Price, delivery time, and retail service sensitive dual-channel supply chain

B. Pal\textsuperscript{a}, L.E. Cárdenas-Barrón\textsuperscript{b,}\textsuperscript{*}, and K.S. Chaudhuri\textsuperscript{c}

\textit{a. Department of Mathematics, The University of Burdwan, Burdwan - 713104, India.}
\textit{b. Department of Industrial and Systems Engineering, School of Engineering and Sciences, Tecnológico de Monterrey. E. Garza Sada 2501 Sur, C. P. 64849, Monterrey, Nuevo León, México.}
\textit{c. Department of Mathematics, Jadavpur University, Kolkata - 700032, India.}

Received 14 November 2018; received in revised form 14 July 2019; accepted 14 September 2019

\begin{abstract}
This study deals with a dual-channel supply chain where the selling price set by each player, delivery time for direct channels, and retail service-dependent demand structures are considered by manufacturers and retailers. In the direct channel, the manufacturer sells products directly to customers within a maximum mentioned delivery time span. The delivery time for products is adjustable according to customers’ demand with an extra delivery charge. In the retail channel, customers are additionally benefited by retail services and direct connection with the products. Selling price in the direct market is considered to be lower than the retail-market selling price. The behavior of the proposed model under the integrated system is analyzed. In the decentralized structure, vertical Nash and manufacturer Stackelberg models are also discussed. The sensitivity of the key parameters is examined to test the feasibility of the model. Finally, a numerical example with graphical illustrations is provided to investigate the proposed model.
\end{abstract}

\textcopyright 2021 Sharif University of Technology. All rights reserved.

\section{Introduction}
Purchasing behavior of consumers is an important element in a company’s decision-making. The use of direct market shopping (also known as online shopping) has been growing swiftly between the consumers. In many ways, companies are now racing to catch up with the market to increase their sales. Nowadays, enjoying the support of new technology and the internet, many people around the world have been buying products online simply by a few clicks at their homes. Most companies already have web pages that allow them to sell products and services via the internet. However, online shopping is subject to a number of issues, e.g., whether the web site is reliable enough or not for making the payment, or if the product quality is as good as its claims on the web site, delivery time, delivery person’s behavior, etc. Normally, shopping online will take a few hours to a few days. Furthermore, it will take several weeks to deliver the items to consumers depending on the delivery terms or the distances. In a physical store, customers may see and feel the products, try them on before purchasing, or talk to the sales associate in person before taking a final decision. Also, in this case, they go to the store, buy the products, and come back home with the products immediately. Therefore, the discussion is so complicated that one needs to find the best answer to: 1) how much a company should spend on its online accessibility? and 2) how much a company should spend on its offline mode? Thus, companies need to find proper business strategies to optimize their profit through a mix of online and offline businesses.
Now, our aim is to study a dual-channel supply chain model comprised of one manufacturer and one retailer considering the selling price, delivery time, and retail service. After reviewing the existing literature related to the study, we found that the channel competition was analyzed through a channel structure with two competing manufacturers and one intermediary that sells both manufacturers’ products by Choi [1]. He investigated the effect of cost differences on equilibrium prices and profits. Also, three noncooperative, two Stackelberg, and one Nash games were analyzed in the mentioned paper. Later, Choi [2] extended the model of Choi [1] and analyzed competitive pricing strategies of duopoly manufacturers who process differentiated products and duopoly retailers who vend both products. Afterward, Chiang et al. [3] built a price setting game for a manufacturer and his/her independent retailer in a dual-channel supply chain. They studied the incentives for a manufacturer to make its own direct channel to contend with the retailer. They found that direct marketing could aid the manufacturer to raise profits by sales through his/her retailer. On the other hand, Cai [4] revealed the effect of channel structures and channel coordination on the supplier, the retailer, and the whole supply chain in two single-channel and two dual-channel supply chains. At the same time, Bin et al. [5] studied the joint decision on production and pricing under information asymmetry in the context of an online dual-channel supply chain. They also discussed whether a single contract or a menu of contracts would be better for the supplier as the leader of dual channels. Hua et al. [6] developed a dual-channel supply chain where the effects of the delivery lead time of the direct channel on the pricing decisions of manufacturer and retailer were analyzed. Afterwards, Huang et al. [7] developed a two-period pricing and production decision model for a manufacturer-retailer dual-channel supply chain where the demand is disrupted in the planning horizon. Basically, they analyzed the price and production decisions to optimize their profit under a disruption scenario. Xu et al. [8] extended the work of Chiang et al. [3] by demonstrating the applicability of the channel configuration strategy to the price and delivery lead time decisions under either the manufacturer-owned or decentralized model. Chen et al. [9] examined a situation where the manufacturer and the retailer both had a preference for a dual-channel supply chain. They also discussed the coordination between the model and different contract policies such that both chain members could be benefited. Dan et al. [10] analyzed the decisions on retail services and price in both centralized and decentralized dual-channel supply chains and studied the impacts of retail services on the manufacturer and retailer’s pricing decisions. Xu et al. [11] investigated a price competition among manufacturers and retailers in a dual-channel supply chain where a two-way revenue sharing contract was proposed to coordinate the supply chains. Besides, many other researches, namely Chen et al. [12], Kurata et al. [13], Cao et al. [14], Wang et al. [15], Kolay [16], Xiao and Shi [17], Matsui [18], Chen et al. [19], Li et al. [20], Wang et al. [21], Mozafari et al. [22], Zhou et al. [23], Jiaping et al. [24], Modares and Shafighi [25], Li et al. [26], Nobil and Taleizadeh [27], Liu et al. [28], Abbasi et al. [29], Yao and Liu [30], Yue and Liu [31], and Nobil et al. [32] have investigated the dual-channel supply chain; we also considered the dual-channel supply chain context in this paper.

Table 1 provides a comparison between this work and past researchers’ works.

This study studies a dual-channel supply chain model consisting of the manufacturer and the retailer where the former sells the product in both direct and retail channels. Additionally, it is assumed that the consumers’ demand is sensitive to the selling price

<table>
<thead>
<tr>
<th>Authors</th>
<th>Demand depending on</th>
<th>Case study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Selling price</td>
<td>Delivery time</td>
</tr>
<tr>
<td>Hua et al. [6]</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Dan et al. [10]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Xu et al. [8]</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Chen et al. [9]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Li et al. [36]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Mozafari et al. [22]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Wang et al. [21]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Chen et al. [19]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Jiaping et al. [24]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Zhou et al. [23]</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>This paper</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
of each player, delivery time for the direct channel, and retail service for the market where the demand in the direct market (online shopping) has negative effect on higher selling price, lengthy delivery time for the direct market, and more retail servicing from the retail market. However, the demand in the retail market has positive effect on lengthy delivery time for the direct market and more retail servicing from the retail market and it has only negative effect on higher selling price. The manufacturer sells the products through the direct channel at the mentioned maximum delivery time, which could be adjustable according to customers' demand with extra delivery charge. We formulated and analyzed the models under integrated, Vertical Nash (VN), and Manufacturer Stackelberg (MS) model scenarios. Our objective is to find the best strategies for selling price, delivery time, and retail service in order to optimize profitability under different scenarios.

The rest of the paper is organized as follows: Section 2 illustrates fundamental assumptions and notations. Section 3 discusses the formulation of the model. Section 4 analyzes numerical analysis. Section 5 presents sensitivity analysis. Finally, Section 6 provides the concluding remarks and other insights.

2. Fundamental assumptions and notation

2.1. Assumptions
The following assumptions are adopted to develop the model:

(i) The model is developed for a single item over two channels: in one channel, the manufacturer sells products directly; in other channels, the manufacturer sells products through retailer;

(ii) Selling price of each player, delivery time for direct channel, service level for the retail channel, and dependent demand structures are considered for both manufacturers and retailers;

(iii) The customers may buy products through direct channel at the maximum mentioned delivery time. The delivery time for products may be adjustable according to customers' demand with extra delivery charge;

(iv) Selling price in the direct channel is less by a percentage than that in the retail channel;

(v) Stock-out situation in each stage is not allowed.

2.2. Notations
The following notations are used throughout the paper:

\( p_r \) Selling price ($/unit) for the retail market
\( p_m \) Selling price ($/unit) for the direct market
\( l \) Delivery time (days) for the direct market
\( t_m \) Maximum mentioned delivery time (days) for the direct market
\( s \) Retail service for the retail market
\( \beta_1, \beta_2 \) Delivery time-sensitive indices
\( \gamma_1, \gamma_2 \) Retail service-sensitive indices
\( \alpha \) Selling price sensitive parameter
\( \phi \) Discount rate on selling price for direct market
\( w \) Wholesale price for the retailer
\( c \) Raw material cost ($/unit) for the manufacturer
\( P_c \) Production cost ($/unit) for the manufacturer
\( d_c \) Delivery charge ($/unit) for the manufacturer if products are delivered upon the mentioned delivery time
\( \eta \) Retail service cost ($/unit) for the retailer
\( \kappa \) Extra delivery cost ($/unit) except \( d_c \) for the manufacturer if products are delivered before the mentioned delivery time
\( A \) The forecast potential demand if the products are free of charge
\( \lambda \) The ratio of forecast demand for the direct channel when the selling prices of the products for both channels are zero
\( \mu \) The percentage of the customers buying the products according to the mentioned delivery time
\( \nu \) The percentage of the customers buying the products according to half of delivery charge condition

3. Formulation of the model
Nowadays, online shopping has absorbed much interest all around the world. Therefore, the marketing strategies of the manufacturing and retailing companies have been changing upon the rising popularity of online marketing. However, there is a difference between online marketing and the marketing from retail shops. In online shopping, each product is entitled to its mentioned delivery time. If one orders a product and receives it before the due time, then an extra delivery cost will be charged according to delivery time. Conversely, in retail shopping, one can obtain the products directly
and retail service is also provided to the customers. It is important to mention that decoration, labor, and rent cost of the shops and holding cost of the products are vital factors associated with the retail shops. Generally, the selling price of products in online marketing is lower than that of the retail shop due to the aforementioned factors. Considering the aforementioned factors, we generate a selling price, delivery time, and retail service-dependent demand structure where the demand in the direct market (online shopping) has a negative effect on higher selling price, lengthy delivery time for direct market, and more retail servicing from the retail market. On the other hand, the demand in the retail market has a negative effect on higher selling price and positive effect on lengthy delivery time for the direct market and more retail servicing from the retail market. We consider direct market selling price
\[ p_m = (1 - \phi) p_r, \]
where \( p_r \) is the selling price in the retail market and \( \phi \) is the discount rate. Hence, the demand structures of the direct market and retail market are respectively as follows:

\[
D_m(p_m, l, s) = \lambda A - \alpha p_m - \beta_1 l - \gamma_1 s, \tag{1}
\]

\[
D_r(p_m, l, s) = (1 - \lambda) A - \alpha p_r + \beta_2 l + \gamma_2 s, \tag{2}
\]

where \( 0 \leq \lambda \leq 1 \), \( \lambda A \), and \( (1 - \lambda) A \) represent the number of customers preferring the direct channel and retail channel, respectively, \( \alpha \) represents the price sensitivity parameter, \( \beta_1 \) and \( \beta_2 \) are delivery time sensitivity indices, and \( \gamma_1 \) and \( \gamma_2 \) are service level sensitivity indices.

3.1. Manufacturer’s individual profit

In this study, manufacturers process products for both direct and retail selling channels. He delivers a portion of manufactured products to the retailer according to the order quantities. They also sell products through the direct online market with some delivery cost conditions. First, the manufacturer declares a maximum delivery time. If the customers order products without restriction on the mentioned maximum delivery time, then no extra delivery cost will be charged. However, if the customers’ orders to deliver the products are fulfilled by a specific day, then a total delivery cost will be charged. When the customers’ orders are delivered in the above two conditions, then half of the delivery cost will be charged. Out of all online market customers, we assume that \( \mu\% \) of customers buy the products according to the mentioned delivery time; \( \nu\% \) of customers buy the products according to half of delivery charge condition; and the rest of customers buy the products according to the delivery charge condition. Then, the profit of the manufacturer is determined by:

\[
\Pi_m = \text{Revenue from sales to retailer} + \text{Revenue from direct sales in market} - \text{Delivery cost} \\
= (w - c) D_r + (p_m - c) D_m - P_c (D_r + D_m) \\
- \left\{ (d_c + \kappa(t_m - l)^2) \left( \mu + \frac{\nu}{2} \right) \right\} D_m. \quad \text{(3)}
\]

where \( P_c \) is production cost and \( d_c \) is delivery cost.

3.2. Retailer’s individual profit

In this model, the retailer and the manufacturer sell products through the retail market and the online channel, respectively. Retailers provide retail servicing to customers. They will try to increase one’s profit with negative effect on higher selling price and positive effect on lengthy delivery time for the direct market and more retail servicing from the retail market. Thus, the profit of the retailer is determined by:

\[
\Pi_r = \text{Revenue from sales} - \text{Retail servicing cost} \\
= (p_r - w) D_r - \frac{\eta s^2}{2}. \quad \text{(4)}
\]

where \( \eta \) is the retail servicing cost.

3.3. Integrated profit

From Eqs. (3) and (4), the integrated profit of the whole supply chain is as follows:

\[
\Pi(p_r, s, l) = p_m D_m + p_r D_r - (P_c + c) (D_r + D_m) \\
- \left\{ (d_c + \kappa(t_m - l)^2) \left( \mu + \frac{\nu}{2} \right) \right\} D_m. \quad \text{(5)}
\]

Now, we optimize the profit function of the supply chain with respect to the retailer’s selling price, delivery time, and retail servicing. Differentiating Eq. (5) with respect to \( p_r \), \( l \), and \( s \), we have:

\[
\frac{\partial \Pi(p_r, s, l)}{\partial p_r} = -2\alpha p_r \left( 1 + \left( 1 - \phi \right)^2 \right) \\
+ l \left( \beta_2 - \left( \beta_1 + 2t_m \alpha \left( \mu + \frac{\nu}{2} \right) \right) \right) \left( 1 - \phi \right)^2 + \alpha \kappa \left( \mu + \frac{\nu}{2} \right) (1 - \phi)^2 \\
+ s (\gamma_2 - \gamma_1 (1 - \phi)) + A (1 - \lambda \phi) \\
+ \alpha \left\{ 2 (c + P_c) \alpha (2 - \phi) + (d_c + \kappa t_m^2) \right\} \\
(2 \mu + \nu)(1 - \phi). \quad \text{(6)}
\]
\[
\frac{\partial \Pi(p_r, s, l)}{\partial l} = p_r \left( \beta_2 - \left\{ \beta_1 + 2(t_m - l)\kappa \left( \mu + \frac{\nu}{2} \right) \right\} \right) \\
(1 - \phi) - 2\kappa l \left\{ 2\beta_1 t_m + A\lambda - s \gamma_1 \right\} \\
\left( \mu + \frac{\nu}{2} \right) + 3\epsilon^2 k\beta_1 \left( \mu + \frac{\nu}{2} \right) \\
- 2\kappa s t_m \gamma_1 \left( \mu + \frac{\nu}{2} \right) + (c + P_r)(\beta_1 - \beta_2) \\
+ \left\{ d_c \beta_1 + k\beta_1 t_m^2 + 2A\kappa s t_m \right\} \\
\left( \mu + \frac{\nu}{2} \right).
\]  

(7)

\[
\frac{\partial \Pi(p_r, s, l)}{\partial s} = p_r \left\{ \gamma_2 - \gamma_1 (1 - \phi) \right\} - s \eta \\
+ \left\{ \rho - 2(t_m) \kappa \gamma_1 \left( \mu + \frac{\nu}{2} \right) \\
+ (c + P_r)(\gamma_1 - \gamma_2) + \left\{ d_c + s t_m^2 \right\} \right\} \\
\gamma_1 \left( \mu + \frac{\nu}{2} \right).
\]

(8)

Now, we check the sufficient condition of optimality.

**Proposition 1**. The integrated profit function \( \Pi(p_r, s, l) \) is jointly concave in \( p_r, l \), and \( s \) if the determinant of the Hessian matrix of the profit function is negative.

**Proof**. By solving the equations \( \frac{\partial \Pi(p_r, s, l)}{\partial p_r} = 0 \), \( \frac{\partial \Pi(p_r, s, l)}{\partial s} = 0 \), and \( \frac{\partial \Pi(p_r, s, l)}{\partial l} = 0 \), let us consider a solution set \( p_r = p_r^*, s = s^*, l = l^* \). The Hessian matrix of the integrated profit function (Eq. (5)) is as follows:

\[
H = \begin{pmatrix}
\frac{\partial^2 \Pi}{\partial p_r^2} & \frac{\partial^2 \Pi}{\partial p_r \partial s} & \frac{\partial^2 \Pi}{\partial p_r \partial l} \\
\frac{\partial^2 \Pi}{\partial p_s \partial p_r} & \frac{\partial^2 \Pi}{\partial p_s^2} & \frac{\partial^2 \Pi}{\partial p_s \partial l} \\
\frac{\partial^2 \Pi}{\partial s \partial p_r} & \frac{\partial^2 \Pi}{\partial s \partial p_s} & \frac{\partial^2 \Pi}{\partial s^2}
\end{pmatrix}
\]

(14)

The profit function is jointly concave if the Hessian matrix (Eq. (14)) is negative definite. \( H \) is negative definite iff all of its three leading principal minors alternate in sign. Now, one leading principal minor of order is \( |H_1| = -2\alpha \left\{ 1 + (1 - \phi) \right\} < 0 \) (from Eq. (9)). With the help of Eqs. (9) and (11), the order two leading principal minors are:

\[
|H_2| = \det \begin{pmatrix}
\frac{\partial^2 \Pi}{\partial p_r^2} & \frac{\partial^2 \Pi}{\partial p_r \partial s} & \frac{\partial^2 \Pi}{\partial p_r \partial l} \\
\frac{\partial^2 \Pi}{\partial p_s \partial p_r} & \frac{\partial^2 \Pi}{\partial p_s^2} & \frac{\partial^2 \Pi}{\partial p_s \partial l} \\
\frac{\partial^2 \Pi}{\partial s \partial p_r} & \frac{\partial^2 \Pi}{\partial s \partial p_s} & \frac{\partial^2 \Pi}{\partial s^2}
\end{pmatrix}
\]

\[= 2\alpha \eta \left\{ 1 + (1 - \phi)^2 \right\} - \left\{ \gamma_2 - \gamma_1 (1 - \phi) \right\}^2 > 0.
\]

as \( \alpha, \eta > \gamma_1 > \gamma_2 \).

Using Eq. (9) to Eq. (10), the three leading principle minors of order are:

\[\det H = -(t_m - l)2\kappa (2\mu + \nu)(\beta_2 - \beta_1 (1 - \phi))\]

\[\left\{ (\alpha \eta - \gamma_1)(1 - \phi) + \gamma_1 \gamma_2 \right\} + (t_m - l)l \]

\[\alpha \kappa^2 (2\mu + \nu)^2 \left\{ 2\gamma_1^2 + 2\gamma_1 \gamma_2 (1 - \phi) \right\} + \kappa (2\mu + \nu) \left\{ 3(1 - \phi) + \eta (\beta_2 - \beta_1 (1 - \phi)) \right\} + \kappa (2\mu + \nu) \left\{ 2\eta (1 - \phi + \phi^2) \right\} \]

\[- \left\{ \gamma_2 - \gamma_1 (1 - \phi) \right\} \]

Hence, the integrated profit function \( \Pi(p_r, s, l) \) is jointly concave in \( p_r, l, \) and \( s \) if \( \det H < 0 \) at \( (p_r^*, s^*, l^*) \).
Proposition 2. The integrated profit function $\Pi(p_r, s, l)$ is increasing with higher values of $A$, $\beta_2$, $\gamma_2$ and decreasing with higher values of $\alpha$, $\lambda$, $\beta_1$, $\gamma_1$, $t_m$, and $\kappa$.

Proof. Differentiating $\Pi(p_r, s, l)$ with respect to $A$, $\alpha$, $\lambda$, $\beta_1$, $\beta_2$, $\gamma_1$, $\gamma_2$, $t_m$, and $\kappa$, we get:

$$\frac{\partial \Pi}{\partial A} = (1 - \lambda \phi)p_r - c - P_c - \lambda(d_c + \kappa(t_m - l)^2)$$

$$\left(\mu + \frac{\nu}{2}\right) > 0,$$

as the selling price is always greater than all costs:

$$\frac{\partial \Pi}{\partial \alpha} = -p_r \left\{ p_r \left( 1 + (1 - \phi)^2 \right) - (c + P_c)(2 - \phi) \right\} + (1 - \phi)\left\{ d_c + \kappa(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) \right\} < 0,$$

$$\frac{\partial \Pi}{\partial \lambda} = -A \left\{ \phi p_r + d_c + \kappa(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) \right\} < 0,$$

$$\frac{\partial \Pi}{\partial \beta_1} = -l \left\{ p_r (1 - \phi) - (c + P_c) - d_c + \kappa(t_m - l)^2 \right\}$$

$$\left( \mu + \frac{\nu}{2} \right) < 0,$$

$$\frac{\partial \Pi}{\partial \beta_2} = l(p_r - c - P_c) > 0,$$

$$\frac{\partial \Pi}{\partial \gamma_1} = -s \left\{ p_r (1 - \phi) - (c + P_c) - d_c + \kappa(t_m - l)^2 \right\}$$

$$\left( \mu + \frac{\nu}{2} \right) < 0,$$

$$\frac{\partial \Pi}{\partial \gamma_2} = s(p_r - c - P_c) > 0,$$

$$\frac{\partial \Pi}{\partial t_m} = -\kappa(t_m - l)(2\mu + \nu) \left\{ \lambda A - \alpha(1 - \phi)p_r - l\beta_1 - s\gamma_1 \right\} < 0,$$

$$\frac{\partial \Pi}{\partial \kappa} = -(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) \left\{ \lambda A - \alpha(1 - \phi)p_r - l\beta_1 - s\gamma_1 \right\} < 0.$$

A function $\Pi$ is increasing or decreasing with $x$ if $\frac{\partial \Pi}{\partial x} > 0$ or $< 0$, respectively. Hence, the proof is complete. $\square$

3.4. Vertical Nash (VN) model

In this model structure, the manufacturer and the retailer make decisions independently. We assume that the retailer sales margin is $m = p_r - w$. Here, the retailer optimizes his individual profit with respect to sale margin ($m$) and retail service ($s$) for given wholesale price ($w$) and manufacturer delivery time ($l$).

Proposition 3. For given $w$ and $l$, the retailer’s best strategies are $m = \frac{\eta (A(1 - \lambda) - \omega w + \beta_2)}{2\omega - \gamma_2}$ and $s = \frac{\gamma_1 (A(1 - \lambda) - \omega w + \beta_1)}{2\omega - \gamma_1}$.

Proof. Putting $p_r = m + w$ in the retailer profit function (Eq. (4)) and then, differentiating the retailer’s profit function with respect to $m$ and $s$, we have:

$$\frac{\partial \Pi_{r_m}^m(m, s)}{\partial m} = -m\alpha - (m + w)\alpha + l\beta_2 + s\gamma_2 + A(1 - \lambda),$$

$$\frac{\partial \Pi_{r_m}^m(m, s)}{\partial s} = m\gamma_2 - s\eta.$$

Solving equations $\frac{\partial \Pi_{r_m}^m(m, s)}{\partial m} = 0$ and $\frac{\partial \Pi_{r_m}^m(m, s)}{\partial s} = 0$, we get $m = \frac{\eta (A(1 - \lambda) - \omega w + \beta_2)}{2\omega - \gamma_2}$ and $s = \frac{\gamma_1 (A(1 - \lambda) - \omega w + \beta_1)}{2\omega - \gamma_1}$.

Now, we check if the solutions are optimal. The Hessian matrix of the profit function is:

$$H_{r_m}^m = \begin{pmatrix} \frac{\partial^2 \Pi_{r_m}^m}{\partial m^2} & \frac{\partial^2 \Pi_{r_m}^m}{\partial m \partial s} \\ \frac{\partial^2 \Pi_{r_m}^m}{\partial s \partial m} & \frac{\partial^2 \Pi_{r_m}^m}{\partial s^2} \end{pmatrix} = \begin{pmatrix} -2\alpha & \gamma_2 \\ \gamma_2 & -\eta \end{pmatrix}.$$

Hence, $\frac{\partial^2 \Pi_{r_m}^m}{\partial m^2} < 0$, $\frac{\partial^2 \Pi_{r_m}^m}{\partial s^2} < 0$ and $det(H_{r_m}^m) = 2\omega - \gamma_2^2 > 0$. Therefore, $m = \frac{\eta (A(1 - \lambda) - \omega w + \beta_2)}{2\omega - \gamma_2}$ and $s = \frac{\gamma_1 (A(1 - \lambda) - \omega w + \beta_1)}{2\omega - \gamma_1}$ are optimal strategies for the retailer.

Here, the manufacturer also optimizes his individual profit function (3) with respect to $w$ and $l$ considering constant retailer’s sale margin $m$ and retail service $s$. Substituting $p_r = m + w$ in the profit function (3), then, differentiating with respect to $w$ and $l$, we have:

$$\frac{\partial \Pi_{m}^m(w, l)}{\partial w} = A(1 - \lambda \phi) - w \alpha + l\beta_2 + s\gamma_2$$

$$+ \alpha \left\{ d_c + (t_m - l)^2 \kappa \left( \mu + \frac{\nu}{2} \right) (1 - \phi) \right\}$$

$$- (l\beta_1 + s\gamma_1)(1 - \phi) + \alpha(P_c + c)(2 - \phi)$$

$$- \alpha(m + w) \left\{ 1 + 2(1 - \phi)^2 \right\},$$

$$\frac{\partial \Pi_{m}^m(w, l)}{\partial l} = P_c (\beta_1 - \beta_2) + (w - c)\beta_2$$

$$+ \beta_1 \left\{ d_c + (t_m - l)^2 \kappa \left( \mu + \frac{\nu}{2} \right) \right\}.$$
\[ w_{vn} = \]
\[
\frac{A(1 - \lambda \phi) + l \beta_2 + s \gamma_2 + \alpha (d_c + (1 - t_m)^2 \kappa) \left( \mu + \frac{\nu}{2} \right)(1 - \phi) + (c + P_c)\alpha(2 - \phi) - (l \beta_1 + s \gamma_1)(1 - \phi) - m \alpha(3 - 2(2 - \phi)\phi)}{2\alpha(1 + (1 - \phi)^2)}.
\]

Box I

\[
- \beta_1 \left\{ (m + w)(1 - \phi) - c \right\}
- \kappa(t_m - l)(2\mu + \nu) \left\{ l \beta_1 + s \gamma_1 - A \lambda \right\}
+ \alpha(m + w)(1 - \phi) \right\}.
\]

The equation \( \frac{\partial \Pi_m(w, l)}{\partial w} = 0 \) gives the equation shown in Box I. Putting the values of \( w_{vn} \) in the equation \( \frac{\partial \Pi_m(w, l)}{\partial l} = 0 \), we have \( l = l_{vn} \). Now, we check the sufficient condition of optimality for supplier strategies. The Hessian matrix of the profit function is:

\[
H_{vn} = \begin{pmatrix}
\frac{\partial^2 \Pi_m}{\partial w^2} & \frac{\partial^2 \Pi_m}{\partial w \partial l} \\
\frac{\partial^2 \Pi_m}{\partial l \partial w} & \frac{\partial^2 \Pi_m}{\partial l^2}
\end{pmatrix},
\]

where:

\[
\frac{\partial^2 \Pi_m}{\partial w^2} = -2\alpha \left\{ 1 + (1 - \phi)^2 \right\} < 0,
\]

\[
\frac{\partial^2 \Pi_m}{\partial w \partial l} = \frac{\partial^2 \Pi_m}{\partial l \partial w} = \beta_2 - \left\{ \beta_1 + 2\alpha \kappa(t_m - l) \right\} \left( \mu + \frac{\nu}{2} \right)(1 - \phi)
\]

and:

\[
\frac{\partial^2 \Pi_m}{\partial l^2} = -[2(t_m - l)\beta_1 + \{ \lambda \alpha - l \beta_1 - s \gamma_1 \}
- \alpha(m + w)(1 - \phi)]2\kappa \left( \mu + \frac{\nu}{2} \right) < 0.
\]

If \( \frac{\partial^2 \Pi_m}{\partial w^2} \times \frac{\partial^2 \Pi_m}{\partial l^2} - \frac{\partial^2 \Pi_m}{\partial w \partial l} \frac{\partial^2 \Pi_m}{\partial l \partial w} > 0 \), then the strategies \( w_{vn} \) and \( l_{vn} \) will be optimal for the manufacturer. Now, solving the four equations:

\[
m = \eta (A(1 - \lambda) - \alpha \alpha + l \beta_2) \frac{2\alpha \eta - \gamma_2}{2\alpha \eta - \gamma_2},
\]

\[
s = \alpha (A(1 - \lambda) - \alpha \alpha + l \beta_2) \frac{2\alpha \eta - \gamma_2}{2\alpha \eta - \gamma_2},
\]

\[
w = w_{vn} \quad \text{and} \quad l = l_{vn},
\]

we get the Nash equilibrium point \( (w'_{vn}, l'_{vn}, m'_{vn}, s'_{vn}) \).

**Proposition 4.** The profit function \( \Pi_m(w, l) \) of the manufacturer is increasing with higher values of \( A, \lambda, \beta_2, \gamma_2 \) and decreasing with higher values of \( \alpha, \beta_1, \gamma_1, t_m, \) and \( \kappa \).

**Proof.** Differentiating \( \Pi(p_r, s, l) \) with respect to \( A, \alpha, \lambda, \beta_1, \beta_2, \gamma_1, \gamma_2, t_m, \) and \( \kappa \), we get:

\[
\frac{\partial \Pi_m}{\partial A} = \lambda(1 - \phi)p_r + (1 - \lambda)w - c - P_c
\]

\[
-\lambda d_c + \kappa(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) > 0,
\]

\[
\frac{\partial \Pi_m}{\partial \alpha} = -p_r \left\{ p_r + w - (c + P_c + \phi p_r)(2 - \phi) \right\} + (1 - \phi)(d_c + \kappa(t_m - l)^2) \left( \mu + \frac{\nu}{2} \right) < 0,
\]

\[
\frac{\partial \Pi_m}{\partial \lambda} = A \left\{ (1 - \phi)p_r - w - \{ d_c + \kappa(t_m - l)^2 \} \right\}
\left( \mu + \frac{\nu}{2} \right) > 0,
\]

\[
\frac{\partial \Pi_m}{\partial \beta_1} = -l \left\{ (1 - \phi)p_r - (c + P_c) - \{ d_c \right. \}
+ \kappa(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) \right\} < 0,
\]

\[
\frac{\partial \Pi_m}{\partial \beta_2} = l(w - c - P_c) > 0,
\]

\[
\frac{\partial \Pi_m}{\partial \gamma_1} = -s \left\{ p_r(1 - \phi) - (c + P_c) - \{ d_c \right. \}
+ \kappa(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) \right\} < 0,
\]

\[
\frac{\partial \Pi_m}{\partial \gamma_2} = s(p_r - c - P_c) > 0,
\]

\[
\frac{\partial \Pi_m}{\partial t_m} = -s(t_m - l)(2\mu + \nu) \left\{ \lambda \alpha - \alpha(1 - \phi)p_r \right\}
- l \beta_1 - s \gamma_1 \right\} < 0,
\]
\[
\frac{\partial \Pi}{\partial \kappa} = -(t_m - l)^2 \left( \mu + \frac{\nu}{2} \right) \left\{ \lambda A - \alpha(1 - \phi) p_r \right. \\
\left. - b_1 - s \gamma_1 \right\} < 0.
\]

A function \( \Pi_m \) is increasing or decreasing with \( x \) if \( \frac{\partial \Pi_m}{\partial x} > 0 \) or \( < 0 \), respectively. Hence the proof is completed. \( \square \)

3.5. Manufacturer Stackelberg (MS) model

In this model structure, the manufacturer takes a decision on wholesale price \( w \) and delivery time \( t \) after observing the decision of the retailer on the retailer's selling price \( p_r \) and retail service \( s \). Therefore, the retailer calculates his selling price \( p_r \) and retail service \( s \) for given wholesale price \( w \) and manufacturer's delivery time \( t \).

Proposition 5. The retailer's profit function \( \Pi^{ms}_r(p_r, s) \) is jointly concave in \( p_r \) and \( s \) for given \( w \) and \( t \).

Proof. Differentiating the retailer's profit function (4) with respect to \( p_r \) and \( s \), we have:

\[
\frac{\partial \Pi_r(p_r, s)}{\partial p_r} = -2 \eta p_r + \gamma_2 s + (1 - \lambda) A + \omega a + \beta_2 l,
\]

\[
\frac{\partial \Pi_r(p_r, s)}{\partial s} = \gamma_2 p_r - \eta s - w \gamma_2.
\]

\[
\frac{\partial^2 \Pi_r(p_r, s)}{\partial p_r^2} = -2 \alpha < 0,
\]

\[
\frac{\partial^2 \Pi_r(p_r, s)}{\partial s^2} = -\eta < 0,
\]

\[
\frac{\partial^2 \Pi_r(p_r, s)}{\partial p_r \partial s} = \frac{\partial^2 \Pi_r(p_r, s)}{\partial s \partial p_r} = \gamma_2.
\]

Solving \( \frac{\partial \Pi_r(p_r, s)}{\partial p_r} = 0 \) and \( \frac{\partial \Pi_r(p_r, s)}{\partial s} = 0 \), we get:

\[
p^{ms}_r = \frac{w (\alpha \eta - \gamma_2^2) + \eta (A(1 - \lambda) + b_2)}{2 \alpha \eta - \gamma_2^2},
\]

\[
s^{ms} = \frac{\gamma_2 (A(1 - \lambda) - \omega a + b_2)}{2 \alpha \eta - \gamma_2^2}.
\]

Now, the profit function \( \Pi^{ms}_r \), will be jointly concave if:

\[
\frac{\partial^2 \Pi_r(p_r, s)}{\partial p_r^2} \frac{\partial^2 \Pi_r(p_r, s)}{\partial s^2} - \left( \frac{\partial^2 \Pi_r(p_r, s)}{\partial p_r \partial s} \right)^2 = 2 \alpha \eta - \gamma_2^2 > 0. \square
\]

Now, the manufacturer takes his decision on his profit after observing the response of the supplier on \( p^{ms}_r \) and \( s^{ms} \). By substituting the values \( p^{ms}_r \) and \( s^{ms} \), the profit of the manufacturer (see Equation (3)) reduces to:

\[
\Pi^{ms}_m = A_0 w^2 + A_1 p_r^2 + A_2 w s + A_3 w + A_4 l + A_5 w + A_6,
\]

where:

\[
A_0 = \frac{1}{2} P \gamma_1 \kappa (2 \mu + \nu) + \frac{1}{2} X \alpha \kappa (2 \mu + \nu)(1 - \phi),
\]

\[
A_1 = -\frac{1}{2} \kappa (2 \alpha (\beta_1 + Q \gamma_1) + A \lambda - R \gamma_1) (2 \mu + \nu)
+ \left( \frac{1}{2} Z \alpha \kappa (2 \mu + \nu) + Y (\beta_1 + Q \gamma_1)
+ t_m \alpha \kappa (2 \mu + \nu) \right) (1 - \phi) - \left( \frac{1}{2} \alpha \kappa (2 \mu + \nu) \right) (1 - \phi),
\]

\[
A_2 = \gamma_2 P - \alpha X - (1 - \phi) \gamma_1 P X - \alpha X^2 (1 - \phi),
\]

\[
A_3 = \beta_2 + \gamma_2 Q - \alpha Y - \gamma_1 \kappa (2 \mu + \nu) P t_m
- \left\{ X \beta_1 + (Q X + P Y) \gamma_1 + t_m X \alpha \kappa (2 \mu + \nu) \right\}
(1 - \phi) - 2 X Y \alpha (1 - \phi),
\]

\[
A_4 = \kappa (\alpha Y - \beta_1 - \beta_2 + Q \gamma_1 - Q \gamma_2)
+ P \kappa (Y \alpha + \beta_1 - \beta_2 + Q \gamma_1 - Q \gamma_2)
+ \left\{ d \beta_1 + d \gamma_1 + t_m^2 \beta_1 \kappa + Q t_m^2 \gamma_1 \kappa \right\}
\times \left( \mu + \frac{\nu}{2} \right) - (R t_m \gamma_1 \kappa - R t_m \alpha \kappa) (2 \mu + \nu)
+ \left\{ Y \left( \alpha \kappa + P \gamma_1 - R \gamma_1 + A \lambda + \frac{\alpha}{2} \right)
+ d \gamma_1 + t_m^2 \kappa \right( 2 \mu + \nu) \right) - \left( Z \beta_1 - Q Z \gamma_1 \right)
- t_m Z \alpha \kappa (2 \mu + \nu)
\right\}
(1 - \phi) - 2 Y Z \alpha (1 - \phi),
\]

\[
A_5 = A (1 - \lambda) + ((c + P \gamma_1 X - Z) \alpha + R \gamma_2)
+ P \left( c + P \gamma_1 (1 - \gamma_2) + d \gamma_1 + t_m^2 \gamma_1 \kappa \right)
\left( \mu + \frac{\nu}{2} \right) + X \left( \alpha \kappa + P \gamma_1 - R \gamma_1 + A \lambda \right)
+ \left( \frac{d \gamma_1}{2} + \frac{t_m^2 \alpha \kappa}{2} \right) (2 \mu + \nu) - P Z \gamma_1
(1 - \phi) - 2 X Z \alpha (1 - \phi),
\]
\[ A_0 = -(c + P_r)(A - Z_o + R(\gamma_2 - \gamma_1)) + \frac{1}{2}(2\mu + \nu) \left[ d_4(R\gamma_1 - A\lambda) + \kappa \left( l_m^k(R\gamma_1 - A\lambda) \right) + \frac{Z}{(Y\alpha + \beta_1 + Q\gamma_1 - Y\alpha\phi)} \right] + \left( Z(c + P_r)\alpha - R\gamma_1 + A\lambda \right) + \left( \frac{d_4 Z\alpha}{2} + \frac{1}{2} l_m^k Z\alpha \kappa \right) \left( 2(\mu + \nu) \right) (1 - \phi) - Z^2\alpha (1 - \phi)^2. \]

\[
X = \frac{\eta^2}{2(\eta - \gamma_2)}, \quad Y = \frac{\eta\beta_2}{2(\eta - \gamma_2)}, \quad Z = \frac{\eta A(1 - \lambda)}{2(\eta - \gamma_2)}, \quad P = -\frac{\gamma_2}{2(\eta - \gamma_2)}, \quad Q = \frac{\gamma_2 A(1 - \lambda)}{2(\eta - \gamma_2)}. \]

**Proposition 6.** The manufacturer’s profit function \( \Pi_m^m(w, l) \) is jointly concave in \( w \) and \( l \) if \( A_2 < 0, A_1 + A_0 w < 0, \) and \( 2A_2(2A_1 + 2A_0 w) - (A_3 + 2A_0 l)^2 > 0. \)

**Proof.** Differentiating Eq. (16) with respect to \( w \) and \( l \), we have:

\[
\frac{\partial \Pi_m^m}{\partial w} = A_0 l^2 + 2A_2 w + A_0 l + A_5, \]

\[
\frac{\partial \Pi_m^m}{\partial l} = 2A_0 l + 2A_1 + 3A_0 w + A_4. \]

\[
\frac{\partial^2 \Pi_m^m}{\partial w^2} = 2A_2, \quad \frac{\partial^2 \Pi_m^m}{\partial l^2} = 2A_1 + 2A_0 w, \]

\[
\frac{\partial^2 \Pi_m^m}{\partial w \partial l} = \frac{\partial^2 \Pi_m^m}{\partial l \partial w} = A_3 + 2A_0 l. \]

Solving \( \frac{\partial^2 \Pi_m^m}{\partial w^2} = 0 \), we get \( w = -\frac{A_0 l^2 + A_2 l + A_5}{2A_1} \). Putting the value of \( w \) in the equation \( \frac{\partial \Pi_m^m}{\partial l} = 0 \), we have:

\[ 2A_3^2 + A_0 A_3 l^2 + (A_1 A_3 - A_0^2 - 2A_0 A_3) l = 0. \]

Solving the equation, we get the value of \( l \). Now, we check the sufficient conditions of optimality at \((w_m^*, l_m^*)\). If the Hessian matrix of the profit function (16) is negative definite at \((w_m^*, l_m^*)\) i.e.:

\[
\frac{\partial^2 \Pi_m^m}{\partial w^2} = 2A_2 < 0, \quad \frac{\partial^2 \Pi_m^m}{\partial l^2} = 2A_1 + 2A_0 w < 0, \]

\[
\left( \frac{\partial^2 \Pi_m^m}{\partial w \partial l} \right)^2 - \left( \frac{\partial \Pi_m^m}{\partial w} \right)^2 A_2 = 2A_1 + 2A_0 w \]

\[ -(A_3 + 2A_0 l)^2 > 0 \] at \((w_m^*, l_m^*)\). Hence, the profit function \( \Pi_m^m(w, l) \) is jointly concave in \( w \) and \( l \) if the above conditions are satisfied. \( \square \)

**4. Numerical example.**

In this section, the model is illustrated numerically to explore the analytical results and study insight behavior of the model. The values of the parameters in appropriate units are considered as follows: \( \lambda = 0.40, \alpha = 12, \beta_1 = 2.0, \gamma_1 = 0.5, \beta_2 = 0.25, \gamma_2 = 1.5, t_m = 10 \text{ unit}, \phi = 0.25, p_m = (1 - \phi) p_r, c = 812, A = 1500, \eta = 7.5, d_c = 1.5, \mu = 0.6, \nu = 0.3, \kappa = 0.2, P_r = 2.5. \) The optimal solutions for the different models discussed are given in Table 2.

Now, we check the sufficient conditions for the optimality of the profit function for different model structures. In the integrated model structure, the Hessian matrix is:

\[
H = \begin{pmatrix} -37.5 & 1.125 & -2.62449 \\ 1.125 & -7.5 & -0.076 \\ -2.624 & -0.076 & -54.78 \end{pmatrix}. \]

The order one leading principal minor is \(-37.5, \) order two leading principal minor is \(279.984, \) and order three leading principal minor is \(-15287.6. \) Hence, the Hessian matrix \( H \) is negative definite as its three leading principal minors alternate in sign.

In the VN model structure, the Hessian matrices for the retailer and manufacturer profit functions are respectively as follows:

\[
H_r^{vn} = \begin{pmatrix} -24 & 1.50 \\ 1.50 & -7.50 \end{pmatrix}, \quad \text{and} \quad H_m^{vn} = \begin{pmatrix} -37.50 & -4.66 \\ -4.66 & -35.35 \end{pmatrix}. \]

The Hessian matrices are negative definite as order one leading principal minors are negative sign and det \( H_r^{vn} = 177.75 \) and det \( H_m^{vn} = 1303.78. \)

In the MS model, the Hessian matrix of the manufacturer’s profit function is:

\[
H_m^{ms} = \begin{pmatrix} -15.40 & -3.76 \\ -3.76 & -24.86 \end{pmatrix}. \]

The Hessian matrix is negative definite as order one leading principal minor is negative sign and det \( H_m^{ms} = 368.99. \)
5. Sensitivity analysis

This section discusses the sensitivity of the key parameters and observes the variation of the decision variables and expected profit for different cases with varying key parameters.

From Figures 1 and 2, we observe that in all the cases, total supply chain profits and profits of the manufacturer and retailer are increasing at higher values of the demand function parameter $A$. The total supply chain profits and profits of the manufacturer and retailer for all the model structure decrease (see Figures 3-6) with increasing values of the demand function parameters $\alpha$ and $\beta_1$. At higher values of the parameter $\lambda$, the total supply chain profits and profits of the retailer for all the models are decreasing; however, the manufacturer’s profits for all the models are decreasing (Figures 7 and 8). When the parameter $\phi$ is increasing, the total supply chain profits and profits of the manufacturer for all the models are increasing, but the retailer’s profits for all the models are decreasing (Figures 9 and 10). Figures 11 to 14 show that the total chain profits for all the cases are gradually increasing at higher values of $\beta_2$ and $\gamma_2$. The retailer’s profit and manufacturer’s profit for all the model structure are increasing at higher values of $\beta_2$; however, the retailer’s profit for all the model structure is increasing and the manufacturer’s profit for all the model structure is slowly decreasing at higher values of $\gamma_2$. When the parameter $\kappa$ is increasing the total profit of the chain for integrated and VN and the profits of the manufacturer for all the model structure are decreasing; however the total profit in MS and profits of the retailer for all the model structure are increasing (see Figures 15 and 16). From the Figures 17 and 18, the total chain profit for integrated and vertical Nash models and profits of retailer are decreasing gradually, and the total chain profit for MS model and the profits of the manufacturer are increasing at higher values of parameter $\eta$.  

### Table 2. Optimal solutions.

<table>
<thead>
<tr>
<th></th>
<th>$p^*_0$</th>
<th>$s^*$</th>
<th>$l^*$</th>
<th>$w^*$</th>
<th>$\Pi_m$ ($)</th>
<th>$\Pi_r$ ($)</th>
<th>$\Pi$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Integrated model</strong></td>
<td>44.23</td>
<td>4.78</td>
<td>9.49</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>14336.90</td>
</tr>
<tr>
<td><strong>Decentralized model</strong></td>
<td>VN: 51.93</td>
<td>4.77</td>
<td>8.74</td>
<td>28.08</td>
<td>6490.13</td>
<td>6739.15</td>
<td>13229.30</td>
</tr>
<tr>
<td></td>
<td>MS: 56.64</td>
<td>3.80</td>
<td>7.67</td>
<td>37.64</td>
<td>7176.64</td>
<td>4276.04</td>
<td>11452.70</td>
</tr>
</tbody>
</table>

![Figure 1. Total profit versus parameter $A$.](image)

![Figure 2. Manufacturer’s and retailer’s profit versus parameter $A$.](image)

![Figure 3. Total profit versus parameter $\alpha$.](image)
6. Conclusion

This study developed a dual-channel supply chain model with one manufacturer and one retailer, where the manufacturer sells products through both of the direct and retail channels. The dual-channel model was formulated and analyzed considering selling price of each player, delivery time for direct channel, and retail service-dependent customer demand pattern where the direct channel had negative effect on higher selling
price, lengthy delivery time for direct market, and more retail servicing from retail market; however, the retail channel had only negative effect on higher selling price and positive effect on lengthy delivery time for direct market and more retail servicing from retail market. In the direct channel, the manufacturer sets maximum delivery time in the beginning; however, if the customers’ orders to deliver a certain product are
fulfilled according to their wishes, the products will be delivered with an extra delivery charge upon the delivery time. In the retail channel, the customers are benefitted by the retail service and direct connection with the products. In this model, the selling price of the products for the direct channel was considered to be lower by a percentage of the selling price of the products in the retail channel due to decoration, huge rent cost for shops, and holding cost of the products for the retail shops. Both of the centralized and decentralized models were formulated which were studied analytically and numerically. In the decentralized scenario, the model was analyzed under vertical Nash and manufacturer Stackelberg scenarios. The model was analyzed with a numerical example and the sensitivity analysis of the key parameters was also carried out graphically to check the existence of the model.

The major contribution of the model is to study a dual-channel supply chain model with a selling price of each player, delivery time for direct channel, and retail service-dependent customer demand pattern. We also studied the model, assuming that the delivery time of the products was adjustable according to customers’ demand with extra delivery charge according to delivery time. The decentralized model was discussed under vertical Nash game and manufacturer Stackelberg game structure.

For future perspectives, the present model can be extended by including multiple rivalry retailers. Our model could be studied with stochastic types of competitive market demand. We may also extend the model by introducing different contract policies among the members. This study can be extended by incorporating return-refund policy.

Acknowledgements

The authors would like to express their gratitude to the editors and referees for their valuable suggestions and corrections to enhance the clarity of the present article. The first author also acknowledges the financial support received from UGC, New Delhi, India through the UGC-BSR Research Start-Up Research Grant (No.F.30-383/2017(BSR)).

References


**Biographies**

**Brojeswar Pal** is an Assistant Professor at the Department of Mathematics, the University of Burdwan,
West Bengal, India. He received his PhD degree from the Jadavpur University, India. He has published several research papers in international journals of repute in the areas of production planning, inventory control, and supply chain management.

Leopoldo Eduardo Cárdenas-Barrón is currently a Professor at the Department of Industrial and Systems Engineering in the School of Engineering and Sciences at Tecnológico de Monterrey, Campus Monterrey, México. He was the Associate Director of the Industrial and Systems Engineering programme from 1999 to 2005. Moreover, he was also the Associate Director of the Department of Industrial and Systems Engineering from 2005 to 2009. His research areas primarily include inventory planning and control, logistics, and supply chain. He has published papers and technical notes in several international journals. He has co-authored one book in the field of Simulation in Spanish.

Kripasindhu Chaudhuri was a Senior Professor since 1983-2008 at the Department of Mathematics, Jadavpur University, India. He was also an UGC & AICTE emeritus fellow at Jadavpur University. He received his BSc degree in Mathematics, an MSc degree in Applied Mathematics, and a PhD in Fluid Mechanics at Visva-Bharati, Santiniketan, India. He was a visiting mathematician to International Centre for Theoretical Physics (ICTP), Trieste, Italy, in 1986, a Fulbright Scholar to the North Carolina State University, Raleigh, USA in 1989, fellow of the National Academy of Sciences (FNASc), India in 1996; fellow of the Institute of Mathematics and its Applications (FIMA), UK in 1999. He has so far guided 48 research scholars and has published 245 papers in international journals of repute in the areas of fluid mechanics, solid mechanics, computer science, atmospheric science, operations research, mathematical ecology, history of mathematics, and marketing science.