Measuring congestion in data envelopment analysis without solving any models

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Abstract

One of the important topics in Data Envelopment Analysis is congestion. Many scholars research in this field and represent their methods. In most of the represented methods, we have to solve lots of models or its used for a special aim like negative data, integer data, different Production Possibility Set and etc. Here we represent our method that measures the congestion without solving a model. It can be used for different Production Possibility Set (different technology) like T_{New} and FDH; different data like negative data and integer data. Also, we can distinguish strongly or weakly congestion of Decision Making Unit. Furthermore, each DMU has congestion, efficient and inefficient, we can measure it by this method. Finally, we represent some numerical example of our purpose method and compare our method with other methods then show the results in tables.

Keywords Data Envelopment Analysis, Congestion, Decision Making Unit, Efficient, Production Possibility Set

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1. Introduction

Data Envelopment Analysis (DEA) models used to estimate the performance of Decision Making Units (DMUs) by measuring the relative efficiency. Charnes et al. [1] expressed CCR model and Banker et al. [2] expressed BCC model. Although other researcher proposed lots of models, these two models are the most important and the most practical models.

For using DEA models, must defined inputs and outputs for DMUs. When augment in one/more inputs cause reduce in one/more outputs, without improving any other inputs or outputs, congestion happen (Cooper et al. [3], [4]). Färe and Grosskopf [5], [6] represented a method to identify the input factors that responsible for the congestion. Brockett et al. [7] believed that congestion of DMU depends on its inputs. Some of the researchers try to represent the method with solving fewer models to measure the congestion (Cooper et al. [8], Khodabakhshi [9], Noura et al. [10]).

Other researchers have represented their models for measuring the congestion in a different situation. Zare-Haghighi et al. [11] measuring the congestion with desirable and undesirable outputs. Mollaeian et al. [12] focused on congestion in the supply chain. Abbasi et al. [13] measuring the congestion in Free Disposal Hull.

congestion has utilization in different cases, such as the economy, industry, energy and so on. Sueyoshi et al. [14], [15] have perused in these cases. They expressed their method by describing desirable and undesirable congestion.

Ebrahimzade Adimi et al. [16] represent a method to find the congestion hyperplane. Khoveyni et al. [17] represent their method for identifying congestion with negative data. Wang et al. [18] present a new definition of congestion. Sole-Ribalta et al. [19] researched on congestion of multiplex networks. The other interesting subjects for the researcher in the congestion topic is energy efficiency (Zhou et al. [20], Zhou et al. [21], Hu et al. [22]).

The remainder of this paper is organized as follows: In section 2, we reviewed some other works in congestion field. Our proposed method is described in section 3. The experimental testing of our proposed method is represented in section 4. The conclusion is represented in section 5.

2. Background

In this section, we briefly define some models for measuring the congestion.

Assume that, the number of DMUs, inputs, and outputs are n, m, and s. The vectors $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{mj})^T$ are the input and output values of DMU_j, $j = 1, \dots, n$, respectively.

Cooper et al. [8] represented this model:

$$
\begin{aligned}
&Max \;\; \varphi+\varepsilon(\sum_{r=1}^{s} s_r^{+}-\varepsilon\sum_{i=1}^{m} s_i^{-c})\\
&s.t.\\
&\sum_{j=1}^{n}\lambda_j x_{ij}+s_{io}^{-c}=x_{io} \qquad , \; i=1,2,\ldots,m\\
&\sum_{j=1}^{n}\lambda_j y_{rj}-s_{ro}^{+}=\varphi_o y_{ro} \quad , \; r=1,2,\ldots,s\\
&\sum_{j=1}^{n}\lambda_j=1\\
&0\le \lambda_j, s_{ro}^{+}, s_{io}^{-c} \qquad , j=1,2,\ldots,n \, , \;\; i=1,2,\ldots,m \, , \;\; r=1,2,\ldots,s
\end{aligned} \tag{1}
$$

$\varepsilon > 0$ is a non-Archimedean element smaller than any positive real number. In fact, it is just used in theory to show two steps model in one model. For more details refer to [7].

The optimal solution of (1) is $(\varphi^*,\; \lambda^*,\; s^{+*},\; s^{-c*})$.

We have congestion if and only if, at least one of the two following conditions is satisfied:

I. $\varphi^* > 1$ and there is at least one $s_i^{-c*} > 0 \quad (i=1,2,\ldots,m)$
II. There is at least one $s_r^{+*} > 0 \quad (r=1,2,\ldots,s)$ and there is at least one $s_i^{-c*} > 0 \quad (i=1,2,\ldots,m)$

Noura et al. [10] represented their method as follow:

First, solve the output-oriented BCC model (Banker et al. [2]):

$$
\begin{aligned}
&Max \;\; \varphi+\varepsilon(\sum_{r=1}^{s} s_r^{+}+\sum_{i=1}^{m} s_i^{-})\\
&s.t.\\
&\sum_{j=1}^{n}\lambda_j x_{ij}+s_{io}^{-}=x_{io} \qquad , \; i=1,2,\ldots,m\\
&\sum_{j=1}^{n}\lambda_j y_{rj}-s_{ro}^{+}=\varphi_o y_{ro} \quad , \; r=1,2,\ldots,s\\
&\sum_{j=1}^{n}\lambda_j=1\\
&0\le \lambda_j, s_{ro}^{+}, s_{io}^{-} \qquad , j=1,2,\ldots,n \, , \;\; i=1,2,\ldots,m \, , \;\; r=1,2,\ldots,s
\end{aligned} \tag{2}
$$

The optimal solution of (2) is $(\varphi^*,\; \lambda^*,\; s^{+*},\; s^{-*})$.

Then define set E as follow:

$$E=\left\{ j \mid \varphi_j^*=1 \right\} \tag{3}$$

$(\varphi_j^* \; is \; \varphi^* \;\; for \; DMU_j)$

A DMU in set E has the highest amounts of ith input component compared with other DMUs is selected.

$$\exists(t \in E) \; ; \; \forall(j \in E) \Rightarrow x_{it} \geq x_{ij} \tag{4}$$

$$x_{it} = x_i^* \quad , \quad i = 1, \ldots, m$$

We have congestion if and only if, at least one of the two following conditions is satisfied:

I. $\varphi^* > 1$ and there is at least one $x_{io} > x_i^* \quad (i = 1, 2, \ldots, m)$

II. There is at least one $s_r^{+*} > 0 \quad (r = 1, 2, \ldots, s)$ and there is at least one $x_{io} > x_i^* \quad (i = 1, 2, \ldots, m)$

Khoveyni et al. [17] represented their method for negative data base on the slack-based DEA approach. Assume that DMU_j, $j = 1, \ldots, n$, is $\mathrm{P}_{\text{Convex}}-$ efficient. Khoveyni et al. [17] represented their method as follow:

$$\begin{aligned}
& Max \; \sum_{r=1}^{s} s_r^+ \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad , \; i = 1, 2, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad , \; r = 1, 2, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad , 0 \leq \lambda_j \; , \; j = 1, 2, \ldots, n \\
& 0 \leq s_i^- \qquad , i = 1, 2, \ldots, m \\
& \varepsilon \leq s_r^+ \qquad , \; r = 1, 2, \ldots, s
\end{aligned} \tag{5}$$

By solving model (5), we have two cases:

1. If the model (5) was feasible, then DMU_o has congestion. For determining the type of its congestion (strong or weak), Khoveyni et al. [17] represented the following model:

$$
\begin{aligned}
& Max \ \sum_{i=1}^{m} s_i^- \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad , \ i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + s_r^{+*} \quad , \ r = 1,2,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad , 0 \le \lambda_j \ , \ j = 1,2,\dots,n \\
& \varepsilon \le s_i^- \qquad , i = 1,2,\dots,m
\end{aligned}
\tag{6}
$$

Where $s_r^{+*}(r=1,2,\dots,s)$ are the obtained optimal solutions from model (5) that are fixed in model (6).

1_a. If the model (6) is feasible, then DMU_o has strong congestion.

1_b. If the model (6) is infeasible, then DMU_o has weak congestion.

2. If the model (5) was infeasible, then DMU_o doesn't have congestion. Then Khoveyni et al. [17] represented the following model:

$$
\begin{aligned}
& Max \ z_k = \sum_{r=1}^{s} s_r^+ \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad , i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad , r = 1,2,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad , 0 \le \lambda_j \ , \ j = 1,2,\dots,n \\
& 0 \le s_i^- \qquad , i = 1,2,\dots,m \\
& 0 \le s_r^+ \qquad , r = 1,2,\dots,s
\end{aligned}
\tag{7}
$$

The optimal solution of (7) is $(z_k{}^*, \ \lambda^*, \ s^{+*}, \ s^{-*})$.

2_a. If $z_k{}^* > 0$, then DMU_o has weak congestion.

2_b. If $z_k{}^* = 0$, then DMU_o doesn't have weak congestion.

Abbasi et al. [13] represented FDH^{-1} output additive model to estimate the congestion in FDH model. At first, consider the following model:

$$
\begin{aligned}
& Max \;\; Z_{FDH} = \sum_{r=1}^{s} s_r^+ \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \qquad , i = 1, 2, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad , r = 1, 2, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad , \lambda_j \in \{0,1\} \, , \; j = 1, 2, \ldots, n \\
& 0 \le s_r^+ \qquad , r = 1, 2, \ldots, s
\end{aligned} \tag{8}
$$

If $Z_{FDH} = 0$ in model (8) then DMU_o is FDH output efficient.

Then consider the following model:

$$
\begin{aligned}
& Max \;\; Z_{FDH^{-1}} = \sum_{r=1}^{s} s_r^+ \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \ge x_{io} \qquad , i = 1, 2, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad , r = 1, 2, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad , \lambda_j \in \{0,1\} \, , \; j = 1, 2, \ldots, n \\
& 0 \le s_r^+ \qquad , r = 1, 2, \ldots, s
\end{aligned} \tag{9}
$$

If $Z_{FDH^{-1}} = 0$ in model (9) then DMU_o is FDH^{-1} output efficient. $DMU_o = (x_o, y_o)$ has congestion if there is $DMU_k = (x_k, y_k)$ that $x_k \le x_o$, $x_k \ne x_o$ and $y_k \ge y_o$, $y_k \ne y_o$. If $x_k < x_o$ and $y_k > y_o$ then DMU_o has strong congestion.

Abbasi et al. [13] expressed that DMU_o has congestion if and only if DMU_o is FDH^{-1} output efficient and it is not FDH output efficient. Therefore, the following steps must be traversed.

1. Solve model (9), the optimal solution of (9) is (λ^*, s^{+*}). Let $y_o = y_o + s^{+*}$. It is obvious that (x_o, y_o) is FDH^{-1} output efficient.
2. Solve model (8) to find (x_o, y_o).
3. If $Z_{FDH} > 0$ then DMU_o has congestion.

3. Proposed method

Based on the Noura et al. [10] method, we propose our method that measure the congestion without solving any model.

Assume that, the number of DMUs, inputs, and outputs are n, m, and s. The vectors $x_j = (x_{1j}, \ldots, x_{mj})^T$ and $y_j = (y_{1j}, \ldots, y_{mj})^T$ are the input and output values of DMU_j, $j = 1, \ldots, n$, respectively.

we will consider the maximum value in each Component of the output.

$For\ DMU_1$ we have:

$$Max\ y_{11} = y_{11}^* \ , \ Max\ y_{21} = y_{21}^* \ , \ \ldots, \ Max\ y_{s1} = y_{s1}^* \tag{10}$$

$For\ DMU_2$ we have:

$$Max\ y_{12} = y_{12}^* \ , \ Max\ y_{22} = y_{22}^* \ , \ \ldots, \ Max\ y_{s2} = y_{s2}^*$$

.

.

.

$For\ DMU_n$ we have:

$$Max\ y_{1n} = y_{1n}^* \ , \ Max\ y_{2n} = y_{2n}^* \ , \ \ldots, \ Max\ y_{sn} = y_{sn}^*$$

Afterwards, we define set F as follow:

$$F = \left\{ y_{rj}^* \ , r = 1, \ldots, s \ , \ j = 1, \ldots, n \right\} \tag{11}$$

A DMU in set F has the highest amounts of ith input component compared with other DMUs is selected.

$$\begin{aligned} &\exists \left(t \in F \right); \forall \left(j \in F \right) \Rightarrow x_{it} \geq x_{ij} \\ &x_{it} = x_i^* \quad , \quad i = 1, \ldots, m \end{aligned} \tag{12}$$

For example, in set F:

DMU_t has the highest amounts of first input component compared with the first input component of the other DMUs:

$$\exists \left(t \in F \right); \forall \left(j \in F \right) \Rightarrow x_{1t} \geq x_{1j} \quad ; \quad x_{1t} = x_1^*$$

DMU_k has the highest amounts of second input component compared with the second input component of the other DMUs:

$$\exists \left(k \in F \right); \forall \left(j \in F \right) \Rightarrow x_{2k} \geq x_{2j} \quad ; \quad x_{2k} = x_2^*$$

and so on.

Definition 1. We have congestion if at least there is one $x_{ip} > x_i^*$ ($i = 1, 2, \ldots, m$)

The congestion in the ith input of DMU_P is

$$s_i^c = x_{ip} - x_i^* \quad , \quad (x_{ip} > x_i^*) \tag{13}$$

For better understanding refer to example 1.

To prove the accuracy of our method, we represent the following theorem.

Theorem 1. If each DMU has the maximum value in each Component of the output, then that DMU is weak efficient and it is on the efficient frontier. Also by solving the output-oriented BCC model, we will have $\varphi^* = 1$.

Proof. Assume for DMU_j we have:

$$\exists r \; ; \; y_{rj} = y_r^*$$

By contradiction suppose that in evaluating DMU_j we have:

$$\varphi^* > 1 \;\; \Rightarrow \begin{pmatrix} x_j \\ \varphi y_j \end{pmatrix} \epsilon \; PPS$$

Then there is another point on PPS (Production Possibility Set) that its rth output is:

$$\varphi^* y_{rj} > y_{rj} = y_r^*$$

That's it a contradiction.

We can develop this theorem as follow:

If each DMU has the maximum value in

- One Component of the output
- Sum of two Component of the output
- Sum of tree Component of the output

.

.

.

- Sum of all Component of the output

then that DMU is weak efficient and it is on the efficient frontier. Also, by solving the output-oriented BCC model, we will have $\varphi^* = 1$.

4. Numerical example

In this section, we apply our method to some numerical examples and compare the results of our method with results of other methods.

4.1 Example 1

Assume that we have 8 DMUs with one input and one output which shown in Fig. 1.

Cooper et al. [8] and Noura et al. [10] solved this example by their models. Here we apply our method to this example and compare the results in Table 1.

Here are 8 DMUs with one input and one output. As you see in figure 1, DMU_B and DMU_C have the maximum value of the output. Set F is:

$$F = \{y_B = y_C = 2\}$$

DMU_C in set F has the highest amounts of input compared with DMU_B. So, we have:

$$x^* = x_C = 3$$

The congestion in the input of DMU_D is:

$$x_D^C = x_D - x^* = 5 - 3 = 2$$

The congestion in the input of DMU_E is:

$$x_E^C = x_E - x^* = 4 - 3 = 1$$

The congestion in the input of DMU_F is:

$$x_F^C = x_F - x^* = 4 - 3 = 1$$

The congestion in the input of DMU_G is:

$$x_G^C = x_G - x^* = 4.5 - 3 = 1.5$$

In Cooper et al. [8] method must solve a model with three steps (model (1)); Infect, we must solve three models to measure the congestion. In Noura et al. [10] method must solve two models (output-oriented BCC (model (2))) then we have to do the computations ((3), (4)). As you see in here there is no need to solve a model in our method and as shown in Table 1, the results of three methods are equal.

4.2 Example 2

Assume that we have 15 DMUs with two inputs and one output which shown in Table 2.

Khoveyni et al. [17] solved this example by their models. Here we apply our method to this example and compare the results in Table 3.

$$F=\{y_C=y_D=y_E=y_F=y_I=1\} \quad , \quad x_1^*=x_{1F}=2 \quad , \quad x_2^*=x_{2E}=x_{2I}=2$$
$$x_G^C=x_{1G}-x_1^*=4-2=2$$
$$x_H^C=x_{2H}-x_2^*=4-2=2$$
$$x_J^C=\begin{cases} x_{1J}-x_1^*=4-2=2 \\ x_{2J}-x_2^*=4-2=2 \end{cases}$$
$$x_K^C=x_{1K}-x_1^*=3-2=1$$
$$x_L^C=x_{2L}-x_2^*=3-2=1$$
$$x_M^C=\begin{cases} x_{1M}-x_1^*=3-2=1 \\ x_{2M}-x_2^*=3-2=1 \end{cases}$$
$$x_O^C=\begin{cases} x_{1O}-x_1^*=3-2=1 \\ x_{2O}-x_2^*=3-2=1 \end{cases}$$

In Khoveyni et al. [17] method must solve at least three models ((5), (6), (7)) to find weakly or strongly congestion efficient DMUs. As you see in here, we measure the congestion in each input and distinguish weakly or strongly congestion of all DMUs not just efficient, without solving a model.

Assume that we had 1500 DMUs instead of 15 DMUs. Using Khoveyni et al. [17] methods means that solving lots of models with immense data, but our purposed method has simple calculating.

4.3 Example 3

Assume that we have 8 DMUs with one input and one output which shown in Table 4.

Abbasi et al. [13] solved this example by their models. Here we apply our method to this example and compare the results in Table 5.

$$F=\{y_D=y_E=4\} \quad , \quad x^*=x_E=7$$
$$x_F^C=x_F-x^*=8-7=1$$
$$x_G^C=x_G-x^*=8-7=1$$
$$x_H^C=x_H-x^*=9-7=2$$

In Abbasi et al. [13] method must solve two mixed-integer programming models ((8), (9)) to measure the congestion. As you see in here there is no need to solve a model in our method and as shown in Table 5, the results of two methods are equal.

5. Conclusion

In this paper, we represented a new method for measuring the congestion without solving a model. It can be used for different Production Possibility Set (different technology) like T_{New} and FDH; different data like negative data and integer data. Also, we can distinguish strongly or weakly congestion of Decision Making Unit. Furthermore, each DMU has congestion efficient and inefficient, we can measure it by this method.

We applied our method in three different situations and the result is shown in the tables. As we discussed in section 4, with the past represented methods, each case has separate models and separate calculating. For example, for FDH Production Possibility Set must use two mixed-integer programming models or for negative data, we must use three complicated models. Assume that we have thousands of DMUs with lots of inputs and outputs; these models take lots of time and need complex calculation. But our method is uncomplicated and we achieve the answer without wasting time. As you see in section 4, our method in comparison with other methods is so easy and we acquire our answer too fast, without solving a model.

For future studies, we can extend our method for fuzzy data and interval data. As there is no need to solving a model in our method; it seems good to use our method for measuring the congestion with these data.

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Captions

Table 1. results of example 1

Table 2. Source: Khoveyni et al. [17]

Table 3. results of example 2

Table 4. Source: Abbasi et al. [13]

Table 5. results of example 3

Fig. 1. Source: Brockett et al. [7] and Noura et al. [10]

Tables

Table 1. results of example 1

DMU	Congestion		
	Cooper et al.’s approach	Noura et al.’s approach	Our new method
A	0	0	0
B	0	0	0
C	0	0	0
D	2	2	2
E	1	1	1
F	1	1	1
G	1.5	1.5	1.5
H	0	0	0

Table 2. Source: Khoveyni et al. [17]

DMU	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Input1	-1	-3	0	-2	-2	2	4	-2	-2	4	3	-2	3	2	3
Input2	-3	-1	-2	0	2	-2	-2	4	2	4	-2	3	3	1	3
Output	-1	-1	1	1	1	1	0	0	1	0	0.5	0.5	0.5	-2	-3

Table 3. results of example 2

DMU	Congestion	
	Khoveyni et al.'s approach	Our new method
G	Weak congestion	Weak congestion
H	Weak congestion	Weak congestion
J	Strong congestion	Strong congestion
K	Weak congestion	Weak congestion
L	Weak congestion	Weak congestion
M	Strong congestion	Strong congestion
O	inefficient	Strong congestion

Table 4. Source: Abbasi et al. [13]

DMU	A	B	C	D	E	F	G	H
Input	2	3	5	6	7	8	8	9
Output	1	3	2	4	4	3	1	2

Table 5. results of example 3

DMU	Congestion	
	Abbasi et al.'s approach	Our new method
G	1	1
F	1	1
H	2	2

Figures

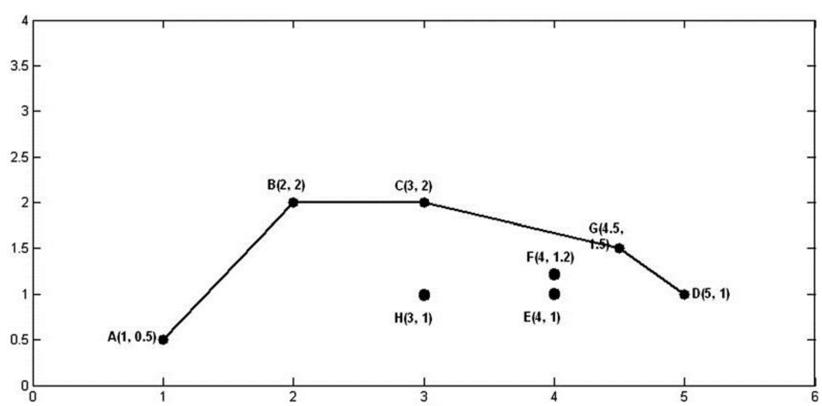

Fig. 1. Source: Brockett et al. [7] and Noura et al. [10]

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