Bridge backwater estimation: A Comparison between artificial intelligence models and explicit equations

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Abstract

Estimation of bridge backwater has been one of practical challenges in hydraulic engineering for decades. In this study, Genetic Programming (GP) has been applied for estimating bridge backwater for the first time based on the conducted literature review. Furthermore, two new explicit equations are developed for predicting bridge afflux using Genetic Algorithm (GA) and hybrid MHBMO-GRG algorithm. The performances of these models are compared with Artificial Neural Network (ANN) and several explicit equations available in the literature considering both laboratory and field data. Based on five considered performance evaluation criteria, the two new explicit equations outperform the ones available in the literature. Furthermore, GP and ANN achieve the best results in favor of four out of five considered criteria for train and test data, respectively. To be more specific, ANN improves the MSE and R^2 values of the explicit equation developed using GA by 44% and 12% for the test data while GP enhances the corresponding values by 62% and 9% for the train data. Finally, the results demonstrate that not only artificial intelligence models considerably improve bridge afflux estimation than the explicit equations but also the suggested equations significantly improve the accuracy of the available explicit ones.

Keywords: Hydraulic structures, bridge backwater estimation, Genetic Programming, explicit equation, Artificial Neural Network.

1 Introduction

Bridges are inevitable useful structures across waterways that connect river sides together for transfer purposes. However, they may confine natural space for water flowing in rivers. Particularly, water surface at the upstream of bridges built over rivers may rise more than its normal limit due to the provided confinement during flood event. This increase of water level with respect to normal water depth is invariantly called bridge afflux or backwater.

Since bridge backwater has important impacts on flood defense schemes and river planning and management projects, searching for better estimation of this phenomenon has been studied for many decades. These efforts can be classified from different facets.

From the type of bridge, they may be grouped as contributions focusing on bridges with either horizontal soffit [1] or arch deck [2-4]. From methodological point of view, some researches applied numerical [5-6], experimental [7-8], and data mining approaches [3, 9-11] to predict bridge backwater. Furthermore, it should be noted that most of methods estimating bridge afflux have been basically developed based on field or laboratory datasets. The numerical efforts may be classified into different kinds of methods to predict bridge backwater including (1) energy method [12-14], (2) momentum method [15], (3) WSPRO [16], (4) Yarnell's method [17], (5) HR method [18], and (6) USBPR method [19]. Although some of these numerical models have been implemented in numerical software simulating rivers and hydraulic structures, like ISIS package [20] and HEC-RAS [12-16], the need for calibration, relatively excessive data requirement, and applicable under certain specified assumptions may be counted as their shortcomings for estimating backwater for different types of bridges.

Since the main focus of this study is estimating backwater for arched bridges, the relevant contributions are particularly reviewed herein. Among studies conducted for this type of bridge, Biery and Delleur's [2] has been the most well-known one. They investigated backwater analysis for single-span arch bridges and recommended an explicit empirical correlation which attributes bridge afflux to Froude number and opening ratio. Brown [18] not only conducted many experimental for different kinds of bridge constrictions but also collected various bridge backwater field data from 1946 to

1983. This contribution yields to HR method, which estimates backwater for arch bridges using normal depth, Froude number, and blockage ratio. Later, this method has been implemented as one of bridge subroutine approaches in the ISIS package program [20]. Although explicit equations have been commonly used for estimating many variables in various water resources applications [21-25], they should be exploited only when their background assumption(s) and valid ranges are applicable. In this regard, the limitations of the available explicit equations for predicting backwater depth include (1) they were not developed by powerful algorithms and (2) their accuracy is not enough to be applied in professional software for analyzing rivers and designing of hydraulic structures. In order to enhance the accuracy of backwater estimation, artificial intelligence (AI) models including radial basis function based neural network (RBNN), multi-layer perceptron (MLP), generalized regression neural networks (GRNN), and adaptive neuro-fuzzy inference system (ANFIS) have been utilized for this purpose using experimental and field databases [3-4, 10-11]. In spite of all these studies, an accurate explicit equation, which takes all involving variables into account and developed by powerful algorithms like GA, has not been proposed based on the current literature. Furthermore, even though several AI models like ANN have been applied to estimate afflux for arched bridges, Genetic programming, which has been successfully used for solving other water-related problems [23, 26-27], has not utilized for this purpose. Since better bridge backwater estimation can yield to more sustainable and reliable schemes for bridge safety during flood events, not only developing new accuracy-improved explicit equations using powerful optimization algorithms but also applying other powerful data mining approaches are still required.

In this paper, two new explicit equations are developed for estimating bridge afflux using Genetic Algorithm (GA) and Modified Honey Bee Mating Optimization-Generalized Reduced Gradient (MHBMO-GRG) Algorithm. These algorithms have been successfully utilized for solving various civil and water engineering problems in the literature [21-22, 28-30] while it is the first time that they have been applied to estimate bridge backwater depth to the authors' knowledge. Moreover, Genetic Programming (GP) has been used for similar purpose for the first time in the literature. The performances of these three models are compared with those of Artificial Neural Network (ANN) and three other explicit equations available in the literature. The results obtained for the afflux prediction using a reliable database demonstrate that new models improve the estimation.

2 Bridge backwater problems

In order to better illustrate this phenomenon, a schematic situation of bridge backwater is depicted in Figure 1. As shown, water suitably flows along the river and under the bridge with normal depth. However, a rise in water surface level may be caused due to bridge constriction during flood events. In Figure 1, dh denotes bridge backwater, and D_1 and D_3 represent normal flow depth at sections 1 and 3, respectively.

According to previous studies [3, 8-10, 31], four parameters have the most significant impacts on bridge backwater. These parameters are (1) the ratio of blockage area of bridge at depth D_1 to flow area at section 1 upstream of bridge (J_1), (2) the ratio of blockage area of bridge at depth D_3 to flow area at section 3 downstream of bridge (J_3), (3) the Froude number at section 3 (F_3), and (4) downstream normal depth (D_3). With the aid of these parameters, bridge afflux, i.e., dh, can be evaluated for arched bridge constrictions in rivers. By applying dimensionless analysis, the bridge backwater can be determined using the following function:

$$\frac{dh}{D_3} = F(J_1, J_3, F_3) \tag{1}$$

where F denotes function.

Three empirical formulas for bridge backwater available in the literature include Biery and Delleur [2], multiple linear regressions (MLR) [3], and multiple nonlinear regressions (MLNR) [3]. The corresponding equations are shown in Eq. 2 to Eq. 4, respectively.

$$\frac{dh}{D_3} = 0.47 \times \left(\frac{F_3}{1 - J_3}\right)^{2.26} \tag{2}$$

$$\frac{dh}{D_3} = 1.62 \times J_1 - 1.54 \times J_3 + 0.429 \times F_3 \quad (3)$$

$$\frac{dh}{D_3} = 1.311 \frac{J_1^{1.8} F_3^{1.23}}{J_3^{0.744}} \quad (4)$$

Among these explicit equations, Biery and Delleur's [2] equation is the best-known empirical formula for bridge backwater. As shown in Eq. 2, the impact of J_1 is not considered in this equation whereas it should be considered as in the general function shown in Eq. 1. According to Biery and Delleur's [2] formula, bridge backwater increases with the increase of Froude number at downstream section and reduces with the reduction of J_3. On the other hand, Eq. 3 and Eq. 4 consider all the independent parameters involved in bridge backwater. The main difference between Eq. 3 and Eq. 4 is that the former is linear while the latter considers nonlinear relationship between involved parameters.

3 Methods and materials

In this section, first the data considered for bridge backwater is introduced. Second, the proposed empirical models are presented. Finally, different methods utilized for estimating backwater depth in this study are briefly described.

3.1 Bridge backwater database

As it was previously mentioned, most of available methods for backwater estimation have been developed using laboratory and/or field data. Likewise, the data considered in this study comprises both experimental and field data sets. The former data was originally pertinent to Hydraulic Research Wallingford experiments conducted regarding investigations on backwater estimation while the latter includes 66 filed data observed between 1946 and 1983 [18]. Moreover, the 202-laboratory data were carried out on two different rectangular flumes incorporating different types of arched bridges. The ranges of J_1, J_3, F_3, and dh/D_3 of these data are summarized in Table 1. Moreover, the variations of dh/D_3 with J_1, J_3, and F_3 are depicted in Figure 2. As shown, most data points in this database have F_3 lower than 0.75 and similar J_1 and J_3 values. Also, Figure 2 depicts that the values of dimensionless bridge backwater depth (dh/D_3) are lower than 0.78 for most of data. Since this database has been already utilized in several studies [3, 10, 18], it is technically reliable for developing accuracy-improved equations for estimating bridge backwater.

3.2 Proposed models for explicit backwater estimation

According to the presented literature review, explicit equation with high precision is still required for estimating backwater occurred in arched bridges. In this regard, after trying many different functions for determining a qualified formula, a new structure for explicit equation is obtained and proposed for afflux estimation in this study. This model, which has a simple structure, is shown as the followings:

$$\frac{dh}{D_3} = \begin{cases} p_1 J_1^{p_2} J_3^{p_3} F_3^{p_4} & for \quad F_3 < p_9 \\ p_5 J_1^{p_6} J_3^{p_7} F_3^{p_8} & for \quad F_3 \geq p_9 \end{cases} \tag{5}$$

where p_i for $i = 1,2,\ldots,9$ are the unknown coefficients of the new proposed model.

As shown in Eq. 5, the proposed model, which has nine coefficients required to be calibrated, divides the whole dataset into two parts. This division is based on the value of the Froude number of the downstream section (F_3). In each part, bridge backwater may be calculated using a simple nonlinear equation obtained by multiplying all involving parameters. All the coefficients of the new proposed model were optimized based on the field and laboratory database. Unlike Biery and Delleur's [2] model (Eq. 2), the proposed model considers all the involved parameters for estimating bridge backwater.

3.3 Artificial Neural Network

The theory of Artificial Neural Network has been presented in many references and has been applied to various problems in different fields, particularly hydraulic and water resources engineering [27, 32-34]. A typical ANN includes some elements invariantly called neurons grouped in layers. The neurons in the input layer take a vector, which consists of input data. These neurons play the role of transmitting the values to the next layer across connections. Each neuron in a layer is connected to all the neurons of the next layer and not connected among them. The data flow through these connections from one neuron to another is multiplied by weights, which control the strength of a passing signal. In the feed-forward network, the data process continues when the output layer is grasped while the data flows exclusively in one direction. However, the Feed Forward Back Propagation (FFBP) utilizes one or more hidden layers while the neurons in this layer intervene between the external input and the network output in favor of improving the network performance.

The data utilized in this study for backwater depth estimation have three input parameters (J_1, J_3, and F_3) and one output parameter (dh/D_3). The considered network has input, output, and hidden layers, which consist of three, ten, and one neurons, respectively. The Levenberg Marquardt optimization technique is used to train the FFBP. According to the literature review, ANN has been already applied for estimating bridge backwater while it is also used here for comparison purposes.

3.4 Genetic Algorithm

GA is a zero-order search-based optimization algorithm that mimics the mechanism of natural selection and evolution. This algorithm has been widely used for numerous applications [22]. Three genetic operators, so called crossover, mutation and selection, are commonly utilized in GA to create new combinations of variables in light of producing better solutions. The crossover operator not only exchanges genetic information among selected population members but also combines the information of selected parents to form new strings. This combination is conducted probabilistically using a swapping process. On the other hand, the mutation operator allows and maintains the diversity of genetic information by changing individual population characteristics randomly. Finally, the selection operator, as its name indicates, chooses between solutions.

In this study, a population size equal to 300 is adopted. Since the proposed model for bridge backwater has nine unknown coefficients, GA was used to calibrate these nine decision variables.

3.5 Genetic Programming

Genetic Programming, which is a random search heuristic method very similar to GA, operates on parse trees whereas GA considers bit strings. In essence, this technique applies a wide range of variables and functions in a flexible tree-structure base. This inevitably makes GP become a powerful tool in favor of finding existing relations that best fit the relation between the input and output variables of a system.

The main steps of GP include initialization, selection, reproduction, and termination. First, it considers an initial population, which consists of randomly generated programs (equations). These programs are basically derived from the random combination of input variables, random numbers and functions. The considered population is subjected to an evolutionary process to find and select individual programs that best describes the relation between the input and output variables. The part of the information between the selected programs is exchanged to create better programs using genetic operators (crossover and mutation). This evolution process is repeated over consecutive generations until symbolic expressions describing the data are reached. Further details on GP may be found in Koza [35] for interested readers. Discipulus [36] software, which has been used for implementing GP in many studies [23, 26-27], was applied in this research.

3.6 The hybrid MHBMO-GRG algorithm

The hybrid MHBMO-GRG algorithm was first suggested by Niazkar and Afzali [28] and has been successfully applied for solving some problems in water engineering field [21-22, 28-30]. This hybrid algorithm comprises search-based and deterministic optimization algorithms used in two consecutive steps.

In the first step, a zero-order optimization algorithm, so called Honey bee mating optimization (MHBMO) algorithm, commences the optimization process after evaluating the controlling parameters of the algorithm. Technically, this algorithm inspires from the mating process of honey bees. The basic steps of the MHBMO algorithm includes: (1) starting mating flight, where a queen (best solution) probabilistically selects drones to create broods, (2) making new broods (trial solutions), (3) conducting local search on new broods (trial solutions) by workers, (4) improving workers fitness, and (5) finding the queen for the next generation by comparing the queen and the best brood [37-39]. The detail of this algorithm is comprehensively presented in the literature [40-41] for interested readers. In this hybrid method, the MHBMO algorithm precedes the optimization process for several numbers of iterations and results of the last iteration will be used as initial guesses for the next step.

In the second step, the problem is implemented in Excel spreadsheet. Afterwards, the GRG algorithm, which is a first-order optimization technique, was utilized for continuing optimization process. This algorithm is one of the features embedded in Excel [42-44]. Although the MHBMO algorithm requires evaluation of five controlling algorithm parameters, the GRG algorithm needs a set of initial guesses, which are the results obtained in the last iteration of the first step in this hybrid method. The final optimum results using the GRG algorithm depend on initial guesses. Therefore, the possibility of achieving a local optimum and initial guess requirement are the substantial shortcomings of the MHBMO and GRG algorithms, respectively. However, the MHBMO-GRG hybrid method overcomes these drawbacks. According to the successful experience of applying this hybrid method for solving several problems [21-22, 28-30], it should be mentioned that not only the new hybrid method enhances the applicability of the MHBMO algorithm in finding global optimum values, but also it adequately provides a set of initial guesses for the GRG algorithm.

4 Application and Results

4.1 Train and test data

First of all, the considered database was randomly divided into the train and test data [21]. Since a generic equation is sought for bridge backwater, both experimental and field data are treated the same in this data-splitting process, as those conducted in the relevant studies [3-4, 10]. As previously mentioned, the utilized database has 202 and 66 laboratory and field data, respectively. From the total 268 data, 80% of the total data (161 experimental and 50 field data) were considered for calibrating the coefficients of new formula while the rest (41 experimental and 16 field data) were kept for comparison purpose. The maximum, minimum, and average values of the involving parameters and dh/D_3 for the total, train, and test data are specified in Table 1. As shown, for all parameters, the maximum of the train is larger than the maximum of the test data and the minimum of the train is lower than the minimum of the test data. In other words, the range of the train data is wider than that of the test data, which clearly indicates that a reasonable and adequate data division has been conducted. Furthermore, Table 1 demonstrates that the new proposed model is applicable for downstream Froude number varying between 0.0075 and 1.8089.

4.2 Proposed models

In this study, two artificial techniques (ANN and GP) and two optimization algorithms (GA and MHBMO-GRG) are calibrated using the train data. To be more specific, the unknown coefficients of the explicit proposed model (Eq. 5) were optimized based on the train data using GA and MHBMO-GRG. On the other hand, ANN and GP estimates bridge backwater without using the proposed model. In this regard, the same objective function are used in the training process for all these models. It is defined as minimizing the root mean square error (RMSE) between the computed and observed ratio of afflux to downstream depth (dh/D_3). The sole constraint of this optimization process was preventing bridge backwater to gain a negative value. The objective function and the sole constraint are shown in Eq. 6 and Eq. 7, respectively.

$$\mathrm{Minimize\,RMSE} = \sqrt{\frac{\sum_{i=1}^{N}[(\frac{dh}{D_3})_{i,observed} - (\frac{dh}{D_3})_{i,calculated}]^2}{N}} \tag{6}$$

$$\mathrm{Subjected\ to}\ \frac{dh}{D_3} > 0 \tag{7}$$

where $(\frac{dh}{D_3})_{i,observed}$ and $(\frac{dh}{D_3})_{i,calculated}$ are the observed and calculated ratio of backwater to downstream water depth for the i^{th} data point and N denotes the total number of training data.

As each data point was randomly allocated into either train or test data, the two artificial models (ANN and GP) are trained for the train data and afterwards applied to estimate test data. On the other hand, the two utilized optimization algorithms (GA and MHBMO-GRG algorithm) were applied to the train data to find the optimum values of p_i coefficients. In this regard, the controlling parameters in the MHBMO algorithm and GA were considered as the ones used in Niazkar and Afzali's [21] and Niazkar el al.'s [22] studies, respectively. As a result, the new explicit equations proposed for bridge backwater estimation using GA and MHBMO-GRG algorithm are introduced in Eq. 8 and Eq. 9, respectively. According to Eq. 8, the J_3 parameter has two different impacts on bridge backwater. To be more specific, bridge backwater increases with the decrease of J_3 for $F_3 < 0.2$ while it reduces with the decrease of J_3 for $F_3 \geq 0.2$. Therefore, J_3 can have positive or negative influence of bridge backwater based on the value of Froude number at the downstream. Additionally, the higher either J_1 or F_3 becomes, the larger bridge afflux may occur. On the other hand, Eq. 9 obtained by GA indicates that it is applicable for most of the considered data while it indicates that dh/D_3 increases with the increase each one of J_1, J_3 or F_3.

$$\frac{dh}{D_3} = \begin{cases} 2.274 J_1^{5.328} J_3^{-0.899} F_3^{0.596} & for \quad F_3 < 0.2 \\ 5.243 J_1^{1.102} J_3^{0.822} F_3^{1.523} & for \quad F_3 \geq 0.2 \end{cases} \tag{8}$$

$$\frac{dh}{D_3}=\begin{cases} 4.49J_1^{1.39}J_3^{0.514}F_3^{1.421} & for \quad F_3<1.179 \\ 4.946J_1^{1.519}J_3^{-0.267}F_3^{-3.242} & for \quad F_3\geq 1.179 \end{cases} \tag{9}$$

The confidence limits of GP, ANN, Eq. 8, and Eq. 9 computed for the measured and calculated dh/D_3 values are depicted in Figure 3. In Figure 3, the ranges of bridge backwater values calculated by each model are compared for the test data. As shown, the range of dh/D_3 values predicted by GP has the lowest minimum, average and maximum values among all the considered methods while the ranges achieved by ANN and Eq. 9 obtained by GA are very close to each other. Moreover, Figure 3 shows that the proposed explicit equations achieved by GA and MHBMO-GRG algorithm can be confidentially used for estimating bridge afflux for dh/D_3 placed within [0.165, 0.258] and [0.158, 0.252], respectively.

4.3 Performance evaluation criteria

In order to better compare the performances of different models for afflux estimation, five performance evaluation criteria are adopted from the literature [3, 10, 45-47]. These criteria include (1) mean square error (MSE), (2) mean absolute error (MAE), (3) mean Absolute Relative Error (MARE), and (4) average of individual ratios (AIR), and (5) coefficient of determination (R^2). These criteria are introduced in Eq. 10 to Eq. 14, respectively as follows:

$$\mathrm{MSE}=\frac{1}{N}\sum_{\mathrm{i=1}}^{\mathrm{N}}[(\frac{dh}{D_3})_{i,observed}-(\frac{dh}{D_3})_{i,calculated}]^2 \tag{10}$$

$$\mathrm{MAE}=\frac{1}{N}\sum_{i=1}^{N}|(\frac{dh}{D_3})_{i,observed}-(\frac{dh}{D_3})_{i,calculated}| \tag{11}$$

$$\mathrm{MARE}=\frac{1}{N}\sum_{i=1}^{N}|\frac{(\frac{dh}{D_3})_{i,observed}-(\frac{dh}{D_3})_{i,calculated}}{(\frac{dh}{D_3})_{i,observed}}|\times 100 \tag{12}$$

$$\mathrm{AIR}=\frac{1}{N}\sum_{i=1}^{N}[\frac{(\frac{dh}{D_3})_{i,observed}}{(\frac{dh}{D_3})_{i,calculated}}] \tag{13}$$

$$\mathrm{R}^2=(\frac{\sum_{\mathrm{i=1}}^{\mathrm{N}}[((\frac{dh}{D_3})_{i,observed}-\frac{\sum_{i=1}^{N}(\frac{dh}{D_3})_{i,observed}}{N})((\frac{dh}{D_3})_{i,calculated}-\frac{\sum_{i=1}^{N}(\frac{dh}{D_3})_{i,calculated}}{N})]}{\sqrt{\sum_{\mathrm{i=1}}^{\mathrm{N}}[((\frac{dh}{D_3})_{i,observed}-\frac{\sum_{i=1}^{N}(\frac{dh}{D_3})_{i,observed}}{N})^2((\frac{dh}{D_3})_{i,calculated}-\frac{\sum_{i=1}^{N}(\frac{dh}{D_3})_{i,calculated}}{N})^2]}})^2 \tag{14}$$

4.4 Comparison results

Using these five evaluation criteria, the performance of different models applied for afflux estimation is compared in Table 2 and Table 3 for the train and test data, respectively. As shown in Table 2, GP achieved the lowest (best) values of MSE, MAE, MARE and the largest (best) values of R^2 in comparison with other ones for the train data while ANN obtains the best value for AIR criterion in Table 2. Among explicit equations listed in Table 2, Eq. 9 determined by GA performs the best in favor of MSE, MAE, and R^2 while Eq. 8 developed by the MHBMO-GRG algorithm achieves the best MARE for the train data. According to Table 3, ANN obtains the lowest (best) values for MSE, MAE and the largest (best) value of R^2 while GP results the best MARE for the test data among all considered models. Furthermore, the new explicit equation calibrated using GA outperforms other ones in Table 3 with respect to MSE, MAE, MARE, and R^2 criteria. The results shown in Table 2 and Table 3 clearly demonstrate that application of available empirical equation, e.g., Biery and Delleur [2], may bring about significant errors in bridge backwater estimation. However, the new explicit equations may be an adequate alternative as they considerably improve afflux estimation in terms of four out of five considered criteria.

Figure 4 and Figure 5 depict measured vs. predicted dh/D_3 values calculated by GP, ANN, Eq. 8, and Eq. 9 for train and test data, respectively. As shown, R^2 values of these models can also be compared using these figures. Based on Figure 4 and Figure 5, GP and ANN achieve the best R^2 values for the train and test data, respectively. Moreover, Figure 6 compares the relative errors calculated by ANN, GP, GA and MHBMO-GRG algorithm for the whole data. As shown, all these four models yield to relative errors close to zero for most of data because most of points depicted in Figure 6 are placed near the horizontal line. According to Figure 6, ANN achieves two considerable negative relative error values whereas other models result to several large positive relative errors. This indicates that ANN significantly underestimates two data points while others overestimate bridge backwater for several data points in the considered data base. Finally, the maximum and minimum relative errors achieves by GA and ANN in Figure 6, respectively.

In order to determine the percentages of bridge afflux calculated within different error ranges and percentage deviations, the maximum, minimum, and average values of percentage deviations between the 45-degree line and measured and calculated bridge backwater are computed using all seven models and listed in Table 4. As shown, the average percentage deviation, i.e., $[\frac{(\frac{dh}{D_3})_{i,calculated} - (\frac{dh}{D_3})_{i,observed}}{(\frac{dh}{D_3})_{i,observed}}] \times 100$, for the test data using Biery and Delleur's [2] equation is -20.57% while the corresponding value calculated by the MHBMO-GRG algorithm is -3.81%. Furthermore, the latter one, which has the lowest average percentage deviation in Table 4, predicted more than 7% and 40% of the corresponding bridge backwater values within $\pm 5\%$ and $\pm 25\%$ error ranges, respectively. However, Biery and Delleur's [2] formula achieved more than 5% and 35% of afflux test data within $\pm 5\%$ and $\pm 25\%$ error ranges, respectively. This comparison also demonstrates that the suggested equations are capable of estimating bridge backwater values within better accuracy range than the available explicit ones.

Furthermore, GP predicts bridge backwater values with the highest percentage within $\pm 5\%$ and $\pm 25\%$ error ranges for the test data.

The explicit equations, like those developed by GA or the hybrid MHBMO-GRG algorithm in this study, do not require calibration process and may be applicable to whatever situation that fits within the ranges of the dataset used for its development. Therefore, these empirical equations may be implemented in river engineering software to estimate bridge backwater depth. Furthermore, AI models, like ANN and GP, give a better estimation of bride backwater depth than all the explicit equations while they need a training process using a reliable dataset. However, once they were trained, they could be exploited for scenarios whose parameter ranges place within the corresponding ranges of the training data. Therefore, the explicit equations and AI models has a valid range of application, which may be considered as their limitation or disadvantages. Obviously, the wider the range of the training data is the more situations they can be used for estimation of bridge afflux. As a result, the applied model not only performs much better than the available explicit formulas for estimating bridge afflux values but also they can be alternatively used in practice within its applicable range of validity.

5 Conclusions

Bridge backwater has been always an inevitable challenge for the safety and management of bridge and channels sides particularly during flood events. Consequently, the necessity for more accurate estimation of bridge afflux has provided an active field of research area in the hydraulic structure engineering field. According to the literature, various models have been recommended based on experimental and field data sets for this purpose. Additionally, the literature review reveals lack of availability of accurate explicit equations developed by applying powerful optimization algorithms like GA. Furthermore, even though several AI models have been already applied for predicting afflux for arched bridges, genetic programming is utilized for this purpose for the first time in this study to the authors' knowledge. In this paper, seven models including ANN, GP, GA, and hybrid MHBMO-GRG algorithm and three explicit equations available in the literature are compared for backwater prediction. The two optimization algorithms were used to develop two new accuracy-improved explicit equations. Comparing the performance of these seven models using five evaluation criteria obviously indicates that the new explicit equations outperform the ones available in the literature while GP and ANN perform as the two best models among all the considered ones. To be more specific, GP achieves R^2 value equal to 0.957 and 0.842 for the train and test data, respectively, while ANN obtains 0.922 and 0.887 for the corresponding criterion. Because of this precision improvement, the applied models may be confidentially altered with the current explicit ones available in the literature for backwater estimation within its range of applicability.

Conflict of interest: None

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List of Table captions:

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Table 1 Comparison of the four statistical criteria for different models

Parameters	J_1	J_3	F_3	dh/D_3
Total data				
maximum	0.803	0.742	1.809	1.805
minimum	0.064	0.047	0.008	0.002
average	0.452	0.385	0.367	0.253
Train data				
maximum	0.803	0.742	1.809	1.805
minimum	0.064	0.047	0.008	0.002
average	0.455	0.388	0.374	0.261
Test data				
maximum	0.746	0.678	1.021	0.685
minimum	0.099	0.097	0.053	0.008
average	0.440	0.374	0.340	0.223

Table 2 Comparison of the performance of different models for the train data

Methods	MSE	MAE	MARE	AIR	R^2
a. Explicit equations					
Biery and Delleur [2]	0.0806	0.103	56.6	3.143	0.559
MLR [3]	0.0498	0.1505	183	0.742	0.587
MLNR [3]	0.0806	0.1144	63	1.235	0.494
MHBMO-GRG (this study)	0.0358	0.0778	53.2	3.377	0.717
GA (this study)	0.0131	0.0724	63.4	1.507	0.876
b. Artificial intelligence models					
GP (this study)	0.0049	0.0346	31	2.079	0.957
ANN	0.0083	0.0427	52.3	1.019	0.922

Table 3 Comparison of the performance of different models for the test data

Methods	MSE	MAE	MARE	AIR	R^2
a. Explicit equations					
Biery and Delleur [2]	0.0140	0.0800	39.9	3.296	0.685
MLR [3]	0.0177	0.1156	123	0.689	0.610
MLNR [3]	0.0138	0.0804	39	1.337	0.669
MHBMO-GRG (this study)	0.0083	0.0637	45.5	4.582	0.782
GA (this study)	0.0077	0.0621	41.1	1.456	0.792
b. Artificial intelligence models					

GP (this study)	0.0066	0.0504	30.6	2.99	0.842
ANN	0.0043	0.0464	32.9	1.034	0.887

Table 4 Comparison of percentages of error ranges and deviation calculated by different models for the test data

Methods	Percentages of calculated dh/D_3 in error ranges					Percentage deviation		
	±5% Errors	±10% Errors	±15% Errors	±20% Errors	±25% Errors	max	min	average
a. Explicit equations								
Biery and Delleur [2]	3	6	9	14	20	66.07	-97.99	-20.57
MLR [3]	4	5	6	7	10	580.62	-44.45	112.55
MLNR [3]	3	5	9	15	21	142.90	-84.57	0.28
MHBMO-GRG (this study)	4	10	18	21	23	219.76	-99.15	-3.81
GA (this study)	3	10	17	23	27	163.15	-86.28	6.34
b. Artificial intelligence models								
GP (this study)	9	17	25	31	33	69.52	-98.21	-17.08
ANN	6	13	19	24	27	139.09	-70.39	12.26

List of Figure captions:

Figure 1 - A schematic illustration of bridge backwater caused by bridge constrictions

Figure 2 - Variation of dh/D_3 with respect to (a) J_1 and F_3, (b) J_1 and J_3, and (c) J_3 and F_3

Figure 3 - Confidence limits of different models for calculating bridge backwater for test data

Figure 4 - Comparison of R^2 values calculated by different models for train data

Figure 5 - Comparison of R^2 values calculated by different models for test data

Figure 6 - Comparison of relative errors calculated by different models

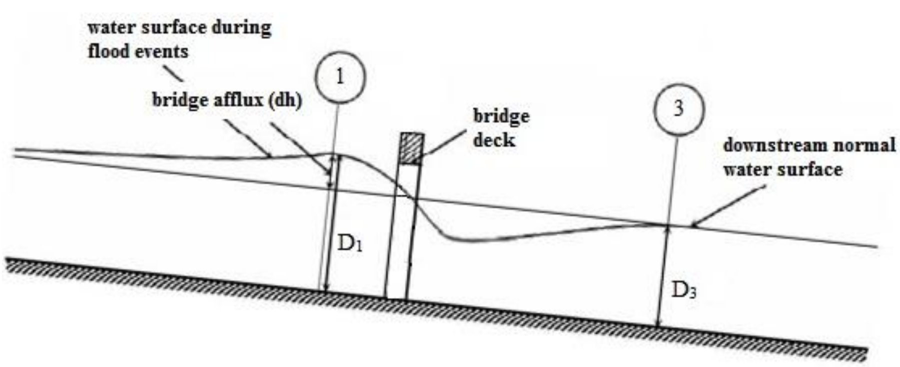
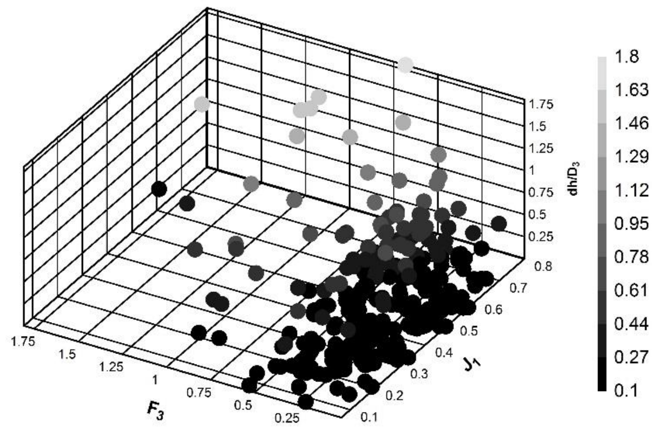
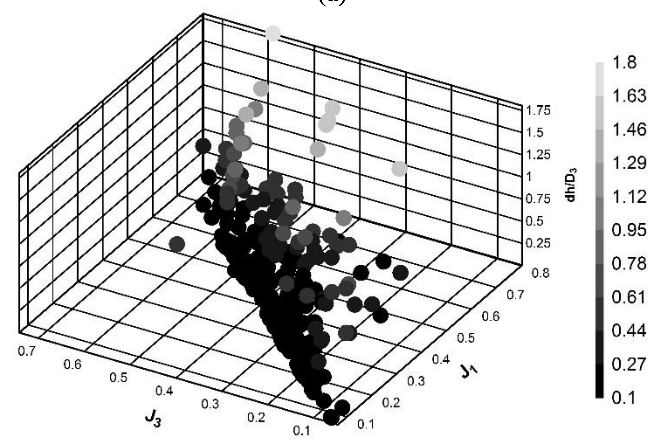


Figure 1 - A schematic illustration of bridge backwater caused by bridge constrictions



(a)



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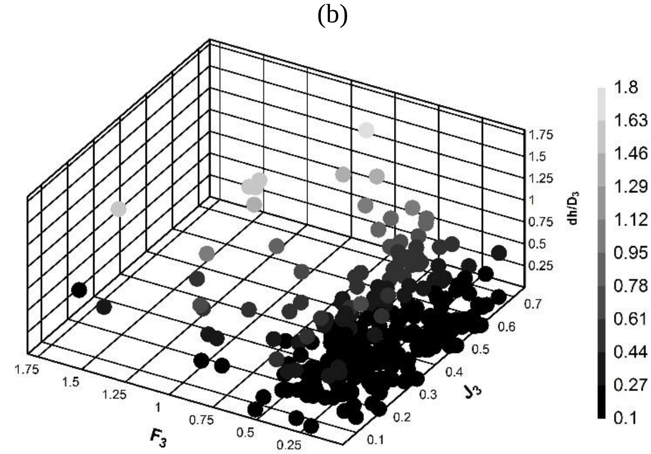

(c)

Figure 2 - Variation of dh/D_3 with respect to (a) J_1 and F_3, (b) J_1 and J_3, and (c) J_3 and F_3

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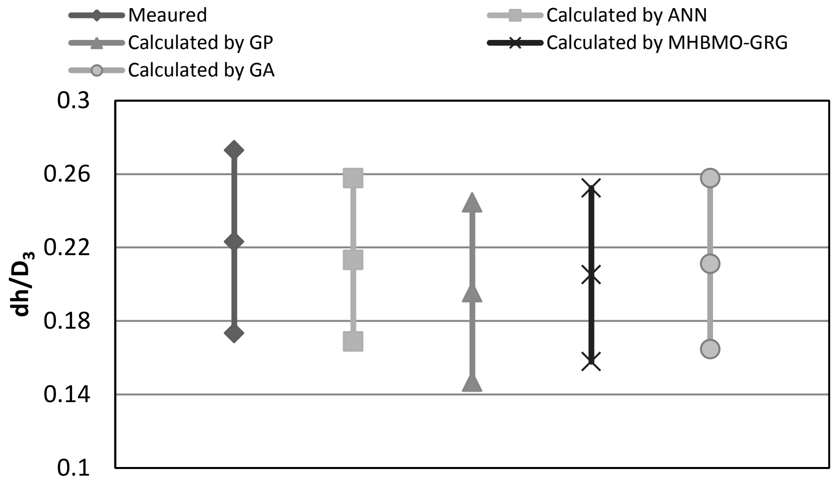
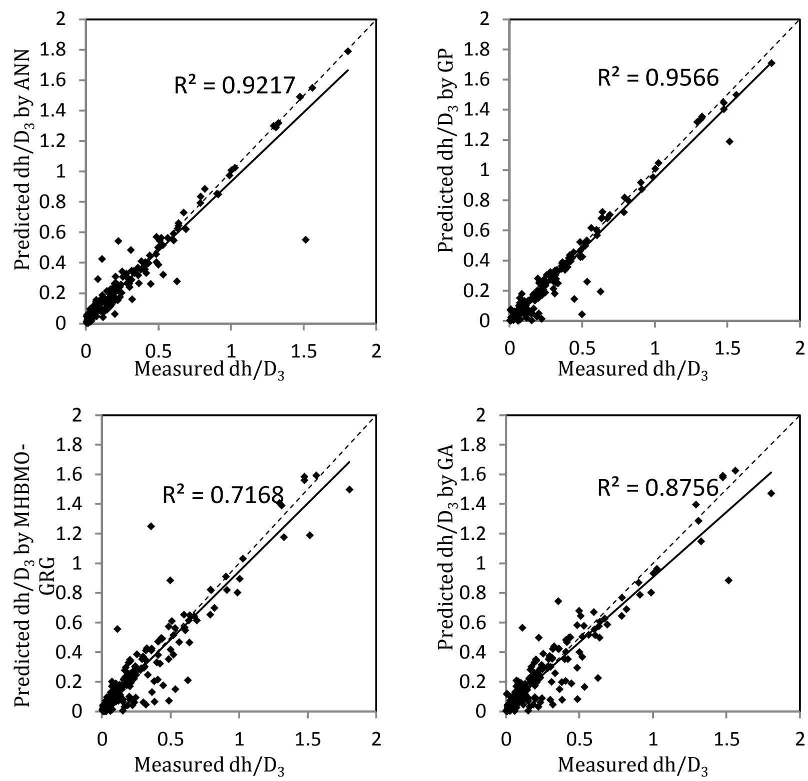
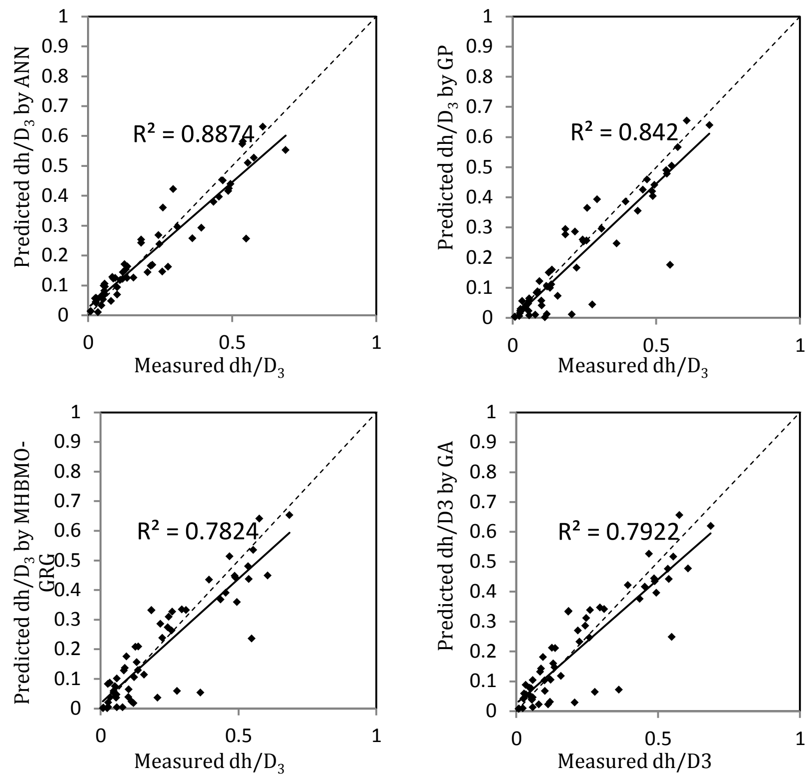


553 Figure 3 - Confidence limits of different models for calculating bridge backwater for test
554 data



555 Figure 4 - Comparison of R^2 values calculated by different models for train data



556 Figure 5 - Comparison of R^2 values calculated by different models for test data

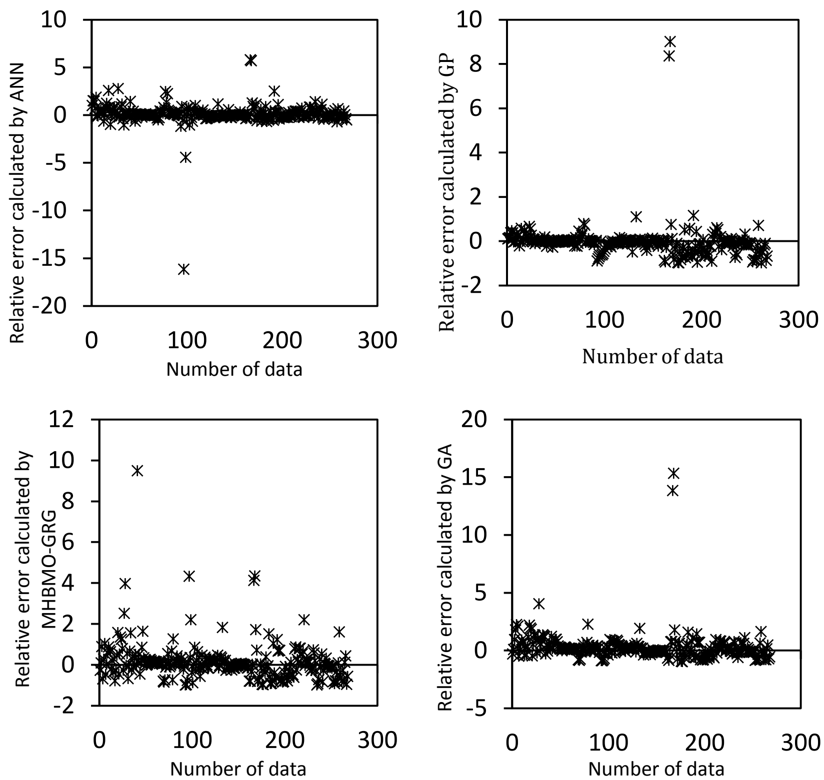


Figure 6 - Comparison of relative errors calculated by different models

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Majid Niazkar received his BS degree in Civil Engineering (2012), MS degree in Hydraulic Structures (2014) and PhD degree is Water Resources all from the Department of Civil and Environmental Engineering, Shiraz University, Shiraz, Iran. He is currently a research assistant in the Department of Civil and Environmental Engineering, Shiraz University, Shiraz, Iran. He has published 18 ISI papers, one ISC paper and 30 papers in international and national conferences. His research interests include hydrology (specifically: flood modeling and routing), hydraulic engineering (specifically: Open-channel hydraulics and channel design), water distribution networks, optimization, numerical modeling, water resources management, critical infrastructure protection, groundwater, soft-computing techniques (ANN and genetic programming) for hydrologic modeling, river engineering and sediment transport, bed roughness prediction, and assessing the impacts of climate change. He also has reviewed some papers for several ISI journals including Water Resources Management, Urban Water Journal, Arabian Journal for Science and Engineering, and Computer Applications in Engineering Education.

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