Scientia Iranica E (2021) 28(6), 3617–3633

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

Price and quality decisions in heterogeneous markets

N. Sedghi and H. Shavandi*

Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran.

Received 22 March 2019; received in revised form 1 January 2020; accepted 22 February 2020

KEYWORDS
Quality level;
Pricing;
Income disparity;
Non-uniform distribution;
Differentiation.

Abstract. The present study analyzes the optimal price and quality decisions of a retailer for its different stores in a heterogeneous market. The consumers are also assumed to be heterogeneous in terms of their willingness to pay for quality who are non-uniformly distributed in the market. This type of heterogeneity which is identified based on income disparity can have important implications for a retailer's optimal policy. The main objective of this study is to determine how the distribution of different types of consumers in the market and how their travel costs affect the optimal setting of price and quality levels among different stores of a retailer. According to the findings of this study, the geographical disparity of willingness to pay plays a significant role in differentiation and targeting strategy of a retailer. Furthermore, a comparative analysis revealed that the widely adopted assumption of uniform distribution of consumers in the literature would lead to non-optimal decisions in which the distribution of consumers is non-uniform in a real-world situation.

1. Introduction

Pricing and quality decisions are two strategic tools for retailers to differentiate themselves in a competitive market environment or to satisfy the needs of heterogeneous consumers. Today, retailers in different sectors such as food, grocery, merchandise or apparel adapt their stores' format to market conditions and characteristics. For example, some traditional supermarkets such as Kroger are operating with multiple price and quality levels. Kroger Inc. covers multiple brands of stores such as Fresh Fare, Kroger, and Food4Less that are operating as high-end stores, traditional supermarket, and price-impact warehouse store, respectively [1]. Other retailers differentiate their stores by introducing an off-price format. For instance, in the high fashion specialty sector, Nordstrom and Saks Fifth Avenue have introduced Nordstrom Rack and Saks OFF 5th as their discount stores [2]. However, apart from the large discounts that these outlet-type formats offer, the quality of these products may slightly differ from the ones in full-price stores. Generally, the products offered in discount stores have lower quality than full-price stores and they are procured from different suppliers [3,4]. Apparel manufacturers usually differentiate their retail and outlet stores through price, variety, and quality level. Retailers also try to match their price strategy with the neighborhood they are operating in. For example, some drugstore chains like Walgreens often charge different prices for the same products in stores located in different places [5]. There is also evidence that sometimes Target stores charge dissimilar prices in different locations [6].

Based on the abovementioned examples, it can be concluded that retailers can differentiate their stores at different levels of quality and price. This

*. *Corresponding author. Tel.: +98 21 66165711*
E-mail addresses: n_sedghi@ie.sharif.edu (N. Sedghi); shavandi@sharif.edu (H. Shavandi)

doi: 10.24200/sci.2020.53210.3113

strategy results from either competition pressure or demographic characteristics of the market. Therefore, one of the most important challenges a retailer (or a firm) must encounter is how to determine the image of its stores operating in new markets; in other words, what price and quality levels are better matched with the consumers' needs and characteristics? Under what conditions should the retailer open identical stores in different neighborhoods and when should the retailer operate in different store formats? These questions gain strategic significance for retailers since, as Gauri et al. [7] stated, the competition of some retail sectors is so fierce that considering an appropriate strategy can ensure either long-term success or failure for a store.

This study takes into account a monopolistic retailer who tends to set up stores in two adjacent neighborhoods and determine the price and quality levels of its stores in order to maximize the overall profit. Consumers in the neighborhoods are heterogeneous in terms of not only their ideal location but also their valuations for quality. As commonly observed in the literature, consumers' willingness to pay can be measured by income level [8], which is one of the publicly available demographic data sets in any neighborhood. More specifically, consumers can be divided into two segments based on their valuations for quality (or price sensitivity). While the high-end segment cares more about the quality, the low-end segment is more price sensitive. In this respect, both uniform and non-uniform distributions of each consumer's type in the market are considered. In addition, it is shown how this heterogeneity in income and location may affect the products' quality and pricing decisions in each store. The proposed model in this study might justify some of the store differences across neighborhoods.

This study provides an analytical framework to highlight the role of income disparity in the retailer's decision. Given that income level is indicative of the willingness to pay for quality, the analysis of its impact on the optimal decision of a retailer is so critical. Several studies have acknowledged that income inequality has been rising in the economy [9,10]. Other studies have also found a gradual spatial shift of lower-income families from the central to suburb residential areas in several US cities, thus resulting in neighborhood income polarization [11,12]. Reardon and Bischoff [13] reported that the percentage of American families living in middle-class neighborhoods fell from 65% in 1970 to 44% in 2009, and this split continued to accelerate which led to residential isolation between high- and low-income families. Gulati and Ray [9] also remarked that this sizable spatial difference in the income level caused a new challenge for educational institutions and healthcare facilities owing to their need for considering both location and income mix of people upon entering a neighborhood and setting the price and quality levels for their products and services.

Accordingly, the main objective of this paper was to provide an analytic framework to answer the following research questions: 1) How does a retailer's decision on price and quality levels depend on the distance of the stores or equivalently the travel cost in the market? 2) What is the effect of non-uniform distribution of consumers' type on the quality and price levels? To this end, heterogeneous consumers with different degrees of willingness to pay for quality and travel costs are taken into account. In addition, Hotelling's modeling framework [14] is employed which is fundamentally defined for showing the horizontal differentiation in a market. Moreover, the impact of geographical income inequality in the market was evaluated by comparing the retailer's decision in the cases of assuming uniform and non-uniform distribution of consumers' types throughout the market. Since the income heterogeneity of consumers is an undeniable fact at least in big cities, the results of this research can help retailers to understand and consider neighborhood heterogeneity in setting up the policies of their stores in the market.

The rest of this paper is organized as follows. Section 2 relates our study to the previous literature. Section 3 introduces our model and basic assumptions. Section 4 provides the analytical results for the model under both uniform and non-uniform distribution of consumers. Section 5 provides numerical results and insights for the retailer's decision. Section 6 concludes the study and offers some ideas for future research. All proofs are included in the Appendix.

2. Related literature

The retailing and operations management literature deals with pricing strategies in different retail channels (e.g., [15]) or store formats (e.g., [7]) under consumer's various characteristics. The present study analyzes the impact of considering setting up physical stores for average quality of products along with the price level decision in order to capture the heterogeneous market tastes in an efficient way. It has been established in the retail marketing literature that differentiation among retailers can be implemented by setting a distinct format for the stores such as product ranges, atmospherics, and price format [7]. Quality-differentiated store formats or channels have been recently explored in several studies. For example, Soysal and Krishnamurthi [16] empirically evaluated the impact of the adoption of a retailer's factory outlet channel on the customers' spending in the traditional retail stores. They found that the retailer could induce customer segmentation through self-selection when the channels were differentiated at price and quality levels. In another empirical study, Ngwe [17] found that when a retailer captured consumers' differences through both regular and outlet stores, it consequently increased its profit through consumers' self-selection. Among the few analytical researches in this field, Li et al. [18] investigated the strategy of a manufacturer in opening an outlet channel with the consideration of the impact of outlet sales on the manufacturer's brand awareness. In this study, a model that could capture the demographic characteristics and their impact on a retailers' store-level decision was proposed. Previous research has established a number of empirical links among the demographic variables such as income levels on the consumer's store choice [19,20]. The environmental characteristics in terms of demographics and competition provide incentives for the retailers to operate under different store formats and commercial names [20–22]. However, the previous literature has not analytically considered non-uniform heterogeneity in the consumers' willingness to pay (as a result of income disparity) at the price and quality levels of a retailer in the horizontal differentiation models.

To capture the demographic differences across neighborhoods, the framework of spatial differentiation models was employed. Of note, the proposed approach differs from that of economics studies that directly model the effect of income disparity on the competition between firms under homogeneous tastes (e.g., [23]). More specifically, the proposed model in this study pertains to the stream of literature that considers two-dimensional consumers' heterogeneity while studying the firms' strategic decisions such as quality, price, or location (e.g., [24–26]). These models consider a market with consumers residing in different places (or have different taste preferences) with different degrees of willingness to pay for quality. More recently, Hernandez [27] studied the impact of transportation cost on the competitive price and quality of products for two symmetric firms. Shi et al. [28] took into account two-dimensional heterogeneity to find the optimal quality in different channel structures and showed that the type of consumer heterogeneity and its distribution in a market could play a substantial role in determining how a channel structure affected product quality. Among these studies, those conducted by Desai [24] and Sedghi et al. [29] were particularly related to our research. Desai [24] studied a product-line design problem in a market of two consumer segments with high and low valuations for quality. These consumers are also heterogeneous in their taste preferences. This study is different from the monopoly model offered by Desai [24] who considered only the vertical differentiation for the products and assumed a uniform distribution of consumers over the Hotelling line. However, in this study, a horizontal differentiation between the stores (as they are in different locations) and a non-uniform income distribution in the market were considered. In this sense, the proposed model in this study is similar to that proposed by Sedghi et al. [29] owing to the different distribution of consumers in each segment. This assumption changes the model analysis substantially because the two dimensions of market heterogeneity are not independent. The analysis carried out by Sedghi et al. [29] is limited to one product, and the focus is on the optimal price and location; however, this study analyzes the impact of two-dimensional heterogeneity on the price and quality levels of two stores in exogenous locations.

Although two-dimensional models of market heterogeneity shed more light on the consumers' characteristics as well as the nature of product differentiation, they are considerably difficult to solve [25]. Accordingly, to make these problem more tractable, most of the spatial models are restricted to one dimension of heterogeneity or they consider simplifying assumptions such as full market coverage or uniform distribution of consumers. Without loss of generality, we assume that these two stores are located at two extremes of the linear market and consequently, the horizontal differentiation is assumed to be exogenous in our model. This assumption allows us to focus on the price and quality levels of the two stores without enforcing limiting assumptions on the distributions of consumers.

Another relevant stream of literature considers the implications of non-uniformity of consumers' distribution in the spatial models. In the competitive location models on a line, there are several papers that relax the uniformity assumption [30–36]. The main focus of these studies is put on the equilibrium location in a Hotelling game with the objective of understanding how the model outcomes change depending on different distributions of consumers. It turns out that this assumption can substantially change the equilibrium location in a competitive setting. Among recent studies that have challenged the uniform assumptions, those proposed by Guo and Lai [37], Sedghi et al. [29], and Shi et al. [28] are more relevant to our study. Guo and Lai [37] analyzed the location and price of brick-and-mortar retailers in a market where non-uniformly-distributed consumers could purchase the product from an online retailer at the cost of a mismatch (for example, a mismatch on size or color). In addition, they considered a homogeneous product in terms of its quality as well as a market consisting of consumers, all with the same willingness to pay for that product. Recently, Benassi et al. [38] proved the existence of a sub-game perfect equilibrium in pure strategies for an uncovered duopoly when the consumers' willingness to pay followed a log-concave distribution. In their model, only the vertical differentiation between forms were taken into consideration. Sedghi et al. [29] explored how a firm would choose its location and price in a non-uniform market. Their analysis was conducted based on a monopoly setting with two types of consumers and a homogeneous product. Shi et al. [28] selected a given location for a product (or firm) and analyzed the implications of non-uniform distribution of consumers (or taste preferences) in a quality and price setting. However, to obtain analytical results for the non-uniform distribution, they considered only one-dimensional heterogeneity.

3. Model

Consider a retailer that is about to set the strategic quality and price levels for its stores in two adjacent neighborhoods. The overall perceived quality of a store can be measured by a variety and average quality of products, sales assistants, store environment, and auxiliary services. The average price level and frequency of promotions can project the price image of that store.

To investigate the conditions that affect the price and quality levels of these two stores, a market where consumers who are both vertically heterogeneous in terms of their willingness to pay for product quality and horizontally heterogeneous in terms of their ideal location is considered. To follow the standard horizontal differentiation literature [14], consider a retailer that wants to open two stores $i \in \{1, 2\}$ in a market with a continuum of consumers distributed along a horizontal [0,1] line. The mass of consumers in the market is normalized to one. Consumers visit any of these two stores if they gain positive utility after considering the quality, price, and cost of travel. We assume that the location of stores is determined in advance and without loss of generality, assume that stores 1 and 2 are located at $x = 0$ and $x = 1$, respectively. This assumption is widely used in the Hotelling framework (e.g., [39–41]) to provide geographical or horizontal differentiation between the firms and products. The retailer should decide on the price and quality levels of both stores. Let p_i and q_i denote the price and quality levels at store $i(i = 1, 2)$.

Consumers in the market are assumed to be heterogeneous in two types of high-valuation consumers (known as H-type) and low-valuation consumers (known as L-type). The H-type consumers exhibiting a higher degree of willingness to pay for quality constitute γ percent of the market, and the remaining $(1 - \gamma)$ percent are L-type consumers with a lower quality valuation. A consumer of type $j(j = L, H)$ derives a utility of $\theta_j q$ from shopping at a store with quality level q. In this study, the model of vertical differentiation was proposed based on the fact that low-income families were more sensitive to price and promotions [8]. This segmentation scheme which is based on the demographic data can be applied by marketing managers. Moreover, one of the patterns of segmentation in the literature is opportunity cost of time. It is common in the literature to regard the income level as a proxy for this measure and assign a higher opportunity cost of time to consumers with higher income (see [42]). Therefore, given the concept of opportunity cost of time as well as the framework presented in [24,41], we can assume that the transportation (travel) cost of segment H is greater than or equal to the travel cost of L-type consumers, i.e., $t_H \geq t_L$.

According to the above framework, a consumer of type j located at x takes the value of the net utility $U_1(\theta_j, x) = \theta_j q_1 - t_j x - p_1$ for buying from store 1 and takes the value of the net utility $U_2(\theta_j, x) = \theta_j q_2 - t_j(1-x) - p_2$ for buying from store 2. We normalize the consumer's net utility derived from an online retailer to zero.

In this study, first, a benchmark case of uniform distribution of consumers in the market was provided. This case determines the retailer's choice when the population and income level of the two neighborhoods are the same. Then, this widely adopted assumption in the literature was challenged using a non-uniform distribution for each consumer's segment. The non-uniform assumption helps model the heterogeneity in the consumers' willingness to pay across the neighborhoods.

The objective of the retailer is to maximize its overall profit from these two stores. The cost of providing quality level q is assumed to be quadratic, $c(q) = \frac{1}{2}q^2$ (e.g., see [24,25]). Therefore, the profit of the retailer is $\pi = \sum_{i=1,2}(p_i - c(q_i))(D_{iH} + D_{iL})$ where D_{ij} is the demand of consumers of type $j(j = L, H)$ for store $i(i = 1, 2)$. Let $r_{ij} = \max\{\frac{\theta_j q_i - p_i}{t_j}, 0\}$ be the coverage radius of store i for the consumers of type j. In other words, the customer of type j is willing to travel at most the distance r_{ij} to shop at store i.

The following analysis emphasizes the case where the H-type consumers are the more profitable consumers for the retailer to serve. This is the reason why in the proposed model we set the reservation utility [see 43] of both segments was set to zero. Therefore, the external options for both types of consumers become similar which makes the H-type consumers more willing to buy from the retailer. The analysis for the relative attractiveness of L-type consumers can also be done in an analogous way.

4. Optimal price and quality levels

In this section, the joint optimization of price and quality levels is analytically taken into consideration when the retailer runs two stores in a heterogeneous market.

Depending on the relative values of unit travel costs (also known as transportation costs) and marginal valuations for qualities $(\theta_{j,j} = L, H)$, the retailer may

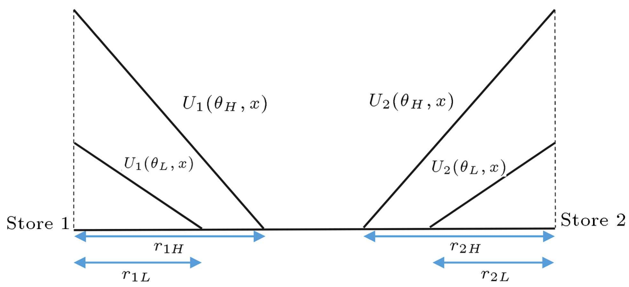

Figure 1. Partial market coverage.

not find it optimal to serve all the consumers in a given segment. Therefore, there are three possibilities for the retailer in serving the consumers in a given segment: full coverage (F); partial coverage (P), and no coverage (N). Based on these possibilities, the following cases appear:

- **Case FF-** The retailer fully covers both segments;
- **Case FP-** The retailer fully covers H-type segment, but it only partially covers the L-type segment;
- **Case FN-** The retailer fully covers H-type segment, but it does not serve any of the L-type segment;
- **Case PF-** The retailer fully covers L-type segment, but it only partially covers H-type segment;
- **Case PP-** The retailer partially covers both segments;
- **Case PN-** The retailer partially covers H-type segment, but it does not serve any of the L-type segment.

Given that the H-type consumers are the most attractive consumers to serve, we omit the cases where the retailer does not serve the H-type consumers.

Figure 1 shows an example of the partial coverage of both segments (Case PP). Note that in this case, there are some consumers in the middle of the market that purchase from none of these stores.

Since the market is characterized by two-dimensional heterogeneity, its parameters can greatly affect the retailer's decision. In this regard, the necessity of these cases to obtain an optimal solution is highlighted. First, the partial coverage of the market was analyzed to obtain some analytical results and insights. Then, the full market coverage was taken into account to numerically show the impact of travel cost on the optimal price and quality decisions. To be specific, we are interested to see how the retailer's decision changes under the non-uniform distribution of consumers' income.

4.1. Partial market coverage

First assume that depending on the market conditions, only Cases PP or PN are in order. These cases are valid when the transportation costs are relatively high and the retailer may not find it optimal to serve all the consumers in any segment. To ensure that the travel cost is high enough to support only partial coverage of the market, it is required to check the condition $r_{1j} + r_{2j} \leq 1$. This condition is equivalent to $t_j \geq \theta_j(q_1+q_2)-(p_1+p_2)$. Hence, the optimal price and quality levels of the stores, as well as the exact conditions that ensure $r_{1j}+r_{2j} \leq 1$ can be obtained. In the following assumption, a condition that guarantees the partial coverage of the market is assumed.

Assumption 1. *When the travel costs in the market are so high that satisfy the following condition, the retailer has no incentive to fully serve any segment of the market:*

$$t_j \geq 2\theta_j\theta_H - {\theta_L}^2, \qquad j = L, H. \tag{1}$$

To see more details of deriving this condition, refer to Appendix A. The optimal price and quality decisions for each store under two different neighborhood conditions is analyzed in this section. To this end, first, consider the case of identical neighborhoods with two segments and uniform distribution of consumers in the market. In the second case, consider a non-uniform distribution of consumers which illustrates the role of income disparity across the two neighborhoods.

4.1.1. Uniform distribution

Assume that travel cost is so high that the two stores cannot fully cover any market segment in the optimal setting. In addition, suppose that consumers of each type are uniformly distributed in the market, implying that the two neighborhoods are identical in terms of consumers' income level. The retailer needs to decide on the price and quality levels for each store to maximize its total profit. This decision depends on the market conditions and it determines whether both segments are served (Case PP) or just the high-valuation segment is targeted (Case PN). It is of significance to determine whether the optimal decision implies vertical differentiation between the stores or only the horizontal differentiation can lead to an optimal decision. According to these conditions under the optimal solution, these two stores fail in complete coverage of the market; therefore, these two stores have no common coverage area and can act as local monopolies. Consequently, in case the distributions of types of consumers are uniform in the market, the price and quality levels of both stores should be identical under an optimal solution.

Of note, high horizontal differentiation between the two stores prevents cannibalization, and the uniform distribution of consumers urges the retailer to set up identical stores, indicating that under the uniform distribution, the existence of different income levels does not encourage the retailer to vertically differen-

Table 1. Optimal decision of the retailer under high travel costs where $s = \frac{(\gamma t_L((1-\gamma)t_H+\gamma t_L)^3)^{1/4}-\gamma t_L}{(1-\gamma)t_H}$.

Condition	Quality	Price	Segment	Profit
$\theta_L \leq s\theta_H$	θ_H	$\frac{3}{4}q^2$	H	$\frac{\gamma {\theta_H}^4}{8t_H}$
$\theta_L > s\theta_H$	$\frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H}$	$\frac{3}{4}q^2$	H,L	$\frac{(\gamma t_L\theta_H+(1-\gamma)t_H\theta_L)^4}{8(\gamma t_L+(1-\gamma)t_H)^3 t_L t_H}$

tiate its stores. However, these identical stores can either target only one type of consumers or serve both types. In the following lemma, a condition is identified that encourages the retailer to serve both consumer segments.

Lemma 1. *When the consumers in each segment are uniformly distributed in the market, and the travel cost is high, the retailer targets both segments if and only if:*

$$\theta_L > \theta_H(\frac{(\gamma t_L((1-\gamma)t_H+\gamma t_L)^3)^{1/4}-\gamma t_L}{(1-\gamma)t_H}).$$

Lemma 1 indicates that when the willingness to pay of the L-type consumers is relatively low, the retailer prefers to set its price and quality to target only the high-valuation segment. For the retailer to have an incentive to target both segments, the willingness to pay for the low-valuation segment should be higher than the threshold. Determination of this threshold requires complete knowledge of the two segments (i.e., willingness to pay, travel costs, and market size). Once this information is available, the retailer can analyze the market and set the target segments.

According to Lemma 1, if θ_L is lower than the threshold, the retailer targets only the H-type consumers. It further leads to the following optimization problem (since the two stores are identical, the subscript i is dropped from the model):

$$\max_{p,q} \quad \pi_1 = \left(p - \frac{q^2}{2}\right) 2\gamma \left(\frac{\theta_H q - p}{t_H}\right). \tag{2}$$

The optimal solution is given by the first-order conditions as $q^* = \theta_H$ and $p^* = \frac{3}{4}\theta_H^2$ (the second derivative test for this solution is provided in Appendix C).

Analogously, when the retailer targets both types of consumers, the optimization problem of a store located at $x = 0$ is as follows:

$$\max_{p,q} \quad \pi_2 = \left(p - \frac{q^2}{2}\right) \left(\gamma(\frac{\theta_H q - p}{t_H}) + (1-\gamma)(\frac{\theta_L q - p}{t_L})\right), \tag{3}$$

which results in $q^* = \frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H}$ and $p^* = \frac{3}{4}q^2$ (the second derivative test for this solution is provided in Appendix C).

Based on a summary of the above solutions, Table 1 presents the optimal decision of the retailer and associated profits. When the retailer targets both types of consumers, it decreases the quality and price levels of its stores. The optimal quality under targeting both market segments is:

$$q^* = \frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H} = \alpha\theta_H + (1-\alpha)\theta_L,$$

where:

$$\alpha = \frac{\gamma t_L}{\gamma t_L+(1-\gamma)t_H}.$$

This quality level is a convex combination of θ_L and θ_H; therefore, it is less than θ_H (the quality level when targeting the H-type consumers only).

4.1.2. Non-uniform distribution

Now, consider a non-uniform distribution for the consumers of each segment in order to model two neighborhoods with different populations and income levels. In addition, assume that consumers of type $j(j = H, L)$ are distributed on the line according to the density function $f_j(.)$. The distributions considered in this section are analogous to those in the study of Sedghi et al. [29] that provide a framework for modeling polarized markets. To show the differences in the neighborhoods, we assume that the consumers on the right side of the market have higher average income than those on the left side. To be more specific, the following assumption on the distribution of consumers should be taken into consideration.

Assumption 2. *The distribution of consumers holds the following conditions:*

1. $f_H(x)$ *is linearly increasing in* x, *while* $f_L(x)$ *is linearly decreasing in* x,
2. $f_L(x) \geq f_H(x), \quad \forall x \leq \frac{1}{2}$,
3. $f_L(x) \leq f_H(x), \quad \forall x \geq \frac{1}{2}$.

This assumption is in line with the neighborhood differences, indicating that in case the number of consumers in both segments is equal, it is more likely for a consumer on the left side of the market to be in the L-type, and vice versa. Enabling us to make analytical comparisons, this assumption can also contribute to analyzing polarized neighborhoods, that is, the types of consumers differ in these two neighborhoods. Moreover, Sedghi et al. [29] found that consideration of linear distribution was a good approximation for modeling these types of heterogeneities.

For ease of exposition, the optimal decisions of the retailer under the uniform and non-uniform distributions of consumers are denoted by superscripts U and N, respectively. The optimization problem that the retailer faces is as follows:

$$\max_{p_1,q_1,p_2,q_2} \pi^N = \left(p_1 - \frac{q_1^2}{2}\right)\left(\gamma \int_0^{r_{1H}} f_H(x)dx + (1-\gamma)\int_0^{r_{1L}} f_L(x)dx\right) + \left(p_2 - \frac{q_2^2}{2}\right) \left(\gamma \int_{1-r_{2H}}^1 f_H(x)dx + (1-\gamma) \int_{1-r_{2L}}^1 f_L(x)dx\right). \quad (4)$$

This section assumes a high transportation cost; therefore, the two stores can be considered as local monopolies. In the above optimization problem, one can separate π^N into two distinct functions for stores 1 and 2. To obtain the optimal quality and price levels for store 1, the first-order conditions are as follows:

$$\frac{\partial \pi^N}{\partial p_1} = \gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L}) - \left(p_1 - \frac{q_1^2}{2}\right) \left(\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})\right) = 0, \quad (5)$$

$$\frac{\partial \pi^N}{\partial q_1} = -q(\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})) + \left(p_1 - \frac{q_1^2}{2}\right) \left(\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})\right) = 0. \quad (6)$$

Eq. (6) results in:

$$q_1 = \left(p_1 - \frac{q_1^2}{2}\right) \frac{\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})}{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}. \quad (7)$$

From Eq. (5), we derive:

$$p_1 - \frac{q_1^2}{2} = \frac{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})}.$$

The left-hand side can be replaced for $p_1 - \frac{q_1^2}{2}$ in Eq. (6) to get:

$$q_1^N = \frac{\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})}. \quad (8)$$

p_1^N is also given by replacing q_1^N in Eq. (5):

$$p_1^N = \frac{q_N^2}{2} + \frac{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})}. \quad (9)$$

As observed in Eqs. (8) and (9), it is not straightforward to obtain the closed-form solution for the optimal price and quality. However, the optimal solution is a root to these equations. Since these equations were derived under a condition where the market coverage for any segment was less than half, some insights were obtained and elaborated in the following propositions.

Analogously, the optimal price and quality levels for store 2 take the following form:

$$q_2^N = \frac{\frac{\gamma\theta_H}{t_H} f_H(1-r_{2H}) + \frac{(1-\gamma)\theta_L}{t_L} f_L(1-r_{2L})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})}, \quad (10)$$

$$p_2^N = \frac{q_N^2}{2} + \frac{\gamma F_H(1-r_{2H}) + (1-\gamma)F_L(1-r_{2L})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})}. \quad (11)$$

As observed above, the optimization problem resulted in different quality and price levels in both stores. This difference cannot be justified by the inequality at the income levels alone. In fact, the non-uniform distribution of incomes (as a representative of willingness to pay) over the neighborhoods plays the main role in obtaining different price and quality levels in the optimal solution. This kind of heterogeneity encourages the retailer to operate at two different quality levels. This finding partly explains the vertical differentiation that the stores of a retailer have in practice. For example, Kroger's Food4Less store offers different price and quality levels from the other Kroger's store, Fresh Fare. The apparel manufactures also set different price and quality levels for their outlet stores and those in the midtown.

It is interesting to see how the qualities differ from the case of uniform distribution. The following proposition shows the comparison between the quality levels.

Proposition 1. *Under high travel costs, the optimal quality level of stores when the consumers are uniformly distributed are identical, and there is a convex combination of differentiated quality levels of the two stores under non-uniform distribution of consumers, i.e.,* $q_1^N \leq q^U \leq q_2^N$.

According to this proposition, the non-uniform distribution of the two market segments leads to lower quality level for store 1, but higher quality level for store 2. Strict inequality holds when the conditions of the market encourage the retailer to target both segments. The conditions for the retailer to partially serve L-type consumers (Case PP) or not to target this segment (Case PN) is expressed in the following lemma.

Lemma 2. *There exists a threshold for θ_L such that in case θ_L is less than that threshold the retailer targets only the H-type consumers.*

This lemma is similar to Lemma 1 in the uniform case. When the willingness to pay for low-valuation segment is higher than a threshold, the retailer will set up stores to target both types of consumers, with the quality and price levels derived from Eqs. (8) to (11).

Proposition 2. *Under the policy of targeting just the high-valuation segment, it is optimal to offer the same quality level in both stores but charge a higher price at store 2 than that at store 1, i.e.,* $q_1^N = q_2^N = \theta_H$ *and* $p_1^N < p_2^N$.

This proposition helps understand one of the reasons why retailers may charge different prices for the same product in different areas. For example, Walgreens drugstores sometimes offer different prices in different neighborhoods. Although this diversity in price could result from several reasons such as competition or rent costs in those areas, as Proposition 2 suggests, the non-uniformity of the distribution of consumers' willingness to pay can also encourage the retailers to offer heterogeneous price levels in stores across different neighborhoods.

4.2. Full market coverage

This section investigates the case of low transportation (travel) cost in the market. This allows the retailer to fully cover one or both segments in the optimal setting. To write the optimization problem that the retailer faces under full market coverage, the location of a consumer of type $j (j = L, H)$ who is indifferent about buying from either stores is determined:

$$\theta_j q_1 - t_j x_j - p_1 = \theta_j q_2 - t_j (1 - x_j) - p_2$$

$$\Rightarrow x_j = \frac{1}{2} - \frac{\theta_j (q_2 - q_1) - (p_2 - p_1)}{2t_j}. \quad (12)$$

Figure 2 shows an example of the full coverage of the H-type consumers (Case FP) and position of x_H in the market.

The optimization problem of the retailer to obtain the optimal policy on both quality and price for the Cases FP and FN is given in the following where the retailer fully covers only the high-end segment. The optimization problems for Cases FF and PF are presented in Appendix C.

$$\max_{p_1, q_1, p_2, q_2} \pi = (p_1 - c(q_1)) \left(\gamma \int_0^{x_H} f_H(x) dx + (1 - \gamma) \int_0^{r_{1L}} f_L(x) dx \right) + (p_2 - c(q_2)) \left(\gamma \int_{x_H}^1 f_H(x) dx + (1 - \gamma) \int_{1 - r_{2L}}^1 f_L(x) dx \right), \quad (13)$$

s.t:

$$x_H = \frac{1}{2} - \frac{\theta_H (q_2 - q_1) - (p_2 - p_1)}{2t_H}, \quad (14)$$

$$r_{iL} = \max \left\{ 0, \frac{\theta_L q_i - p_i}{t_L} \right\}, \qquad i = 1, 2, \quad (15)$$

$$\theta_H q_1 - t_H x_H - p_1 \geq 0, \quad (16)$$

$$p_i, q_i \geq 0, \qquad i = 1, 2. \quad (17)$$

In the Objective function (Eq. (13)), if the market follows a uniform distribution of income levels, we have $f_j(x) = 1, (j = L, H)$. Constraint (14) shows the position of the indifferent consumer. Constraint (15) is the definition of coverage radius and it ensures that the coverage is non-negative. These two constraints clarify the optimization problem, and they can be replaced in the objective function. Constraint (16) states that the net utility of the H-type consumer who is indifferent about buying from either store should be nonnegative. In fact, this constraint ensures that the high-end segment is fully covered. The analysis of the optimal solution depends on the values of r_{iL} in Constraint (15) which shows the targeting policy of the retailer in each of the stores. Since the parametric analysis of these cases provides little insights for the optimal solution, we numerically analyze the optimal decision of the retailer in the next section.

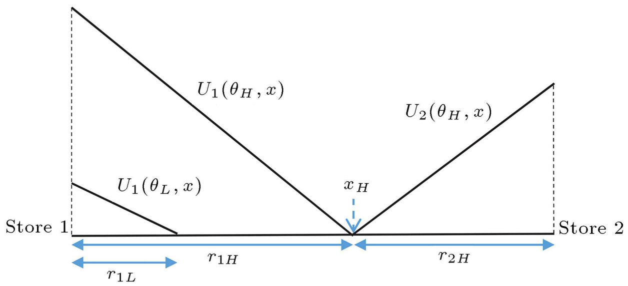

Figure 2. An example of optimal coverage under uniform distribution of consumers.

5. Numerical results

This section provides several insights into the optimal price and quality levels that the retailer sets for its stores. We are especially interested in the impact of travel cost on the retailer's decision. To be specific, these questions are posed: How does the strategy of the retailer differ in the case of high and low transportation costs? How does the distribution of consumers affect this strategy?

To answer these questions, the optimal solution of all cases that can emerge at different levels of travel cost should be taken into account. To simultaneously show the effect of travel costs of both segments on the retailer's decision, assume that the travel cost in each segment is proportional to the marginal willingness to pay for quality in that segment. In other words, we assume that $t_j = k\theta_j$ $(j = L, H)$. This assumption allows altering the travel cost for both segments proportionally while preserving the $t_H \geq t_L$ assumption. We call k the travel cost factor and show the effect of this factor on the retailer's decisions. In the following analysis, we set $\theta_H = 1$, $\theta_L = 0.7$, and $\gamma = 0.5$.

5.1. Optimal price levels

5.1.1. Uniform distribution of consumers

First, assume that the consumers of each types are uniformly distributed in the neighborhoods. Figure 3 illustrates how the optimal price of the retailer changes with respect to the travel cost factor. Note that the retailer uses different pricing strategies depending on the level of travel cost in the market. In addition, since relatively high valuation for quality was assigned to L-type consumers in this numerical study, according to Lemma 1, Case PN did not emerge in the optimal strategy of the retailer.

According to Figure 3, there are four strategies that the retailer may choose. The discontinuity on the price curves results from adopting a different strategy according to the range of travel cost factors. The numerical results are interesting because given that consumers are uniformly distributed in the neighborhoods, one might conclude that the retailer has an incentive to set up identical stores in the market. However, according to Figure 3, this is not the case for all travel cost levels. There are some ranges of the travel costs in which the retailer prefers to differentiate between its stores even in the case of uniform distribution of different types of consumers. One of the examples of such a setting and its optimal coverage is illustrated in Figure 2. In this case, the retailer applies different targeting strategies for its two stores. However, as Figure 3 shows, high and low travel costs will encourage the retailer to set up identical stores and follow a similar targeting strategy in the stores.

In case the travel cost is low ($k < 0.45$), the retailer fully serves all the H-type consumers and sets high prices in the market (Case FN). In contrast to the case of competition between retailers where the low travel cost results in a Bertrand competition and reduces the prices, as the pricing in the two stores is regulated by a single retailer here, the low travel cost results in high identical prices for the two stores. With these high prices, the retailer targets only H-type consumers.

When the travel cost is moderate in the market ($0.45 \leq k \leq 0.63$), the retailer changes its strategy to serve both types of consumers (Case FP). However, whether the retailer differentiates between its stores or just sets up identical stores, it might charge different prices. As observed in Figure 3, Case FP has two different parts. When $0.45 \leq k \leq 0.53$, the retailer fully covers all the H-type consumers, but differentiates between its two stores. In that case, store 1 serves both types of consumers, but store 2 targets only the high-end segment. This situation is depicted in Figure 2. However, when $0.53 < k \leq 0.63$, the retailer targets both consumers with identical stores. Therefore, it reduces the price of store 2 to serve the low-end consumers as well. In the case of identical stores, the retail price is increasing in the market mainly because the quality levels of the stores are also increasing.

Finally, when the travel cost is high enough ($k > 0.63$), the retailer partially serves both types of consumers. The analytic results for this case are presented in Section 4.1.1. As the analysis in that section suggests, the retailer opens up two identical stores with high travel costs.

5.1.2. Non-uniform distribution of consumers

Now, assume that the consumers of each type are non-uniformly distributed in the market. In addition, assume that $f_H(x) = 2x$ and $f_L(x) = 2 - 2x$. While satisfying Assumption 2, these distributions also approximately represent the income disparity observed in large cities. Figure 4 illustrates the optimal pricing strategy of the retailer.

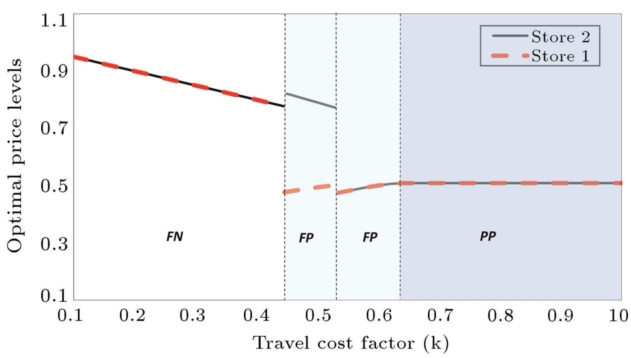

Figure 3. Optimal price levels for the two stores of the retailer with respect to travel costs under uniform distribution of consumers.

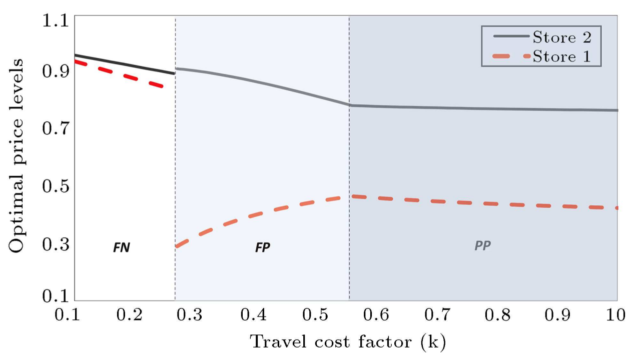

Figure 4. Optimal price levels for the two stores of the retailer with respect to travel costs under non-uniform distribution of consumers.

Note that in contrast to the uniform case, the retailer differentiates between the stores, but the level of differentiation depends on the travel cost in the market.

When the travel cost is very low ($k < 0.27$), the retailer targets only H-type consumers (Case FN) with slightly different prices. Moderate levels of travel cost ($0.27 \leq k \leq 0.55$) make the retailer apply a higher level of differentiation between its stores and target both types of consumers (Case FP). In this case, while store 1 targets both types of consumers, store 2 serves only H-type consumers. Moreover, the price level of store 1 is increasing with respect to the travel cost factor. This happens because the quality level also increases in the travel cost factor for that store.

When the travel cost is relatively high ($k \geq 0.55$), each store can operate only in a limited local market. Since the retailer cannot fully serve H-type consumers, it reduces its price in store 2 to increase the coverage radius for the high-end segment.

5.2. Comparison between uniform and non-uniform markets

To see the effect of consumers' types distribution on the optimal decision of the retailer, we compare the optimal prices in Figures 3 and 4. Several insights emerge. In case of uniform distribution, the travel cost factor needs to be higher than non-uniform distribution case in order to encourage the retailer to target the low-end segment as well. Therefore, it can be concluded that the income disparity in the neighborhoods is more in favor of low-end segment in terms of having access to the stores. However, as Figure 5(a) shows, the quality that the L-type consumers receive is low. Moreover, in the non-uniform market, store 1 generally offers lower quality levels than the uniform setting, except for a small range of travel costs ($0.53 < k < 0.6$). As shown in Proposition 1, for high transportation costs, the inequality $q_1^N < q_1^U = q_2^U < q_2^N$ holds at the quality levels. This disparity also encourages the retailer to provide higher quality levels on the right side of the market (Figure 5(b)). Another interesting observation is that low travel cost would result in the same quality level and targeting strategy under uniform and non-uniform distributions of consumer types. Therefore, when the unit travel cost across the market is lower than the threshold, uniform distribution can be a valid assumption even under disparity in consumers' willingness to pay. However, for higher travel cost in the market, uniform assumption is employed to simplify the analysis, thus leading to the retailer's non-efficient decisions.

The maximum profit that the retailer earns in uniform and non-uniform markets is presented in Figure 6. When the travel cost is relatively low, the retailer's profits are similar in the two settings; however, when the travel cost gets relatively high, the retailer can gain greater profit by differentiating the stores. The income disparity in the market provides the retailer with more opportunities for store differentiation without facing the threat of cannibalization between its stores. However, the quality differentiation is not always an available option for retailers. Small retailers that usually operate under one-brand name and format do not have such a kind of flexibility to change the quality level of their stores due to the reputation they want to set in the market. In that case, they need to consider their stores' location in homogeneous neighborhoods.

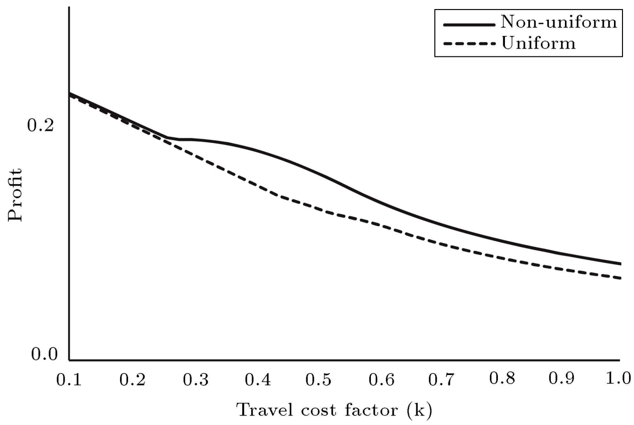

Figure 6. Maximum profit of the retailer in uniform and non-uniform markets.

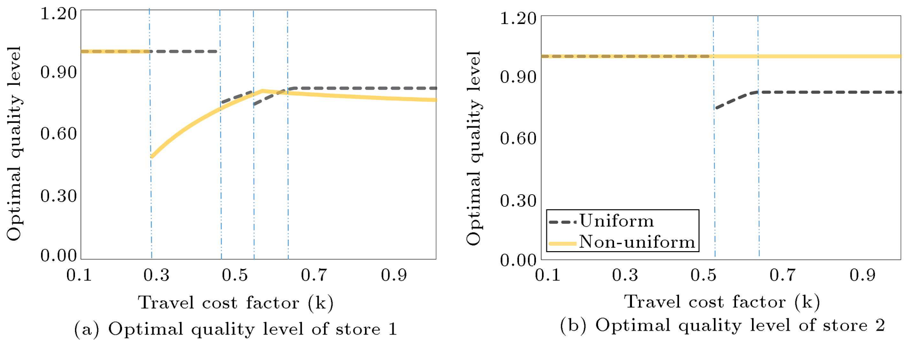

Figure 5. Optimal quality level with respect to travel cost factor under uniform and non-uniform distributions of consumer's types.

Table 2. Optimal price levels for the retailer's stores under uniform and non-uniform distributions of consumer types.

	$\gamma = 0.3$			$\gamma = 0.5$				$\gamma = 0.7$		
θ_L	p_1^N	p_2^N	p_1^U	p_1^N	p_2^N	p_1^U	p_2^U	p_1^N	p_2^N	p_1^U
0.2	0.70	0.80	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.3	0.70	0.80	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.4	0.70	0.80	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.5	0.22	0.81	0.75	0.70	0.80	0.75	0.75	0.70	0.80	0.75
0.6	0.30	0.82	0.75	0.34	0.82	0.75	0.75	0.70	0.80	0.75
0.7	0.41	0.82	0.40	0.45	0.82	0.49	0.79	0.51	0.82	0.75
0.8	0.53	0.80	0.52	0.56	0.81	0.57	0.57	0.60	0.81	0.65

Table 3. Optimal quality levels for the retailer's stores under uniform and non-uniform distributions of consumer types.

	$\gamma = 0.3$			$\gamma = 0.5$				$\gamma = 0.7$		
θ_L	q_1^N	q_2^N	q_1^U	q_1^N	q_2^N	q_1^U	q_2^U	q_1^N	q_2^N	q_1^U
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	0.53	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	0.62	1.00	1.00	0.66	1.00	1.00	1.00	1.00	1.00	1.00
0.7	0.73	1.00	0.70	0.76	1.00	0.78	1.00	0.82	1.00	1.00
0.8	0.84	1.00	0.80	0.87	1.00	0.82	0.82	0.91	1.00	0.90

However, larger companies or retailers that operate under different names (like Kroger Inc.) can choose among different formats and store characteristics when locating in different neighborhoods to increase their market share.

The numerical results were extended by exhibiting the impact of variations in θ_L (marginal valuation of L-type consumers) and γ (percentage of H-type population in the market) on the price and quality levels of stores for both uniform and non-uniform distributions of types of consumers. In this regard, we set $\theta_H = 1$, $t_L = 0.35$, and $t_H = 0.5$. Of note, these levels of travel costs can be obtained by setting $k = 0.5$ in the numerical results of Section 5 and they lie in the range to encourage the retailer to fully serve the H-types in the market. Therefore, both FN and FP strategies were detected in the following results. They were then compared in both uniform and non-uniform cases. The optimal prices for different levels of θ_L and γ are presented in Table 2. The proposed range for θ_L is set to make a distinction between these two classes meaningful. Since the price level in the uniform case is identical for both stores, only one price is shown in Table 2 when $\gamma = 0.3$ and $\gamma = 0.7$. For $\gamma = 0.5$, the retailer sets different price levels in its stores and this is consistent with the price given in Figure 3 for $k = 0.5$. As observed in Table 2, at low levels of θ_L, the retailer follows FN strategy because it does not serve any consumer in the L-segment. When θ_L is greater than the threshold level, the retailer changes its strategy to FP and sets a low price in store 1 to serve the L-type consumers too. This threshold increases by γ in the market, indicating that an increase in the population of L-type consumers gives an incentive to the retailer to serve this segment. Table 2 confirms the results from Figures 3 and 4 and shows that in some markets, the price differentiation between the stores of a retailer is quite different in uniform and non-uniform cases. Ignoring the distribution of consumer types (or equivalently the income distribution in the neighborhood) may result in the profit loss for the retailer.

Table 3 shows the quality level of these stores at different levels of θ_L and γ. The results indicate that in the non-uniform case, the retailer always sets the highest quality level for store 2. However, the quality level in store 1 depends on the threshold level for θ_L that provides the retailer with an incentive to serve the L-segment. For different levels of γ, we have $q_1^N \leq q^U \leq q_2^N$.

Table 4 shows the maximum profit of the retailer in the uniform and non-uniform markets. Since the non-uniform distribution of incomes geographically separates the H-type from L-type segments, it weakens the cannibalization effect. Therefore, the higher profit for the retailer under the nonuniform distribution of incomes can be justified by the retailer's ability to differentiate its stores and expand its market without

Table 4. Optimal profit of the retailer under uniform and non-uniform distributions of consumer types.

	$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.7$	
θ_L	π^U	π^N	π^U	π^N	π^U	π^N
0.2	0.075	0.079	0.125	0.132	0.175	0.185
0.3	0.075	0.079	0.125	0.132	0.175	0.185
0.4	0.075	0.079	0.125	0.132	0.175	0.185
0.5	0.075	0.080	0.125	0.132	0.175	0.185
0.6	0.075	0.098	0.125	0.137	0.175	0.185
0.7	0.102	0.124	0.130	0.160	0.175	0.198
0.8	0.156	0.158	0.174	0.187	0.201	0.217

cannibalization threat. According to Table 4, nonuniform distribution in heterogeneous markets provides potential profit for the retailers that are able to operate under different brand names and target different segments in the market.

6. Conclusion

The present study investigated the problem of setting price and quality levels for the stores of a retailer in two adjacent neighborhoods. Consumers in the market were different in terms of their willingness to pay for quality as well as their location. Consumers were segmented in two types based on their valuations for quality (or price sensitivity). The high-end segment cared more about the quality, while the low-end segment was more price sensitive. Both uniform and non-uniform distributions of each consumer segment in the market were taken into consideration. It was also shown how this spatial heterogeneity in willingness to pay, resulting from geographic income disparity, could affect the retailer's decision. In this study, the Hotelling's modeling framework was employed to derive the demands of the stores in the spatial market.

The analysis in this study revealed that the distance of the stores (or equivalently the travel cost) played a key role in the retailer's decision on price and quality levels offered in its stores. High travel costs gave the retailer an incentive to differentiate the stores when the two types of consumers were non-uniformly distributed in the market. The spatial income disparity that has been recently amplified in big cities gave the retailers the incentive to differentiate between their stores in terms of price and quality levels. Therefore, non-uniform distribution of incomes in neighborhoods could change the optimal strategy in targeting a consumer segment that should be considered by retailers upon entry in a new market. This study also pointed out that failure to account for neighborhoods heterogeneity would lead to non-efficient decisions on the quality and price levels among the stores.

The competition between two retailers in such markets can be an interesting subject for future research to examine how the strategies of the retailers at the quality and price levels vary under competition with two types of customers with non-uniform distribution in the market. This study assumed exogenous locations for the stores. Another avenue for research is the joint optimization of location, price, and quality levels of stores in a heterogeneous, non-uniform market.

References

1. Hansen, K. and Singh, V. "Market structure across retail formats", *Marketing Science*, **28**(4), pp. 656–673 (2009).
2. Petro, G. "Department store pricing no easy task (part 4 of 4)", https://www.forbes.com/sites/gregpetro/2013/01/10/department-store-pricing-no-easy-task-part-4-of-4/#6121d87a103a (2013).
3. Lieber, C. "Buyer beware: What you're actually getting at outlet stores", https://www.racked.com/2014/10/8/7573957/outlet-mall-stores (2014).
4. Kestenbaum, R. "Soaring off-price retail chains may be set for a fall", https://www.forbes.com/sites/richardkestenbaum/2017/11/15/the-off-price-business-is-more-troubled-than-it-looks/#782c25871554 (2017).
5. NCL. "New study reveals disturbing price variation within chain retailers", https://www.nclnet.org/new_study_reveals_disturbing_price_variation_within_chain_retailers (2013).
6. Steinhauer, J. "Cost of necessities rises in new york", https://www.nytimes.com/2005/02/08/ nyregion/ cost-of-necessities-rises-in-new-yo rk.html?_r=0 (2005).
7. Gauri, D.K., Trivedi, M., and Grewal, D. "Understanding the determinants of retail strategy: an empirical analysis", *Journal of Retailing*, **84**(3), pp. 256–267 (2008).
8. Gupta, S. and Chintagunta, P.K. "On using demographic variables to determine segment membership in logit mixture models", *Journal of Marketing Research*, **31**(1), pp. 128–136 (1994).

9. Gulati, N. and Ray, T. "Inequality, neighbourhoods and welfare of the poor", *Journal of Development Economics*, **122**, pp. 214–228 (2016).

10. Booza, J., Cutsinger, J., and Galster, G. "Where did they go?: The decline of middle-income neighborhoods in metropolitan America", *Metropolitan Policy Program*, Brookings Institution (2006).

11. Berube, A. and Frey, W., *A Decade of Mixed Blessings*, Washington, DC: Brookings Institution (2002).

12. Cooke, T. and Marchant, S. "The changing intrametropolitan location of high-poverty neighbourhoods in the us, 1990–2000", *Urban Studies*, **43**(11), pp. 1971–1989 (2006).

13. Reardon, S.F. and Bischo, K. "Growth in the residential segregation of families by income, 1970–2009", US 2010 Project (2011).

14. Hotelling, H. "Stability in competition", *The Economic Journal*, **39**(153), pp. 41–57 (1929).

15. Li, Z., Lu, Q., and Talebian, M. "Online versus bricks-and-mortar retailing: a comparison of price, assortment and delivery time", *International Journal of Production Research*, **53**(13), pp. 3823–3835 (2015).

16. Soysal, G. and Krishnamurthi, L. "How does adoption of the outlet channel impact customers' spending in the retail stores: Conflict or synergy?", *Management Science*, **62**(9), pp. 2692–2704 (2015).

17. Ngwe, D. "Why outlet stores exist: Averting cannibalization in product line extensions", *Marketing Science*, **36**(4), pp. 523–541 (2017).

18. Li, Z., Ryan, J.K., and Sun, D. "Selling through outlets: The impact of quality, product development risk, and market awareness", *International Journal of Production Economics*, **186**, pp. 71–80 (2017).

19. Carpenter, J.M. and Moore, M. "Consumer demographics, store attributes, and retail format choice in the us grocery market", *International Journal of Retail & Distribution Management*, **34**(6), pp. 434–452 (2006).

20. Fox, E.J., Montgomery, A.L., and Lodish, L.M. "Consumer shopping and spending across retail formats", *The Journal of Business*, **77**(S2), pp. S25–S60 (2004).

21. Hoch, S.J., Kim, B.-D., Montgomery, A.L., and Rossi, P.E. "Determinants of store-level price elasticity", *Journal of Marketing Research*, **32**(1), pp. 17–29 (1995).

22. Gonzalez-Benito, O., Munoz-Gallego, P.A., and Kopalle, P.K. "Asymmetric competition in retail store formats: Evaluating inter-and intra-format spatial effects", *Journal of Retailing*, **81**(1), pp. 59–73 (2005).

23. Gabszewicz, J.J. and Thisse, J.-F. "Price competition, quality and income disparities", *Journal of Economic Theory*, **20**(3), pp. 340–359 (1979).

24. Desai, P.S. "Quality segmentation in spatial markets: When does cannibalization affect product line design?", *Marketing Science*, **20**(3), pp. 265–283 (2001).

25. Rochet, J.-C. and Stole, L.A. "Nonlinear pricing with random participation", *The Review of Economic Studies*, **69**(1), pp. 277–311 (2002).

26. Tyagi, R.K. "Technological advances, transaction costs, and consumer welfare", *Marketing Science*, **23**(3), pp. 335–344 (2004).

27. Hernandez, M.A. "Nonlinear pricing and competition intensity in a hotelling-type model with discrete product and consumer types", *Economics Letters*, **110**(3), pp. 174–177 (2011).

28. Shi, H., Liu, Y., and Petruzzi, N.C. "Consumer heterogeneity, product quality, and distribution channels", *Management Science*, **59**(5), pp. 1162–1176 (2013).

29. Sedghi, N., Shavandi, H., and Abouee-Mehrizi, H. "Joint pricing and location decisions in a heterogeneous market", *European Journal of Operational Research*, **261**(1), pp. 234–246 (2017).

30. Neven, D.J. "On hotelling's competition with nonuniform customer distributions", *Economics Letters*, **21**(2), pp. 121–126 (1986).

31. Tabuchi, T. and Thisse, J.-F. "Asymmetric equilibria in spatial competition", *International Journal of Industrial Organization*, **13**(2), pp. 213–227 (1995).

32. Gupta, B., Pal, D., and Sarkar, J. "Spatial cournot competition and agglomeration in a model of location choice", *Regional Science and Urban Economics*, **27**(3), pp. 261–282 (1997).

33. Anderson, S.P., Goeree, J.K., and Ramer, R. "Location, location, location", *Journal of Economic Theory*, **77**(1), pp. 102–127 (1997).

34. Calvó-Armengol, A. and Zenou, Y. "The importance of the distribution of consumers in horizontal product di erentiation", *Journal of Regional Science*, **42**(4), pp. 793–803 (2002).

35. Liu, Y. and Weinberg, C.B. "The welfare and equity implications of competition in television broadcasting: the role of viewer tastes", *Journal of Cultural Economics*, **30**(2), pp. 127–140 (2006).

36. Meagher, K.J., Teo, E.G., and Wang, W. "A duopoly location toolkit: Consumer densities which yield unique spatial duopoly equilibria", *The BE Journal of Theoretical Economics*, **8**(1), pp. 1–23 (2008).

37. Guo, W.-C. and Lai, F.-C. "Prices, locations and welfare when an online retailer competes with heterogeneous brick-and-mortar retailers", *The Journal of Industrial Economics*, **65**(2), pp. 439–468 (2017).

38. Benassi, C., Chirco, A., and Colombo, C. "Vertical di erentiation beyond the uniform distribution", *Journal of Economics*, **126**(3), pp. 221–248 (2019).

39. Liu, Y. and Tyagi, R.K. "Outsourcing to convert fixed costs into variable costs: A competitive analysis", *International Journal of Research in Marketing*, **34**(1), pp. 252–264 (2017).

40. Xin, M. and Choudhary, V. "It investment under competition: The role of implementation failure", *Management Science*, **65**(4), pp. 1909–1925 (2018).

41. Ishibashi, I. and Matsushima, N. "The existence of low-end firms may help high-end firms", *Marketing Science*, **28**(1), pp. 136–147 (2009).
42. Galata, G., Bucklin, R.E., and Hanssens, D.M. "On the stability of store format choice", Research Paper, Anderson Graduate School of Management, UCLA (1999).
43. Lacourbe, P. "A model of product line design and introduction sequence with reservation utility", *European Journal of Operational Research*, **220**(2), pp. 338–348 (2012).

Appendix A

Derivation of condition in Assumption 1

To derive the conditions for travel costs to ensure partial market coverage, both uniform and non-uniform distributions for consumers' types in the market were considered. Partial coverage implies that the total coverage by the two stores is less than the whole market size for each consumer segment, i.e., $r_{1j}+r_{2j}<1$. Since the stores in the uniform distribution are identical, this condition reduces to $r_j<1/2$, i.e., $t_j \geq 2(\theta_j q - p)$. Since $q \leq \theta_H$ and $p \geq \frac{3}{4}\theta_L^2$ (refer to Table 1):

$$(\theta_j q - p) < 2(\theta_j\theta_H - \frac{3}{4}\theta_L^2). \tag{A.1}$$

Therefore, in the uniform case $t_j \geq 2(\theta_j\theta_H - \frac{3}{4}\theta_L^2)$ ensures that the market is partially covered. For the non-uniform distribution, partial market coverage is equivalent to $r_{1j}+r_{2j}<1$, which results in $\theta_j(q_1^N + q_2^N)-(p_1^N+p_2^N)<t_j$. On the other hand, Proposition 2 implies that $q_1^N \leq q_2^N$ and $p_1^N \leq p_2^N$. Therefore we have the following condition:

$$\theta_j(q_1^N+q_2^N)-(p_1^N+p_2^N) \leq 2(\theta_j q_2^N - p_1^N). \tag{A.2}$$

Eqs. (9) and (10) were used to confirm that $q_1^N \leq \theta_H$ and $p_1^N \geq \theta_L^2/2$. Based on Eq. (10) we have:

$$q_2^N = \frac{\frac{\gamma\theta_H}{t_H}f_H(1-r_{2H}) + \frac{(1-\gamma)\theta_L}{t_L}f_L(1-r_{2L})}{\frac{\gamma}{t_H}f_H(1-r_{2H}) + \frac{1-\gamma}{t_L}f_L(1-r_{2L})} < \frac{\frac{\gamma\theta_H}{t_H}f_H(1-r_{2H}) + \frac{(1-\gamma)\theta_H}{t_L}f_L(1-r_{2L})}{\frac{\gamma}{t_H}f_H(1-r_{2H}) + \frac{1-\gamma}{t_L}f_L(1-r_{2L})} = \theta_H. \tag{A.3}$$

To show that $p_1^N > \theta_L^2/2$, we should first prove that $q_1^N > \theta_L$.

$$q_1^N = \frac{\frac{\gamma\theta_H}{t_H}f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L}f_L(r_{1L})}{\frac{\gamma}{t_H}f_H(r_{1H}) + \frac{1-\gamma}{t_L}f_L(r_{1L})} > \frac{\frac{\gamma\theta_L}{t_H}f_H(r_{1H}) + \frac{(1-\gamma)\theta_L}{t_L}f_L(r_{1L})}{\frac{\gamma}{t_H}f_H(r_{1H}) + \frac{1-\gamma}{t_L}f_L(r_{1L})} = \theta_L. \tag{A.4}$$

Base on Eq. (9) we have:

$$p_1^N = \frac{q_N^2}{2} + \frac{\gamma F_H(r_{1H}) + (1-\gamma)F_L(r_{1L})}{\frac{\gamma}{t_H}f_H(r_{1H}) + \frac{1-\gamma}{t_L}f_L(r_{1L})} > \frac{q_N^2}{2} > \frac{\theta_L^2}{2}. \tag{A.5}$$

Inequalities (A.3) and (A.4) are employed to show that the right side of Inequality (A.2), which provides a lower bound for the travel costs (i.e., $t_j \geq 2\theta_j\theta_H - \theta_L^2$), results in the partial market coverage under non-uniform case. To use the same travel cost threshold for the uniform and non-uniform distributions, consider $t_j \geq 2\theta_j\theta_H - \theta_L^2$ as the condition that ensures partial market coverage under uniform and non-uniform distributions of consumers' types.

Appendix B

Proofs

Proof of lemma 1. We first obtain the retailer's optimal profit when it targets only H-type consumers (Case PN). Then, we compare it with the optimal profit under partial coverage of both segments (Case PP) to specify the threshold value for θ_L. In Case PN, as proposed in Section 4.1.1, the retailer's optimization problem is:

$$\max_{p,q} \quad \pi_1 = 2\gamma\left(p - \frac{q^2}{2}\right)\left(\frac{\theta_H q - p}{t_H}\right), \tag{B.1}$$

which yields the optimal quality and price levels of $q=\theta_H, p=\frac{3}{4}\theta_H^2$.

When the retailer targets two segments (Case PP), and the travel costs are high the retailer symmetric store configuration with the following optimization can be measured by:

$$\max_{p,q} \quad \pi_2 = 2\left(p-\frac{q^2}{2}\right)\left(\gamma\left(\frac{\theta_H q - p}{t_H}\right) + (1-\gamma)(\frac{\theta_L q - p}{t_L})\right), \tag{B.2}$$

which results in $q=\frac{\gamma t_L\theta_H+(1-\gamma)t_H\theta_L}{\gamma t_L+(1-\gamma)t_H}$ and $p=\frac{3}{4}q^2$.

We then obtain the optimal profits under Cases PN and PP:

$$\pi_1^* = \frac{\gamma{\theta_H}^4}{8t_H}, \tag{B.3}$$

$$\pi_2^* = \frac{(\gamma t_L\theta_H + (1-\gamma)t_H\theta_L)^4}{8(\gamma t_L + (1-\gamma)t_H)^3 t_L t_H}. \tag{B.4}$$

Comparing Eqs. (B.3) and (B.4), we get $\pi_1^* \leq \pi_2^*$ if and only if:

$$\theta_L \geq \theta_H(\frac{(\gamma t_L((1-\gamma)t_H+\gamma t_L)^3)^{1/4} - \gamma t_L}{(1-\gamma)t_H}).\square$$

Proof of Proposition 1. We first show that $q_1^N \leq q^U$. Note that q^U can be written as $q^U = \alpha\theta_H + (1-\alpha)\theta_L$. Based on Eq. (8), we regard q_1^N as a convex combination of θ_H and θ_L. More specifically, $q_1^N = \alpha_1\theta_H + (1-\alpha_1)\theta_L$ where:

$$\alpha_1 = \frac{\frac{\gamma}{t_H} f_H(r_{1H})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})}.$$

If $\alpha_1 \leq \alpha$ then $q_1^N \leq q^U$.

$$\alpha_1 \leq \alpha \Longleftrightarrow \frac{\frac{\gamma}{t_H} f_H(r_{1H})}{\frac{\gamma}{t_H} f_H(r_{1H}) + \frac{1-\gamma}{t_L} f_L(r_{1L})}$$
$$\leq \frac{\frac{\gamma}{t_H}}{\frac{\gamma}{t_H} + \frac{1-\gamma}{t_L}} \Longleftrightarrow f_H(r_{1H}) \leq f_L(r_{1L}). \quad \text{(B.5)}$$

In case the travel costs are high enough, each store of the retailer can only serve less than half of the market, thus resulting in $r_{1L} < \frac{1}{2}$ and $r_{1H} < \frac{1}{2}$. According to Assumption 2, $f_H(r_{1H}) \leq f_L(r_{1L})$ based on which the argument in Inequality (B.5) results in $q_1^N \leq q^U$.

The same proof can apply to $q_2^U \geq q^N$. In an analogous way, we can write q_2^N as a convex combination of θ_L and θ_H, i.e., $q_2^N = \alpha_2\theta_H + (1-\alpha_2)\theta_L$ where:

$$\alpha_2 = \frac{\frac{\gamma\theta_H}{t_H} f_H(1-r_{2H})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})}.$$

In this case we show that $\alpha_2 \geq \alpha$.

$$\alpha_2 \geq \alpha \Longleftrightarrow \frac{\frac{\gamma}{t_H} f_H(1-r_{2H})}{\frac{\gamma}{t_H} f_H(1-r_{2H}) + \frac{1-\gamma}{t_L} f_L(1-r_{2L})}$$
$$\geq \frac{\frac{\gamma}{t_H}}{\frac{\gamma}{t_H} + \frac{1-\gamma}{t_L}} \Longleftrightarrow f_H(1-r_{2H}) \geq f_L(1-r_{2L}). \quad \text{(B.6)}$$

According to Assumption 2, if $r_{2L} \leq r_{2H} \leq \frac{1}{2}$, $f_H(1-r_{2H}) \geq f_L(1-r_{2L})$. Since we assumed that the market is partially covered, the coverage radius for the upscale store should be lower than half. Therefore, $r_{2L} \leq r_{2H} \leq \frac{1}{2}$ which completes the proof.□

Proof of Lemma 2. Let us denote the profit of the retailer under Cases PN and PP by $\pi_1^N(\theta_L)$ and $\pi_2^N(\theta_L)$, respectively. To prove that a threshold θ_L^N exists so that $\pi_1^N(\theta_L) \leq \pi_2^N(\theta_L)$, $\forall\theta_L \geq \theta_L^N$ we use the Bolzano's theorem. Under Case PN, we have:

$$\max_{p_1,q_1,p_2,q_2} \pi_1^N = (p_1 - c(q_1))\left(\gamma\int_0^{r_{1H}} f_H(x)dx\right)$$
$$+(p_2 - c(q_2))\left(\gamma\int_{1-r_{2H}}^1 f_H(x)dx\right). \quad \text{(B.7)}$$

Under Case PP, we have:

$$\max_{p_1,q_1,p_2,q_2} \pi_2^N = (p_1 - c(q_1))\Bigg(\gamma\int_0^{r_{1H}} f_H(x)dx$$
$$+(1-\gamma)\left(\int_0^{r_{1L}} f_L(x)dx\right)\Bigg) + (p_2 - c(q_2))$$
$$(\gamma\int_{1-r_{2H}}^1 f_H(x)dx + (1-\gamma)$$
$$\left(\int_{1-r_{2L}}^1 f_L(x)dx)\right). \quad \text{(B.8)}$$

Note that π_1^N is independent of θ_L because we assumed that in Case PN, the retailer targets H-type consumers. However, since π_2^N denotes the profit under serving both segments, it depends on the value of θ_L. We show that π_2^N is increasing in θ_L. By using the Envelope Theorem, we have:

$$\frac{\partial\pi_2^N}{\partial\theta_L}|_{(p_1^N,p_2^N,q_1^N,q_2^N)} = (p_1^N - c(q_1^N))\frac{(1-\gamma)}{t_L} f_L(r_{1L})$$
$$+(p_2^N - c(q_1^N))\frac{(1-\gamma)}{t_L} f_L(r_{2L}) > 0. \quad \text{(B.9)}$$

The above derivative is positive since $p_1^N - c(q_1^N)$ and $p_2^N - c(q_2^N)$ are positive in the optimal setting. Therefore, we showed that $\frac{\partial\pi_2^N}{\partial\theta_L} > \frac{\partial\pi_1^N}{\partial\theta_L} = 0$. Moreover, we know that $\pi_1^N(0) > \pi_2^N(0)$ and $\pi_1^N(\theta_H) < \pi_2^N(\theta_H)$. In this setting, Bolzano's theorem implies that there exists a $\theta_L^N \in (0,\theta_H)$ such that $\pi_2^N(\theta_L^N) = \pi_1^N$. Since $\frac{\partial\pi_2^N}{\partial\theta_L} > 0$, then for $\theta_L > \theta_L^N$ we have $\pi_2^N(\theta_L^N) > \pi_1^N$. Therefore, for $\theta_L > \theta_L^N$ the retailer can increase its profit by targeting both segments.□

Proof of Proposition 2. Based on Lemma 2, when θ_L is less than the threshold level θ_L^N the retailer targets only the H-type consumers. Under Case PN, we know that the market is only partially covered, thus the demand of the stores does not overlap. In this case, the decision of each store can be made independently of the other store. The profit function of the retailer is:

$$\pi^N = \left(p_1 - \frac{q_1^2}{2}\right)\gamma\int_0^{r_{1H}} f_H(x)dx$$
$$+\left(p_2 - \frac{q_2^2}{2}\right)\gamma\int_{1-r_{2H}}^1 f_H(x)dx. \quad \text{(B.10)}$$

The derivation of the optimal solution is analogous to the general case presented in Section 4.1.2.

$$\frac{\partial\pi^N}{\partial p_1} = \gamma F_H(r_{1H}) - \left(p_1 - \frac{q_1^2}{2}\right)\frac{\gamma}{t_H} f_H(r_{1H}) = 0. \quad \text{(B.11)}$$

$$\frac{\partial\pi^N}{\partial q_1} = -q\gamma F_H(r_{1H}) + \left(p_1 - \frac{q_1^2}{2}\right)\frac{\gamma\theta_H}{t_H} f_H(r_{1H}) = 0. \quad \text{(B.12)}$$

which results in:

$$q_1^N = \theta_H, \quad p_1^N = \frac{\theta_H^2}{2} + \frac{t_H F_H(r_{1H})}{f_H(r_{1H})}. \tag{B.13}$$

Solving the first-order conditions for store 2 is also similar:

$$q_2^N = \theta_H, \quad p_2^N = \frac{\theta_H^2}{2} + \frac{t_H \bar{F}_H(1-r_{2H})}{f_H(1-r_{2H})}. \tag{B.14}$$

Based on Eqs. (B.13) and (B.14), it can be concluded that $q_1^N = q_2^N = \theta_H$. To compare the prices, assume that $f_H(x)$ is increasing in x, thus we have $F_H(x) \leq x f_H(x)$ and $\bar{F}_H(x) \geq x f_H(x)$, which results in:

$$\frac{F_H(r_{1H})}{f_H(r_{1H})} \leq r_{1H}, \qquad \frac{\bar{F}_H(1-r_{1H})}{f_H(1-r_{2H})} \geq r_{2H}. \tag{B.15}$$

Now, suppose that $p_1^N \geq p_2^N$ which leads to $r_{1H} \leq r_{2H}$. According to Inequality (B.15), we have:

$$p_1^N = \frac{\theta_H^2}{2} + \frac{t_H F_H(r_{1H})}{f_H(r_{1H})} \leq \frac{\theta_H^2}{2} + \theta_H r_{1H}$$

$$\leq \frac{\theta_H^2}{2} + \theta_H r_{2H} \leq \frac{\theta_H^2}{2} + \frac{t_H \bar{F}_H(1-r_{2H})}{f_H(1-r_{2H})} = p_2^N,$$

which results in a contradiction. Therefore, $p_1^N < p_2^N$.□

Appendix C

Second partial derivative test

To check the optimality of the solutions obtained in Section 4.1.1, Hessian matrix for function $\pi_1(p,q)$ is used in the following:

$$H(p,q) = \begin{pmatrix} \frac{\partial^2 \pi_1}{\partial p^2} & \frac{\partial^2 \pi_1}{\partial p \partial q} \\ \frac{\partial^2 \pi_1}{\partial q^2} & \frac{\partial^2 \pi_1}{\partial q \partial p} \end{pmatrix} \Rightarrow$$

$$H(p^*,q^*) = \begin{pmatrix} -\frac{2\gamma}{t_H} & \frac{2\gamma\theta_H}{t_H} \\ \frac{2\gamma\theta_H}{t_H} & -\frac{9\gamma\theta_H^2}{4t_H} \end{pmatrix}. \tag{C.1}$$

The determinant of $H(p^*,q^*)$ is $\frac{\gamma^2\theta_H^2}{2t_H^2}$ which is positive. Since $\frac{\partial^2 \pi_1}{\partial p^2}$ is negative, the solution $(p^*,q^*) = (\frac{3}{4}\theta_H^2, \theta_H)$ is a local maximum.

Hessian matrix for the π_2 at the point:

$$(p^*,q^*) = (\frac{3}{4}(\frac{\gamma t_L\theta_H + (1-\gamma)t_H\theta_L}{\gamma t_L + (1-\gamma)t_H})^2,$$

$$\frac{\gamma t_L\theta_H + (1-\gamma)t_H\theta_L}{\gamma t_L + (1-\gamma)t_H})$$

is shown in Eq. (C.2):

$$H(p^*,q^*) = \begin{pmatrix} -\frac{2\gamma}{t_H} - \frac{2(1-\gamma)}{t_L} & \frac{2\gamma\theta_H t_L + 2(1-\gamma)\theta_L t_H}{t_L t_H} \\ \frac{2\gamma\theta_H t_L + 2(1-\gamma)\theta_L t_H}{t_L t_H} & -\frac{9(\gamma\theta_H t_L + (1-\gamma)\theta_L t_H)^2}{4t_H t_L(\gamma t_L + (1-\gamma)t_H)} \end{pmatrix}. \tag{C.2}$$

The determinant of Matrix (C.2) is:

$$|H(p^*,q^*)| = \frac{(\gamma\theta_H t_L + (1-\gamma)\theta_L t_H)^2}{2(t_L t_H)^2}. \tag{C.3}$$

Since Eq. (C.3) is positive and $\frac{\partial^2 \pi_1}{\partial p^2}$ is negative, the solution (p^*,q^*) is a local maximum.

Appendix D

Optimization problem under full market coverage of low-end segment

This section proposes the mathematical model for analyzing Cases F1 and F4 that also represents the retailer's full market coverage of L-type consumers. Based on Eq. (12), the location of an L-type consumer who is indifferent about buying from either store can be determined. Therefore, the optimization problem for Case F1 is:

$$\max_{p_1,q_1,p_2,q_2} \pi = (p_1 - c(q_1))\Big(\gamma \int_0^{x_H} f_H(x)dx$$

$$+(1-\gamma)\int_0^{x_L} f_L(x)dx\Big) + (p_2 - c(q_2))$$

$$\Big(\gamma \int_{x_H}^1 f_H(x)dx + (1-\gamma)\int_{x_L}^1 f_L(x)dx\Big), \tag{D.1}$$

s.t:

$$x_j = \frac{1}{2} - \frac{\theta_j(q_2-q_1) - (p_2-p_1)}{2t_j}, \qquad j = L,H, \tag{D.2}$$

$$\theta_j q_1 - t_j x_j - p_1 \geq 0, \qquad j = L,H, \tag{D.3}$$

$$p_i, q_i \geq 0, \qquad i = 1,2. \tag{D.4}$$

The optimization problem under Case F4 (full coverage of the low-end segment and partial coverage of the high-end segment) is as follows:

$$\max_{p_1,q_1,p_2,q_2} \pi = (p_1 - c(q_1))\Big(\gamma \int_0^{r_{1H}} f_H(x)dx$$

$$+(1-\gamma)\int_0^{x_L} f_L(x)dx\Big) + (p_2 - c(q_2))$$

$$\Big(\gamma \int_{1-r_{2H}}^1 f_H(x)dx + (1-\gamma)\int_{x_L}^1 f_L(x)dx\Big), \tag{D.5}$$

s.t:

$$x_L = \frac{1}{2} - \frac{\theta_L(q_2 - q_1) - (p_2 - p_1)}{2t_L}, \tag{D.6}$$

$$r_{iH} = \max\left\{0, \frac{\theta_H q_i - p_i}{t_H}\right\}, \qquad i = 1, 2, \tag{D.7}$$

$$\theta_L q_1 - t_L x_L - p_1 \geq 0, \tag{D.8}$$

$$p_i, q_i \geq 0, \qquad i = 1, 2. \tag{D.9}$$

Biographies

Nafiseh Sedghi is currently a PhD Candidate at Sharif University of Technology, Tehran, Iran. She received her BS and MS degrees in Industrial Engineering from Sharif University of Technology in 2005 and 2008, respectively. Her research lies at the intersection of operations management, marketing, and economics with the focus on product differentiation and heterogeneous markets.

Hassan Shavandi is currently an Associate Professor at Sharif University of Technology, Tehran, Iran. He received his PhD in Industrial Engineering from Sharif University of Technology in 2005. His research is mainly in the area of pricing and revenue management and applied operations research.