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Price, delivery time, and retail service sensitive dual-channel supply chain

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Abstract. This study deals with a dual-channel supply chain where the selling price set by each player, delivery time for direct channels, and retail service-dependent demand structures are considered by manufacturers and retailers. In the direct channel, the manufacturer sells products directly to customers within a maximum mentioned delivery time span. The delivery time for products is adjustable according to customers' demand with an extra delivery charge. In the retail channel, customers are additionally benefited by retail services and direct connection with the products. Selling price in the direct market is considered to be lower than the retail-market selling price. The behavior of the proposed model under the integrated system is analyzed. In the decentralized structure, vertical Nash and manufacturer Stackelberg models are also discussed. The sensitivity of the key parameters is examined to test the feasibility of the model. Finally, a numerical example with graphical illustrations is provided to investigate the proposed model.

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1. Introduction

Purchasing behavior of consumers is an important element in a company's decision-making. The use of direct market shopping (also known as online shopping) has been growing swiftly between the consumers. In many ways, companies are now racing to catch up with the market to increase their sales. Nowadays, enjoying the support of new technology and the internet, many people around the world have been buying products online simply by a few clicks at their homes. Most companies already have web pages that allow them to sell products and services via the internet. However, online shopping is subject to a number of issues, e.g., whether the web site is reliable enough or not for making the payment, or if the product quality is as good as its claims on the web site, delivery time, delivery person's behavior, etc. Normally, shopping online will take a few hours to a few days. Furthermore, it will take several weeks to deliver the items to consumers depending on the delivery terms or the distances. In a physical store, customers may see and feel the products, try them on before purchasing, or talk to the sales associate in person before taking a final decision. Also, in this case, they go to the store, buy the products, and come back home with the products immediately. Therefore, the discussion is so complicated that one needs to find the best answer to: 1) how much a company should spend on its online accessibility? and 2) how much a company should spend on its offline mode? Thus, companies need to find proper business strategies to optimize their profit through a mix of online and offline businesses.

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Now, our aim is to study a dual-channel supply chain model comprised of one manufacturer and one retailer considering the selling price, delivery time, and retail service. After reviewing the existing literature related to the study, we found that the channel competition was analyzed through a channel structure with two competing manufacturers and one intermediary that sells both manufacturers' products by Choi [1]. He investigated the effect of cost differences on equilibrium prices and profits. Also, three noncooperative, two Stackelberg, and one Nash games were analyzed in the mentioned paper. Later, Choi [2] extended the model of Choi [1] and analyzed competitive pricing strategies of duopoly manufacturers who process differentiated products and duopoly retailers who vend both products. Afterward, Chiang et al. [3] built a price setting game for a manufacturer and his/her independent retailer in a dual-channel supply chain. They studied the incentives for a manufacturer to make one's own direct channel to contend with the retailer. They found that direct marketing could aid the manufacturer to raise profits by sales through his/her retailer. On the other hand, Cai [4] revealed the effect of channel structures and channel coordination on the supplier, the retailer, and the whole supply chain in two single-channel and two dual-channel supply chains. At the same time, Bin et al. [5] studied the joint decision on production and pricing under information asymmetry in the context of an online dual-channel supply chain. They also discussed whether a single contract or a menu of contracts would be better for the supplier as the leader of dual channels. Hua et al. [6] developed a dual-channel supply chain where the effects of the delivery lead time of the direct channel on the pricing decisions of manufacturer and retailer were analyzed. Afterwards, Huang et al. [7] developed a two-period pricing and production decision model for a manufacturer-retailer dual-channel supply chain where the demand is disrupted in the planning horizon. Basically, they analyzed the price and production decisions to optimize their profit under a disruption scenario. Xu et al. [8] extended the work of Chiang et al. [3] by demonstrating the applicability of the channel configuration strategy to the price and delivery lead time decisions under either the manufacturer-owned or decentralized model. Chen et al. [9] examined a situation where the manufacturer and the retailer both had a preference for a dual-channel supply chain. They also discussed the coordination between the model and different contract policies such that both chain members could be benefitted. Dan et al. [10] analyzed the decisions on retail services and price in both centralized and decentralized dual-channel supply chains and studied the impacts of retail services on the manufacturer and retailer's pricing decisions. Xu et al. [11] investigated a price competition among manufacturers and retailers in a dual-channel supply chain where a two-way revenue sharing contract was proposed to coordinate the supply chains. Besides, many other researches, namely Chen et al. [12], Kurata et al. [13], Cao et al. [14], Wang et al. [15], Kolay [16], Xiao and Shi [17], Matsui [18], Chen et al. [19], Li et al. [20], Wang et al. [21], Mozafari et al. [22], Zhou et al. [23], Jiaping et al. [24], Modarres and Shafiei [25], Li et al. [26], Nobil and Taleizadeh [27], Liu et al. [28], Abbasi et al. [29], Yao and Liu [30], Yue and Liu [31], and Nobil et al. [32] have investigated the dual-channel supply chain; we also considered the dual-channel supply chain context in this paper.

Table 1 provides a comparison between this work and past researchers' works.

This study studies a dual-channel supply chain model consisting of the manufacturer and the retailer where the former sells the product in both direct and retail channels. Additionally, it is assumed that the consumers' demand is sensitive to the selling price

Table 1. A comparison table between the present work and other related research works.

Authors	Demand depending on			Case study		
	Selling price	Delivery time	Service	Integrated	Vertical Nesh	Stackelberg
Hua et al. [6]	√	√	×	√	×	√
Dan et al. [10]	√	×	√	√	×	√
Xu et al. [8]	√	√	×	√	×	√
Chen et al. [9]	√	×	×	√	×	√
Li et al. [26]	√	×	×	√	×	√
Mozafari et al. [22]	√	×	√	√	×	√
Wang et al. [21]	√	×	√	√	×	√
Chen et al. [19]	√	×	×	√	√	√
Jiaping et al. [24]	√	×	√	√	×	√
Zhou et al. [23]	√	×	√	√	×	√
This paper	√	√	√	√	√	√

of each player, delivery time for the direct channel, and retail service for the market where the demand in the direct market (online shopping) has negative effect on higher selling price, lengthy delivery time for the direct market, and more retail servicing from the retail market. However, the demand in the retail market has positive effect on lengthy delivery time for the direct market and more retail servicing from the retail market and it has only negative effect on higher selling price. The manufacturer sells the products through the direct channel at the mentioned maximum delivery time, which could be adjustable according to customers' demand with extra delivery charge. We formulated and analyzed the models under integrated, Vertical Nash (VN), and Manufacturer Stackelberg (MS) model scenarios. Our objective is to find the best strategies for selling price, delivery time, and retail service in order to optimize profitability under different scenarios.

The rest of the paper is organized as follows: Section 2 illustrates fundamental assumptions and notations. Section 3 discusses the formulation of the model. Section 4 analyzes numerical analysis. Section 5 presents sensitivity analysis. Finally, Section 6 provides the concluding remarks and other insights.

2. Fundamental assumptions and notation

2.1. Assumptions

The following assumptions are adopted to develop the model:

(i) The model is developed for a single item over two channels: in one channel, the manufacturer sells products directly; in other channels, (s)he sells products through retailer;

(ii) Selling price of each player, delivery time for direct channel, service level for the retail channel, and dependent demand structures are considered for both manufacturers and retailers;

(iii) The customers may buy products through direct channel at the maximum mentioned delivery time. The delivery time for products may be adjustable according to customers' demand with extra delivery charge;

(iv) Selling price in the direct channel is less by a percentage than that in the retail channel;

(v) Stock-out situation in each stage is not allowed.

2.2. Notations

The following notations are used throughout the paper:

$D(m)$	Demand rate of the customer for the manufacturer
$D(r)$	Demand rate of the customer for the retailer
p_r	Selling price (\$/unit) for the retail market
p_m	Selling price (\$/unit) for the direct market
l	Delivery time (days) for the direct market
t_m	Maximum mentioned delivery time (days) for the direct market
s	Retail service for the retail market
β_1, β_2	Delivery time-sensitive indices
γ_1, γ_2	Retail service-sensitive indices
α	Selling price sensitive parameter
ϕ	Discount rate on selling price for direct market
w	Wholesale price for the retailer
c	Raw material cost (\$/unit) for the manufacturer
P_c	Production cost (\$/unit) for the manufacturer
d_c	Delivery charge (\$/unit) for the manufacturer if products are delivered upon the mentioned delivery time
η	Retail service cost (\$/unit) for the retailer
κ	Extra delivery cost (\$/unit) except d_c for the manufacturer if products are delivered before the mentioned delivery time
A	The forecast potential demand if the products are free of charge
λ	The ratio of forecast demand for the direct channel when the selling prices of the products for both channels are zero
μ	The percentage of the customers buying the products according to the mentioned delivery time
ν	The percentage of the customers buying the products according to half of delivery charge condition

3. Formulation of the model

Nowadays, online shopping has absorbed much interest all around the world. Therefore, the marketing strategies of the manufacturing and retailing companies have been changing upon the rising popularity of online marketing. However, there is a difference between online marketing and the marketing from retail shops. In online shopping, each product is entitled to its mentioned delivery time. If one orders a product and receives it before the due time, then an extra delivery cost will be charged according to delivery time. Conversely, in retail shopping, one can obtain the products directly and retail service is also provided to the customers. It is important to mention that decoration, labor, and rent cost of the shops and holding cost of the products are vital factors associated with the retail shops. Generally, the selling price of products in online marketing is lower than that of the retail shop due to the above-mentioned factors. Considering the aforementioned factors, we generate a selling price, delivery time, and retail service-dependent demand structure where the demand in the direct market (online shopping) has a negative effect on higher selling price, lengthy delivery time for direct market, and more retail servicing from the retail market. On the other hand, the demand in the retail market has a negative effect on higher selling price and positive effect on lengthy delivery time for the direct market and more retail servicing from the retail market. We consider direct market selling price $p_m = (1-\phi)p_r$, where p_r is the selling price in the retail market and ϕ is the discount rate. Hence, the demand structures of the direct market and retail market are respectively as follows:

$$D_m(p_m, l, s) = \lambda A - \alpha p_m - \beta_1 l - \gamma_1 s, \tag{1}$$

$$D_r(p_m, l, s) = (1-\lambda)A - \alpha p_r + \beta_2 l + \gamma_2 s, \tag{2}$$

where $0 \leq \lambda \leq 1$, λA, and $(1-\lambda)A$ represent the number of customers preferring the direct channel and retail channel, respectively, α represents the price sensitivity parameter, β_1 and β_2 are delivery time sensitivity indices, and γ_1 and γ_2 are service level sensitivity indices.

3.1. Manufacturer's individual profit

In this study, manufacturers process products for both direct and retail selling channels. He delivers a portion of manufactured products to the retailer according to the order quantities. They also sell products through the direct online market with some delivery cost conditions. First, the manufacturer declares a maximum delivery time. If the customers order products without restriction on the mentioned maximum delivery time, then no extra delivery cost will be charged. However, if the customers' orders to deliver the products are fulfilled by a specific day, then a total delivery cost will be charged. When the customer's orders are delivered in the above two conditions, then half of the delivery cost will be charged. Out of all online market customers, we assume that $\mu\%$ of customers buy the products according to the mentioned delivery time; $\nu\%$ of customers buy the products according to half of delivery charge condition; and the rest of customers buy the products according to the delivery charge condition. Then, the profit of the manufacturer is determined by:

$$\begin{aligned}\Pi_m =& \text{ Revenue from sales to retailer}\\ &+\text{Revenue from direct sales in market}\\ &-\text{Production cost} - \text{Delivery cost}\\ &= (w-c)D_r + (p_m - c)D_m - P_c(D_r + D_m)\\ &-\{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right)D_m, \end{aligned} \tag{3}$$

where P_c is production cost and d_c is delivery cost.

3.2. Retailer's individual profit

In this model, the retailer and the manufacturer sell products through the retail market and the online channel, respectively. Retailers provide retail servicing to customers. They will try to increase one's profit with negative effect on higher selling price and positive effect on lengthy delivery time for the direct market and more retail servicing from the retail market. Thus, the profit of the retailer is determined by:

$$\begin{aligned}\Pi_r &= \text{Revenue from sales} - \text{Retail servicing cost}\\ &= (p_r - w)D_r - \frac{\eta s^2}{2}, \end{aligned} \tag{4}$$

where η is the retail servicing cost.

3.3. Integrated profit

From Eqs. (3) and (4), the integrated profit of the whole supply chain is as follows:

$$\begin{aligned}\Pi(p_r, s, l) =& p_m D_m + p_r D_r - (P_c + c)(D_r + D_m)\\ &-\{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right)D_m - \frac{\eta s^2}{2}. \end{aligned} \tag{5}$$

Now, we optimize the profit function of the supply chain with respect to the retailer's selling price, delivery time, and retail servicing. Differentiating Eq. (5) with respect to p_r, l, and s, we have:

$$\begin{aligned}\frac{\partial \Pi(p_r, s, l)}{\partial p_r} =& -2\alpha p_r\left(1 + (1-\phi)^2\right)\\ &+ l\left(\beta_2 - \left\{\beta_1 + 2t_m\kappa\alpha\left(\mu + \frac{\nu}{2}\right)\right\}\right.\\ &\left.(1-\phi)\right) + \alpha\kappa\left(\mu + \frac{\nu}{2}\right)(1-\phi)l^2\\ &+ s(\gamma_2 - \gamma_1(1-\phi)) + A(1-\lambda\phi)\\ &+ \frac{\alpha}{2}\Big\{2(c + P_c)\alpha(2-\phi) + (d_c + \kappa t_m^2)\\ &(2\mu + \nu)(1-\phi)\Big\}, \end{aligned} \tag{6}$$

$$\frac{\partial \Pi(p_r,s,l)}{\partial l} = p_r\Bigg(\beta_2 - \left\{\beta_1 + 2(t_m-l)\kappa\alpha\left(\mu+\frac{\nu}{2}\right)\right\}(1-\phi)\Bigg) - 2\kappa l\{2\beta_1 t_m + A\lambda - s\gamma_1\}\left(\mu+\frac{\nu}{2}\right) + 3l^2\kappa\beta_1\left(\mu+\frac{\nu}{2}\right) - 2\kappa s t_m\gamma_1\left(\mu+\frac{\nu}{2}\right) + (c+P_c)(\beta_1-\beta_2) + \{d_c\beta_1 + \kappa\beta_1 t_m^2 + 2A\lambda\kappa t_m\}\left(\mu+\frac{\nu}{2}\right). \tag{7}$$

$$\frac{\partial \Pi(p_r,s,l)}{\partial s} = p_r\{\gamma_2 - \gamma_1(1-\phi)\} - s\eta + \{l^2 - 2lt_m\}\kappa\gamma_1\left(\mu+\frac{\nu}{2}\right) + (c+P_c)(\gamma_1-\gamma_2) + \{d_c + \kappa t_m^2\}\gamma_1\left(\mu+\frac{\nu}{2}\right), \tag{8}$$

$$\frac{\partial^2 \Pi(p_r,s,l)}{\partial p_r^2} = -2\alpha\{1+(1-\phi)^2\},$$

$$\frac{\partial^2 \Pi(p_r,s,l)}{\partial s^2} = -\eta, \tag{9}$$

$$\frac{\partial^2 \Pi(p_r,s,l)}{\partial l^2} = -2\kappa(2t_m\beta_1 - s\gamma_1 + A\lambda)\left(\mu+\frac{\nu}{2}\right) + 6l\beta_1\kappa\left(\mu+\frac{\nu}{2}\right) + 2p_r\alpha\kappa\left(\mu+\frac{\nu}{2}\right)(1-\phi), \tag{10}$$

$$\frac{\partial^2 \Pi(p_r,s,l)}{\partial p_r \partial s} = \frac{\partial^2 \Pi(p_r,s,l)}{\partial s \partial p_r} = \gamma_2 - \gamma_1(1-\phi), \tag{11}$$

$$\frac{\partial^2 \Pi(p_r,s,l)}{\partial s \partial l} = \frac{\partial^2 \Pi(p_r,s,l)}{\partial l \partial s} = -2(t_m-l)\gamma_1\kappa\left(\mu+\frac{\nu}{2}\right), \tag{12}$$

$$\frac{\partial^2 \Pi(p_r,s,l)}{\partial p_r \partial l} = \frac{\partial^2 \Pi(p_r,s,l)}{\partial l \partial p_r} = \beta_2 - \left\{\beta_1 + 2(t_m-l)\alpha\kappa\left(\mu+\frac{\nu}{2}\right)\right\}(1-\phi). \tag{13}$$

Now, we check the sufficient condition of optimality.

Proposition 1. *The integrated profit function $\Pi(p_r, s, l)$ is jointly concave in p_r, l, and s if the determinant of the Hessian matrix of the profit function is negative.*

Proof. By solving the equations $\frac{\partial \Pi(p_r,s,l)}{\partial p_r} = 0$, $\frac{\partial \Pi(p_r,s,l)}{\partial s} = 0$, and $\frac{\partial \Pi(p_r,s,l)}{\partial l} = 0$, let us consider a solution set $p_r = p_r^*$, $s = s^*$, and $l = l^*$. The Hessian matrix of the integrated profit function (Eq. (5)) is as follows:

$$H = \begin{pmatrix} \frac{\partial^2\Pi}{\partial p_r^2} & \frac{\partial^2\Pi}{\partial p_r \partial l} & \frac{\partial^2\Pi}{\partial p_r \partial s} \\ \frac{\partial^2\Pi}{\partial l \partial p_r} & \frac{\partial^2\Pi}{\partial l^2} & \frac{\partial^2\Pi}{\partial l \partial s} \\ \frac{\partial^2\Pi}{\partial s \partial p_r} & \frac{\partial^2\Pi}{\partial s \partial l} & \frac{\partial^2\Pi}{\partial s^2} \end{pmatrix}. \tag{14}$$

The profit function is jointly concave if the Hessian matrix (Eq. (14)) is negative definite. H is negative definite iff all of its three leading principal minors alternate in sign. Now, one leading principal minor of order is $|H_1| = -2\alpha\{1+(1-\phi)^2\} < 0$ (from Eq. (9)). With the help of Eqs. (9) and (11), the order two leading principal minors are:

$$|H_2| = \det\begin{pmatrix} \frac{\partial^2\Pi}{\partial p_r^2} & \frac{\partial^2\Pi}{\partial p_r \partial s} \\ \frac{\partial^2\Pi}{\partial s \partial p_r} & \frac{\partial^2\Pi}{\partial s^2} \end{pmatrix} = 2\alpha\eta\{1+(1-\phi)^2\} - \{\gamma_2 - \gamma_1(1-\phi)\}^2 > 0,$$

as $\alpha, \eta > \gamma_1 > \gamma_2$.

Using Eq. (9) to Eq. (10), the three leading principle minors of order are:

$$\det H = -(t_m-l)2\kappa(2\mu+\nu)(\beta_2-\beta_1(1-\phi))\Big\{(\alpha\eta-\gamma_1^2)(1-\phi)+\gamma_1\gamma_2\Big\} + (t_m-l)^2\alpha\kappa^2(2\mu+\nu)^2\Big\{2\gamma_1^2 + 2\gamma_1\gamma_2(1-\phi) + \alpha\eta(1-\phi)^2\Big\} + \eta(\beta_2-\beta_1(1-\phi))^2 + \kappa(2\mu+\nu)\Big\{3l\beta_1 - 2t_m\beta_1 + s\gamma_1 - A\lambda + \alpha p_r(1-\phi)\Big\}\Big\{2\alpha\eta(2-2\phi+\phi^2) - (\gamma_2-\gamma_1(1-\phi))^2\Big\}.$$

Hence, the integrated profit function $\Pi(p_r,s,l)$ is jointly concave in p_r, l, and s if $\det H < 0$ at (p_r^*, s^*, l^*).□

Proposition 2. *The integrated profit function $\Pi(p_r, s, l)$ is increasing with higher values of A, β_2, γ_2 and decreasing with higher values of α, λ, β_1, γ_1, t_m, and κ.*

Proof. Differentiating $\Pi(p_r, s, l)$ with respect to A, α, λ, β_1, β_2, γ_1, γ_2, t_m, and κ, we get:

$$\frac{\partial \Pi}{\partial A} = (1-\lambda\phi)p_r - c - P_c - \lambda\{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right) > 0,$$

as the selling price is always greater than all costs:

$$\frac{\partial \Pi}{\partial \alpha} = -p_r\Big\{p_r(1+(1-\phi)^2) - (c+P_c)(2-\phi) +(1-\phi)\{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right)\Big\} < 0,$$

$$\frac{\partial \Pi}{\partial \lambda} = -A\Big\{\phi p_r + \{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right)\Big\} < 0,$$

$$\frac{\partial \Pi}{\partial \beta_1} = -l\Big\{p_r(1-\phi) - (c+P_c) - \{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right)\Big\} < 0,$$

$$\frac{\partial \Pi}{\partial \beta_2} = l(p_r - c - P_c) > 0,$$

$$\frac{\partial \Pi}{\partial \gamma_1} = -s\Big\{p_r(1-\phi) - (c+P_c) - \{d_c + \kappa(t_m - l)^2\}\left(\mu + \frac{\nu}{2}\right)\Big\} < 0,$$

$$\frac{\partial \Pi}{\partial \gamma_2} = s(p_r - c - P_c) > 0,$$

$$\frac{\partial \Pi}{\partial t_m} = -\kappa(t_m - l)(2\mu + \nu)\Big\{\lambda A - \alpha(1-\phi)p_r -l\beta_1 - s\gamma_1\Big\} < 0,$$

$$\frac{\partial \Pi}{\partial \kappa} = -(t_m - l)^2\left(\mu + \frac{\nu}{2}\right)\Big\{\lambda A - \alpha(1-\phi)p_r -l\beta_1 - s\gamma_1\Big\} < 0.$$

A function Π is increasing or decreasing with x if $\frac{\partial \Pi}{\partial x} > 0$ or < 0, respectively. Hence, the proof is complete.□

3.4. Vertical Nash (VN) model

In this model structure, the manufacturer and the retailer make decisions independently. We assume that the retailer sales margin is $m = p_r - w$. Here, the retailer optimizes his individual profit with respect to sale margin (m) and retail service (s) for given wholesale price (w) and manufacturer delivery time (l).

Proposition 3. *For given w and l, the retailer's best strategies are $m = \frac{\eta(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2}$ and $s = \frac{\gamma_2(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2}$.*

Proof. Putting $p_r = m + w$ in the retailer profit function (Eq. (4)) and then, differentiating the retailer's profit function with respect to m and s, we have:

$$\frac{\partial \Pi_r^{vn}(m,s)}{\partial m} = -m\alpha - (m+w)\alpha + l\beta_2 + s\gamma_2 + A(1-\lambda),$$

$$\frac{\partial \Pi_r^{vn}(m,s)}{\partial s} = m\gamma_2 - s\eta.$$

Solving equations $\frac{\partial \Pi_r^{vn}(m,s)}{\partial m} = 0$ and $\frac{\partial \Pi_r^{vn}(m,s)}{\partial s} = 0$, we get $m = \frac{\eta(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2}$ and $s = \frac{\gamma_2(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2}$. Now, we check if the solutions are optimal. The Hessian matrix of the profit function is:

$$H_r^{vn} = \begin{pmatrix} \frac{\partial^2 \Pi_r^{vn}}{\partial m^2} & \frac{\partial^2 \Pi_r^{vn}}{\partial m \partial s} \\ \frac{\partial^2 \Pi_r^{vn}}{\partial s \partial m} & \frac{\partial^2 \Pi_r^{vn}}{\partial s^2} \end{pmatrix} = \begin{pmatrix} -2\alpha & \gamma_2 \\ \gamma_2 & -\eta \end{pmatrix}.$$

Hence, $\frac{\partial^2 \Pi_r^{vn}}{\partial m^2} < 0$, $\frac{\partial^2 \Pi_r^{vn}}{\partial s^2} < 0$ and $\det H_r^{vn} = 2\alpha\eta - \gamma_2^2 > 0$. Therefore, $m = \frac{\eta(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2}$ and $s = \frac{\gamma_2(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2}$ are optimal strategies for the retailer.

Here, the manufacturer also optimizes his individual profit function (3) with respect to w and l considering constant retailer's sale margin m and retail service s. Substituting $p_r = w+m$ in the profit function (3), then, differentiating with respect to w and l, we have:

$$\frac{\partial \Pi_m^{vn}(w,l)}{\partial w} = A(1-\lambda\phi) - w\,\alpha + l\beta_2 + s\gamma_2 + \alpha\Big\{d_c + (t_m - l)^2\kappa\Big\}\left(\mu + \frac{\nu}{2}\right)(1-\phi) - (l\beta_1 + s\gamma_1)(1-\phi) + \alpha(P_c + c)(2-\phi) - \alpha(m+w)\left\{1 + 2(1-\phi)^2\right\},$$

$$\frac{\partial \Pi_m^{vn}(w,l)}{\partial l} = P_c(\beta_1 - \beta_2) + (w-c)\beta_2 + \beta_1\left\{d_c + (t_m - l)^2\kappa\right\}\left(\mu + \frac{\nu}{2}\right)$$

$$w_{vn} = \frac{A(1-\lambda\phi)+l\beta_2+s\gamma_2+\alpha\left(d_c+(l-t_m)^2\kappa\right)\left(\mu+\frac{\nu}{2}\right)(1-\phi)+(c+P_c)\alpha(2-\phi)-(l\beta_1+s\gamma_1)(1-\phi)-m\alpha(3-2(2-\phi)\phi)}{2\alpha\left(1+(1-\phi)^2\right)}.$$

Box I

$$-\beta_1\Big\{(m+w)(1-\phi)-c\Big\}$$

$$-\kappa(t_m-l)(2\mu+\nu)\Big\{l\beta_1+s\gamma_1-A\lambda$$

$$+\alpha(m+w)(1-\phi)\Big\}.$$

The equation $\frac{\partial\Pi_m^{vn}(w,l)}{\partial w}=0$ gives the equation shown in Box I. Putting the values of w_{vn} in the equation $\frac{\partial\Pi_m^{vn}(w,l)}{\partial l}=0$, we have $l=l_{vn}$. Now, we check the sufficient condition of optimality for supplier strategies. The Hessian matrix of the profit function is:

$$H_m^{vn}=\begin{pmatrix}\frac{\partial^2\Pi_m^{vn}}{\partial w^2} & \frac{\partial^2\Pi_m^{vn}}{\partial w\partial l}\\ \frac{\partial^2\Pi_m^{vn}}{\partial l\partial w} & \frac{\partial^2\Pi_m^{vn}}{\partial l^2}\end{pmatrix},$$

where:

$$\frac{\partial^2\Pi_m^{vn}}{\partial w^2}=-2\alpha\{1+(1-\phi)^2\}<0,$$

$$\frac{\partial^2\Pi_m^{vn}}{\partial w\partial l}=\frac{\partial^2\Pi_m^{vn}}{\partial l\partial w}=\beta_2-\Big\{\beta_1+2\alpha\kappa(t_m-l)$$

$$\left(\mu+\frac{\nu}{2}\right)\Big\}(1-\phi)$$

and:

$$\frac{\partial^2\Pi_m^{vn}}{\partial l^2}=-[2(t_m-l)\beta_1+\{\lambda A-l\beta_1-s\gamma_1$$

$$-\alpha(m+w)(1-\phi)\}]2\kappa\left(\mu+\frac{\nu}{2}\right)<0.$$

If $\frac{\partial^2\Pi_m^{vn}}{\partial w^2}\times\frac{\partial^2\Pi_m^{vn}}{\partial l^2}-\frac{\partial^2\Pi_m^{vn}}{\partial w\partial l}\frac{\partial^2\Pi_m^{vn}}{\partial l\partial w}>0$, then the strategies w_{vn} and l_{vn} will be optimal for the manufacturer. Now, solving four equations:

$$m=\frac{\eta(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2},$$

$$s=\frac{\gamma_2(A(1-\lambda)-w\alpha+l\beta_2)}{2\alpha\eta-\gamma_2^2},$$

$$w=w_{vn}\quad\text{and}\quad l=l_{vn},$$

we get the Nash equilibrium point $(w_{vn}^*,\ l_{vn}^*,\ m_{vn}^*,\ s_{vn}^*)$.

Proposition 4. *The profit function $\Pi_m(w,l)$ of the manufacturer is increasing with higher values of A, λ, β_2, γ_2 and decreasing with higher values of α, β_1, γ_1, t_m, and κ.*

Proof. Differentiating $\Pi(p_r,s,l)$ with respect to A, α, λ, β_1, β_2, γ_1, γ_2, t_m, and κ, we get:

$$\frac{\partial\Pi_m}{\partial A}=\lambda(1-\phi)p_r+(1-\lambda)w-c-P_c$$

$$-\lambda\{d_c+\kappa(t_m-l)^2\}\left(\mu+\frac{\nu}{2}\right)>0,$$

$$\frac{\partial\Pi_m}{\partial\alpha}=-p_r\Big\{p_r+w-(c+P_c+\phi p_r)(2-\phi)$$

$$+(1-\phi)\{d_c+\kappa(t_m-l)^2\}\left(\mu+\frac{\nu}{2}\right)\Big\}<0,$$

$$\frac{\partial\Pi_m}{\partial\lambda}=A\Big\{(1-\phi)p_r-w-\{d_c+\kappa(t_m-l)^2\}$$

$$\left(\mu+\frac{\nu}{2}\right)\Big\}>0,$$

$$\frac{\partial\Pi_m}{\partial\beta_1}=-l\Big\{(1-\phi)p_r-(c+P_c)-\{d_c$$

$$+\kappa(t_m-l)^2\}\left(\mu+\frac{\nu}{2}\right)\Big\}<0,$$

$$\frac{\partial\Pi_m}{\partial\beta_2}=l(w-c-P_c)>0,$$

$$\frac{\partial\Pi_m}{\partial\gamma_1}=-s\Big\{p_r(1-\phi)-(c+P_c)-\{d_c$$

$$+\kappa(t_m-l)^2\}$$

$$\left(\mu+\frac{\nu}{2}\right)\Big\}<0,$$

$$\frac{\partial\Pi_m}{\partial\gamma_2}=s(p_r-c-P_c)>0,$$

$$\frac{\partial\Pi}{\partial t_m}=-\kappa(t_m-l)(2\mu+\nu)\Big\{\lambda A-\alpha(1-\phi)p_r$$

$$-l\beta_1-s\gamma_1\Big\}<0,$$

$$\frac{\partial \Pi}{\partial \kappa} = -(t_m - l)^2 \left(\mu + \frac{\nu}{2}\right) \Big\{\lambda A - \alpha(1-\phi)p_r$$
$$-l\beta_1 - s\gamma_1\Big\} < 0.$$

A function Π_m is increasing or decreasing with x if $\frac{\partial \Pi_m}{\partial x} > 0$ or < 0, respectively. Hence the proof is completed.□

3.5. Manufacturer Stackelberg (MS) model

In this model structure, the manufacturer takes a decision on wholesale price w and delivery time l after observing the decision of the retailer on the retailer's selling price p_r and retail service s. Therefore, the retailer calculates his selling price p_r and retail service s for given wholesale price w and manufacturer's delivery time l.

Proposition 5. *The retailer's profit function* $\Pi_r^{ms}(p_r, s)$ *is jointly concave in* p_r *and* s *for given* w *and* l.

Proof. Differentiating the retailer's profit function (4) with respect to p_r and s, we have:

$$\frac{\partial \Pi_r(p_r,s)}{\partial p_r} = -2\alpha p_r + \gamma_2 s + (1-\lambda)A + w\alpha + \beta_2 l,$$

$$\frac{\partial \Pi_r(p_r,s)}{\partial s} = \gamma_2 p_r - \eta s - w\gamma_2,$$

$$\frac{\partial^2 \Pi_r(p_r,s)}{\partial p_r^2} = -2\alpha < 0,$$

$$\frac{\partial^2 \Pi_r(p_r,s)}{\partial s^2} = -\eta < 0,$$

$$\frac{\partial^2 \Pi_r(p_r,s)}{\partial p_r \partial s} = \frac{\partial^2 \Pi_r(p_r,s)}{\partial s \partial p_r} = \gamma_2.$$

Solving $\frac{\partial \Pi_r(p_r,s)}{\partial p_r} = 0$ and $\frac{\partial \Pi_r(p_r,s)}{\partial s} = 0$, we get:

$$p_r^{ms} = \frac{w\left(\alpha\eta - \gamma_2^2\right) + \eta(A(1-\lambda) + l\beta_2)}{2\alpha\eta - \gamma_2^2},$$

$$s^{ms} = \frac{\gamma_2(A(1-\lambda) - w\alpha + l\beta_2)}{2\alpha\eta - \gamma_2^2}. \tag{15}$$

Now, the profit function, Π_r^{ms}, will be jointly concave if:

$$\frac{\partial^2 \Pi_r(p_r,s)}{\partial p_r^2}\frac{\partial^2 \Pi_r(p_r,s)}{\partial s^2} - \left(\frac{\partial^2 \Pi_r(p_r,s)}{\partial p_r \partial s}\right)^2$$
$$= 2\alpha\eta - \gamma_2^2 > 0.□$$

Now, the manufacturer takes his decision on his profit after observing the response of the supplier on p_r^{ms} and s^{ms}. By substituting the values p_r^{ms} and s^{ms}, the profit of the manufacturer (see Equation (3)) reduces to:

$$\Pi_m^{ms} = A_0 wl^2 + A_1 l^2 + A_2 w^2 + A_3 wl + A_4 l$$
$$+ A_5 w + A_6, \tag{16}$$

where:

$$A_0 = \frac{1}{2}P\gamma_1\kappa(2\mu+\nu) + \frac{1}{2}X\alpha\kappa(2\mu+\nu)(1-\phi),$$

$$A_1 = -\frac{1}{2}\kappa(2t_m(\beta_1 + Q\gamma_1) + A\lambda - R\gamma_1)(2\mu+\nu)$$
$$+ \Bigg(\frac{1}{2}Z\alpha\kappa(2\mu+\nu) - Y(\beta_1 + Q\gamma_1$$
$$+ t_m\alpha\kappa(2\mu+\nu))\Bigg)(1-\phi) - Y^2\alpha(1-\phi)^2,$$

$$A_2 = \gamma_2 P - \alpha X - (1-\phi)\gamma_1 PX - \alpha X^2(1-\phi)^2,$$

$$A_3 = \beta_2 + \gamma_2 Q - \alpha Y - \gamma_1\kappa(2\mu+\nu)Pt_m$$
$$- \Big\{X\beta_1 + (QX+PY)\gamma_1 + t_m X\alpha\kappa(2\mu+\nu)\Big\}$$
$$(1-\phi) - 2XY\alpha(1-\phi)^2,$$

$$A_4 = c(\alpha Y + \beta_1 - \beta_2 + Q\gamma_1 - Q\gamma_2)$$
$$+ P_c(Y\alpha + \beta_1 - \beta_2 + Q\gamma_1 - Q\gamma_2)$$
$$+ \Big\{d_c\beta_1 + d_c Q\gamma_1 + t_m^2\beta_1\kappa + Qt_m^2\gamma_1\kappa\Big\}$$
$$\times \left(\mu + \frac{\nu}{2}\right) - (Rt_m\gamma_1\kappa - At_m\kappa\lambda)(2\mu+\nu)$$
$$+ \Bigg\{Y\Bigg(c\alpha + P_c\alpha - R\gamma_1 + A\lambda + \frac{\alpha}{2}$$
$$\left(d_c + t_m^2\kappa\right)(2\mu+\nu)\Bigg) - Z\beta_1 - QZ\gamma_1$$
$$- t_m Z\alpha\kappa(2\mu+\nu)\Bigg\}(1-\phi) - 2YZ\alpha(1-\phi)^2,$$

$$A_5 = A(1-\lambda) + ((c+P_c)X - Z)\alpha + R\gamma_2$$
$$+ P\Bigg((c+P_c)(\gamma_1 - \gamma_2) + \left(d_c\gamma_1 + t_m^2\gamma_1\kappa\right)$$
$$\left(\mu + \frac{\nu}{2}\right)\Bigg) + \Bigg(X\Bigg(c\alpha + P_c\alpha - R\gamma_1 + A\lambda$$
$$+ \left(\frac{d_c\alpha}{2} + \frac{1}{2}t_m^2\alpha\kappa\right)(2\mu+\nu)\Bigg) - PZ\gamma_1\Bigg)$$
$$(1-\phi) - 2XZ\alpha(1-\phi)^2,$$

$$A_6 = -(c+P_c)(A - Z\alpha + R(\gamma_2-\gamma_1)) + \frac{1}{2}(2\mu+\nu)\Bigg[d_c(R\gamma_1 - A\lambda) + \kappa\Big\{t_m^2(R\gamma_1 - A\lambda) + l^3(Y\alpha + \beta_1 + Q\gamma_1 - Y\alpha\phi)\Big\}\Bigg] + \Bigg(Z((c+P_c)\alpha - R\gamma_1 + A\lambda) + \left(\frac{d_c Z\alpha}{2} + \frac{1}{2}t_m^2 Z\alpha\kappa\right)(2\mu+\nu)\Bigg)(1-\phi) - Z^2\alpha(1-\phi)^2,$$

$$X = \frac{\alpha\eta - \gamma_2^2}{2\alpha\eta - \gamma_2^2}, \qquad Y = \frac{\eta\beta_2}{2\alpha\eta - \gamma_2^2},$$

$$Z = \frac{\eta A(1-\lambda)}{2\alpha\eta - \gamma_2^2}, \qquad P = -\frac{\gamma_2\alpha}{2\alpha\eta - \gamma_2^2},$$

$$Q = \frac{\gamma_2\beta_2}{2\alpha\eta - \gamma_2^2}, \qquad R = \frac{\gamma_2 A(1-\lambda)}{2\alpha\eta - \gamma_2^2}.$$

Proposition 6. *The manufacturer's profit function* $\Pi_m^{ms}(w,l)$ *is jointly concave in* w *and* l *if* $A_2 < 0$, $A_1 + A_0 w < 0$, *and* $2A_2(2A_1 + 2A_0 w) - (A_3 + 2A_0 l)^2 > 0$.

Proof. Differentiating Eq. (16) with respect to w and l, we have:

$$\frac{\partial \Pi_m^{ms}}{\partial w} = A_0 l^2 + 2A_2 w + A_3 l + A_5,$$

$$\frac{\partial \Pi_m^{ms}}{\partial l} = 2A_0 l w + 2A_1 l + A_3 w + A_4.$$

$$\frac{\partial^2 \Pi_m^{ms}}{\partial w^2} = 2A_2, \qquad \frac{\partial^2 \Pi_m^{ms}}{\partial l^2} = 2A_1 + 2A_0 w,$$

$$\frac{\partial^2 \Pi_m^{ms}}{\partial w \partial l} = \frac{\partial^2 \Pi_m^{ms}}{\partial l \partial w} = A_3 + 2A_0 l.$$

Solving $\frac{\partial \Pi_m^{ms}}{\partial w} = 0$, we get $w = -\frac{A_0 l^2 + A_3 l + A_5}{2A_2}$. Putting the value of w in the equation $\frac{\partial \Pi_m^{ms}}{\partial l} = 0$, we have:

$$2A_0^2 l^2 + 3A_0 A_3 l^2 - (4A_1 A_3 - A_3^2 - 2A_0 A_5)l - (2A_2 A_4 - A_3 A_5) = 0.$$

Solving the equation, we get the value of l. Now, we check the sufficient conditions of optimality at (w_{ms}^*, l_{ms}^*). If the Hessian matrix of the profit function (16) is negative definite at (w_{ms}^*, l_{ms}^*) i.e.:

$$\frac{\partial^2 \Pi_m^{ms}}{\partial w^2} = 2A_2 < 0, \qquad \frac{\partial^2 \Pi_m^{ms}}{\partial l^2} = 2A_1 + 2A_0 w < 0,$$

$$\frac{\partial^2 \Pi_m^{ms}}{\partial w^2}\frac{\partial^2 \Pi_m^{ms}}{\partial l^2} - \left(\frac{\partial^2 \Pi_m^{ms}}{\partial w \partial l}\right)^2 = 2A_2(2A_1 + 2A_0 w) - (A_3 + 2A_0 l)^2 > 0 \quad \text{at} \quad (w_{ms}^*, l_{ms}^*),$$

then the solution will be optimal at (w_{ms}^*, l_{ms}^*). Hence, the profit function $\Pi_m^{ms}(w,l)$ is jointly concave in w and l if the above conditions are satisfied.□

4. Numerical example

In this section, the model is illustrated numerically to explore the analytical results and study insight behavior of the model. The values of the parameters in appropriate units are considered as follows: $\lambda = 0.40$, $\alpha = 12$, $\beta_1 = 2.0$, $\gamma_1 = 0.5$, $\beta_2 = 0.25$, $\gamma_2 = 1.5$, $t_m = 10$ unit, $\phi = 0.25$, $p_m = (1-\phi)p_r$, $c = \$12$, $A = 1500$, $\eta = \$7.5$, $d_c = \$1.5$, $\mu = 0.6$, $\nu = 0.3$, $\kappa = 0.2$, $P_c = \$2.5$. The optimal solutions for the different models discussed are given in Table 2.

Now, we check the sufficient conditions for the optimality of the profit function for different model structures. In the integrated model structure, the Hessian matrix is:

$$H = \begin{pmatrix} -37.5 & 1.125 & -2.62449 \\ 1.125 & -7.5 & -0.076 \\ -2.624 & -0.076 & -54.78 \end{pmatrix}.$$

The order one leading principal minor is -37.5, order two leading principal minor is 279.984, and order three leading principal minor is -15287.6. Hence, the Hessian matrix H is negative definite as its three leading principal minors alternate in sign.

In the VN model structure, the Hessian matrices for the retailer and manufacturer profit functions are respectively as follows:

$$H_r^{vn} = \begin{pmatrix} -24 & 1.50 \\ 1.50 & -7.50 \end{pmatrix}, \text{ and}$$

$$H_m^{vn} = \begin{pmatrix} -37.50 & -4.66 \\ -4.66 & -35.35 \end{pmatrix}.$$

The Hessian matrices are negative definite as order one leading principal minors are negative sign and $\det H_r^{vn} = 177.75$ and $\det H_m^{vn} = 1303.78$.

In the MS model, the Hessian matrix of the manufacturer's profit function is:

$$H_m^{ms} = \begin{pmatrix} -15.40 & -3.76 \\ -3.76 & -24.86 \end{pmatrix}.$$

The Hessian matrix is negative definite as order one leading principal minor is negative sign and $\det H_m^{ms} = 368.99$.

Table 2. Optimal solutions.

		p_r^*	s^*	l^*	w^*	Π_m (\$)	Π_r (\$)	Π (\$)
Integrated model		44.23	4.78	9.49	-	-	-	14336.90
Decentralized model	VN	51.93	4.77	8.74	28.08	6490.13	6739.15	13229.30
	MS	56.64	3.80	7.67	37.64	7176.64	4276.04	11452.70

5. Sensitivity analysis

This section, discusses the sensitivity of the key parameters and observes the variation of the decision variables and expected profit for different cases with varying key parameters.

From Figures 1 and 2, we observe that in all the cases, total supply chain profits and profits of the manufacturer and retailer are increasing at higher values of the demand function parameter A. The total supply chain profits and profits of the manufacturer and retailer for all the model structure decrease (see Figures 3–6) with increasing values of the demand function parameters α and β_1. At higher values of the parameter λ, the total supply chain profits and profits of the retailer for all the models are decreasing; however, the manufacturer's profits for all the models are decreasing (Figures 7 and 8). When the parameter ϕ is increasing, the total supply chain profits and profits of the manufacturer for all the models are increasing, but the retailer's profits for all the models are decreasing (Figures 9 and 10). Figures 11 to 14, show that the total chain profits for all the cases are gradually increasing at higher values of β_2 and γ_2. The retailer's profit and manufacturer's profit for all the model structure are increasing at higher values of β_2; however, the retailer's profit for all the model structure is increasing and the manufacturer's profit for all the model structure is slowly decreasing at higher values of γ_2. When the parameter κ is increasing the total profit of the chain for integrated and VN and the profits of the manufacturer for all the model structure are decreasing; however the total profit in MS and profits of the retailer for all the model structure are increasing (see Figures 15 and 16). From the Figures 17 and 18, the total chain profit for integrated and vertical Nash models and profits of retailer are decreasing gradually, and the total chain profit for MS model and the profits of the manufacturer are increasing at higher values of parameter η.

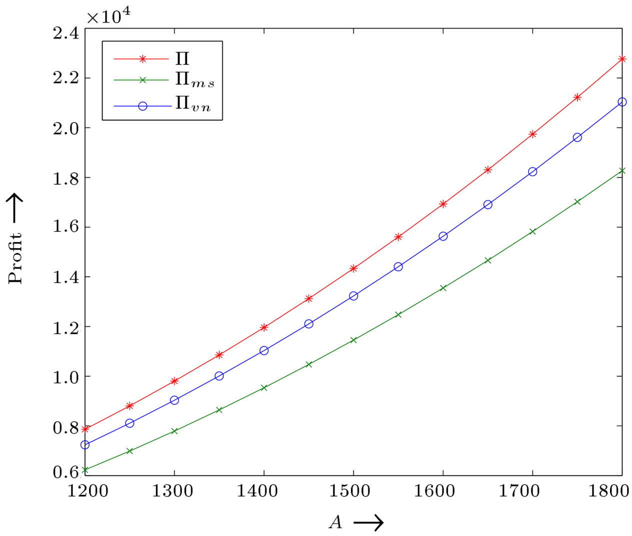

Figure 1. Total profit versus parameter A.

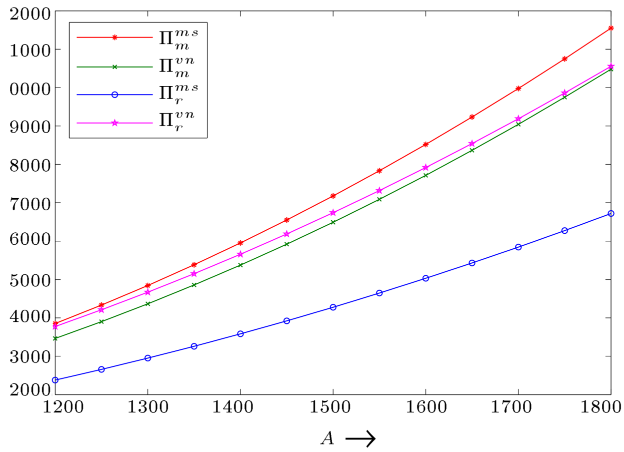

Figure 2. Manufacturer's and retailer's profit versus parameter A.

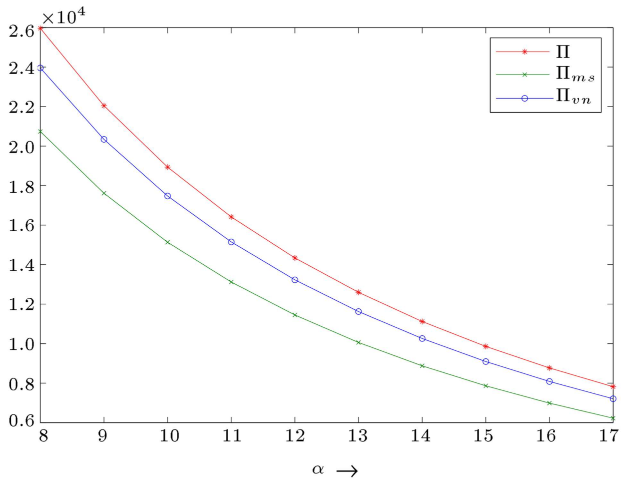

Figure 3. Total profit versus parameter α.

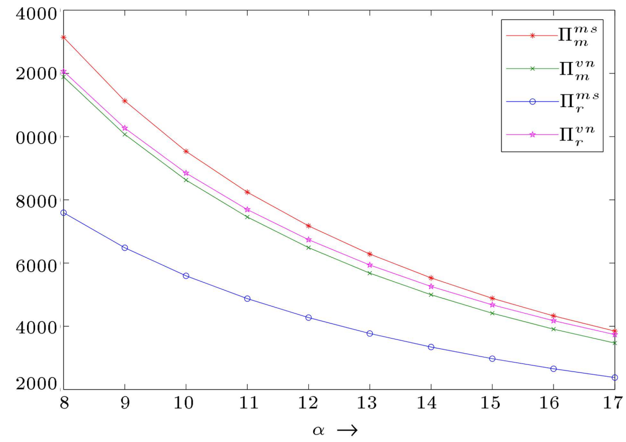

Figure 4. Manufacturer's and retailer's profit versus parameter α.

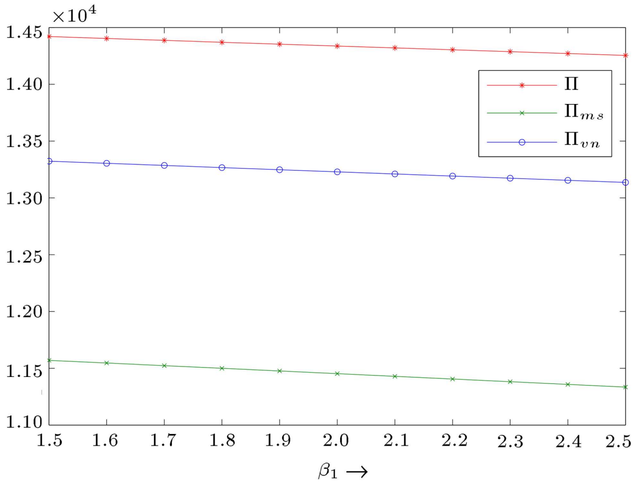

Figure 5. Total profit versus parameter β_1.

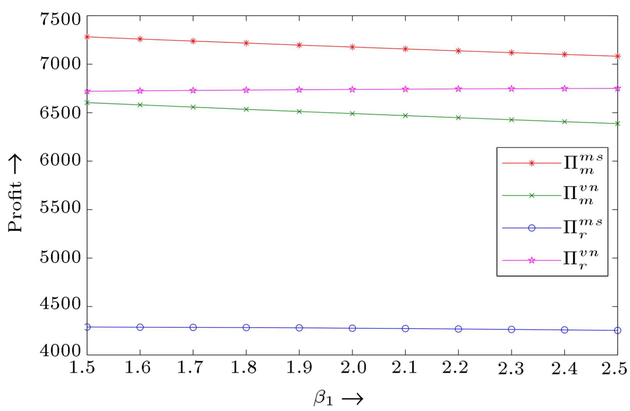

Figure 6. Manufacturer's and retailer's profit versus parameter β_1.

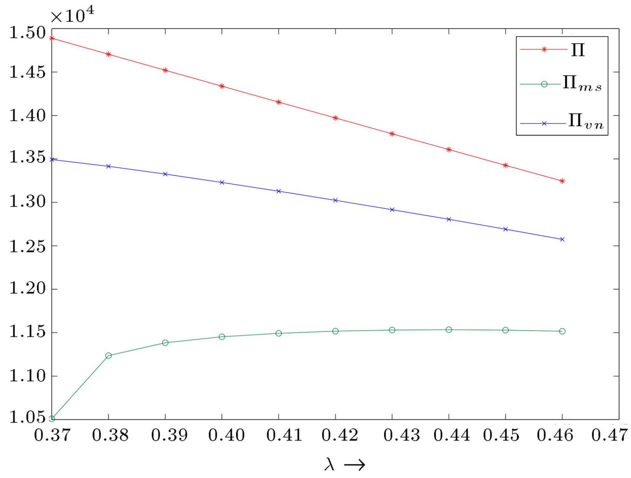

Figure 7. Total profit versus λ.

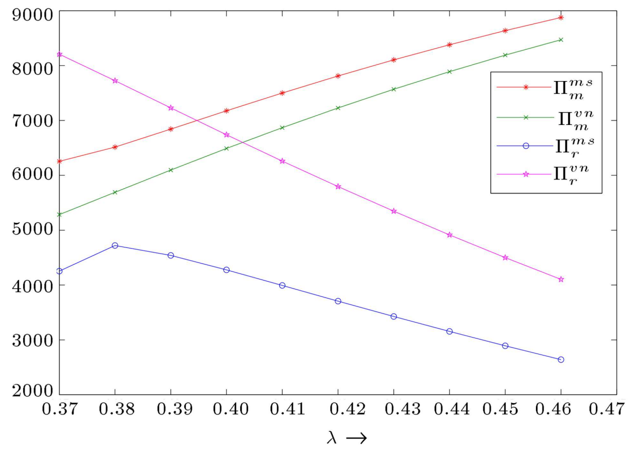

Figure 8. Manufacturer's and retailer's profit versus λ.

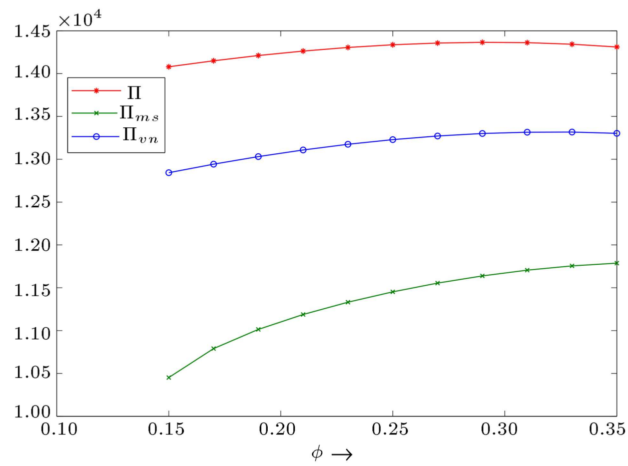

Figure 9. Total profit versus ϕ.

6. Conclusion

This study developed a dual-channel supply chain model with one manufacturer and one retailer, where the manufacturer sells products through both of the direct and retail channels. The dual-channel model was formulated and analyzed considering selling price of each player, delivery time for direct channel, and retail service-dependent customer demand pattern where the direct channel had negative effect on higher selling price, lengthy delivery time for direct market, and more retail servicing from retail market; however, the retail channel had only negative effect on higher selling price and positive effect on lengthy delivery time for direct market and more retail servicing from retail market. In the direct channel, the manufacturer sets maximum delivery time in the beginning; however, if the customers' orders to deliver a certain product are

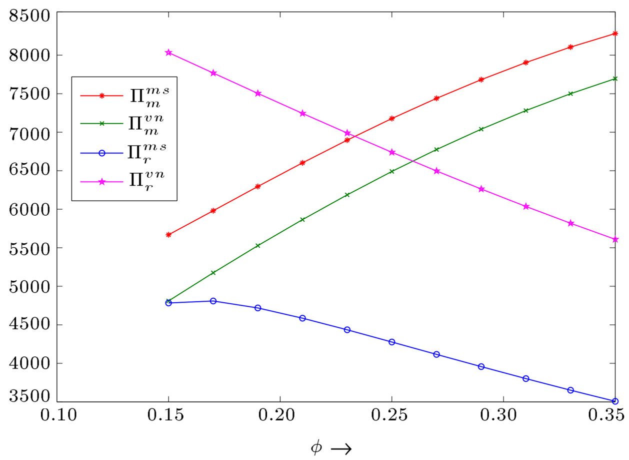

Figure 10. Manufacturer's and retailer's profit versus ϕ.

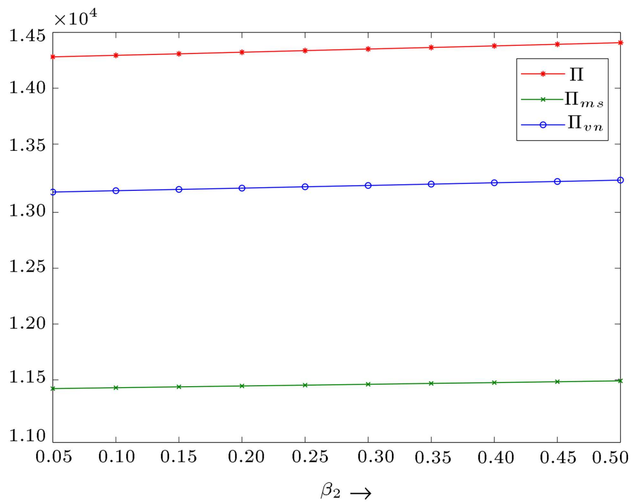

Figure 11. Total profit versus β_2.

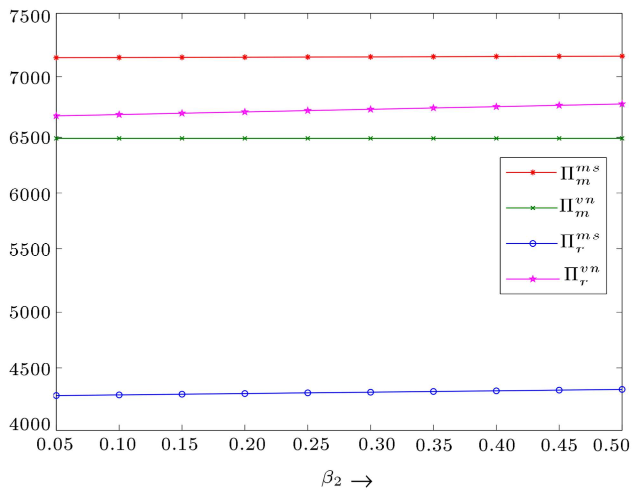

Figure 12. Manufacturer's and retailer's profit versus β_2.

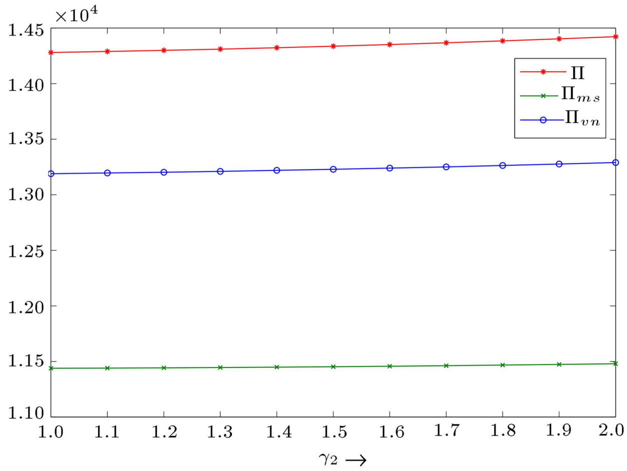

Figure 13. Total profit versus γ_2.

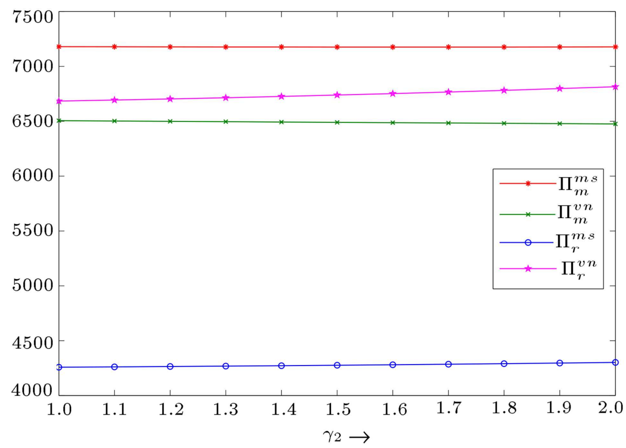

Figure 14. Manufacturer's and retailer's profit versus γ_2.

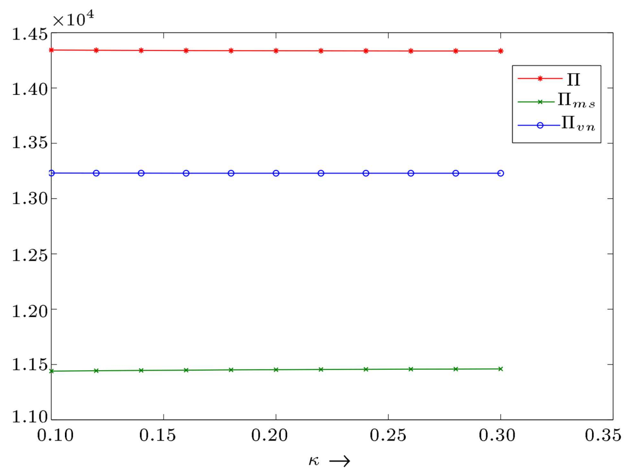

Figure 15. Total profit versus κ.

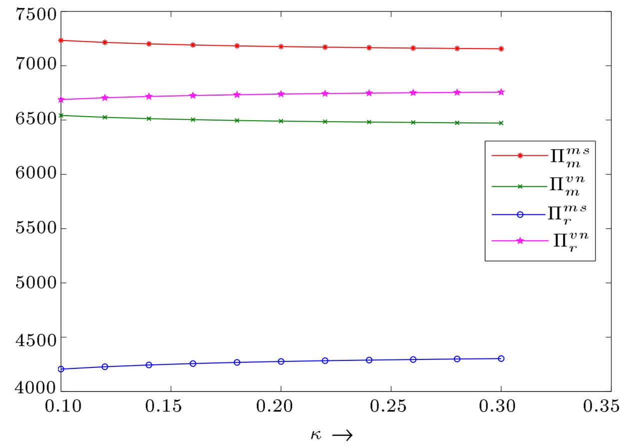

Figure 16. Manufacturer's and retailer's profit versus κ.

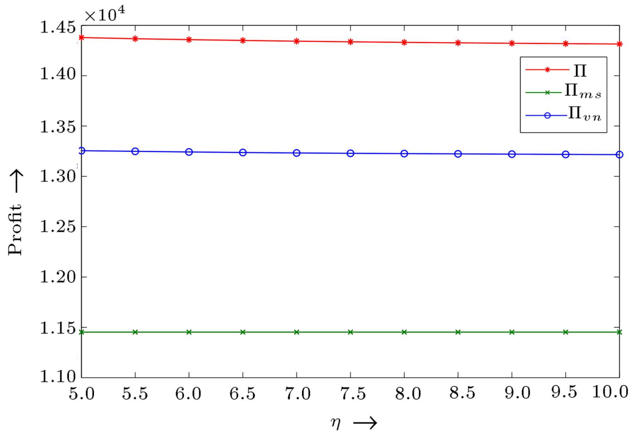

Figure 17. Total profit versus η.

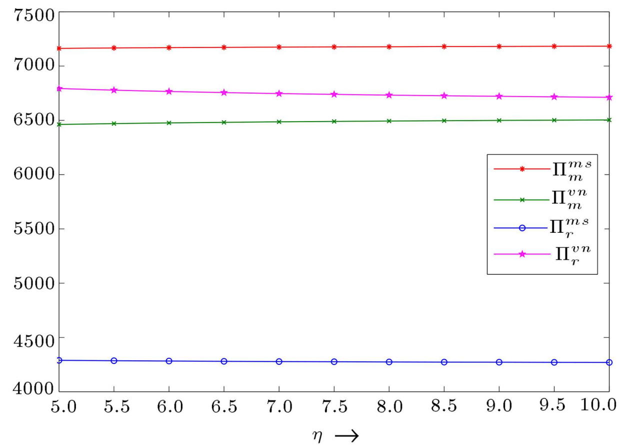

Figure 18. Manufacturer's and retailer's profit versus η.

fulfilled according to their wishes, the products will be delivered with an extra delivery charge upon the delivery time. In the retail channel, the customers are benefitted by the retail service and direct connection with the products. In this model, the selling price of the products for the direct channel was considered to be lower by a percentage of the selling price of the products in the retail channel due to decoration, huge rent cost for shops, and holding cost of the products for the retail shops. Both of the centralized and decentralized models were formulated which were studied analytically and numerically. In the decentralized scenario, the model was analyzed under vertical Nash and manufacturer Stackelberg scenarios. The model was analyzed with a numerical example and the sensitivity analysis of the key parameters was also carried out graphically to check the existence of the model.

The major contribution of the model is to study a dual-channel supply chain model with a selling price of each player, delivery time for direct channel, and retail service-dependent customer demand pattern. We also studied the model, assuming that the delivery time of the products was adjustable according to customers' demand with extra delivery charge according to delivery time. The decentralized model was discussed under vertical Nash game and manufacturer Stackelberg game structure.

For future perspectives, the present model can be extended by including multiple rivalry retailers. Our model could be studied with stochastic types of competitive market demand. We may also extend the model by introducing different contract policies among the members. This study can be extended by incorporating return-refund policy.

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