A linear matrix inequality approach to discrete-time finite impulse response controller design for integrating time-delay processes

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Abstract. The present study proposes a short-term memory discrete-time Finite Impulse Response (FIR) controller design along with an optimized tuning method. To this end, the loop shaping scheme was employed in the framework of Linear Matrix Inequalities (LMIs) to adjust some characteristics of the open-loop frequency response such as phase margin and bandwidth to the desired values at appropriate frequencies. Unlike the conventional methods whose functions are based on state-space models, the proposed procedure generates LMIs directly in the frequency domain. The proposed controller design procedure was applied to several integrating time-delay systems to illustrate its performance, and the obtained results were compared with the results of some other competing methods.

1. Introduction

An integrating system is a process whose transfer function consists of at least one pole at the origin. Such systems are difficult to control because the process output develops persistently over time in response to a step change in the input. Level control in distillation column [1,2], boiler steam drum [3], bio-reactors [4], and DC motors [5] are a few examples of integrating systems.

For several years, much effort has been devoted to the study of methods for controlling the integrating systems. For example, Pai et al. [6] investigated the problem of designing PID controllers for integrating time-delay systems by minimizing the Integral of Abso-lute Error (IAE) criterion and using a direct synthesis method for disturbance rejection. In [7], tuning formulas for PID controllers cascaded by a first-order noise filter were introduced to stable/integrating/unstable systems equipped with dead-time and oscillatory poles to achieve the satisfactory disturbance rejection. Mercader and Banos [8] proposed a method for tuning PI controllers in integrating time-delay systems with parametric uncertainty by considering the constraints on the sensitivity magnitude and complementary sensitivity functions to guarantee the optimized disturbance rejection. Studies [9,10] focused on the design of a Model Predictive Control (MPC) for integrating time-delay systems with model uncertainty.

However, to the best of our knowledge, the problem of designing controllers based on loop shaping in the Linear Matrix Inequality (LMI) framework for integrating systems has not been investigated in the previous researches. Loop shaping techniques have been used for designing controllers in several studies (see [11,12]). Hara et al. [11] presented a PID controller

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design procedure to satisfy multiple frequency domain constraints. Grassi et al. [12] proposed a method for tuning PID controllers to control the temperature of a three-zone industrial diffusion furnace on the basis of loop shaping.

In recent years, a number of studies have concentrated on the design of controllers using LMI approach [13–16]. Ojaghi et al. [13] designed a robust MPC controller for nonlinear systems with state-dependent uncertainties. To this end, they employed an LMI approach to minimizing the upper bound of the infinite horizon cost function. Argha et al. [14] designed a robust discrete-time sliding mode controller for uncertain discrete-time systems. In this respect, they developed a new framework to design a sliding function which was linear in terms of states. Wang et al. [15] tuned multi-loop PID controllers by developing a computationally efficient method on the basis of LMIs. Wu et al. [16] designed an Multi-Input Multi-Output (MIMO) PID controller for discrete time systems by developing algorithms in the framework of LMIs.

Today, almost any controller is realized through a microprocessor; in other words, the designer should discretize the transfer function of the controller if it is designed in the s-domain. In fact, two basic approaches are adopted while designing a discrete-time controller for a continuous-time plant. In the first approach, the plant is first discretized and then, a controller is directly designed in the discrete-time domain. For instance, Wang et al. [17] derived the discrete-time model of integrating and unstable processes and then, designed a discrete-time two-Degree-Of-Freedom (2DOF) controller for them. In the second approach, a continuous-time controller is designed for the plant and then, its discrete-time counterpart is calculated. However, discretizing a continuous-time controller may yield some undesirable effects such as loss of the controller optimality, decrease in the phase margin, and even instability of the feedback control system. Recently, Merrikh-Bayat et al. [18] proposed a new discrete-time Fractional-Order PID (FOPID) controller for continuous-time processes and showed that while discretizing a continuous-time controller may cause instability in the closed-loop system, directly tuning a discrete-time controller can eliminate this problem.

The present study aims to introduce a new discrete-time controller as well as a new method for tuning its parameters based on the LMI approach. More precisely, the transfer function of the proposed controller is equal to the sum of the positive integer powers of z^{-1}, i.e., it is characterized by the structure of a causal Finite Impulse Response (FIR) filter. Application of FIR filter as a controller has been investigated for the first time in this paper. From the computational point of view, the major advantage of this controller

is that regardless of its order, it is a linear function of tuning parameters. Therefore, the values for these parameters can be efficiently calculated using LMIs, which are applicable only when the problem under consideration is linear in variables. From the practical point of view, the main advantage of the proposed controller is that it has quite a simple structure, which can be realized with no difficulty. However, it is shown later in this paper that an FIR controller tuned by the proposed LMI approach operates considerably better (at least, in dealing with the numerical examples under consideration) than the advanced PIDs with the same or even more tuning parameters when the process is integrating with time-delay.

The rest of this study is organized as follows. Section 2 discusses the formulation of the problem. Section 3 presents the simulation results. Finally, Section 4 concludes the paper.

2. Problem formulation

The transfer function of some controllers is a linear function of tuning parameters which can be written as $C(s) = W(s)X$, where X is the vector contains tuning parameters (i.e., variables of the problem) and $W(s)$ is a weight vector whose entries are functions of s. The vectors X and $W(s)$ are specified according to the type of the controller under consideration. For example, a PID controller with transfer function is considered:

$$C(s) = K_p + \frac{K_i}{s} + K_d s, \qquad (1)$$

where K_p, K_i, and K_d are the design parameters. Here, the vectors $W(s)$ and X are as follows:

$$W(s) = \begin{bmatrix} 1 & 1/s & s \end{bmatrix}, \quad X = \begin{bmatrix} K_p & K_i & K_d \end{bmatrix}^T. \qquad (2)$$

As another example, by using Tustin method, the structure of the digital PID controller takes the following form:

$$C(z) = K_p + K_i \frac{T}{2} \frac{1 + z^{-1}}{1 - z^{-1}} + K_d \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}, \qquad (3)$$

where T is the sampling period. Therefore, the vectors $W(z)$ and X are determined as follows:

$$W(z) = \begin{bmatrix} 1 & \frac{T}{2} \frac{1+z^{-1}}{1-z^{-1}} & \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} \end{bmatrix},$$

$$X = \begin{bmatrix} K_p & K_i & K_d \end{bmatrix}^T. \qquad (4)$$

Finally, the FOPID controller with the following transfer function is considered:

$$C(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu. \qquad (5)$$

Given that λ and μ are assumed to be constant, the vectors $W(s)$ and X can be considered as follows:

$$W(s) = \begin{bmatrix} 1 & \frac{1}{s^\lambda} & s^\mu \end{bmatrix}, \quad X = \begin{bmatrix} K_p & K_i & K_d \end{bmatrix}^T. \quad (6)$$

The idea of considering the transfer function of the controller as $C(s) = W(s)X$ was first proposed in [19], where it was used for tuning the parameters of a FOPID.

The controller proposed in this paper is characterized by the structure of a discrete-time FIR filter with short-term memory. To be specific, the transfer function of this controller can be written as follows:

$$C(z) = C_0 + C_1 z^{-1} + C_2 z^{-2} + \cdots + C_n z^{-n}, \quad (7)$$

where $C_0, C_1, \cdots, C_n$ are the real parameters to be tuned. The motivation that lies behind the proposition of this structure for controller is that the Laurent series expansion of the transfer function of any causal discrete-time controller around the origin is generally in the form of $C(z) = \sum_{k=0}^{\infty} C_k z^{-k}$ (keep in mind that the necessary condition for causality is $\lim_{z\to\infty} |C(z)| < \infty$; this is the reason why the positive powers of z do not appear in the Laurent series of $C(z)$). In this regard, the transfer function of the proposed controller, as given in Eq. (7), can approximate any causal transfer function with arbitrary precision by assigning a sufficiently large number to n. However, compared to the original controller, the proposed structure enjoys the advantage of linearity in tuning parameters.

Assume the vector of variables as follows:

$$X = \begin{bmatrix} C_0 & C_1 & \cdots & C_n \end{bmatrix}^T. \quad (8)$$

Then, the proposed controller can be written as:

$$\begin{aligned} C(z) =& C_0 + C_1 z^{-1} C_2 z^{-2} + \cdots + C_n z^{-n} \\ =& \begin{bmatrix} 1 & z^{-1} & \cdots & z^{-n} \end{bmatrix} X = W(z)X. \end{aligned} \quad (9)$$

In the following, a method is developed for calculating C_0, C_1, ..., C_n. Consider the closed-loop system shown in Figure 1, where ZOH is the Zero-Order Hold and $C(z) = W(z)X$ is the proposed discrete-time controller. The main objective here is to determine X such that the frequency response of the open-loop system satisfies the following three properties adopted from [20]:
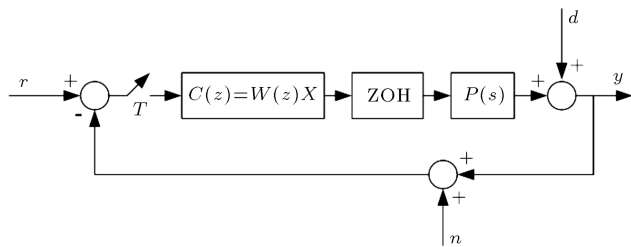
Figure 1. Block diagram of the closed-loop system.

1. The phase margin of the feedback system, ϕ_m, equals the desired value at the given frequency ω_c. In other words, the equality:

$$\arg \left\{ C(e^{j\omega_c}) P(j\omega_c) \right\} = -\pi + \phi_m, \quad (10)$$

is met for the given ϕ_m and ω_c. The above problem is equivalent to calculating X from the following optimization problem:

$$\begin{aligned} \min_{C(e^{j\omega_c})} & \left\| C\left(e^{j\omega_c}\right) P\left(j\omega_c\right) - e^{j\varphi} \right\| \\ &= \min_X \left\| W\left(e^{j\omega_c}\right) XP(j\omega_c) - B \right\|, \end{aligned} \quad (11)$$

where φ is the open-loop phase angle that forms the desired phase margin (i.e., $\varphi := -\pi + \phi_m$) and $B := e^{j\varphi}$. Ideally, the solution to the optimization problem in Eq. (11) is obtained through $C(e^{j\omega_c})P(j\omega_c) = e^{j\varphi}$. However, such a solution is not desired since the controller must also satisfy some other properties, as it will be discussed later.

In the following, an equivalent LMI representation for the optimization problem in Eq. (11) is proposed. By introducing the new scalar variable β, Eq. (11) can be expressed as Eq. (12):

$$\min \quad \beta,$$
$$\text{subject to} \quad \left\| W(e^{j\omega_c}) XP(j\omega_c) - B \right\| < \beta, \quad (12)$$

which can also be written as follows:

$$\min \quad \beta,$$
$$\begin{aligned} \text{subject to} \quad & \left(W\left(e^{j\omega_c}\right) XP(j\omega_c) - B \right)^H \\ & \left(W\left(e^{j\omega_c}\right) XP(j\omega_c) - B \right) < \beta^2 I. \end{aligned} \quad (13)$$

According to the Schur complement lemma [21] stating that the following matrix inequalities in Eqs. (14) and (15):

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \prec 0, \qquad \Phi_{22} \prec 0, \quad (14)$$

$$\Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^T \prec 0, \quad (15)$$

are equivalent, the norm minimization in Problem (13) corresponds to the LMI problem (Inequality (16)), shown in Box I. This problem can be written in the form of a Generalized Eigenvalue Problem (GEVP) (Inequality (17)), shown in Box II. The only difficulty with Inequality (17) is that it contains complex matrices, while trivial LMI solvers accept only real matrices. In order to eliminate this trouble, the complex-valued LMIs theorem was

$$\min \quad \beta,$$

$$\text{subject to} \quad \begin{bmatrix} \beta I & W(e^{j\omega_c})XP(j\omega_c) - B \\ \left(W(e^{j\omega_c})XP(j\omega_c) - B\right)^H & \beta I \end{bmatrix} \succ 0. \tag{16}$$

<div align="center">Box I</div>

$$\min \quad \beta,$$

$$\text{subject to} \quad \begin{bmatrix} 0 & B - W(e^{j\omega_c})XP(j\omega_c) \\ \left(B - W(e^{j\omega_c})XP(j\omega_c)\right)^H & 0 \end{bmatrix} \prec \beta I. \tag{17}$$

<div align="center">Box II</div>

employed [22], stating that a Hermitian matrix $M(x)$ satisfies $M(x) \prec 0$ if and only if:

$$\begin{bmatrix} \text{Re}(M(x)) & \text{Im}(M(x)) \\ -\text{Im}(M(x)) & \text{Re}(M(x)) \end{bmatrix} \prec 0. \tag{18}$$

Applying this theorem to Inequality (17) would yield, Inequality (19) as shown in Box III, where the subscripts R and I denote the real and imaginary parts, respectively. Of note, all of the frequency-dependent terms in Inequality (19) are evaluated at $\omega = \omega_c$. Given that the matrix in Inequality (19) is symmetric, only the entries above the main diagonal are represented and the other entries are shown by $*$.

2. The closed-loop system shows an acceptable level of robustness to uncertainties in the gain of the process. This requirement can be achieved by satisfying the following equality:

$$\left. \frac{d\left(\angle C\left(e^{j\omega}\right) P\left(j\omega\right)\right)}{d\omega} \right|_{\omega = \omega_c} = 0. \tag{20}$$

Clearly, satisfying the above equality points to the flatness of the Bode phase plot around $\omega = \omega_c$.

Unfortunately, Eq. (20) is nonlinear in X and it cannot be represented by LMIs. One approximate approach to making the Bode phase plot of $C(e^{j\omega})P(j\omega)$ almost flat at frequencies around $\omega = \omega_c$ is to calculate X such that the following equation holds:

$$\angle \left\{ C(e^{j\omega_0})P(j\omega_0) \right\} = \varphi, \tag{21}$$

where $\varphi := -\pi + \phi_m = \angle\{C(e^{j\omega_c})P(j\omega_c)\}$ and ω_0 is a frequency close to ω_c. Substitution of $C(e^{j\omega_0}) = W(e^{j\omega_0})X$ into Eq. (21) yields:

$$\angle \left\{ W\left(e^{j\omega_0}\right) X \right\} = \varphi - \angle P(j\omega_0). \tag{22}$$

By taking $W(e^{j\omega_0}) = W_R(e^{j\omega_0}) + jW_I(e^{j\omega_0})$ into account, one can write:

$$\tan^{-1} \frac{W_I\left(e^{j\omega_0}\right) X}{W_R\left(e^{j\omega_0}\right) X} = \varphi - \angle P\left(j\omega_0\right), \tag{23}$$

where taking $\tan(\cdot)$ from both sides of Eq. (23) yields:

$$\left(W_I\left(e^{j\omega_0}\right) - \tan(\varphi - \angle P(j\omega_0))W_R\left(e^{j\omega_0}\right)\right)X = 0. \tag{24}$$

The LMI representation of Eq. (24) is as follows:

$$\min \quad \beta,$$

$$\text{subject to} \quad \begin{bmatrix} 0 & B_R - W_R X P_R + W_I X P_I & 0 & B_I - W_R X P_I - W_I X P_R \\ * & 0 & -B_I + X^T W_R^T P_I + X^T W_I^T P_R & 0 \\ * & * & 0 & B_R - W_R X P_R + W_I X P_I \\ * & * & * & 0 \end{bmatrix} \prec \beta I. \tag{19}$$

<div align="center">Box III</div>

$$-\varepsilon < \left(W_I \left(e^{j\omega_0} \right) - \tan(\varphi - \angle P(j\omega_0)) W_R \left(e^{j\omega_0} \right) \right) X < \varepsilon, \tag{25}$$

where ε is quite a small positive real constant. Note that Inequality (25) consists of two LMI constraints.

3. The feedback system reduces the high frequency noise. This is accomplished if the following inequality is satisfied:

$$\left| \frac{C \left(e^{j\omega} \right) P(j\omega)}{1 + C \left(e^{j\omega} \right) P(j\omega)} \right| \leq D \quad \text{dB} \qquad \omega \geq \omega_d \quad \text{rad/s}, \tag{26}$$

where D and ω_d are given constants. Clearly, Inequality (26) is satisfied if the following inequality:

$$\left| C \left(e^{j\omega_d} \right) P \left(j\omega_d \right) \right| \leq \alpha, \tag{27}$$

holds for an appropriate α. Substituting $C(e^{j\omega_d}) = W(e^{j\omega_d})X$ in Inequality (27) yields:

$$-\alpha^2 + W \left(e^{j\omega_d} \right) X \left| P \left(j\omega_d \right) \right|^2 X^T W^H \left(e^{j\omega_d} \right) < 0. \tag{28}$$

Although Inequality (28) is nonlinear in X, through the Schur complement lemma, it can be written as:

$$\begin{bmatrix} -\alpha^2 & W \left(e^{j\omega_d} \right) X \\ X^T W^H \left(e^{j\omega_d} \right) & -\left| P \left(j\omega_d \right) \right|^{-2} \end{bmatrix} \prec 0, \tag{29}$$

which is linear in X. Finally, based on Inequality (18), the complex-valued LMI in Inequality (29) can be represented in the following equivalent real-valued form:

$$\begin{bmatrix} -\alpha^2 & W_R(e^{j\omega_d})X & 0 & W_I(e^{j\omega_d})X \\ * & -\left| P(j\omega_d) \right|^{-2} & -X^T W_I^T(e^{j\omega_d}) & 0 \\ * & * & -\alpha^2 & W_R(e^{j\omega_d})X \\ * & * & * & -\left| P(j\omega_d) \right|^{-2} \end{bmatrix}$$

$$\prec 0. \tag{30}$$

2.1. Stability analysis

According to the aforementioned discussions in the previous section, the vector of unknown variables, X, can be calculated by Inequality (19) subject to Inequalities (25) and (30). However, in dealing with some problems, the controller obtained in this manner may lead to an unstable feedback system since the notion of stability is not considered in the formulation of problem (remember that in order to achieve stability, we just adjust one point on the frequency response of the open-loop system by setting the phase margin to the desired value). In such cases, additional (approximate) linear constraints can be added to the problem formulation to preserve stability. These stability constraints can be obtained from the Jury stability test, to be discussed below.

For example, consider a unity feedback control system in which the transfer functions of the Integrating Process with Time Delay (IPTD) and controller are $P(s) = K e^{-Ls}/s$ and $C(z) = C_0 + C_1 z^{-1}$, respectively. Given that $T = 0.5L$, where T is the sampling period, the characteristic equation of the closed-loop system is obtained as follows:

$$\Delta(z) = z^3 - z^2 + KC_0 z + KC_1 = 0. \tag{31}$$

For the sake of simplicity, assume that $K = 1$. From the Jury stability test, it can be concluded that a feedback system with characteristic Eq. (31) is stable if and only if the following three inequalities hold simultaneously:

$$-1 < C_1 < 1, \tag{32}$$

$$0 < (1 - C_1^2)^2 - (C_0 + C_1)^2, \tag{33}$$

$$0 < (1 - C_1^2)^2 - (C_0 + C_1)^2$$
$$- (1 + C_0 C_1)^2 (C_1^2 + C_0 + C_1 - 1). \tag{34}$$

Unfortunately, Inequalities (33) and (34) are nonlinear in $X = \begin{bmatrix} C_0 & C_1 \end{bmatrix}^T$. The proposed approximate method for deriving LMIs from Inequalities (33) and (34) aims to plot the region defined by these inequalities in $C_0 - C_1$ plane and approximate it by a convex polygon. Figure 2 shows the region defined through Inequalities (32)–(34) in red and a convex polygon used to approximate it in black. Of note, infinitely many polygons with different side numbers can be employed to approximate this region where the accuracy of approximation would increase upon increasing the number of sides of the polygon. Similarly, in case the discrete-time controller under consideration has three parameters, the region of stability can be
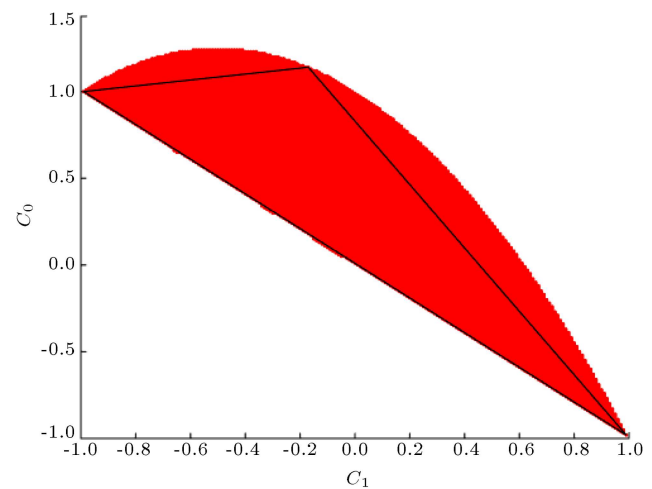
Figure 2. The region of stability obtained from Inequalities (34)–(36) and its polygon approximation.

approximated by a polyhedron. Of note, in case the approximating polygon or polyhedron is non-convex, it can be partitioned into several convex sets and then, the controller can be designed for each set separately and the optimal controller can be chosen.

3. Illustrative examples

The results of MATLAB simulations for two different integrating time-delay processes are summarized in this section.

Example 1. Consider the following First-Order Delayed Integrating Process (FODIP) taken from [23–25]:

$$P(s) = \frac{0.2}{s(4s+1)}e^{-s}. \tag{35}$$

The proposed controller is designed for this process to reach $\omega_c = 0.5$ rad/s, $\phi_m = 60°$, $\omega_d = 100$ rad/s, and $D = -40$ dB. Table 1 summarizes the values obtained for two- and three-parameter controllers. To measure the robustness of the resulting feedback system when the proposed controller is used, a perturbation of $+10\%$ is applied simultaneously to the time delay and process gain, and the simulations are repeated. Figures 3 and 4 show the step responses of the closed-loop systems with nominal and perturbed process models, respectively, when different controllers are employed (more details of the controllers used for comparison can be found in Table 2). Note that in both of these figures, a -50% disturbance is applied at $t = 30$ s. As observed, the propounded controllers suppress other controllers in terms of set-point tracking and disturbance rejection. Furthermore, as observed, the proposed controller with the term z^{-2} yielded the best response. Table 3 represents the IAE, overshoot, and phase margin indices for the proposed controller and the controllers calculated in [23–25] (the stepinfo command in MATLAB is used for computing the maximum overshoots). In this example, the controllers are evaluated regardless of the stability constraints. However, it was observed that consideration of the stability constraints when the controller had two tuning parameters made no changes in the results.
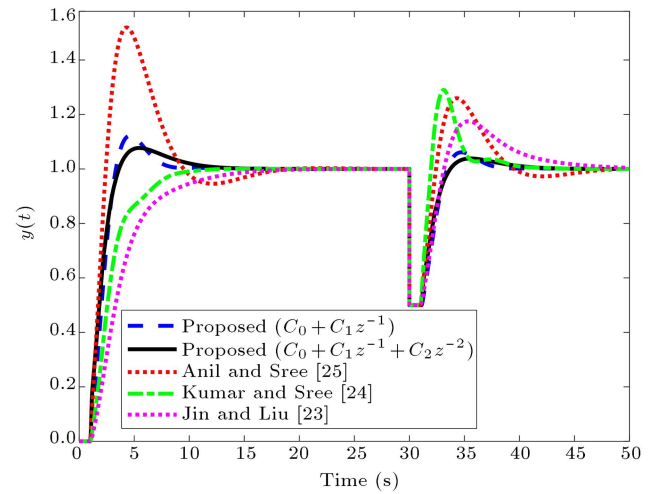
Figure 3. Step responses of the closed-loop system with the nominal model of process, corresponding to Example 1.
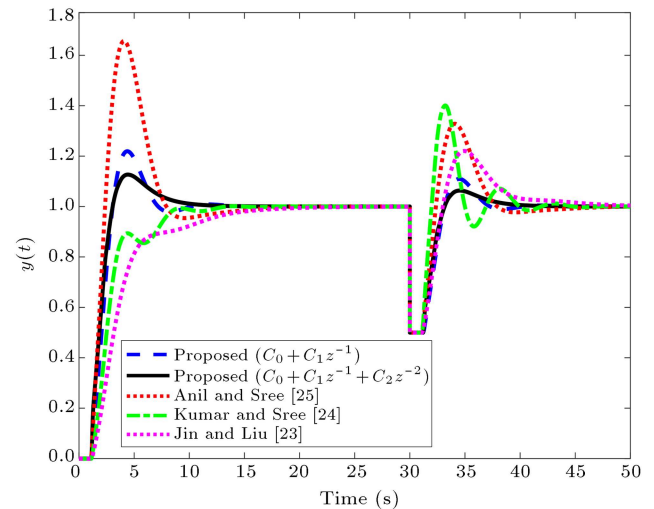
Figure 4. Step responses of the closed-loop system with the $+10\%$ perturbed model of process, corresponding to Example 1.

Example 2. Consider the following Double Integrating Process with Time Delay (DIPTD) discussed in [23,25,26]:

$$P(s) = \frac{1}{s^2}e^{-s}. \tag{36}$$

The proposed controller is designed for this process to

Table 1. Parameters measured for the proposed controller.

| Example	Proposed controller	C_0	C_1	C_2	Sampling period (T)
Ex. 1	$C_0 + C_1 z^{-1}$	101.4935	−98.6551	—	0.1
$P(s) = \frac{0.2e^{-s}}{s(4s+1)}$	$C_0 + C_1 z^{-1} + C_2 z^{-2}$	339.2258	−574.0294	237.8912	0.1
Ex. 2	$C_0 + C_1 z^{-1}$	7.0849	−7.0842	—	0.1
$P(s) = \frac{e^{-s}}{s^2}$	$C_0 + C_1 z^{-1} + C_2 z^{-2}$	30.6793	−54.3350	23.6565	0.1

Table 2. Parameter settings for comparing controllers with transfer function $K_c \left(1 + \frac{1}{T_i s} + T_d s\right) \frac{\alpha s + 1}{\beta s + 1}$.

Example	Proposed controller	K_c	T_i	T_d	α	β	Set-point filter
	Anil and Sree [25]	5.74	5.90	1.95	0.63	0.49	—
Ex. 1	Kumar and Sree [24]	7.41	7.80	1.94	0.50	0.19	$\frac{0.7s+1}{3.8s+1}$
	Jin and Liu [23]	3.68	10.39	2.47	—	—	$\frac{9.8s^2+6.2s+1}{25.7s^2+10.4s+1}$
	Lee et al. [26]	0.14	7.07	3.53	—	—	$\frac{6.25s^2+5s+1}{25s^2+10s+1}$
Ex. 2	Anil and Sree [25]	0.13	9.72	3.82	1.07	1.04	—
	Jin and Liu [23]	0.05	21.38	7.25	—	—	$\frac{46.2s^2+13.6s+1}{155.1s^2+21.4s+1}$

**Table 3.** Comparison of different methods in terms of performance and robustness.

Example	Method	Nominal model			+10% perturbed model		
		IAE	Overshoot	ϕ_m	IAE	Overshoot	ϕ_m
	$C_0 + C_1 z^{-1}$	3.69	12.22	54.66	4.00	21.90	48.49
	$C_0 + C_1 z^{-1} + C_2 z^{-2}$	3.51	7.69	58.57	3.58	12.67	53.55
Ex. 1	Anil and Sree [25]	5.97	51.83	28.97	5.96	65.69	22.93
	Kumar and Sree [24]	2.88	29.00	30.05	3.19	40.13	20.85
	Jin and Liu [23]	4.43	17.00	37.62	4.37	21.55	34.39
	$C_0 + C_1 z^{-1}$	3.84	28.04	45.28	4.84	43.39	36.37
	$C_0 + C_1 z^{-1} + C_2 z^{-2}$	2.64	10.66	57.65	3.16	25.24	49.42
Ex. 2	Lee et al. [26]	5.91	38.21	18.44	5.64	44.55	13.79
	Anil and Sree [25]	8.06	67.76	20.39	8.09	81.62	14.69
	Jin and Liu [23]	6.05	18.30	25.97	5.66	19.31	26.25

reach $\omega_c = 0.73$ rad/s, $\phi_m = 60°$, $\omega_d = 100$ rad/s, and $D = -40$ dB. Table 1 summarizes the values obtained for two- and three-parameter controllers. To measure the robustness of the resulting feedback system when the proposed controller is used, a perturbation of +10% is applied to the time delay and process gain, and the simulations are repeated. Figures 5 and 6 show the step responses of the closed-loop systems with nominal and perturbed process models, respectively, when different controllers are employed (more details of the controllers used for comparison can be found in Table 2). Note that in both of these figures, a -50% disturbance is applied at $t = 50$ s. It was observed that the proposed controller provided a lower overshoot value than that proposed by Anil and Sree [25]. In addition, the proposed controller provided a faster response and better disturbance rejection than those introduced by Jin and Liu [23] and Lee et al. [26]. The interesting point here is that the proposed controller is characterized by a lower order and less tuning parameters than the controllers designed in [23,25,26]. The corresponding performance indices are listed in Table 3. Note that in this example, the discrete-time controllers were designed without considering the approximate linear stability constraints developed in Section 2.1.
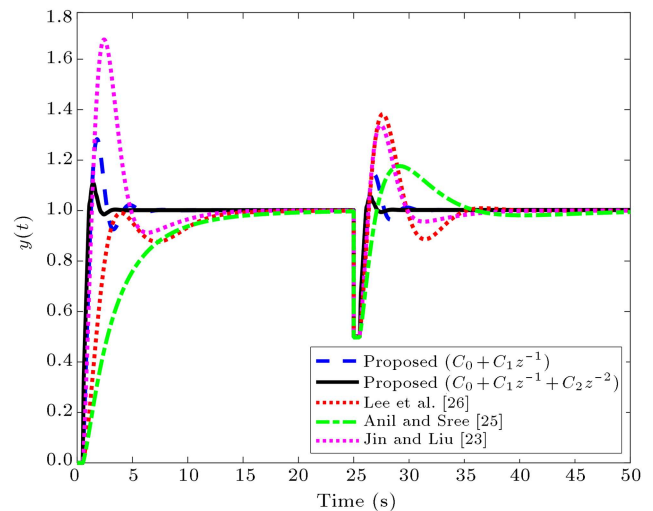
Figure 5. Step responses of the closed-loop system with the nominal model of process, corresponding to Example 2.

4. Conclusion

The present study aimed to introduce a new discrete-time controller and an LMI-based method for tuning its parameters. The structure of the proposed controller is similar to that of an Finite Impulse Response (FIR)
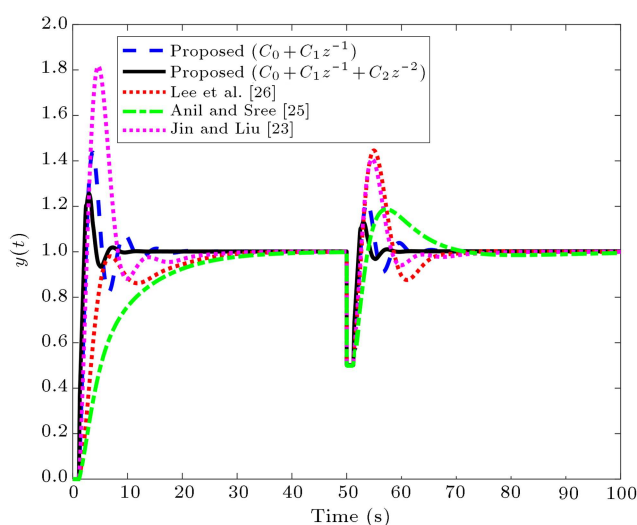
Figure 6. Step responses of the closed-loop system with the +10% perturbed model of process, corresponding to Example 2.

filter, and the proposed tuning method measured the controller parameters by open-loop shaping to obtain the desired phase margin and bandwidth. Moreover, robustness of the closed-loop system to uncertainties in the process model was considered in the formulation of the algorithm. This study contributed to developing approximate linear constraints used for achieving the closed-loop stability when the order of controller was at most equal to three. All of the control objectives and stability constraints were formulated using Linear Matrix Inequalities (LMIs) and then, they were readily solved using MATLAB. The propounded approach was applied to two different integrating processes with time delay. The results of simulations showed the superiority of the proposed structure and tuning method over some already existing methods in the considered examples.

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