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Determination of product acceptance for two suppliers with linear profiles

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Abstract. In the supplier management, it is necessary to compare the performances of two suppliers using linear profiles. In this study, the determination procedure for product acceptance was designed using the Exponentially Weighted Moving Average (EWMA) statistic based on the process-yield index applied to the linear profiles of two suppliers. The design parameters of the proposed plan were also determined to satisfy both the producer's and consumer's risks. The efficiency of the proposed sampling plan was compared to that of the sampling plan developed based on Wang's test statistic in terms of the sample size required for selecting a better supplier. To this end, a real example was presented to explain the proposed sampling plan.

1. Introduction

Quality is a critical factor in selection of supplier, evaluation of manufacturing firms, reduction of rework and operation costs, and promotion of the share of companies in the market [1,2]. The process yield which is based on the Process Capability Index (PCI) is employed to judge the process performance of a supplier [3]. A yield index is used as one of the available tools for judging the quality or performance of two or more suppliers. A higher index results in higher quality and a smaller fraction of nonconforming product. According to Wang [4], "if the index value of one supplier can be shown to be significantly greater than that of another supplier, then the supplier with higher index value will incur lower costs". In [5–9], some studies were introduced that evaluated and determined a better supplier including different PCIs under normal distribution. Lin and Pearn [10] compared multiple suppliers using PCI. Lin and Kuo [11] presented a method for multiple comparisons based on PCI.

A functional relationship between the dependent and explanatory variables is called a profile. Profile monitoring has drawn considerable attention in recent years owing to its wide applications in quality engineering. A review of the linear and nonlinear profiles was provided in [12]. More details about this type of studies can be seen in [13–20].

An acceptance sampling plan is one of the tools for inspecting products at the final stage [21]. The sampling plan is also used to select a better supplier to provide a good-quality product. Assume that according to the null hypothesis, the product from Suppler 1 is better than that from Supplier 2. A random sample with a particular size is selected from the submitted lot of each supplier, and the decision on the hypothesis can be made based on a suitable statistic. Wang [4] developed a difference test statistic for two suppliers using linear profile under a normal distribution. In this study, a sampling plan was designed based on

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this test statistic; however, this test statistic should be further improved by the Exponentially Weighted Moving Average (EWMA) statistic.

Usually, a sampling plan provides the decision about the submitted product using the current state only. Application of EWMA statistic in a sampling plan can increase the accuracy of the decision on the acceptance of many products. The EWMA statistic enables the engineer to use the current and past information to make the final decision on supplier selection. According to Montgomery [3], this statistic weights the sample in an exponentially decreasing order. Aslam et al. [22] designed a sampling plan using EWMA statistic when the quality of interest followed the normal distribution. More details on the applications of the sampling plans can be seen in [23–32]. Moreover, some of the sampling plans have been developed based on EWMA statistic (see for example [33–43].

The present study aimed to improve the test statistic proposed by Wang [4] using an EWMA scheme in the presence of the available profile data on two suppliers. A sampling plan is also designed using the proposed EWMA statistic. The efficiency of the proposed sampling plan is compared with that based on Wang's test statistic in terms of the sample size required for the selection of a better supplier among the two. The application of the proposed sampling plan is illustrated through an industrial example.

2. Design of proposed EWMA plan

Let x_i denote the ith level of the independent variable of interest and y_{ij} the jth sample of the response variable at the fixed level x_i. It is assumed that the following linear relationship of x_i and y_{ij} holds:

$$y_{ij} = \beta_0 + \beta_1 x_i + \varepsilon_{ij},$$

$$i = 1, 2, \cdots, n; \qquad j = 1, 2, \cdots, k, \tag{1}$$

where n is the number of levels of the independent variable, k is the number of observations (or sample size), and β_0 and β_1 are the coefficients of the linear profile. Here, ε_{ij} is the error term that follows a normal distribution with the mean of 0 and variance of σ^2. Note that $(y_{1j}, y_{2j}, \cdots, y_{nj})$ is called the j-th profile.

Wang [4] proposed the following process-yield index for the response variable at level x_i:

$$S_{pk_i} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{USL_i - \mu_i}{\sigma_i}\right) + \frac{1}{2}\Phi\left(\frac{\mu_i - LSL_i}{\sigma_i}\right)\right]$$

$$= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{1 - c_{dr_i}}{c_{dp_i}}\right) + \frac{1}{2}\Phi\left(\frac{1 + c_{dr_i}}{c_{dp_i}}\right)\right], \tag{2}$$

where USL_i and LSL_i are the upper and lower specification limits of the response variable at x_i, μ_i and σ_i are the mean and standard deviation, respectively, of the response variable at x_i, $c_{dr_i} = (\mu_i - m_i)/d_i$, $c_{dri} = \sigma_i/d_i$, $m_i = (USL_i + LSL_i)/2$, and $d_i = (USL_i - LSL_i)/2$. Here, $\Phi(x)$ is the cumulative distribution function of a standard normal distribution.

Wang [4] considered the following estimator of the process-yield index for a simple linear profile model in Eq. (1) as:

$$S_{pk_i} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{1 - \hat{c}_{dr_i}}{\hat{c}_{dp_i}}\right) + \frac{1}{2}\Phi\left(\frac{1 + \hat{c}_{dr_i}}{\hat{c}_{dp_i}}\right)\right], \tag{3}$$

where $\hat{c}_{dr_i} = (\hat{\mu}_i - m_i)/d_i$ and $\hat{c}_{dr_i} = \hat{\sigma}_i/d_i$.

Assume that there are two suppliers: Supplier 1 and Supplier 2. The problem here is whether to accept a lot from Supplier 1 or 2 who can provide a better quality. Suppose that Supplier 2 claims that their products are of better quality with a higher process-yield index than those provided by Supplier 1. Based on their claim, we set the null and alternative hypotheses as follows:

$$\begin{cases} H_0 : S_{pkA_2} - S_{pkA_1} \geq 0 \\ H_1 : S_{pkA_2} - S_{pkA_1} < 0 \end{cases}$$

In case the sample information supports H_0, it can be concluded that Supplier 2 outperforms Supplier 1; therefore, a lot of products supplied by Supplier 2 should be accepted. Otherwise, a lot of products provided by Supplier 1 should be accepted. It is assumed that k_1 profiles are available at n_1 levels of the independent variable for Supplier 1, and k_2 profiles are available at n_2 levels of the independent variable for Supplier 2.

Wang [4] derived the approximate normal distribution of the difference statistic $\hat{D}$ between Supplier 1 and Supplier 2 as:

$$\hat{D} = \hat{S}_{pkA_2} - \hat{S}_{pkA_1} \sim N\left(S_{pkA_2} - S_{pkA_1}, \sigma_{s_2}^2 + \sigma_{s_1}^2\right), \tag{4}$$

where:

$$\sigma_{s_1}^2 = \frac{G_1^2[\phi(3G_1^2)]^2}{2n_1^2 k_1 \left[\phi\left(3S_{pkA_1}\right)\right]^2},$$

$$G_1 = \frac{1}{3}\Phi^{-1}\left\{\frac{n_1\left[2\Phi\left(3S_{pkA_1}\right) - 1\right] - (n_1 - 2)}{2}\right\},$$

$$\sigma_{s_2}^2 = \frac{G_2^2[\phi(3G_2^2)]^2}{2n_2^2 k_2 \left[\phi\left(3S_{pkA_2}\right)\right]^2},$$

$$G_2 = \frac{1}{3}\Phi^{-1}\left\{\frac{n_2\left[2\Phi\left(3S_{pkA_2}\right) - 1\right] - (n_2 - 2)}{2}\right\}.$$

In this study, the sampling plan was proposed using the EWMA scheme of the above difference test statistic. The proposed plan is given as follows:

Step 1: At time t, obtain k_1 random profiles at n_1 levels of the independent variable for Supplier 1 and k_2 random profiles at n_2 levels of the independent variable for Supplier 2.

Step 2: Calculate the difference statistic $\hat{D}_t = \hat{S}_{pkA_2} - \hat{S}_{pkA_1}$. Then, compute the following EWMA statistic:

$$\hat{D}_t^{EWMA} = \lambda \hat{D}_t + (1-\lambda)\,\hat{D}_{t-1}^{EWMA}, \tag{5}$$

where λ is a smoothing constant ranging from 0 to 1. At $t = 1$, we set $\hat{D}_t^{EWMA} = \hat{D}_1$.

Step 3: Accept the lot by Supplier 1 if $\hat{D}_t^{EWMA} \geq c$, or accept the lot by Supplier 2 instead, where c is the acceptance constant to be determined.

The proposed sampling plan is based on the number of profiles (sample size) for each supplier (k_1 and k_2) and the acceptance constant c when the number of levels is specified. The acceptance constant c should be determined to minimize the sample size while satisfying the producer's and the consumer's risks.

The smoothing constant λ determines the rate at which the "previous lots" enter into the calculation of the EWMA statistic. A value of $\lambda = 1$ implies that only the most recent measurement can affect the statistic. Hence, a large value of λ gives more weight to the recent data and less weight to the older data. Further, a small value of λ gives more weight to older data. The value of λ ranging between 0.1 and 0.3 is recommended in practice [3].

First, the EWMA statistic for sufficiently large t follows the normal distribution given as:

$$\hat{D}_t^{EWMA} \sim N\left(S_{pkA_2} - S_{pkA_1}, (\lambda/2-\lambda)\left(\sigma_{s_2}^2 + \sigma_{s_1}^2\right)\right).$$

Therefore, the Operating Characteristic (OC) function of the proposed plan is derived as shown in Box I, where Z is the standard normal random variable. Finally, the lot acceptance probability is given as shown in Box II.

Assume that α is the producer's risk and β the consumer's risk. The producer is interested in guaranteeing that the lot acceptance probability for a good lot should be greater than his confidence level, $1-\alpha$, at Acceptable Quality Level (AQL). The consumer requires that the lot acceptance for a bad lot be smaller than his/her risk at the Lot Tolerance Percent Defective (LTPD). Let C_{AQL_1} be the AQL value for Supplier 1 and C_{AQL_2} the AQL value for Supplier 2, C_{LTPD_1} the LTPD value for Supplier 1, and C_{LTPD_2} the LTPD value for Supplier 2. For simplicity, it is assumed that $n_1 = n_2 = n$ and $k_1 = k_2 = k$. The plan parameters of the proposed plan based on EWMA statistic are determined by solving the non-linear optimization problem in Eq. (7) as shown in Box III.

Here, we consider the cases where the quality level for Supplier 2 is higher than that for Supplier 1, i.e., $\Delta C_{AQL} = C_{AQL_2} - C_{AQL_1} > 0$ and $\Delta C_{LTPD} = C_{LTPD_2} - C_{LTPD_1} > 0$. There are many combinations of $(C_{AQL_2}, C_{LTPD_2}, C_{AQL_1}, C_{LTPD_1})$ such as (1.5, 1.3, 1.0, 0.9) and (1.6, 1.4, 1.1, 1.0). It should be noted that $C_{AQL} > C_{LTPD}$.

Tables 1 and 2 present the plan parameters of the proposed sampling plan for different values of smoothing constant and numbers of independent variable levels when $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$. Two combinations of ($C_{AQL_2} = 1.5$, $C_{LTPD_2} = 1.3$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$) and ($C_{AQL_2} = 1.6$, $C_{LTPD_2} = 1.4$, $C_{AQL_1} = 1.1$, $C_{LTPD_1} = 1.0$) are considered in this study. The producer's and the

$$P\left(\hat{D}_t^{EWMA} \geq c\right) = P\left(Z \geq \frac{c - \left(S_{pkA_2} - S_{pkA_1}\right)}{\sqrt{\left(\lambda/\left(2-\lambda\right)\right)\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n_1^2 k_1\left[\phi\left(3S_{pkA_1}\right)\right]^2} + \frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n_2^2 k_2\left[\phi\left(3S_{pkA_2}\right)\right]^2}\right]}}\right).$$

Box I

$$P\left(\hat{D}_t^{EWMA} \geq c\right) = 1 - \Phi\left(\frac{c - \left(S_{pkA_2} - S_{pkA_1}\right)}{\sqrt{\left(\lambda/\left(2-\lambda\right)\right)\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n_1^2 k_1\left[\phi\left(3S_{pkA_1}\right)\right]^2} + \frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n_2^2 k_2\left[\phi\left(3S_{pkA_2}\right)\right]^2}\right]}}\right). \tag{6}$$

Box II

$$\text{Minimize } k$$

$$\text{subject to :} \tag{7a}$$

$$1-\Phi\left(\frac{c-\left(C_{AQL_2}-C_{AQL_1}\right)}{\sqrt{\left(\lambda/\left(2-\lambda\right)\right)\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{AQL_1}\right)\right]^2}+\frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{AQL_2}\right)\right]^2}\right]}}\right)\geq 1-\alpha, \tag{7b}$$

$$1-\Phi\left(\frac{c-\left(C_{LTPD_2}-C_{LTPD_1}\right)}{\sqrt{\left(\lambda/\left(2-\lambda\right)\right)\left[\frac{G_1^2\left[\phi\left(3G_1^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{LTPD_1}\right)\right]^2}+\frac{G_2^2\left[\phi\left(3G_2^2\right)\right]^2}{2n^2k\left[\phi\left(3C_{LTPD_2}\right)\right]^2}\right]}}\right)\leq \beta. \tag{7c}$$

Box III

consumer's risks are chosen by $\alpha = 0.05$ and $\beta = 0.10$ for all tables.

According to Tables 1 and 2, for all other same values, as n changes from 2 to 30, there is a decreasing trend in k. As expected, smaller sample size (or profile) is required as a larger number of variables are used. It is also observed that the sample size becomes smaller when using smaller smoothing constant.

Tables 3 and 4 are reported when $\Delta C_{AQL} = 0.6$ and $\Delta C_{LTPD} = 0.5$. Here, again, two combinations of ($C_{AQL_2} = 1.6$, $C_{LTPD_2} = 1.4$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$) and ($C_{AQL_2} = 1.7$, $C_{LTPD_2} = 1.5$, $C_{AQL_1} = 1.1$, $C_{LTPD_1} = 1.0$) are considered.

As observed, the values of k increased for these cases as compared with the ones in Tables 1 and 2.

3. Comparative study

A sampling plan is designed for the comparison purpose using Wang's difference statistic given in Eq. (4). In fact, the plan based on Wang's test statistic is a special case of the proposed sampling plan with $\lambda = 1$. In this section, the proposed sampling plan is compared with the plan developed based on Wang's testing procedure in terms of the required sample size. To compare the efficiency of both sampling plans, the same values of all specified parameters were selected. The plan parameters of the sampling plan proposed by Wang [4] are placed in the last columns of Table 2 (when $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$) and Table 4 (when $\Delta C_{AQL} = 0.6$ and $\Delta C_{LTPD} = 0.5$).

A comparison between these plans and those with λ value smaller than 1 showed that the proposed EWMA plan could provide smaller values for k than those of Wang's sampling plan. For example, when $C_{AQL_2} = 1.5$, $C_{LTPD_2} = 1.3$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$, and $n = 5$, the value of k for the proposed plan with $\lambda = 0.1$ is 32, while it is 589 for the sampling plan based on Wang's test statistic. Therefore, the proposed EWMA sampling plan looks more efficient than the sampling plan without the EWMA scheme. Practically, the values of λ lying between 0.1 and 0.5 are preferable in industries. Therefore, the proposed plan is more efficient in this range.

4. Application of the proposed plan

In this section, the application of the proposed sampling plan in the leather industry [44] is discussed. Wang [4] used the leather industry data to discuss the application of his difference statistic. According to Wang [4], "the quality performance of leather dyeing process is characterized by a relationship between the leather color effluent and temperature. The corresponding color effluent was examined in 150 ml water at five different temperatures including 25, 32, 39, 46, and 53°C".

Assume that the industry employs the proposed sampling plan with $\lambda = 0.29$ and $n = 5$ by specifying the values of $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$. Then, according to Table 1, $k = 100$ and $c = 0.4300$. Therefore, it is necessary to collect data for 100 profiles from two suppliers. Table 5 presents the specification limits as well as means and standard deviations (from both Suppliers 1 and 2) at five levels of the independent variable.

Based on the data listed in Table 5, the EWMA statistic based on Eq. (5) can be obtained

Table 1. The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$.

n	$\lambda = 0.10$		$\lambda = 0.15$		$\lambda = 0.20$		$\lambda = 0.29$	
	k	c	k	c	k	c	k	c
($C_{AQL_2} = 1.5$, $C_{LTPD_2} = 1.3$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$)								
2	34	0.4243	53	0.4246	73	0.4249	109	0.4228
4	32	0.4288	50	0.4287	70	0.4298	105	0.4286
5	32	0.4306	48	0.4299	66	0.4298	100	0.4300
10	26	0.4306	40	0.4306	57	0.4316	85	0.4311
15	24	0.4299	34	0.4291	48	0.43	71	0.4293
20	20	0.4277	30	0.427	41	0.4278	61	0.4273
25	18	0.4254	27	0.4262	36	0.4255	55	0.4253
30	17	0.425	24	0.4244	34	0.4255	49	0.4241
($C_{AQL_2} = 1.6$, $C_{LTPD_2} = 1.4$, $C_{AQL_1} = 1.1$, $C_{LTPD_1} = 1.0$)								
2	30	0.4105	46	0.4105	62	0.4107	96	0.4108
4	30	0.419	47	0.4187	62	0.4174	95	0.4183
5	30	0.4211	47	0.4192	64	0.4214	95	0.4197
10	29	0.4254	44	0.4253	61	0.4253	93	0.4254
15	27	0.4278	42	0.428	56	0.4272	86	0.4271
20	26	0.4271	39	0.4279	54	0.4277	84	0.4281
25	25	0.4292	37	0.4279	50	0.4281	77	0.4282
30	23	0.4272	35	0.4282	47	0.4274	72	0.4276

Table 2. The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.5$ and $\Delta C_{LTPD} = 0.4$.

	$\lambda = 0.50$		$\lambda = 0.75$		Wang's test statistic	
n	k	c	k	c	k	c
($C_{AQL_2} = 1.5$, $C_{LTPD_2} = 1.3$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$)						
2	212	0.4229	381	0.4231	630	0.4233
4	204	0.4289	365	0.4288	609	0.4289
5	198	0.4300	356	0.4302	589	0.4298
10	166	0.4305	298	0.4310	491	0.4306
15	139	0.4294	252	0.4296	417	0.4292
20	121	0.4273	216	0.4274	368	0.4276
25	109	0.4262	195	0.4263	322	0.4254
30	100	0.4243	173	0.4237	289	0.4239
($C_{AQL_2} = 1.6$, $C_{LTPD_2} = 1.4$, $C_{AQL_1} = 1.1$, $C_{LTPD_1} = 1.0$)						
2	186	0.4106	333	0.4110	551	0.4108
4	191	0.4178	331	0.4176	562	0.4175
5	184	0.4199	338	0.4199	553	0.4196
10	179	0.4255	327	0.4255	543	0.4255
15	170	0.4275	305	0.4274	504	0.4272
20	159	0.4280	291	0.4276	480	0.4278
25	149	0.4278	268	0.4279	446	0.4278
30	141	0.4276	251	0.4274	426	0.4276

Table 3. The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.6$ and $\Delta C_{LTPD} = 0.5$.

	$\lambda = 0.10$		$\lambda = 0.15$		$\lambda = 0.20$		$\lambda = 0.29$	
n	k	c	k	c	k	c	k	c
($C_{AQL_2} = 1.6$, $C_{LTPD_2} = 1.4$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$)								
2	39	0.5254	57	0.5221	77	0.5223	116	0.5222
4	35	0.5275	55	0.5277	76	0.5273	116	0.5275
5	35	0.5286	53	0.5286	77	0.5295	110	0.5287
10	30	0.5297	46	0.5296	61	0.5289	95	0.5292
15	27	0.5291	40	0.5278	53	0.5274	80	0.5277
20	23	0.526	35	0.5256	47	0.5254	72	0.5258
25	20	0.5233	31	0.5234	42	0.5243	66	0.5250
30	18	0.5225	28	0.5217	39	0.5237	58	0.5226
($C_{AQL_2} = 1.7$, $C_{LTPD_2} = 1.5$, $C_{AQL_1} = 1.1$, $C_{LTPD_1} = 1.0$)								
2	33	0.5104	53	0.51	68	0.5104	106	0.5112
4	33	0.5181	52	0.5193	68	0.5174	105	0.5170
5	33	0.5184	50	0.5195	69	0.5197	104	0.5191
10	34	0.5262	50	0.5236	68	0.5243	103	0.5243
15	30	0.5264	49	0.528	63	0.5259	97	0.5261
20	29	0.5272	44	0.5262	60	0.526	90	0.5263
25	27	0.5267	43	0.5261	58	0.5272	86	0.5265
30	26	0.5259	41	0.5261	55	0.5254	81	0.5257

Table 4. The plan parameters for the proposed plan when $\Delta C_{AQL} = 0.6$ and $\Delta C_{LTPD} = 0.5$.

	$\lambda = 0.50$		$\lambda = 0.75$		Wang's test statistic	
n	k	c	k	c	k	c
($C_{AQL_2} = 1.6$, $C_{LTPD_2} = 1.4$, $C_{AQL_1} = 1.0$, $C_{LTPD_1} = 0.9$)						
2	234	0.5223	408	0.5222	682	0.5222
4	223	0.5274	399	0.5276	667	0.5276
5	216	0.5285	396	0.5286	647	0.5284
10	185	0.5289	337	0.5298	551	0.5290
15	159	0.5281	286	0.5275	473	0.5276
20	139	0.5256	249	0.5255	417	0.5258
25	124	0.5238	226	0.5246	373	0.5243
30	114	0.5220	203	0.5221	340	0.5221
($C_{AQL_2} = 1.7$, $C_{LTPD_2} = 1.5$, $C_{AQL_1} = 1.1$, $C_{LTPD_1} = 1.0$)						
2	51	0.5111	361	0.5102	605	0.5103
4	50	0.5169	366	0.5170	608	0.5170
5	50	0.5191	365	0.5189	608	0.5191
10	49	0.5248	363	0.5246	597	0.5246
15	48	0.5253	336	0.5258	572	0.5265
20	45	0.5265	320	0.5264	533	0.5261
25	41	0.5263	305	0.5264	503	0.5264
30	40	0.5273	291	0.5259	482	0.5257

Table 5. Leather industry data with two suppliers.

Level	X_i (Temp.)	LSL_i	USL_i	$Target_i$	Supplier 1			Supplier 2		
					Mean	S.D.	$\hat{S}_{pk_i}$	Mean	S.D.	$\hat{S}_{pk_i}$
1	25	0.00400	0.06600	0.03500	0.03498	0.01249	0.8274	0.03453	0.01020	1.0120
2	32	0.00600	0.10600	0.05600	0.05570	0.02675	0.6230	0.05508	0.01134	1.4650
3	39	0.00800	0.16600	0.08700	0.08657	0.03405	0.7734	0.08543	0.01120	2.3312
4	46	0.01600	0.20000	0.11000	0.11002	0.01806	1.6881	0.10998	0.01645	1.8517
5	53	0.02000	0.24000	0.13000	0.12808	0.01853	1.9687	0.12934	0.01598	2.2926

as $\hat{D}_t^{EWMA} = 0.4896541$. According to the proposed plan, since $\hat{D}_t^{EWMA} > c = 0.4300$, it can be concluded that Supplier 2 has significantly outperformed Supplier 1. Given this, the leather product by Supplier 2 is accepted and a lot of leather products by Supplier 1 are rejected.

5. Concluding remarks

The main objective of this study was to propose a designing methodology for a sampling plan based on Exponentially Weighted Moving Average (EWMA) statistic to inspect products supplied by two suppliers. The tables for different profiles were taken into consideration for practical purposes. The performance of the proposed sampling plan was compared with that based on the Wang's test statistic in terms of the required sample size. A comparison of both sampling plans revealed that the proposed plan based on EWMA statistic required a small sample size when determining the lot of products by two suppliers. The application of the proposed plan was also illustrated through the industrial data. Application of the proposed plan in the industry for product acceptance made it possible for the product by two suppliers to be inspected at the same time. The proposed sampling plan with multiple suppliers can be considered as the future research.

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