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A multi-attribute decision-making method based on the third-generation prospect theory and grey correlation degree

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Abstract. Based on the third-generation Prospect Theory (3-PT) and Grey Correlation Analysis (GRA), a method was proposed that considers the uncertainty of the natural state and convenience of calculation to solve Multi-Attribute Decision-Making (MADM) problems, in which the attributes were described by the Linguistic Intuitionistic Fuzzy Numbers (LIFNs). First, the LIFNs were transformed into a belief structure comprising the identity value and belief degree. Then, the evaluation information represented by belief structure was calculated using the 3-PT, and the prospect matrix was given. The alternatives were ranked using GRA. Finally, the proposed method was employed to calculate the example under study and compare it with other methods to prove its effectiveness and superiority.

1. Introduction

Multi-Attribute Decision-Making (MADM) [1–5] is regarded as the problem of ranking the finite alternatives or selecting the best one from multiple alternatives with multiple attributes. Nowadays, MADM, quite a common problem in everyday life, has drawn the attention of many researchers. For instance, Kannan et al. [6] studied MADM methods for green supplier selection and Mardani et al. [7] examined the application of MADM techniques in the field of sustainable and renewable energy. However, in real decision-making, many problems cannot be described by accurate numbers, while they can expressed by fuzzy numbers. Zadeh [8] introduced Fuzzy Sets (FSs) as useful tools to describe the fuzzy information. However, they have a drawback, i.e., they can describe only Membership Degree (MD), not the Non-Membership Degree (NMD). Then, Intuitionistic Fuzzy Set (IFS) proposed by Atanassov [9] was proposed to overcome this shortcoming. Both MD and NMD in IFS were real numbers defined at [0, 1] which could describe the quantitative attributes and, yet, could not express the qualitative attributes well. Therefore, Chen et al. [10] combined IFS with Linguistic Variables (LVs) with the proposed Linguistic IFS (LIFS). Followed by the introduction of LIFS, it was carefully studied and developed by a number of scholars. Zhang et al. [11] presented the distance formula of LIFNs as well as an extended outranking approach to solve MADM problems. Liu et al. [12] extended the partitioned Heronian Mean (HM) to deal with LIFNs. Ou et al. [13] proposed the TOPSIS method for LIFS. Peng et al. [14] proposed a linguistic intuitionistic MADM approach based on the Heronian operator with Frank operations and employed it to evaluate the safety of coal mines. Based on some new operational laws and entropy, Li et

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al. [15] proposed an extended VIKOR method to solve decision-making problems of attribute values as LIFNs.

On the basis of empirical analysis, Kahneman and Tversky [16] presented the Prospect Theory (PT) which was a combination of psychology, behavior, and game theory. PT took into account the bounded rationality of Decision-Makers (DMs) and it was more in line with the actual decision-making behavior of DMs. They also [17] proposed Cumulative PT (CPT). Value function, weight function, and parameters of PT were significant research contents. Tversky and Fox [18] and Fox and Tversky [19] proposed a two-stage method to determine the decision weight. In the first stage, the DMs judged the probability of the event based on its randomness. In the second stage, the probability weight function was employed to convert the probability into the decision weight. Zeng [20] designed an experiment to determine the parameters of the value and weight functions. A comparison between the obtained results and those by Kahneman and Tversky [16] showed that different types of DMs had different parameters for the value function. Ma and Sun [21] improved the value function and extended the parameter range on the basis of Zeng's experiment [20] and further suggested that different risk attitudes of DMs could determine the parameters for the value function. Wakker and Zank [22] studied the simple preference foundation of CPT. In addition, different DMs might choose different reference points from different perspectives; therefore, choosing reference points had a significant impact on the PT. However, in PT and CPT, the reference points were fixed and could not be changed in the state. Therefore, on the basis of PT and CPT, Schmidt et al. [23] put forward the third-generation Prospect Theory (3-PT) and introduced the dynamic reference point considering the uncertainty of natural state. In recent years, the application of PT has received a great deal of attention. Birnbaum [24] conducted an empirical evaluation of 3-PT. Xiang and Ma [25] proposed an MADM method under risk based on 3-PT. Wu et al. [26] evaluated renewable power sources based on CPT. Jin et al. [27] proposed a method for MADM under uncertainty using evidential reasoning and PT. Zhang et al. [28] proposed different situational emergency decision-making methods based on game theory and PT. Phochanikorn and Tan [29] proposed an integrated model based on PT for green supplier selection under uncertain environment.

Grey Relational Analysis (GRA) is a significant method of MADM in grey system theory [30]. It is widely employed to solve MADM problems for several reasons: First, it does not need a large sample size; the calculation volume is small, and it is easy to combine it with other decision methods. Liu [31] described the steps of MADM through the grey relational method in detail. Liu et al. [32] conducted GRA and grey cluster analysis on key indicators of the remanufacturing industry in China. Having employed GRA, Zhan et al. [33] studied factors that influenced consumer loyalty towards geographical indication products.

Given that it is easy to combine GRA with other methods, the 3-PT fully takes into account the subjective preference of DMs and introduces dynamic reference points. To this end, this paper combines 3-PT and GRA to propose a new MADM method and solve the MADM problem expressed by LIFNs. In the decision-making process, first, the LIFNs are transformed into belief structure and then, the formula of the 3-PT is used to get the prospect matrix. Finally, the grey correlation method is employed to rank the alternatives and find the optimal one. The proposed method brings about three main advantages:

1. It is easier and more accurate for DMs to evaluate the decision-making problems in the form of LIFNs;
2. It can consider the uncertainty of the natural state and subjective preferences of the human by 3-PT;
3. It is simple and more reasonable by the GRA based on the positive and negative ideal alternatives.

The present study aims to attain the following objectives:

1. Proposing a transformation method to transform the LIFNs into belief structure;
2. Proposing a new MADM method based on 3-PT and GRA to calculate the example under study;
3. Determining the effectiveness and superiority of the proposed method by comparing it with other methods.

The rest of this paper is organized as follows. In the second section, some preliminaries including LIFNs, 3-PT, and GRA are discussed. In the third section, a method for transforming the LIFNs into belief structure is introduced. In the fourth section, based on PT and GRA, the MADM method is given. In the fifth section, an example is calculated using the proposed method, and the effectiveness and superiority of the proposed method are proved through a comparison between this method and two others. The last section concludes this study.

2. Preliminaries

2.1. LIFNs

Definition 1 [10]. Let $s_p,\ s_q \in \bar{S}$, and $\gamma = (s_p, s_q)$; if $p + q \leq t$, then γ is called a LIFN. Herein, $\bar{S}$ is a set of the continuous Linguistic Terms (LTs) based on the discrete LTs $S = \{s_0, s_1, \cdots, s_t\}$. In addition, s_t is the upper limit of LTs. Generally, $\Gamma_{[0,t]}$ is employed to express the set of all LIFNs.

Remark 1 [34]. The uncertain LVs (ULVs) are equivalent to the LIFNs. If $\tilde{s} = [s_p, s_q]$ is an ULV, where $p, q \in [0, t]$, and $p \leq q$, then LIFN (s_p, s_{t-q}) is subsequently equivalent to $\tilde{s} = [s_p, s_q]$.

Remark 2 [35]. Suppose that there is a linguistic set $S = \{s_i | i = 0, 1, 2 \cdots t\}$. When $\theta_i \in [0, 1]$ is a numerical value, the Linguistic Scale Function (LSF) is mapped from s_i to θ_i $(i = 0, 1, \cdots t)$. Based on subscript functions $sub\ (s_i) = i$, LSF is $f(s_i) = \theta_i = \frac{i}{t}(i = 1, 2, \cdots t)$.

Through LSF, LVs can be converted to LIFNs and ULVs to real numbers.

2.2. Third-generation prospect theory

In 1979, Kahneman and Tversky [16] first proposed the concept of PT. PT is selected and employed by the prospect value (V) calculated by value function (ν) and decision weight (ω). Then, Tversky and Kahneman [17] proposed Cumulative PT (CPT) in 1992. In PT or CPT, how to select the reference point is a challenging research problem. In this regard, Schmidt et al. [23,36] introduced the concept of dynamic reference point based on PT and CPT and proposed the 3-PT

Definition 2 [23,36]. Suppose that $ST = \{st_b \,| b = 1, 2, \cdots B\}$ represents a collection of natural states and the probability of occurrence for it is $P = \{p_b | b = 1, 2, \cdots B\}$, $0 \leq p_b \leq 1$, $\sum\limits_{b=1}^{B} p_b = 1$. The result of state ST at probability P is $X = \{x_b | b = 1, 2, \cdots B\}$ then, $\{x_1, p_1; x_2, p_2; \cdots ; x_b, p_b; \cdots ; x_B, p_B\}$ suggests that the result x_b can be obtained at probability p_b. Suppose that $\forall b \in \{1, 2, \cdots B\}$, $f(st_b) \in x$, $h(st_b) \in x$, h is the reference point, and the value function is as follows:

$$\nu(\Delta(f, h)) = \begin{cases} \varepsilon^+(\Delta(f, h))^\mu, & \Delta(f, h) \geq 0 \\ -\varepsilon^-(-\Delta(f, h))^\nu, & \Delta(f, h) < 0 \end{cases} \tag{1}$$

$\Delta(f, h) = f(st_b) - h(st_b)$. $\varepsilon^+ \varepsilon^-$ represent the sensitivity of DMs to gains or losses. If the DMs are more sensitive to the gain than the loss, then $\varepsilon^+ > 1$ and $\varepsilon^- = 1$. If the loss is more sensitive than the gain, then $\varepsilon^+ = 1$ and $\varepsilon^- > 1$. DMs can be divided into three types of conservative, neutral, and risky, and the values of μ and ν are different depending on DMs' attitude towards risk. For conservative DMs, $\mu > 1, \nu > 1$; for neutral DMs, $\mu = \nu = 1$; and for adventurous DMs, $\mu < 1, \nu < 1$.

The events are sorted according to the value function, satisfying if and only if $\nu(\Delta(f_m, h_m)) > \nu(\Delta(f_n, h_n))$, $m > n$. Further, $\Delta\,(f_b, h_b) < 0$ indicates that there is a strict loss in state s_b; s^- is the number of states of strict loss; however, $\Delta\,(f_b, h_b) \geq 0$ indicates that there is a weak gain in state s_b; s^+ is the number of states of weak gain; and $s^+ + s^- = B$. Then, the decision weight is defined as follows:

$$\omega(s_b;\ f, h) = \begin{cases} \omega^+(p_b) \qquad b = B \\ \omega^+\left(\sum\limits_{t \geq b} p_t\right) - \omega^+\left(\sum\limits_{t > b} p_t\right) \\ \qquad s^- + 1 \leq b \leq B \\ \omega^-\left(\sum\limits_{t \leq b} p_t\right) - \omega^-\left(\sum\limits_{t < b} p_t\right) \\ \qquad 1 \leq b \leq s^- \\ \omega^-(p_b) \qquad b = 1 \end{cases} \tag{2}$$

$\omega(p)$ is the probability weight function, and

$$\omega^+(p) = \begin{cases} \exp\left\{-\xi^+\left[-\ln(p)\right]^{\tau^+}\right\} & p \neq 0 \\ 0 & p = 0 \end{cases} \tag{3}$$

$$\omega^-(p) = \begin{cases} \exp\left\{-\xi^-\left[-\ln(p)\right]^{\tau^-}\right\} & p \neq 0 \\ 0 & p = 0 \end{cases} \tag{4}$$

In $\omega(p)$, p represents the probability of event x occurring, and $\xi^+, \xi^- > 0$. For conservative DMs, $0 < \tau^- < \tau^+ < 1$; for neutral DMs, $0 < \tau^- = \tau^+ < 1$; and for adventurous DMs, $0 < \tau^+ < \tau^- < 1$. The prospect value is calculated below:

$$V = \sum_{b=1}^{B} \nu\left(\Delta\left(f_b, h_b\right)\right) \omega\left(s_b; f_b, h_b\right). \tag{5}$$

2.3. Grey correlation analysis

Deng [30] first proposed the grey system theory in 1983. Liu [31] described the steps of MADM using GRA method in detail.

Assuming that the jth attribute value of the ith alternative is $g_{ij}^*\ (i = 1, 2, \cdots m;\ j = 1, 2, \cdots n)$, the weight vector of the attributes is $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ with $\omega_j \in [0, 1]$ $(j = 1, 2, \cdots, n)$, $\sum_{j=1}^{n} \omega_j = 1$. The decision steps are given in the following.

Step 1. Get a normalized decision matrix $G = (g_{ij})_{m \times n}$.

Step 2. Determine the ideal solution and negative ideal solution as follows.

$$G_j^+ = \max_i(g_{ij}),\ G_j^- = \min_i(g_{ij}),\ j = 1, 2, \cdots n. \tag{6}$$

Step 3. Calculate the grey relational degree between the ith and ideal solutions on the jth index.

3-1. First, calculate the grey relational coefficient:

$$q_{ij}^+ = \frac{m + \eta M}{\Delta_{ij}^+ + \eta M}, \quad \eta \in (0, 1), \tag{7}$$

Of note, $\Delta_{ij}^+ = |G_j^+ - g_{ij}|$, $m = \underbrace{\min}\underbrace{\min}\Delta_{ij}^+$, $M = \underbrace{\max}\underbrace{\max}\Delta_{ij}^+$, η is the coefficient, and $\eta = 0.5$.

3-2. Then, the coefficient matrix Q^+ of each alternative and the ideal solution is given as follows:

$$Q^+ = \begin{pmatrix} q_{11}^+ & q_{12}^+ & \cdots & q_{1n}^+ \\ q_{21}^+ & q_{22}^+ & \cdots & q_{2n}^+ \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1}^+ & q_{m2}^+ & \cdots & q_{mn}^+ \end{pmatrix}. \quad (8)$$

3-3. The grey relational degree between each alternative and the ideal solution is Q_i^+ $(i = 1, 2, \cdots m)$.

$$Q_i^+ = \sum_{j=1}^{n} \omega_j \ q_{ij}^+ (i = 1, 2, \cdots m). \quad (9)$$

Step 4. Calculate the grey relational degree of the ith and negative ideal solution on the jth index.

4-1. First, calculate the grey relational coefficient:

$$q_{ij}^- = \frac{m + \eta M}{\Delta_{ij}^- + \eta M}, \eta \in (0, 1) \quad (10)$$

Given that $\Delta_{ij}^- = |G_j^- - g_{ij}|$, $m = \underbrace{\min}\underbrace{\min}\Delta_{ij}^-$, $M = \underbrace{\max}\underbrace{\max}\Delta_{ij}^-$, η is the coefficient, and we usually take $\eta = 0.5$.

4-2. Then, the coefficient matrix of each alternative and the negative ideal solution is shown in the following:

$$Q^- = \begin{pmatrix} q_{11}^- & q_{12}^- & \cdots & q_{1n}^- \\ q_{21}^- & q_{22}^- & \cdots & q_{2n}^- \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1}^- & q_{m2}^- & \cdots & q_{mn}^- \end{pmatrix}. \quad (11)$$

4-3. Finally, the grey relational degree between each alternative and the negative ideal solution is $Q_i^- (i = 1, 2, \cdots m)$.

$$Q_i^- = \sum_{j=1}^{n} \omega_j q_{ij}^- \ (i = 1, 2, \cdots m). \quad (12)$$

Step 5. Calculate the relative closeness of each alternative.

$$C_i = \frac{Q_i^+}{Q_i^+ + Q_i^-} \ (i = 1, 2, \cdots m). \quad (13)$$

Step 6. Rank all alternatives.

All alternatives should be ranked according to the relative closeness. The better the alternative, the higher the relative closeness.

3. Transformation method of belief structure

Shortliffe and Buchanan [37] proposed the certainty factor in the MYCIN expert system. Based on the MYCIN certainty factor, a new form called belief structure, is proposed by Jin et al. [27]. Belief structures are employed to describe the uncertainty of events and the uncertainty of human perceptions.

Definition 3 [27]. Suppose that there is an event λ with a value of [0, 1] according to the subjective cognition or objective analysis, indicating that the event λ is true. This value is called the belief degree of λ, recorded as $cd(\lambda)$.

Definition 4 [38]. Given that there is an event λ and the belief degree of λ is $cd(\lambda) \in [0, 1]$, then $(\lambda, cd(\lambda))$ is called the belief structure. λ is called the identity value that can be numerical, fuzzy numbers, linguistic variables, etc.

Jin et al. [27] proposed belief structure transformation methods of real numbers, interval numbers, IFNs, and linguistic variables. But how the LIFNs would be translated into belief structure has not yet been thoroughly discussed. Any LIFN $\gamma = (s_p, s_q) \in \Gamma_{[0,t]}$ consists of two parts: MD and N-MD. However, the identity value in the belief structure is determined by a number. The degree of certainty can be measured by the similarity (already proved in [38]), and the method of transforming the interval value is also proposed. Based on the two abovementioned points, the method for transforming the LIFNs is proposed in the following:

1. **Identity value.** First, according to Remark 1, we change the LIFNs into ULVs. Then, $\gamma = (s_p, s_q)$ becomes $\gamma^* = [s_p, s_{t-q}]$.

   Then, the linguistic scale function is employed to convert the ULVs into interval numbers. Based on LSF $f(s_i) = \theta_i = \frac{i}{t} (i = 1, 2, \cdots t)$, $\gamma^* = [s_p, s_{t-q}]$ is coverted into $\gamma^{**} = [\frac{p}{t}, \frac{t-q}{t}]$.

   Finally, the midpoint value of the interval number, i.e., the belief degree, is determined.

$$\lambda = \frac{p - q + t}{2t}. \quad (14)$$

2. **Belief degree.** The degree of certainty can be measured by the similarity (already proved in [38])] obtained through the distance formula. To this end, the distance of the LIFNs should be calculated first. Here, the distance is defined as the difference between the attribute values ($\gamma = (s_p, s_q)$) and the optimal attribute value ($\gamma' = (s_m, s_n)$). In case of the benefit attributes, the optimal value is $\gamma' = (s_t, s_0)$, and the cost attributes can be converted to a benefit type. Liu and Qin [39] proposed a distance formula for LIFNs. Let $\gamma_1 = (s_{p1}, s_{q1})$, and $\gamma_2 = (s_{p2}, s_{q2}) \in \Gamma_{[0,t]}$. The distance between γ_1 and γ_2 is given below:

$$d(\gamma_1,\gamma_2)=\frac{|p_1-p_2|+|q_1-q_2|}{2t}. \tag{15}$$

Liu's method is subject to some minor drawbacks; for instance, $\gamma_1=(s_7,s_1)$, $\gamma_2=(s_4,s_4)$, $\gamma_3=(s_2,s_2)$, $t=8$, and $d(\gamma_1,\gamma_2)=d(\gamma_1,\gamma_3)=0.375$. Obviously, this is not reasonable because the hesitation of LIFNs has not been considered. Referring to the Hamming distance of the IFNs proposed by Szmidt and Kacprzyk [40], we proposed a distance formula for the LIFNs.

Definition 5. Let $\gamma_1=(s_{p1},s_{q1})$, and $\gamma_2=(s_{p2},s_{q2})\in\Gamma_{[0,t]}$ be any two LIFNs; in this case, $s_{\pi1}$ and $s_{\pi2}$ represent the degrees of hesitation of γ_1 and γ_2, respectively, $\pi1=t-p1-q1$, $\pi2=t-p2-q2$, and the distance between γ_1 and γ_2 can be measured as follows:

$$d(\gamma_1,\gamma_2)=\frac{|p1-p2|+|q1-q2|+|\pi1-\pi2|}{2t}=\frac{|p1-p2|+|q1-q2|+|p2-p1+q2-q1|}{2t}. \tag{16}$$

Based on Eq. (16), the similarity between the required attribute value $\gamma=(s_p,s_q)$ and the optimal attribute value $\gamma'=(s_m,s_n)$ can be calculated as follows:

$$S_{(\gamma,\gamma')}=1-d(\gamma,\gamma')=1-\frac{|p-m|+|q-n|+|m-p+n-q|}{2t}, \tag{17}$$

and this is called the belief degree.

Example 1. In the MADM problem, there is a benefit-type attribute c_1 and the evaluation value of c_1 is $\gamma_1=(s_4,s_3)$, $t=8$. Then, we can convert it into the belief structure $(\lambda,cd(\lambda))$.

For identity value, $\lambda=\frac{p-q+t}{2t}=\frac{4-3+8}{2*8}=0.5625$; for belief degree, the optimal attribute value $\gamma'=(s_8,s_0)$; therefore:

$$S_{(\gamma,\gamma')}=1-d(\gamma,\gamma')=1-\frac{|4-8|+|3-0|+|8-4+0-3|}{2*8}=0.5.$$

Then, the final result is $(\lambda,cd(\lambda))=(0.5625,0.5)$.

4. An MADM method based on prospect theory and grey relational analysis

In this section, a method is proposed to solve the MADM problem described by the form of LIFNs based on the PT, grey relational degree, and belief structure. In the following, this specific method is described in detail.

Suppose that $A=\{a_1,a_2,\cdots,a_m\}$ is a set of alternatives, and $C=\{c_1,c_2,\cdots,c_n\}$ is a collection of attributes. The weight vector of the attributes is $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ with $\omega_j\in[0,1]$ $(j=1,2,\cdots,n)$, $\sum_{j=1}^n\omega_j=1$. Further, assume that there are three natural states in the process of selecting a plan, namely good, medium, and poor, with each different state having distinct impact on the final benefit. The three states are represented by $ST=\{st_1,st_2,st_3\}$, and the probability of occurrence is $W=\{w_1,w_2,w_3\}$, satisfying $w_1,w_2,w_3\in[0,1]$ and $w_1+w_2+w_3=1$. In the state st_b, the attribute value c_j of each alternative a_i is represented by the form of LIFN $\gamma_{ij}^b=\left(s_{p_{ij}^b},s_{q_{ij}^b}\right)$ $(i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ b=1,2,3)$ and then, the decision matrix $R_b^*=[\gamma_{ij}^b]_{m\times n}$ $(b=1,2,3)$ is constructed. The decision objective here is to give a ranking of all alternatives.

In the following, the decision-making steps are elaborated.

Step 1. *Normalize the decision matrix.* Given there are two types of attributes, i.e., cost or benefit types, the cost type should be first converted into benefit type. The standardized matrix is $\tilde{R}_b\left[\tilde{\gamma}_{ij}^b\right]_{m\times n}$ $(b=1,2,3)$. The specific conversion method is shown as follows:

$$\tilde{\gamma}_{ij}^b=\begin{cases}\left(s_{p_{ij}^b},s_{q_{ij}^b}\right) & \text{for benefit}-\text{type attribute } c_j\\ \left(s_{q_{ij}^b},s_{p_{ij}^b}\right) & \text{for cost}-\text{type attribute } c_j\end{cases} \tag{18}$$

Step 2. *Convert the standardized LIFNs into the belief structure.* Convert the LIFNs to the belief structure according to Eqs. (14) and (17) given in Section 3, and the decision matrix is changed from $\tilde{R}_b=\left[\tilde{\gamma}_{ij}^b\right]_{m\times n}=\left[\left(s_{p_{ij}^b},s_{q_{ij}^b}\right)\right]_{m\times n}$ to $R_b=\left[\left(\lambda_{ij}^b,\ cd\left(\lambda_{ij}^b\right)\right)\right]_{m\times n}$. In addition, λ_{ij}^b represents the identity value of the attribute c_j of the alternative a_i in the state st_b and $cd\left(\lambda_{ij}^b\right)$ denotes the degree to which the attribute c_j of the alternative a_i in the state st_b takes the value λ_{ij}^b.

Step 3. *Calculate the prospect matrix.*

3-1. Get the belief structure attribute values that take into account the future state uncertainties. The belief degree $cd\left(\lambda_{ij}^b\right)$ reflects the uncertainty of the attribute value and the probability w_b represents the uncertainty of the future state; they are both independent of each other; therefore, their product was used in this study to combine the uncertainties of both attribute value and future state [2]. The calculation formula is given in the following:

$$cd\left(\lambda_{ij}^b\right)^*=cd\left(\lambda_{ij}^b\right)\times w_b\quad(b=1,2,3). \tag{19}$$

The belief structure is changed from $\left(\lambda_{ij}^b, cd\left(\lambda_{ij}^b\right)\right)$ to $\left(\lambda_{ij}^b, cd(\lambda_{ij}^b)^*\right)$.

3-2. Calculate the value function via Eq. (1).

3-3. Calculate the decision weights through Eqs. (2), (3), and (4).

3-4. Calculate the prospect value through Eq. (5) and obtain the prospect matrix.

Step 4. *The final ordering of the alternatives is obtained using the grey relational method.*

4-1. Calculate the ideal and negative ideal solutions of the prospect matrix.

4-2. Calculate the grey relational degree of the ith and ideal solution on the jth index.

4-3. Calculate the grey relational degree of the ith and negative ideal solutions on the jth index.

4-4. Calculate the relative closeness of each alternative and get the final order. The higher the relative closeness, the better the alternative.

5. An illustrate example

5.1. decision-making problem

In this part, a practical problem was solved using the method presented in the previous section. Assume that an investment company intends to select an investment target from four candidate companies $A = \{a_1, a_2, a_3, a_4\}$ evaluated from five different aspects $C = \{c_1, c_2, c_3, c_4, c_5\}$: economic benefits, risk controllable analysis, social impact analysis, company policy, and development sustainability. The attribute weights are equal, i.e., $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5 = 0.2$. In the investment process, there are three possible natural states $ST = \{st_1, st_2, st_3\}$, i.e., good, medium, and poor, and the probability of occurrence is $W = \{w_1, w_2, w_3\}$ and, specifically, $w_1 = 0.3, w_2 = 0.5$, and $w_3 = 0.2$. Based on the linguistic set, $S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor, $s_3 =$ slightly poor, $s_4 =$ fair, $s_5 =$ slightly good, $s_6 =$ good $s_7 =$ very good, $s_8 =$ extremely good$\}$ in different natural states, the evaluation values of each attribute of each company are different and expressed by the form of LIFNs. The decision matrix $R_b^* = \left[\gamma_{ij}^b\right]_{m\times n}$ $(b = 1, 2, 3)$ is shown in Tables 1–3.

A specific calculation process is presented in the following.

Table 1. Decision matrix R_1^*.

	c_1	c_2	c_3	c_4	c_5
a_1	$(s_7,\ s_1)$	$(s_6,\ s_2)$	$(s_6,\ s_1)$	$(s_7,\ s_1)$	$(s_5,\ s_2)$
a_2	$(s_6,\ s_2)$	$(s_5,\ s_2)$	$(s_6,\ s_1)$	$(s_6,\ s_2)$	$(s_7,\ s_1)$
a_3	$(s_6,\ s_1)$	$(s_5,\ s_3)$	$(s_7,\ s_1)$	$(s_5,\ s_2)$	$(s_6,\ s_2)$
a_4	$(s_5,\ s_2)$	$(s_7,\ s_1)$	$(s_5,\ s_3)$	$(s_6,\ s_1)$	$(s_6,\ s_2)$

Table 2. Decision matrix R_2^*.

	c_1	c_2	c_3	c_4	c_5
a_1	$(s_7,\ s_1)$	$(s_4,\ s_4)$	$(s_6,\ s_2)$	$(s_5,\ s_2)$	$(s_3,\ s_5)$
a_2	$(s_7,\ s_1)$	$(s_5,\ s_1)$	$(s_6,\ s_1)$	$(s_5,\ s_2)$	$(s_4,\ s_3)$
a_3	$(s_5,\ s_2)$	$(s_6,\ s_1)$	$(s_7,\ s_1)$	$(s_5,\ s_3)$	$(s_4,\ s_4)$
a_4	$(s_6,\ s_2)$	$(s_4,\ s_3)$	$(s_5,\ s_2)$	$(s_7,\ s_1)$	$(s_5,\ s_3)$

Table 3. Decision matrix R_3^*.

	c_1	c_2	c_3	c_4	c_5
a_1	$(s_5,\ s_3)$	$(s_4,\ s_4)$	$(s_7,\ s_1)$	$(s_5,\ s_1)$	$(s_4,\ s_2)$
a_2	$(s_6,\ s_1)$	$(s_6,\ s_2)$	$(s_6,\ s_1)$	$(s_5,\ s_2)$	$(s_6,\ s_1)$
a_3	$(s_5,\ s_2)$	$(s_3,\ s_4)$	$(s_6,\ s_2)$	$(s_3,\ s_3)$	$(s_5,\ s_2)$
a_4	$(s_4,\ s_3)$	$(s_5,\ s_1)$	$(s_4,\ s_2)$	$(s_6,\ s_2)$	$(s_5,\ s_2)$

Step 1. *Standardize the evaluation matrix.* Because all attributes are classified as benefit type, standardization of the matrix $R_1^* \sim R_3^*$ is not required.

Step 2. *Convert the standardized LIFNs into the belief structure.* The identity value and the belief degree are separately transformed using Eqs. (14) and (17). The matrices $R_1^* \sim R_3^*$ are transformed from the LIFNs to the form of belief structure. More details are presented in Tables 4–6.

Step 3. *Calculate the prospect matrix.*

3-1. Belief structure is calculated using Eq. (19).

The probability of occurrence for the three states is $w_1 = 0.3$, $w_2 = 0.5$, and $w_3 = 0.2$. Considering the probability of occurrence of the natural state, we get the new belief structures, as shown in Tables 7–9.

3-2. Use Eq. (1) to get the value function.

It is believed that the reference points of each attribute in the same state are the same, and the reference points of each state after the transformation are consequently presented. In other words, $h(st_1) = 0.8125$, $h(st_2) = 0.75$, and $h(st_3) = 0.6875$. Assume that decision-makers are neutral, yet more sensitive to losses than earnings; then, parameters $\varepsilon^+ = 1$, $\varepsilon^- = 2.25$, and $u = 1$, and $\nu = 1$ [41] should be taken into account to obtain the value function, as shown in Table 10.

3-3. Calculate the decision weights according to Eqs. (2)-(4).

The alternatives should be sorted in descending order in terms of the size of the value function and then, the decision weights should be calculated, the results of which are shown in Table 11, where the parameters $\xi^+ = 1$, $\xi^- = 1$, $\tau^+ = 0.604$, and $\tau^- = 0.604$ given by Prelec were utilized [42].

Table 4. Decision matrix R_1.

	c_1	c_2	c_3	c_4	c_5
a_1	$(0.875, 0.875)$	$(0.75, 0.75)$	$(0.8125, 0.75)$	$(0.875, 0.875)$	$(0.6875, 0.625)$
a_2	$(0.75, 0.75)$	$(0.6875, 0.625)$	$(0.8125, 0.75)$	$(0.75, 0.75)$	$(0.875, 0.875)$
a_3	$(0.8125, 0.75)$	$(0.625, 0.625)$	$(0.875, 0.875)$	$(0.6875, 0.625)$	$(0.75, 0.75)$
a_4	$(0.6875, 0.625)$	$(0.875, 0.875)$	$(0.625, 0.625)$	$(0.8125, 0.75)$	$(0.75, 0.75)$

Table 5. Decision matrix R_2.

	c_1	c_2	c_3	c_4	c_5
a_1	$(0.875, 0.875)$	$(0.5, 0.5)$	$(0.75, 0.75)$	$(0.6875, 0.625)$	$(0.375, 0.375)$
a_2	$(0.875, 0.875)$	$(0.75, 0.625)$	$(0.8125, 0.75)$	$(0.6875, 0.625)$	$(0.5625, 0.5)$
a_3	$(0.6875, 0.625)$	$(0.8125, 0.75)$	$(0.875, 0.875)$	$(0.625, 0.625)$	$(0.5, 0.5)$
a_4	$(0.75, 0.75)$	$(0.5625, 0.5)$	$(0.6875, 0.625)$	$(0.875, 0.875)$	$(0.625, 0.625)$

Table 6. Decision matrix R_3.

	c_1	c_2	c_3	c_4	c_5
a_1	$(0.625, 0.625)$	$(0.5, 0.5)$	$(0.875, 0.875)$	$(0.75, 0.625)$	$(0.625, 0.5)$
a_2	$(0.8125, 0.75)$	$(0.75, 0.75)$	$(0.8125, 0.75)$	$(0.6875, 0.625)$	$(0.8125, 0.75)$
a_3	$(0.6875, 0.625)$	$(0.4375, 0.375)$	$(0.75, 0.75)$	$(0.5, 0.375)$	$(0.6875, 0.625)$
a_4	$(0.5625, 0.5)$	$(0.75, 0.625)$	$(0.625, 0.5)$	$(0.75, 0.75)$	$(0.6875, 0.625)$

Table 7. Belief structure matrix under st_1.

	c_1	c_2	c_3	c_4	c_5
a_1	$(0.875, 0.2625)$	$(0.75, 0.225)$	$(0.8125, 0.225)$	$(0.875, 0.2625)$	$(0.6875, 0.1875)$
a_2	$(0.75, 0.225)$	$(0.6875, 0.1875)$	$(0.8125, 0.225)$	$(0.75, 0.225)$	$(0.875, 0.2625)$
a_3	$(0.8125, 0.225)$	$(0.625, 0.1875)$	$(0.875, 0.2625)$	$(0.6875, 0.1875)$	$(0.75, 0.225)$
a_4	$(0.6875, 0.1875)$	$(0.875, 0.2625)$	$(0.625, 0.1875)$	$(0.8125, 0.225)$	$(0.75, 0.225)$

Table 8. Belief structure matrix under st_2.

	c_1	c_2	c_3	c_4	c_5
a_1	$(0.875, 0.4375)$	$(0.5, 0.25)$	$(0.75, 0.375)$	$(0.6875, 0.3125)$	$(0.375, 0.1875)$
a_2	$(0.875, 0.4375)$	$(0.75, 0.3125)$	$(0.8125, 0.375)$	$(0.6875, 0.3125)$	$(0.5625, 0.25)$
a_3	$(0.6875, 0.3125)$	$(0.8125, 0.375)$	$(0.875, 0.4375)$	$(0.625, 0.3125)$	$(0.5, 0.25)$
a_4	$(0.75, 0.375)$	$(0.5625, 0.25)$	$(0.6875, 0.3125)$	$(0.875, 0.4375)$	$(0.625, 0.3125)$

Table 9. Belief structure matrix under st_3.

	c_1	c_2	c_3	c_4	c_5
a_1	$(0.625, 0.125)$	$(0.5, 0.1)$	$(0.875, 0.175)$	$(0.75, 0.125)$	$(0.625, 0.1)$
a_2	$(0.8125, 0.15)$	$(0.75, 0.15)$	$(0.8125, 0.15)$	$(0.6875, 0.125)$	$(0.8125, 0.15)$
a_3	$(0.6875, 0.125)$	$(0.4375, 0.075)$	$(0.75, 0.15)$	$(0.5, 0.075)$	$(0.6875, 0.125)$
a_4	$(0.5625, 0.1)$	$(0.75, 0.125)$	$(0.625, 0.1)$	$(0.75, 0.15)$	$(0.6875, 0.125)$

Table 10. Value function matrix.

	c_1	c_2	c_3	c_4	c_5
	0.0625	−0.1406	0	0.0625	−0.2813
a_1	0.125	−0.5625	0	−0.1406	−0.8438
	−0.1406	−0.4219	0.1875	0.6225	−0.1406
	0.0625	−0.2813	0	−0.1406	0.0625
a_2	0.125	0	0.0625	−0.1406	−0.4219
	0.125	0.0625	0.125	0	0.125
	0	−0.4219	0.0625	−0.2813	−0.1406
a_3	−0.1406	0.0625	0.125	−0.2813	−0.5625
	0	−0.5625	0.0625	−0.4219	0
	−0.2813	0.0625	−0.4219	0	−0.1406
a_4	0	−0.4219	−0.1406	0.125	−0.2813
	−0.2813	0.0625	−0.1406	0.0625	0

Table 11. Decision weight matrix.

	c_1	c_2	c_3	c_4	c_5
	0.1688	0.2799	0.1413	0.3036	0.1280
a_1	0.4101	0.2958	0.3722	0.3343	0.2554
	0.2110	0.0620	0.2469	0.0757	0.1911
	0.2799	0.2554	0.2799	0.1461	0.1608
a_2	0.4101	0.1933	0.2336	0.3343	0.2958
	0.0922	0.2294	0.2294	0.2110	0.2294
	0.2799	0.1164	0.1655	0.2554	0.1461
a_3	0.3343	0.3722	0.4101	0.1933	0.2958
	0.0757	0.1692	0.2294	0.1692	0.2110
	0.1164	0.3036	0.2554	0.2799	0.1461
a_4	0.3722	0.2958	0.3343	0.4101	0.3343
	0.1911	0.0764	0.0605	0.0922	0.2110

3-4. Calculate the prospect value according to Eq. (5), and the prospect matrix is shown in Table 12.

Step 4. The final ordering of the alternatives is obtained using the grey relational method.

4-1. The ideal and negative ideal solutions should be determined.

$$G^+ = (0.0453, -0.0575, 0.0759, 0.0570, -0.0861),$$

$$G^- = (-0.0865, -0.2319, -0.1633, -0.1976, -0.2783).$$

4-2. Calculate the grey relational degree of the ith and ideal solutions on the jth index.

$$Q^+ = (0.6302, 0.8602, 0.6276, 0.6801).$$

4-3. Calculate the grey relational degree of the ith and negative ideal solutions on the jth index.

Table 12. Prospect matrix.

	c_1	c_2	c_3	c_4	c_5
a_1	0.0321	−0.2319	0.0463	−0.0233	−0.2783
a_2	0.0453	−0.0575	0.0433	−0.0676	0.0861
a_3	−0.0470	−0.1210	0.0759	−0.1976	−0.1869
a_4	−0.0865	−0.1010	−0.1633	0.0570	−0.1146

$$Q^- = (0.6635, 0.4375, 0.6454, 0.6528).$$

4-4. Calculate the relative closeness of each alternative and get the final order. After calculation, the relative closeness of each alternative is $C_{a_1} = 0.4871$, $C_{a_2} = 0.6629$, $C_{a_3} = 0.4930$, and $C_{a_4} = 0.5103$, and the final ordering is $a_2 \succ a_4 \succ a_3 \succ a_1$. Further, the optimal solution is a_2.

In order to elaborate on the influence of the types of decision-makers on the final decision results, the parameters of the value function as well as the weight function were changed to recalculate this example. Six representative combinations of parameters were obtained, as shown in Table 13.

The value function parameters $\varepsilon^+ = 1$, $\varepsilon^- = 2.25$, $u = 0.89$, and $\nu = 0.92$ were given by Tversky and Kahneman [17]. Xu et al. [43] obtained $\varepsilon^+ = 1$, $\varepsilon^- = 1.51$, $u = 0.37$, and $\nu = 0.59$. Zeng [20] obtained the parameters $\varepsilon^+ = 1$, $\varepsilon^- = 2.25$, $u = 1.21$, and $\nu = 1.02$ through experiments. Four different weight function parameters $\xi^+ = 0.938$, $\xi^- = 0.9381$, $\tau^+ = 0.603$, $\tau^- = 0.605$; $\xi^+ = 1.083$, $\xi^- = 1.083$, $\tau^+ = 0.533$, $\tau^- = 0.535$; $\xi^+ = 0.938$, $\xi^- = 0.938$, $\tau^+ = 0.605$, $\tau^- = 0.603$; $\xi^+ = 1.083$, $\xi^- = 1.083$, $\tau^+ = 0.535$, $\tau^- = 0.533$ were the research results obtained by Prelec [42] and Bleichrodt and Pinto [44].

According to the decision method presented above, six kinds of parameter combinations were used, and the final orders were obtained, as shown in Table 14.

According to Tables 13 and 14, the ordering obtained through the proposed MADM method is basically the same for different parameters. To be specific, the results obtained by five combinations were the same as those of the previous calculation; in other words, $a_2 \succ a_4 \succ a_3 \succ a_1$. In case the parameters are $\varepsilon^+ = 1$, $\varepsilon^- = 1.51$, $u = 0.37$, and $\nu = 0.59$, the ordering becomes $a_2 \succ a_3 \succ a_4 \succ a_1$; in other words, there are two alternatives in the middle of the ranking exchange positions. However, the optimal solution would remain unchanged. The author holds the view that the ordering is stable at a certain reference value.

5.2. Comparison with other methods

In this section, two other methods are employed to calculate the examples mentioned above and analyze

Table 13. Representative parameter combination.

	Parameter of value function	Parameter of weight function
1	$\varepsilon^+ = 1, \varepsilon^- = 2.25$ $u = 0.89, \nu = 0.92$	$\xi^+ = 0.938, \xi^- = 0.938$ $\tau^+ = 0.603, \tau^- = 0.605$
2	$\varepsilon^+ = 1, \varepsilon^- = 2.25$ $u = 0.89, \nu = 0.92$	$\xi^+ = 1.083, \xi^- = 1.083$ $\tau^+ = 0.533, \tau^- = 0.535$
3	$\varepsilon^+ = 1, \varepsilon^- = 2.25$ $u = 1.21, \nu = 1.02$	$\xi^+ = 0.938, \xi^- = 0.938$ $\tau^+ = 0.605, \tau^- = 0.603$
4	$\varepsilon^+ = 1, \varepsilon^- = 2.25$ $u = 1.21, \nu = 1.02$	$\xi^+ = 1.083, \xi^- = 1.083$ $\tau^+ = 0.535, \tau^- = 0.533$
5	$\varepsilon^+ = 1, \varepsilon^- = 1.51$ $u = 0.37, v = 0.59$	$\xi^+ = 0.938, \xi^- = 0.938$ $\tau^+ = 0.603, \tau^- = 0.605$
6	$\varepsilon^+ = 1, \varepsilon^- = 1.51$ $u = 0.37, \nu = 0.59$	$\xi^+ = 1.083, \xi^- = 1.083$ $\tau^+ = 0.535, \tau^- = 0.533$

Table 14. Calculation results and ranking.

	Parameter of value function	Parameter of weight function
1	$C_{a_1} = 0.4939$ $C_{a_2} = 0.6503$ $C_{a_3} = 0.4961$ $C_{a_4} = 0.5093$	$a_2 \succ a_4 \succ a_3 \succ a_1$
2	$C_{a_1} = 0.4944 C_{a_2} = 0.6507$ $C_{a_3} = 0.4951$ $C_{a_4} = 0.5091$	$a_2 \succ a_4 \succ a_3 \succ a_1$
3	$C_{a_1} = 0.4989 C_{a_2} = 0.6604$ $C_{a_3} = 0.5018 C_{a_4} = 0.5217$	$a_2 \succ a_4 \succ a_3 \succ a_1$
4	$C_{a_1} = 0.4995 C_{a_2} = 0.6604$ $C_{a_3} = 0.5009 C_{a_4} = 0.5215$	$a_2 \succ a_4 \succ a_3 \succ a_1$
5	$C_{a_1} = 0.4868 C_{a_2} = 0.6120$ $C_{a_3} = 0.4914 C_{a_4} = 0.4913$	$a_2 \succ a_3 \succ a_4 \succ a_1$
6	$C_{a_1} = 0.4870 C_{a_2} = 0.6130$ $C_{a_3} = 0.4906 C_{a_4} = 0.4911$	$a_2 \succ a_4 \succ a_3 \succ a_1$

the orderings. The first comparison method is a stochastic intuitionistic fuzzy decision-making method based on the PT proposed by Li et al. [45], and the second one suggests using the dynamic multiple-attribute grey relational decision model proposed by Dang et al. [46].

Since Li et al.'s method [45] utilizes the form of IFNs and our example under study gives the evaluation matrix in the form of LIFNs, the LSF (introduced in Section 2.1) should be employed to convert the LIFNs into IFNs before calculation. Furthermore, the parameter values of the PT formula presented in Li's paper [45] were $\gamma = 0.604$, $\delta = 1.21$, $\beta = 1.02$, and $\sigma = 2.25$. Dang et al.'s method [46] considers and

Table 15. Ranking results by different methods.

Methods	Values	Ranking
Li et al.'s method [45] based on PT	$W_1 = 0.3292, W_2 = 0.4778$ $W_3 = 0.3137, W_4 = 0.3554$	$a_2 \succ a_4 \succ a_1 \succ a_3$
Dang et al.'s method [46] based on grey correlation method	$u_1 = 0.4226, u_2 = 0.6797$ $u_3 = 0.4623, u_4 = 0.4819$	$a_2 \succ a_4 \succ a_3 \succ a_1$
The method proposed in this paper	$C_{a_1} = 0.4871, C_{a_1} = 0.6629$ $C_{a_1} = 0.4930, C_{a_1} = 0.5103$	$a_2 \succ a_4 \succ a_3 \succ a_1$

uses the evaluation value only as a real number. The identity value in the belief structure mentioned in this paper is the real number obtained through subjective cognition or objective evaluation. To this end, the belief structure transformation method was used to convert the LIFNs into real numbers. The final results and ordering from calculation are shown in Table 15.

In the following, a detailed analysis of the results is presented in Table 15.

The ordering of Li et al.'s method [45] is roughly the same as that of the method proposed in this paper. To be specific, the ordering of Li is $a_2 \succ a_4 \succ a_1 \succ a_3$, and our obtained ordering is $a_2 \succ a_4 \succ a_3 \succ a_1$. The optimal solution obtained by these two methods was the same and both were a_2, which could explain the effectiveness of the proposed method. Moreover, the proposed method enjoys several advantages as follows:

1. Li's method [45] evaluates the alternatives by IFNs, and LIFNs are consequently used. In the real world, it is difficult to evaluate some alternatives with accurate numbers; therefore, it is more convenient to use LIFNs. When evaluating the qualitative attributes, Li's method is likely to cause more difficulty than our proposed method;
2. Although both methods use PT, the proposed method in this paper uses 3-PT. While the reference point of the 3-PT is dynamic, those in different states can be different. In Li's method, the reference points in different cases are the same, and the value is zero. In this respect, the mentioned author is of the idea that his approach seems more reasonable;
3. While Li's method uses only PT, the proposed method combines PT with GRA method, which is more reasonable in the final ranking result.

In summary, the proposed method is reasonable and superior to Li's.

The order obtained by the method in [46] is the same as that obtained in this paper, indicating the rationality of the proposed method. The superiority of the method in this paper is mainly manifested in two aspects:

1. The method of Dang et al. [46] solves the problem of MADM in which the information form represent a real number, but in the real problems, many qualitative attributes cannot be evaluated with accurate figures. The proposed method solves the MADM problem in which the information form is LIFNs and the scope of application is wider. Via LSF, the LIFNs can be converted into the IFNs. In case the NMD is zero, the IFNs are converted into real numbers. Therefore, compared with the method of Dang et al. [46], our method can better deal with a variety of information forms of MADM problems with the scope of adaptation being wider;
2. Further, the method adopted by Dang et al. [46] benefits from only the GRA method, not the 3-PT, in evaluating the dynamic multi-indicator problem. Although the DM is a limited rational person, he/she does not always seek to maximize the effect, but to choose a satisfactory plan according to the actual situation. The proposed method in this study employs the 3-PT to take into account different actual situations in reference points in different natural states. At the same time, different parameters can be selected according to the sensitivity of decision-makers to risks and benefits. At this point, our proposed approach is also more reasonable.

In summary, a comparison of this method and other two existing methods points to the superiority and sensibility of the proposed method.

6. Conclusion

The MADM method based on 3-PT and GRA was proposed in this study to solve the problem with dynamic reference points. To this end, first, the proposed transformation method was employed to convert the LIFNs into a form of belief structure. Then, the decision matrix in different states was calculated using the formula of 3-PT to obtain a prospect matrix. Finally, the GRA method was employed to calculate the prospect matrix, and the alternatives were ranked. The effectiveness and superiority of the proposed method were proved by comparing its results with those of the other two methods. On the contrary, the form of information used in this study was the LIFNs, which made it feasible for DMs to evaluate the qualitative attributes of both MD and NMD. In addition, LIFNs represented a generalized form of information that could be transformed into other forms. However, the 3-PT was adopted in the proposed method, taking into account the limited rationality of DMs and dynamic reference point, which was more in line with the actual situation. In the future, we hope that our method can be successfully applied to practical problems such as medical diagnosis, supplier selection, or extension of methods from MADM to multi-attribute group decision making [47,48].

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