr>
<td>$q_{2lt}$</td>
<td>5,548</td>
<td>5,644</td>
<td>4,906</td>
<td>7,783</td>
<td>8,108</td>
<td>8,059</td>
</tr>
<tr>
<td>$Q_{1t}$</td>
<td>23,425</td>
<td>23,188</td>
<td>22,171</td>
<td>20,198</td>
<td>19,992</td>
<td>16,957</td>
</tr>
<tr>
<td>$Q_{2t}$</td>
<td>7,543</td>
<td>7,670</td>
<td>6,937</td>
<td>11,989</td>
<td>11,448</td>
<td>10,916</td>
</tr>
<tr>
<td>Total Profit</td>
<td></td>
<td></td>
<td></td>
<td>809,007,474,563</td>
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</tbody>
</table>
Table 4. Obtained results for reference price ($r_t$)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>15877314</td>
<td>15921438</td>
<td>14974242</td>
<td>14516418</td>
<td>14139994</td>
<td>13813416</td>
<td>13500223</td>
<td>13136833</td>
<td></td>
</tr>
<tr>
<td>$t = 3$</td>
<td>16303640</td>
<td>16607282</td>
<td>16208185</td>
<td>15370783</td>
<td>15007139</td>
<td>14555638</td>
<td>14087691</td>
<td>13426942</td>
<td></td>
</tr>
<tr>
<td>$t = 4$</td>
<td>16321540</td>
<td>16659900</td>
<td>1629623</td>
<td>15836194</td>
<td>15697307</td>
<td>15110087</td>
<td>14625369</td>
<td>13689432</td>
<td></td>
</tr>
<tr>
<td>$t = 5$</td>
<td>20607783</td>
<td>19847676</td>
<td>18656123</td>
<td>18414456</td>
<td>17748913</td>
<td>16690467</td>
<td>15870418</td>
<td>14353023</td>
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</tr>
<tr>
<td>$t = 6$</td>
<td>26415669</td>
<td>25956267</td>
<td>24592661</td>
<td>23599193</td>
<td>22567458</td>
<td>19767484</td>
<td>18132018</td>
<td>15543436</td>
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</tbody>
</table>

Second Manufacturer (Saipa)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td>12920000</td>
<td></td>
</tr>
<tr>
<td>$t = 2$</td>
<td>15214048</td>
<td>15078214</td>
<td>14711695</td>
<td>14424970</td>
<td>13867327</td>
<td>13618067</td>
<td>13356424</td>
<td>13068008</td>
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</tr>
<tr>
<td>$t = 3$</td>
<td>15624712</td>
<td>15744582</td>
<td>15450808</td>
<td>15338377</td>
<td>14603882</td>
<td>14287626</td>
<td>1383394</td>
<td>13314922</td>
<td></td>
</tr>
<tr>
<td>$t = 4$</td>
<td>15763690</td>
<td>15968766</td>
<td>15797394</td>
<td>16238165</td>
<td>15916341</td>
<td>14781251</td>
<td>14266210</td>
<td>13595785</td>
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<tr>
<td>$t = 5$</td>
<td>17845780</td>
<td>17275240</td>
<td>17374451</td>
<td>16925230</td>
<td>16795609</td>
<td>16152735</td>
<td>15596856</td>
<td>14971128</td>
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</tr>
<tr>
<td>$t = 6$</td>
<td>18183455</td>
<td>17802268</td>
<td>18010545</td>
<td>17505284</td>
<td>17420788</td>
<td>17013900</td>
<td>16345420</td>
<td>15678260</td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Total profits of games in each step

<table>
<thead>
<tr>
<th>Ratio</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coop.</td>
<td>8.56E+11</td>
<td>8.56E+11</td>
<td>8.52E+11</td>
<td>8.52E+11</td>
<td>8.51E+11</td>
<td>8.3E+11</td>
<td>8.24E+11</td>
<td>8.1E+11</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.9</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stack.</td>
<td>6.74E+11</td>
<td>6.51E+11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coop.</td>
<td>7.9E+11</td>
<td>7.8E+11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6. Optimal amount of \( w_{2t} \) in disruption periods

<table>
<thead>
<tr>
<th>Disruption period</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
<td>([16,17])</td>
<td>([17,19])</td>
<td>([17,20])</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>([16,17])</td>
<td>([17,19])</td>
<td>([20,23])</td>
</tr>
</tbody>
</table>

**Appendix (A)**

\[ p_{mt}d_{mt} = p_{mt}d_{mt} \left( \int_{\infty}^{\infty} f(x) \, dx + \int_{-\infty}^{\infty} f(x) \, dx \right) \]  
(A-1)

\[ \int_{\infty}^{\infty} p_{mt} f(x) \, dx + \int_{-\infty}^{\infty} p_{mt} \int_{\infty}^{\infty} f(x) \, dx = - \int_{\infty}^{\infty} p_{mt} (z_{mt} - x) f(x) \, dx + p_{mt} z_{mt} \]  
(A-2)

\[ p_{mt} z_{mt} = p_{mt} \int_{-\infty}^{\infty} f(x) \, dx + \int_{\infty}^{\infty} f(x) \, dx - p_{mt} \int_{\infty}^{\infty} (z_{mt} - x) f(x) \, dx - \int_{\infty}^{\infty} (x - z_{mt}) f(x) \, dx + E(\varepsilon_{mt}) \]  
(A-3)

Replacing Eq. (A-3) with (A-2), Eq. (A-4) is obtained as

\[ \int_{\infty}^{\infty} p_{mt} f(x) \, dx + \int_{-\infty}^{\infty} p_{mt} z_{mt} f(x) \, dx = - p_{mt} \int_{\infty}^{\infty} (x - z_{mt}) f(x) \, dx + p_{mt} E(\varepsilon_{mt}) \]  
(A-4)

\( w_{0} \) is obtained in the same way. Finally, the retailer’s profit function is written as

\[ E(\Pi_{mt}) = \sum_{i=1}^{2} \left( (p_{mt} - w_{0})d_{mt} - ((w_{0} + \beta_{i}) \Phi(z_{mt}) + (p_{mt} + b_{i} - w_{0}) \Theta(z_{mt})) + ((p_{mt} + w_{0}) E(\varepsilon_{mt}) + I_{mt}^{*} w_{0} \right) \]  
(A-5)

**Appendix (B)**

The Hessian Matrix of \( \pi_{mt} \) with respect to \( p_{mt} \) and \( p_{3-i,mt} \) is

\[
H = \begin{bmatrix}
\frac{\partial^2 \pi_{mt}}{\partial p_{mt}^2} & \frac{\partial^2 \pi_{mt}}{\partial p_{mt} \partial p_{3-i,mt}} \\
\frac{\partial^2 \pi_{mt}}{\partial p_{3-i,mt} \partial p_{mt}} & \frac{\partial^2 \pi_{mt}}{\partial p_{3-i,mt}^2}
\end{bmatrix}
\]
(B-1)

\[
\frac{\partial^2 \Pi_{mt}}{\partial p_{mt}^2} = \frac{\partial d_{mt}}{\partial p_{mt}} = -(\beta_{i} + g_{i,1,1,2}) < 0
\]
(B-2)

\[
\frac{\partial^2 (\Pi_{mt})}{\partial p_{3-i,mt} \partial p_{mt}} = \frac{\partial d_{3-i,mt}}{\partial p_{mt}} = y_{mt} > 0
\]
(B-3)
It is obvious from Eq. (A-2) that the sign of first subdeterminant of the Hessian Matrix is negative. To determine the sign of determinant of the Hessian Matrix, some algebraic equation should be written as

$$\frac{\partial^2 (\Pi_{\alpha})}{\partial p_{\alpha}^2} \frac{\partial^2 (\Pi_{\alpha})}{\partial p_{\alpha}^2} \frac{\partial^2 (\Pi_{\alpha})}{\partial p_{\alpha}^2} \frac{\partial^2 (\Pi_{\alpha})}{\partial p_{\alpha}^2} = \frac{\partial d_{\alpha \alpha}}{\partial p_{\alpha}} \frac{\partial d_{\alpha \alpha}}{\partial p_{\alpha}} - \frac{\partial d_{\alpha \alpha}}{\partial p_{\alpha}} \frac{\partial d_{\alpha \alpha}}{\partial p_{\alpha}} > 0$$

(B-6)

The sign of determinant of the Hessian Matrix is positive considering Eq. (7). The negative sign of first subdeterminant and positive sign of the main determinant prove the strict concavity of $\Pi_{\alpha}$ with respect to $p_{\alpha \alpha}$ and $p_{\alpha \alpha}$.

**Biography of Authors**

**Farnia Zarouri**

Farnia Zarouri is a Ph.D. Candidate in Industrial Engineering at Kharazmi University, Iran. She received her MSc. in Industrial Engineering from Tarbiat Modares University, Iran in 2015, and she holds a BSc. in Industrial Engineering from Urmia University of Technology (UUT), Iran in 2013. She was ranked as the top Student in her BSc. Course. Her main research interests are pricing, disruption management, coordination and contracts in supply chain, inventory management (especially VMI inventory system) and revenue management.

**Seyed Hesameddin Zegordi**

S.H. Zegordi is a Professor of Industrial Engineering in the Faculty of Industrial & Systems Engineering at Tarbiat Modares University, Iran. He received his PhD from the department of Industrial Engineering and Management at Tokyo Institute of Technology, Japan in 1994. He holds an MSc in Industrial Engineering and Systems from Sharif University of Technology, Iran and a BSc in Industrial Engineering from Isfahan University of Technology, Iran. His main areas of teaching and research interests include production planning and scheduling, multi-objective optimization problems, meta-heuristics, quality management and productivity. He has published several articles in international conferences and academic journals including European Journal of Operational Research, International Journal of Production Research, Journal of Operational Research Society of Japan, Computers & Industrial Engineering, Transportation Research Part E, International Journal of Advanced Manufacturing Technology, Decision Support Systems, Scientia Iranica - International Journal of Science and Technology, and Amirkabir Journal of Science and Engineering.

**Ali Husseinzadeh Kashan**

Dr. Ali Husseinzadeh Kashan holds degrees in Industrial Engineering from Amirkabir University of Technology, Iran. He worked as a postdoctoral research fellow at the department of Industrial Engineering and Management Systems. Dr. Kashan is currently an assistant professor in the Department of Industrial and Systems Engineering, Tarbiat Modares University.
His research focuses on modeling and solving hard combinatorial optimization problems in areas such as logistics and supply networks, revenue management and pricing, resource scheduling, grouping problems, financial engineering, etc. He has introduced several intelligent optimization procedures, which inspire from natural phenomena, such as League Championship Algorithm, Optics Inspired Optimization Grouping Evolution Strategies. Dr. Kashan has published over 70 peer-reviewed journal and conference papers, and has served as a referee for several outstanding journals such as: IEEE Transactions on Evolutionary Computations, Omega, Computers & Operations Research, Computers & Industrial Engineering, International Journal of Production Research, Information Sciences, Applied Soft Computing, etc.