A Response-Based Approach for Online Prediction of Generating Unit Angular Stability

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Abstract

In this paper, first, a rotor angle trajectory model based on polynomial functions is proposed. Afterwards, a response-based approach for online prediction of power system angular instability is presented. The proposed method utilizes bus phase angle data measured by phasor measurement unit at the point of common coupling of power plant transformer to the bulk power grid. In the prediction process, by computing the second order derivative of post-fault data, the starting point of the calculation data window is determined. Next, a fifth-degree polynomial curve is fitted on the designated data window to predict the angular curve of generating unit. Based on the sign of the first order derivative of predicted curve, the angular stability of generating unit is judged. This approach is testified on the western system coordinating council standard test bed under different operation and fault type scenarios. Taking into account various fault conditions and their associated occurrence probability, a probabilistic index is also defined to sum up the overall performance of the new method. Simulation results confirm that the proposed method outperforms the existing ones in terms of both accuracy and speed. Prediction results could be used in generator rejection schemes to prevent severe power plant outages.

KEYWORDS: Angular stability; response-based approach; phasor measurement unit (PMU).

1. INTRODUCTION

1.1. Motivation

The ability of the power network to maintain its synchronization in the face of intense events is referred to as transient stability [1]. Transient stability is categorized into twofold major classes: assessment and prediction [2-4]. In the transient stability assessment, the results, of which the critical clearing time (CCT) is the most vital one, are obtained basically according to the power system equipment models. Time consuming calculations, heavy computational burden, and the need to accurate power equipment models are some barriers against the prosperous implementation of transient stability assessment techniques. On the opposite, the prediction methods are mainly based on the response and behavior of the power system. Prediction of power system transient stability status is the
ultimate goal of these techniques.

Trajectory of some important characteristics such as frequency, voltage, rotor angle, and rotor speed can reflect wide-area response of the power system in facing with severe faults. Analysis and process of these features, before occurrence of the system instability, would provide invaluable data to activate the emergency control of power system and diminish the severity of disturbances. In the power system transient stability phenomenon, the rotor angle of generating unit is the most important characteristic. Prediction of the rotor angle trajectory is the first and most prominent category of transient stability prediction approaches founded on the real-time wide-area phasor measurements. This category consists of super real-time simulation methods, curve fitting extrapolations, and angular velocity prediction techniques. The second category is the transient instability detection. This category, in contrast, uses geometric properties of the response curve along with threshold value criteria to anticipate the future angular stability of the generating unit [5]. The use of hybrid response-based methods which requires short prediction time, has an acceptable level of accuracy, and can be simplicity in application are always noteworthy.

1.2. Literature review

Some research efforts on the transient instability prediction method through synchrophasor measurements have been reported in the literature. In [6-8], a polynomial curve fitting method was used to predict the future trend of bus phase angle. This method has been compared to other curve fitting methods such as trigonometric function and auto-regressive model in [9]. In [10], a suit of criteria based on the generator angle security, frequency deviation, load bus voltage magnitude, and load bus voltage angle have been introduced to be used for transient stability prediction. In [11, 12], artificial neural network as an intelligent method has been applied for the angular instability prediction. A machine learning approach using the linear support vector and decision tree is proposed to predict transient stability condition of power system [13]. In [14, 15], the effects of quality, availability, and uncertainty on the transient stability prediction have been investigated based on the decision tree method. In [16], an adaptive equivalent of power system based on a ball-on-concave-surface mechanics system has been used to determine stability status of the monitored variable such as angle/ frequency difference between two areas. An adaptive method was used closely with an auto-regressive model in [17] to predict the future trend of generator power angle. The outcome of this method is estimating the transient behavior of generating unit. In [18], Generator instability is predicted with PMU data based on a novel predictive out of step protection approach. In this technique, by comparing the acceleration
areas respect to the fault time and deceleration areas corresponding to the post-fault condition, transient instability of generating unit is predicted. Transient stability of generator by using rotor speed-acceleration \((w-a)\) curve based on the PMU data is predicted in [19]. An adaptive generator out of step prediction scheme based on the Bayesian technique is presented in [20]. In this scheme Bayesian technique is applied on the measured data to extracted proper features. Tripping signal of proposed scheme is estimated based on these features. In [21], transient stability is predicted through three successive steps by comparing real-time relative angle measured by PMU and predefined thresholds calculated with offline simulations. In [22], transient stability margin after a fault clearance is estimated based on the energy function through PMU data. Then, a look-up table consists different fault conditions data is used to predict the proper stable or unstable situations.

Two functional aspects of all transient stability prediction techniques are their accuracy and speed. These features will be much critical if the output of prediction method stimulates control or protection actions. Low speed or inaccurate methods can lead to incorrect actions and consequently adverse effects on the security and reliability of power system [23].

Special Protection Schemes (SPSs) as part of defense plans are designed in order to minimize the effects of severe disturbances. Generator Rejection System (GRS) as a sort of SPSs uses the outcomes of transient instability prediction methods to keep the whole power plants away from sudden outages [8]. The proper operation of GRS directly depends on the accuracy and speed of the feeding transient stability prediction method. In addition, the GRS desirable or adverse performance has economic effects on the power system operation.

1.3. Contribution

This paper presents a response-based approach for the online prediction of power system angular stability to overcome the shortcoming of existing methods, which are time consuming and complex computations as well as requiring equipment models and network configuration changes. The proposed method uses phasor measurement data captured at the point of common coupling (PCC) of power plant transformer to the bulk power grid by phasor measurement unit (PMU). The information is used to predict the angular stability status of generating unit in a very short time based on the Second order Derivative Method (SDM). The advantages of this method include requiring only bus voltage phase angle, very low computation burden, short prediction time, not requiring equipment models and system configuration and operation status, high accuracy and dependability against fault types, and simplicity in
application. The overall performance of the new method is measured through a probabilistic index incorporating various fault types and probabilities.

1.4. Organization

The paper is followed by modeling of rotor angle trajectory in Section 3. The main aspects of the second order derivative method is presented in Section 4. Prediction of power system angular stability by using SDM is explained in Section 5. In Section 6, numerical results obtained by examining this approach on a well-known test systems are presented. The drawn conclusions are outlined in Section 7.

2. MODELING OF ROTOR ANGLE TRAJECTORY

The rotor angle trajectory of synchronous generators following a severe disturbance would be one of the cases displayed in Figure 1. As pointed out in [24], case 1 is referred to as the first-swing stable, although it is oscillatory unstable. Cases 2 and 3 illustrate unstable trajectory due to the continuous increase of rotor angle, and case 4 represents the stable situation as a result of the oscillation damping out at the post-fault rotor angle trajectory. As mentioned previously, the goal of transient stability prediction methods is to determine that the future response of generating unit would be similar to which of four cases illustrated in Figure 1. Majority of efforts in this context have focused on the prediction of stability or instability of disturbed power network only in the first swing time duration [8], [25, 26].

FIGURE 1.

To get a better insight into rotor angle trajectory modeling, it is therefore interesting to take a look at one of the well-known transient stability methods called SIngle Machine Equivalent (SIME) which rely on a one machine infinite bus (OMIB) equivalent and depicted in Figure 2.

FIGURE 2.

The accelerating power associated with OMIB model is expressed as [3], [27]:

\[ P_a = P_m - P_c \]  \hspace{1cm} (1)
\[ P_a = a\delta^2 + b\delta + c \]  \hspace{1cm} (2)
The temporal characteristic of rotor angle could be shown as [28]:

\[ \delta(t) = A \sin(w_d t + \theta) \]  

(4)

Inserting Equation (4) in the right side of Equation (2) and replacing the terms \( aA^2, bA, \) and \( wdt+\theta \) by Relation (5) yields Equation (6):

\[ P_a = a(A \sin(w_d t + \theta))^2 + b(A \sin(w_d t + \theta)) + c = a_i(\sin(x_i))^2 + b_i \sin(x_i) + c \]  

(5)

with the first three orders of McLaurin expansion, \( \sin(x) \) could be expressed as Equation (7) [28], and inserting Equation (7) in Equation (6) yields Equation (8)

\[ \sin(x_i) = x_i - \frac{x_i^3}{3!} + \frac{x_i^5}{5!} \]  

(7)

\[ P_a = a_i(x_i - \frac{x_i^3}{3!} + \frac{x_i^5}{5!})^2 + b_i(x_i - \frac{x_i^3}{3!} + \frac{x_i^5}{5!}) + c = a_i(x_i^2 + \frac{x_i^6}{(3!)^2} + \frac{x_i^{10}}{(5!)^2} - \frac{2x_i^4}{3!} + \frac{2x_i^6}{5!} + \frac{2x_i^8}{3!5!}) + b_i(x_i - \frac{x_i^3}{3!} + \frac{x_i^5}{5!}) + c \]  

(8)

According to Relation (5), \( x_1 \) is a function of \( t, w_d, \) and \( \theta \). In a real power system it could be assumed that \( w_d \) and \( \theta \) are equal to 8 rad/sec and 0.2 rad, respectively. Based on the sampling and reporting rate of measured data, \( t \) is at the scale of \( 10^{-2} \) sec. In this case, possible values of \( x_1 \) are in the range of 0.3 to 1 rad. Neglecting the higher orders with small coefficients, \( P_a \) can be expressed as follows:

\[ P_a = -\frac{b_i w_d^3}{3!} t^3 + (a_i w_d^2 - \frac{b_i}{2!} w_d^2 \theta) t^2 + (a_i w_d \theta - \frac{b_i}{2!} 3w_d \theta^2) t + b_i \theta + a_i \theta^2 - \frac{b_i}{3!} \theta^3 + c \]  

(9)

Equation (2) could be expressed as Equation (10) by replacing coefficient of Equation (9) by Relation (11) as follow:

\[ P_a = et^3 + ft^2 + gt + d \]  

(10)

where

\[ -\frac{b_i}{3!} w_d^3 = e, (a_i w_d^2 - \frac{b_i}{2!} w_d^2 \theta) = f, (a_i w_d \theta - \frac{b_i}{2!} 3w_d \theta^2) = g, b_i \theta + a_i \theta^2 - \frac{b_i}{3!} \theta^3 + c = d \]  

(11)

So, Equation (3) could be recast as follow:

\[ et^3 + ft^2 + gt + d = \frac{W_o}{2H} \frac{d^2 \delta}{dt^2} + D \frac{d \delta}{dt} \]  

(12)

In addition, the right side of Equation (12) could be extended as follow:
with the first two orders of McLaurin expansion of \( \sin(x_i) \) the last term of Equation (13) is written as:

\[
DA \int \sin(w \cdot t + \theta) = DA \int ((w \cdot t + \theta) - \frac{(w \cdot t + \theta)^3}{3!}) + c_4 t^4 + c_3 t^3 + c_2 t^2 + c t + c_0
\]

where

\[
c_4 = -\frac{DA}{4} w^3_d, c_3 = -\frac{DA}{3} w^2_d \theta, c_2 = DA \left( \frac{w_d}{2} - \frac{1}{2 \times 2!} w^2_d \theta^2 \right), c_1 = DA \left( \theta - \frac{1}{3!} \theta^3 \right)
\]

With Equation (14), Equation (13) can be recast as:

\[
\frac{W_0}{2H} \delta(t) + c_4 t^4 + c_3 t^3 + c_2 t^2 + c t + c_0
\]

Similar (twice integration in the time domain) on the left side of the Equation (12) yields:

\[
et^3 + ft^2 + gt + d \int e t^4 + \frac{f}{3} t^3 + \frac{g}{2} t^2 + dt + h \int e \frac{t^5}{20} + \frac{f}{12} t^4 + \frac{g}{6} t^3 + \frac{d}{2} t^2 + ht + i
\]

Equalization (14) and Equation (17), and simplification yields:

\[
\delta(t) = A_5 t^5 + A_4 t^4 + A_3 t^3 + A_2 t^2 + A_1 t + A_0
\]

where \( A_i (i=0 \rightarrow 5) \) are as follow:

\[
A_5 = \frac{2H}{W_0} e, A_4 = \frac{2H}{W_0} (f - c_4), A_3 = \frac{2H}{W_0} (g - c_3), A_2 = \frac{2H}{W_0} (d - c_2), A_1 = \frac{2H}{W_0} (h - c_1), A_0 = \frac{2H}{W_0} (i - c_0)
\]

Finally, Equation (20) can be used to predict the rotor angle

\[
\hat{\delta}(t) = \sum_{i=0}^{n} A_i t^i
\]

where \( \hat{\delta}(t) \) is the estimated rotor angle at time \( t \). \( A_i (i=0 \rightarrow n) \) and \( n \) are polynomial coefficients and the order of rotor angle polynomial model, respectively. According to equation (14) and (17), and the aforementioned assumptions about Equation (8) and (9), 5 is selected as final value for \( n \). Polynomial coefficients are expressed in the form of a parameter vector as:

\[
A_N = [A_0, A_1, A_2, ..., A_n]^T
\]

Similarly, the observation vector is:

\[
O(N) = [\delta(0), \delta(\Delta t), \delta(2(\Delta t)), ..., \delta(N(\Delta t))]^T
\]

where \( \Delta t \) is the time duration between two consecutive samples. If samples are provided by PMUs, \( \Delta t \) should be in accordance with the reporting rate of PMUs. Eventually, the parameter vector \( (A_N) \) could be obtained by following equations based on the least square method:
\[
O(N) = A_N \cdot T(N)
\]
\[A_N = F_N T^T(N) \cdot O(N)
\]

Where \(T(N)\) and \(F_N\) are:

\[
T(N) = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
1 & \Delta t & (\Delta t)^2 & \cdots & (\Delta t)^n \\
1 & 2(\Delta t) & (2(\Delta t))^2 & \cdots & (2(\Delta t))^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & N(\Delta t) & (N(\Delta t))^2 & \cdots & (N(\Delta t))^n
\end{bmatrix}
\]

\[
F_N = [T^T(N)T(N)]^{1/2}
\]

\(T(N)\) and \(F_N\) are two constant matrices since they are independent from measurements and are only in terms of the time duration between two consecutive measured datasets. Having calculated \(A_N\), \(m\) samples of rotor angle future quantity can be computed by Equation (20) replacing \(i\Delta t\) instead of \(t\) in which \(i\) takes \(N+1, N+2, \ldots, N+m\). Rolling prediction method can be used to calculate \(O(N+1)\) and \(A_{N+1}\) through Equation (22) and Equation (24), respectively, when a new rotor angle measurement is accessible.

Coefficients \(A_i (i = 0 \rightarrow n)\) are obtained by minimizing the sum of squared differences of the actual and estimated angle values formulated as:

\[
M = \sum_{j=1}^{\text{NS}} (\delta_j - \hat{\delta}_j)^2
\]

where \(\delta_j\) and \(\hat{\delta}_j\) are \(j^{th}\) actual value obtained from PMU sampling and estimated value from Equation (20), respectively, and \(\text{NS}\) is the number of samples in the curve fitting DW.

3. THEORETICAL AND MATHEMATICAL ASPECTS OF THE SECOND ORDER DERIVATIVE METHOD (SDM)

In [8] it was shown that a wider data window (DW) used for transient stability prediction does not necessarily enhance the accuracy of prediction. Alongside the DW length, the start point of DW is another important feature affecting the method performance. In order to investigate the start point of DW, this paper proposes a curve fitting-based rotor angle trajectory prediction method enhanced by geometrics properties of rotor angle graph. The proposed method is called SDM.

The curve fitting-based rotor angle trajectory prediction does not need the knowledge of power system properties such as network topology, equipment models, and dynamic network equivalence [5]. On the other hand, the geometrical characteristics of rotor angle curve are very illustrative in selecting an appropriate start point of the curve.
fitting DW. Aggregation of all these properties in SDM enables it as a simple, accurate, and fast method tailored to be used in the online GRSs.

SDM uses a moving fixed-length DW beside the fifth-degree curve fitting method for the prediction of angular stability of power network. DW is moving because its start point is specified in terms of the second order derivative of incoming data transmitted by PMUs. Having fully accumulated the DW, all subsequent processing to predict the angular stability begins and it is updated upon receiving a new dataset.

Figure 3 shows the local maximum, minimum, and inflection points of a typical fifth-degree polynomial angular curve. In this figure, if \( X_0 \) is assumed to be the start time of the fault, the portion between \( X_0 \) and \( X_4 \) is considered as a stable rotor angle first swing and \( F(X_2) \) is the maximum value of angular position. In this piece, a point such as \( X_i \) will definitely exist in which the sign of curve concavity is changed from positive to negative. SDM highlights the role of this point in improving the efficiency of angular stability prediction.

The local maximum, minimum, and inflection points of fifth-degree polynomial are the roots of its first and second order derivatives given as follows:

\[
\frac{d \hat{\delta}(t)}{dt} = 5A_t t^4 + 4A_t t^3 + 3A_t t^2 + 2A_t t + A_1 = 0 \tag{28}
\]

\[
\frac{d^2 \hat{\delta}(t)}{dt^2} = 20A_t t^3 + 12A_t t^2 + 6A_t t + 2A_2 = 0 \tag{29}
\]

\( X_0, X_2 \) and, \( X_4 \) as local minimum and maximum points of the fifth-degree polynomial curve are obtained from Equation (28) and \( X_i \) and \( X_j \) as inflection points are the zeros of Equation (29).

It should be noted that the aforementioned equations are expressed in the continuous space; while PMUs measure and report data based on their technical specifications in the discrete space. Accordingly, the first and second order derivatives in the discrete space are derived as:

\[
\frac{\Delta (\hat{\delta}(t))}{\Delta t} = \frac{\hat{\delta}(t) - \hat{\delta}(t-\Delta t)}{\Delta t} , \frac{\Delta (\hat{\delta}(t-\Delta t))}{\Delta t} = \frac{\hat{\delta}(t-\Delta t) - \hat{\delta}(t-2\Delta t)}{\Delta t} \tag{30}
\]

\[
\frac{\Delta^2 (\hat{\delta}(t))}{\Delta t^2} = \frac{\delta(t) - \hat{\delta}(t-\Delta t) - \hat{\delta}(t-\Delta t) - \hat{\delta}(t-2\Delta t)}{\Delta t} = \frac{\hat{\delta}(t) - 2\hat{\delta}(t-\Delta t) + \hat{\delta}(t-2\Delta t)}{(\Delta t)^2} \tag{31}
\]

where \( \Delta t \) is time difference between two successive PMU reported data. Referring to Equation (30) and Equation (31), calculation of the first and second order derivatives requires to be elapsed one and two PMU reporting time,
respectively. In other words, calculation of second order derivative at each time step is based on two time steps ago. These calculations also valid for the first swing of the fifth-degree polynomial.

4. PREDICTION OF POWER SYSTEM ANGULAR STABILITY BASED ON SDM

Rotor angle of generating unit is the main data for the transient stability prediction. Rotor angle can be computed by solving the swing equation and the electrical output power of generator [6],[26]. The shortcomings of such techniques include determination of the initial value of rotor angle, lack of online access to the inertia constants, and variations in inertia constant by physical and geometrical structural changes [26], [29]. Generator rotor angle and the associated terminal bus phase angle have a same variation trend. Even though they are not exactly equal, in some transient stability prediction methods, the generator rotor angle is hence estimated with voltage phase angle at PCC of power plant transformer to the bulk power grid [8]. In this paper, we have this assumption. Bus phase angle is measured by PMU synchronized with the high accuracy Global Positioning System (GPS). In terms of sampling and reporting rates of PMUs, the DW required for the prediction of angular stability is specified.

Figure 4 depicts the general scheme of the SDM for the prediction of power system angular stability. The algorithm consists of four main steps explained in the following:

- **Step 1:** Transient stability prediction methods are stimulated if a fault is detected on the power system. On this basis, the first step of the SDM consists of fault detection on transmission system and fault clearing time. These requirements can be fulfilled based on PMU data [30]. At the fault clearing time, the next step begins.

- **Step 2:** After the fault clearance, the voltage phase angle at the PCC of power plant transformers to the bulk power grid, which is continuously measured by PMU, are loaded in the SDM algorithm. This process continues until the third data arrives. From this point, Equation (31) will be calculated for all subsequent data. When the second derivative became zero or negative, the calculation stops. At this data point, the curve concavity changes from positive to negative. The third step will start at this point.

- **Step 3:** Based on the result of Step 2, calculation DW starts to fill up with PMU data. The length of DW is in terms of the PMU reporting rate. According to IEEE Standard for Synchrophasor Measurements (IEEE std C37.118) [31], PMU shall support various data reporting rates associated with the nominal frequency of power systems. For 50 Hz system, supported rates are 10, 25, 50 Hz while 100 Hz reporting rate is permissible as well.
Without lack of generality, 100 ms DW is adopted here. This DW consists of 10 post-fault samples measured by PMU. When DW is filled out, the fifth-degree polynomial curve fitting is estimated. The obtained curve is used as an input data for the final step of processing.

- **Step 4:** In the final step, there are two complementary parts which ultimately predict the angular stability of power network. In the first part, the first order derivative of the resultant curve known as the curve slope is calculated by Equation (28). This part starts from the second predicted dataset. In the next part, the slope sign is to be determined at each data point. This process is applied only on predicted rotor angle value less than or equal to 180 degrees. 180 degrees is the boundary rotor angle value; because the system is unstable if the predicted value exceeds this threshold. For predicted rotor angle less than 180 degrees, two scenario are expected as follows:
  - Unstable cases: if the slope sign is always positive, it clarifies that the rotor angle is constantly growing and definitely passes the boundary value (Cases 2 and 3 in Figure 1).
  - Stable cases: positive and negative signs reflect the growth and then decline of predicted curve. Rotor angle grows after the fault clearance but its growth stops before crossing the stability boundary. As a result, the system stability is maintained at the first swing (Cases 1 and 4 in Figure 1).

**FIGURE 4.**

5. **NUMERICAL STUDY**
This section examines the proposed scheme for a practical SIME as shown at Figure 2. This system incorporates generating unit rated at 128 MVA and 115 MW with an inertia constant of 3.18 sec. which is connected to the 400 kV substation through one 12.7/400 kV transformer. A PMU device is installed at the high voltage side of the substation. The sampling and reporting rates of the PMU are 10 kHz and 100 sample/s, respectively. The subsystem is connected to the external grid as infinite bus through two high voltage transmission lines. For the sake of simulation, the DIgSILENT software package [32] is used. Data analysis process is performed in the MATLAB environment. The three phase fault (LLL) is supposed to be located at 0.1% of L2 that occurs at t = 0. The Critical Clearing Time (CCT) is 0.329 sec. So, the fault is cleared by opening the line circuit breaker after t=0.07, 0.328, 0.330, and 0.35 to simulate the stable, border stable, border unstable, and unstable cases, respectively. The simulation results for SIME is depicted in Figure 5. Here, model (5\textsuperscript{th} order) has been able to accurately follow the rotor angle trajectory in all cases and the validity of the model especially for the first swing is proven. For the sake of
validation, the simulations for different Fault locations, fault duration, and loading levels are performed. The results show that the 5th order model has accuracy and closest value with respect to actual rotor angle trajectory. So, this model is selected to be used in the rotor angle stability prediction process.

FIGURE 5.

The proposed methodology predicts transient instability status based on the local data and characteristics of the system will have no effect on the process and its implementation. So, this approach in addition to the SIME is examined on the WSCC nine-bus test system, depicted in Figure 6. G1 is a salient pole generator and deemed to be the reference generator at WSCC test bed. G2 and G3 are of the round rotor type and operating in the PV mode. The excitation system of all generators is IEEE DC1 and represented by a full order model. The conditions stimulating the angular stability cases are as follows:

- **Fault type, duration, and location**: symmetrical and asymmetrical faults with two different durations, 120 ms and 100 ms, are simulated on various locations on lines 7-8 and 7-5. The faults resistance is assumed to be 0.8 Ω.

- **Pre-fault system operation status**: generation level of power plant G2 changes while the system configuration is kept unchanged. This level decreases to the point at which the generating unit remains stable even for 120 ms three phases to ground (LLLG) fault at the beginning of each line. Keeping the system load unchanged, generator G1 as the reference generator compensates the reduced generation.

- **Special cases**: in a real power network when an asymmetrical fault occurs in a three-phase line, the associated circuit breaker on each phase clears fault independently of the other phases. Although the system angular stability behavior will change after opening each phase, the fault clearing time should be specified based on the time of last opened phase.

Table 1 presents the obtained results in terms of 100 ms and 120 ms LLLG fault located on various locations along the line 7-5. The generation level of G2 as pre-fault system operation status is kept equal to 145 MW. In this table, S and U denote stable and unstable predictions, respectively, and the actual column is determined based on numerical stability assessment methods. Results in column 2 or 4 demonstrate that system is angularly unstable for the fault location near bus 2 (power plant bus). According to Equal Area Criterion (EAC), it can be proven that for the fault closer to the bus, the effective impedance between fault and power plant bus is reduced and the acceleration area increases. In such a stressed situation, the system will more likely be unstable. The comparison of fault time
durations shows that the longer fault duration (more stressed situation) raises the probability of system instability for a fault at the same location. This conclusion can also be proven based on EAC. The results of Table 1 show the correctness of SDM prediction in unstable situation. However, in middle stressed conditions at which the power system maintains its stability, the probability of wrong prediction increases.

The time required to carry out all SDM processing steps and predicting the angular stability, after completion of DW is less than only 1 ms. The reason is that SDM predicts the stability status of power system based on two simple equations (30) and (31). The simplicity, low computational burden, and very short processing time are unprecedented features for real world implementation of the proposed methodology.

In order to evaluate performance of the proposed method, SDM is compared with the curve fitting-based rotor angle trajectory (CF) method with fixed DW. For all simulation studies, both aforementioned methods use 100 ms calculation DW after clearing the fault for prediction of the angular stability. The difference between these two methods is the start point of DW. SDM exploits a moving start point but CF uses a fixed start point which is the first data right after the fault clearance.

**FIGURE 6.**

Table 1.

To see the fault location effect on the accuracy of both prediction methods, the LLLG fault as the severe one is applied for 120 ms on 3% and 20% of line 5-7 in the G2 generation level equal to 145 MW. In Figure 7, the fault is examined on the 3% of line 5-7. Since the fault location is too close to bus 2, it inflicts more angular stability effect on the system. In addition to the actual phase angle of power system as benchmark curve obtained from the time domain simulation, resultant curves of CF and SDM prediction processes are depicted. According to the actual curve, the system becomes unstable after the fault clearance since the bus phase angle is constantly growing with a positive slope and finally passes the boundary value (180°). According to Figure 7, both CF and SDM can correctly predict the system angular instability after clearing the fault. However, the SDM resultant curve is closer to the actual phase angle in bus 2.

In another simulation, the fault is applied on 20% of line 5-7. The actual bus angle, CF, and SDM predicted curves are shown in Figure 8. According to this figure, the actual bus phase angle value returns back after clearing
the fault and the system is stable within the first swing duration. As can be seen in Figure 8, CF predicted curve is continuously increasing and shows instability. However, SDM which uses moving DW returns back and predicts a stable case correctly.

In Figure 9, the actual, CF, and SDM phase angle curves for 120 ms LG fault located on 0% of line 7-5 at PG2=150 MW are shown. This figure represents that SDM predicted curve is more accurate than the CF one in terms of following the actual path of rotor angle. However, both of them predict stability of power system correctly.

**FIGURE 7.**

**FIGURE 8.**

**FIGURE 9.**

SDM as a response-based approach does not require equipment models and system configuration and operation status. However, performance of the response-based angular stability prediction method depends on not only on the fault location but also the power plant generation level, fault duration, and fault type. Table 2 shows the results associated with various generation levels of G2 as pre-fault system operation status, fault duration, fault type, and fault location on line 7-5. Columns 2 to 5, 6 to 9, and 10 to 13 show prediction error of each method in stability/instability situations for LLLG, LLG, and LG fault types, respectively. Prediction error percentage of both CF and SDM is obtained by the following steps: 1) Select fault type and duration, 2) Chose generation levels of power plant G2, 3) Locate fault on every percent of each line length, and eventually 4) At each step of these simulations, if actual and predicted curves show opposite system stability status, prediction error is increased by 1%. This process is carried out for all fault types and durations, generation levels, and fault locations along lines 7-5 and 7-8. For the most stressed situation (LLLG fault, high generation level, 120 ms fault duration) results show that SDM outperforms CF method. In other words, angular stability prediction results are improved by 5% in the worst situation for PG2= 135 MW and by 22% in PG2=150 MW generation level in the best situation.

In order to further investigate the performance of two aforementioned methods, special cases consisting of LLLG, LLG, and LG fault located in 0% of lines 7-5 and 7-8 at PG2= 150 MW are simulated. In these studies, faulted or healthy phases are opened at different times and CCT is considered as the opening time of the last phase. For example, consider an LLLG fault where phases A, B, and C opens after 110, 115, and 120 ms, respectively. In this
case, CCT is 120 ms. In these cases, SDM and CF can correctly predict the stability/instability of power system angular behavior.

As mentioned in Table 2, in addition to the same performance of SDM and CF in many case studies, in some situations SDM is more accurate than CF to predict angular stability and in other cases vice versa. However, in order to have a comprehensive judgment on SDM performance and a complete comparison between performances these two angular stability prediction methods, two quantitative and one qualitative comparisons are conducted in the following

**MSE comparison:** this study gives numerical indices which determine the mismatch of predicted and actual responses [29]. In order to make quantitative comparisons between SDM and CF in prediction of the generating unit rotor angle values, the Mean Squared Errors (MSE) of actual and predicted responses are calculated as follows:

\[
MSE = \frac{1}{DS} \sum_{p=1}^{DS} (\hat{COA}_p - COA_p)^2
\]

(32)

where \(p\) is the index for the sample associated with the predicted and actual responses, respectively, \(DS\) is the number of all data points, and \(\hat{COA}_p\) and \(COA_p\) are the Center Of inertia Angle (COA) associated with the predicted and actual responses. COA as an important index in power system transient stability studies is formulated as:

\[
COA = \frac{\sum_{g=1}^{NG} (H_g \cdot \delta_g)}{\sum_{g=1}^{NG} H_g}
\]

(33)

where \(H\) is the inertia constant and \(NG\) is the total number of generators in power system.

MSE as a quantitative index is calculated for 18 different case studies and given in Table 3. According to these results, for the time duration equal to 300 ms after the DW is filled out, the final predicted curves obtained by SDM has closer performance to the actual measurements, although in rare cases, the response of the CF has lower MSE index. Generally, we can conclude that the rotor angles predicted by SDM are of higher accuracies.

Table 2.

Table 3.

**Probabilistic comparison:** the wrong prediction of system angular behavior based on SDM and CF are presented
in Table 2. These results can be used to calculate the generation level-based wrong prediction probability index relaxing the type, time, and location of the faults. The generation level-based wrong prediction probability is:

\[ P_{gb} = \sum_{l=1}^{2} \left( \sum_{e=1}^{3} P(W_l \cap E_e) \right) \]  

(34)

where \( P(W \cap E) \) is probability that wrong prediction \( W \) and fault \( E \) occur together. The line number \((1, 2)\) and fault type \((LG, LLG, \) and \( LLLG)\) are denoted by \( l \) and \( e \), respectively. According to probability rules, \( P(W \cap E) \) can be expanded as:

\[ P(W_e \cap E_e) = P(W_e | E_e) \times P(E_e) \]  

(35)

where \( P(W_e | E_e) \) is the probability of wrong prediction in case of event \( E_e \) occurrence and is shown in Table 2 and 3. In [33], the \( P(E_e) \) is defined to 80\%, 15\%, and 5\% for LG, LLG, and LLLG fault, respectively. Having used these values and applying Equation (34) and Equation (35), the generation level-based wrong prediction probability is achieved. In Figure 10, the \( P_{gb} \) of CF and SDM are demonstrated. The results show that in simulated scenarios, SDM performance is better than CF. In addition, results of Figure 10 reveal greater accuracy of proposed angular stability prediction approach compared to CF method.

**FIGURE 10.**

✓ **Qualitative comparison:** this comparison is on the basis that both prediction and system actual behaviors have similar trajectories, say having stable or unstable angular curves. A marvelous result is that both methods can predict correctly the instability situations and there is no error when the power system is more stressed. In accordance with the power system protection principles, in angular unstable situations, proper function of SPS is which stimulated based on the output of these methods is guaranteed. Therefore, the dependability of protective equipment will be perfect. Dependability means the certainty that the SPS operates when its function is required. In all circumstances such as angular instability, which may cause the system collapse, this performance is required [23].

The other aspect of protection system is security. Security is the certainty that the SPS will not operate when its function is not required. According to obtained result, although security is not quite satisfied but it is acceptable. Dependability and security together mean that protection in terms of system stability will operate properly. With
respect to the qualitative considerations, the prediction scheme of SDM and CF are confirmed.

6. CONCLUSIONS
A response-based approach using PMU measurement data, polynomial rotor angle trajectory model, and second order derivative method has been proposed in this paper for the angular stability prediction of power system. The performance of the new method has been testified on the WSCC 9 bus test bed. Based on comprehensive suite of simulations, the proposed method was compared by actual responses and the results an existing angular stability prediction method based on curve fitting. It was demonstrated that the proposed method has significant features including requiring only bus voltage phase angle, very low computation burden and prediction time, being independent of equipment models and system configuration and operation status, simplicity in application, and better generation level-based wrong prediction probability. Considering these features, utilization of this method in SPS and GRS could be promising although more improvements in the security of the method are still requested. Furthermore, according to the new fast communication media such as optic fiber, it is proposed to investigate the feasibility of using SDM in wide-area measurement, protection, and control system (WAMPAC).

7. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_N$</td>
<td>Parameter vector.</td>
</tr>
<tr>
<td>$C_k$</td>
<td>Polynomial coefficients.</td>
</tr>
<tr>
<td>COA</td>
<td>Center Of inertia Angle.</td>
</tr>
<tr>
<td>$D$</td>
<td>Damping factor (MW/Hz.).</td>
</tr>
<tr>
<td>$DS$</td>
<td>Number of all data points.</td>
</tr>
<tr>
<td>$F_N$</td>
<td>Equation matrix.</td>
</tr>
<tr>
<td>$g$</td>
<td>Index of generators.</td>
</tr>
<tr>
<td>$H$</td>
<td>Inertia constant.</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of future time instants.</td>
</tr>
<tr>
<td>$j$</td>
<td>Index of sample points.</td>
</tr>
<tr>
<td>$k$</td>
<td>Index of polynomial coefficients.</td>
</tr>
<tr>
<td>$l$</td>
<td>Index of line.</td>
</tr>
<tr>
<td>$m$</td>
<td>Index of future sample points.</td>
</tr>
<tr>
<td>$MSE$</td>
<td>Mean Squared Errors.</td>
</tr>
<tr>
<td>$n$</td>
<td>Rotor angle polynomial order.</td>
</tr>
<tr>
<td>$NG$</td>
<td>Total number of generators.</td>
</tr>
<tr>
<td>$NS$</td>
<td>Number of samples in data window.</td>
</tr>
<tr>
<td>$O(N)$</td>
<td>Observation vector.</td>
</tr>
<tr>
<td>$p$</td>
<td>Index of predicted and actual samples.</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Accelerating power (p.u.).</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Output electrical power (p.u.).</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Input mechanical power (p.u.).</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (sec.).</td>
</tr>
<tr>
<td>$T(N)$</td>
<td>Time matrix.</td>
</tr>
<tr>
<td>$w_{d}$</td>
<td>Angular velocity of machine small-signal swings (Rad/sec).</td>
</tr>
<tr>
<td>$W_0$</td>
<td>Nominal synchronous speed (Rad/sec).</td>
</tr>
</tbody>
</table>


Special protection schemes in electric power systems, EEE-Power Systems Laboratory, pp. 1-22 (2002).


Figure and Table Captions

FIGURE 1. Different rotor angle trajectories of a synchronous generator following a severe disturbance.

FIGURE 2. Single Machine Equivalent (SIME).

FIGURE 3. The local maximum, minimum, and inflection points of the fifth-degree polynomial curve.

FIGURE 4. Flowchart of SDM for the prediction of power system angular stability.

FIGURE 5. Rotor angle modeling results for SIME and LLLG fault at 1% of L2.


Table 1. Angular stability prediction results for PG2=145 MW and LLLG fault along line 7-5.

FIGURE 7. Actual, CF, and SDM angular stability curves associated with an LLLG fault on 3% of line 5-7, PG2=145 MW.

FIGURE 8. Actual, CF, and SDM angular stability curves associated with an LLLG fault on 20% of line 5-7, PG2=145 MW.

FIGURE 9. Actual, CF, and SDM angular stability curves associated with an LG fault on 0% of line 5-7, PG2=150 MW.

Table 2. Prediction error of CF and SDM methods in various simulation scenarios when the fault is on line 7-5.

Table 3. Comparison of MSE of CF and SDM.

FIGURE 10. Generation level-based wrong prediction probability of CF and SDM.
Figure and Table

FIGURE 1.

FIGURE 2.
FIGURE 3.

**Fault Detection**

- **Step 1**
  - Receive data recorded by PMU
  - Is the 3rd dataset received?
    - No: Receiving next data
    - Yes: Proceed to Step 2

**Data Processing**

- **Step 2**
  - Is the second derivative based on the last dataset less than or equal to zero?
    - No: System is angularly unstable
    - Yes: Receiving next data

**Curve Fitting**

- **Step 3**
  - DW begins on this point with the length of 100 ms
  - Fifth-degree polynomial curve fitting is executed on the DW

**Predictive Analysis**

- **Step 4**
  - Is the $j$th predicted rotor angle greater than or equal to 180º?
    - Yes: System is angularly unstable
    - No: Proceed to Step 5
  - Is the first derivative of $j$th predicted rotor angle calculated by (28) greater than 0?
    - Yes: System is angularly stable
    - No: Proceed to Step 4
FIGURE 4.

(a) Stable case

(b) Border stable

(c) Border unstable

(d) Unstable

FIGURE 5.

FIGURE 6.

Table 1.

<table>
<thead>
<tr>
<th>Fault location (%)</th>
<th>Fault Time Duration (ms)</th>
<th>120</th>
<th>100</th>
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<tr>
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<td>SDM</td>
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<td>U</td>
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Table 2.

<table>
<thead>
<tr>
<th>Generation Level (MW)</th>
<th>LLGG fault</th>
<th>LLGG fault</th>
<th>LG fault</th>
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<td>Fault Time Duration (ms)</td>
<td>Fault Time Duration (ms)</td>
<td>Fault Time Duration (ms)</td>
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<tr>
<td></td>
<td>120</td>
<td>100</td>
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<tr>
<td></td>
<td>CF error (%)</td>
<td>SDM error (%)</td>
<td>CF error (%)</td>
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<td>35</td>
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<td>135</td>
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1- Two phases to ground (LLG)
2- Single phase to ground (LG)

Table 3.

<table>
<thead>
<tr>
<th>PG2 (MW)</th>
<th>Fault Type</th>
<th>Location (0% Line)</th>
<th>Duration (ms)</th>
<th>MSE CF</th>
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<td>7-5</td>
<td>100</td>
<td>0.4942</td>
<td>1.3093</td>
</tr>
</tbody>
</table>

FIGURE 10.
Biographies

Ali A. Hajnoroozi was born in Golpayegan, Iran, in 1987. He received the B.Sc. degree from the Isfahan University of Technology, Isfahan, Iran, in 2009, and the M.Sc. (Hons.) degree from the Iran University of Science and Technology, Tehran, Iran, in 2011, both in electrical engineering. He is currently with Iran University of Science and Technology, Tehran, as Ph.D. student. His current research interests include wide-area measurement systems, transient and voltage stability, special protection schemes, and model validation.

Heidar Ali Shayanfar received the B.S. and M.S.E. degrees in electrical engineering in 1973 and 1979, respectively, and the Ph.D. degree in electrical engineering from Michigan State University, Lansing, MI, USA, in 1981. He is currently a Full Professor with the Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran. His current research focuses on the application of artificial intelligence to power system, dynamic load modeling, voltage collapse, and smart grid.