A linear matrix inequality approach to discrete-time finite impulse response controllr design for integrating time-delay processes

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ABSTRACT

Short-term memory discrete-time finite impulse response (FIR) controller design along with an optimized tuning method is presented in this paper. For this purpose, the loop shaping scheme is employed in the linear matrix inequalities (LMIs) framework for adjusting some characteristics of the open-loop frequency response such as phase margin and bandwidth to the desired values at appropriate frequencies. Unlike the conventional methods which work based on state-space models, the proposed procedure generates LMIs directly in the frequency domain. The proposed controller design procedure was applied to several integrating time-delay systems to illustrate its performance and the results were compared with some other competing methods.

Keywords: Discrete-time FIR controller, Linear matrix inequality (LMI), Loop shaping, Integrating time-delay process, Robustness

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1. INTRODUCTION

An integrating system is a process whose transfer function consists of at least one pole at the origin. Such systems are difficult to control because the process output increases persistently over time in response to a step change in input. Level control in distillation column [1, 2], boiler steam drum [3], bio-reactors [4], and DC motors [5] are a few examples of integrating systems.

For several years, much effort has been devoted to the study of methods for controlling integrating systems. For example, Pai et al. [6] investigated the problem of designing PID controllers for integrating time-delay systems by minimizing the integral of the absolute value of the error (IAE) criterion and using a direct synthesis method for disturbance rejection. In [7], tuning formulas for PID controllers cascaded with a first-order noise filter were presented for stable/integrating/unstable systems which has dead-time and oscillatory poles such that satisfactory disturbance rejection is achieved. Mercader and Baños [8] introduced a method for tuning PI controllers for integrating time-delay systems with parametric uncertainty by taking into account the constraints on the magnitude of the sensitivity and complementary sensitivity functions to guarantee optimized disturbance rejection. Studies [9, 10] focused on the design of a model predictive control (MPC) for integrating time-delay systems with model uncertainty.

However, to the best of our knowledge, the previous research has not considered the problem of designing controllers based on loop shaping in the linear matrix inequality (LMI) framework for integrating systems. Loop shaping techniques have been used for designing controllers in several studies; see e.g. [11] and [12]. In fact, in Hara et al. [11], a PID controller design procedure was presented to satisfy multiple frequency domain constraints. Grassi et al. [12] proposed a method for tuning PID controllers to control the temperature of a three-zone industrial diffusion furnace on the basis of loop shaping.

In recent years, many studies have concentrated on the design of controllers via LMI approach [13-16]. Ojaghi et al. [13] designed a robust MPC controller for nonlinear systems with state-dependent uncertainties. To this end, they used an LMI approach to minimize the upper bound of the infinite horizon cost function. Argha et al. [14] designed a robust discrete-time sliding mode controller for uncertain discrete-time systems. For this purpose, they developed a new framework to design a sliding function which is linear with respect to states. Wang et al. [15] tuned multi-loop PID controllers by developing a computationaly efficient method on the basis of LMIs. Wu et al. [16] designed a MIMO PID controller for discrete-time systems by developing algorithms in the framework of LMIs.

Today, almost any controller is realized by using a microprocessor, which means that the designer has to discretize the transfer function of controller if it is designed in *s*-domain. Actually, two basic approaches are adopted when designing a discrete-time controller for a continuous-time plant is aimed. One is to discretize the plant and then to design a controller directly in the discrete-time domain. For instance, Wang et al. [17] derived the discrete-time model of integrating and unstable processes and then designed a discrete-time two-degrees-of-freedom (2DOF) controller for them. The other approach is to design a continuous-time controller for the plant and then to calculate its discrete-time counterpart. However, discretizing a continuous-time controller may result in some undesirable effects such as losing the optimality of the controller or decreasing the phase margin or even instability of the feedback control system. Recently, Merrikh-Bayat et al. [18] proposed a new discrete-time fractional-order PID (FOPID) controller for continuous-time processes and showed that discretizing a continuous-time controller may lead to an unstable closed-loop system, whereas directly tuning a discrete-time controller can eliminate this problem.

The present paper introduces a new discrete-time controller and a new method for tuning its parameters based on the LMI approach. More precisely, the transfer function of the proposed controller is equal to the sum of the positive integer powers of z^{-1}, i.e. it has the structure of a causal finite impulse response (FIR) filter. Application of FIR filter as a controller is studied for the first time in this paper. From the computational point of view, the big advantage of this controller is that regardless of its order it is a linear function of tuning parameters. So, the value of these parameters can be calculated very effectively using LMIs, which are applicable only when the problem under consideration is linear in variables. From the practical point of view, the advantage of the proposed controller is that it has a very simple structure which can be realized very easily. On the other hand, it will be shown later in this paper that an FIR controller tuned by using the proposed LMI approach works considerably better (at least in dealing with the numerical examples under consideration) than advanced PIDs with the same or even more number of tuning parameters when the process is integrating with time-delay.

The rest of this article is organized as follows. Formulation of the problem is discussed in Section 2. Section 3 shows the simulation results. Lastly, Section 4 concludes the paper.

2. PROBLEM FORMULATION

The transfer function of some controllers is a linear function of tuning parameters which can be written as $C(s) = W(s)X$, where X is the vector contains tuning parameters (i.e., variables of the problem) and $W(s)$ is a weight vector whose entries are functions of s. The vectors X and $W(s)$ are specified according to the type of the controller under consideration. For example, consider a PID controller with transfer function

$$C(s) = K_p + \frac{K_i}{s} + K_d s, \tag{1}$$

where K_p, K_i and K_d are the design parameters. Here the vectors $W(s)$ and X are as follows:

$$W(s) = [1 \quad 1/s \quad s], \quad X = [K_p \quad K_i \quad K_d]^T. \tag{2}$$

As another example, by using Tustin method the structure of the digital PID controller takes the following form:

$$C(z) = K_p + K_i \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} + K_d \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}, \tag{3}$$

where T is the sampling period. Thus, the vectors $W(z)$ and X are determined as follows:

$$W(z) = \left[1 \quad \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \quad \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \right], \quad X = [K_p \quad K_i \quad K_d]^T. \tag{4}$$

Finally, consider the FOPID controller with the following transfer function:

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}. \tag{5}$$

Assuming that λ and μ are constant, the vectors $W(s)$ and X can be considered as follows:

$$W(s) = \left[1 \quad \frac{1}{s^{\lambda}} \quad s^{\mu} \right], \quad X = [K_p \quad K_i \quad K_d]^T. \tag{6}$$

The idea of considering the transfer function of controller as $C(s) = W(s)X$ was first proposed in [19] where it was used for tuning the parameters of a fractional-order PID.

The controller proposed in this paper has the structure of a discrete-time FIR filter with short-term memory. More precisely, the transfer function of this controller can be written as follows:

$$C(z) = C_0 + C_1 z^{-1} + C_2 z^{-2} + \cdots + C_n z^{-n}, \tag{7}$$

where C_0, C_1, …, C_n are the real parameters to be tuned. The motivation for proposing this structure for controller is the fact that the Laurent series expansion of the transfer function of any causal discrete-time controller around the origin is, in general, in the form of $C(z) = \sum_{k=0}^{\infty} C_k z^{-k}$ (recall that the necessary condition for causality is $\lim_{z\to\infty} |C(z)| < \infty$; that is why the positive powers of z do not appear in the Laurent series of $C(z)$). Hence, the transfer function of the proposed controller as given in Equation 7 can approximate any causal transfer function with arbitrary precision by assigning a sufficiently large number to n. However, compared to the original controller the proposed structure has the advantage of being linear in tuning parameters.

Assuming the vector of variables as:

$$X = [C_0 \quad C_1 \quad \cdots \quad C_n]^T, \tag{8}$$

the proposed controller can be written as

$$C(z)=C_0+C_1z^{-1}+C_2z^{-2}+\cdots+C_nz^{-n}=[1\ \ z^{-1}\ \ \cdots\ \ z^{-n}]X=W(z)X\ . \tag{9}$$

In the following we develop a method for calculating C_0, C_1, …, C_n. Consider the closed-loop system of Figure 1, where ZOH is the zero-order hold, and $C(z)=W(z)X$ denotes the proposed discrete-time controller. Our aim here is to determine X in a way that the frequency response of the open-loop system satisfies the following three properties which are adopted from [20].

1) The phase margin of the feedback system, ϕ_m, equals the desired value at the given frequency ω_c. In other words, the equality

$$\arg\left\{C(e^{j\omega_c})P(j\omega_c)\right\}=-\pi+\phi_m\,, \tag{10}$$

is met for the given ϕ_m and ω_c. The above problem is equivalent to calculating X from the following optimization problem

$$\min_{C(e^{j\omega_c})}\left\|C(e^{j\omega_c})P(j\omega_c)-e^{j\varphi}\right\|=\min_X\left\|W(e^{j\omega_c})XP(j\omega_c)-B\right\|, \tag{11}$$

where φ is the open-loop phase angle that leads to the desired phase margin (i.e., $\varphi:=-\pi+\phi_m$) and $B:=e^{j\varphi}$. Ideally, the solution of the optimization problem in Equation 11 is obtained via $C(e^{j\omega_c})P(j\omega_c)=e^{j\varphi}$. But, such a solution is not desired since the controller must also satisfy some other properties as it will be discussed later.

In the following we propose an equivalent LMI representation for the optimization problem in Equation 11. By introducing the new scalar variable β, Equation 11 can be expressed as Equation 12:

$$\min\beta$$

$$\text{subject to }\left\|W(e^{j\omega_c})XP(j\omega_c)-B\right\|<\beta\,, \tag{12}$$

which can also be written as

$$\min\beta$$

$$\text{subject to }\left(W(e^{j\omega_c})XP(j\omega_c)-B\right)^H\left(W(e^{j\omega_c})XP(j\omega_c)-B\right)<\beta^2 I\ . \tag{13}$$

According to the Schur complement lemma [21], which states that the matrix inequalities in Relations 14 and 15 as given below:

$$\Phi=\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22}\end{bmatrix}\prec 0,\qquad \Phi_{22}\prec 0\,, \tag{14}$$

$$\Phi_{11}-\Phi_{12}\Phi_{22}^{-1}\Phi_{12}^T\prec 0\,, \tag{15}$$

are equivalent, the norm minimization in Problem 13 corresponds to the following LMI problem:

$$\min\beta$$

$$\text{subject to} \begin{bmatrix} \beta I & W(e^{j\omega_c})XP(j\omega_c)-B \\ \left(W(e^{j\omega_c})XP(j\omega_c)-B\right)^H & \beta I \end{bmatrix} \succ 0. \tag{16}$$

The above problem can be written in the form of a generalized eigenvalue problem (GEVP) problem:

$$\min \beta$$

$$\text{subject to} \begin{bmatrix} 0 & B-W(e^{j\omega_c})XP(j\omega_c) \\ \left(B-W(e^{j\omega_c})XP(j\omega_c)\right)^H & 0 \end{bmatrix} \prec \beta I. \tag{17}$$

The only difficulty with Inequality 17 is that it contains complex matrices while trivial LMI solvers accept only real matrices. In order to remove this trouble we can employ the complex-valued LMIs theorem [22], which states that a Hermitian matrix $M(x)$ satisfies $M(x) \prec 0$ if and only if

$$\begin{bmatrix} \mathrm{Re}\left(M(x)\right) & \mathrm{Im}\left(M(x)\right) \\ -\mathrm{Im}\left(M(x)\right) & \mathrm{Re}\left(M(x)\right) \end{bmatrix} \prec 0. \tag{18}$$

Applying this theorem to Inequality 17 yields:

$$\min \beta$$

$$\text{subject to} \begin{bmatrix} 0 & B_R - W_R X P_R + W_I X P_I & 0 & B_I - W_R X P_I - W_I X P_R \\ * & 0 & -B_I + X^T W_R^T P_I + X^T W_I^T P_R & 0 \\ * & * & 0 & B_R - W_R X P_R + W_I X P_I \\ * & * & * & 0 \end{bmatrix} \prec \beta I, \tag{19}$$

where the subscripts R and I denote the real and imaginary part, respectively. It should be noted that all of the frequency-dependent terms in Inequality 19 are evaluated at $\omega = \omega_c$. Considering the fact that the matrix in Inequality 19 is symmetric, only the entries above the main diagonal are represented and the other entries are shown by $*$.

2) The closed-loop system shows an acceptable level of robustness to uncertainties in the gain of process. This requirement can be achieved by satisfying the following equality:

$$\left. \frac{\mathrm{d}\left(\measuredangle C\left(e^{j\omega}\right)P\left(j\omega\right)\right)}{\mathrm{d}\omega} \right|_{\omega=\omega_C} = 0. \tag{20}$$

Clearly, satisfaction of the above equality means flatness of the Bode phase plot around $\omega = \omega_c$. Unfortunately, Equation 20 is nonlinear in X and cannot be represented by LMIs. One approximate approach to make the Bode phase plot of $C\left(e^{j\omega}\right)P\left(j\omega\right)$ almost flat at frequencies around $\omega = \omega_c$ is to calculate X such that equation:

$$\measuredangle\left\{C\left(e^{j\omega_0}\right)P\left(j\omega_0\right)\right\} = \varphi, \tag{21}$$

holds, where $\varphi := -\pi + \phi_m = \measuredangle\left\{C\left(e^{j\omega_c}\right)P\left(j\omega_c\right)\right\}$ and ω_0 is a frequency close to ω_c. Substitution of $C\left(e^{j\omega_0}\right) = W\left(e^{j\omega_0}\right)X$ in Equation 21 yields

$$\measuredangle\left\{W\left(e^{j\omega_0}\right)X\right\} = \varphi - \measuredangle P\left(j\omega_0\right). \tag{22}$$

Considering $W\left(e^{j\omega_0}\right) = W_R\left(e^{j\omega_0}\right) + jW_I\left(e^{j\omega_0}\right)$, one can write

$$\tan^{-1}\frac{W_I\left(e^{j\omega_0}\right)X}{W_R\left(e^{j\omega_0}\right)X} = \varphi - \measuredangle P\left(j\omega_0\right). \tag{23}$$

Taking $\tan(.)$ from both sides of Equation 23 yields

$$\left(W_I\left(e^{j\omega_0}\right) - \tan\left(\varphi - \measuredangle P\left(j\omega_0\right)\right)W_R\left(e^{j\omega_0}\right)\right)X = 0. \tag{24}$$

The LMI representation of Equation 24 is as follows

$$-\varepsilon < \left(W_I\left(e^{j\omega_0}\right) - \tan\left(\varphi - \measuredangle P\left(j\omega_0\right)\right)W_R\left(e^{j\omega_0}\right)\right)X < \varepsilon, \tag{25}$$

where ε is a very small positive real constant. Note that Inequality 25 consists of two LMI constraints.

3) The feedback system reduces the high frequency noise. This is accomplished if the following inequality is satisfied:

$$\left|\frac{C\left(e^{j\omega}\right)P\left(j\omega\right)}{1+C\left(e^{j\omega}\right)P\left(j\omega\right)}\right| \le D \text{ dB} \qquad \omega \ge \omega_d \text{ rad/s}, \tag{26}$$

where D and ω_d are given constants. Clearly, Inequality 26 is satisfied if inequality:

$$\left|C\left(e^{j\omega_d}\right)P\left(j\omega_d\right)\right| \le \alpha, \tag{27}$$

holds for an appropriate α. Substituting $C\left(e^{j\omega_d}\right) = W\left(e^{j\omega_d}\right)X$ in Inequality 27 yields

$$-\alpha^2 + W\left(e^{j\omega_d}\right)X\left|P\left(j\omega_d\right)\right|^2 X^T W^H\left(e^{j\omega_d}\right) < 0. \tag{28}$$

Inequality 28 is nonlinear in X, but using the Schur complement lemma it can be written as

$$\begin{bmatrix} -\alpha^2 & W\left(e^{j\omega_d}\right)X \\ X^T W^H\left(e^{j\omega_d}\right) & -\left|P\left(j\omega_d\right)\right|^{-2} \end{bmatrix} \prec 0, \tag{29}$$

which is linear in X. Finally, considering Inequality 18, the complex-valued LMI in Inequality 29 can be represented in the following equivalent real-valued form:

$$\begin{bmatrix} -\alpha^2 & W_R\left(e^{j\omega_d}\right)X & 0 & W_I\left(e^{j\omega_d}\right)X \\ * & -\left|P\left(j\omega_d\right)\right|^{-2} & -X^T W_I^T\left(e^{j\omega_d}\right) & 0 \\ * & * & -\alpha^2 & W_R\left(e^{j\omega_d}\right)X \\ * & * & * & -\left|P\left(j\omega_d\right)\right|^{-2} \end{bmatrix} \prec 0. \tag{30}$$

2.1. Stability Analysis

According to the discussions of previous section, the vector of unknown variables, X, can be calculated from Inequality 19 subject to Inequalities 25 and 30. But, in dealing with some problems the controller obtained in this manner may lead to an unstable feedback system since the notion of stability is not considered in the formulation of problem (recall that in order to achieve stability we just adjust one point on the frequency response of the open-loop system by setting phase margin to the desired value). In such cases we can add some extra (approximate) linear constraints to the formulation of problem to preserve stability. These stability constraints can be obtained from the Jury stability test as discussed below.

For example, consider a unity feedback control system where the transfer functions of the integrating process with time delay (IPTD) and controller are $P(s)=K\,e^{-Ls}/s$ and $C(z)=C_0+C_1z^{-1}$, respectively. Assuming $T=0.5L$, where T is the sampling period, characteristic equation of the closed-loop system is obtained as follows

$$\Delta(z)=z^3-z^2+KC_0z+KC_1=0. \tag{31}$$

For the sake of simplicity assume $K=1$. It is concluded from the Jury stability test that a feedback system with characteristic Equation 31 is stable if and only if the following three inequalities hold simultaneously:

$$-1<C_1<1,, \tag{32}$$

$$0<\left(1-C_1^2\right)^2-\left(C_0+C_1\right)^2, \tag{33}$$

$$0<\left(1-C_1^2\right)^2-\left(C_0+C_1\right)^2-\left(1+C_0C_1\right)^2\left(C_1^2+C_0+C_1-1\right). \tag{34}$$

Unfortunately Inequalities 33 and 34 are nonlinear in $X=\begin{bmatrix}C_0 & C_1\end{bmatrix}^T$. The approximate method proposed in this paper to derive LMIs from Inequalities 33 and 34 is to plot the region defined by these inequalities in C_0-C_1 plane and approximate it by a convex polygon. Figure 2 shows the region defined through Inequalities 32-34 in red and a convex polygon used to approximate it in black. Of course, infinitely many polygons with different number of sides can be used to approximate this region where the accuracy of approximation is increased by increasing the number of sides of the polygon. Similarly, when the discrete-time controller under consideration has three parameters, the region of stability can be approximated by a polyhedron. Note that when the approximating polygon or polyhedron is non-convex, one can partition it to several convex sets and then design the controller for each set separately and finally choose the optimal controller.

3. ILLUSTRATIVE EXAMPLES

The results of MATLAB simulations for two different integrating time-delay processes are summarized in this section.

Example 1. Consider the following first-order delayed integrating process (FODIP) taken from [23-25]:

$$P(s)=\frac{0.2}{s(4s+1)}e^{-s}. \tag{35}$$

The proposed controller is designed for this process to reach $\omega_c=0.5\,\mathrm{rad/s}$ rad/s, $\phi_m=60^\circ$, $\omega_d=100\,\mathrm{rad/s}$, and $D=-40\,\mathrm{dB}$. Table 1 summarizes the values obtained for two- and three-parameter controllers. To measure the robustness of the resulting feedback system when the proposed controller is used, a perturbation of +10% is applied simultaneously to the time delay and process gain, and the simulations are repeated. Figures 3 and 4 show the step responses of the closed-loop systems with nominal and perturbed process models, respectively when different controllers are employed (details of the controllers used for comparison can be found in Table 2). Note that in both of these figures a -50% disturbance is applied at

$t = 30\text{s}$. As it can be observed, the propounded controllers suppress other controllers in terms of set-point tracking and disturbance rejection. Additionally, it is observed that the proposed controller with the term z^{-2} yields the best response. Table 3 represents the IAE, overshoot, and phase margin indices for the proposed controller and the controllers calculated in [23-25] (the stepinfo command in MATLAB is used for computing the maximum overshoots). It should be noted that in this example the controllers are calculated without the need to taking into account the stability constraints. However, it was observed that considering the stability constraints when the controller has two tuning parameters makes no changes in the results.

Example 2. Consider the following double integrating process with time delay (DIPTD) discussed in [23], [25] and [26]

$$P(s) = \frac{1}{s^2} e^{-s} \tag{36}$$

The proposed controller is designed for this process to reach $\omega_c = 0.73\,\text{rad/s}$, $\phi_m = 60^\circ$, $\omega_d = 100\,\text{rad/s}$, and $D = -40\text{dB}$. Table 1 summarizes the values obtained for two- and three-parameter controllers. To measure the robustness of the resulting feedback system when the proposed controller is used, a perturbation of +10% is applied to the time delay and process gain, and the simulations are repeated. Figures 5 and 6 show the step responses of the closed-loop systems with nominal and perturbed process models, respectively when different controllers are employed (details of the controllers used for comparison can be found in Table 2). Note that in both of these figures a -50% disturbance is applied at $t = 50$ s. It is observed that the proposed controller can provide a lower overshoot compared to the controller proposed by Anil and Sree [25] and a faster response and a better disturbance rejection compared to the controllers introduced by Jin and Liu [23] and Lee et al. [26]. Interesting point is that the proposed controller has a lower order and less tuning parameters compared to the controllers designed in [23], [25] and [26]. The corresponding performance indices are listed in Table 3 for comparison. Note that in this example the discrete-time controllers were designed without considering the approximate linear stability constraints developed in Section 2.1.

4. CONCLUSION

This paper introduced a new discrete-time controller and an LMI-based method for tuning its parameters. Structure of the proposed controller is like an FIR filter and the proposed tuning method calculates the parameters of controller by open-loop shaping such that the desired phase margin and bandwidth are obtained. Moreover, robustness of the closed-loop system to uncertainties in the process model is considered in the formulation of algorithm. The other contribution of paper is developing approximate linear constraints to achieve closed-loop stability when the order of controller is at most equal to three. All of the control objectives and stability constraints were formulated using LMIs and then readily solved using MATLAB. The propounded approach was applied to two different integrating processes with time delay. The results of simulations showed the superiority of the proposed structure and tuning method over some existing methods in the considered examples.

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Figure Captions

Figure 1. Block diagram of the closed-loop system.

Figure 2. The region of stability obtained from Inequalities 34-36 and its polygon approximation.

Figure 3. Step responses of the closed-loop system with the nominal model of process, corresponding to Example 1.

Figure 4. Step responses of the closed-loop system with the +10% perturbed model of process, corresponding to Example 1.

Figure 5. Step responses of the closed-loop system with the nominal model of process, corresponding to Example 2.

Figure 6. Step responses of the closed-loop system with the +10% perturbed model of process, corresponding to Example 2.

Table Captions

Table 1. Parameters calculated for the proposed controller

Table 2. Parameter settings for comparing controllers with transfer function $K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{\alpha s+1}{\beta s+1}$

Table 3. Comparison of different methods in terms of performance and robustness

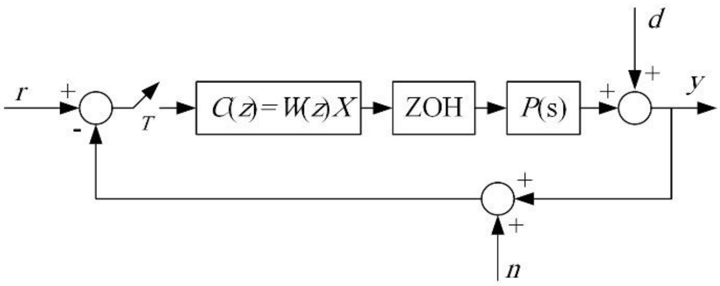

Figure 1

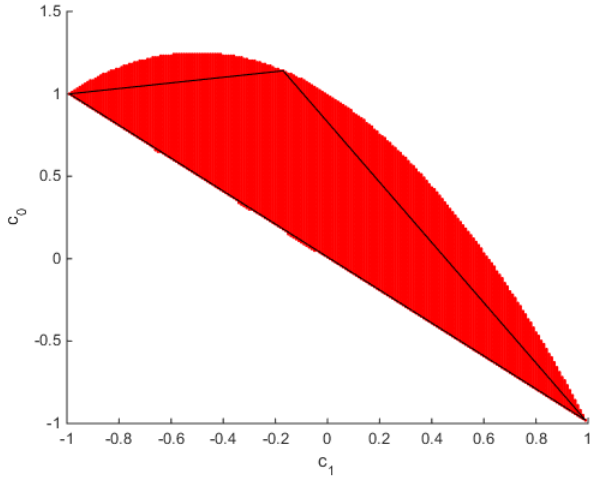

Figure 2

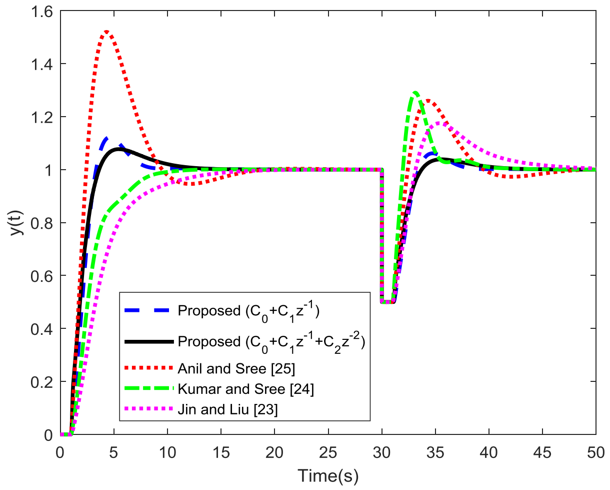

Figure 3

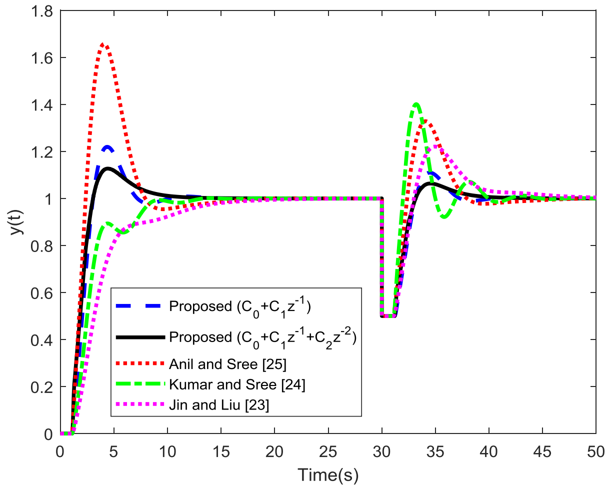

Figure 4

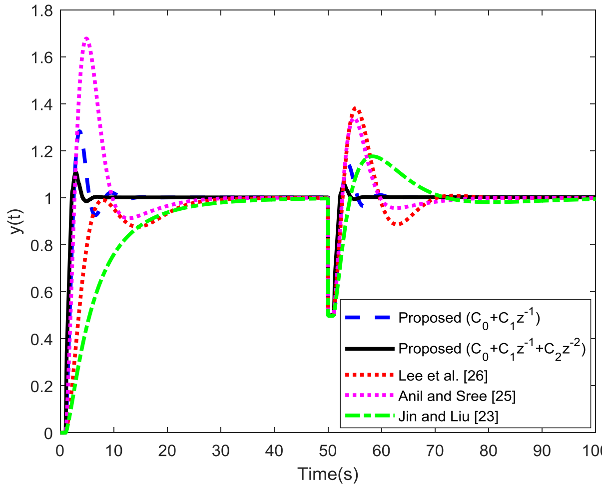

Figure 5

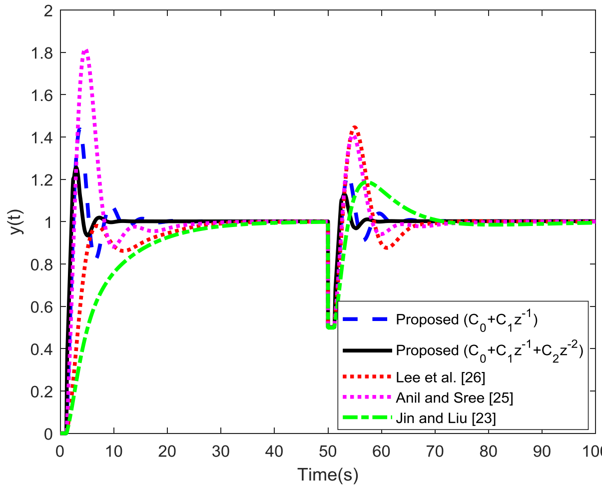

Figure 6

Table 1

Example	Proposed controller	C_0	C_1	C_2	Sampling period (T)
Ex. 1 $P(s)=\dfrac{0.2e^{-s}}{s(4s+1)}$	$C_0+C_1z^{-1}$	101.4935	-98.6551	-	0.1
	$C_0+C_1z^{-1}+C_2z^{-2}$	339.2258	-574.0294	237.8912	0.1
Ex. 2 $P(s)=\dfrac{e^{-s}}{s^2}$	$C_0+C_1z^{-1}$	7.0849	-7.0842	-	0.1
	$C_0+C_1z^{-1}+C_2z^{-2}$	30.6793	-54.3350	23.6565	0.1

Table 2

Example	Proposed controller	K_c	T_i	T_d	α	β	Set-point filter
Ex. 1	Anil and Sree [25]	5.74	5.90	1.95	0.63	0.49	-
	Kumar and Sree [24]	7.41	7.80	1.94	0.50	0.19	$\frac{0.7s+1}{3.8s+1}$
	Jin and Liu [23]	3.68	10.39	2.47	-	-	$\frac{9.8s^2+6.2s+1}{25.7s^2+10.4s+1}$
Ex. 2	Lee et al. [26]	0.14	7.07	3.53	-	-	$\frac{6.25s^2+5s+1}{25s^2+10s+1}$
	Anil and Sree [25]	0.13	9.72	3.82	1.07	1.04	-
	Jin and Liu [23]	0.05	21.38	7.25	-	-	$\frac{46.2s^2+13.6s+1}{155.1s^2+21.4s+1}$

Table 3. Comparison of different methods in terms of performance and robustness

Example	Method	Nominal model			+10% perturbed model		
		IAE	Overshoot	ϕ_m	IAE	Overshoot	ϕ_m
Ex. 1							
	$C_0+C_1z^{-1}$	3.69	12.22	54.66	4.00	21.90	48.49
	$C_0+C_1z^{-1}+C_2z^{-2}$	3.51	7.69	58.57	3.58	12.67	53.55
	Anil and Sree [25]	5.97	51.83	28.97	5.96	65.69	22.93
	Kumar and Sree [24]	2.88	29.00	30.05	3.19	40.13	20.85
	Jin and Liu [23]	4.43	17.00	37.62	4.37	21.55	34.39
Ex. 2							
	$C_0+C_1z^{-1}$	3.84	28.04	45.28	4.84	43.39	36.37
	$C_0+C_1z^{-1}+C_2z^{-2}$	2.64	10.66	57.65	3.16	25.24	49.42
	Lee et al. [26]	5.91	38.21	18.44	5.64	44.55	13.79
	Anil and Sree [25]	8.06	67.76	20.39	8.09	81.62	14.69
	Jin and Liu [23]	6.05	18.30	25.97	5.66	19.31	26.25