Product Acceptance Determination for Two Suppliers with Linear Profiles

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Abstract

In the management of suppliers, it is an important task to compare the performance of two suppliers using the linear profiles. In this paper, the product acceptance determination procedure is designed using a EWMA statistic based on the process-yield index applied to the linear profiles of two suppliers. The design parameters of the proposed plan are determined to satisfy both the producer's and consumer's risks. The efficiency of the proposed sampling plan is compared with the sampling plan developed based on the Wang's test statistic in terms of the sample size required for the selection of a better supplier. A real example is given to explain the proposed sampling plan.

Key words:

Sampling plan; EWMA statistic; linear profile; process-yield index; difference statistic

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1 Introduction

Quality is the critical factor for supplier selection, evaluation of manufacturing firms, reducing rework and operation costs and to increase share of the companies in the market (Weber et al. [1], Olhager and Selldin [2]). The process yield which is based on the process capability index (PCI) is used to judge process performance of a supplier (Montgomery [3]). A yield index is used as one of the tools to judge the quality or performance of two or more suppliers. A higher index results in higher quality and a smaller fraction of nonconforming product. According to Wang [4] “if the index value of one supplier can be shown to be significantly greater than that of another supplier, then the supplier with higher index value will incur lower costs”. Some works to evaluate and to determine a better supplier including various PCIs under a normal distribution can be seen in [5-9]. Lin and Pearn [10] worked on comparing multiple suppliers using PCI. Lin and Kuo [11] presented a method for multiple comparisons based on PCI.

A functional relationship between the dependent variable and explanatory variables is called a profile. Profile monitoring has attracted the researchers in recent years because of its wide applications in quality engineering. A review on the linear and nonlinear profiles is provided by [12]. More details about this type of studies can be seen in [13-20].

An acceptance sampling plan is one of the tools for the inspection of the products at the final stage (Aslam et al. [21]). The sampling plan is also used to select a better supplier to provide a good quality product. Suppose that the null hypothesis is that the product from suppler 1 is better than the product from supplier 2. A random sample with a particular size is selected from the submitted lot of each supplier and the decision on the hypothesis can be made based on a suitable statistic. Wang [4] developed a difference test statistic for two suppliers using linear profile under a normal distribution. We may design a sampling plan based on this test statistic, however this test statistic has to be improved further by using exponentially weighted moving average (EWMA) statistic.

Usually a sampling plan provides the decision about the submitted product using the current state only. The use of EWMA statistic in a sampling plan increases the accuracy in decision about the acceptance of a lot of products. The EWMA statistic enables the engineer to use the current and the past information to make the final decision about the selection of supplier. According to Montgomery [3], this statistic weights sample in exponentially decreasing order. Aslam et al. [22] designed a sampling plan using EWMA statistic when quality of interest follows the normal distribution. More details on the applications of the sampling plans can be seen in [23-32]. Also, some of the sampling plans have been developed based on EWMA statistic (see for example [33-43].

In this paper, we improve the test statistic proposed by Wang [4] using a EWMA scheme when the profile data are available from two suppliers. The designing of a sampling plan is also given using the proposed EWMA statistic. The efficiency of the proposed sampling plan is compared with the one based on the Wang's test statistic in terms of the sample size required for the selection of a better supplier among two. The application of the proposed sampling plan is discussed with the help of an industrial example.

2 Design of Proposed EWMA Plan

Let x_i denote the i-th level of the independent variable of interest and y_{ij} denote the j-th sample of the response variable at the fixed level x_i. It is assumed that the following linear relationship of x_i and y_{ij} holds:

$$y_{ij} = \beta_0 + \beta_1 x_i + \varepsilon_{ij} \qquad i = 1, 2, \ldots, n;\ \ j = 1, 2, \ldots, k \qquad (1)$$

where n is the number of levels of the independent variable and k is the number of observations (or sample size) while β_0 and β_1 are coefficients of the linear profile. Here, ε_{ij} is the error term and follows a normal distribution with mean 0 and variance σ^2. Note that $(y_{1j}, y_{2j}, \ldots, y_{nj})$ is called the j-th profile.

Wang [4] proposed the following process-yield index for the response variable at level x_i:

$$S_{pk_i} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{USL_i-\mu_i}{\sigma_i}\right)+\frac{1}{2}\Phi\left(\frac{\mu_i-LSL_i}{\sigma_i}\right)\right] = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{1-c_{dr_i}}{c_{dp_i}}\right)+\frac{1}{2}\Phi\left(\frac{1+c_{dr_i}}{c_{dp_i}}\right)\right] \quad (2)$$

where $USL_i$and LSL_i are the upper and the lower specification limits of the response variable at x_i, μ_i and σ_i are the mean and standard deviation, respectively of the response variable at x_i, $c_{dr_i} = (\mu_i - m_i)/d_i$, $c_{dr_i} = \sigma_i/d_i$, $m_i = (USL_i + LSL_i)/2$ and $d_i = (USL_i - LSL_i)/2$. Here, $\Phi(x)$ is the cumulative distribution function of a standard normal distribution.

Wang [4] considered the following estimator of the process-yield index for a simple linear profile model in Eq. (1) as

$$S_{pk_i} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{1-\hat{c}_{dr_i}}{\hat{c}_{dp_i}}\right) + \frac{1}{2}\Phi\left(\frac{1+\hat{c}_{dr_i}}{\hat{c}_{dp_i}}\right)\right] \quad (3)$$

where $\hat{c}_{dr_i} = (\hat{\mu}_i - m_i)/d_i$ and $\hat{c}_{dr_i} = \hat{\sigma}_i/d_i$.

Assume that there are two suppliers, supplier 1 and supplier 2, say. Our problem is whether to accept a lot from supplier 1 or 2 providing better quality. Suppose that the supplier 2 claims that their products are of better quality having a higher process-yield index than the products provided by supplier 1. Based on the claim by supplier 2, we set the following null and alternative hypotheses.

$$\begin{cases} H_0: S_{pkA_2} - S_{pkA_1} \geq 0 \\ H_1: S_{pkA_2} - S_{pkA_1} < 0 \end{cases}$$

If sample information supports H_0, then it is concluded that supplier 2 is better than supplier 1 and therefore, a lot of products supplied by the supplier 2 should be accepted. Otherwise, a lot of products provided by the supplier 1 should be accepted. It is assumed that k_1 profiles are available at n_1 levels of the independent variable for supplier 1 and k_2 profiles are available at n_2 levels of the independent variable for supplier 2.

Wang [4] derived the approximate normal distribution of the difference statistic $\widehat{D}$ between supplier 1 and suppler 2 as

$$\widehat{D} = \hat{S}_{pkA_2} - \hat{S}_{pkA_1} \sim N\left(S_{pkA_2} - S_{pkA_1}, \sigma_{S_2}^2 + \sigma_{S_1}^2\right) \quad (4)$$

where

$$\sigma_{S_1}^2 = \frac{G_1^2\left[\emptyset(3G_1^2)\right]^2}{2n_1^2 k_1\left[\emptyset(3S_{pkA_1})\right]^2},\ G_1 = \frac{1}{3}\Phi^{-1}\left\{\frac{n_1\left[2\Phi(3S_{pkA_1})-1\right]-(n_1-2)}{2}\right\}$$

$$\sigma_{S_2}^2 = \frac{G_2^2\left[\emptyset(3G_2^2)\right]^2}{2n_2^2 k_2\left[\emptyset(3S_{pkA_2})\right]^2},\ G_2 = \frac{1}{3}\Phi^{-1}\left\{\frac{n_2\left[2\Phi(3S_{pkA_2})-1\right]-(n_2-2)}{2}\right\}$$

We propose the sampling plan using the EWMA scheme of the above difference test statistic. The proposed plan is stated as follows:

Step-1: At time t, obtain k_1 random profiles at n_1 levels of the independent variable for supplier 1 and k_2 random profiles at n_2 levels of the independent variable for supplier 2.

Step-2: Calculate the difference statistic $\widehat{D}_t = \hat{S}_{pkA_2} - \hat{S}_{pkA_1}$. Then, compute the following EWMA statistic:

$$\widehat{D}_t^{EWMA} = \lambda \widehat{D}_t + (1-\lambda)\widehat{D}_{t-1}^{EWMA} \tag{5}$$

where λ is a smoothing constant ranging from 0 to 1. At t=1, we set $\widehat{D}_t^{EWMA} = \widehat{D}_1$.

Step-3: Accept the lot by supplier 1 if $\widehat{D}_t^{EWMA} \geq c$ or accept the lot by supplier 2, otherwise, where c is the acceptance constant to be determined.

The proposed sampling plan is based on the number of profiles (sample size) for each supplier (k_1 and k_2) and the acceptance constant c when the number of levels is specified. We need to determine the acceptance constant c so as to minimize the sample size while satisfying the producer's and the consumer's risks.

The smoothing constant λ determines the rate at which "previous lots" enter into the calculation of the EWMA statistic. A value of λ=1 implies that only the most recent measurement influences the statistic. Thus, a large value of λ gives more weight to recent data and less weight to older data; a small value of λ gives more weight to older data. The value of λ ranging between 0.1 and 0.3 is recommended in practice [3].

First, the EWMA statistic for sufficiently large t follows the normal distribution given as

$$\widehat{D}_t^{EWMA} \sim N\left(S_{pkA_2} - S_{pkA_1}, (\lambda/2-\lambda)\left(\sigma_{S_2}^2 + \sigma_{S_1}^2\right)\right)$$

Therefore, the operating characteristic (OC) function of the proposed plan is derived as follows

$$P\left(\widehat{D}_t^{EWMA} \geq c\right) = P\left(Z \geq \frac{c-\left(S_{pkA_2}-S_{pkA_1}\right)}{\sqrt{(\lambda/(2-\lambda))\left[\frac{G_1^2\left[\emptyset\left(3G_1^2\right)\right]^2}{2n_1^2k_1\left[\emptyset\left(3S_{pkA_1}\right)\right]^2}+\frac{G_2^2\left[\emptyset\left(3G_2^2\right)\right]^2}{2n_2^2k_2\left[\emptyset\left(3S_{pkA_2}\right)\right]^2}\right]}}\right)$$

where Z is the standard normal random variable. Finally, the lot acceptance probability is given as

$$P\left(\widehat{D}_t^{EWMA} \geq c\right) = 1 - \Phi\left(\frac{c-\left(S_{pkA_2}-S_{pkA_1}\right)}{\sqrt{(\lambda/(2-\lambda))\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n_1^2k_1\left[\phi\left(3S_{pkA_1}\right)\right]^2}+\frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n_2^2k_2\left[\phi\left(3S_{pkA_2}\right)\right]^2}\right]}}\right) \quad (6)$$

Let α be the producer's risk and β be the consumer's risk. The producer is interested in guaranteeing that the lot acceptance probability for a good lot should be larger than his confidence level, 1–α, at acceptable quality level (AQL). The consumer requires that the lot acceptance for a bad lot should be smaller than his risk at lot tolerance percent defective (LTPD). Let C_{AQL_1} be the AQL value of supplier 1 and C_{AQL_2} be the AQL value of supplier 2, while C_{LTPD_1} be the LTPD value of supplier 1 and C_{LTPD_2} be the LTPD value of supplier 2. For the simplicity, it is assumed that $n_1 = n_2 = n$ and $k_1 = k_2 = k$. The plan parameters of the proposed plan using EWMA statistic will be determined by solving the following non-linear optimization problem:

Minimize k (7a)

Subject to

$$1 - \Phi\left(\frac{c-\left(C_{AQL_2}-C_{AQL_1}\right)}{\sqrt{(\lambda/(2-\lambda))\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{AQL_1}\right)\right]^2}+\frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{AQL_2}\right)\right]^2}\right]}}\right) \geq 1-\alpha \quad (7b)$$

$$1 - \Phi\left(\frac{c-\left(C_{LTPD_2}-C_{LTPD_1}\right)}{\sqrt{(\lambda/(2-\lambda))\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{LTPD_1}\right)\right]^2}+\frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{LTPD_2}\right)\right]^2}\right]}}\right) \leq \beta \quad (7c)$$

Here, we consider the cases where the quality level for supplier 2 is higher than that for supplier 1, that is, $\Delta C_{AQL} = C_{AQL_2} - C_{AQL_1} > 0$ and $\Delta C_{LTPD} = C_{LTPD_2} - C_{LTPD_1} > 0$. There are many combinations of $\left(C_{AQL_2}, C_{LTPD_2}, C_{AQL_1}, C_{LTPD_1}\right)$ such as (1.5, 1.3, 1.0, 0.9), (1.6, 1.4, 1.1, 1.0). It should be noted that $C_{AQL} > C_{LTPD}$.

Tables 1-2 present the plan parameters of the proposed sampling plan for various values of smoothing constant as well as the number of independent variable levels when

$\Delta C_{AQL} = 0.5$ and ΔC_{LTPD}= 0.4. Two combinations of (C_{AQL_2}= 1.5, C_{LTPD_2}=1.3, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9) and (C_{AQL_2}= 1.6, C_{LTPD_2}=1.4, C_{AQL_1}= 1.1, C_{LTPD_1}= 1.0) are considered. The producer's and the consumer's risks are chosen by α=0.05 and β=0.10 for all tables.

Tables1-2 are around here

From Tables 1-2, it is noted that for all other same values, as n changes from 2 to 30, there is a decreasing trend in k. As expected, smaller sample size (or profile) is required as a larger number of variables is used. It is also seen that the sample size becomes smaller as a smaller smoothing constant is used.

Tables 3-4 are reported when ΔC_{AQL} =0.6 and ΔC_{LTPD} =0.5. Here again two combinations of (C_{AQL_2}=1.6, C_{LTPD_2}=1.4, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9) and (C_{AQL_2}= 1.7, C_{LTPD_2}=1.5, C_{AQL_1}= 1.1, C_{LTPD_1}= 1.0) are considered.

Tables 3-4 are around here

It is observed that the values of k are increased for these cases as compared with the ones in Tables 1-2.

3 Comparative Study

We may design a sampling plan for the comparison purpose using the Wang's difference statistic given in Eq. (4). In fact, the plan based on the Wang's test statistic is a special case of the proposed sampling plan with $\lambda = 1$. In this section, we compare the proposed sampling plan with the plan developed based on Wang's testing procedure in terms of the sample size required. To compare the efficiency of both sampling plans, we select same values of all specified parameters. The plan parameters of the sampling plan by Wang [4] are placed in the last columns of Table 2 (when ΔC_{AQL} = 0.5 and ΔC_{LTPD} = 0.4) and Table 4 (when ΔC_{AQL} = 0.6 and ΔC_{LTPD} = 0.5).

By comparing these with the ones having λ's smaller than 1, we note that the proposed EWMA plan provides smaller values of k as compared to the Wang's sampling plan. For example, when C_{AQL_2}= 1.5, C_{LTPD_2}=1.3, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9 and n = 5, the value of k for the proposed plan with λ = 0.1 is 32 and it is 66 with λ = 0.2, while it is 589 for the sampling plan based on Wang's test statistic. So, the proposed EWMA sampling plan looks more efficient than the sampling plan without using EWMA scheme. Practically, the values of λ lie between 0.1 and 0.5 are preferable in the industries. So, the proposed plan is more efficient in this range.

4 Application of the Proposed Plan

In this section, we discuss an application of the proposed sampling plan in the leather industry [44]. Wang [4] used the leather industry data to discuss the application of his difference statistic. According to Wang [4] "the quality performance of leather dyeing process is characterized by a relationship between the leather color effluent and temperature. The corresponding color effluent was examined in 150 ml water at five different temperatures, including 25, 32, 39, 46, and 53 °C".

Suppose the industry is using the proposed sampling plan with λ = 0.29 and n=5 by specifying the values of ΔC_{AQL} =0.5 and ΔC_{LTPD} = 0.4. Then, from Table 1, we find k=100 and c = 0.4300. So, we need to collect data for 100 profiles from two suppliers. Suppose now that the specification limits, means and standard deviations (from both supplier 1 and supplier 2) at the five levels of the independent variable are given in Table 5.

Table 5 is around here

Based on the data given in Table 5, the EWMA statistic based on Eq. (5) can be obtained, as $\widehat{D}_t^{EWMA}$ = 0.4896541. According to the proposed plan, since $\widehat{D}_t^{EWMA} > c$=0.4300, we conclude that the supplier 2 has significantly better capability than supplier 1. So, we accept the leather product by supplier 2 and reject the lot of leather products supplied by supplier 1.

5 Concluding Remarks

In this paper, the designing methodology of a sampling plan based on EWMA statistic has been proposed for the inspection of products supplied by two suppliers. The tables for various profiles have been provided for practical use. The performance of the proposed sampling plan has been compared with the plan based on the Wang's test statistic in terms of the sample size required. By comparing the both sampling plans, it is concluded that the proposed plan using EWMA statistic requires a small sample size when deciding about the lot of products by two suppliers. The application of the proposed plan has also been given with the help of industrial data. By applying the proposed plan in the industry for product acceptance, the product by two suppliers can be inspected at the same time. The proposed sampling plan considering multiple suppliers can be considered as future research.

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References

[1] Weber, C.A., Current, J.R. and Benton, W. "Vendor selection criteria and methods", *European Journal of Operational Research*, 50, pp.2-18 (1991).

[2] Olhager, J. and Selldin, E. "Supply chain management survey of Swedish manufacturing firms", *International Journal of Production Economics*, 89, pp. 353-361 (2004).

[3] Montgomery, D.C. *Introduction to statistical quality control*: John Wiley & Sons, New York (2007).

[4] Wang, F.K. "The difference test statistic for two suppliers with linear profiles", *Quality and Reliability Engineering International*, 32, pp. 69-78 (2016).

[5] Daniels, L., Edgar, B., Burdick, R.K. and Hubele, N.F. "Using confidence intervals to compare process capability indices", *Quality Engineering*, 17, pp. 23-32 (2004).

[6] Hubele, N.F., Berrado, A. and Gel, E.S. “A Wald test for comparing multiple capability indices”, *Journal of Quality Technology*, 37, pp. 304-307 (2005).

[7] Polansky, A.M. “Permutation methods for comparing process capabilities”, *Journal of Quality Technology*, 38, pp. 254-266 (2006).

[8] Lin, C.J. and Pearn, W. “Process selection for higher production yield based on capability index Spk”, *Quality and Reliability Engineering International*, 26, pp. 247-258. (2010).

[9] Wu, C.-W., Liao, M.-Y. and Yang, T.-T. “Efficient methods for comparing two process yields–strategies on supplier selection”, *International Journal of Production Research*, 51, pp. 1587-1602. (2013).

[10] Lin, C.-J. and Pearn, W.L. “Group selection for production yield among *k* manufacturing lines”, *Journal of Statistical Planning and Inference*, 141, pp. 1510-1518 (2011).

[11] Lin, C. J. and Kuo, H.H. “Multiple comparisons with the best for supplier selection”, *Quality and Reliability Engineering International*, 30, pp. 1083-1092. (2014).

[12] Woodall, W.H. “Current research on profile monitoring”, *Produção*, 17, pp. 420-425 (2007).

[13] Noorossana, R., Saghaei, A. and Amiri, A. *Statistical analysis of profile monitoring*: John Wiley & Sons; (2012).

[14] Hosseinifard, S.Z. and Abbasi, B. “Evaluation of process capability indices of linear profiles”, *International Journal of Quality & Reliability Management*, 29, pp. 162-176 (2012).

[15] Hosseinifard, S.Z. and Abbasi, B. “Process capability analysis in non normal linear regression profiles”, *Communications in Statistics-Simulation and Computation*, 41, pp. 1761-1784. (2012).

[16] Ebadi, M. and Amiri, A. “Evaluation of process capability in multivariate simple linear profiles”, *Scientia Iranica*, 19, pp. 1960-1968. (2012).

[17] Ebadi, M. and Shahriari, H. “A process capability index for simple linear profile”, *The International Journal of Advanced Manufacturing Technology*, 64, pp. 857-865

(2013).
[18] Wang, F.K. “A process yield for simple linear profiles”, *Quality Engineering*, 26, pp. 311-318 (2014).
[19] Wang, F.K. and Guo, Y.C. “Measuring process yield for nonlinear profiles”, *Quality and Reliability Engineering International*, 30, pp. 1333-1339 (2014).
[20] Wang, F.K. “Measuring the process yield for simple linear profiles with one-sided specification”, *Quality and Reliability Engineering International*, 30, pp. 1145-1151 (2014).
[21] Aslam, M., Azam, M. and Jun, C.-H. “Multiple dependent state sampling plan based on process capability index”, *Journal of Testing and Evaluation*, 41, pp. 340-346 (2013).
[22] Aslam, M., Azam, M. and Jun, C.-H. “A new lot inspection procedure based on exponentially weighted moving average”, *International Journal of Systems Science*, 46, pp. 1329-1400 (2015).
[23] Arif, O.H., Aslam, M. and Jun, C.-H. “Acceptance sampling plan for multiple manufacturing lines using EWMA process capability index”, *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, 11, pp. 1-11 (2017).

[24] Aslam, M., Azam, M. and Jun, C.-H. “Acceptance sampling plans for multi-stage process based on time-truncated test for Weibull distribution”, *The International Journal of Advanced Manufacturing Technology*, 79, pp. 1779-1785 (2015).
[25] Aslam, M., Azam, M. and Jun, C.-H. “A new sampling plan under the exponential distribution”, *Communications in Statistics-Theory and Methods*, 46, pp. 644-652 (2017).
[26] Balamurali, S., Aslam, M. and Liaquat, A. “Determination of a new mixed variable lot-size multiple dependent state sampling plan based on the process capability index”, *Communications in Statistics-Theory and Methods*, 47, pp. 615-627 (2018).
[27] Butt, K., Aslam, M., Wang, F.K., Lee, H. and Jun, C.-H. “Evaluation of two process yields in acceptance sampling plans”, *Journal of Testing and Evaluation*, 46, pp. 756-763 (2018).

[28] Tamirat, Y. and Wang, F.K. "Sampling plan based on the exponentially weighted moving average yield index for autocorrelation within linear profiles", *Quality and Reliability Engineering International*, 32, pp. 1757-1768 (2016).

[29] Wang FK, Tamirat Y, Lo SC and Aslam M. "Dependent mixed and mixed repetitive sampling plans for linear profiles", *Quality and Reliability Engineering International*, 33, pp. 1669-1683 (2017).

[30] Nezhad, M.F. and Nasab, H.H. "A new Bayesian acceptance sampling plan considering inspection errors", *Scientia Iranica*, 19, pp. 1865-1869 (2012).

[31] Fallahnezhad, M., Qazvini, E. and Abessi, M. "Designing an economical acceptance sampling plan in the presence of inspection errors based on maxima nomination sampling method", *Scientia Iranica*, 25, pp. 1701-1711 (2018).

[32] Aslam, M., Niaki, S., Rasool, M. and Fallahnezhad, M. "Decision rule of repetitive acceptance sampling plans assuring percentile life", *Scientia Iranica*. 19, pp. 879-884 (2012).

[33] Balamurali, S. "Determination of a new sampling system based on exponentially weighted moving average", *Sequential Analysis* 37, pp. 158-173 (2018).

[34] Aslam, M., Azam, M. and Jun, C.-H. "Improved acceptance sampling plan based on EWMA statistic", *Sequential Analysis*, 34, pp. 406-422 (2015).

[35] Aslam, M., Azam, M. and Jun, C.-H. "Mixed sampling plan based on exponentially weighted moving average statistic", *Communications in Statistics-Theory and Methods*, 45, pp. 6709-6719 (2016).

[36] Aslam, M., Azam, M. and Jun, C.-H. "Sampling plan using EWMA statistic of regression estimator", *Iranian Journal of Science and Technology, Transactions A*: *Science*, 42, pp. 115-127 (2018).

[37] Azam, M., Arif, O.H., Azam, M. and Ejaz, W. "Repetitive acceptance sampling plan based on exponentially weighted moving average regression estimator", *Journal of Computational and Theoretical Nanoscience*, 13, pp. 4413-4426 (2016).

[38] Khan, N., Aslam, M., Jun, C.-H. and Hussain, J. "Design of acceptance sampling plan using a modified EWMA statistic", *Communications in Statistics-Theory and Methods*, 47, pp. 2881-2891 (2018).

[39] Tamirat, Y. and Mulatu, M.A. "Exponentially weighted moving average yield index for lot sentencing in first order autoregressive between linear profiles", *Industrial Engineering and Management Systems*, 17, pp. 796-804 (2018).

[40] Tamirat, Y. and Wang, F.K. "Acceptance sampling plans based on EWMA yield index for the first autoregressive process", *Journal of the Operational Research Society*, DOI: 10.1080/01605682.2018.1487819 (2019).

[41] Wang, F.K. "A single sampling plan based on exponentially weighted moving average model for linear profiles", *Quality and Reliability Engineering International*, 32, 1795-1805 (2016).

[42] Wang, F.K. and Tamirat, Y. "Acceptance sampling plan based on an exponentially weighted moving average statistic with the yield index for autocorrelation between polynomial profiles", *Communications in Statistics-Theory and Methods*, 47, pp. 4859-4871 (2018).

[43] Yen, C.-H., Aslam, M. and Jun, C.-H. "A lot inspection sampling plan based on EWMA yield index", *The International Journal of Advanced manufacturing Technology*, 75, pp. 861-868 (2014).

[44] Amiri, A.Z. and A, Soudbakhsh D, editors. Monitoring simple linear profiles in the leather industry (a case study). Proceedings of the 2nd International Conference on Industrial Engineering and Operations Management, Kuala Lumpur, Malaysia, January; 2011.

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Table 1: The plan parameters for the proposed plan when ΔC_{AQL} = 0.5 and ΔC_{LTPD} = 0.4

n	λ = 0.10		λ = 0.15		λ = 0.20		λ = 0.29	
	k	c	k	c	k	c	k	c
(C_{AQL_2}= 1.5, C_{LTPD_2}=1.3, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9)								
2	34	0.4243	53	0.4246	73	0.4249	109	0.4228
4	32	0.4288	50	0.4287	70	0.4298	105	0.4286
5	32	0.4306	48	0.4299	66	0.4298	100	0.4300
10	26	0.4306	40	0.4306	57	0.4316	85	0.4311
15	24	0.4299	34	0.4291	48	0.43	71	0.4293
20	20	0.4277	30	0.427	41	0.4278	61	0.4273
25	18	0.4254	27	0.4262	36	0.4255	55	0.4253
30	17	0.425	24	0.4244	34	0.4255	49	0.4241
(C_{AQL_2}= 1.6, C_{LTPD_2}=1.4, C_{AQL_1}= 1.1, C_{LTPD_1}= 1.0)								
2	30	0.4105	46	0.4105	62	0.4107	96	0.4108
4	30	0.419	47	0.4187	62	0.4174	95	0.4183
5	30	0.4211	47	0.4192	64	0.4214	95	0.4197
10	29	0.4254	44	0.4253	61	0.4253	93	0.4254
15	27	0.4278	42	0.428	56	0.4272	86	0.4271
20	26	0.4271	39	0.4279	54	0.4277	84	0.4281
25	25	0.4292	37	0.4279	50	0.4281	77	0.4282
30	23	0.4272	35	0.4282	47	0.4274	72	0.4276

Table 2: The plan parameters for the proposed plan when ΔC_{AQL} = 0.5 and ΔC_{LTPD} = 0.4

n	λ = 0.50		λ = 0.75		Wang's test statistic	
	k	c	k	c	k	c
(C_{AQL_2}= 1.5, C_{LTPD_2}=1.3, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9)						
2	212	0.4229	381	0.4231	630	0.4233
4	204	0.4289	365	0.4288	609	0.4289
5	198	0.4300	356	0.4302	589	0.4298

10	166	0.4305	298	0.4310	491	0.4306
15	139	0.4294	252	0.4296	417	0.4292
20	121	0.4273	216	0.4274	368	0.4276
25	109	0.4262	195	0.4263	322	0.4254
30	100	0.4243	173	0.4237	289	0.4239
(C_{AQL_2}= 1.6, C_{LTPD_2}=1.4, C_{AQL_1}= 1.1, C_{LTPD_1}= 1.0)						
2	186	0.4106	333	0.4110	551	0.4108
4	191	0.4178	331	0.4176	562	0.4175
5	184	0.4199	338	0.4199	553	0.4196
10	179	0.4255	327	0.4255	543	0.4255
15	170	0.4275	305	0.4274	504	0.4272
20	159	0.4280	291	0.4276	480	0.4278
25	149	0.4278	268	0.4279	446	0.4278
30	141	0.4276	251	0.4274	426	0.4276

Table 3: The plan parameters for the proposed plan when ΔC_{AQL} = 0.6 and ΔC_{LTPD} = 0.5

n	λ = 0.10		λ = 0.15		λ = 0.20		λ = 0.29	
	k	*c*	*k*	*c*	*k*	*c*	*k*	*c*
(C_{AQL_2}= 1.6, C_{LTPD_2}=1.4, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9)								
2	39	0.5254	57	0.5221	77	0.5223	116	0.5222
4	35	0.5275	55	0.5277	76	0.5273	116	0.5275
5	35	0.5286	53	0.5286	77	0.5295	110	0.5287

10	30	0.5297	46	0.5296	61	0.5289	95	0.5292
15	27	0.5291	40	0.5278	53	0.5274	80	0.5277
20	23	0.526	35	0.5256	47	0.5254	72	0.5258
25	20	0.5233	31	0.5234	42	0.5243	66	0.5250
30	18	0.5225	28	0.5217	39	0.5237	58	0.5226
(C_{AQL_2}= 1.7, C_{LTPD_2}=1.5, C_{AQL_1}= 1.1, C_{LTPD_1}= 1.0)								
2	33	0.5104	53	0.51	68	0.5104	106	0.5112
4	33	0.5181	52	0.5193	68	0.5174	105	0.5170
5	33	0.5184	50	0.5195	69	0.5197	104	0.5191
10	34	0.5262	50	0.5236	68	0.5243	103	0.5243
15	30	0.5264	49	0.528	63	0.5259	97	0.5261
20	29	0.5272	44	0.5262	60	0.526	90	0.5263
25	27	0.5267	43	0.5261	58	0.5272	86	0.5265
30	26	0.5259	41	0.5261	55	0.5254	81	0.5257

Table 4: The plan parameters for the proposed plan when ΔC_{AQL} = 0.6 and ΔC_{LTPD} = 0.5

n	λ = 0.50		λ = 0.75		Wang's test statistic	
	k	*c*	*k*	*c*	*k*	*c*
(C_{AQL_2}= 1.6, C_{LTPD_2}=1.4, C_{AQL_1}= 1.0, C_{LTPD_1}= 0.9)						
2	234	0.5223	408	0.5222	682	0.5222
4	223	0.5274	399	0.5276	667	0.5276
5	216	0.5285	396	0.5286	647	0.5284
10	185	0.5289	337	0.5298	551	0.5290
15	159	0.5281	286	0.5275	473	0.5276
20	139	0.5256	249	0.5255	417	0.5258
25	124	0.5238	226	0.5246	373	0.5243

30	114	0.5220	203	0.5221	340	0.5221
(C_{AQL_2}= 1.7, C_{LTPD_2}=1.5, C_{AQL_1}= 1.1, C_{LTPD_1}= 1.0)						
2	51	0.5111	361	0.5102	605	0.5103
4	50	0.5169	366	0.5170	608	0.5170
5	50	0.5191	365	0.5189	608	0.5191
10	49	0.5248	363	0.5246	597	0.5246
15	48	0.5253	336	0.5258	572	0.5265
20	45	0.5265	320	0.5264	533	0.5261
25	41	0.5263	305	0.5264	503	0.5264
30	40	0.5273	291	0.5259	482	0.5257

Table 5: Comparison table

Level	X_i (Temp.)	LSL_i	USL_i	$Target_i$	Supplier 1			Supplier 2		
					Mean	S.D.	$\hat{S}_{pk_i}$	Mean	S.D.	$\hat{S}_{pk_i}$
1	25	0.00400	0.06600	0.03500	0.03498	0.01249	0.8274	0.03453	0.01020	1.0120
2	32	0.00600	0.10600	0.05600	0.05570	0.02675	0.6230	0.05508	0.01134	1.4650
3	39	0.00800	0.16600	0.08700	0.08657	0.03405	0.7734	0.08543	0.01120	2.3312
4	46	0.01600	0.20000	0.11000	0.11002	0.01806	1.6881	0.10998	0.01645	1.8517
5	53	0.02000	0.24000	0.13000	0.12808	0.01853	1.9687	0.12934	0.01598	2.2926