Design of optimum vibration absorbers for a bus vehicle to suppress unwanted vibrations against the harmonic and random road excitations

Alireza Rezazadeh, Hamed Moradi *

Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran, PO. BOX: 11155-9567, Tel: (+98) 21-66165545, Fax: (+98) 21-66000021
* hamedmoradi@sharif.edu ; hamedmoradi@asme.org

Abstract
Unwanted vibrations of the vehicles are regarded as harmful threats to the human health from various biomechanical and psychophysical aspects. Road roughness has been considered as the main cause of unwanted vibrations in bus vehicles. Vertical seat vibrations have been found via simulation of a ten degree of freedom (10-DOF) model of an intercity bus vehicle under harmonic and random excitations caused by road roughness. To suppress undesirable vibrations, mass-spring-damper passive absorbers are proposed in a thirteen degrees of freedom (13-DOFs) model of the bus. By optimizing the characteristics of the embedded passive absorbers under each seat, and implementation of the designed absorbers, it is observed that the vertical displacement amplitudes in the frequency response of the seats are reduced especially near the bus resonant frequencies. In addition, the vertical displacement and acceleration amplitudes are decreased in the random excitation of the road roughness. According to the results, optimized mass-spring-damper absorbers are suggested as a practical solution to suppress the unwanted vibration effects in the bus vehicle.

Keywords: Intercity bus vehicle; 13-DOFs model; Road roughness; Harmonic & random excitations; Unwanted vibrations; Seat comfort; Tunable vibration absorbers

1. Introduction

Professional bus drivers, as occupants operating vehicles for a considerable amount of driving hours, and passengers experience unwanted vibrations particularly because of the road surface excitations [1]. The unwanted vibrations in vehicles are concentrated in relatively low frequencies (0.5-25 Hz), exposing the driver and the passengers to a whole-body-vibration (WBV) condition [2-4].
WBV at low frequencies can be harmful to the human body since the resonant conditions have been reported to occur at natural frequencies of the viscera which are distributed in the low-frequency range of 4-10 Hz [5, 6]. Several studies showed that WBV encompass a wide range of health risks; from biomechanical damages (such as musculoskeletal, cardiovascular, nervous, gastrointestinal, etc.) to psychological damages (such as fatigue, stress, sleep disorder, etc.) [5, 7-9].

Also, exposure to the WBV leads to the low-back-pain (LBP) in the long run [10-12] and LBP has been repeatedly reported as a dominant health risk among professional bus drivers [13-15]. Moreover, LBP is the main reason of long sick leaves in general [7, 10, 11, 16]. The exposure-response relationship between WBV and LBP has been extracted in numerous studies [17-22]; however, the mechanism by which WBV leads to the LBP has not been identified thoroughly [5, 23, 24].

Since vibration isolation in vehicles is more efficient when achieved locally as opposed to globally [25], various main strategies have been proposed in studies to isolate the vibration and reduce the transmission of vibration effects on the seats. In different vehicles, these methods can include approaches such as the passive control, semi-active control and active control.

Maciejewski et al. developed a modified passive suspension system for working machine seats through modification of a viscous-elastic passive seat using air-springs and shock absorbers. In their study, the acceleration of operator and the relative displacement of seat suspension are two opposing criteria to obtain optimal solutions in the frequency range of 0-4 Hz [26].

In a study by Verros et al., a single degree of freedom (SDOF) and a 2-DOFs non-linear and linear quarter models of the vehicle were studied. Models proposed in their study were subjected to the random road excitations to extract a methodology for optimizing the passive suspension damping and stiffness characteristics [27].

Optimization of the passive vibration absorbers can be difficult because of the varying conditions of a vehicle and the amplification of the vibrations near the resonant frequencies [28]. Conversely, in semi-active approaches, the seat suspension can be achieved more sufficiently in various conditions of a vehicle [29]. Eason et al. [30] presented a dynamic absorber with passive and semi-active tuned mass in series to reduce the vibration in 3DOFs system in a wider range of
frequencies. Choi et al. studied the application of magneto-rheological (MR) dampers in vehicles such as trucks [31].

In spite of the capability of optimizable functionality in varying conditions, semi-active systems are known to have a poor performance in relatively low frequencies [32]. In addition, ideal models of the semi-active vibration isolation are not applicable in actual systems due to some attributes of the real operating conditions such as hysteresis of dampers and time delays [1].

Active seat suspension, which is characterized by utilizing an external energy source, leads to a better vibration isolation performance at low frequencies [29]. Numerous studies have been conducted to incorporate this approach in vehicle suspension. Stein [33] proposed a driver’s seat with electro-pneumatic active suspension system using a pneumatic spring. Maciejewski et al. [34] designed an active seat suspension using an adaptive control strategy with a high robustness against the varying load. Gan et al. [29] presented an active seat suspension system considering the time-varying and non-linearity of the system for isolating single and multiple frequency excitations.

Although many studies demonstrated the advantages of the active control for the seat vibration isolation [35-37], technically an active system requires measurements of the velocity and relative displacement. Therefore, it creates unnecessary complications for the system and results in a lower reliability. Moreover, using pneumatic systems in active control suspension can cause shortcomings such as the need to implement a large energy source to maintain the required air pressure and less reliability due to the complex controllability of the active system [38].

In some studies, the three aforementioned control strategies have been compared. Orečný et al. [39] reported that the idealized semi-active control of the seat vibration isolation is improved about 7% compared to the passive seat. Bouazara et al. [40] performed a comparison between passive, semi-active and active vehicle models and concluded that the suspension is improved approximately 50% in the active and semi-active methods.

Choosing a control strategy can become more of a trade-off problem in the actual design. While, according to the comparisons, active control seat suspension may result in a better vibration cancellation, implementing those systems on an intercity bus seats is not feasible from a commercial aspect. Complexity of the system and costs of the design and operation of semi-
active and active methods can be compromised with simplicity and practicability of passive approaches at the expense of a lower vibration cancellation in some cases.

In this research, dynamics of an intercity is modeled using a 10-DOFs mechanism. Vibrations of three specific seats, including the driver’s seat and two passengers’ seat, are evaluated with harmonic and random excitations caused by the asphalt road profile. The multiple degrees of freedom model of the problem provides a more accurate simulation of the real condition to examine the vibration transmission responses to the seats. Afterwards, additional mass-spring-damper passive absorbers are tuned for each seat with optimized characteristics to reduce the vibration transmission effects to the seat leading to a modified model of the bus with passive absorbers including thirteen degrees of freedom.

Efficacy of the optimized passive absorbers is then evaluated by two criteria, including the relative vibrations and accelerations of the seats. Implemented passive seat absorbers decrease the relative vibration amplitudes in random excitations and near the natural frequencies of the bus (under resonant conditions). Reduction in accelerations is also achieved in random excitations, resulting in a lower force transmission to the seats.

According to the model, vibration amplitudes and accelerations are decreased more in passengers’ seats compared to the driver’s seat. Therefore, vibration cancellation with passive absorbers tends to be a more pragmatic and low-cost solution for passengers’ seat in a bus and it is highly recommended to increase the comfort of the rides for passengers. However, passive control strategy for the driver’s seat is not recommended considering long working shifts and more frequent exposure to the unwanted vibrations although vibration enhancements are also achieved using passive absorbers for the driver’s seat.

2. Mathematical modeling of the intercity bus vehicle with vibration absorbers

2.1 Dynamics of the intercity bus vehicle as 10-DOFs model

To study the dynamics and vibrations of the bus, a 10-DOFs model is used in the simulations according to the mathematical model proposed by Dragan Sekulic’ et al. [41]. The bus is considered to have the stiff front axis and the rear axis attached to the vehicle body by two suspension air bags and four telescopic shock absorbers in the front axle in alongside with four suspension air bags and four telescopic shock absorbers on the rear axle. A pneumatic elastic
suspension and a shock absorber are embedded in the driver’s seat and the passengers’ seats are stiff-suspended. The bus has two wheels on the front axle and four wheels on the rear axle which is common in most of the heavy road vehicles. The bus is considered to be fully loaded.

Three locations for studying the vibration effects on the driver and two passengers are selected in a way that these locations represent three different parts of a bus with different experience of the transmitted vibrations in the front overhang, rear overhang and the middle part of the bus (Fig. 1).

Ten independent motions of this model which are consisted of six vertical and four rotational motions, are described as follows (as also demonstrated in Fig. 2):

**Vertical motions related to:**

1- Driver’s seat located in the front overhang ($Z_v$)
2- Passenger’s seat located in the middle of the bus ($Z_{p1}$)
3- Passenger’s seat located in the rear overhang ($Z_{p2}$)
4- Center of gravity of the bus suspension system ($Z$)
5- Center of gravity of the front axle ($Z_1$)
6- Center of gravity of the rear axle ($Z_2$)

**Rotational motions describing the orientation of a Cartesian frame located at the:**

7- Center of gravity of the bus suspension system about the x axis ($\varphi$)
8- Center of gravity of the bus suspension system about the y axis ($\theta$)
9- Center of gravity of the front axle about the x1 axis ($\varphi_1$)
10- Center of gravity of the front axle about the x2 axis ($\varphi_2$)
For simplification, the model is assumed to have a constant speed movement along the x-axis with steady contact between the wheels and road. Front and rear axles are considered to be rigid bodies and the vibration characteristics of the model are linear. All sizes and vibrational characteristics of the model have been extracted from the catalogue of technical data of the bus [42]. Geometrical parameters, mass properties and oscillatory parameters of the model are given in Tables 1, 2 and 3, respectively.

By applying the Lagrange’s equations, the motion equations can be extracted as follows (Equation 1-10) for the vertical and rotational degrees of freedom:

For vertical motions:

\[ Z_v : \quad m_v \ddot{z}_v + b_v \dot{z}_v + c_v z_v - b_v \dot{z}_v - c_v z_v - \dot{s}_z b_v \dot{\phi} - s_i b_v \phi + s_z b_v \dot{\phi} + s_z c_v \theta = 0 \]  \hspace{1cm} (1)

\[ Z_{p1} : \quad m_{p1} \ddot{z}_{p1} + b_{p1} \dot{z}_{p1} + c_{p1} z_{p1} - b_{sp1} \dot{z}_{p1} - c_{sp1} z_{p1} - s_{p1} b_{sp1} \dot{\phi} - s_{p1} c_{sp1} \phi - s_{p1} b_{sp1} \dot{\phi} - s_{p1} c_{sp1} \theta = 0 \]  \hspace{1cm} (2)

\[ Z_{p2} : \quad m_{p2} \ddot{z}_{p2} + b_{p2} \dot{z}_{p2} + c_{p2} z_{p2} - b_{sp2} \dot{z}_{p2} - c_{sp2} z_{p2} - s_{p2} b_{sp2} \dot{\phi} - s_{p2} c_{sp2} \phi - s_{p2} b_{sp2} \dot{\phi} - s_{p2} c_{sp2} \theta = 0 \]  \hspace{1cm} (3)

\[ Z : \quad m \ddot{z} + (b_v + b_{sp1} + b_{sp2} + 2b_p + 2b_c) \dot{z} \\
+ (s rb_v + s rb_{sp1} + s rb_{sp2}) \dot{\phi} \\
+ (s c_v - s c_{sp1} + s c_{sp2}) \phi \\
- (s z b_v + s z b_{sp1} + s z b_{sp2} + 2a b_p - s b c_\zeta) \dot{\theta} \\
- (s c_{p1} - s c_{sp2} + 2a c_p - s b c_\zeta) \theta \\
- b_v \dot{z}_v - c_v \dot{z}_v - b_{sp1} \dot{z}_{p1} - c_{sp1} \dot{z}_{p1} - b_{sp2} \dot{z}_{p2} - c_{sp2} \dot{z}_{p2} - 2b_p \dot{z}_1 - z_1 - 2b_c \dot{z}_2 = 0 \]  \hspace{1cm} (4)

\[ Z_1 : \quad m_{p} \ddot{x}_1 + 2(b_p + b_{pp}) \dot{x}_1 + (c_p + c_{pp}) \dot{z}_1 - 2b_p \dot{z}_1 - 2c_p \zeta + 2b c_\zeta + 2 b c_\zeta = \\
b_{pp} \ddot{e}_{pl} + c_{pp} \ddot{e}_{pl} + b_{pp} \dot{e}_{pl} + c_{pp} \dot{e}_{pl} \]  \hspace{1cm} \hspace{1cm} (5)

\[ Z_2 : \quad m_{p} \ddot{x}_2 + 2(b_c + b_{cp}) \dot{x}_2 + (c_c + c_{cp}) \dot{z}_2 - 2b_c \dot{z}_2 - 2c_c \zeta + 2a b_p \dot{\theta} + 2 a c_\theta \]  \hspace{1cm} \hspace{1cm} (6)

For Rotational motions:
\[ \varphi: \quad J_\varphi \ddot{\varphi} + (s_1^2 b_{s_1} + s_2^2 b_{s_2} + s_3^2 b_{s_3} + 2e_1^2 b_p + 2e_2^2 b_c) \varphi \\
+ (s_1^2 c_{s_1} + s_2^2 c_{s_2} + s_3^2 c_{s_3} + 2e_1^2 c_p + 2e_2^2 c_c) \varphi \\
-s_1 b_{s_1} \ddot{z}_{s_1} - s_2 b_{s_2} \ddot{z}_{s_2} + s_3 b_{s_3} \ddot{z}_{s_3} -s_4 b_{s_4} \ddot{z}_{s_4} - s_5 b_{s_5} \ddot{z}_{s_5} + s_6 b_{s_6} \ddot{z}_{s_6} +
(\dot{s}_{s_1} b_{s_1} - s_3 b_{s_3} + s_5 b_{s_5}) \ddot{z} + (s_1 c_{s_1} - s_3 c_{s_3} + s_5 c_{s_5}) z
\]

\[ - (s_1 s_2 b_{s_1} + s_2 s_3 b_{s_3} + s_3 s_4 b_{s_4} + s_4 s_5 b_{s_5} + s_5 s_6 b_{s_6}) \ddot{\theta} - (s_1 s_2 c_{s_1} + s_3 s_4 c_{s_3} + s_5 s_6 c_{s_6}) \varphi \\
-2e_1^2 b_p \ddot{\varphi} - 2e_1^2 c_p \varphi - 2e_2^2 b_c \varphi - 2e_2^2 c_c \varphi = 0 \] (7)

\[ \theta: \quad J_\theta \ddot{\theta} + (s_1^2 b_{s_1} + s_2^2 b_{s_2} + s_3^2 b_{s_3} + 2a_1^2 b_p + 2a_2^2 b_c) \dot{\theta} \\
+ (s_1^2 c_{s_1} + s_2^2 c_{s_2} + s_3^2 c_{s_3} + 2a_1^2 c_p + 2a_2^2 c_c) \theta \\
+ s_1 b_{s_1} \ddot{z}_{s_1} + s_2 b_{s_2} \ddot{z}_{s_2} + s_3 b_{s_3} \ddot{z}_{s_3} + s_4 b_{s_4} \ddot{z}_{s_4} + s_5 b_{s_5} \ddot{z}_{s_5} - s_6 b_{s_6} \ddot{z}_{s_6} +
(\dot{s}_{s_1} b_{s_1} - s_3 b_{s_3} + s_5 b_{s_5}) \ddot{z} + (s_1 c_{s_1} - s_3 c_{s_3} + s_5 c_{s_5}) z
\]

\[ - (s_1 s_2 b_{s_1} + s_2 s_3 b_{s_3} + s_3 s_4 b_{s_4} + s_4 s_5 b_{s_5} + s_5 s_6 b_{s_6}) \ddot{\varphi} - (s_1 s_2 c_{s_1} + s_3 s_4 c_{s_3} + s_5 s_6 c_{s_6}) \varphi \\
-2ab_1 \ddot{z}_{z_1} - 2ac_1 \ddot{z}_{z_2} - 2bc_1 \ddot{z}_{z_2} = 0 \] (8)

After extracting a system of differential equations by the Lagrange’s equations for each degree of freedom, these equations can be written in a matrix form as:

\[ \begin{bmatrix} [M] & [C] & [K] \end{bmatrix} \{\ddot{x}\} + \{\dot{x}\} + \{x\} = \{F\} \] (11)

In this matrix form, \([M]_{10 \times 10}\), \([C]_{10 \times 10}\) and \([K]_{10 \times 10}\) are the mass, damping and spring stiffness coefficient matrices. Also, \(\{\ddot{x}\}_{10 \times 1}\), \(\{\dot{x}\}_{10 \times 1}\) and \(\{x\}_{10 \times 1}\) denote the acceleration, velocity and displacement of the degrees of freedom. \(\{F\}_{10 \times 1}\) matrix is the external force effect, which represents the effect of displacement excitation due to the road surface profile in this problem. These equations can be solved in modal space; for instance, by means of the Runge-Kutta numerical method. Natural frequencies of the intercity bus vehicle model are given in Table 4.

2.2. Dynamics of the intercity bus vehicle with vibration absorbers
To suppress the unwanted vibrations in the bus, a passive strategy is used in this study. A mass-spring-damper is embedded under each seat of the model (as shown in Fig. 3). By inclusion of the three vertical vibration absorbers under the driver’s seat ($Z_{ex\;v}$) and the each of two other passengers’ seats ($Z_{ex\;p1}$, $Z_{ex\;p2}$), the model becomes 13-DOFs.

Dynamic equations of the intercity bus with passive absorbers (as 13-DOFs model) can be obtained via Lagrange’s equations (Equation 12-17). Additional terms appear in the equations due to the increase in the system’s degrees of freedom. These new terms are distinguished in the following expressions (with blue color):

For vertical motions of each seat:

$$Z_v: \quad m_v \ddot{z}_v + c_{sv} \dot{z}_v + k_{sv} z_v - c_{sv} \dot{z}_v - k_{sv} z_v = s_i c_{sv} \dot{\phi} - s_i k_{sv} \phi + s c_{sv} \dot{\theta} + s_2 k_{sv} \theta - c_{ex\;v} z_{ex\;v} - c_{ex\;v} k_{ex\;v} = 0$$  \hfill (12)

$$Z_{p1}: \quad m_{p1} \ddot{z}_{p1} + c_{sp1} \dot{z}_{p1} + k_{sp1} z_{p1} - c_{sp1} \dot{z}_{p1} - k_{sp1} z_{p1} = s_i c_{sp1} \dot{\phi} + s_2 k_{sp1} \theta + s c_{sp1} \dot{\theta} + s_2 k_{sp1} \theta - c_{ex\;p1} z_{ex\;p1} - k_{ex\;p1} z_{ex\;p1} = 0$$  \hfill (13)

$$Z_{p2}: \quad m_{p2} \ddot{z}_{p2} + c_{sp2} \dot{z}_{p2} + k_{sp2} z_{p2} - c_{sp2} \dot{z}_{p2} - k_{sp2} z_{p2}$$
$$+ s_i c_{sp2} \dot{\phi} + s_2 k_{sp2} \theta + s c_{sp2} \dot{\theta} + s_2 k_{sp2} \theta - c_{ex\;p2} z_{ex\;p2} - k_{ex\;p2} z_{ex\;p2} = 0$$  \hfill (14)

For vertical motions of the damper under each seat:

$$Z_v: \quad m_{ex\;v} \ddot{z}_{ex\;v} + c_{ex\;v} \dot{z}_{ex\;v} + k_{ex\;v} z_{ex\;v} - c_{ex\;v} \dot{z}_{ex\;v} - k_{ex\;v} z_{ex\;v} = 0$$  \hfill (15)

$$Z_{p1}: \quad m_{ex\;p1} \ddot{z}_{ex\;p1} + c_{ex\;p1} \dot{z}_{ex\;p1} + k_{ex\;p1} z_{ex\;p1} - c_{ex\;p1} \dot{z}_{ex\;p1} - k_{ex\;p1} z_{ex\;p1} = 0$$  \hfill (16)

$$Z_{p2}: \quad m_{ex\;p2} \ddot{z}_{ex\;p2} + c_{ex\;p2} \dot{z}_{ex\;p2} + k_{ex\;p2} z_{ex\;p2} - c_{ex\;p2} \dot{z}_{ex\;p2} - k_{ex\;p2} z_{ex\;p2} = 0$$  \hfill (17)

For the rotational motions, the equations are similar to those of the 10-DOFs model (Eqs. (7) through (10)). Similar to the 10-DOFs system, differential equations can be written in a matrix form and can be then solved via a numerical approach; for instance, the Runge-Kutta method in modal space.

$$[M]_{13\times13} \{\ddot{x}\}_{13\times1} + [C]_{13\times13} \{\dot{x}\}_{13\times1} + [K]_{13\times13} \{x\}_{13\times1} = \{F\}_{13\times1}$$  \hfill (18)
2.3. Random model of the road profile

Road roughness profile can be modeled as a stochastic phenomenon. The stochastic model by Johannesson [43] is used in this study to define the vertical displacement of a single point on the road track, as follows:

\[ \xi(t) = \sum_{i=1}^{N} A_i \cos(2\pi\Omega_i t + \alpha_i) \]  

(19)

In this formulation, \( \xi \) is the temporal displacement of a single point on the road, which has been described as a summation of harmonic functions with \( A_i \) amplitudes, \( \Omega_i \) frequencies and \( \alpha_i \) phase degrees which are distributed randomly in \([0-2\pi]\). Frequencies (\( \Omega_i \)) are calculated as follows:

\[ \Omega_i = \Omega + \left( i - \frac{1}{2} \right) \Delta \Omega \]  

(20)

\[ \Delta \Omega = \frac{\Omega - \Omega_l}{N} \]  

(21)

where \( \Delta \Omega \) is the frequency step; also, frequency bounds are considered to be \( \Omega_u = 50 \text{ Hz} \) for the upper frequency bound and \( \Omega_l = 0.5 \text{ Hz} \) for the lower frequency bound. The amplitudes (\( A_i \)) are described as follows:

\[ A_i = \sqrt{2\Phi_\xi(\Omega_i) \Delta \Omega} \]  

(22)

where \( \Phi_\xi(\Omega) \) denotes the road roughness coefficient and is calculated as follows by means of the Power Spectral Density (PSD):

\[ \Phi_\xi(\Omega) = V^{-w-1} \Phi_\xi(v_0) \left( \frac{v_0}{\Omega} \right)^w \]  

(23)

In this expression, \( \Phi_\xi(v_0) \) is the road roughness coefficient for the reference spatial frequency and \( w \) is the fitted PSD exponent. These parameters’ values are extracted for an asphalt concrete road with a good condition as \( w = 2.2 \) and \( \Phi_\xi(v_0) = 1.3 \) for \( v_0 = 1 \). Also, the speed is assumed to be \( V = 80 \text{ Km/h} = 22.23 \text{ m/s} \) in the simulations.
After generating a random profile for vertical displacements of two front wheels \((\xi_{pd}, \xi_{pl})\) in the bus, rear wheels’ vertical displacements can be calculated with a time delay of \(l/V\) seconds due to the constant speed of the bus. This time shift is demonstrated in Fig. 4 for a randomly generated profile.

3. Dynamic Response of the Intercity Bus Model without Passive Absorbers

3.1. Time response of the 10-DOFs model under the random excitation

Dynamics study of three degrees of freedom of the model is of more importance which are the driver’s seat located in the front overhang \((Z_d)\), a passenger’s seat located in the middle of the bus \((Z_{p1})\) and a passenger’s seat located in the rear overhang \((Z_{p2})\). The related time responses of these locations under a random excitation of the road are shown in Fig. 5.

3.2. Time response of the 10-DOFs model under the harmonic excitation

In a different approach from the random excitation of the model, a harmonic excitation of the road profile can be used to extract the maximum amplitude of each degree of freedom’s displacement in the frequency domain which is called the frequency response of the model. In this approach, the amplitude of the harmonic excitation is assumed to be 2 \(cm\) which is approximately equal to the maximum vertical displacement in the random profile of the road roughness (which was shown in Fig. 4).

Similar to the random profile of the road roughness, the harmonic profile is calculated for the front wheels on each side of the vehicle and then the rear wheels’ profiles are calculated with a time delay of \(l/V\) seconds.

To clearly study the effects of designed absorbers on the vibrations of driver and passengers, two distinct models are considered in this study. In the next simulations, one model is in the presence of the bus suspension parameters (damped model) and the other one is in the absence of the suspension absorption parameters (undamped model) for which the characteristics of the suspension system’s absorption \(b_1, b_p, b_2, b_z\) are assumed to be zero.

The natural frequencies of the system are distributed in a frequency range of 0-100 Hz (as listed in Table 4). Frequency responses of the 10-DOFs model without suspension, for driver’s seat and passengers’ seats are demonstrated in Fig. 6.
It can be concluded from Fig. 6 that the resonance phenomena occur in the lower frequency range of 0-30 Hz as expected. Therefore, further simulations are concentrated in this frequency domain of the system. A comparison between the frequency responses of the 10-DOFs model in the presence and absence of the suspension system is shown in Fig. 7 (for the frequency range of 0-30 Hz).

Effect of the bus suspension system is obvious in Fig. 7. As it is observed, the vibration amplitudes transmitted to the driver and the passengers are reduced in the presence of the suspension system; especially near the natural frequencies where the resonance occurs. One objective of this study is to reduce these amplitudes more by means of the embedded mass-spring-damper absorbers to lower the health risk and increase the comfort of the driver and passengers.

4. Optimum Design of Tunable Passive Vibration Absorbers

Optimization of the absorbers’ characteristics can be performed through various methods. 13-DOFs dynamic model of the intercity bus including the three passive mass-spring-damper shock absorbers embedded under each of the three seats is considered. The design method used in this study is an iterative optimization method to decrease the vertical vibration amplitudes in the driver’s seat and the passengers’ seats. Three vibrational characteristics exist including the mass, spring stiffness and damping. It should be mentioned that the damping factor has not been optimized due to the fact that by increasing this factor there is always a decrease in the vertical vibration amplitudes of the seats. Thus, the damping amount has been set to a common used value of $c = 10 \text{ Ns/m}$ in practice.

The algorithm employed in this paper is first, choosing initial values for mass ($m = 10 \text{ Kg}$) and damping factor ($c = 10 \text{ Ns/m}$) for each of the three seats. Then the maximum amplitudes of the vertical displacement for each of the three seats have been recorded for various values of spring stiffness $k$ in a domain of 100-1000 N/m by running the harmonic simulation of the undamped model in the frequency range of 0-30 Hz. The spring stiffness which resulted in the minimum vertical displacement amplitude is then reported as the optimum value of the stiffness of the absorber for each seat. Fig. 8 represents the optimum point of spring stiffness for each three seats’ absorbers. Optimization algorithm in this stage leads to the optimum stiffness value
equals to 410 N/m for the driver’s seat absorber and the stiffness values of 620 N/m for the passengers’ seats.

After extracting the spring stiffness, the same optimization algorithm is performed for the mass of the absorbers to find the mass values which result in the minimum vertical displacement of the seats. This iterative algorithm for optimizing stiffness and mass is practicable if the final vibrational characteristics are not significantly different from the initial values. Fig. 9 demonstrates the optimum absorber mass for each of the three seats in the undamped model.

Results shows an optimum mass value equals to 15 Kg for the driver’s seat absorber and mass values equal to 10 Kg for the passengers’ seat absorbers. Note that the final values of the absorbers’ mass are not significantly different from the initial values of 10 Kg, so the iterative optimization method is practicable and the optimum values for vibrational characteristics are as follows:

\[
\begin{align*}
    m_{\text{ex,v}} &= 15 \text{ Kg} , m_{\text{ex,p1}} = m_{\text{ex,p2}} = 10 \text{ Kg} \\
    k_{\text{ex,v}} &= 410 \text{ N/m} , k_{\text{ex,p1}} = k_{\text{ex,p2}} = 620 \text{ N/m} \\
    c_{\text{ex,v}} &= c_{\text{ex,p1}} = c_{\text{ex,p2}} = 10Nsm
\end{align*}
\]

(23)

5. The Effect of Designed Optimum Absorbers in Suppression of Vibrations

Effects of passive absorbers embedded under the three seats can be evaluated by comparing frequency responses, time responses of the vertical displacements and vertical accelerations of the driver’s and the passengers’ seats in the 10-DOFs and 13-DOFs models.

5.1. The effect of designed optimum absorbers on frequency response of the system

By applying the optimum vibrational characteristics of the absorbers in the 13-DOFs model, harmonic responses can be calculated for both of the damped and undamped models. Fig. 10 shows the effect of embedding absorbers on the response of the undamped model and Fig. 11 shows that effect on the response of the damped model.

According to the comparison between the model with the absorbers and the model without the absorbers, in both damped and undamped cases; embedding passive absorbers under the seats reduce the maximum amplitude of the seats’ vibrations especially near the natural
frequencies. Although the amount of reduction is considerable for all seats, it is more significant for the passengers’ seats compared to the driver’s seat.

5.2. The effect of designed absorbers on time response of the system under random excitation

The effect of implementing passive absorbers can also be evaluated under the random excitation caused by the road roughness. Fig. 12 shows the effect of designed absorbers in vibrations’ suppression of the driver’s seat and the passengers’ seats.

It should be mentioned that in the simulation results, similar random functions of the road excitation are used in the models with/without absorbers to provide the same condition, resulting in a correct comparison. Fig. 12 demonstrates a total reduction in the amplitudes of displacements in random excitation. The reduction is more significant in passengers’ seats compared to the driver’s seat.

5.3. The effect of designed absorbers on vertical acceleration of the system under random excitation

By calculating the second time derivative of the vertical displacements of the seats under the random excitation, vertical accelerations of the seats are calculated. Vertical acceleration corresponds to the vertical force transmitted from the seat to the human body. The comparison between the vertical acceleration of the driver’s seat and the passengers’ seats of the undamped model with/without absorbers is shown in Fig. 13.

After embedding the absorbers and according to these significant reductions in the amplitudes of the vertical accelerations, it is expected that lower vertical forces will be transmitted from the seat to the human body.

6. Conclusions

A passive vibration isolation method is proposed in this paper in order to reduce the unwanted vibrational effects of the road roughness on passengers of an intercity bus vehicle. To provide a more accurate simulation of the real conditions and to investigate the transmitted
vibrations to the seats, a 10-DOFs model of the bus vehicle is considered. After inclusion of the optimized mass-spring-damper absorbers for the seats, a 13-DOFs model is developed.

Implementation of the passive absorbers under the two passengers’ seats and the driver's seat, leads to a considerable reduction in vertical vibration amplitudes under the harmonic excitation of the road roughness. The amount of reduction is more significant near the natural frequencies of the vehicle where the resonance phenomenon occurs.

Under the random excitations of the road roughness, it is shown that the vibration amplitudes and accelerations of the driver’s and passengers’ seats are also decreased by the seat passive absorbers. For both cases of the harmonic and random excitations, the reduction values are higher in the passengers’ seats compared to the driver’s seat.

According to the results obtained, using the passive mass-spring-damper absorbers is suggested for vibrations’ suppression in the bus vehicle. This method is also more economically pragmatic for the public transportation vehicles to enhance the ride comfort and reduce the health risks of the unwanted vibrations. Lower costs and efforts of designing and embedding the passive seat absorbers weigh against the advantages of the semi-active and active methods for reducing the unwanted vibrations in a bus vehicle. However, according to the results, the passive method is less effective for the driver’s seat (in comparison with passengers’ seats). Although the vibration enhancements are also achieved using passive absorbers for the driver’s seat, it is suggested that an active or semi-active absorber is used there due to the longer exposure to the unwanted vibrations. In future work we hope to enhance our modeling approach to take into account the mechanical characteristics of the human body. Mechanistic modeling of human body has been explored in a diverse line of work [44-46] and would lead to a more solid design of vibration absorbers.

7. Conflict of Interest

The authors declare that they have no conflict of interest.
8. References


Figures and Tables Caption

**Fig. 1:** Schematic view of three objective seat locations for vibrational study in the bus model [41]

**Fig. 2:** 10-DOFs model of the bus including the vertical and rotational motions [41]

**Fig. 3:** Schematic view of the passive absorber used for the seats

**Fig. 4:** A random generated vertical displacement of the front wheels (——) and rear wheels (---) for left (upper profile) and right (lower profile) wheels sets

**Fig. 5:** Vertical displacement of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) under a random excitation of the road profile

**Fig. 6:** Frequency response of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) in 10-DOFs model. Vertical lines are the natural frequencies of the model

**Fig. 7:** Frequency response of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) in 10-DOFs model in $0 - 30 \, Hz$ for the undamped model (——) and the damped mode (---)

**Fig. 8:** Maximum vertical displacement of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) by changing the absorber’s stiffness value. The vertical line shows the optimum point.

**Fig. 9:** Maximum vertical displacement of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) by changing the absorber’s mass value. The vertical line shows the optimum point.
**Fig. 10:** The effect of passive absorbers on the frequency response of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) for undamped models without absorber (—) and with absorber (—), vertical lines are natural frequencies of the model.

**Fig. 11:** The effect of passive absorbers on the frequency response of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) for damped models without absorber (—) and with absorber (—).

**Fig. 12:** The effect of passive absorbers on the vertical displacement of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) for undamped models without absorber (—) and with absorber (—) (under random excitation).

**Fig. 13:** The effect of passive absorbers on the vertical acceleration of (a) the driver’s seat, (b) the passenger’s seat P1 (middle seat) and (c) the passenger’s seat P2 (rear seat) for undamped models without absorber (—) and with absorber (—) (under random excitation).

**Table 1:** Geometrical parameters of the model

**Table 2:** Mass properties of the model

**Table 3:** Oscillatory parameters of the model

**Table 4:** Natural frequencies of the intercity bus vehicle
Figures

Fig. 1

Fig. 2
Fig. 5
Fig. 6
Fig. 7
Fig. 8

Fig. 9
Fig. 10
Fig. 12
Fig. 13
### Tables

#### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$l$</td>
<td>Distance from the front wheel to rear wheel</td>
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</tr>
<tr>
<td>$a$</td>
<td>Distance from the front axle to the bus center of gravity</td>
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<td>$b$</td>
<td>Distance from the rear axle to the bus center of gravity</td>
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<td>Distance from the right and left wheel to the front axle center of gravity</td>
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<td>Distance from the right and left wheel to the rear axle center of gravity</td>
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#### Table 2

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<td>The front axle mass</td>
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Table 4

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