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Prediction of primary energy consumption using NDGM(1,1,*k*,*c*) model with Simpson formula

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Abstract. Energy consumption plays a key role in economic development for all countries. Keeping up with the future trend of energy consumption is essential for governments and energy companies. In this research, the primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam is systematically studied using different forecasting models. Based on the actual data derived from the year 2006 to 2016, a novel discrete grey forecasting model termed NDGM$_S$(1,1,k,c) is proposed where the Simpson numerical integration formula is applied to construct the background value. The expressions of the present model are all derived and then, its unbiased property is proved. As demonstrated by the results, the NDGM$_S$(1,1,k,c) model can achieve better prediction accuracy than other forecasting models, and it is quite applicable to predicting a sequence based on homogeneous/non-homogeneous exponential law.

1. Introduction

Primary energy is an energy resource found in nature that has not been subjected to any human engineered conversion or transformation process. It is also called natural energy contained in oil, coal, natural gas, water energy, and modern renewable energy used to generate electricity. The *BP Statistical Review of World Energy 2017* [1] demonstrated that global primary energy consumption expanded by 1% in 2016, following the growth of 0.9% in the year 2015 and 1% in the year 2014. This trend is measured with the 10-year average of 1.8% a year. Primary energy consumption in Saudi Arabia grew by 1.9% in 2016 corresponding to 266.5 million tonnes of oil equivalent (Mtoe), which is 2.0% of the total international primary energy consumption. In addition, the annual growth rate during the years 2005 to 2015 is 5.1%. For India's primary energy consumption, a growth is 5.4% in 2016 corresponding to 723.9 Mtoe, which is 5.5% of the total global consumption. From 2005 to 2015, primary energy consumption grew at an incredible average rate of 5.1%. In Philippines, it grew by 11.3% in 2016 corresponding to 42.1 Mtoe, which is 0.3% of the total global consumption. The annual growth rate from 2005 to 2015 is 3.6%. For Vietnam, the mentioned energy consumption growth was 1.5% in

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2016 corresponding to 64.8 Mtoe, which is 0.5% of the total global consumption. From the year 2005 to 2015, this consumption grew at an incredible average rate of 7.5%. Presently, energy markets are accommodated and the near-term strength may continuously ease. It is important for decision-markers and government departments to develop a better understanding and judgment of the energy resource plan scientifically and formulate appropriate energy plans.

Energy is and has been receiving remarkable attention over a long time because of its importance all over the world. A variety of methods and techniques have been advanced for energy forecast utilization such as cointegrated panel analysis [2], artificial neural network [3], time series analysis [4,5], coupling mathematical models [6,7], hybrid forecasting models [8,9], grey models [10–14], etc. Among those excellent methods/techniques, grey system theory that was presented by Deng [15] is a feasible and efficient prediction technique to analyze uncertain problems. In his work, the first-order linear model with single variable termed GM(1,1) model was discussed in detail. The main advantage of grey models is that they require a small number of samples to describe the system. Over the past three decades, the GM(1,1) model has significantly generalized with the following aspects: the univariate linear grey models [16,17], the univariate nonlinear grey models [18–20], and the multivariate grey models [21–23].

Recently, Cui et al. [24] studied the continuous non-homogeneous grey model named NGM(1,1,k) model where bk is grey action quantity. The yearly amount of concave soil in Xuyi of China and the CSI 300 index specimen data were used to illustrate the NGM(1,1,k) model and their optimized model was effective. However, Chen and Yu's work [25] identified that the parameters of the NGM(1,1,k) model had a fatal flaw that badly affected the application value. Based on Cui's work, a modified model named NGM(1,1,k,c) was proposed in which $bk+c$ was grey action quantity. This model is truly feasible for simulation and forecasting of approximate non-homogeneous exponential sequence and can achieve outstanding prediction accuracy. Meanwhile, Xie et al. [26] developed an NDGM model where the background value was derived from the trapezoid formula and the initial point was optimized. The expression of this model was obtained and the prediction precision was found to be dependent on the pure non-homogeneous index sequence.

Encouraged by these works [24–26] and considering the non-homogenous exponential sequence existing in the real world [27,28], the present study focuses on the non-homogenous discrete NGM(1,1,k,c) model called NDGM$_S$(1,1,k,c) where the background value is computed using the Simpson numerical integration formula. Its solutions, properties, and applications are derived in this paper. Meantime, we prove the new model is able to simulate a linear sequence and a homogeneous/non-homogeneous exponential sequence without error. Further, the primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam is calculated by grey models, Auto-Regressive Integrated Moving Average model (ARIMA), and Support Vector Machines (SVMs). It is noted that the NDGM$_S$(1,1,k,c) model presents high accuracy in the primary energy consumption.

This paper is organized below. A brief introduction to the NGM(1,1,k,c) model is given in Section 2. Detailed discussions of the NDGM$_S$(1,1,k,c) model are given in Section 3. Applications of the primary energy consumption are arranged in Section 4. The last section concludes the paper.

2. The existing NGM model

Next, a brief analysis of the continuous NGM model is conducted based on the Trapezoid formula.

2.1. Grey NGM(1,1,k) model

It is assumed that an original non-negative data sequence with n entries is $X^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$, where $x^{(0)}(k)$ stands for the role of the data at the time index k.

Let $X^{(1)}=(\sum_{i=1}^{1}x^{(0)}(i),\sum_{i=1}^{2}x^{(0)}(i),\cdots,\sum_{i=1}^{n}x^{(0)}(i))$ be the first accumulated generating operation (1-AGO) sequence of $X^{(0)}$.

We denote $Z^{(1)}$ as the background value of the grey forecasting model where $Z^{(1)}=(z^{(1)}(2),z^{(1)}(3),\cdots,z^{(1)}(n))$ with $z^{(1)}(k)=\frac{1}{2}x^{(1)}(k)+\frac{1}{2}x^{(1)}(k-1)$, $k=2,3,\cdots,n$.

From Ref. [24], the mathematical model of the NGM(1,1,k) is as follows:

$$\frac{dx^{(1)}(t)}{dt}+ax^{(1)}(t)=bt, \tag{1}$$

which is a linear differential equation, a is the developing coefficient, and $bt+c$ is the grey action quantity.

The values for the unknown parameters a and b of NGM(1,1,k) model are computed by the least squares estimation:

$$\begin{pmatrix}a\\b\end{pmatrix}=\left(\Lambda^{\mathcal{T}}\Lambda\right)^{-1}\Lambda^{\mathcal{T}}\eta, \tag{2}$$

where:

$$\Lambda=-\begin{pmatrix}z^{(1)}(2) & -2\\ z^{(1)}(3) & -3\\ \vdots & \vdots\\ z^{(1)}(n) & -n\end{pmatrix},\qquad \eta=\begin{pmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{pmatrix}.$$

The time response function of the NGM(1,1,k) model is:

$$\hat{x}^{(1)}(k)=\left(x^{(0)}(1)+\frac{b}{a^2}-\frac{b}{a}\right)e^{-a(k-1)}+\frac{b}{a}k-\frac{b}{a^2},$$
$$k=2,3,\cdots,n. \quad (3)$$

The restored values of $\hat{x}^{(0)}(k)$ are:

$$\hat{x}^{(0)}(k)=\left(x^{(0)}(1)+\frac{b}{a^2}-\frac{b}{a}\right)(1-e^{a})e^{-a(k-1)}+\frac{b}{a},$$
$$k=1,2,\cdots,n. \quad (4)$$

2.2. Grey NGM(1,1,k,c) model

From Ref. [25], the mathematical form of the NGM(1,1,k,c) model is:

$$\frac{dx^{(1)}(t)}{dt}+ax^{(1)}(t)=bt+c, \quad (5)$$

where a is the developing coefficient and $bt+c$ is the grey action quantity.

The unknown parameters a, b, and c of the NGM(1,1,k,c) model are determined by the least squares estimation:

$$\begin{pmatrix}a\\b\\c\end{pmatrix}=(\Lambda^{\mathcal{T}}\Lambda)^{-1}\Lambda^{\mathcal{T}}\eta, \quad (6)$$

where:

$$\Lambda=-\begin{pmatrix}z^{(1)}(2) & -2 & -1\\ z^{(1)}(3) & -3 & -1\\ \vdots & \vdots & \vdots\\ z^{(1)}(n) & -n & -1\end{pmatrix}, \qquad \eta=\begin{pmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{pmatrix}.$$

The time response function of the NGM(1,1,k,c) model is:

$$\hat{x}^{(1)}(k)=\left(x^{(0)}(1)-\frac{c}{a}-\frac{b}{a}+\frac{b}{a^2}\right)e^{-a(k-1)}+\frac{b}{a}k$$
$$-\frac{b}{a^2}+\frac{c}{a}, \qquad k=2,3,\cdots,n. \quad (7)$$

The restored values of $\hat{x}^{(0)}(k)$ are:

$$\hat{x}^{(0)}(k)=\left(x^{(1)}(1)-\frac{c}{a}-\frac{b}{a}+\frac{b}{a^2}\right)(1-e^{a})e^{-a(k-1)}$$
$$+\frac{b}{a}, \qquad k=2,3,\cdots,n. \quad (8)$$

3. The NDGM$_S$(1,1,k,c) model

3.1. Representation of the NDGM$_S$(1,1,k,c) model

In this subsection, we plan to derive the discrete NDGM$_S$(1,1,k,c) model from the Simpson numerical integration formula. Considering the integration of Eq. (5) at the interval $[k-1,\ k+1]$, it follows that:

$$\int_{k-1}^{k+1}dx^{(1)}(t)+a\int_{k-1}^{k+1}x^{(1)}(t)dt=b\int_{k-1}^{k+1}tdt+c\int_{k-1}^{k+1}dt. \quad (9)$$

From Eq. (9), we have:

$$x^{(1)}(k+1)-x^{(1)}(k-1)+a\int_{k-1}^{k+1}x^{(1)}(t)dt=2kb+2c. \quad (10)$$

Applying the Simpson numerical integration formula, we realize that:

$$\int_{k-1}^{k+1}x^{(1)}(t)dt=\frac{1}{3}x^{(1)}(k-1)+\frac{4}{3}x^{(1)}(k)+\frac{1}{3}x^{(1)}(k+1). \quad (11)$$

By substituting Eq. (11) into Eq. (10), it turns to be:

$$(3+a)x^{(1)}(k+1)+4ax^{(1)}(k)-(3-a)x^{(1)}(k-1)$$
$$=6kb+6c. \quad (12)$$

It follows from Eq. (12) that:

$$x^{(1)}(k+1)-wx^{(1)}(k)$$
$$=\frac{a-3}{w(a+3)}\left[x^{(1)}(k)-wx^{(1)}(k-1)\right]$$
$$+\frac{6b}{a+3}k+\frac{6c}{a+3}, \quad (13)$$

where $w=\frac{\sqrt{3a^2+9}-2a}{3+a}$. Iterating Eq. (13) by itself, we obtain that:

$$x^{(1)}(k+1)-wx^{(1)}(k)$$
$$=\frac{a-3}{w(a+3)}\left\{\frac{a-3}{w(a+3)}\left[x^{(1)}(k-1)-wx^{(1)}(k-2)\right]\right\}$$
$$+\frac{a-3}{w(a+3)}\left[(k-1)\frac{6b}{a+3}+\frac{6c}{a+3}\right]$$
$$+k\frac{6b}{a+3}+\frac{6c}{a+3}$$
$$=\left(\frac{a-3}{w(a+3)}\right)^2\left[x^{(1)}(k-1)-wx^{(1)}(k-2)\right]$$
$$+\frac{6b}{a+3}\sum_{m=0}^{1}\left[\left(\frac{a-3}{w(a+3)}\right)^m(k-m)\right]$$
$$+\frac{6c}{a+3}\sum_{m=0}^{1}\left(\frac{a-3}{w(a+3)}\right)^m$$
$$=\left(\frac{a-3}{w(a+3)}\right)^{k-1}\left[x^{(1)}(2)-wx^{(1)}(1)\right]$$

$$+\frac{6c}{a+3}\sum_{m=0}^{k-2}\left(\frac{a-3}{w(a+3)}\right)^m$$

$$+\frac{6b}{a+3}\sum_{m=0}^{k-2}\left[\left(\frac{a-3}{w(a+3)}\right)^m(k-m)\right]$$

$$=\alpha^{k-1}\left[x^{(1)}(2)-wx^{(1)}(1)\right]$$

$$+\frac{6b}{a+3}\sum_{m=0}^{k-2}\alpha^m(k-m)+\frac{6c}{a+3}\sum_{m=0}^{k-2}\alpha^m, \quad (14)$$

with $\alpha=\frac{a-3}{w(a+3)}$. Note that:

$$x^{(1)}(k+1)-w^{k-1}x^{(1)}(2)$$

$$=\sum_{i=0}^{k-2}w^i\left[x^{(1)}(k+1-i)-wx^{(1)}(k-i)\right]$$

$$=\sum_{i=0}^{k-2}w^i\alpha^{k-i-1}\left[x^{(1)}(2)-wx^{(1)}(1)\right]$$

$$+\sum_{i=0}^{k-2}w^i\frac{6b}{a+3}\sum_{m=0}^{k-i-2}\alpha^m(k-i-m)$$

$$+\sum_{i=0}^{k-2}w^i\frac{6c}{a+3}\sum_{m=0}^{k-i-2}\alpha^m$$

$$=\frac{\alpha^{k-1}-(w\alpha^2)^{k-1}}{1-w\alpha}\left[x^{(1)}(2)-wx^{(1)}(1)\right]$$

$$+\frac{6b}{a+3}\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}w^i\alpha^m(k-i-m)$$

$$+\frac{6c}{a+3}\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}w^i\alpha^m. \quad (15)$$

The 1-AGO sequence $\hat{X}^{(1)}$ of discrete NDGM$_S$(1,1,k,c) is:

$$\hat{x}^{(1)}(k+1)=w^{k-1}x^{(1)}(2)$$

$$+\frac{\alpha^{k-1}-(w\alpha^2)^{k-1}}{1-w\alpha}\left[x^{(1)}(2)-wx^{(1)}(1)\right]$$

$$+\frac{6b}{a+3}\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}w^i\alpha^m(k-i-m)$$

$$+\frac{6c}{a+3}\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}w^i\alpha^m, \qquad k=1,2,\cdots,n-1. \quad (16)$$

The IAGO on $\hat{X}^{(1)}$ is applied to obtain:

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k),$$

$$k=1,2,\cdots,n-1. \quad (17)$$

3.2. Parameters estimation of the discrete NDGM$_S$(1,1,k,c) model

Based on the definition of 1-AGO, we obtain:

$$x^{(1)}(k+1)-x^{(1)}(k-1)=\sum_{i=1}^{k+1}x^{(0)}(i)-\sum_{i=1}^{k}x^{(0)}(i)$$

$$=x^{(0)}(k+1)+x^{(0)}(k).$$

Upon employing the Simpson numerical integration formula, the background value of $X^{(1)}$ is provided below:

$$z^{(1)}(k)=\frac{1}{3}x^{(1)}(k-1)+\frac{4}{3}x^{(1)}(k)+\frac{1}{3}x^{(1)}(k+1),$$

$$k=2,3,\ldots,n.$$

Substituting $z^{(1)}(k)$ into Eq. (10), we have that:

$$x^{(0)}(k)+x^{(0)}(k+1)+az^{(1)}(k)=2kb+2c. \quad (18)$$

It follows from Eq. (18) that:

$$\begin{cases} x^{(0)}(2)+x^{(0)}(3)=-az^{(1)}(2)+4b+2c,\\ x^{(0)}(3)+x^{(0)}(4)=-az^{(1)}(3)+6b+2c,\\ \vdots \\ x^{(0)}(n-1)+x^{(0)}(n)=-az^{(1)}(n-1)+2(n-1)b+2c. \end{cases} \quad (19)$$

By applying the least squares estimation method, the model parameters $\hat{\xi}=(a,b,c)^{\mathcal{T}}$ of the NDGM$_S$(1,1,k,c) are:

$$\hat{\xi}=(B^{\mathcal{T}}B)^{-1}B^{\mathcal{T}}Y, \quad (20)$$

where:

$$B=-\begin{pmatrix} z^{(1)}(2) & -4 & -2\\ z^{(1)}(3) & -6 & -2\\ \vdots & \vdots & \vdots\\ z^{(1)}(n-1) & -2(n-1) & -2 \end{pmatrix},$$

$$Y=\begin{pmatrix} x^{(0)}(3)+x^{(0)}(2)\\ x^{(0)}(4)+x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)+x^{(0)}(n-1) \end{pmatrix}.$$

Here, we give a short explanation to demonstrate that $\hat{\xi}$ is the least squares estimation of the model. It is known that to determine the least squares estimation of the $\text{NDGM}_\text{S}(1,1,k,c)$ model is to pursuit an $\hat{\xi}$, thus making the subsequent equation minimum:

$$s(\xi)=\sum_{i=2}^{n-1}\left(x^{(0)}(i)+x^{(0)}(i+1)+az^{(1)}(i)-2ib-2c\right)^2$$
$$=(Y-B\xi)^T(Y-B\xi)=\|Y-B\xi\|^2. \qquad (21)$$

If $\hat{\xi}$ is the least squares estimation of the model, there must be $s(\xi)\geq s(\hat{\xi})$ for any ξ. Let ξ' be the solution of Eq. (20), that is, $\xi'=(B^TB)^{-1}B^TY$.

For any values of ξ, we have:

$$s(\xi)=(Y-B\xi'+B\xi'-B\xi)^T(Y-B\xi'+B\xi'-B\xi)$$
$$=(Y-B\xi')^T(Y-B\xi')+(Y-B\xi')^T(B\xi'-B\xi)$$
$$+(B\xi'-B\xi)^T(Y-B\xi')+(B\xi'-B\xi)^T(B\xi'-B\xi)$$
$$=s(\xi')+(Y^TB-\xi'^TB^TB)(\xi'-\xi)$$
$$+(\xi'-\xi)^T(B^TY-B^TB\xi')$$
$$+(B\xi'-B\xi)^T(B\xi'-B\xi)=s(\xi')+\|B(\xi'-\xi)\|^2. \qquad (22)$$

This means that ξ' is the least square estimation of the $\text{NDGM}_\text{S}(1,1,k,c)$ model. Furthermore, taking $\hat{\xi}$ into Eq. (22), we acquire that:

$$s\left(\hat{\xi}\right)=s(\xi')+\left\|B\left(\xi'-\hat{\xi}\right)\right\|^2. \qquad (23)$$

It follows from Eq. (23) that $s(\hat{\xi})\geq s(\hat{\xi})$. As $\hat{\xi}$ is the least square estimation of the model, we know that $s(\hat{\xi})\leq s(\hat{\xi})$. Thus, $s(\hat{\xi})=s(\hat{\xi})$ and $B(\xi'-\hat{\xi})=0$. Then, $B^TB\hat{\xi}=B^TB\xi'=B^TY$ which leads to $\hat{\xi}=(B^TB)^{-1}B^TY$.

3.3. Modeling evaluation criteria

To examine the forecasting correctness of the $\text{NDGM}_\text{S}(1,1,k,c)$ model, the Absolute Percentage Error (APE), the mean absolute simulation percentage error (MAPE_simu), the mean absolute prediction percentage error (MAPE_pred), and the overall mean absolute percentage error (MAPE_over) are applied. In general, the APE, MAPE_simu, MAPE_pred, and MAPE_over are defined as follows:

$$\text{APE}(k)=\left|1-\frac{\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%, \qquad k=2,3,\cdots,n, \qquad (24)$$

$$\text{MAPE}_\text{simu}=\frac{1}{\nu-1}\sum_{k=2}^{\nu}\left|1-\frac{\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%, \qquad (25)$$

$$\text{MAPE}_\text{pred}=\frac{1}{n-\nu}\sum_{k=\nu+1}^{n}\left|1-\frac{\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%, \qquad (26)$$

$$\text{MAPE}_\text{over}=\frac{1}{n-1}\sum_{k=2}^{n}\left|1-\frac{\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%. \qquad (27)$$

3.4. Unbiased property of the $\text{NDGM}_\text{S}(1,1,k,c)$ model

In this subsection, we prove that the $\text{NDGM}_\text{S}(1,1,k,c)$ model is unbiased to simulate a linear sequence and a homogeneous/non-homogeneous exponential sequence without inaccuracy.

3.4.1. Simulate a linear sequence

Suppose that a linear sequence is $X^{(0)}=\{rk+\theta, k=1,2,\cdots,n\}$. Then, we obtain:

$$x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)=\frac{1}{2}(k+1)kr+k\theta. \qquad (28)$$

The 1-AGO of $X^{(0)}$ is stated by:

$$X^{(1)}=\left\{r+\theta, 3r+2\theta, 6r+3\theta, \frac{(n+1)n}{2}r+n\theta\right\}. \qquad (29)$$

By using these expressions into the matrix B and Y, it can be found that:

$$B=\begin{pmatrix}-19r/3-4\theta & 4 & 2\\ -37r/3-6\theta & 6 & 2\\ \vdots & \vdots & \vdots\\ -r(3n^2-3n+1)/3-(2n-2)\theta & 2(n-1) & 2\end{pmatrix},$$

$$Y=\begin{pmatrix}5r+2\theta\\ 7r+2\theta\\ \vdots\\ (2n-1)r+2\theta\end{pmatrix}.$$

After some calculations, we acquire:

$$\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\left(B^TB\right)^{-1}B^TY=\begin{pmatrix}0\\ r\\ r/2+\theta\end{pmatrix}. \qquad (30)$$

Then, we can easily get:

$$w=1, \qquad \alpha=-1.$$

Substituting these values into Eq. (16), we have:

$$\hat{x}^{(1)}(k+1)=(3r+2\theta)+\frac{(-1)^{k-1}-1}{2}(2r+\theta)$$
$$+2r\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}(-1)^m(k-i-m)$$
$$+(r+2\theta)\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}(-1)^m$$
$$=(3r+2\theta)+\frac{(-1)^{k-1}-1}{2}(2r+\theta)$$

$$+ 2r\left(\frac{k^2}{4}+\frac{3}{8}(-1)^k+\frac{k}{2}-\frac{3}{8}\right)$$

$$+ (r+2\theta)\left(\frac{k}{2}-\frac{1}{4}+\frac{1}{4}(-1)^k\right)$$

$$=\left(\frac{k^2}{2}r+kr+\frac{k}{2}r+r\right)+(k+1)\theta$$

$$+\left(2r+\frac{3}{4}(-1)^k r-\frac{3}{4}r-\frac{1}{4}r+\frac{r}{4}(-1)^k+(-1)^{k-1}r-r\right)$$

$$+\left(\theta-\frac{1}{2}\theta+\frac{\theta}{2}(-1)^k+\frac{\theta}{2}(-1)^{k-1}-\frac{\theta}{2}\right)$$

$$=\frac{(k+2)(k+1)}{2}r+(k+1)\theta=x^{(1)}(k+1). \qquad (31)$$

From Eq. (31), the proposed $\mathrm{NDGM_S}(1,1,k,c)$ model can simulate a linear sequence without errors.

3.4.2. Simulation of a homogeneous/non-homogeneous exponential sequence

Assume that a non-homogeneous exponential sequence is $X^{(0)}=\{rq^k+\theta, k=1,2,\cdots,n\}$. Then, we possess:

$$x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)=\frac{rq(1-q^k)}{1-q}+k\theta,$$

$$k=1,2,\cdots,n. \qquad (32)$$

The 1-AGO of $X^{(0)}$ is given by:

$$X^{(1)}=\left\{rq+\theta,\frac{rq(1-q^2)}{1-q}+2\theta,\frac{rq(1-q^3)}{1-q}\right.$$

$$\left.+3\theta,\cdots,\frac{rq(1-q^n)}{1-q}+n\theta\right\}. \qquad (33)$$

Substituting these expressions into the matrices B and Y, the equation shown in Box I is obtained. After some calculations, we obtain:

$$\begin{pmatrix}a\\b\\c\end{pmatrix}=(B^{\mathcal{T}}B)^{-1}B^{\mathcal{T}}Y=\begin{pmatrix}\frac{3(1-q^2)}{1+4q+q^2}\\ \frac{3\theta(1-q^2)}{1+4q+q^2}\\ \frac{3rq(1+q)+\theta(1+4q+q^2)}{1+4q+q^2}\end{pmatrix}. \qquad (34)$$

Then, we can easily get:

$$w=q, \qquad \alpha=-\frac{q+2}{2q+1}.$$

Further, we have:

$$\frac{6b}{a+3}=\frac{3\theta(1-q^2)}{2q+1},$$

$$\frac{6c}{a+3}=\frac{\theta(1+4q+q^2)+3rq(1+q)}{2q+1}.$$

Substituting these values into Eq. (16), we have:

$$\hat{x}^{(1)}(k+1)=q^{k-1}(rq+rq^2+2\theta)$$

$$+(rq+2\theta-q\theta)\sum_{i=0}^{k-2}q^i\alpha^{k-i-1}$$

$$+\sum_{i=0}^{k-2}q^i\left(\frac{6b(k-i)}{a+3}+\frac{6c}{a+3}\right)\sum_{m=0}^{k-i-2}\alpha^m$$

$$-\sum_{i=0}^{k-2}q^i\frac{6b}{a+3}\sum_{m=0}^{k-i-2}m\alpha^m$$

$$=q^{k-1}(rq+rq^2+2\theta)$$

$$+(rq+2\theta-q\theta)\sum_{i=0}^{k-2}q^i\alpha^{k-i-1}$$

$$+\sum_{i=0}^{k-2}q^i\frac{3\theta(1-q^2)}{2q+1}(k-i)\frac{1-\alpha^{k-i-1}}{1-\alpha}$$

$$+\sum_{i=0}^{k-2}q^i\frac{\theta(1+4q+q^2)+3rq(1+q)}{2q+1}\frac{1-\alpha^{k-i-1}}{1-\alpha}$$

$$+\sum_{i=0}^{k-2}q^i\frac{3\theta(1-q^2)(q+2)}{(2q+1)^2}\left(\frac{1-\alpha^{k-i-1}}{(1-\alpha)^2}\right.$$

$$\left.-\frac{(k-i-1)\alpha^{k-i-2}}{(1-\alpha)}\right)$$

$$B=\begin{pmatrix}-(6+5q+q^2)-12\theta & 4 & 2\\ -(6+6q+5q^2+q^3)-18\theta & 6 & 2\\ \vdots & \vdots & \vdots\\ -\left(6+6\sum\limits_{k=1}^{n-3}q^k+5q^{n-2}+q^{n-1}\right)-6(n-1)\theta & 2(n-1) & 2\end{pmatrix}, \qquad Y=\begin{pmatrix}rq^2+rq^3+2\theta\\ rq^3+rq^4+2\theta\\ \vdots\\ rq^{n-1}+rq^n+2\theta\end{pmatrix}.$$

Box I

Table 1. Results of the NGM(1,1,k,c) and NDGM$_S$(1,1,k,c) models for a nonhomogeneous exponential sequence with $r = 0.06$, $q = 2.25$, and $\theta = 3$.

k	Actual value	NGM(1,1,k,c) model		NDGM$_S$(1,1,k,c) model	
		$\hat{x}^{(0)}(k)$	APE(k)%	$\hat{x}^{(0)}(k)$	APE(k)%
1	3.1350	3.1350	0	3.1350	0
2	3.3038	5.0186	51.9055	3.3038	0
3	3.6834	7.3563	99.7130	3.6834	4.8346×10^{-11}
4	4.5377	12.4014	173.2942	4.5377	8.8666×10^{-11}
5	6.4599	23.2891	260.5184	6.4599	1.5553×10^{-10}
6	10.7848	46.7861	**333.8160**	10.7848	$\mathbf{2.0291 \times 10^{-10}}$
7	20.5158	97.4950	375.2202	20.5158	2.3222×10^{-10}
8	42.4105	206.9302	**387.9227**	42.4105	2.3950×10^{-10}
9	91.6735	443.1029	383.3489	91.6735	2.3462×10^{-10}
10	202.5154	952.7886	370.4771	202.5154	2.2340×10^{-10}
11	451.9097	2052.7440	354.2377	451.9097	2.0963×10^{-10}
12	1013.0470	4426.5639	336.9556	1013.0467	$\mathbf{1.9434 \times 10^{-10}}$
MAPE$_{\text{simu}}$(%)			183.8494		9.9090×10^{-11}
MAPE$_{\text{pred}}$(%)			368.0270		2.2228×10^{-10}
MAPE$_{\text{over}}$(%)			284.3099		1.6629×10^{-10}

$$= q^{k-1}(rq + rq^2) + rq\sum_{i=0}^{k-2} q^i \alpha^{k-i-1}$$

$$+ \sum_{i=0}^{k-2} q^i \frac{3rq(1+q)}{2q+1}\frac{1-\alpha^{k-i-1}}{1-\alpha} + 2\theta q^{k-1}$$

$$+ \theta \sum_{i=0}^{k-2} q^i \Bigg\{ (2-q)\alpha^{k-i-1}$$

$$+ (1-q)(k-i)(1-\alpha^{k-i-1})$$

$$+ \left[\frac{1+4q+q^2}{2q+1} + \frac{(1-q)(q+2)}{2q+1}\right]\frac{1-\alpha^{k-i-1}}{1-\alpha}$$

$$- (k-i-1)\alpha^{k-i-2}\frac{(1-q)(q+2)}{2q+1}\Bigg\}$$

$$= \frac{rq(1-q^{k+1})}{1-q} + 2\theta q^{k-1} + \theta\sum_{i=0}^{k-2} q^i \left\{(2-q)\alpha^{k-i-1}\right.$$

$$+ (1-q)(k-i)(1-\alpha^{k-i-1})$$

$$\left. + (1-q)(k-i-1)\alpha^{k-i-1}\right\}$$

$$= \frac{rq(1-q^{k+1})}{1-q} + 2\theta q^{k-1}$$

$$+ \theta\sum_{i=0}^{k-2} q^i[1 + (1-q)(k-i)]$$

$$= \frac{rq(1-q^{k+1})}{1-q} + (k+1)\theta = x^{(1)}(k+1). \tag{35}$$

From Eq. (35), the proposed NDGM$_S$(1,1,k,c) model can attain an unbiased simulation of a homogeneous/non-homogeneous exponential sequence.

Next, we here provide a numerical experiment to illustrate the accuracy of the NGM(1,1,k,c) and NDGM$_S$(1,1,k,c) models to simulate and predict the non-homogeneous index sequence. Let $x^{(0)}(k) = rq^k + \theta$, $k = 1, 2, \cdots, 12$, $r > 0$. For ease of referencing, the following notation is defined:

$$\varepsilon = |\hat{a} - a| + \left|\hat{b} - b\right|, \tag{36}$$

where $\hat{a}$ and $\hat{b}$ are approximated parameters of NGM(1,1,k,c) and NDGM$_S$(1,1,k,c) models. In addition, parameters a and b are determined using Eq. (34).

Table 1 gives results for $r = 0.06$, $q = 2.25$, and $\theta = 3$. It can be seen in Table 1 that the maximum APEs for simulation of NGM(1,1,k,c) and NDGM$_S$(1,1,k,c) are 333.8160% and 1.7146×10^{-11}% and those for prediction are 387.9227% and 2.0291×10^{-10}%. The MAPE$_{\text{simu}}$, MAPE$_{\text{pred}}$, and MAPE$_{\text{over}}$ for the NGM(1,1,k,c) are 183.8494%, 368.0270%, and

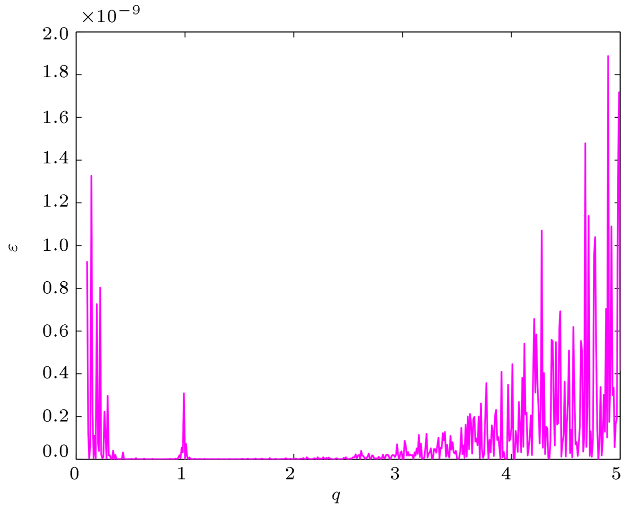

Figure 1. The values of ε for NDGM$_S$(1,1,k,c) model under different values of r, q and θ.

284.3099%; those for the NDGM$_S$(1,1,k,c) are 9.9090×10^{-11}%, 2.2228×10^{-10}%, and 1.6629×10^{-10}%, respectively. Clearly, the APEs of the NDGM$_S$(1,1,k,c) model are caused by the round-off error of computer, while the APEs of the NGM(1,1,k,c) model are caused by its inconsistency.

Further, we select the parameter q given at the interval [0.1,5.0] by the step 0.01 and the parameters r and θ randomly generated at the interval [1,15] and [1,5] by the discrete uniform distribution, respectively. Computational results are depicted in Figure 1. According to Figure 1, the maximum ε is only 1.8867×10^{-9}, which is obviously a truncation error occurring in the computer program process.

4. Applications

In this part, the NDGM$_S$(1,1,k,c) model is utilized to predict the primary energy consumption in Saudi Arabia, India, Philippines, and Vietnam. Outcomes are compared to those of discrete DGM(1,1) model, non-homogeneous NGM(1,1,k) model, NGM(1,1,k,c) model, ARIMA, and SVMs.

The raw data of the primary energy consumption belonging to Saudi Arabia, India, Philippines, and Vietnam are announced from the *BP Statistical Review of World Energy 2017*. These observations are divided into two categories: The observations from 2006 to 2013 utilized to construct different prediction models and the observations from 2014 to 2016 used to verify and differentiate the forecasting results. Raw observation of the primary energy consumption are given in Table 2.

Table 2. Raw data of the primary energy consumption.

Year	Saudi Arabia	India	Philippines	Vietnam
2006	164.5	414.0	25.6	28.1
2007	171.4	450.2	26.7	30.6
2008	186.9	475.7	27.6	38.2
2009	196.5	513.2	28.0	39.3
2010	216.1	537.1	28.8	44.3
2011	222.2	568.7	29.5	50.3
2012	235.7	611.6	30.5	52.5
2013	237.4	621.5	32.5	54.8
2014	252.1	663.6	34.4	59.8
2015	260.8	685.1	37.7	63.7
2016	266.5	723.9	42.1	64.8

4.1. *The primary energy consumption in Saudi Arabia*

We first take the NDGM$_S$(1,1,k,c) model as an example to explain how to build and calculate the simulation and prediction values. From Table 2, the values of $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ of the Saudi Arabia are given below:

$$X^{(0)} = (164.5, 171.4, 186.9, 196.5, 216.1, 222.2, 235.7, 237.4),$$

$$X^{(1)} = (164.5, 335.9, 522.8, 719.3, 935.4, 1157.6, 1393.3, 1630.7),$$

$$Z^{(1)} = (676.9667, 1048.8, 1445.1333, 1872.8333, 2319.7, 2787.1667).$$

It follows from Subsection 3.2 that:

$$B = \begin{pmatrix} -676.9667 & 4 & 2 \\ -1048.8 & 6 & 2 \\ -1445.1333 & 8 & 2 \\ -1872.8333 & 10 & 2 \\ -2319.7 & 12 & 2 \\ -2787.1667 & 14 & 2 \end{pmatrix}, \qquad Y = \begin{pmatrix} 358.3 \\ 383.4 \\ 412.6 \\ 438.3 \\ 457.9 \\ 473.1 \end{pmatrix}.$$

The system parameters can be further resolved to:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \left(B^{\mathcal{T}} B\right)^{-1} B^{\mathcal{T}} Y = \begin{pmatrix} 0.1262 \\ 38.4235 \\ 144.1316 \end{pmatrix}.$$

Moreover, the expression of the NDGM$_S$(1,1,k,c) model is:

$$\hat{x}^{(1)}(k+1) = (0.8815)^{k-1} x^{(1)}(2) + \frac{(-1.0429)^{k-1} - (0.9587)^{k-1}}{1.9193} \times \left(x^{(1)}(2) - 0.8815 x^{(1)}(1)\right)$$

$$+73.7425\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}(0.8815)^i(-1.0429)^m(k-i-m)$$

$$+ 276.6278\sum_{i=0}^{k-2}\sum_{m=0}^{k-i-2}(0.8815)^i(-1.0429)^m,$$

$$k = 1, 2, \dots, 10. \quad (37)$$

Finally, the values of $\hat{x}^{(0)}(k)$ are obtained through Eqs. (17) and (37).

Similarly, the expressions of DGM(1,1), NGM(1, 1,k), NGM(1,1,k,c), and ARIMA models are provided below:

The DGM(1,1) model:

$$\hat{x}^{(1)}(k+1) = 1.0554^k x^{(1)}(1) - \frac{168.0427}{0.0554}(1 - 1.0554^k). \quad (38)$$

The NGM(1,1,k) model:

$$\hat{x}^{(1)}(k+1) = (x^{(1)}(1) + 94.0293)e^{-0.7090k} + 229.0920k - 94.0293. \quad (39)$$

The NGM(1,1,k,c) model:

$$\hat{x}^{(1)}(k+1) = (x^{(1)}(1) + 842.0723)e^{-0.1472k} + 288.7666k - 1130.8389. \quad (40)$$

The ARIMA model:

$$(1-0.9661\mathcal{B})(1-\mathcal{B})x^{(0)}(k) = (1 - 0.5342\mathcal{B})\varepsilon_k, \quad (41)$$

where $\mathcal{B}$ and ε_k are the lag operator and error terms, respectively.

The outcomes of the primary energy consumption in Saudi Arabia are tabulated in Table 3 and Figure 2. The errors are listed in Table 4 and Figure 3.

From Table 3 and Figure 2, We can notice that DGM (1,1), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA (1,1,1), and SVMs models successfully catch the tendency of the primary energy consumption in Saudi Arabia. The numerical outcomes by the NDGM$_S$(1,1,k,c) model are usually closer to the raw data than the outcomes of the other models.

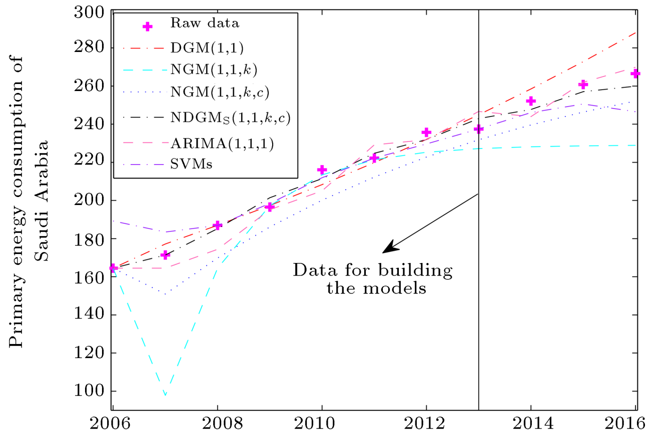

Figure 2. Computational result of the primary energy consumption for Saudi Arabia by DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Table 3. Computational results of the primary energy consumption for Saudi Arabia by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Year	Data	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(1,1,1)	SVMs
2006	164.5	164.5	164.5	164.5	164.5	164.5	189.2396
2007	171.4	177.1631	97.7947	150.9690	171.4	164.5	183.4264
2008	186.9	186.9856	164.4757	169.8332	185.2058	174.3795	186.9510
2009	196.5	197.3527	197.2919	186.1150	201.4061	195.1848	198.4368
2010	216.1	208.2945	213.4420	200.1677	211.4817	205.0715	211.9135
2011	222.2	219.8430	221.3901	212.2967	224.7475	229.1427	222.2509
2012	235.7	232.0318	225.3016	222.7653	231.8679	231.8020	229.6540
2013	237.4	244.8963	227.2266	231.8007	242.9132	246.6592	237.4510
2014	252.1	258.4741	228.1740	239.5992	247.6754	243.9889	246.0452
2015	260.8	272.8047	228.6402	246.3301	257.0601	261.9677	250.5585
2016	266.5	287.9299	228.8697	252.1396	259.9228	269.8285	246.5711

Table 4. Errors of the primary energy consumption for Saudi Arabia by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Year	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(1,1,1)	SVMs
2006	0	0	0	0	0	15.0393
2007	3.3624	42.9436	11.9200	0	4.0257	7.0166
2008	0.0458	11.9980	9.1315	0.9065	6.6990	0.0273
2009	0.4339	0.4030	5.2850	2.4967	0.6693	0.9856
2010	3.6120	1.2299	7.3726	2.1371	5.1034	1.9373
2011	1.0608	0.3645	4.4569	1.1465	3.1245	0.0229
2012	1.5563	4.4117	5.4878	1.6258	1.6538	2.5651
2013	3.1577	4.2853	2.3586	2.3223	3.9003	0.0215
2014	2.5284	9.4907	4.9587	1.7551	3.2174	2.4017
2015	4.6030	12.3312	5.5483	1.4340	0.4477	3.9270
2016	8.0412	14.1202	5.3885	2.4680	1.2490	7.4780
MAPE$_{simu}$	1.8898	9.3766	6.5732	**1.5193**	3.5966	1.7966
MAPE$_{pred}$	5.0576	11.9807	5.2985	1.8857	**1.6380**	4.6022
MAPE$_{over}$	2.8402	10.1578	6.1908	**1.6292**	3.0090	2.6383

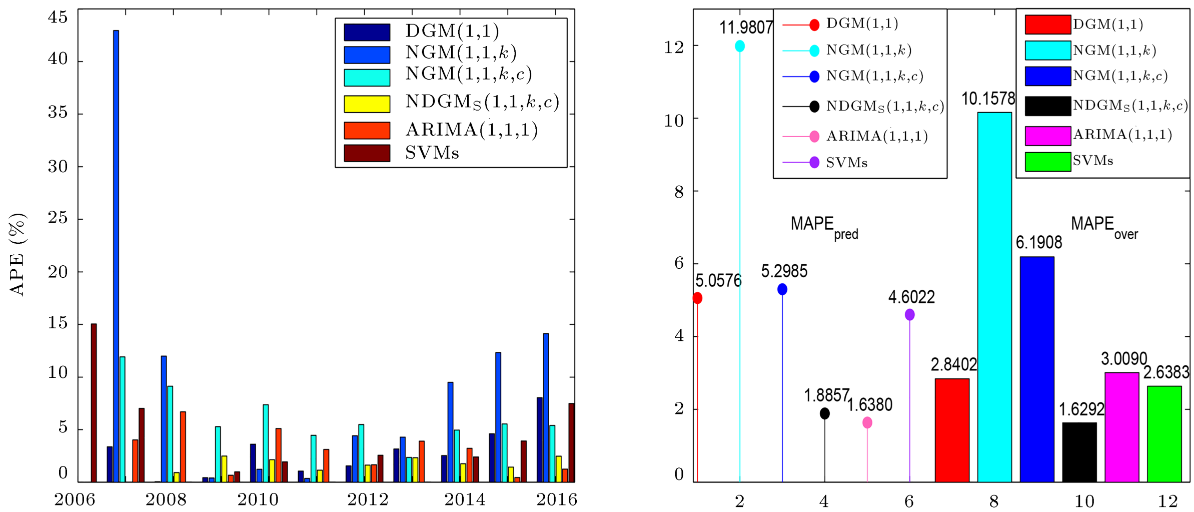

Figure 3. Error values of the primary energy consumption for Saudi Arabia by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

As shown in Table 4, MAPE$_{pred}$ and MAPE$_{over}$ for the NDGM$_S$(1,1,k,c) model are 1.8857% and 1.6292%; those for the DGM(1,1) model are 5.0576% and 2.8402%; those for the NGM(1,1,k) model are 11.9807% and 10.1578%; those for the NGM(1,1,k,c) model are 5.2985% and 6.1908%; those for the ARIMA model are 1.6380% and 3.0090%; and those for the SVM model are 4.6022% and 2.6383%, respectively.

It can be concluded that the new model exceeds other models in this application.

4.2. The primary energy consumption of India

This subsection studies the performance of DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA, and SVMs models in predicting the primary energy consumption in India. Computation results and raw data are shown in Tables 5 and 6 and Figures 4 and 5.

It can be seen in Table 6 that MAPE$_{simu}$, MAPE$_{pred}$, and MAPE$_{over}$ for the DGM(1,1) are 1.0430%, 2.5142%, and 1.4844%; those for the NGM (1,1,k) model are 9.9016%, 14.6999%, and 11.3411%; those for the NGM(1,1,k,c) model are 3.8010%, 3.8784%, and 3.8242%; those for the NDGM$_S$ (1,1,k,c) model are 0.7936%, 0.7412%, and 0.7778%; those for the ARIMA(1,1,1) model are 3.0268%, 2.9249%, and 2.9962%; and those for the SVMs are 1.4642%, 5.1280%, and 2.5633%, respectively.

According to Tables 5 and 6 as well as Figures 4 and 5, the predicted values from the NDGM$_S$(1,1,k,c) model are closer to raw samples than other prediction models. The computation results illustrate that the

Table 5. Computational results of the primary energy consumption for India by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Year	Data	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(1,1,1)	SVMs
2006	414	414	414	414	414	414	482.4519
2007	450.2	454.3107	253.8983	425.7561	450.2	414	471.0947
2008	475.7	480.1950	422.8578	458.4002	475.8445	477.7045	485.5684
2009	513.2	507.5540	506.8931	490.1910	512.0522	498.6237	513.0450
2010	537.1	536.4717	548.6897	521.1506	537.7097	544.6501	541.1383
2011	568.7	567.0371	569.4781	551.3010	573.9289	559.3171	568.5450
2012	611.6	599.3439	579.8176	580.6631	599.5994	595.5762	595.0908
2013	621.5	633.4914	584.9602	609.2577	635.8302	647.6630	621.6551
2014	663.6	669.5845	587.5180	637.1048	661.5137	633.6752	648.4537
2015	685.1	707.7339	588.7901	664.2239	697.7562	697.1583	663.1702
2016	723.9	748.0569	589.4228	690.6341	723.4526	705.7656	652.2294

Table 6. Errors of the primary energy consumption for India by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Year	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(1,1,1)	SVMs
2006	0	0	0	0	0	16.5343
2007	0.9131	43.6032	5.4296	0	8.0409	4.6412
2008	0.9449	11.1083	3.6367	0.0304	0.4214	2.0745
2009	1.1002	1.2289	4.4834	0.2237	2.8403	0.0302
2010	0.1169	2.1578	2.9695	0.1135	1.4057	0.7519
2011	0.2924	0.1368	3.0594	0.9195	1.6499	0.0273
2012	2.0039	5.1966	5.0583	1.9622	2.6200	2.6993
2013	1.9294	5.8793	1.9698	2.3058	4.2097	0.0250
2014	0.9018	11.4650	3.9926	0.3144	4.5095	2.2824
2015	3.3037	14.0578	3.0472	1.8473	1.7601	3.2010
2016	3.3370	18.5768	4.5954	0.0618	2.5051	9.9006
MAPE$_{simu}$	1.0430	9.9016	3.8010	**0.7936**	3.0268	1.4642
MAPE$_{pred}$	2.5142	14.6999	3.8784	**0.7412**	2.9249	5.1280
MAPE$_{over}$	1.4844	11.3411	3.8242	**0.7778**	2.9962	2.5633

NDGM$_S$(1,1,k,c) model outperforms the DGM(1,1), GNM(1,1,k), NGM(1,1,k,c), ARIMA(1,1,1), and SVMs models; in addition, the NGM(1,1,k) has the worst performance.

4.3. The primary energy consumption of Philippines

The simulation and forecasting results of the primary energy consumption of Philippines are tabulated in Table 7 and Figure 6, while the errors are tabulated in Table 8 and Figure 7.

From Table 7 and Figure 6, we can notice that DGM(1,1), NGM(1,1,k), NDGM$_S$(1,1,k,c), ARIMA(2,1,2), and SVMs models successfully catch the tendency of the primary energy consumption in Philippines. The numerical results of the NDGM$_S$(1,1,k,c) model are closer to the raw data than the results of the other models.

As shown in Table 8, MAPE$_{simu}$, MAPE$_{pred}$, and MAPE$_{over}$ for the DGM(1,1) model are 1.1396%, 10.6679%, and 3.9981%; those for the NGM(1,1,k) model are 9.6444%, 21.0239%, and 13.0582%; those for the NGM(1,1,k,c) model are 30.9143%, 76.0195%, and 44.4459%; those for the NDGM$_S$(1,1,k,c) model are 0.8213%, 3.2315%, and 1.5443%; those for the ARIMA model are 3.0352%, 3.2165%, and 3.0896%; and those for the SVMs model are 0.3125%, 4.7765%, and 1.6517%, respectively. Obviously, according to

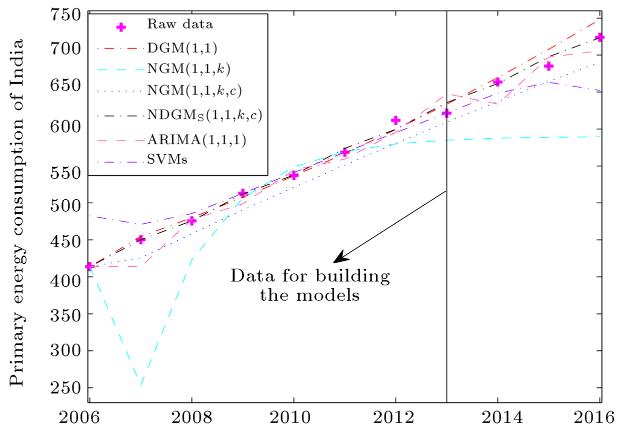

Figure 4. Computational result of the primary energy consumption for India by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

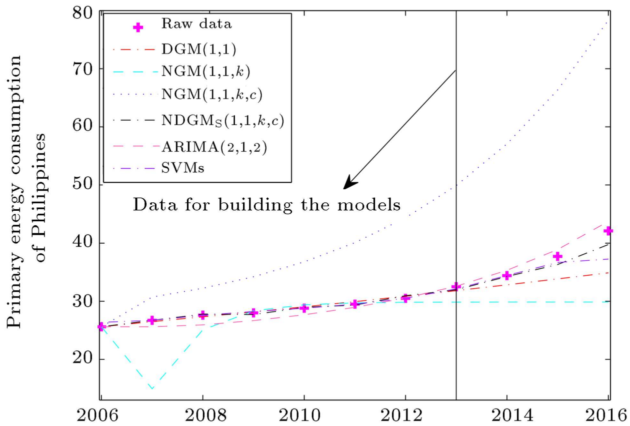

Figure 6. Computational results of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(2,1,2), and SVMs models.

Table 8 and Figure 7, the proposed model outperforms the other models in the case.

4.4. The primary energy consumption in Vietnam

This subsection studies the performance of DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA, and SVMs models in predicting the primary energy consumption in Vietnam. Computation results and raw data are shown in Tables 9 and 10 as well as Figures 8 and 9.

As can be seen in Table 10, the MAPE$_{\text{simu}}$, MAPE$_{\text{pred}}$, and MAPE$_{\text{over}}$ for the DGM(1,1) are 3.7971%, 8.4716%, and 5.1995%; those for the NGM(1,1,k) model are 10.3366%, 14.0915%, and 11.4631%; those for the NGM(1,1,k,c) model are 7.4440%, 6.7516%, and 7.2363%; those for the NDGM$_S$(1,1,k,c) model are 2.1315%, 3.1512%, and 2.4374%; those for the ARIMA(1,1,2) model are 10.6703%, 9.0352%, and 10.1798%; and those of the SVMs are 4.0866%, 3.6375%, and 3.9519%, respectively. Based on Tables 9 and 10 as well as Figures 8 and 9, the predicted values by the NDGM$_S$(1,1,k,c) model are much closer to the raw data than the other models. The computation results illustrate that the NDGM$_S$(1,1,k,c) model exceeds the DGM(1,1), GNM(1,1,k), NGM(1,1,k,c), ARIMA(1,1,1), and SVMs models; besides, the NGM(1,1,k) has the bad performance.

4.5. Discussions and suggestions

The primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam is systematically discussed in this paper by using the DGM(1,1), NGM (1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA, and SVMs models and based on the actual date from 2006

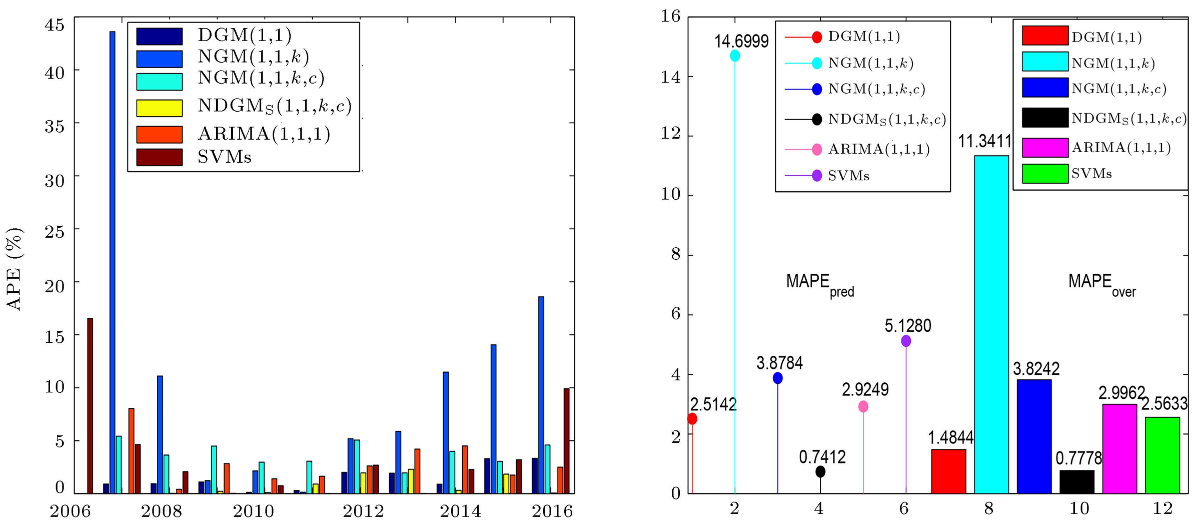

Figure 5. Error values of the primary energy consumption for India by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Table 7. Computational results of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(2,1,2), and SVMs models.

Year	Data	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(2,1,2)	SVMs
2006	25.6	25.6	25.6	25.6	25.6	25.6	26.3584
2007	26.7	26.4819	14.9267	30.7052	26.7	25.6	26.7165
2008	27.6	27.3055	25.0760	32.2316	27.7639	25.9164	27.5835
2009	28.0	28.1546	28.3283	34.2071	27.7091	26.6613	28.2206
2010	28.8	29.0302	29.3705	36.7638	28.9468	27.6595	28.7835
2011	29.5	29.9329	29.7045	40.0728	29.3329	28.9488	29.4929
2012	30.5	30.8638	29.8116	44.3552	30.9366	30.5619	30.5165
2013	32.5	31.8236	29.8459	49.8976	31.9773	32.5695	32.1285
2014	34.4	32.8133	29.8568	57.0707	34.2522	35.3050	34.4165
2015	37.7	33.8337	29.8604	66.3542	36.3111	38.8936	36.6577
2016	42.1	34.8859	29.8615	78.3689	39.7505	43.7219	37.2514

Table 8. Errors of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(2,1,2), and SVMs models.

Year	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(2,1,2)	SVMs
2006	0	0	0	0	0	2.9625
2007	0.8168	44.0947	15.0007	0	4.1199	0.0618
2008	1.0672	9.1451	16.7812	0.5937	6.1000	0.0598
2009	0.5521	1.1726	22.1682	1.0389	4.7812	0.7879
2010	0.7991	1.9811	27.6522	0.5096	3.9601	0.0572
2011	1.4676	0.6933	35.8399	0.5666	1.8684	0.0239
2012	1.1928	2.2572	45.4270	1.4316	0.2031	0.0540
2013	2.0812	8.1666	53.5312	1.6084	0.2137	1.1430
2014	4.6126	13.2069	65.9032	0.4296	2.6309	0.0480
2015	10.2555	20.7948	76.0057	3.6840	3.1662	2.7647
2016	17.1357	29.0701	86.1494	5.5809	3.8525	11.5169
MAPE$_{simu}$	1.1396	9.6444	30.9143	0.8213	3.0352	**0.3125**
MAPE$_{pred}$	10.6679	21.0239	76.0195	3.2315	**3.2165**	4.7765
MAPE$_{over}$	3.9981	13.0582	44.4459	**1.5443**	3.0896	1.6517

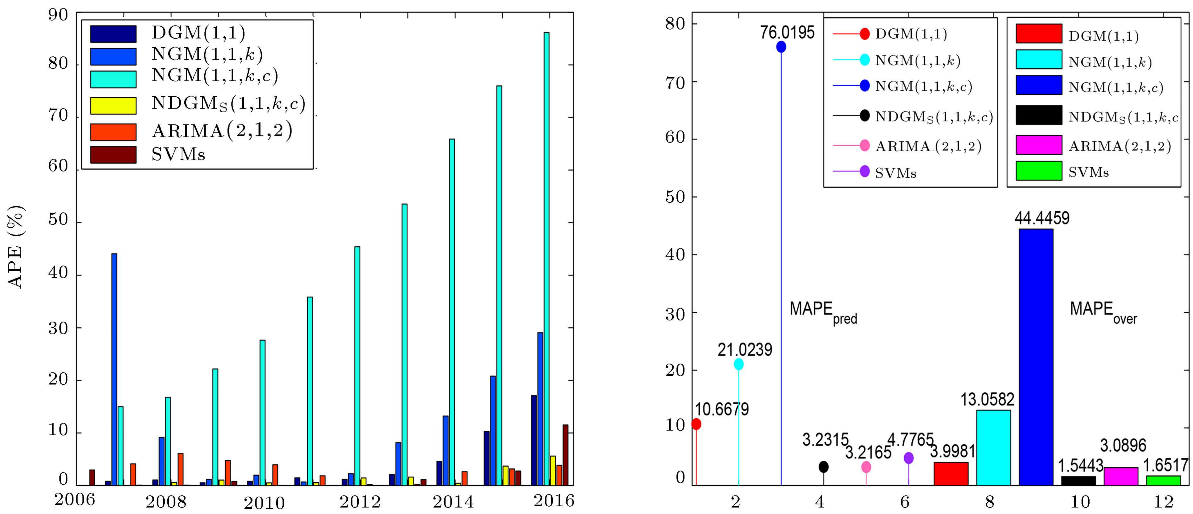

Figure 7. Error values of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,1), and SVMs models.

Table 9. Computational results of the primary energy consumption for Vietnam by DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,2), and SVMs models.

Year	Data	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(1,1,2)	SVMs
2006	28.1	28.1	28.1	28.1	28.1	28.1	38.7583
2007	30.6	33.4598	18.7478	26.9497	30.6	28.1	37.5079
2008	38.2	36.5603	30.8373	32.5806	37.8519	33.2028	38.1755
2009	39.3	39.9482	38.9456	37.6267	39.3964	43.6051	40.3060
2010	44.3	43.6500	44.3837	42.1488	46.2959	35.4189	44.3244
2011	50.3	47.6948	48.0309	46.2012	47.3175	53.7805	49.0752
2012	52.5	52.1145	50.4770	49.8328	53.9123	47.6229	52.4755
2013	54.8	56.9437	52.1176	53.0872	54.4497	58.2137	55.2775
2014	59.8	62.2204	53.2180	56.0036	60.7831	52.2579	58.9776
2015	63.7	67.9860	53.9559	58.6171	60.8705	68.3572	61.6825
2016	64.8	74.2860	54.4509	60.9593	66.9822	60.1459	60.6723

Table 10. Errors of the primary energy consumption for Vietnam by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_S$(1,1,k,c), ARIMA(1,1,2), and SVMs models.

Year	DGM(1,1)	NGM(1,1,k)	NGM(1,1,k,c)	NDGM$_S$(1,1,k,c)	ARIMA(1,1,2)	SVMs
2006	0	0	0	0	0	37.9298
2007	9.3457	38.7327	11.9291	0	8.1699	22.5747
2008	4.2923	19.2740	14.7104	0.9112	13.0817	0.0640
2009	1.6494	0.9018	4.2577	0.2452	10.9546	2.5597
2010	1.4673	0.1889	4.8560	4.5054	20.0475	0.0552
2011	5.1792	4.5111	8.1487	5.9294	6.9195	2.4350
2012	0.7343	3.8532	5.0804	2.6901	9.2897	0.0466
2013	3.9118	4.8948	3.1256	0.6393	6.2294	0.8714
2014	4.0474	11.0068	6.3485	1.6440	12.6123	1.3752
2015	6.7285	15.2968	7.9794	4.4419	7.3111	3.1673
2016	14.6388	15.9709	5.9271	3.3676	7.1823	6.3700
MAPE$_{simu}$	3.7971	10.3366	7.4440	**2.1315**	10.6703	4.0866
MAPE$_{pred}$	8.4716	14.0915	6.7516	**3.1512**	9.0352	3.6375
MAPE$_{over}$	5.1995	11.4631	7.2363	**2.4374**	10.1798	3.9519

to 2016. The computational results show that the NDGM$_S$(1,1,k,c) model outperforms the other prediction models in primary energy consumption.

The *BP Statistical Review of World Energy* states that the energy mix inches towards cleaner, lower carbon fuels determined by the environment needs and the technological progress. This result points out that the growth of worldwide primary energy consumption remained low in 2016. This growth is below average in all states except Europe (Saudi Arabia) & Eurasia (India, Philippines, Vietnam). As known, the fossil energy can produce harmful gases that pollute the environment and lead to ecological problems. Moreover, the government will play its role in meeting the dual challenge of supplying the energy for the nation's needs to grow and prosper and reducing carbon emissions. The mentioned entity should reduce the traditional energy consumption and greatly increase clean energy consumption in the future. We hope that our computational results can provide a guidance for the government to formulate and adjust energy policies.

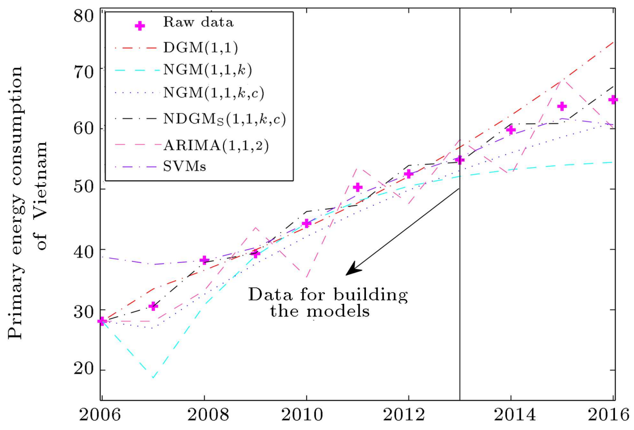

Figure 8. Computational results of the primary energy consumption for Vietnam by DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_{\mathrm{S}}$(1,1,k,c), ARIMA(1,1,2), and SVMs models.

5. Conclusions

This research study investigated the discrete NDGM$_{\mathrm{S}}$(1,1,k,c) model with Simpson formula. Mathematical analysis was carried out to determine the properties of the proposed model. Further, the primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam was carried out to verify the performance of our model with the DGM (1,1), NGM(1,1,k), NGM(1,1,k,c), Auto-Regressive Integrated Moving Average (ARIMA), and the Support Vector Machines (SVMs) models. The results showed that the new NDGM$_{\mathrm{S}}$(1,1,k,c) model had high potential in the primary energy consumption with higher accuracy than the other models.

It needs to be pointed out that the GM(1,1), DGM(1,1), and their generalized models are homogeneous exponential models. However, it is difficult to meet data sequences with the significant growth of homogeneous exponent in real situations. This result illustrates that the homogeneous exponent models are inapplicable. According to the analysis of the NDGM$_{\mathrm{S}}$(1,1,k,c) model, it is known that the new model can be used as either a homogeneous exponent model or an non-homogenous model, which has a wide range of applications in the real world. Moreover, the proposed model is suitable for simulation and prediction data sequences with only a few samples (not less than four). However, the time series analysis and the computational intelligence technology require a large amount of data. It is sometimes impossible to get as many as observed samples in the real world.

In the future, the new NDGM$_{\mathrm{S}}$(1,1,k,c) model can be used for data forecasting such as nuclear energy consumption, the production of shale gas, etc. Further, the method for the NDGM$_{\mathrm{S}}$(1,1,k,c) model can be applied to analyze other grey models such as GM(1, n) or Verhulst models.

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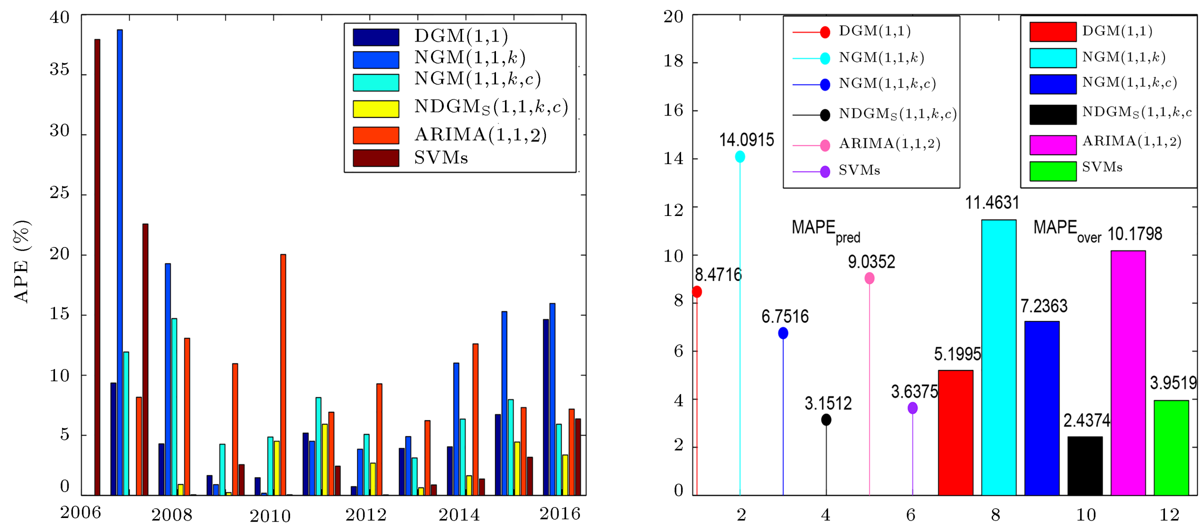

Figure 9. Errors of the primary energy consumption for Vietnam by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM$_{\mathrm{S}}$(1,1,k,c), ARIMA(1,1,2), and SVMs models.

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