A new bi-objective integrated vehicle transportation model considering simultaneous pick-up and split delivery

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Abstract

Nowadays, global competition causes that the companies are dealing with the issue of cost reduction besides increasing productivity in business network more than ever. Because of that, today, both industrial practitioners and researchers are focusing on the supply chain structure issues. In order to achieve the real word objectives, the aim of the paper is to improve the performance of the supply chain network via not only taking into account simultaneous pick-up and split delivery but also minimizing the total costs as well as maximizing customer services alongside multi-period and multi-products. Besides, to accumulate the data of parameters, a food industry is considered for a case study which is in the north of Iran. Eventually, the proposed mixed-integer linear programming model is addressed by a ε-constraint method. Finally, related results of this solution are analyzed and also is compared with simple vehicle routing problem (VRP).

Keyword: Vehicle routing problem; Simultaneous pick-up and split delivery; Production planning; Integration; Mixed-integer programming.

1. Introduction and Literature Review

The syntax of supply chain arrangement are complex, thus it always characterized via myriad merits diffusing over numerous functions and settings. Also, we can define logistics as a method which obtains the right quantity of defined commodities at the right time as well as to the right site, so it can be suggested to supply chain network problems [1]. The problem arises when the supplies of service industries are asked to devote or apply in an efficient way. This usually happens since the service industries must taking account the transportation route and also improve the vehicle loading rate [2].

The mean of shipping expenses in Iran is between 1.7 and 2 times that of the international average base on the report of the Iran Chamber of Commerce Industries, Mines, and Agriculture and the World Bank as shown in Fig. 1, 2, 3 [3]. Therefore, if modern companies want to remind in global arena not only should look for innovate methods to enhance logistics and distribution activities to satisfy customer demands, but also should try to reduce cost of product [4].

Fig. 1

Fig. 2

Fig. 3

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The purpose of traditional vehicle routing problem (VRP) is to determine the minimum length itineraries to visit a collection of dispersed customers with carrier machines in a way the capacity of each carrier machine is considered and every client should be met one time. Moreover, there are three different groups to categorize the attributes of the distribution manner. Warehousing, cross-docking and direct shipping. Here, authors want to briefly define the purpose of the mentioned terms: Warehousing is needed, so the stock could pile up with goods and transferred to customers in the time of need. Cross-docking reduces the need for large stocks by using a conveyance procedure named just-in-time and direct shipping used to distribute things directly from suppliers to customers [5].

Many researchers have crave for to attend distribution planning alongside integrated production under supply chain structure over the past few years. Some researchers claim that two principal causes are responsible for integrating production and distribution planning: A) Desirable aspect of the increase in income in the supply chain. B) Plus points on a decrease in lead times and showing prompt responses to trade changes [6-7]. Similarly, some papers pointed out integrating production and distribution planning may lead to (A) the expense of operating a production-distribution method is reduced also (B) the buyer's order fulfills more effectively [8-10].

Liu and Papageorgiou [11] presented a mathematical model for production, distribution, and capacity planning in a global supply chain. In this paper, three objectives broadened as total expenses, lost sale and flow time alongside two strategies extending the capacity of manufacturing. The core of a systemized scattering is the VRP. Thousands of corporations acting in occupations such as collecting, shipment, conveyance and manufacturing are dealing with this issue permanently. Since the conditions are different for a system to another one, the objectives and the constraints of the VRP is very diverse.

The VRP was firstly proposed by Ramser and Dantzig [12]. A case study was considered being the distribution of gasoline from a terminal to a vast number of customers. In this paper and other traditional VRP were utilized Euclidean distance; nevertheless, this assumption is in contrast with the real condition of roads. With considering this assumption, estimated expenses are upper than real that one, because the expenses related to the traveling time are greatly ignored. In a model presented by Polimeni and Vitetta [13], the total time that takes until vehicles visit nodes which are included clients and warehouses is dependent not only on the distance among clients but also the time of day that vehicles moving for taking into account a real situation like weather conditions and traffic jams (the velocity of the fleet depends on travel speed). Besides, the cumulative capacitated vehicle routing problem proposed by Ozsoydan and Sipahioglu [14]. In this paper, authors ignored the whole tour cost of the supply chain instead they would like to minimize the whole of arrival times at points (clients and depots). After a special time such as a natural disaster, this kind of routing issue popular because the delivery of momentous commodities is in priority. Although considering a single depot in the VRP subjects is interesting and simple until now, corporations having more than one depot need a new model. A model has been introduced by Bertazzi and Speranza [15] with a change in the classic inventory shipping model. Firstly, this model started with a vehicle that is responsible for transferring commodities. Then a lot of vehicles add to the previous fleet. Allahyari et al. [16] focused on a model called Multi-Depot Vehicle Routing Problem where each depot was responsible for a cluster of determined customers and each vehicle came back to the same depot that started from that.

Baldacci et al. [17] have expressed the other applications of VRP such as food delivery, web connections, letterboxes distribution, and waste collect planning. Moreover, Golmohamadi et al. [18] proposed a model that the commodities were shipped in boxes in a constant expense transportation issue. In another study, Fathollahi-Fard et al. [19] developed and coordinated the integrated shipping
and production scheming issue alongside time windows and due date time to minimize the total cost. In traditional VRP, final level of supply chain deals with the delivery of goods. Although sometimes we understand that goods should be picked up beside the delivery of those which is named Vehicle Routing Problem with Simultaneous Pick-up and Delivery (VRPSPD) presented by Min [20] for the first time. Salhi and Nagy [21] designed a new model related to the shipping organize which utilized an array of types of VRPs. The number of depots and distribution centers was added to the previous paper of Nagy and Salhi to expand it [22]. Dethloff [23] investigated a model based on an emphasis on reverse logistics in a VRPSPD and the connection among VRPSPD and other shipping issues was studied. Gendreau et al. [24] developed a transportation shipping issue when they came across with dynamic requests from their clients. In the following, a model considered the time window for VRPSPD presented by Wang and Chen [25]. A VRPSPD model alongside with non-homogeneous vehicles was introduced by Avci and Topaloglu [26] solved with metaheuristic algorithm. In another study, Hosseini-Motlagh and et al. [27] showed a pollution routing problem is discussed using simultaneous pickup and delivery. The authors declared the aims of this issue minimized the fuel and emissions consumption as well as using scheduling and routing customers. A new VRP with developing various manner to develop and pick-up was expanded by Ting et al. [28]. This innovation method was made with help of finding the shortages path from one point (depots and clients) to another one which was optimal alongside ordinary constraints such as fleet volume and length of road. Recently the new topic like that environment matter has added to VRP such as Wang and Li [29] and Sousa et al. [30]. They focused on Green supply chain and routing issues with aiming to minimize pollution such as a decrease in carbon emissions. Abad et al. [31] have proposed a bi-objective mathematical programming model. The authors wanted to consider consolidation manners in cross-dock matters alongside with coordinate the VRP with the integration of delivery and pick-up. Shi et al. [32] considered a Home Health Care (HHC) shipping issue with stochastic service times and travel coming from the logistics practice of HHC corporates.

In most VRPs, one of the supposals being significant is fleet just one time visit each customer. It is not always a realistic assumption because sometimes the customer’s demand passes the vehicle’s volume. Therefore, the constraint that causes this assumption is relaxed in this paper. As a result, a customer could be met by several transportation vehicles. Split Delivery Vehicle Routing Problem (SDVRP) was firstly defined by Dror and Trudeau, [33]. Besides, Archetti et al. [34] have shown the application of this method to keep the traveled distance up to 50 percent. Transferring the products to customers using SDVRP, often causes two kinds of dilemmas: interference and additional job. To overcome this, Gulkczynaski et al. [35] introduced Split Delivery Vehicle Routing Problem with Minimum Delivery Amounts (SDVRP-MDA) model for the first time. In this model, minimum of predetermined requests containing up to 50 percent of buyer requests are shipped in each meeting that results in less disorder and difficulty. Moreover, Archetti’s SDVRP format have been changed to SDVRP-MDA by Han et al [36] and presented an innovation heuristic approach. A mathematical model with a bi-objective function as well as the unique kind of split deliveries proposed by Xia et al. [37]. Moreover, the only way to split the customer’s request is to divide it by backpack and the innovation heuristic algorithm like Tabu Search was suggested to solve the problem. Another paper aiming to minimize cost for capacitated vehicle problem was proposed by Hernández-Pérez et al [38]. They addressed this problem by proposing a model via the optimization of goods' transportation and the other product among an array of the buyer. They allowed two new and infrequent characteristics in the pickup and delivery area. The first characteristic was related to the number of customers' visits. In fact, buyers may be met a lot of times. Besides, in the second one, the buyer is turned to the temporary and middle location to collect and deliver products. Qiu et al. [39] proposed a model namely Vehicle Routing Problem with Discrete Split Deliveries and Pickups (VRPDSDP) and designed a TS algorithm with particularly batch mixture and item initiating function. Abdi et al. [40]
introduced new effective VRP model in order to reduce the total cost of the proposed network. The model considers not only greenness, but also other environmental factors. The model solved using some meta-heuristics. The problem of using VMI policy with perishable products addressed by Salehi Amiri et al. [41]. The authors designed two-echelon supply chain network and utilized multiple meta-heuristics to work out the issue.

Fathollahi-Fard et al. [42-43] presented a bi-objective which was introduced location-allocation-distribution model in HHC logistics service that in those caregivers begun from a pharmacy to ills’ homes are planned and shipped to do divers care services. Finally, they move to their lab to improve the ills’ health background. Moreover, they employed heuristics and a hybrid constructive metaheuristic in their paper. Feng et al [44] optimized hybrid disassembly and proposed end-of-life processes of commodity improvement alongside both maximizing the improvement benefit and minimizing the surrounding effect. They dealt with the operation planning and end-of-life decision-making issue of commodity improvement. Tian et al. [45-46] showed an innovation AND/OR-graph-based disassembly sequence planning issue by supposing the uncertain element quality and varying disassembly operational expense. While in these papers, an innovation hybrid intelligent algorithm integrating fuzzy simulation and artificial bee colony was presented to work out them. Fu et al. [47] addressed a two-agent stochastic flow shop deteriorating scheduling problem with the objectives of minimizing the make span of the first agent and the total tardiness of the second agent. Then, they used a hybrid multi-objective evolutionary algorithm to solve it. Fu et al. [48] proposed sustainable and effective production operation alongside the scheduling model that can make an interplay between the manufacturing expense and energy consumption.

The papers reviewed above were looking at the issues merely from the point of view of the supply chain especially VRP aspects. In issues about the supply chain, regardless of shipping restrictions, there is usually a uniform manner of shipping and transportation problems merely discuss routing deals. Objective functions of transportation models are following as: A- Minimizing the number of fleets needed to give service to the buyers, B- Stabilizing the journey from the point of view of travel time and vehicle load, C- Considering the deadline of the agreement and be caution about the earliness and tardiness, and D- Minimizing the whole journey length.

In this paper, a VRP problem has been extended to get better supply chain efficiency. Using the aforementioned cases and literature reviews, one can infer that, simultaneous pick-up and delivery issues and also split delivery issues have been studied separately. To the best of our knowledge, a few papers have been reported these two separated issues. Thus, this raise the important aspect of our study and necessity to integrate the pre-mentioned decisions and considerations. Accordingly, we propose an innovative formulation, which aims to detect the best track from the perspective of a decrease in the journeyed space and minimizing the number of fleets needed to dispatch goods from the distribution centers to the buyers. In addition, production planning is another significant decision to the optimization so that is determined the amount of plant production on the horizon, amount of inventory in the plant, lost sales for each buyer and dispatching point. Also, with the help of an epsilon-constraint via the GAMS, the mathematical model will be solved.

Contribution of this study compared to other papers, based on the top comments, are summed up in the forthcoming bullet points:

- As seen in Fig. 4, both warehousing and direct shipping for dispatching manners are considered.
- A bi-objective simultaneous pick-up and split delivery VRP mathematical model alongside taking into account the multi-period and multi-product, is developed.
- Adding the production planning into the VRP problem in order to expand a practical framework of the organized operations, at the same time.
The subsequent sections are subdivided into the following. In the upcoming section, not only we explain a framework for the problem and describe the objective function, but also the mathematical model is elaborated. A numerical example, validation, computerized results, and sensitivity analysis are shown in section 3. Finally, in section 4, the conclusion is presented.

2. The Problem Formulation and Explanations

Fig. 4 depict the framework of the introduced supply chain structure in this paper. The integrated planning of shipping and production is simultaneously conducted in this system. As are shown, a plant warehouse (upcoming is named warehouse) and a Distribution Center (DC) are considered.

Fig. 4.

The aim of this research is to detect the best quantity of commodities output, as well as inventories of each product at the warehouse and DC in the planning horizon, vehicle routing in supply chain network by considering transportation costs and increasing the level of buyer fulfillment by minimizing the number of lost sales. The objective function includes the production, distribution, and inventory holding costs being the total cost of the chain.

In this study, some significant features of routing and production like that A) the number of requests for multi-commodities in the scheming horizon, B) constant and floating charge of manufacture, C) capacity peak of plant manufacture, D) constant charge of routing incurred for each track utilized in the solution, besides the floating charge of routing being adequate to both the number of all goods on fleet and traveled distance, E) the capacity peak of fleet, F) the quantity of picked-up commodities, G) the quantity of inventory and H) the capacity peak of warehouse and DC are incorporated.

Significant assumptions:
- Lost sales occur when customers' requests are unsatisfied and model do not let postpone to another time.
- Both warehousing and direct shipping for sharing out are considered.
- Each vehicle is a member of either warehouse or DC, they start and finish from the same point.
- Buyers like that fleet are assigned to either DC or the warehouse.

In the following, the mathematical model formulation is detailed out.

Indices:

\[ j, j' \in \{1,2,\ldots,J\} \] Index of buyers allocated to DC
\[ k, k' \in \{1,2,\ldots,K\} \] Index of buyers allocated to the warehouse
\[ n \in \{1,2,\ldots,N\} \] Index of fleet allocates to DC
\[ m \in \{1,2,\ldots,M\} \] Index of fleet allocated the warehouse
\[ p \in \{1,2,\ldots,P\} \] Index of commodity
\[ t \in \{1,2,\ldots,T\} \] Index of time period
Parameters:

- **U**: Large amount
- **a_p**: Size of commodity \( p \)
- **V_m**: The volume of fleet \( m \)
- **V_n**: The volume of fleet \( n \)
- **FC_{pt}**: Constant charge of applied fleet \( m \) at time \( t \)
- **FC_{nt}**: Constant charge of applied fleet \( n \) at time \( t \)
- **d_{s_k}**: Space between the factory to buyer \( k \)
- **d_{s_j}**: Space between DC to buyer \( j \)
- **d_{j_{kk}}**: Space between buyer \( k \) and \( k' \)
- **d_{j_{jj}}**: Space between buyer \( j \) and \( j' \)
- **C_{pm}**: Charge of shipping between the factory to DC via fleet \( m \) per unit of commodity \( p \)
- **CW_{nj}**: Charge of delivery between DC to buyer \( j \) via fleet \( n \) per unit of commodity \( p \)
- **CW_{nj_{jj}}**: Charge of shipping between buyer \( j \) to buyer \( j' \) via fleet \( n \) per unit of commodity \( p \)
- **CE_{pmk}**: Charge of shipping between the factory to buyer \( k \) via fleet \( m \) per unit of commodity \( p \)
- **CE_{pm_{kk}}**: Charge of shipping between buyer \( k \) to buyer \( k' \) via fleet \( m \) per unit of commodity \( p \)
- **IC_{pt}**: Charge of inventory at the warehouse per unit of commodity \( p \) at time \( t \)
- **IC_{pt}**: Charge of inventory at DC per unit of commodity \( p \) at time \( t \)
- **INV_{max}**: The peak of the warehouse volume
- **INV_{max}**: The peak of DC volume
- **CAP_{pt}**: Volume of production for commodity \( p \) at time \( t \)
- **D_{pkt}**: The quantity of commodity \( p \) at time \( t \) demanded via buyer \( k \)
- **D_{p_{jj}}**: The quantity of commodity \( p \) at time \( t \) demanded via buyer \( j \)
- **P_{pkt}**: The quantity of commodity \( P \) at time \( t \) which is picked-up from buyer \( k \)
- **P_{p_{jj}}**: The quantity of commodity \( P \) at time \( t \) which is picked-up from buyer \( j \)
- **FFC_{pt}**: Constant production charge of commodity \( P \) at time \( t \)
- **VFC_{p}**: The floating charge of commodity \( p \)

Integer and Binary variables:

- **QD_{pm_{tt}}**: The quantity of the commodity \( p \) via fleet \( m \) at time \( t \) transferred of the warehouse to DC
- **QP_{pm_{tt}}**: The quantity of the picked-up commodity \( p \) via fleet at time \( t \) transferred of DC to the warehouse \( m \)
The quantity of the commodity \( p \) via fleet \( n \) at time \( t \) transferred of DC to buyer \( j \)

The quantity of the commodity \( p \) via fleet \( n \) at time \( t \) picked-up of buyer \( j \)

The quantity of the commodity \( p \) at time \( t \) via fleet \( n \) picked-up of the buyer \( j' \) when the fleet journeyed of buyer \( j \) to buyer \( j' \)

The quantity of the commodity \( p \) at time \( t \) via fleet \( n \) transferred of buyer \( j \) to buyer \( j' \)

The quantity of the commodity \( p \) at time \( t \) via fleet \( m \) transferred of the warehouse to buyer \( k \)

The quantity of the commodity \( p \) at time \( t \) via fleet \( m \) picked-up of buyer \( k \)

The quantity of the commodity \( p \) at time \( t \) via fleet \( m \) picked-up of buyer \( k' \) when the fleet journeyed of buyer \( k \) to buyer \( k' \)

The quantity of the commodity \( p \) at time \( t \) via fleet \( n \) transferred of buyer \( k \) to buyer \( k' \)

The quantity of the commodity \( p \) stayed on fleet \( n \) instantly after transferring to buyer \( j \) at time \( t \)

The quantity of the commodity \( p \) stayed on fleet \( m \) instantly after transferring to buyer \( k \) at time \( t \)

The quantity of the commodity \( p \) at time \( t \) in the warehouse put on fleet \( m \)

The quantity of the commodity \( p \) at time \( t \) in DC put on fleet \( n \)

The quantity of the commodity \( p \) at time \( t \) came back to the warehouse of DC

The quantity of the commodity \( p \) at time \( t \) came back to the warehouse picked-up of all buyer which is allocated to DC

Inventory level of commodity \( p \) in the warehouse at the end of time \( t \)

Inventory level of commodity \( p \) in DC at the end of time \( t \)

Lost sale amount of commodity \( p \) at time \( t \) to buyer \( j \)

Lost sale amount of commodity \( p \) at time \( t \) to buyer \( k \)

The quantity of commodity \( p \) at time \( t \) manufactured

\( \{1; \text{If fleet } m \text{ at time } t \text{ moves to DC from the warehouse. 0; Otherwise}\} \)

\( \{1; \text{If fleet } n \text{ at time } t \text{ moves to buyer } j \text{ from DC. 0; Otherwise}\} \)

\( \{1; \text{If fleet } n \text{ at time } t \text{ moves to buyer } j' \text{ from buyer } j. 0; \text{Otherwise}\} \)

\( \{1; \text{If fleet } n \text{ at time } t \text{ moves to DC from buyer } j. 0; \text{Otherwise}\} \)
\[ Y_{\text{mk}t} \begin{cases} 1; \text{If fleet } m \text{ at time } t \text{ moves to buyer } k \text{ from the warehouse.} & 0; \text{Otherwise} \end{cases} \]

\[ Y_{\text{mkk} \text{t}} \begin{cases} 1; \text{If fleet } m \text{ at time } t \text{ moves to buyer } k \text{ from buyer } k. & 0; \text{Otherwise} \end{cases} \]

\[ Z_{\text{mk}t} \begin{cases} 1; \text{If fleet } m \text{ at time } t \text{ moves to the warehouse from buyer } k. & 0; \text{Otherwise} \end{cases} \]

\[ W_{pt} \begin{cases} 1; \text{If commodity } p \text{ at time } t \text{ is produced.} & 0; \text{Otherwise} \end{cases} \]

\[ \text{Model:} \]

\[ \text{Min } Z_1 = \text{WPE + WSE + WIE} \] (1)

\[ \text{WPE} = \sum \sum_{p,t} FFC_{pt} \cdot W_{pt} + \sum \sum_{p,t} VFC_{pt} \cdot AP_{pt} \] (1.1)

\[ \text{WSE} = \sum \sum_{m,t} FC_{mt} \cdot Y_{mt} + \sum \sum_{k,t} FC_{mk} \cdot Y_{mk} + \sum \sum_{n,j,t} FC_{nj} \cdot Y_{nj} + \sum \sum_{m,k,n,t} ds_{kj} \cdot Y_{mk} + \sum \sum_{n,j,t} YW_{nj} + \sum \sum_{m,k,n,t} \sum_{k} \sum_{t} di_{kk} \cdot Y_{mk} \] (1.2)

\[ \text{WIE} = \sum \sum_{p,t} IC_{pt} \cdot Inv_{pt} + \sum \sum_{p,t} I_C_{pt} \cdot Inv'_{pt} \] (1.3)

\[ \text{Min } Z_2 = \sum \sum_{p,k,t} LS_{pkt} + \sum \sum_{p,j,t} LS'_{pjt} + \sum \sum_{p,k,t} LP_{pkt} + \sum \sum_{p,j,t} LP'_{pjt} \] (2)

The first objective function \( Z_1 \) consists of three components as described at the following.

Element (1.1) show the Whole Production Expenses (WPE) calculating the constant as well as floating charge of production for the commodities in the horizon of planning. Element (1.2) show the Whole Shipping Expenses (WSE) calculating the constant track expenses for transferring the fleet, the floating shipping expenses as cause fuel consumption and fleet depreciation relevant the journeyed length by the fleet and the quantity of shipped commodities. Element (1.3) show the Whole Inventory Expenses (WIE) calculating holding expenses of commodities. Another target is \( Z_2 \) consisting of the whole quantity of came back commodities named lost sales.

Subject to:

\[ \sum_{p} QWP_{pjt} \leq U \cdot YW_{njt} \quad \forall n, j, t \] (3)

\[ \sum_{p} QEP_{pmtk} \leq U \cdot YE_{mkt} \quad \forall m, k, t \] (4)

In limitation (3) and (4) pick-up condition from buyers \( j \) and \( k \) is that fleet \( n \) and \( m \) travel from DC and the warehouse to buyers \( j \) and \( k \).
\[
\sum_{p} QWP_{\text{p}n} \leq U \cdot YW_{n} \quad \forall n, j, j' (j \neq j'), t
\] (5)

\[
\sum_{p} QEP_{\text{p}m} \leq U \cdot YE_{m} \quad \forall m, k, k' (k \neq k'), t
\] (6)

Limitations (5) and (6) are similar to earlier constraints with small deference that the fleet must visit another buyer before visiting a new buyer.

\[
\sum_{p} a_{p} \cdot QD_{\text{p}m} \leq V_{m} \cdot Y_{mt} \quad \forall m, t
\] (7)

Limitation (7) shows the condition of transfer from the warehouse towards DC. Besides, we have to consider the maximum fleet volume.

\[
\sum_{p} L'_{\text{p}n} \cdot a_{p} \leq V'_{n} \quad \forall n, t
\] (8)

\[
\sum_{p} L_{\text{p}m} \cdot a_{p} \leq V_{m} \quad \forall m, t
\] (9)

Limitation (8) and (9) state the capacity condition of the fleet when they decided to transfer commodities.

\[
\sum_{p} L'_{\text{p}j} \cdot a_{p} \leq V'_{n} \quad \forall n, j, t
\] (10)

\[
\sum_{p} L_{\text{p}m} \cdot a_{p} \leq V_{m} \quad \forall m, k, t
\] (11)

Limitation (10) and (11) are similar to (8,9) with a difference in which constraints (8,9) start from depots but these constraints before visiting the new buyers visit the other buyers.

\[
L'_{\text{p}n} = \sum_{j} QWD'_{\text{p}j} \cdot YW_{n} + \sum_{j} \sum_{j'} QWD'_{\text{p}n_{j} j_{i}} \cdot YW_{n_{j_{i}}} \quad \forall p, n, t
\] (12)

This equation figures out the loaded quantity on fleet when it exits from DC.

\[
L'_{\text{p}j} \geq L'_{\text{p}n} - QWD_{\text{p}j} + QWP_{\text{p}j} - U \cdot (1 - YW'_{\text{n}}) \quad \forall p, n, j, t
\] (13)

\[
L'_{\text{p}j} \leq L'_{\text{p}n} - QWD_{\text{p}j} + QWP_{\text{p}j} + U \cdot (1 - YW'_{\text{n}}) \quad \forall p, n, j, t
\] (14)

Constraints (13) and (14) figure out the quantity of remained load on fleet after visiting the first buyer which is related to DC.

\[
L'_{\text{p}j} \geq L'_{\text{p}n} - QWD_{\text{p}j} + QWP_{\text{p}j} - U \cdot (1 - YW'_{\text{n}}) \quad \forall p, n, j, j' (j \neq j'), t
\] (15)
\[ L'_{pnt} \leq L'_{pny} - QWD_{pny} + QWP_{pny} + U \cdot (1 - YW_{njy}) \quad \forall p, n, j, j'(j \neq j'), t \] (16)

Constraints (15) and (16) are similar to (13, 14) with a difference in which constraints (13, 14) start from depots but these constraints before visiting the new buyers visit the other buyers.

\[ L'_{pnt} \leq U \cdot (YW_{nj} + \sum_{j' \neq j} YW_{njy}) \quad \forall p, n, j, t \] (17)

Edition of the loaded quantities is declared in Inequality (17).

\[ L_{pmk} = \sum_k QED'_{pmkt} \cdot YE'_{mk} + \sum_k \sum_{k' \neq k} QED''_{pmkkt} \cdot YE''_{mkkt} \quad \forall p, m, t \] (18)

Constraint (18) calculates the loaded quantity on the fleet when the vehicles leave from the warehouse.

\[ L_{pmkt} \geq L_{0pmkt} - QED'_{pmkkt} + QEP'_{pmkkt} - U \cdot (1 - YE'_{mkkt}) \quad \forall p, m, k, t \] (19)

\[ L_{pmkt} \leq L_{0pmkt} - QED''_{pmkkt} + QEP''_{pmkkt} + U \cdot (1 - YE''_{mkkt}) \quad \forall p, m, k, t \] (20)

Similar to Constraints (13, 14), Constraint (19) and (20) figure out the loaded quantity stayed on fleet after visiting the first buyer.

\[ L_{pmkt} \geq L_{pmk} - QED''_{pmkkt} + QEP''_{pmkkt} - U \cdot (1 - YE''_{mkkt}) \quad \forall p, m, k, k'(k \neq k'), t \] (21)

\[ L_{pmkt} \leq L_{pmk} - QED''_{pmkkt} + QEP''_{pmkkt} + U \cdot (1 - YE''_{mkkt}) \quad \forall p, m, k, k'(k \neq k'), t \] (22)

Constraints (21) and (22) are similar to constraints (15, 16), compute the rest of the quantity being loaded on fleet after visiting the buyers.

\[ L_{pmkt} \leq U \left( YE'_{mk} + \sum_{k' \neq k} YE''_{mkkt} \right) \quad \forall p, m, k, t \] (23)

Similar to limitation (17), limit (23) declares the condition of the loaded quantities for the fleet being responsible for the warehouse.

\[ YW_{njy} + \sum_{j' \neq j} YW_{njy} = \sum_{j' \neq j} YW_{njy} + ZW_{njt} \quad \forall n, j, t \] (24)

\[ YE''_{mkkt} + \sum_{k' \neq k} YE''_{mkkt} = \sum_{k' \neq k} YE''_{mkkt} + ZE_{mkkt} \quad \forall m, k, t \] (25)
\[ \sum_{j}^{Y W} \leq 1 \quad \forall n, t \]  
\[ \sum_{k}^{Y E} + Y_m \leq 1 \quad \forall m, t \]  
\[ \sum_{j \neq j}^{Y W} + ZW_{njt} \leq 1 \quad \forall n, j, t \]  
\[ \sum_{k}^{Y E} + ZE_{mkt} \leq 1 \quad \forall m, k, t \]  
\[ \sum_{j=k}^{Y W} \leq U \sum_{j}^{Y W'} \quad \forall n, t \]  
\[ \sum_{k \neq k}^{Y E} \leq U \sum_{k}^{Y E'} \quad \forall m, t \]

Limitations (24)-(31) elaborate on both sub-tours and routing constraints.

\[ INV_{pt}^{'} = \sum_{m}^{QD_{pmt}} + INV_{p(t-1)}^{'} - \sum_{j \neq j}^{\sum_{j}^{QW_{pjt}}} - \sum_{j \neq j}^{\sum_{j}^{QW_{pj}}_i} \quad \forall p, t > 1 \]  
\[ INV_{pt}^{'} = \sum_{m}^{QD_{pmt}} - \sum_{j \neq j}^{\sum_{j}^{QWD_{pjt}}} - \sum_{j \neq j}^{\sum_{j}^{QWD_{pj}}_i} \quad \forall p, t = 1 \]  
\[ INV_{pt} = INV_{p(t-1)} + AP_{pt} - \sum_{m}^{QD_{pmt}} - \sum_{k \neq k}^{\sum_{k}^{QED_{pmkt}}} - \sum_{k \neq k}^{\sum_{k}^{QED_{pmk}}_i} \quad \forall p, t > 1 \]  
\[ INV_{pt} = AP_{pt} - \sum_{m}^{QD_{pmt}} - \sum_{k \neq k}^{\sum_{k}^{QED_{pmkt}}} - \sum_{k \neq k}^{\sum_{k}^{QED_{pmk}}_i} \quad \forall p, t = 1 \]

Limitation (32 - 35) calculate the amount of inventory in the warehouse and DC in the planning horizon. Have been presumed at the beginning of the planning horizon, depots have no commodities.

\[ LS_{pjt}^{'} = D_{pjt}^{'} - \sum_{j \neq j}^{\sum_{j}^{QWD_{pjt}}} - \sum_{j \neq j}^{\sum_{j}^{QWD_{pj}}_t} \quad \forall p, j, t \]  
\[ LS_{pkt}^{'} = D_{pkt}^{'} - \sum_{k \neq k}^{\sum_{k}^{QED_{pmkt}}} - \sum_{k \neq k}^{\sum_{k}^{QED_{pmk}}_t} \quad \forall p, k, t \]

Constraints (36) and (37) figure out the number of lost sales occurring for buyers assigned the warehouse and DC.
Constraints (38) and (39) calculate lost pick-up amounts for each buyer in the planning horizon allocated to DC and the warehouse.

\[
LP = P - \sum_n QWP_{pjt} - \sum_j QWP'_{pjt} \quad \forall p, j, t \tag{38}
\]

\[
LP = P - \sum_m QEP_{pmkt} - \sum_k QEP'_{pmk't} \quad \forall p, k, t \tag{39}
\]

Equation (43) calculates the number of products which is produced by the factory.

Since the proposed model is nonlinear due to multiplying of decision variables in Constraints (12) and (18), it is necessary to convert it into a linear counterpart. Therefore, the following auxiliary variables \( bE'_{pmkt} \), \( bE'_{pmk'k} \), \( bW'_{pjt} \) and \( bW'_{pjtj} \) are defined and Constraints (44)-(51) are added to the main model:

\[
bE'_{pmkt} \leq (1 - YE'_{mk't}) \cdot U \quad \forall p, m, k, t \tag{44}
\]

\[
QED'_{pmkt} \leq U \cdot YE'_{mk't} \quad \forall p, m, k, t \tag{45}
\]

\[
bE''_{pmk'k} \leq (1 - YE''_{mk'k}) \cdot U \quad \forall p, m, k, k' (k \neq k'), t \tag{46}
\]

\[
QED''_{pmk'k} \leq U \cdot YE''_{mk'k} \quad \forall p, m, k, k' (k \neq k'), t \tag{47}
\]

\[
bW'_{pjt} \leq (1 - YW'_{pjt}) \cdot U \quad \forall p, n, j, t \tag{48}
\]
\[ QWD_{pnj}^{'} \leq U \cdot YW_{nj}^{'} \quad \forall p, n, j, t \tag{49} \]

\[ bW_{pnj}^{'} - (1 - YW_{nj}^{''}) \cdot U \leq QWD_{pnj}^{'} \leq bW_{pnj}^{''} - (1 - YW_{nj}^{''}) \cdot \forall p, n, j, j' (j \neq j') , t \tag{50} \]

\[ QWD_{pnj}^{'} \leq U \cdot YW_{nj}^{''} \quad \forall p, n, j, j', t \tag{51} \]

Considering Constraints (44)-(51), Constraints (12) and (18) can be rewritten as follows:

\[ L_{0pn} = \sum_{j} bW_{pnj}^{'} + \sum_{j \neq j'} bW_{pnj'}^{''} \quad \forall p, n, t \tag{52} \]

\[ L_{0pmk} = \sum_{k} bE_{pmk}^{'} + \sum_{k \neq k'} bE_{pmk'}^{''} \quad \forall p, m, t \tag{53} \]

### 3. A case study and sensitivity analysis

In this part, is presented an instance for a case study to validate the proposed model. A newly-established factory in the northern region of Iran has been considered as the case study. Fig. 5 illustrates the location of the related case study. As it is shown, \( D_1 \) is the location of the factory and its warehouse being responsible for saving and maintaining raw material and commodities. Besides, this point transfer commodities to DC and some buyer assigned to the warehouse. In addition, \( D_2 \) is a place that dispatches commodities to the buyers which are assigned to DC. Moreover, DC receives picked-up goods from those customers. Circl determine boundary of customers that related to each depot. In this model, the aim is to convince the request of buyers who purchase in bulk from the factory, such as big department restaurants and etc., to figure out an optimal output for two-week in the planning horizon.

Model validation is a critically important step in deploying the real world models. To obtain the appropriate result, at first we designed a mathematical model based on some assumptions inspired from a real case and then solved it by CPLEX solver to obtain optimal solutions. Then, we have examined the results to make sure the accuracy of the model.

Actually, to make the solution analysis easier, a small illustrative example as first trial is extracted from the real case by reducing problem size. In first trial, there are only five customers from east of Mazandaran province with numbers 21 to 25, and five from the west of Mazandaran province with numbers 1 to 5. As an example, the total quantity of pick-up and delivery for all buyers in east and west during two days (1 and 11) is presented in Table 1. Three and two homogeneous transporting vehicles with freight capacity of 30 units are considered for the warehouse and DC, respectively. Also, the price of first and second commodity are 100 and 2000 monetary units.

Fig. 6 details the shipping of the fleet and conveyed deliveries and picks-up by the fleet. The demand quantity of delivery and pick-up defined by each customer shown respectively with sings \( D \)
(first commodity, second commodity) and \( P \) (first commodity, second commodity) with black color. Also, the actual amount of delivery and pick up transported by a vehicle for a customer is shown by signs \( D \) (first commodity, second commodity) and \( P \) (first commodity, second commodity) with red color and purple color for tenth and eleventh days, respectively.

\{\text{Fig. 5 (a)}\}
\{\text{Fig. 5(b)}\}
\{\text{Table 1}\}

The obtained results of the model solution are described by Figs. 6 - 7 depicting a schematic of how to allocate vehicles to transportation, the fleet shipping, and the quantity of delivery, pick-up to or from each buyer. Also, the forthcoming table (Table 2) shows the amount of the objective function.

\{\text{Fig. 6.}\}
\{\text{Fig. 7.}\}
\{\text{Table 2}\}

As can be seen from Figs. 6 - 7, the first category of Constraints (3-7) as well as (12), (17), (18), and (23) are satisfied so that if the fleet moves from the warehouse to DC or the buyers, it can dispatch or pick up the commodities to or from that DC or the buyers.

The second category of Constraints (8-11) and (12-23) do not let the fleet departing from the warehouse carries more than its peak volume. Similarly, the amount of load delivered and picked up at the time of meeting the customer is so that after leaving that customer, the loaded quantity on the fleet is less than its peak volume. For instance, when a fleet departs from the warehouse towards the buyer 21 on the first day, it has loaded 30 quantity being equal to its peak volume and delivers 10 quantity to the range of customers from 21 to 23. When it meets customer 21, it delivers 5 quantity of the first commodity and 5 quantity of the second commodity. Afterward, it picks up 1 quantity of each commodity as items to be picked up and returned to the factory. The fleet is transferring 22 quantity of commodities just departing from buyer 21, being less than its volume. Another category of limitation (24-31) as shipping ones act appropriately prevents sub-tours to occur.

On the start day, we encounter 20 units of deficiency for product of type 2 demanded by customers 2, 3, 4, and 5 allocated to warehouse but on the eleventh day the whole of requests are satisfied. The reason is that, at the first day, there were 50 demands from the customers assigned to DC and there is only one vehicle having the volume of 30 quantity for delivering the commodities from the factory to DC. As a result, there is 20 units deficiency.

Now, two important questions arise: First of all, why does the solution provide commodities for the farther buyers like fourth and fifth buyer rather than convincing the request of nearby ones (e.g., the second commodity for the second and third buyer)? Does it occur for diverse transportation expenses in the objective function? Equally importantly, if there were more fleets moving from the factory to DC, will the buyers which are allocated to the warehouse face any commodity shortage?

The first question's answer is that due to the variable production costs expressed in the objective function, and considering the fact that the variable production costs for the first and second product is 100 and 2000 units respectively, the model comes to the conclusion that the production costs
outweigh transportation costs for the second product according to the constraints of fleet volume in the first day and comparing the floating shipping expenses to the manufacture ones, as well as considering lost sales being in the second objective function which should be minimized so costs are less in this situation. Therefore, the production of product 1 is more affordable and less expensive.

To validate such an assertion moreover to be aware of the model accurate performance, is supposed that the value of the second commodity as the same as the first one (2000 units) and solve the first trial again. The results have only changed in the first day as shown in Fig. 8.

As shown in Fig. 8, by considering the prices of two commodities equal and according to the expenses in the objective function, the mathematical model decides to give no services to the customers 4 and 5, instead to meet all demands of buyer 2 and 3. When the costs of production and inventory holding are equal for both products, based on transportation costs, service is considered for the nearby buyers.

For answering the second question, we have observed in the first trial that due to volume constraints and number of shipping fleet of the warehouse, we have encountered product deficiency in the customer assigned to DC.

In the second trial, an increase in the number of fleets assigned to the warehouse from three to four to be convinced the model accuracy. So we expect that with this growth we do not face any deficiency. Table 3 illustrates the second trial with its parameters.

Table 3 - 7 include the whole parameters that are considered to present a case study. Table 5 represents the space among the buyers. Table 6 shows the space among the factory and buyers.
moreover, Table 7 depicts the amount of demand by each buyer on the second day. It is worth mentioning that all pack of first and second products includes 250 and 24 units with the same volume, respectively.

Next, the routes traveled by the transportation vehicles in the second day of planning horizon in a city in the north of Iran (Qaemshahr) are depicted in Fig. 10.

Table 7
Fig. 10.
Table 8

The paths of transportation vehicles are depicted in Fig. 10. The dotted path illustrates the travel from the warehouse to the customers, and the solid line depicts the travel of transportation fleet when it leaves the last customer and returns to the warehouse. Table 8 elaborates on the fleet movement.

In Table 8, the buyers whose requests have not been satisfied totally by the corresponding fleet belong to the deficient buyers’ column. Moreover, when the request of the buyer has been covered by more than one fleet, the buyer belongs to the shared buyers’ column. For instance, as shown in Table 8 fleet C and E are responsible for covering the sixth buyer.

Fig. 11 illustrates the obtained Pareto graph.

As mentioned in the introduction, the mean of shipping expenses in Iran is between 1.7 and 2 times that of the international average base on the report of the Iran Chamber of Commerce Industries, Mines, and Agriculture and the World Bank. Transportation, inventory, and management cost which this paper focus on these relate to logistic cost. The outputs illustrated when managers are going to gain better efficiency of managing routing planning from the daily operations in all over the supply chain, they can count on the developed model for this model is a practical appliance for decision-makers in making operational decisions and tactical. In addition, the outcome of Pareto solution helps the manager to decide better choices in different situations and a trade-off between costs and lost sales in various conditions.

In this section, the effect of split delivery on the model performance is evaluated through addition of limitations (54)-(55) that causes each customer is visited at most by only vehicle in one day of planning horizon and leads to omitting of split delivery feature.

For more simplicity, only the demand for the products in one day of planning horizon is considered.

\[ \sum_{k} Y_{mk}^{r} + \sum_{m} \sum_{k} Y_{mk'}^{n} \leq 1 \quad \forall k, t \]  

(54)
Now, we compare the model performance in two VRP and SDVRP states with five different scenarios. In the third scenario, the whole quantity of delivery to the customers is exactly matching to total capacity of vehicles. In the first, second, fourth, and fifth scenario, the demands are considered to 0.8, 0.9, 1.1 and 1.2 of total vehicles capacity, respectively. Keep in mind that the fleet capacity in all scenarios is 90 units.

\[
\sum_{j} Y_{n}W_{nj} + \sum_{n} \sum_{j \neq j'} Y_{n}W_{nj'} \leq 1, \quad \forall j, t
\]  \hspace{1cm} (55)

The results obtained with different scenarios for SDVRP and VRP represented in Table 9 are depicted by Figs. 12-17.

As it can be seen from Fig. 12 and was predictable in advance, the value of shortage of demand for products delivery increases in VRP state compared to SDVRP. The relative gap of products shortages in VRP and SDVRP states are shown in Fig. 13. In the third scenario, where the customers demand is matching to the peak volume of fleet, the highest gap happens.

Total costs of VRP and SDVRP states for five scenario is presented in Fig. 14. This figure shows that in SDVRP state, due to more effective utilization of transportation vehicles and higher production level for better services to the customers which results in decreasing the amount of lost sales, supply chain costs increase. The relative gap of costs for each scenario is given in Fig. 15.

In Fig. 16, the unit cost per delivered product is shown by splitting the whole costs of supply chain to the quantity of products delivered to the customers. In the third scenario, total costs in SDVRP state is about 7% more than VRP one, whereas, according to Fig. 17, unit cost of product delivered to customers in SDVRP state is 7% less than VRP one. To conclude, it reveals that SDVRP results in more demand satisfaction with less unit cost per delivered products.

4. Conclusions

The main goal of the current study was to determine the model focusing on the supply chain network issues by utilizing the mixed-integer programming approach. The objectives were to
minimize both the whole expenses consisting transportation, production and inventory as well as the lost sales. An \( \varepsilon \)-constraint approach was used to work out the mathematical model by GAMS software. Then, the model was analyzed to ensure the validation of model. A real case in an Iranian food industry was placed to verify the results and presented the vehicle routings in a day.

Another purpose which was considered for this paper, was to investigate the performance of the considered supply chain and its productivity. In the small-sized instances, by changing the parameters, five scenarios are designed to compare the performance of the model in both VRP and SDVRP states. As a result, in all scenarios, in case of SDVRP, lost sale is reduced, while by using just traditional VRP, total cost is decreased. Besides, the results have shown that unit cost per delivered product for each scenario in SDVRP is less than or equal to VRP. The case study’s outcomes prove the absolute efficiency of employing the SDVRP in comparison with the VRP. Indeed, unit cost of product delivered to customers in SDVRP state is 7\% less than VRP one. The outcomes depicted when managers are going to gain better efficiency of managing routing planning from the daily operations in all over the supply chain, they can count on the developed model for this model is a practical appliance for decision-makers in making operational decisions and tactical.

For future studies, comprehensive analyses can be evaluated by some other techniques employing for large scales such as metaheuristic algorithms. In this regard, using the mentioned model in common practical like fresh fruits, food and etc. can be one of the research space for the future studies. Besides, some real constraints can be employed to expand the mathematical model, such as, stochastic models with uncertain parameters, cross-dock scheduling and etc.

References


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Captions of figure

Fig. 1. Logistic costs.

Fig. 2. Percent of the price of finished product.

Fig. 3. Percent of the price of finished product.

Fig. 4. The schematic flow of material.

Fig. 5 (a, b). The span of implementation of the case study.

Fig. 6. The graphic solution on the first day for the first trial.

Fig. 7. The graphic solution on tenth and eleventh day for the first trial.

Fig. 8. The graphic solution after an alteration in the value of the first commodity on the first day.

Fig. 9. The graphic solution on the first day for the second trial.

Fig. 10. The fleet shipping in Qaemshahr on the second day.

Fig. 11. Pareto front for case study.

Fig. 12. The amount of lost sales for five scenarios.

Fig. 13. The relative gap of load deficiency delivered to customers between Vehicle Routing Problem (VRP) and Split Delivery Vehicle Routing Problem (SDVRP) states for five scenario.

Fig. 14. Total costs for five scenario.

Fig. 15. The relative gap of total costs between Vehicle Routing Problem (VRP) and Split Delivery Vehicle Routing Problem (SDVRP) states for five scenario.

Fig. 16. Unit cost per delivered product for each scenario.
The relative gap of unit costs per product delivered to customers between Vehicle Routing Problem (VRP) and Split Delivery Vehicle Routing Problem (SDVRP) states.

Captions of table

Table 1. The whole quantity of delivery and pick-up for the First trial.
Table 2. The outcome of two objective function for the first trial.
Table 3. Second trial’s data.
Table 4. The outcome of two objective function for the second trial.
Table 5. Distances between the customers.
Table 6. The distance between the factory and the buyers allocated to the warehouse.
Table 7. Buyers’ demands.
Table 8. The detail of fleet shipping in the Qaemshahr on the second day.
Table 9. The obtained results with different scenarios for Split Delivery Vehicle Routing Problem (SDVRP) and Vehicle Routing Problem (VRP) states.

Figures:
Fig. 2

![Bar Chart]

GLOBAL AVERAGE

IRAN

Logistic Costs

Other Costs

Fig. 3

![Diagram]

Fig. 4
Fig. 5 (a).

Fig. 5 (b).
Fig. 6.

Fig. 7.
Fig. 8.

Fig. 9.
Fig. 10.

Fig. 11.
Tables:

Table 1

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|                  | 4 homogeneous shipping fleet                    |
|                  | (The tonnage of each vehicle is 30 units)       |
| Warehouse        | 2 homogeneous shipping fleet                    |
|                  | (The tonnage of each vehicle is 30 units)       |

|                  | Value of first commodity, 100 units             |
|                  | Value of second commodity, 2000 units           |

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