Multi-echelon green open-location-routing problem: A robust-based stochastic optimization approach

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Abstract

In recent years, considering the environmental competencies could help the companies/countries to improve their industries successfully regarding the sustainable development. In this study, a green open location-routing problem with simultaneous pickup and delivery (GOLRPSPD) is considered to minimize the overall costs. In addition to cost minimization, the objective function is provided the environmental competencies regarding the costs of CO_2 emissions and fuel consumptions. Meanwhile, in a complex situation, considering the precise information could lead the results to unreliable in which considering the uncertainty theories could prevent the data loss. In this respect, this study considered the pickup and delivery demand and the travel time as probabilistic parameters. To address the issue, a robust stochastic programming approach is developed to decrease the deviations of imprecise information. Moreover, the proposed approach is implemented based on five scenarios to decide the best decision in different situations. In addition, a practical example of the multi-echelon open-location-routing model is provided to represent the feasibility and applicability of the presented robust stochastic programming approach. Finally, a comparative and sensitivity analysis is considered to indicate the validity of the proposed approach, and also represent the robustness and sensitiveness of the obtained results regarding some significant parameters, respectively.

Keywords: Open-location-routing problem, Green logistic, stochastic programming, robust optimization, uncertainty.

1. introduction

Green location-routing problem covers classic location-routing with attention to minimizing the cost of fuel consumption and reducing greenhouse gases in the atmosphere, particularly CO_2 sent out by human resource activities. Increasing CO_2 causes a critical problem for the depletion of the ozone layer and human health. Thus, organization and companies are recognizing the need to reduce and asses the environmental effect of operations and services [1]. In this respect, some authors focused on environmental competencies to solve their location and routing problems.

Thereby, Schneider et al. [2] developed green vehicle routing problem (GVRP) with time windows to solve green logistics problems in the electric vehicles industry. Erdogan and Miller-Hooks [3] presented a novel formulates and conceptualizes for GVRP regarding the proposed density-based clustering and modified Clarke and Wright savings heuristic algorithms. Salimifard and Raeesi [4] developed a new routing problem that accounts for optimizing consumption fuel cost and CO_2

emissions by considering main and cleaner alternative fuel. Tiwari and Chang [5] proposed a block recombination model for the GVRP by the goal of minimizing the distance traveled from depot to distribution center. Montaya et al. [6] presented an extension of the green vehicle routing problem in which has renewable fuel consumption and duration constraints. Dukkanci et al. [7] extended the classical location-routing problem by considering all of environmental and social sides effects on greenhouse gas(GHG) emissions and fuel consumption into the mixed integer programming formulations. According to the importance of environmental issues and presenting an efficient scheme for the locating depots and routing of vehicles, OLRPSPD has become a key element of supply chain management.

In many complex GVRP problems, imprecise inherent of information lead us to defined them based on uncertain theory. In this sake, the uncertainty theories such as probabilistic theory is a powerful tool that can assist the managers or experts in the GVRP in overcoming the uncertain environment. Therefore, utilizing the probabilistic theory and their solving tools could be considered as interesting tools for authors to solve the GVRP under imprecise information in practice. Furthermore, considering the probabilistic information in the procedure of extending the multi-echelon open-location-routing could suitability deal with the possible uncertain situation in the real cases. Meanwhile, the survey of the literature represented that the authors focused on multi-echelon location-routing problem based on precise and imprecise information.

In the field of precise information, Hemmelmayr, et al. [8] proposed a heuristic solution for two-echelon vehicle routing problem in two-level transportation systems of city logistics. Contardo et al. [9] introduced a branch-and-cut and an adaptive large-neighborhood search Meta heuristic for the two-echelon capacitated location-routing problem. Rahmani et al. [10] presented a mixed integer linear programming formulation for modeling multi products location-routing with pick-up and delivery by considering a two-echelon distribution scenario. Perwira Redi et al. [11] addressed the open vehicle routing problem with time windows and presented a heuristic algorithm to solve the problem (VSN). Marinakis and Marinaki [12] presented an improved version of the Bumble Bees Mating algorithm for solving the open vehicle routing problem and tested the proposed algorithms by using two sets of benchmark instances. Rahmani et al. [13] presented a mathematical programming model for the two-echelon multi products location-outing problem with pick-up and delivery.

To solve the problem, two types of local search algorithm are presented. Koc et al. [14] extended the location-routing problem by considering time windows and heterogeneous fleet and presented mixed integer programming formulations and solved the problem by using a developed hybrid evolutionary search algorithm. Tajbakhsh and Shamsi [15] extended a bootstrap data envelopment analysis framework with undesirable factors for capacitated facility location problem based on multi-sourcing constraints which are implemented to the United States energy sector. Halek et al. [16] presented a model based on the GIS (Geospatial Information System) to obtain the approximate amount of particular matter (PM) in the critical part of Tehran.

Capelle et al. [17] modeled the location-routing problem with pickup and delivery by integer programming formulation and validated the model by implementation the column generation. Brandao [18] presented the open vehicle routing problem by considering time concentrate and solved the problem by an iterated local search algorithm. Pishka et al. [19] addressed mixed-integer linear programming for two-echelon open location-routing problem, in this case for satisfying the open routes, considered third party logistics providers. Shen et al. [20] proposed the open vehicle routing problem with time windows that considers low carbon trading policies. Wang et al. [21] developed a bi-objective model for two-echelon location-routing problems with time concentrates by a three steps customer clustering-based approach. Dai et al. [22] proposed two approaches for multi-echelon location-routing problems which obtain the solution for two location-routing problems in less time. Queiroz [23] proposed two heuristics algorithms for solving the capacitated location-routing problem. Hosseini et al. [24] addressed the capacitated location-routing problem for a company that collects return products from the customer by designing a collection network. Zhou et al. [25] introduced two-echelon vehicle routing problem of e-commerce distribution network which is happened in the last mile of delivery option. For solving this problem, the effective heuristic algorithm is provided.

In the field of imprecise information, Ghaffari Nasab et al. [26] presented a different stochastic programming model for the capacitated location-routing problem with probabilistic travel times and presented bi-objective mathematical programming. Zarandi et al. [27] presented a location-routing problem with time windows, in which has been assumed that travel times and demands of customers are fuzzy variables. Bagherinejad and Dehghani [28] proposed a robust optimization for multi-objective capacitated location-allocation model and considered the customer demand as an uncertain parameter. Mousavi et al. [29] presented a fuzzy possibilistic-stochastic programming model for the location of cross-docking and vehicle routing schedul. Tajik et al. [30] addressed a new robust model for pollution routing problem with time windows and simultaneous pick-up and delivery by considering reduction of greenhouse emissions and the amount of fuel consumption in objective function. Cheref et al. [31] presented a new robust optimization approaches for a production scheduling and delivery routing problem. Schiffer and Walther [32] proposed a robust approach for the location-routing problem for strategic network design of electric supply chain fleet by considering uncertain customer pattern. Shahparvari and Abbasi [33] proposed robust stochastic modeling for vehicle routing and scheduling problem based on imprecise time windows, evacuee population, and bushfire propagation in Australia.

Wu et al. [34] proposed integer linear scenario-based models under uncertainty by considering travel time as an uncertain parameter and developed a new robust method for the vehicle routing problem. Braaten et al. [35] introduced a robust model of the vehicle routing problem with time windows by considering travel times as uncertainty parameters. Nadizade and Kafash [36] addressed the capacitated location-routing with simultaneous pickup and delivery demands in which the pickup and delivery demands of the customer are fuzzy variables. Lu and Gzara [37] addressed the vehicle routing problem with time windows in the imprecise environment by considering only uncertain demand parameter and presented robust optimization for modeling the problem and solved the problem with branch and price and cut. Hu et al. [38] addressed the vehicle routing problem by modeling a robust optimization based on new route-dependent uncertainty sets, in this case demand and travel time uncertainty. Veysmoradi et al. [39] offered a mixed integer nonlinear open location-routing model for relief distribution network by considering the event of a disaster as an uncertain situation such as earthquake and flood.

The investigation of the literature indicates that for the importance of the GOLRPSPD, presenting the robust stochastic model regarding the environmental competencies could cope with imprecise/incomplete information in which there are few papers that considering the robust optimization and stochastic programming approach for the multi-echelon open-location-routing problem, simultaneously. This paper aims to bring GOLRPSPD problem, closer to the real world, so the GOLRPSPD is modeled by stochastic programming and robust optimization in which the travel time and customer demands consist of pick-up, and delivery demands are assumed to be probabilistic that in location-routing problem literature has been little attention. By considering the literature of location-routing problems, this paper finds a gap in the literature.

So far, GOLRPSPD in this paper, introduced CO_2 emission cost in objective function whit total cost of system in the model that is the first work has been considered all costs consist of CO_2 emission cost in one objective function in order to reduce the amount of fuel consumptions, although it seems to be applicable in the real world. Also, this paper has been considered to have probabilistic both pick-up and delivery demands simultaneously and travel time by applying both probabilistic and robust optimization as a solution method in the GOLRPSPD, then compare two models to get the best result. By taking into account the scenario based concept for two models to deal with different situations. Regarding Table1, there is a gap in considering the robust stochastic approach in uncertain situation for solving the open location-routing problem by environmental consideration. All this consideration in this paper, make the mathematical models closer to the real world. However, this paper can be applied in the situations especially in the distribution management like perishable commodities.

The remainder of this paper is organized as follows: In section 2, the problem definition and the stochastic and robust mathematical formulations for GOLRPSPD are presented. Besides, a numerical example is considered to represent the implementation procedure of the proposed approach in section 3. Moreover, in section 4, the comparative and sensitivity analysis are performed to represent the powerfulness of the proposed robust stochastic approach. Finally, some concluding remarks and suggestions for future research are manipulated in section 5.

{Please insert Table 1 here.}

2. Multi-echelon open-location-routing model

In this section, the proposed robust stochastic mathematical model for green open location-routing problem with simultaneous pickup and delivery (GOLRPSPD) is established. In this respect, the problem description of the multi-echelon open-location-routing problem is provided. Then, the assumptions for constructing the proposed model are expressed. Moreover, the stochastic and robust mathematical models for the multi-echelon open-location-routing problem are developed.

2.1. Problem definition

This study focuses on designing the two-echelon open-location-routing problem including warehouse centers, customers, and recycling centers. In this respect, the goal of this research is to optimize the location of warehouse centers as well as the service routes for delivery of customers demand. These two decisions are optimized by minimizing the total routing costs (e.g., fuel consummation cost) and the warehouse locations costs. In the open-location-routing problem, two customer demands consist of pick-up and delivery demand are provided in which the delivery demand is the demand for products that shipped from warehouse centers to customers. Also, each customer has several used and returned products (e.g., empty soda bottles, etc.) in which should be shipped to the recycling center by the same vehicle that is called pick-up demand. Each route starts from a warehouse center and after supplying the customer's delivery demand, load the pick-up demands from customers for shipping to the recycling center. In this respect, the output of the recycling center is considered as a material of other industries. On the other hand, the recycling center is provided as supplied of them in which the open-location-routing problem is established when a company does not have its transportation system or servicing all the customer with its fleet is almost impossible because of the lack in the fleet of vehicle. So these companies usually use the 3PL company to distribute their commodity due to cost saving and efficient solution. In this respect, this paper considers the open route for transportation system which starts its tour in the depot, and after servicing the last customer does not come back the depot where starts its tour.Meanwhile, the delivery and pick-up demand and the travel time are considered as imprecise parameters.

To address the issue, the robust stochastic programming method is provided regarding the scenario-based approach. Indeed, the strategic decisions such as establishing a warehouse center are considered in first stage of proposed approach, and then the tactical decisions such as routing optimization are provided in the second stage according to scenario-based approach. However, the structure of the GOLRPSPD problem is demonstrated in Figure 1.

{Please insert Figure 1 here.}

2.2. Assumptions

Some assumptions for extending the multi-echelon open-location-routing model is explained as follows:

- The pick-up and delivery demand and the travel time are uncertain.
- The vehicle routing problem is open in which the output of the recycling center is considered for other industries.
- The supply chain is two-echelon includes warehouses, customers, and a recycling center.
- There is one-off problem and decisions are taken for a period in the planning horizon.
- The capacity of vehicles has been considered different.
- Each customer is serviced by only one warehouse.
- The warehouses have limitation on supplying.
- Backorder is not allowable.
- In each scenario, some customers may not service, so cost of non-covering is considered.
- There is no limitation on travel time.
- The pick-up and delivery are considered, simultaneously.
- The different sequences of future events are considered as the number of scenarios.

2.3. Nomenclature

In this section, the notations include sets, parameters, and variables are defined as follows:

Sets

$N=\{1,\ldots,N\}$	Set of all nodes, $N=N_c\cup N_o\cup N_r$
$N_c(N_c\subset N)$	Set of customer nodes ($j\in N_c$)
$N_o(N_o\subset N)$	Set of possible warehouse center nodes **(** $i\in N_o$ **)**
$N_r(N_r\subset N)$	Set of recycling center nodes
$K=\{1,\ldots,K\}$	Set of vehicles
$E=\{(i,j)\mid i,j\in N\}$	Set of edges
$S=\{1,\ldots,s\}$	Set of scenarios

Parameters

O_i	Fixed cost of establishing warehouse center in location/node i
F_k	Fixed cost of using vehicle k
c_k	Transportation cost per unit of time by vehicle type k
c'_k	Cost of CO_2 emission per unit of time by vehicle type k
dis_{ij}	The distance between nodes i and j
t_{ijs}	Transportation time in edge (i,j) of scenario s ; $t_{ijs}=dis_{ij}\times\alpha_s$ where α_s is a balance factor in scenario s
CD_i	Maximum capacity of warehouse center in location/node i
Q_k	Maximum capacity of vehicle type k
p_{js}	Pick-up demand of customer j in scenario s
d_{js}	Delivery demand of customer in scenario s

b_k	Number of available vehicles type k
λ	Coefficient of deviation from the average cost of the second stage in the robust model
ω	Robustness as defined by coefficient of non-covering the demand
θ_s	Positive deviation from the mean value of SSC_s .
$\Pr_s$	Probability of scenario s
π	Cost of non-covering one unit of delivery demand
π'	Cost of non-covering one unit of pick-up demand
M	A large number

Decision variables

U_{jks}	The amount of products delivered by vehicle k before serving customer j of scenario s
V_{jks}	The amount of products collected by vehicle k before serving customer j of scenario s
Z_i	1, if a warehouse center is established in location/node i ; otherwise 0 .
Y_{ijs}	1, if the delivery demand of customer j is fulfilled by warehouse center i of scenario *s*; otherwise 0.
X_{ijks}	1, if vehicle k goes from node i to node j of scenario s ; otherwise 0.
Cov_{js}	1, if node j of scenario s is not fulfilled; otherwise 0.
T_{is}	Vehicle arrival time to node i of scenario s
SSC_s	Costs of the second stage related to scenario s

2.4. *Stochastic mathematical Formulation*

One of the most commonly possibilistic models is the stochastic programming scenario-based approach. Thus, the most important feature of this modeling approach is to divide decisions into two stages that the decision maker takes a decision in the first stage and then a random event may occure in which the second stage decisions are taken to compensate the adverse effects of the first stage decisions. In this approach, it is not necessary to make decisions of first and second stages at the same time. Indeed, the second stage decisions can be postponed until they eliminated the existence uncertainties. Moreover, the decisions of choosing the best route and transportation fleet can be postponed until one of the considered scenarios is occurred. So, problem formulation is presented as a stochastic programming in which imprecise parameters are considered in the forms of scenarios in the model. For example, when traffic, the vehicle failure, the climate change, the lack of timely delivery by suppliers and the constant changing of customer's requirements occur, it affects travel time and demand imprecise information. These factors represent the source of uncertainty and they are considered as the criteria of scenarios. However, the mathematical model of the multi-echelon open-location-routing problem is developed regarding the aforementioned nomenclature as follows:

$$\text{Min} \left[\sum_{i \in N_o} o_i z_i + \sum_{s \in S} Pr_s . SSC_s + \sum_{i \in N_c} Pr_s . Cov_{is} . (d_{is} . \pi + p_{is} . \pi') \right] \tag{1}$$

where:

$$SSC_s = \sum_{i\in N_o}\sum_{j\in N_C}\sum_{k\in K} F_k X_{ijks} + \sum_{i\in N}\sum_{j\in N}\sum_{k\in K} C_k.t_{ijs}.X_{ijks} + \sum_{i\in N}\sum_{j\in N}\sum_{k\in K} C'_k.t_{ijs}.X_{ijks} \qquad \forall s\in S \tag{2}$$

Subject to:

$$\sum_{j\in N}\sum_{k\in K} X_{ijks} + Cov_{is} = 1 \qquad \forall i\in N_C, i\neq j, s\in S \tag{3}$$

$$\sum_{j\in N} X_{ijks} - \sum_{j\in N} X_{jiks} = 0 \qquad \forall k\in K, i\in N_C, s\in S \tag{4}$$

$$\sum_{j\in N} X_{ijks} = 0 \qquad \forall k\in K, i\in N_r, s\in S \tag{5}$$

$$\sum_{k\in K} X_{ijks} \leq Y_{ijs} \qquad \forall i\in N_o, j\in N_c, s\in S \tag{6}$$

$$\sum_{j\in N_c} X_{ijks} \leq b_k Z_i \qquad \forall k\in K, i\in N_o, s\in S \tag{7}$$

$$\sum_{i\in N_o} Y_{ijs} + Cov_{is} = 1 \qquad \forall j\in N_c, s\in S \tag{8}$$

$$\sum_{j\in N_c} d_{js} Y_{ijs} \leq CD_i Z_i \qquad \forall i\in N_o, s\in S \tag{9}$$

$$U_{jks} - U_{iks} + Q_k X_{ijks} + (Q_k - d_{is} - d_{js})\mathrm{X}_{jiks} \leq Q_k - d_{js} + (1 - \mathrm{X}_{jiks} - X_{ijks}).\mathrm{M} \qquad \forall k\in K \;\; j,i\in N_c, j\neq i, s\in S \tag{10}$$

$$V_{iks} - V_{jks} + Q_k X_{ijks} + (Q_k - p_{is} - p_{js})\mathrm{X}_{jiks} \leq Q_k - p_{js} + (1 - \mathrm{X}_{jiks} - X_{ijks}).\mathrm{M} \qquad \forall k\in K \;\; j,i\in N_c, j\neq i, s\in S \tag{11}$$

$$U_{jks} + V_{jks} - d_{js} \leq Q_k \qquad \forall j\in N_c, k\in K, s\in S \tag{12}$$

$$V_{iks} \geq p_{is}\sum_{j\in N, i\neq j} X_{ijks} + \sum_{j\in N_c, i\neq j} p_j X_{jiks} \qquad \forall i\in N_c, k\in K, s\in S \tag{13}$$

$$U_{iks} \geq d_{is}\sum_{j\in N, i\neq j} X_{ijks} + \sum_{j\in N_c, j\neq i} d_{js} X_{ijks} \qquad \forall i\in N_c, k\in K, s\in S \tag{14}$$

$$U_{iks} \leq Q_k - (Q_k - d_{is})\left(\sum_{j\in N_s} X_{ijks}\right) \qquad \forall i\in N_c, k\in K, s\in S \tag{15}$$

$$V_{iks} \leq Q_k - (Q_k - p_{is})\left(\sum_{j\in N_o} X_{jiks}\right) \qquad \forall i\in N_c, k\in K, s\in S \tag{16}$$

$$T_{is} \leq \sum_{k\in K} t_{jis}.X_{jiks} + T_{js} + (1 - \sum_{k\in K} X_{jiks}).\mathrm{M} \qquad \forall i\in N_c, \mathrm{j}\in \mathrm{N}, j\neq i, s\in S \tag{17}$$

$$T_{is} = 0 \qquad \forall i\in N_o, s\in S \tag{18}$$

$$V_{iks}, U_{iks}, T_{is} \geq 0 \qquad \forall i\in N, k\in K, s\in S \tag{19}$$

$$Z_i, Y_{ijs}, X_{ijks}, Cov_{is} \in \{0,1\} \qquad \forall i,j\in N, k\in K, s\in S \tag{20}$$

Eq. (1) shows the objective function that minimizes the cost of establishing warehouse centers as well as the expected costs based on different scenarios. Thereby, Eq. (2) established based on three parts which are defined as routing costs including fixed cost of using vehicles and transportation costs, cost of CO_2 emission and cost of non-covering the customer demands, respectively. According to constraint (3), guarantee that each customer must be serviced exactly once by vehicle type *k*. The constraint (4) ensure the balance between entering and existing edge of each node.

Constraint (5) ensures that, in each scenario, there is no edge exiting from recycling center, all paths end in the recycling center. Constraints (6) and (7) forbid infeasible routes. On the other hand, constraint (6) makes sure each customer is assigned to a warehouse. Constraint (7) guarantees that if a warehouse is established, only the routes between that warehouse and customers can be activated. Constraint (8) ensures each customer is assigned to exactly a warehouse. Constraint (9) the total loading limited to the maximum capacity of the warehouse. Constraint (10) implies that supplying the customer demands is related to the warehouse capacity. Constraint (11) ensures that any vehicle that is assigned to a customer load its' pick-up demand. Constraint (12) states that the load of the vehicle must not exceed of vehicle capacity. Constraints (12) to (16) define the domain of variables that are associated with pick-up and delivery products. These constraints, along with constraints (10) and (11) that determine the exact value of pick-up and delivery. Constraint (17) expresses the arrival time to customers. Constraint (18) guarantees that arrival time for each warehouse node is zero. Constraints (19) and (20) indicates the positive and binary variables, respectively.

2.5. *Robust mathematical model*

The robust model exactly examines the planning risk exposure and mitigates the effect of pessimistic state on the results of the system. The robustness has a lower sensitivity of the models' results to the variation of scenario parameters, so it facilitates the application of this model in practice and real life. Hence, the robust programming approach that is developed by Yu and Li [40] and Leung Tsang et al [41] is considered in this study. The objective function consists of three terms: the first term shows the costs associated with the first stage decisions that are independent of the scenario, the second term minimizes the average costs of the second stage decisions regarding the scenario-based approach, and the last term estimates variation of uncertain parameters and minimizes deviations from the mean value to create the robustness. Moreover, the value of coefficients in the last term of the objective function (i.e., coefficient of the average cost (λ) and coefficient of deviation from the average cost (ω)) depends on experts' opinion. In fact, there is a trade-off between robustness and cost saving. However, robustness of solutions minimizes the variation of uncertainties, but on the other hand, it increases the total cost of the system. However, the objective function of the robust model is provided as below:

$$\text{Min} \begin{bmatrix} \sum_{i\in N_o} o_i z_i + \sum_{s\in S} Pr_s.SSC_s + \lambda.\sum_{s\in S} Pr_s.\left(\left|SSC_s - \sum_{s'\in S} Pr_{s'}.SSC_{s'}\right|\right) \\ +\,\omega.\sum_{i\in N_c} Pr_s.Cov_{is}.(d_{is}.\pi + p_{is}.\pi') \end{bmatrix} \tag{21}$$

As it was proposed by Yu and Li [40], the standard deviation is replaced by average absolute deviation. Furthermore, the Eq. (21) should be replaced with Eqs. (22) and (23) to linearize the proposed model in which the considered modifications are represented as follows:

$$\text{Min} \begin{bmatrix} \sum_{i\in N_o} o_i z_i + \sum_{s\in S} Pr_s.SSC_s + \lambda.\sum_{s\in S} Pr_s.\left(SSC_s - \sum_{s'\in S} Pr_{s'}.SSC_{s'} + 2\theta_s \right) \\ +\omega.\sum_{i\in N_c} Pr_s.Cov_{is}.(d_{is}.\pi + p_{is}.\pi') \end{bmatrix} \tag{22}$$

Subject to:

$$SSC_s - \sum_{s' \in S} Pr_{s'}.SSC_{s'} + \theta_s \geq 0 \qquad \forall s \in S. \tag{23}$$

Constraint (23) states that if SSC_s is greater than the mean value, the θ_s should be equal to the positive deviation from the mean value of SSC_s. In contrast, if SSC_s is less than the mean value, θ_s should be equal to the negative deviation from the mean value of SSC_s as Eqs. (24) and (25). Finally, the robust mathematical model is established as follows:

$$\text{Min}\left[\begin{array}{l} \sum_{i \in N_o} o_i z_i + \sum_{s \in S} Pr_s.SSC_s + \lambda.\sum_{s \in S} Pr_s.\left(SSC_s - \sum_{s' \in S} Pr_{s'}.SSC_{s'} + 2\theta_s\right) \\ \qquad + \omega.\sum_{i \in N_c} Cov_{is}.(d_{is}.\pi + p_{is}.\pi') \end{array}\right] \tag{24}$$

where:

$$\text{SSC}_s = \sum_{i \in N_o} \sum_{j \in N_C} \sum_{k \in K} F_k X_{ijks} + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} C_k.t_{ijs}.X_{ijks} + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} C'_{ik}.t_{ijs}.X_{ijks} \qquad \forall s \in S. \tag{25}$$

Subject to:

Equations (3)-(20).

3. Experimental example

In this section, an experimental example is provided to confirm the feasibility and validity of the proposed robust stochastic approach. In this case, assume that there are 15 nodes that nodes of 1-6 are possible locations for establishing warehouse center, nodes 7-14 belongs to customers and node 15 is a recycling center. Furthermore, there are 12 vehicles which are divided into three types of vehicle that used to move giant product and each vehicle is capable of moving one or two pieces regarding their capacity. The cost of non-covering a demand is denoted by $\pi' = \pi = 10$, and also five different scenarios are considered for uncertain parameters. It should be noted that the proposed model was solved by GAMS CPLEX 10.1 optimization software and the results were obtained based on a 3 GHz computer with 4 GB RAM. In this respect, the probability of occurrences is defined as 0.15, 0.2, 0.3, 0.2, and 0.15, respectively. Moreover, some other parameters such as delivery demand, pick-up demand, and the distance between each node are represented for instances in Tables 2 to 4, respectively. In next sections, the results of solving the experimental example are reported regarding the implementation of stochastic and robust approaches, respectively, then Figures 2 and 3 illustrated the routes of the solutions to different models and compared.

{Please insert Table 2 here.}

{Please insert Table 3 here.}

{Please insert Table 4 here.}

In figure 2, the selected fleet of vehicles in the third scenario is shown. As seen, in this scenario by considering the stochastic parameters, two vehicles of type 2 and two vehicles of type 3 for shipment is selected. Also, it is obvious that the establishment of depot number 4 has additional cost for the system in the third scenario. The other hand in figure 3, the selected fleet of vehicles in the third scenario are consist of two vehicles of type 1 and two vehicles of type 3. And depots number 2, 4, 6, give service to customers and the depot number 5 is inactive. As result, the selected rout in figure 2 and 3 is completely different due to minimizing the cost of system in each model. Also establishment of depot in two figure is different from each other for the reason is above-mentioned.

{Please insert Figure 2 here.}

{Please insert Figure 3 here.}

3.1. The results of stochastic approach

In this section, the results of applying the stochastic approach are represented. In this respect, the total value of objective function is 16700.9, and nodes 2, 4, and 6 are considered for establishing the warehouse centers. In this respect, as explained before, the considered approaches are analyzed based on two stages. In the first stage, the total cost of establishing of warehouse centers is 9000, and in the latter one, the total cost is reported based on five scenarios in Table 5. Meanwhile, the fifth scenario has the highest demand, and the nodes of 7 and 12 of customers are not covered in which the value of objective function is increased by 2982. It is worthwhile to note that the total running time of stochastic approach is 5.52 minutes.

{Please insert Table 5 here.}

The results of robust approach

In this section, the obtained results from implementation of robust approach are demonstrated. Meanwhile, the deviation coefficient from the average cost (λ) and the robustness (ω) are considered to be 2, simultaneously. In this respect, the value of objective function is 18036.7, and nodes 2, 4, 5, and 6 are allocated for establishing warehouse centers. Furthermore, the cost of establishing the warehouse centers at first and second stages are reported in Table 6. Moreover, the total running time of robust approach is 45 seconds less than the implementation of stochastic approach.

{Please insert Table 6 here.}

4. Comparative analysis, validation approach, and sensitivity analysis

4.1. Comparative analysis

In this section, the results of the robust model are compared with deterministic and stochastic models to represent the validity of this approach. Meanwhile, the proposed approach is determined the value of decision variables for future practice in which the most suitable decision have best value in objective function. Consequently, the events that are likely occurred in the future are simulated to validate the proposed model and analyze the obtained results. As stated in assumptions, the number of scenarios is considered as the different sequences of events that may be occurred in the future. Consequently, the parameters of each scenario can be accurately determined. Thus, the scenarios with different probabilities are generated for simulating the future and then considered as input parameters of deterministic model. The deterministic model provides the amount of cost that each decision will be made in reality. Indeed, the first-stage variables are constant and equal to those decisions that we will want to make. However, the deterministic model is presented as follows:

$$Min:\ C_{real}.X^{*} + C'_{real}.Y + \Pi.R \tag{26}$$

$$A_{real}.X^{*} + A'_{real}.Y - R \geq B_{real} \quad (27)$$

Where $C_{real}, C'_{real}, A_{real}, A'_{real}$, and B_{real} in Eqs. (26) And (27) are defined as the definite values of non-deterministic parameters. Moreover, X^{*} is the constant value of the first stage variables, and Y is the second stage variable of the model that is determined when the event occurs. However, the following steps are considered to implement the validation procedure:

Step 1. Solve the deterministic, stochastic, and robust models based on simulation inputs.

Step 2. Store the obtained results of the proposed models as $X_1^{*}, X_2^{*}, X_3^{*}$, respectively.

Step 3. Select a scenario randomly and consider its parameters as input data of the deterministic model.

Step 4. Solve the deterministic, stochastic, and robust models for each X^{*} and store the obtained value of objective functions.

Step 5. Repeat steps 3 and 4 for definite times (*N*).

Step 6. Compute the average, variance, and standard deviation of obtained *N* values for each objective function of proposed model.

However, the implementation process of validation approach is provided, and simulation results of deterministic, stochastic and robust models are reported in Table 7. As represented in this Table, the standard deviation of robust model is significantly less than the deterministic and stochastic approaches that could confirm the validity of the robust model. Furthermore, the average of objective functions indicated that the cost of applying the stochastic model is lower than the robust model. Because minimizing the deviation of imprecise information in environment of the system imposed the costs that have higher objective function value in robust model is reasonable and verified both stochastic and robust model. As mentioned before, the stochastic model minimizes the average value of costs, but the robust model minimizes the deviation from the average value of costs. Therefore, the results show that the average value of costs in robust model is somewhat higher than the stochastic model. In this respect, the trend of validation approach for deterministic, stochastic, and robust models is depicted in Figure 4.

{Please insert Table 7 here.}
{Please insert Figure 4 here.}

The proposed robust-based stochastic model can appropriately deal with to uncertain situation regarding the customer demand and travel time as imprecise parameters. However, Lu and Gzara [37] and Hu et al. [38] studies as relevant methods to this study are considered to compare the results of the proposed methods. Therefore, advantages and disadvantages of these approaches vs. our methods are expressed in Table 8.

{Please insert Table 8 here.}

4.2. Validation approach

In this section, to show the treatment of the method and prove the validation of the proposed model in this manuscript, the method which proposed in Lu and Gzara [37] and Hu et al. [38] is solved by the instances in this manuscript. The obtained results are compared and reported in Figure 5. As shown in Figure 5, there is no significant differences between the results of three compared models.

Regarding Figure 6, the performance of the models is similar to each other. Meanwhile the standard deviation and variance of the proposed model are less than the robust model Vs. Lu and Gzara [37] and Hu et al. [38]. That proves the robustness of the proposed model based on stochastic is more reliable than Lu and Gzara [37] and Hu et al. [38] and confirmed the validity of the robust proposed model.

{Please insert Figure 5 here.}

{Please insert Figure 6 here.}

4.3. Sensitivity analysis

In this section, the sensitivity analysis is carried out on some parameters to represent the robustness and sensitiveness of them for ensuring the advantages and effectiveness of the proposed approach and giving an insight into this section. In addition, the performance of the model is also investigated regarding by the robustness of model under variation of key parameters which are the critical parameters that affected logistic systems and also the critical parameters in this model such as lead time, customer demand, fixed costs, Co_2 emission costs and etc., which are controlled by the important coefficient of the robust model like non-covering of customer demand ω that strongly manage the robustness of the model. In this respect, Table 9 shows the sensitivity analysis under variation of ω that is defined as the coefficient of non-covering of customer demand in objective function or robustness coefficient. In this analysis, the value of ω increases by 0.3 in each epoch. As shown in this Table, by increasing the cost of non-coverage (ω), the number of warehouses for serving the customers is increased and also the number of non-covered nodes is decreased, simultaneously. Furthermore, for $\omega > 1.2$, the number of warehouse, non-covered nodes, and the value of objective function are fixed. In addition, the schematically representation of changing the non-coverage demands is represented in Figure 7.

{Please insert Table 9 here.}

{Please insert Figure 7 here.}

Furthermore, Table 5 shows the sensitivity analysis under the variation of λ which is defined as the optimal robust coefficient. Meanwhile, the value of λ increases by 0.5 in each epoch. As reported in Table 10 and depicted in Figure 8, the standard deviation from the mean value of second stage costs decreases when the value of λ is increased. In other words, the behavior of the system is more robust by increasing the value of λ.

{Please insert Table 10 here.}

{Please insert Figure 8 here.}

Moreover, Table 11 shows the sensitivity analysis under variation of warehouses capacity centers. In this sake, the results show that the number of established warehouse is directly related to warehouse capacity in which when the warehouse capacity is increased, the number of established warehouse is decreased. Consequently, all warehouse centers available for customers when the warehouse capacity is not considered in the process of decision making. Finally, the obtained results are demonstrated in Figure 9.

{Please insert Table 11 here.}

{Please insert Figure 9 here.}

5. Conclusions and future directions

In recent years, green logistics get more attention from companies and countries regarding the importance of environmental competencies in human life. Consequently, logistics strategies should be sustainable and consider environmental effects in distribution and production decisions. In this work, two different scenario-based mathematical programming formulations were introduced for the green open-location-routing problem with stochastic travel time and stochastic pick-up and delivery demand simultaneously which are named probabilistic programming and robust optimization, respectively. In this respect, the proposed robust stochastic mathematical model is implemented to an experimental example for representing the feasibility and applicability of the proposed approach. Hence, the results show that the first and fifth scenarios of stochastic and robust models are selected as the lowest CO_2 emissions cost regarding all scenarios, respectively. However, although the stochastic model has lower CO_2 emissions cost than the robust model, the standard deviation of imprecise variables for robust model is minimized. The sensitivity analysis is provided to investigate the performance of the robust model regarding the variation of some key parameters. In this respect, the computational results show that both stochastic and robust models are verified.

Furthermore, a comparative analysis is considered based on the deterministic, stochastic, and robust models to indicate the efficiency of these methods. Meanwhile, the comparative results based on objective functions' value represent that the stochastic model has minimum value. In addition, the robust mathematical model has lower standard deviation for obtained results regarding two other approaches. However, selecting each of stochastic and robust models is related to experts who are sensitive to fluctuating results or desired the minimum cost of CO_2 emissions. Due to the difficulties of the problem, the proposed model was able only to cope small-sized instances which is the limitation of this study that strongly suggested for future researches.

All in all, extending the proposed approach based on inventory decisions could lead up the obtained results in a realistic manner. Moreover, the metaheuristic solving approaches could be appropriately applied to solve the GOLRPSPD for large size problems. Finally, the proposed approach could be implemented for wide range of applicable problems especially for distribution management of perishable products.

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Figures' captions

Figure 1. The schematically representation of the open-location-routing problem

Figure 2. The routes of the solution to the stochastic model for 3th scenario

Figure 3. The routes of the solution to the robust model for 3th scenario

Figure 4. The results of implementation procedure of validation approach for N=20

Figure 5. The Comparative result of the proposed deterministic model and Lu and Gzara [37] and Hu et al [38]

Figure 6. the Comparative result of the proposed robust model and Lu and Gzara [37] and Hu et al [38].

Figure 7. The result of changing the non-coverage demands

Figure 8. The result of changing the optimal stability coefficient

Figure 9. The result of changing the warehouse capacity

Tables' captions

Table 1. Categories of studies on the open location-routing

Table 2. The amount of delivery demand for each scenario

Table 3. The amount of pick-up demand for each scenario

Table 4. Distance between node *i* and node *j*

Table 5. The obtained results from the stochastic approach

Table 6. The obtained results from the robust approach

Table 7. The results of implementation procedure of validation approach

Table 8. Summarized comparative analysis of the proposed approach Vs Lu and Gzara [37] and Hu et al. [38]

Table 9. The results of changing the non-coverage coefficient of demand

Table 10. The results of changing the optimality stability coefficient

Table 11. The results of changing the warehouse capacity (CD_i)

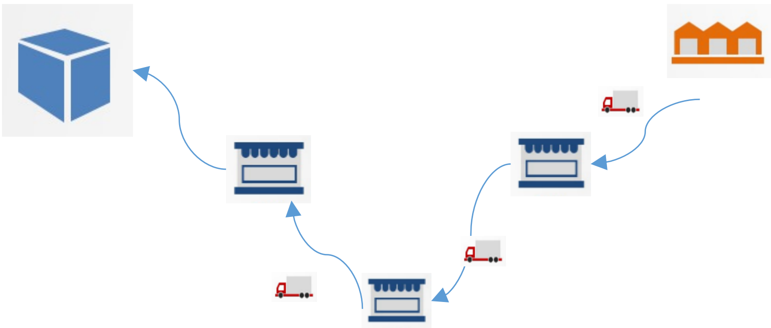


Figure 1.

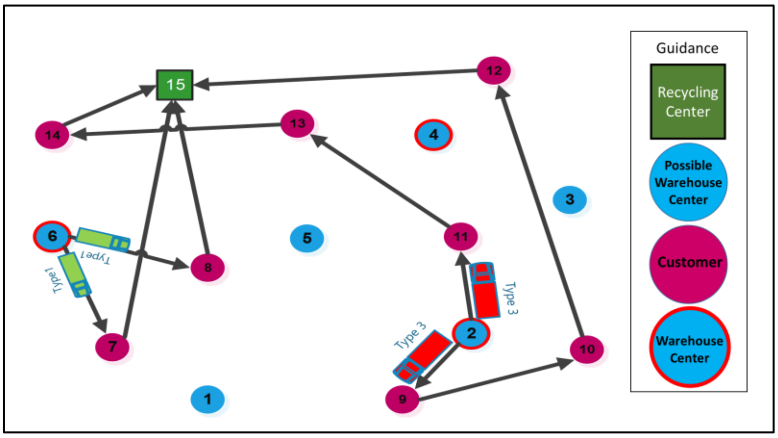
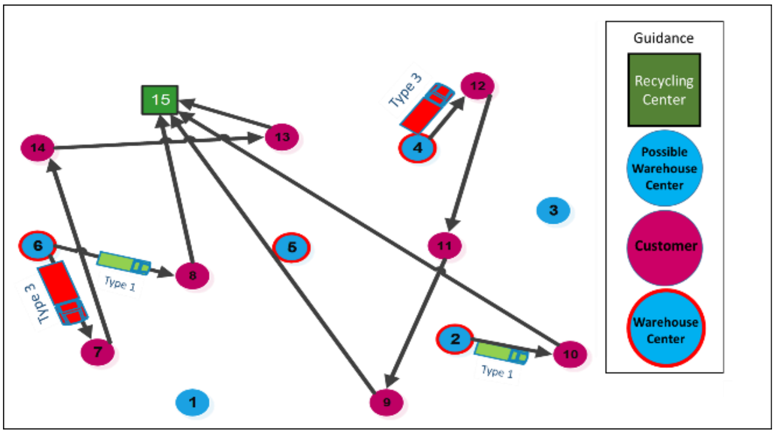
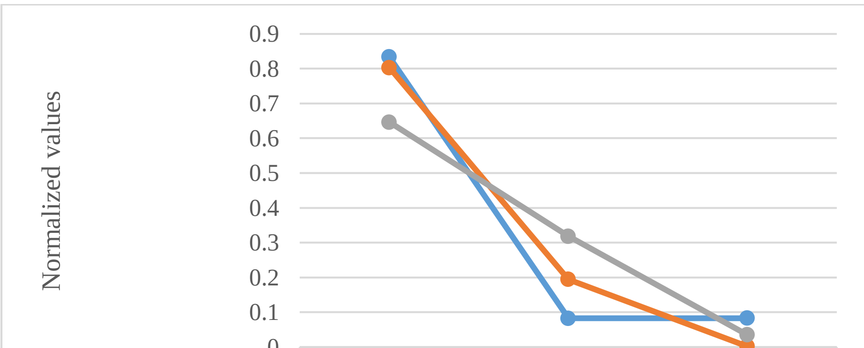


Figure 2.

Figure 3.





	Deterministic	Stochastic	Robust
Average	0.833846738	0.082742479	0.083410782
Variance	0.802811783	0.19480661	0.002381607
Standard deviation	0.646384634	0.318407557	0.035207809

Figure 4.

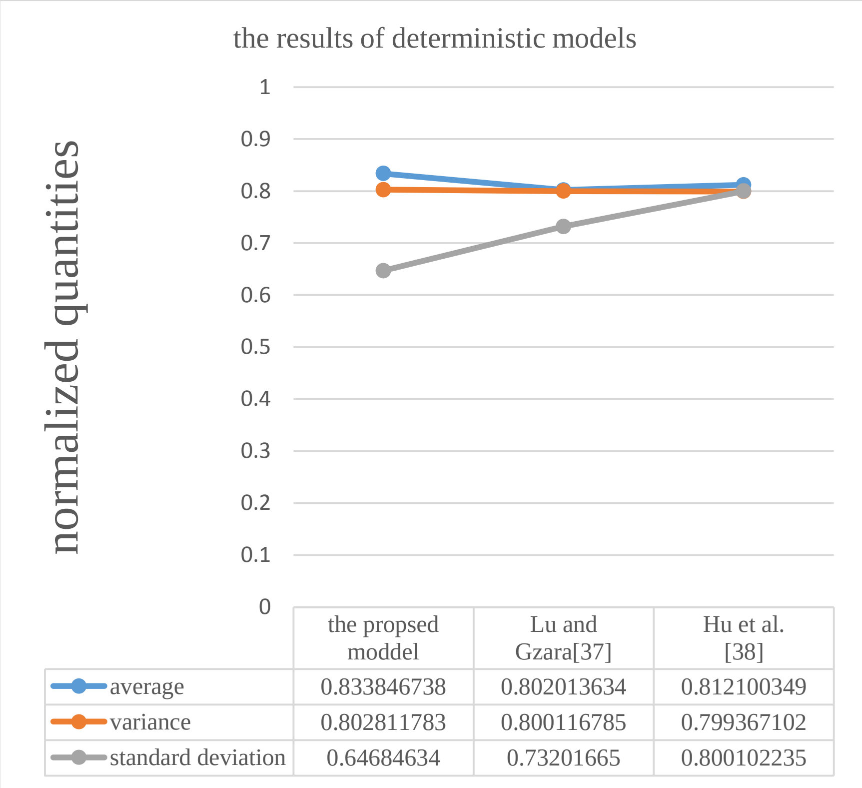


Figure 5.

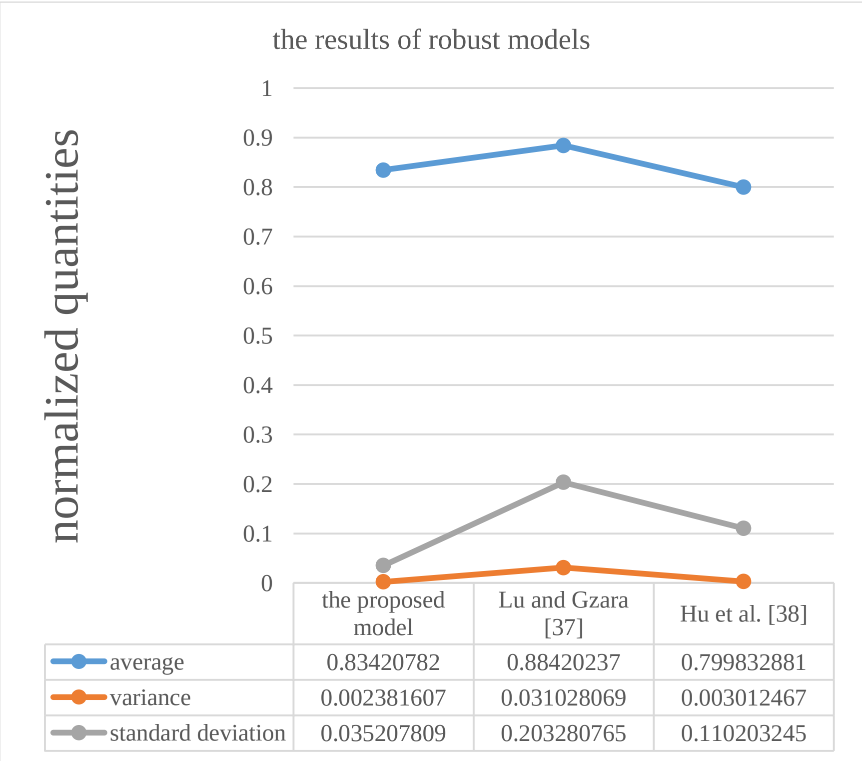
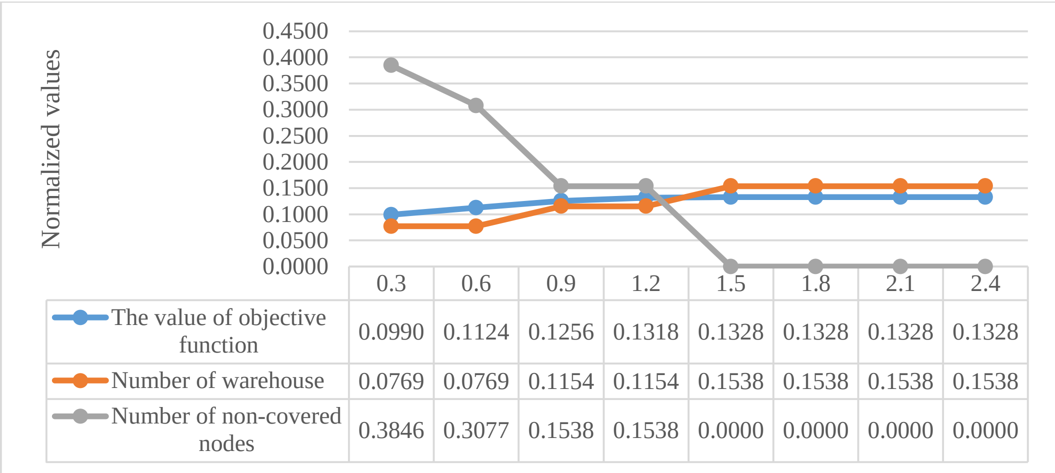
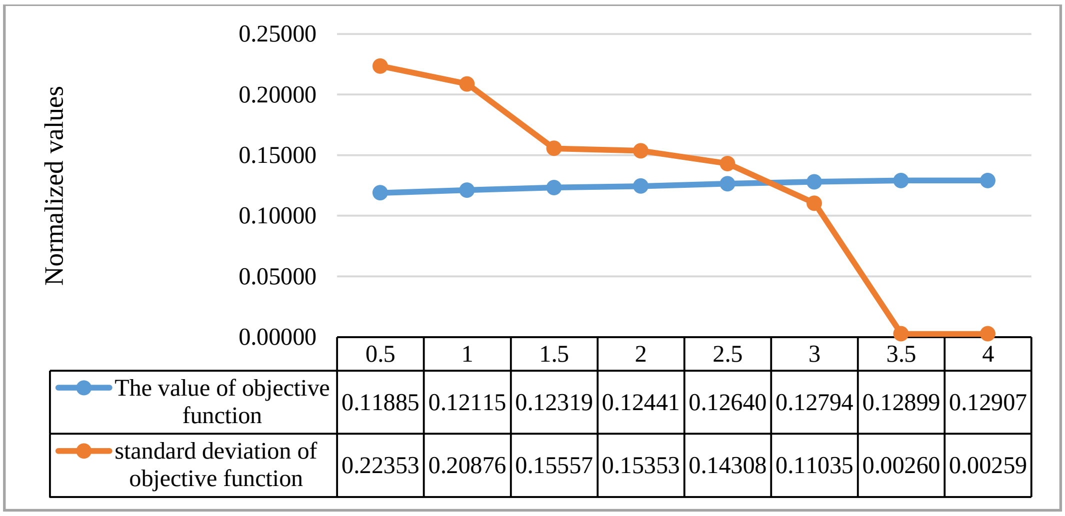


Figure 6.



	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4
The value of objective function	0.0990	0.1124	0.1256	0.1318	0.1328	0.1328	0.1328	0.1328
Number of warehouse	0.0769	0.0769	0.1154	0.1154	0.1538	0.1538	0.1538	0.1538
Number of non-covered nodes	0.3846	0.3077	0.1538	0.1538	0.0000	0.0000	0.0000	0.0000

Figure 7.



	0.5	1	1.5	2	2.5	3	3.5	4
The value of objective function	0.11885	0.12115	0.12319	0.12441	0.12640	0.12794	0.12899	0.12907
standard deviation of objective function	0.22353	0.20876	0.15557	0.15353	0.14308	0.11035	0.00260	0.00259

Figure 8.

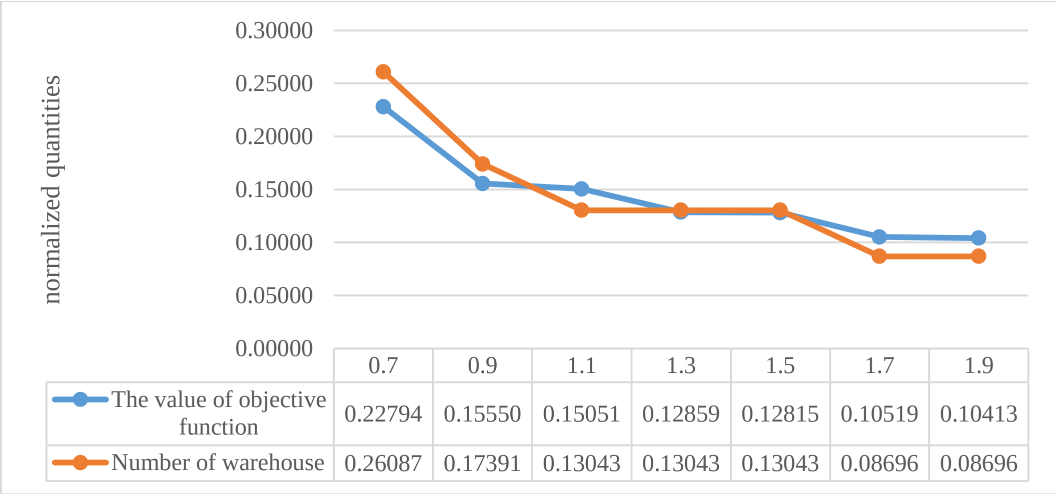


Figure 9.

Table 1.

Ref.	location	Routing	Single objective	open	green	deterministic	stochastic	robust
[17]	✓	✓	✓			✓		
[18]		✓	✓	✓		✓		
[19]	✓	✓	✓	✓		✓		
[22]	✓	✓	✓			✓		
[32]	✓	✓	✓			✓		✓
[37]		✓	✓			✓		✓
[38]		✓	✓			✓		✓
Current research	✓	✓	✓	✓	✓	✓	✓	✓

Table 2.

d_{js}		Scenarios (S)				
		1	2	3	4	5
Customers nodes (j)	7	158	221	414	349	699
	8	337	411	537	542	1044
	9	236	478	288	276	657
	10	324	444	500	545	780
	11	278	231	418	605	871
	12	246	386	395	356	721
	13	207	272	423	384	872
	14	196	216	342	457	1063

Table 3.

p_{js}		Scenarios (S)				
		1	2	3	4	5
Customers nodes (j)	7	219	340	379	431	279
	8	280	199	286	302	275
	9	199	333	246	325	458
	10	192	231	320	321	422
	11	219	285	314	403	400
	12	220	218	253	343	289
	13	169	213	212	291	378
	14	275	308	307	275	409

Table 4.

dis_{ij}		Node (j)														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Node (i)	1	0	121	190	172	96	110	53	71	86	168	141	215	143	156	167
	2	121	0	83	107	87	190	162	121	46	51	52	139	137	211	185
	3	190	83	0	69	116	227	219	162	128	79	51	76	135	229	183
	4	172	107	69	0	78	176	185	122	141	132	55	42	72	168	117
	5	96	87	116	78	0	112	107	46	95	135	68	121	57	125	100
	6	110	190	227	176	112	0	63	70	176	241	180	213	110	54	97
	7	53	162	219	185	107	63	0	63	134	212	168	226	135	115	143
	8	71	121	162	122	46	70	63	0	111	172	113	163	75	97	97
	9	86	46	128	141	95	176	134	111	0	85	90	179	152	208	194
	10	168	51	79	132	135	241	212	172	85	0	81	152	179	260	228
	11	141	52	51	55	68	180	168	113	90	81	0	89	100	187	149
	12	215	139	76	42	121	213	226	163	179	152	89	0	104	196	140
	13	143	137	135	72	57	110	135	75	152	179	100	104	0	96	50
	14	156	211	229	168	125	54	115	97	208	260	187	196	96	0	60
	15	167	185	183	117	100	97	143	97	194	228	149	140	50	60	0

Table 5.

$Z^* = 16700.9$		Total cost of first stage =9000			
SSCs (total cost of second stage)	S=1	S=2	S=3	S=4	S=5
	3740.6	4305.9	4906.1	5049.6	5432.6
Cost of CO2 emission	446.9	743.1	769.0	878.4	491.4

Table 6.

$Z^* = 18036.7$		Total cost of first stage =12000			
SSCs (total cost of second stage)	S=1	S=2	S=3	S=4	S=5
	3770.2	4854.3	4997.6	5049.6	6693.6
Cost of CO2 emission	758.7	966.9	798.8	878.4	569.7

Table 7.

N=20	Deterministic model	Stochastic model	Robust model
Average	171185.4	16986.7	17123.9
Variance	229131562.7	55600009.8	679737.5
Standard deviation	15137.1	7456.5	824.5

Table 8.

Parameters of the comparison	**The results of comparisons**
Uncertainty modeling	Because of considering the robust stochastic based, these 2 methods are adequate to cope with uncertain situation in the vehicle routing problem which part of the problem face uncertainty. However, proposed approach is tailored based on scenario which could appropriately elaborate the imprecision and close the results near the real-life by considering the uncertainty under scenarios.
Robustness of model	The proposed approach determined the robustness of model by considering the non-coverage customer demand and feasible rout as the result of the robustness in stochastic robust-based model to comparison deterministic model. The results show the high rate of the robustness in proposed model by analyzing the standard deviation which has less amount among three models. While, the methods which in the Lu and Gzara [37] and Hu et al [38], only compared the deterministic and robust model by analyzing the feasibility ratio of robust model and non-coverage customer demand, respectively. So there is a lack to conclude the robustness of the model in comparison to other studies.

Reliability	The proposed approach considered the deterministic results and compare those with stochastic and robust results. So give the chance to have enough insight which is appropriate. The Lu and Gzara [37] and Hu et al. [38] studies do not consider this concept; therefore, the obtained results from the proposed method of this study are more reliable.
Time complexity	The time complexity is connected to computational size of method. Lu and Gzara [37] and Hu et al. [38] methods perform better than the proposed approach. Due to, determining the factors examined such as, imprecise travel time and demand, time windows, vehicle capacity, warehouse capacity, and considering these factors through the process of the proposed scenario-based robust stochastic optimization approach, increases the size of required computation

Table 9.

ω	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4
The value of objective function	13439.7	15264	17055.2	17893.3	18036.7	18036.7	18036.7	18036.7
Number of warehouse	2	2	3	3	4	4	4	4
Number of non-covered nodes	5	4	2	2	0	0	0	0

Table 10.

λ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
The value of objective function	17230.4	17564.0	17859.1	18036.7	18325.0	18548.1	18700.9	18711.4
Standard deviation of objective function	1841.0	1719.4	1281.3	1264.5	1178.4	908.9	21.4	21.3

Table 11. The results of changing the warehouse capacity (CD_i)

CD_i	0.7	0.9	1.1	1.3	1.5	1.7	1.9
The value of objective function	26851.4	18317.4	17729.7	15147.8	15096.6	12391.4	12266.2
Number of warehouse	6	4	3	3	3	2	2