Price and Quality Decisions in Heterogeneous Markets

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Abstract

This paper analyzes the optimal price and quality decisions of a retailer for its different stores in a heterogeneous market. The consumers are assumed to be heterogeneous in their willingness to pay for quality and are non-uniformly distributed in the market. This type of heterogeneity which is identified based on income disparity can have important implications for a retailer's optimal policy. The specific objective of this paper is to investigate how the distribution of consumers' types in the market and their travel costs affect the optimal setting of price and quality levels among different stores of a retailer. Our results express that the geographical disparity of willingness to pay plays a significant role in the differentiation and targeting strategy of a retailer. Comparative analysis shows that the widely adopted assumption of uniform distribution of consumers in the literature leads to non-optimal decisions where the distribution of consumers is non-uniform in a real-world situation.

Keywords: Quality level, Pricing, Income disparity, Non-uniform distribution, Differentiation.

1 Introduction

Pricing and quality decisions are two strategic tools for retailers to differentiate themselves in a competitive market or to satisfy the needs of heterogeneous consumers. Today, we witness that retailers in different sectors such as food, grocery, merchandise or apparel, adapt their stores' format with market characteristics. For example, some traditional supermarkets like Kroger are operating with multiple price and quality levels. Kroger, Inc. has multiple brands of stores such as Fresh Fare, Kroger and Food4Less that are operating as high-end store, traditional supermarket, and price-impact warehouse store, respectively [1]. Other retailers have differentiated their stores by introducing an off-price format. For example in the high fashion specialty sector, Nordstrom and

Saks Fifth Avenue have introduced Nordstrom Rack and Saks OFF 5th as their discount stores [2]. However, apart from the large discounts that these outlet-type formats offer, the quality of the products may slightly differ from the ones in the full-price stores. Generally, the products offered in discount stores have lower quality than full-price stores and are procured from different suppliers [3, 4]. Apparel manufacturers usually differentiate their retail and outlet stores through price, variety, and quality level. Retailers also try to match their price strategy with the neighborhood they are operating in. For example, some drugstore chains like Walgreens often charge different prices for the same product in stores located in different places [5]. There is also evidence that sometimes Target stores charge dissimilar prices in different locations [6].

The above examples show that retailers can differentiate their stores through different levels of quality and price. This strategy could be due to the competition pressure or the demographic characteristics of the market. Therefore, one of the most important questions a retailer (or a firm) must deal with is how to determine the image of its stores operating in new markets. What price and quality levels are better matched with the consumers' needs and characteristics? Under what conditions the retailer should open identical stores in different neighborhoods and when should the retailer operate under different store formats? These questions are of strategic importance for retailers, since as Gauri et al. [7] state, the competition in some retail sectors is so fierce that considering an appropriate strategy can determine the longterm success or failure of a store.

We consider a monopolistic retailer who wants to set up stores in two adjacent neighborhoods and determine the price and quality levels of its stores in order to maximize the overall profit. Consumers in the neighborhoods are heterogeneous not only in terms of their ideal location but also in their valuations for quality. As it is common in the literature, consumers' willingness to pay can be measured by the income level [8], which is one of the publicly available demographic data at any neighborhood. More specifically, consumers can be divided in two segments based on their valuations for quality (or price sensitivity). The high-end segment cares more about the quality while the low-end segment is more price sensitive. We consider both uniform and non-uniform distributions of each consumer's type in the market. We show how this heterogeneity in income and location affects the products' quality and pricing decisions in each store. Our model might rationale some of the store differences across neighborhoods.

In this study we provide an analytical framework to highlight the role of income disparity in the retailer's decision. Since the income level can represent the willingness to pay for quality, analyzing its impact on the optimal decision of a retailer is so critical. Several studies acknowledge that income inequality is rising in the economy [9, 10]. Other studies also find a gradual spatial shift of lower-income families from the central to suburb residential areas in several US cities, resulting in neighborhood income polarization [11, 12]. Reardon and Bischoff [13] reported that the percentage of American families living in middle-class neighborhoods fell from 65% in 1970 to 44% in 2009, and that split is accelerating, which results in residential isolation between high and low-income families. Gulati and Ray [9] also highlight that this sizable spatial difference in the income level poses a new challenge for educational institutions and health care facilities as they need to consider both the location and income mix of people upon entering a neighborhood and setting the price and quality levels for their products and services.

Accordingly, the goal of this paper is to provide an analytic framework to answer the following research questions: 1) How a retailer's decision on price and quality levels depends on the distance of the stores or equivalently the travel cost in the market? 2) What is the effect of non-uniform distribution of consumers' type on the quality and price levels? To that end, we consider heterogeneous consumers that differ in their willingness to pay for quality and travel costs. We adopt the Hotelling's modeling framework [14] which is defined fundamentally for showing the horizontal differentiation in a market. We investigate the impact of geographical income inequality in the market by comparing the decision of the retailer in cases of assuming uniform and non-uniform distribution of consumers' types over the market. Since the income heterogeneity of consumers is an observed fact at least in big cities, the results of this research can help retailers to understand and consider neighborhood heterogeneity in setting up their stores' policies in the market.

The remainder of this paper is organized as follows. Section 2 relates our work to the prior literature. Section 3 introduces our model and the basic assumptions. Section 4 provides the analytical results for the model under both uniform and non-uniform distribution of consumers. Section 5 provides numerical results and insights for the retailer's decision. Section 6 concludes the paper and offers some ideas for future research. All proofs are included in the Appendix.

2 Related Literature

The retailing and operations management literature have dealt with pricing strategies alongside different retail channels (e.g., [15]) or store formats (e.g., [7]) under various consumer's characteristics. In this paper we analyze the impact of considering physical stores' setup for average quality of products along with the price level decision in order to capture the market heterogeneous tastes in an efficient way. In the retail marketing literature it has been established that differentiation among retailers can be implemented through setting distinct format for the stores such as product ranges, atmospherics and price format [7]. Quality differentiated store formats or channels have been recently explored by several papers. For example, [16] empirically investigate the impact of the adoption of a retailer's factory outlet channel on the customers' spending in the retailer's traditional stores. They find out that the retailer can induce customer segmentation through self-selection when the channels are differentiated in price and quality levels. In another empirical study, Ngwe [17] shows that when a retailer captures consumers differences through both regular and outlet stores, it can increase its profit by consumers' self-selection. Among the few analytical works in this regard, Li et al. [18] investigate the strategy of a manufacturer in opening an outlet channel when outlet sales may impact on the manufacturer's brand awareness. In our study, we provide a model that captures the demographic characteristics and their influence on a retailers' store-level decision. Previous research has established a number of empirical links between demographic variables such as income levels on the consumer's store choice [19, 20]. The environmental characteristics in terms of demographics and competition provide incentives for the retailers to operate under different store formats and commercial names [20–22]. However, the prior literature has not analytically considered the non-uniform heterogeneity in the consumers willingness to pay (as a result of income disparity) on the price and quality levels of a retailer in the horizontal differentiation models.

To capture the demographic differences across neighborhoods we apply the framework of spatial differentiation models. Therefore, our approach is different from the economics studies that directly model the effect of income disparity on the competition between firms under homogeneous tastes (e.g., [23]). More specifically our model is related to the stream of literature that considers two-dimensional consumers' heterogeneity to study the firms' strategic decisions such as quality, price or location (e.g., [24–26]). These models consider a market with consumers who are located in different places (or have different taste preferences) and also differ in willingness to pay for quality. More recently, Hernandez [27] studies the impact of transportation cost on the competitive price and quality of products for two symmetric firms. Shi et al. [28] consider the two-dimensional heterogeneity to find the optimal quality in different channel structures and show that the type of consumer heterogeneity and its distribution in a market, play a substantial role in determining how a channel structure affects product quality. Among this stream of research, Desai [24] and Sedghi et al. [29] are particularly related to our paper. Desai [24] studies a product-line design problem in a market consisting of two consumer segments with high and low valuations for quality. These consumers are also heterogeneous in their taste preferences. Our paper is different from the monopoly model offered in Desai [24] as he considers only the vertical differentiation for the products and assumes a uniform distribution of consumers over the Hotelling line. On the other hand, we assume a horizontal differentiation between the stores (as they are in different locations) and a non-uniform income distribution in the market. In that sense our model is similar to Sedghi et al. [29] since consumers in each segment have a different distribution. This assumption changes the analysis of the model substantially, because the two dimensions of market heterogeneity are not independent. However, the analysis in Sedghi et al. [29] is limited to one product and the focus is on the optimal price and location, but this article is to analyze the impact of two-dimensional heterogeneity on the price and quality levels of the two stores in exogenous locations.

Although two-dimensional models of market heterogeneity shed more light on the consumers characteristics and the nature of products differentiation, they are considerably difficult to solve [25]. Accordingly, most of the spatial models are restricted to one-dimension of heterogeneity or they consider simplifying assumptions such as full market coverage or uniform distribution of consumers to make the problem more tractable. Without loss of generality, we assume that the two stores are located at the two extremes of the linear market and, therefore, the horizontal differentiation is assumed to be exogenous in our model. This assumption allows us to focus on the price and quality levels of the two stores without enforcing limiting assumptions on the distributions of consumers.

Another relevant stream of literature considers the implications of non-uniformity of consumers' distribution in the spatial models. In the competitive location models on a line, there are several papers that relax the uniformity assumption [30–36]. The focus of these studies is on the equilibrium location in a Hotelling game with the target of understanding how the model outcomes change under different distributions of consumers. It turns out that this assumption can substantially change the equilibrium location in the competitive setting. Among the recent papers that have challenged the uniform assumption Guo and Lai [37], Sedghi et al. [29] and Shi et al. [28] are more relevant to our work. Guo and Lai [37] analyze the location and price of brick-and-mortar retailers in a market that consumers are non-uniformly distributed and can purchase the product from an Online retailer at the cost of a mismatch (for example a mismatch on size or color). However, they consider a homogeneous product with respect to quality and a market consisting of consumers that all have the same willingness to pay for that product. Recently, Benassi et al. [38] proved the existence of a sub-game perfect equilibrium in pure strategies for an uncovered duopoly when the willingness to pay of consumers follow a log-concave distribution. However, their model only considers the vertical differentiation between firms. Sedghi et al. [29] explore how a firm chooses its location and price in a non-uniform market. Their analysis is based on a monopoly setting with two types of consumers but a homogeneous product. Shi et al. [28] consider a given location for a product (or firm) and analyze the implications of non-uniform distribution of consumers (or taste preferences) on the quality and price setting. However, to obtain analytical results for the non-uniform distribution they consider only one dimensional heterogeneity.

3 Model

Consider a retailer that wants to set the strategic quality and price levels for its stores in two adjacent neighborhoods. The overall perceived quality of a store can be measured by the variety and average quality of products, sales assistants, the store environment and auxiliary services. The average price level and the frequency of promotions can project the price image of that store.

To investigate the conditions that affect the two stores' price and quality levels, we consider a market where consumers are vertically heterogeneous with respect to their willingness to pay for product quality and horizontally heterogeneous with respect to their ideal location. To follow the standard horizontal differentiation literature [14], consider a retailer that wants to open two stores $i \in \{1, 2\}$ in a market with a continuum of consumers distributed along a horizontal $[0, 1]$ line. The mass of consumers in the market is normalized to one. Consumers visit any of the two stores if they gain positive utility after considering the quality, price and the cost of travel. We assume that the location of stores are determined in advance and without loss of generality, assume that store 1 and store 2 are located at $x = 0$ and $x = 1$ respectively. This assumption is widely used in the Hotelling framework (e.g., [39], [40], [41]) to provide geographical or horizontal differentiation between the firms and products. The retailer has to decide on the price and quality levels at both stores. Let p_i and q_i denote the price and quality levels at store $i (i = 1, 2)$.

Consumers in the market are assumed to be heterogeneous in two types of high-valuation consumers (known as H-type) and low-valuation consumers (known as L-type). The H-type consumers have a higher willingness to pay for quality and constitute γ percent of the market and the remaining $(1 - \gamma)$ percent are L-type consumers who have a lower valuation for quality. A consumer of type $j(j = L, H)$ derives a utility of $\theta_j q$ from shopping at a store with quality level q. Our model of vertical differentiation is based on the fact that low-income families are more sensitive to price and promotions [8]. This segmentation scheme which is based on the demographic data is attainable and can be considered by marketing managers. Moreover, one of the patterns of segmentation in the literature is the opportunity cost of time. It is common in the literature to consider income level as a proxy for this measure and assign higher opportunity cost of time to consumers with higher income (e.g. see [42]). Therefore, by considering the concept of opportunity cost of time and the framework presented in [24] and [41] we assume that the transportation (travel) cost of segment H is greater than or equal to the travel cost of L-type consumers; i.e., $t_H \geq t_L$.

With the above framework, a consumer of type j, located at x derives the net utility $U_1(\theta_j, x) = \theta_j q_1 - t_j x - p_1$ for buying from store 1 and derives the net utility $U_2(\theta_j, x) = \theta_j q_2 - t_j(1-x) - p_2$ for buying from store 2. We normalize the consumer's net utility derived from an online retailer to zero.

We first provide a benchmark case of uniform distribution of consumers types in the market. This case shows the retailer's choice when the population and income level of the two neighborhoods are the same. We then challenge this widely adopted assumption in the literature by a non-uniform distribution for each consumer's segment. The non-uniform assumption helps us to model the heterogeneity in the consumers' willingness to pay across the neighborhoods.

The objective of the retailer is to maximize its overall profit from the two stores. The cost of providing quality level q is assumed to be quadratic, $c(q) = \frac{1}{2}q^2$ (e.g., see [24, 25]). Therefore, the profit of the retailer is $\pi = \sum_{i=1,2}(p_i - c(q_i))(D_{iH} + D_{iL})$ where D_{ij} is the demand of consumers of type $j (j = L, H)$ for store $i (i = 1, 2)$. Let $r_{ij} = \max\{\frac{\theta_j q_i - p_i}{t_j}, 0\}$ be the coverage radius of store i for the consumers of type j. This means that a customer of type j is willing to travel at most the distance r_{ij} to shop at store i.

In the following analysis we direct our attention to the case that the H-type consumers are the more profitable consumers for the retailer to serve. For that, in our model we set the reservation utility [see 43] of both segments to zero. Therefore, the external options for both types of consumers become similar which makes the H-type consumers more willing to buy from the retailer. The analysis for the relative attractiveness of L-type consumers can also be done in an analogous way.

4 Optimal Price and Quality Levels

In this section we analytically consider the joint optimization of price and quality levels when the retailer operates with two stores in a heterogeneous market.

Depending on the relative values of unit travel costs (also known as transportation costs) and the marginal valuations for qualities ($\theta_j, j = L, H$), the retailer may not find it optimal to serve all the consumers in a given segment. Therefore, there are three possibilities for the retailer in serving the consumers in a given segment: full coverage(F); partial coverage (P); and no coverage(N). Based on these possibilities we identify the following cases:

Case FF- The retailer fully covers both segments.

Case FP -The retailer fully covers H-type segment but only partially covers the L-type segment.

Case FN -The retailer fully covers H-type segment but does not serve any of the L-type segment.

Case PF-The retailer fully covers L-type segment but only partially covers H-type segment.

Case PP-The retailer partially covers both segments.

Case PN-The retailer partially covers H-type segment but does not serve any of the L-type segment.

Since we assume that the H-type consumers are the more attractive consumers to serve, we omit the cases that the retailer does not serve the H-type consumers.

Figure 1 shows an example of partial coverage of both segments (Case PP). Note that in this case there are some consumers in the middle of the market that purchase from none of these stores.

[Insert Figure 1 about here.]

Since the market has two-dimensional heterogeneity, its parameters can greatly affect the decision of the retailer. Therefore, considering these cases is necessary to obtain the optimal solution. We first analyze the partial coverage of the market and derive some analytical results and insights. Then, we consider the full market coverage and numerically show the impact of travel cost on the optimal price and quality decisions. Specifically, we are interested to see how the decision of the retailer changes under the non-uniform distribution of consumers' income.

4.1 Partial Market Coverage

We first assume that market conditions imply that only Cases PP or PN is in order. These cases are valid when the transportation costs are relatively high and the retailer may not find it optimal to serve all the consumers in any segment. To ensure that the travel cost is high enough to encourage only partial coverage of the market we need to check the condition $r_{1j} + r_{2j} \leq 1$. This condition is equivalent to $t_j \geq \theta_j(q_1 + q_2) - (p_1 + p_2)$. Thus, when we have the optimal price and quality levels of the stores, exact conditions that ensure $r_{1j} + r_{2j} \leq 1$ can be obtained. In the following assumption we specify a condition that guarantees the partial coverage of the market.

Assumption 1. *When the travel costs in the market are so high that satisfy the following condition, the retailer has no incentive to fully serve any segment of the market:*

$$t_j \geq 2\theta_j\theta_H - \theta_L{}^2, \qquad j = L, H. \tag{1}$$

Please refer to Appendix A to see the detail of deriving this condition.In this section we analyze the optimal price and quality decisions for each store under two different neighborhood conditions. First, we consider the case of identical neighborhoods with two segments but uniform distribution of consumers in the market. In the second case, we consider a non-uniform distribution of consumers which represents the role of income disparity across the two neighborhoods.

4.1.1 Uniform Distribution

Assume that travel cost is high enough so that the two stores cannot fully cover any market segment in the optimal setting. We also assume that consumers of each type are uniformly distributed in the market. This implies that the two neighborhoods are identical in terms of consumers' income level. The retailer needs to decide on the level of price and quality for each store to maximize its total profit. This decision depends on the market conditions and will determine whether both segments are served (Case PP) or just the high-valuation segment is targeted (Case PN). This is also important to see whether the optimal decision implies vertical differentiation between the stores or just the horizontal differentiation can lead to the optimal decision. Since the conditions imply that under the optimal solution the two stores have incomplete coverage of the market then, the two stores have no common coverage area and can act as local monopolies. Therefore, when the distributions of consumers' types are uniform in the market, it is clear that in the optimal solution the stores' price and quality levels should be identical.

Therefore, high horizontal differentiation between the two stores, prevents cannibalization and along with the uniform distribution of consumers leads the retailer to set up identical stores. This shows that under the uniform distribution, the existence of different income levels does not encourage the retailer to vertically differentiate its stores. However, these identical stores can target only one type of consumers or serve both types. In the following lemma we identify the condition that encourages the retailer to serve both consumer segments.

Lemma 1. *When the consumers in each segment are uniformly distributed in the market and the travel cost is high, the retailer targets both segments if and only if* $\theta_L > \theta_H(\frac{(\gamma t_L((1-\gamma)t_H+\gamma t_L)^3)^{1/4}-\gamma t_L}{(1-\gamma)t_H})$

Lemma 1 indicates that when the willingness to pay of L-type consumers is relatively low, the retailer prefers to set its price and quality to target only the high-valuation segment. For the retailer to have an incentive to target both segments, the willingness to pay of low-valuation segment should be higher than a threshold. Determining this threshold needs complete knowledge of the two segments (i.e., willingness to pay, travel costs and the market size). As far as this information is available, the retailer can analyze the market and set the target segments.

According to Lemma 1 if θ_L is less than the threshold then the retailer targets only H-type consumers. It leads to the following optimization problem (since the two stores are identical, the subscript i is dropped from the model):

$$max_{p,q} \quad \pi_1 = (p - \frac{q^2}{2})2\gamma(\frac{\theta_H q - p}{t_H}) \tag{2}$$

The optimal solution is given by the first-order conditions as $q^* = \theta_H$ and $p^* = \frac{3}{4}\theta_H^2$ (the second derivative test for this solution is provided in Appendix C).

Analogously, when the retailer targets both types of consumers, the optimization problem of a store located at $x = 0$ is as follows:

$$max_{p,q} \quad \pi_2 = (p - \frac{q^2}{2})\left(\gamma(\frac{\theta_H q - p}{t_H}) + (1-\gamma)(\frac{\theta_L q - p}{t_L})\right) \tag{3}$$

Which results in $q^* = \frac{\gamma t_L \theta_H + (1-\gamma)t_H\theta_L}{\gamma t_L + (1-\gamma)t_H}$ and $p^* = \frac{3}{4}q^2$ (the second derivative test for this solution is provided in Appendix C).

[Insert Table 1 about here.]

Summarizing the above solutions, the optimal decision of the retailer and the associated profits are shown in Table 1. When the retailer targets both types of consumers, it decreases the quality and price levels of its stores. The optimal quality under targeting both market segments, is

$q^* = \frac{\gamma t_L \theta_H + (1-\gamma) t_H \theta_L}{\gamma t_L + (1-\gamma) t_H} = \alpha \theta_H + (1-\alpha)\theta_L$, where $\alpha = \frac{\gamma t_L}{\gamma t_L + (1-\gamma) t_H}$. This quality level is a convex combination of θ_L and θ_H, and thus it is less than θ_H (the quality level when targeting H-type consumers only).

4.1.2 Non-Uniform Distribution

Now, we consider a non-uniform distribution for the consumers of each segment in order to model two neighborhoods with different populations and income levels. We assume that consumers of type j ($j = H, L$) are distributed on the line according to a density function $f_j(.)$. The distributions we consider in this section are analogous to Sedghi et al. [29] which provides a framework for modeling polarized markets. To show the differences in the neighborhoods, we assume that the consumers in the right side of the market have higher average income than the consumers in the left side. To be more specific, we make the following assumption on the distribution of consumers.

Assumption 2. *The distribution of consumers holds the following conditions:*

1. *$f_H(x)$ is linearly increasing in x while $f_L(x)$ is linearly decreasing in x,*
2. *$f_L(x) \geq f_H(x), \quad \forall x \leq \frac{1}{2}$,*
3. *$f_L(x) \leq f_H(x), \quad \forall x \geq \frac{1}{2}$.*

This assumption is in line with the neighborhood differences and states that when the number of consumers in both segments are equal, it is more likely that a consumer in the left side of the market to be of L-type and vise versa. This assumption while enabling us to make analytical comparisons, can contribute to analyzing polarized neighborhoods, in that the types of consumers differ across the two neighborhoods. Moreover, Sedghi et al. [29] show that considering linear distribution is a good approximation for modeling these types of heterogeneities.

For the ease of exposition, we show the optimal decision of the retailer under the uniform and non-uniform distributions of consumers by superscript U and N, respectively. The optimization problem that the retailer faces is as follows:

$$\max_{p_1, q_1, p_2, q_2} \pi^N = (p_1 - \frac{q_1^2}{2})(\gamma \int_0^{r_{1H}} f_H(x)dx + (1-\gamma)\int_0^{r_{1L}} f_L(x)dx) \\ + (p_2 - \frac{q_2^2}{2})(\gamma \int_{1-r_{2H}}^1 f_H(x)dx + (1-\gamma)\int_{1-r_{2L}}^1 f_L(x)dx) \tag{4}$$

Since in this section we assume a high transportation cost, the two stores can be considered as local monopolies. Therefore, in the above optimization problem one can separate π^N into two distinct functions for stores 1 and 2. To obtain the optimal quality and price levels for store 1 we write the first order conditions:

$$\frac{\partial \pi^N}{\partial p_1} = \gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L}) - (p_1 - \frac{q_1^2}{2})(\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})) = 0. \quad (5)$$

$$\frac{\partial \pi^N}{\partial q_1} = -q(\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})) + (p_1 - \frac{q_1^2}{2})(\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})) = 0. \quad (6)$$

Equation (6) results in:

$$q_1 = (p_1 - \frac{q_1^2}{2})\frac{\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})}{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})} \quad (7)$$

From Equation (5), we derive $p_1 - \frac{q_1^2}{2} = \frac{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})}$. The left-hand side can be replaced for $p_1 - \frac{q_1^2}{2}$ in Equation (6) to get:

$$q_1^N = \frac{\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})} \quad (8)$$

p_1^N is also given by replacing q_1^N in Equation (5):

$$p_1^N = \frac{q_N^2}{2} + \frac{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})} \quad (9)$$

As observed in Equations (8) and (9), it is not straightforward to obtain the closed-form solution for the optimal price and quality. However, the optimal solution is a root to these equations. Since we derived these equations under the condition that the market coverage for any segment is less than half, we can obtain some insights, which we state in the following propositions.

Analogously, the optimal price and quality levels for store 2 have the following form:

$$q_2^N = \frac{\frac{\gamma\theta_H}{t_H} f_H(1-r_{2H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(1-r_{2L})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})}, \quad (10)$$

$$p_2^N = \frac{q_N^2}{2} + \frac{\gamma F_H(1-r_{2H}) + (1-\gamma)F_L(1-r_{2L})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})}. \quad (11)$$

As seen above, the optimization problem results in different quality and price levels for the two stores. This difference cannot be justified by the inequality in the income levels alone. In fact, the non-uniform distribution of incomes (as a representative of willingness to pay) over the neighborhoods plays the main role in obtaining different price and quality levels at the optimal solution. This kind of heterogeneity encourages the retailer to operate under two different quality levels. This result may partly explain the vertical differentiation that the stores of a retailer have in practice. For example, Kroger's Food4Less store offers different price and quality levels compared to the other Kroger's store, Fresh Fare. The apparel manufactures also set different price and quality levels for their outlet stores and their stores in the midtown.

It is interesting to see how the qualities differ from the case of uniform distribution. The following proposition shows the comparison between the quality levels.

Proposition 1. *Under high travel costs, the optimal quality level of stores when the consumers are uniformly distributed are identical and a convex combination of differentiated quality levels of the two stores under non-uniform distribution of consumers, i.e.,* $q_1^N \leq q^U \leq q_2^N$.

This proposition states that the non-uniform distribution of the two market segments leads to lower quality level for store 1 but higher quality level for store 2. The strict inequality holds when the conditions of the market encourages the retailer to target both segments. The conditions for the retailer to partially serve the L-type consumers (Case PP) or not to target this segment (Case PN) is in the following lemma.

Lemma 2. *There exists a threshold for* θ_L *such that when* θ_L *is less than that threshold the retailer targets only the H-type consumers.*

This lemma is similar to Lemma 1 in the uniform case. When the willingness to pay of low-valuation segment is higher than a threshold the retailer will set up stores to target both types of consumers, with the quality and price levels derived from Relations (8) to (11).

Proposition 2. *Under the policy of targeting just the high-valuation segment, it is optimal to offer the same quality level in both stores but charge a higher price at store 2 compared to store 1, i.e.,* $q_1^N = q_2^N = \theta_H$ *and* $p_1^N < p_2^N$.

This proposition helps us to understand one of the reasons why retailers may charge different prices for the same product in different areas. For example, Walgreens drugstores sometimes offer different prices in different neighborhoods. Although, this diversity in price could be due to various reasons such as competition or rent costs in those areas, as Proposition 2 suggests, the non-uniformity of the distribution of consumers' willingness to pay can also encourage the retailers to offer heterogeneous price levels in stores across different neighborhoods.

4.2 Full Market Coverage

In this section we investigate the case of low transportation (travel) cost in the market. This allows the retailer to fully cover one or both segments in the optimal setting. To write the optimization problem that the retailer faces under full market coverage, we first determine the location of a consumer of type $j(j = L, H)$ who is indifferent between buying from either stores:

$$\theta_j q_1 - t_j x_j - p_1 = \theta_j q_2 - t_j(1 - x_j) - p_2 \Rightarrow x_j = \frac{1}{2} - \frac{\theta_j(q_2 - q_1) - (p_2 - p_1)}{2t_j}. \tag{12}$$

Figure 2, shows an example of the full coverage of the H-type consumers (Case FP) and the position of x_H in the market.

[Insert Figure 2 about here.]

In the following we write the optimization problem of the retailer to obtain the optimal policy on quality and price for the cases FP and FN in which the retailer fully covers only the high-end segment. The optimization problems for cases FF and PF are presented in Appendix C.

$$\begin{aligned} \max_{p_1,q_1,p_2,q_2} \pi = & (p_1 - c(q_1))(\gamma \int_0^{x_H} f_H(x)dx + (1-\gamma)\int_0^{r_{1L}} f_L(x)dx) \\ & + (p_2 - c(q_2))(\gamma \int_{x_H}^1 f_H(x)dx + (1-\gamma)\int_{1-r_{2L}}^1 f_L(x)dx) \end{aligned} \tag{13}$$

s.t:

$$x_H = \frac{1}{2} - \frac{\theta_H(q_2 - q_1) - (p_2 - p_1)}{2t_H}, \tag{14}$$

$$r_{iL} = \max\{0, \frac{\theta_L q_i - p_i}{t_L}\}, \qquad i = 1, 2, \tag{15}$$

$$\theta_H q_1 - t_H x_H - p_1 \geq 0, \tag{16}$$

$$p_i, q_i \geq 0, \qquad i = 1, 2. \tag{17}$$

In the objective function (13) if the market follows a uniform distribution of income levels, then we have $f_j(x) = 1, (j = L, H)$. Constraint (14) shows the position of the indifferent consumer. Constraint (15) is the definition of coverage radius and it ensures that the coverage is non-negative. These two constraints are written to clarify the optimization problem and they can be replaced in the objective function. Constraint (16) states that the net utility of the H-type consumer who is indifferent between buying from either stores, should be nonnegative. In fact, this constraint ensures that the high-end segment is fully covered. The analysis of the optimal solution depends on the values of r_{iL} in Constraint (15) which show the targeting policy of the retailer in each of the stores. Since the parametric analysis of these cases provides little insights for the optimal solution, in the next section we numerically analyze the optimal decision of the retailer.

5 Numerical Results

In this section we provide several insights about the optimal price and quality levels that the retailer sets for its stores. We are especially interested in the impact of travel cost on the decision of the retailer. Specifically, how the strategy of the retailer differs in case of high and low transportation cost and how the distribution of consumers affects this strategy?

To answer these questions we consider the optimal solution of all cases that can emerge by different levels of travel cost. To simultaneously show the effect of travel costs of both segments on the retailer's decision, we assume that the travel cost in each segment is proportional to the marginal willingness to pay for quality in that segment. In other words, we assume that $t_j = k\theta_j$ $(j = L, H)$. This assumption allows us to alter the travel cost for both segments proportionally while preserving the $t_H \geq t_L$ assumption. We call k the travel cost factor and show the effect of this factor on the retailer's decisions. In the following analysis, we set $\theta_H = 1,\ \theta_L = 0.7$ and $\gamma = 0.5$.

5.1 Optimal Price Levels

Uniform Distribution of Consumers. We first, assume that consumers' types are uniformly distributed in the neighborhoods. Figure 3 illustrates how the optimal price of the retailer changes with respect to the travel cost factor. Note that the retailer uses different pricing strategies depending on the level of travel cost in the market. Also, note that since in our numerical study we assigned a relatively high valuations for quality to L-type consumers, according to Lemma 1, Case PN does not emerge in the optimal strategy of the retailer.

Figure 3 shows that there are four strategies that the retailer may choose. The discontinuity on the price curves is because of adapting a different strategy according to the range of travel cost factor. The numerical results are interesting because considering that consumers are uniformly distributed in the neighborhoods one might conclude that the retailer has an incentive to set up identical stores in the market. However, Figure 3 shows that this is not the case for all levels of travel cost. There exists some ranges for the travel cost within which the retailer prefers to differentiate between its stores even in the case of uniform distribution of consumers' types. One of the examples of such setting and its optimal coverage is depicted in Figure 2. In this case the retailer applies different targeting strategies for its two stores. However, as Figure 3 shows, high and low travel costs will encourage the retailer to set up identical stores and follow similar targeting strategy in the stores.

[Insert Figure 3 about here.]

When the travel cost is low ($k < 0.45$) the retailer fully serves all the H-type consumers and sets high prices in the market (Case FN). In contrast to the case of competition between retailers where the low travel cost results in a Bertrand competition and reduces the prices, here, as the pricing in the two stores is regulated by a single retailer, the low travel cost results in high identical prices for the two stores. With these high prices the retailer targets only the H-type consumers.

When the travel cost is moderate in the market ($0.45 \leq k \leq 0.63$), the retailer changes its strategy to serve both types of consumers (Case FP). However, based on the fact that whether it differentiates between its stores or just sets up identical stores, it might charges different prices. As seen in Figure 3, Case FP has two different parts. When $0.45 \leq k \leq 0.53$, the retailer fully covers all the H-type consumers but differentiates between its two stores. In that case, store 1 serves both types of consumers but store 2 targets only the high-end segment. This situation is depicted in Figure 2. However, when $0.53 < k \leq 0.63$ the retailer targets both consumers with identical stores. Therefore, it reduces the price of store 2 to serve the low-end consumers as well. In the case of identical stores the retail price is increasing in the market. This is because the quality levels of the stores are also increasing.

Finally, when the travel cost is high enough ($k > 0.63$), the retailer partially serves both types of consumers. The analytic results for this case are presented in Section 4.1.1. As the analysis in that section also suggests, the retailer opens up two identical stores under high travel cost.

Non-uniform Distribution of Consumers. Now, we assume that the consumers of each type are non-uniformly distributed in the market. Specifically, we assume $f_H(x) = 2x$ and $f_L(x) = 2 - 2x$. While satisfying Assumption 2, these distributions also approximately represent the income disparity that can be seen in large cities. Figure 4, illustrates the optimal pricing strategy of the retailer.

Note that in contrast to the uniform case, the retailer differentiates between the stores, but the level of differentiation depends on the travel cost in the market.

[Insert Figure 4 about here.]

When the travel cost is very low ($k < 0.27$), the retailer targets only H-type consumers (Case FN) but with slightly different prices. Moderate levels of travel cost ($0.27 \leq k \leq 0.55$) leads the retailer to apply a higher level of differentiation between its stores and target both types of consumers (Case FP). In this case, store 1 targets both types of consumers while store 2 serves only H-type consumers. Moreover, the price level of store 1 is increasing with respect to the travel cost factor. This happens because the quality level also increases in travel cost factor for that store.

When the travel cost is relatively high ($k \geq 0.55$) each store can operate only in a limited local market. Therefore, as the retailer cannot fully serve H-type consumers, it reduces its price in store 2 to increase the coverage radius for the high-end segment.

[Insert Figure 5a and 5b about here.]

Comparison between uniform and non-uniform markets. To see the effect of consumers' types distribution on the optimal decision of the retailer, we first compare the optimal prices in Figures 3 and 4. Several insights emerge. In case of uniform distribution, the travel cost factor needs to be higher than non-uniform distribution case in order to encourage the retailer to target the low-end segment as well. Therefore, one can say that the income disparity in the neighborhoods is more in favor of low end segment in terms of having access to the stores. However, as Figure 5a shows, the quality that the L-type consumers receive is low. Moreover, in the non-uniform market, store 1 generally offers lower quality levels compared to the uniform setting, except for a small range of travel costs ($0.53 < k < 0.6$). For high transportation costs as we showed in Proposition 1 the inequality $q_1^N < q_1^U = q_2^U < q_2^N$ holds for the quality levels. This disparity also encourages the retailer to provide higher quality levels in the right side of the market (Figure 5b). Another interesting observation is that the low travel cost will result in the same quality level and targeting strategy under uniform and non-uniform distribution of consumers' types. Therefore, when the unit travel cost across the market is lower than a threshold, uniform distribution can be a valid assumption even under disparity in willingness to pay of consumers. However, for higher travel cost in the market, using uniform assumption to simplify the analysis can lead to non-efficient decisions by the retailer.

[Insert Figure 6 about here.]

The maximum profit that the retailer earns in uniform and non-uniform markets is depicted in Figure 6. When the travel cost is relatively low, the profit of the retailer is similar under the two settings but when the travel cost gets relatively high the retailer can gain more profit by differentiating the stores. The income disparity in the market provides the retailer with more opportunity for store differentiation without facing the threat of cannibalization between its stores. However, the quality differentiation is not always an available option for the retailers. Small retailers that usually operate under one-brand name and format, do not have such kind of flexibility to change the quality level of their stores due to the reputation they want to set in the market. In that case they need to consider their stores' location in homogeneous neighborhoods. However, larger companies or retailers that operate under different names (like Kroger Inc.) can choose among the different formats and store characteristics when locating in different neighborhoods to increase their market share.

We extend the numerical results by showing the impact of variation in θ_L (marginal valuation of L-type consumers) and γ (percentage of H-type population in the market) on the price and quality levels of stores under both uniform and non-uniform distribution of consumers' types. To that regard, we set $\theta_H = 1$, $t_L = 0.35$ and $t_H = 0.5$. Note that these levels of travel costs can be found by setting $k = 0.5$ in the numerical results of Section 5 and lies in the range to encourage the retailer to fully serve the H-types in the market. Therefore in the following results, we see FN and FP strategy. We compare them under uniform and non-uniform cases. The optimal price levels for different levels of θ_L and γ can be seen in Table 2. The proposed range for θ_L is set to make the distinction between the two classes meaningful. For the uniform case, since the price level is identical in both stores when $\gamma = 0.3$ and $\gamma = 0.7$, only one price has been shown in Table 2. For $\gamma = 0.5$ the retailer sets different price levels at its stores and this is consistent with the price in Figure 3 for $k = 0.5$. As seen in Table 2, for low levels of θ_L the retailer follows FN strategy, in that it does not serve any consumer in the L-segment. When θ_L is more than a threshold level, the retailer changes its strategy to FP and sets a low price in Store 1 to serve the L-type consumers too. It can also be seen that this threshold is increasing by γ in the market. The intuition behind this result is that the increase in the population of L-type consumers provides an incentive for the retailer to serve this segment. Table 2 confirms the results in Figures 3 and 4 and shows that in some markets the price differentiation between the stores of a retailer is very different under uniform and non-uniform case. Ignoring the distribution of consumer types (or equivalently the income distribution in the neighborhood) may result in the profit loss for the retailer.

[Insert Table 2 about here.]

Table 3 shows the quality level of these stores for different levels of θ_L and γ. The results show that in the non-uniform case, the retailer always sets the highest quality level for Store 2. However, the quality level in Store 1 depends on the threshold level for θ_L that provides the retailer with an incentive to serve the L-segment. It is also apparent that $q_1^N \leq q^U \leq q_2^N$ for different levels of γ.

[Insert Table 3 about here.]

Table 4 shows the maximum profit of the retailer in the uniform and non-uniform markets. Since the non-uniform distribution of incomes geographically separates the H-type and L-type segments, it weakens the cannibalization effect. Therefore, the higher profit for the retailer under the non-uniform distribution of incomes can be justified by the ability of the retailer to differentiate its stores and expand its market without the cannibalization threat. Table 4 shows that the non-uniform distribution in heterogeneous markets provides potential profit for the retailers that are able to operate under different brand names and target different segments in the market.

[Insert Table 4 about here.]

6 Conclusion

In this paper we considered the problem of setting price and quality levels for the stores of a retailer in two adjacent neighborhoods. Consumers in the market are different in terms of their willingness to pay for quality and their location. Consumers can be segmented in two types based on their valuations for quality (or price sensitivity). The high-end segment cares more about the quality while the low-end segment is more price sensitive. We considered both uniform and non-uniform distributions of each consumers' type in the market. We show how this spatial heterogeneity in willingness to pay, which can be due to the geographic income disparity, can affect the retailer's decision. We adopted the Hotelling's modeling framework to derive the stores demand in the spatial market.

Our analysis shows that the distance of the stores (or equivalently the travel cost) can play a key role in the retailer's decision on price and quality levels offered in its stores. High travel costs provide an incentive for the retailer to differentiate the stores when the consumers' types follow a non-uniform distribution in the market. The spatial income disparity that has been recently amplified in big cities, can provide an incentive for retailers to differentiate between their stores in terms of price and quality levels. Therefore, non-uniform distribution of incomes in neighborhoods, can change the optimal strategy in targeting a consumer segment and should be considered by retailers upon entry in a new market. We demonstrated that failure to account for neighborhoods heterogeneity can lead to non-efficient decisions on the quality and price levels among the stores.

Future research could consider the competition between two retailers in such markets to examine how the strategies of the retailers in quality and price level vary under competition when two types of customers with non-uniform distribution exist in the market. In this article, we also assumed exogenous locations for the stores. Another avenue for research is the joint optimization of location, price and quality levels of stores in a heterogeneous, non-uniform market.

Appendix A Derivation of condition in Assumption 1

To derive the conditions for travel costs to ensure partial market coverage, we consider both uniform and non-uniform distribution for consumers' types in the market. The partial coverage implies that the total coverage by the two stores is less than the whole market size for each consumer segment, i.e., $r_{1j} + r_{2j} < 1$. Under the uniform distribution, since the stores are identical this condition reduces to $r_j < 1/2$, which implies $t_j \geq 2(\theta_j q - p)$. Since $q \leq \theta_H$ and $p \geq \frac{3}{4}\theta_L^2$ (please refer to Table 1) we have:

$$2(\theta_j q - p) < 2(\theta_j \theta_H - \frac{3}{4}\theta_L^2). \tag{A.1}$$

Therefore, for the uniform case $t_j \geq 2(\theta_j \theta_H - \frac{3}{4}\theta_L^2)$ ensures that the market is partially covered. For the non-uniform distribution, partial market coverage is equivalent to $r_{1j} + r_{2j} < 1$, which results in $\theta_j(q_1^N + q_2^N) - (p_1^N + p_2^N) < t_j$. On the other hand, Proposition 2 implies that $q_1^N \leq q_2^N$ and $p_1^N \leq p_2^N$. Therefore we have the following condition:

$$\theta_j(q_1^N + q_2^N) - (p_1^N + p_2^N) \leq 2(\theta_j q_2^N - p_1^N) \tag{A.2}$$

We use Relations (10) and (9) to show that $q_1^N \leq \theta_H$ and $p_1^N \geq \theta_L^2/2$. By using (10) we have:

$$q_2^N = \frac{\frac{\gamma\theta_H}{t_H} f_H(1-r_{2H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(1-r_{2L})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})} < \frac{\frac{\gamma\theta_H}{t_H} f_H(1-r_{2H}) + \frac{(1-\gamma)\theta_H}{t_L} f_L(1-r_{2L})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})} = \theta_H. \quad \text{(A.3)}$$

To show that $p_1^N > \theta_L^2/2$ we first prove that $q_1^N > \theta_L$.

$$q_1^N = \frac{\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})} > \frac{\frac{\gamma\theta_L}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})} = \theta_L. \quad \text{(A.4)}$$

Using Relation (9) we have:

$$p_1^N = \frac{q_N^2}{2} + \frac{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})} > \frac{q_N^2}{2} > \frac{\theta_L^2}{2}. \quad \text{(A.5)}$$

By using Inequalities (A.3) and (A.4) we can show that the right side of Inequality (A.2), which provides a lower bound for the travel costs (i.e., $t_j \geq 2\theta_j\theta_H - \theta_L^2$) results in the partial market coverage under non-uniform case. To use the same travel cost threshold for the uniform and non-uniform distribution we use $t_j \geq 2\theta_j\theta_H - \theta_L^2$ as the condition that ensures partial market coverage under uniform and non-uniform distribution of consumers' types.

Appendix B Proofs

Proof of lemma 1. We first obtain the optimal profit of the retailer when it targets only H-type consumers (Case PN). Then we compare it with the optimal profit under partial coverage of both segments (Case PP) to specify the threshold value for θ_L. In Case PN, as proposed in Section 4.1.1 the retailer's optimization problem is:

$$max_{p,q} \quad \pi_1 = 2\gamma(p - \frac{q^2}{2})(\frac{\theta_H q - p}{t_H}), \quad \text{(B.1)}$$

which yields to the optimal quality and price levels of $q = \theta_H, p = \frac{3}{4}\theta_H^2$.

When the retailer targets two segments (Case PP) and the travel costs are high the retailer symmetric store configuration with the following optimization:

$$max_{p,q} \quad \pi_2 = 2(p - \frac{q^2}{2})\left(\gamma(\frac{\theta_H q - p}{t_H}) + (1-\gamma)(\frac{\theta_L q - p}{t_L})\right), \quad \text{(B.2)}$$

which results in $q = \frac{\gamma t_L\theta_H + (1-\gamma)t_H\theta_L}{\gamma t_L + (1-\gamma)t_H}$ and $p = \frac{3}{4}q^2$.

We then obtain the optimal profits under cases PN and PP.

$$\pi_1^* = \frac{\gamma{\theta_H}^4}{8t_H} \quad \text{(B.3)}$$

$$\pi_2^* = \frac{(\gamma t_L\theta_H + (1-\gamma)t_H\theta_L)^4}{8(\gamma t_L + (1-\gamma)t_H)^3 t_L t_H} \quad \text{(B.4)}$$

Comparing Relations (B.3) and (B.4), we get $\pi_1^* \leq \pi_2^*$ if and only if $\theta_L \geq \theta_H(\frac{(\gamma t_L((1-\gamma)t_H+\gamma t_L)^3)^{1/4}-\gamma t_L}{(1-\gamma)t_H})$.

$\square$

Proof of Proposition 1. We first show that $q_1^N \leq q^U$. Note that q^U can be written as $q^U = \alpha\theta_H + (1-\alpha)\theta_L$. From Equation (8) we can also write q_1^N as a convex combination of θ_H and θ_L. More specifically, $q_1^N = \alpha_1\theta_H + (1-\alpha_1)\theta_L$ where $\alpha_1 = \frac{\frac{\gamma}{t_H}f_H(r_{1H})}{\frac{\gamma}{t_H}f_H(r_{1H})+\frac{1-\gamma}{t_L}f_L(r_{1L})}$. If $\alpha_1 \leq \alpha$ then $q_1^N \leq q^U$.

$$\alpha_1 \leq \alpha \iff \frac{\frac{\gamma}{t_H}f_H(r_{1H})}{\frac{\gamma}{t_H}f_H(r_{1H})+\frac{1-\gamma}{t_L}f_L(r_{1L})} \leq \frac{\frac{\gamma}{t_H}}{\frac{\gamma}{t_H}+\frac{1-\gamma}{t_L}} \iff f_H(r_{1H}) \leq f_L(r_{1L}). \tag{B.5}$$

When the travel costs are high enough, each store of the retailer can only serve less than half of the market, which results in $r_{1L} < \frac{1}{2}$ and $r_{1H} < \frac{1}{2}$. Therefore, Assumption (2) implies that $f_H(r_{1H}) \leq f_L(r_{1L})$ which by the argument in (B.5) results in $q_1^N \leq q^U$.

The same proof can apply for $q_2^U \geq q^N$. In an analogous way, we can write q_2^N as a convex combination of θ_L and θ_H, i.e., $q_2^N = \alpha_2\theta_H + (1-\alpha_2)\theta_L$ where $\alpha_2 = \frac{\frac{\gamma\theta_H}{t_H}f_H(1-r_{2H})}{\frac{\gamma}{t_H}f_H(1-r_{2H})+\frac{1-\gamma}{t_L}f_L(1-r_{2L})}$. In this case we show that $\alpha_2 \geq \alpha$.

$$\alpha_2 \geq \alpha \iff \frac{\frac{\gamma}{t_H}f_H(1-r_{2H})}{\frac{\gamma}{t_H}f_H(1-r_{2H})+\frac{1-\gamma}{t_L}f_L(1-r_{2L})} \geq \frac{\frac{\gamma}{t_H}}{\frac{\gamma}{t_H}+\frac{1-\gamma}{t_L}} \iff f_H(1-r_{2H}) \geq f_L(1-r_{2L}) \tag{B.6}$$

Assumption (2) implies that if $r_{2L} \leq r_{2H} \leq \frac{1}{2}$ then $f_H(1-r_{2H}) \geq f_L(1-r_{2L})$. Since we assumed that the market is partially covered, the coverage radius for the upscale store should be lower than the half. Therefore, $r_{2L} \leq r_{2H} \leq \frac{1}{2}$ which completes the proof.

$\square$

Proof of Lemma 2. Let us denote the profit of the retailer under cases PN and PP by $\pi_1^N(\theta_L)$ and $\pi_2^N(\theta_L)$ respectively. To prove that a threshold θ_L^N exists so that $\pi_1^N(\theta_L) \leq \pi_2^N(\theta_L), \quad \forall\theta_L \geq \theta_L^N$ we use the Bolzano's theorem. Under Case PN we have:

$$\begin{aligned}\max_{p_1,q_1,p_2,q_2} \pi_1^N = (p_1 - c(q_1))(\gamma \int_0^{r_{1H}} f_H(x)dx) \\ + (p_2 - c(q_2))(\gamma \int_{1-r_{2H}}^{1} f_H(x)dx).\end{aligned} \tag{B.7}$$

Under Case PP we have:

$$\begin{aligned}\max_{p_1,q_1,p_2,q_2} \pi_2^N = (p_1 - c(q_1))(\gamma \int_0^{r_{1H}} f_H(x)dx + (1-\gamma)(\int_0^{r_{1L}} f_L(x)dx)) \\ + (p_2 - c(q_2))(\gamma \int_{1-r_{2H}}^{1} f_H(x)dx + (1-\gamma)(\int_{1-r_{2L}}^{1} f_L(x)dx)).\end{aligned} \tag{B.8}$$

Note that π_1^N is independent of θ_L because we assumed that in Case PN the retailer targets H-type consumers. But, since π_2^N denotes the profit under serving both segments, it depends on the value of θ_L. We show that π_2^N is increasing in θ_L. By the Envelope Theorem we have:

$$\frac{\partial \pi_2^N}{\partial \theta_L}|_{(p_1^N,p_2^N,q_1^N,q_2^N)} = (p_1^N - c(q_1^N))\frac{(1-\gamma)}{t_L}f_L(r_{1L}) + (p_2^N - c(q_1^N))\frac{(1-\gamma)}{t_L}f_L(r_{2L}) > 0. \tag{B.9}$$

The above derivative is positive since in the optimal setting $p_1^N - c(q_1^N)$ and $p_2^N - c(q_2^N)$ are positive. Therefore, we showed that $\frac{\partial \pi_2^N}{\partial \theta_L} > \frac{\partial \pi_1^N}{\partial \theta_L} = 0$. Moreover, we know that $\pi_1^N(0) > \pi_2^N(0)$ and $\pi_1^N(\theta_H) < \pi_2^N(\theta_H)$. In this setting the Bolzano's theorem implies that there exists a $\theta_L^N \in (0, \theta_H)$ such that $\pi_2^N(\theta_L^N) = \pi_1^N$. Since $\frac{\partial \pi_2^N}{\partial \theta_L} > 0$, then for $\theta_L > \theta_L^N$ we have $\pi_2^N(\theta_L^N) > \pi_1^N$. Therefore, for $\theta_L > \theta_L^N$ the retailer can increase its profit by targeting both segments. □

Proof of Proposition 2. Based on Lemma 2 when θ_L is less than the threshold level θ_L^N the retailer targets only the H-type consumers. Under Case PN, we know that the market is only partially covered, thus the demand of the stores do not overlap. In this case the decision of each store can be made independent of the other store. profit function of the retailer is:

$$\pi^N = (p_1 - \frac{q_1^2}{2})\gamma \int_0^{r_{1H}} f_H(x)dx + (p_2 - \frac{q_2^2}{2})\gamma \int_{1-r_{2H}}^1 f_H(x)dx \tag{B.10}$$

The derivation of optimal solution is analogous to the general case presented in Section 4.1.2.

$$\frac{\partial \pi^N}{\partial p_1} = \gamma F_H(r_{1H}) - (p_1 - \frac{q_1^2}{2})\frac{\gamma}{t_H} f_H(r_{1H}) = 0. \tag{B.11}$$

$$\frac{\partial \pi^N}{\partial q_1} = -q\gamma F_H(r_{1H}) + (p_1 - \frac{q_1^2}{2})\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) = 0. \tag{B.12}$$

Which results in:

$$q_1^N = \theta_H, \quad p_1^N = \frac{\theta_H^2}{2} + \frac{t_H F_H(r_{1H})}{f_H(r_{1H})}. \tag{B.13}$$

Solving the first-order conditions for store 2 is also similar and results in:

$$q_2^N = \theta_H, \quad p_2^N = \frac{\theta_H^2}{2} + \frac{t_H \bar{F}_H(1-r_{2H})}{f_H(1-r_{2H})}. \tag{B.14}$$

From Relations (B.13) and (B.14) it is derived that $q_1^N = q_2^N = \theta_H$. To compare the prices, note that we assumed that $f_H(x)$ is increasing in x, thus we have $F_H(x) \leq x f_H(x)$ and $\bar{F}_H(x) \geq x f_H(x)$, which results in:

$$\frac{F_H(r_{1H})}{f_H(r_{1H})} \leq r_{1H}, \qquad \frac{\bar{F}_H(1-r_{1H})}{f_H(1-r_{2H})} \geq r_{2H}. \tag{B.15}$$

Now, suppose that $p_1^N \geq p_2^N$ which leads to $r_{1H} \leq r_{2H}$. By using Inequality (B.15) it is derived that:

$$p_1^N = \frac{\theta_H^2}{2} + \frac{t_H F_H(r_{1H})}{f_H(r_{1H})} \leq \frac{\theta_H^2}{2} + \theta_H r_{1H} \leq \frac{\theta_H^2}{2} + \theta_H r_{2H} \leq \frac{\theta_H^2}{2} + \frac{t_H \bar{F}_H(1-r_{2H})}{f_H(1-r_{2H})} = p_2^N.$$

Which results in a contradiction. Therefore, $p_1^N < p_2^N$. □

Appendix C Second Partial Derivative Test

To check the optimality of the solutions obtained in Section 4.1.1 we write the Hessian matrix for function $\pi_1(p,q)$ in the following:

$$H(p,q)=\begin{pmatrix}\frac{\partial^2\pi_1}{\partial p^2} & \frac{\partial^2\pi_1}{\partial p\partial q}\\ \frac{\partial^2\pi_1}{\partial q^2} & \frac{\partial^2\pi_1}{\partial q\partial p}\end{pmatrix}\Rightarrow H(p^*,q^*)=\begin{pmatrix}-\frac{2\gamma}{t_H} & \frac{2\gamma\theta_H}{t_H}\\ \frac{2\gamma\theta_H}{t_H} & -\frac{9\gamma\theta_H^2}{4t_H}\end{pmatrix} \tag{C.1}$$

The determinant of $H(p^*,q^*)$ is $\frac{\gamma^2\theta_H^2}{2t_H^2}$ which is positive. Since $\frac{\partial^2\pi_1}{\partial p^2}$ is negative, the solution $(p^*,q^*)=(\frac{3}{4}\theta_H^2,\theta_H)$ is a local maximum.

Now, we write the Hessian matrix for the π_2 at the point $(p^*,q^*)=(\frac{3}{4}(\frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H})^2,\frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H})$:

$$H(p^*,q^*)=\begin{pmatrix}-\frac{2\gamma}{t_H}-\frac{2(1-\gamma)}{t_L} & \frac{2\gamma\theta_H t_L+2(1-\gamma)\theta_L t_H}{t_L t_H}\\ \frac{2\gamma\theta_H t_L+2(1-\gamma)\theta_L t_H}{t_L t_H} & -\frac{9(\gamma\theta_H t_L+(1-\gamma)\theta_L t_H)^2}{4t_H t_L(\gamma t_L+(1-\gamma)t_H)}\end{pmatrix} \tag{C.2}$$

The determinant of matrix (C.2) is:

$$|H(p^*,q^*)|=\frac{(\gamma\theta_H t_L+(1-\gamma)\theta_L t_H)^2}{2(t_L t_H)^2} \tag{C.3}$$

Since (C.3) is positive and $\frac{\partial^2\pi_1}{\partial p^2}$ is negative, the solution (p^*,q^*) is a local maximum.

Appendix D Optimization problem under full market coverage of low-end segment

In this section we provide the mathematical model for analyzing cases F1 and F4, that represent the full market coverage of L-type consumers by the retailer. With the help of Equation (12) we can obtain the location of an L-type consumer who is indifferent between buying from either stores.

Therefore, the optimization problem for Case F1 is:

$$\max_{p_1,q_1,p_2,q_2} \pi = (p_1 - c(q_1))(\gamma \int_0^{x_H} f_H(x)dx + (1-\gamma)\int_0^{x_L} f_L(x)dx) \tag{D.1}$$
$$+ (p_2 - c(q_2))(\gamma \int_{x_H}^1 f_H(x)dx + (1-\gamma)\int_{x_L}^1 f_L(x)dx)$$

s.t:

$$x_j = \frac{1}{2} - \frac{\theta_j(q_2-q_1)-(p_2-p_1)}{2t_j}, \qquad j = L, H, \tag{D.2}$$

$$\theta_j q_1 - t_j x_j - p_1 \geq 0, \qquad j = L, H, \tag{D.3}$$

$$p_i, q_i \geq 0, \qquad i = 1, 2. \tag{D.4}$$

The optimization problem under Case F4 (the full coverage of low-end segment but partial coverage of high-end segment) is as follows:

$$\max_{p_1,q_1,p_2,q_2} \pi = (p_1 - c(q_1))(\gamma \int_0^{r_{1H}} f_H(x)dx + (1-\gamma)\int_0^{x_L} f_L(x)dx) \tag{D.5}$$
$$+ (p_2 - c(q_2))(\gamma \int_{1-r_{2H}}^1 f_H(x)dx + (1-\gamma)\int_{x_L}^1 f_L(x)dx)$$

s.t:

$$x_L = \frac{1}{2} - \frac{\theta_L(q_2-q_1)-(p_2-p_1)}{2t_L}, \tag{D.6}$$

$$r_{iH} = \max\{0, \frac{\theta_H q_i - p_i}{t_H}\}, \qquad i = 1, 2, \tag{D.7}$$

$$\theta_L q_1 - t_L x_L - p_1 \geq 0, \tag{D.8}$$

$$p_i, q_i \geq 0, \qquad i = 1, 2. \tag{D.9}$$

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Figure 4- Optimal price levels for the two stores of the retailer with respect to travel costs under non-uniform distribution of consumers.

Figure 5- Optimal quality level with respect to travel cost factor under uniform and non-uniform distributions of consumer's types.

Figure 6- Maximum profit of the retailer in uniform and non-uniform markets.

Table 1

Condition	Quality	Price	Segment	Profit
$\theta_L \leq s\theta_H$	θ_H	$\frac{3}{4}q^2$	H	$\frac{\gamma\theta_H{}^4}{8t_H}$
$\theta_L > s\theta_H$	$\frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H}$	$\frac{3}{4}q^2$	H,L	$\frac{(\gamma t_L\theta_H+(1-\gamma)t_H\theta_L)^4}{8(\gamma t_L+(1-\gamma)t_H)^3 t_L t_H}$

Table 2

	$\gamma = 0.3$			$\gamma = 0.5$				$\gamma = 0.7$		
θ_L	p_1^N	p_2^N	p_1^U	p_1^N	p_2^N	p_1^U	p_2^U	p_1^N	p_2^N	p_1^U
0.2	0.70	0.80	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.3	0.70	0.80	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.4	0.70	0.80	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.5	0.22	0.81	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.6	0.30	0.82	0.75	0.34	0.82	0.75	0.75	0.70	0.80	0.75
0.7	0.41	0.82	0.40	0.45	0.82	0.49	0.79	0.51	0.82	0.75
0.8	0.53	0.80	0.52	0.56	0.81	0.57	0.57	0.60	0.81	0.65

Table 3

	$\gamma = 0.3$			$\gamma = 0.5$				$\gamma = 0.7$		
θ_L	q_1^N	q_2^N	q_1^U	q_1^N	q_2^N	q_1^U	q_2^U	q_1^N	q_2^N	q_1^U
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	0.53	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	0.62	1.00	1.00	0.66	1.00	1.00	1.00	1.00	1.00	1.00
0.7	0.73	1.00	0.70	0.76	1.00	0.78	1.00	0.82	1.00	1.00
0.8	0.84	1.00	0.80	0.87	1.00	0.82	0.82	0.91	1.00	0.90

Table 4

	$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.7$	
θ_L	π^U	π^N	π^U	π^N	π^U	π^N
0.2	0.075	0.079	0.125	0.132	0.175	0.185
0.3	0.075	0.079	0.125	0.132	0.175	0.185
0.4	0.075	0.079	0.125	0.132	0.175	0.185
0.5	0.075	0.080	0.125	0.132	0.175	0.185
0.6	0.075	0.098	0.125	0.137	0.175	0.185
0.7	0.102	0.124	0.130	0.160	0.175	0.198
0.8	0.156	0.158	0.174	0.187	0.201	0.217

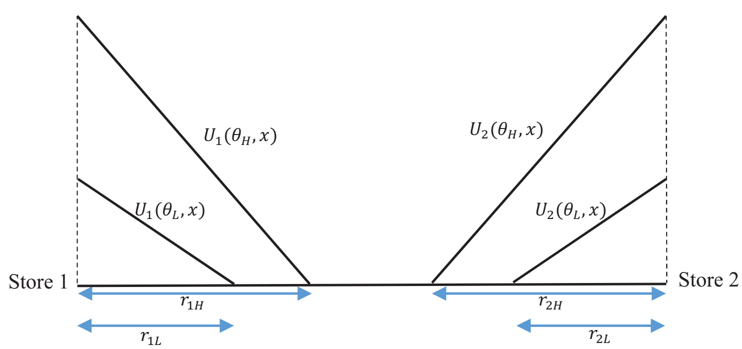


Figure 1

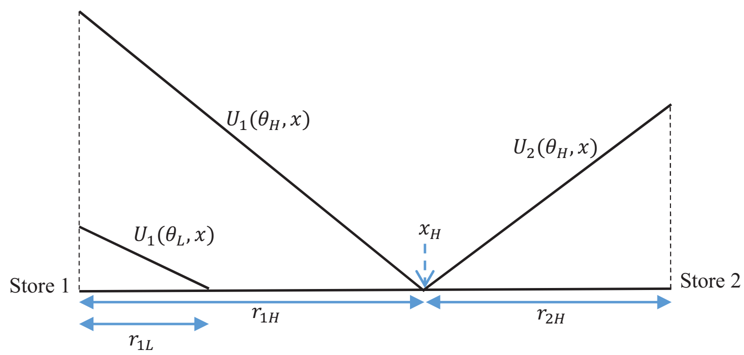


Figure 2

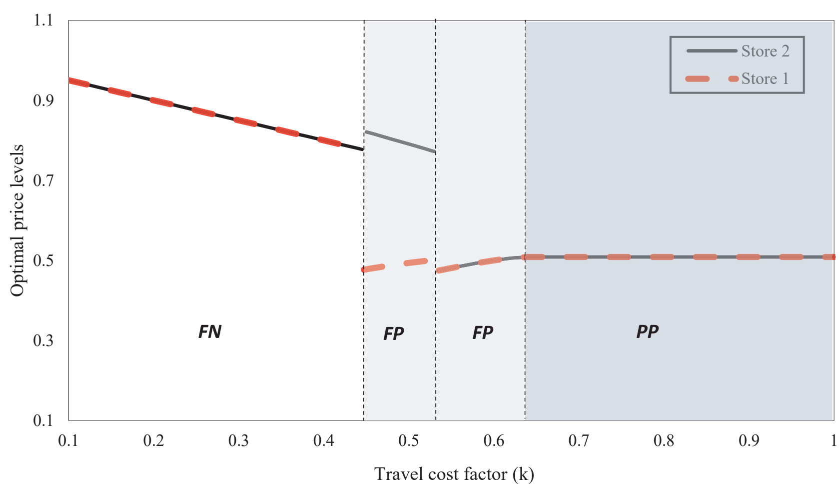


Figure 3

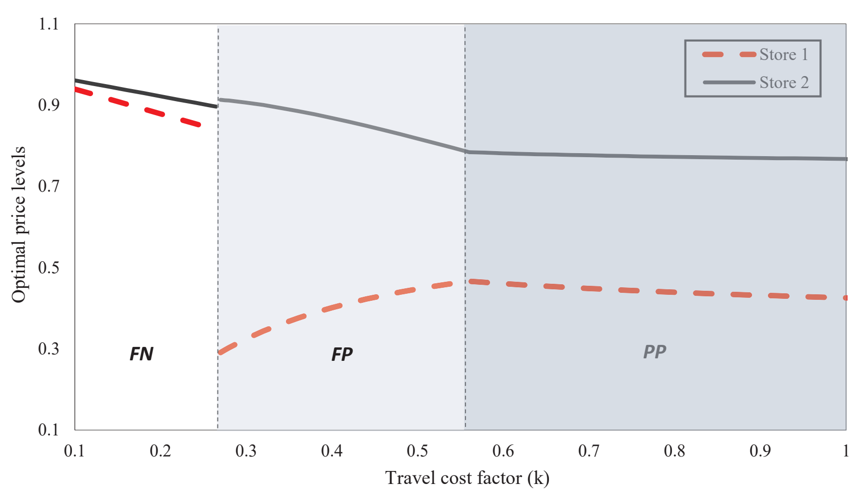
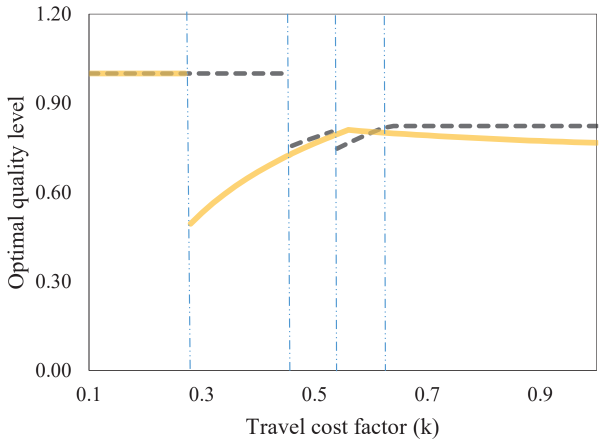
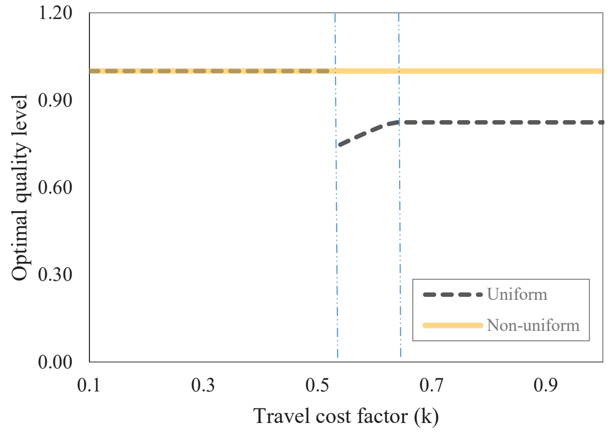


Figure 4



(a) Optimal quality level of store 1



(b) Optimal quality level of store 2.

Figure 5

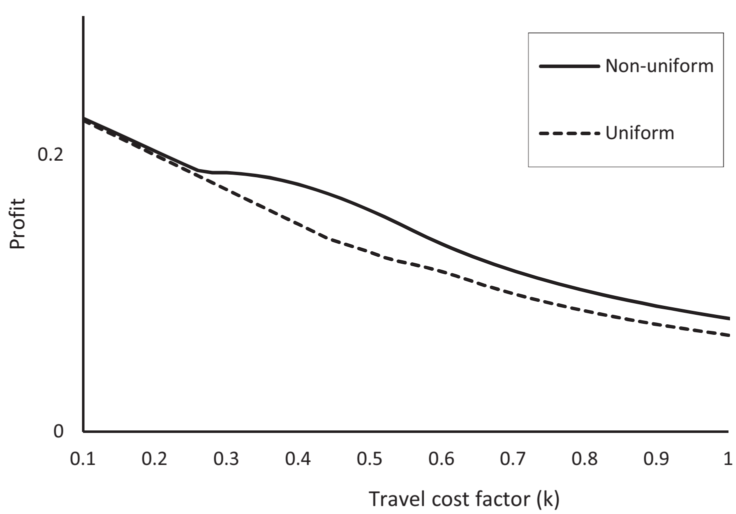


Figure 6