An Efficient Mixed-Memory-Type Control Chart for Normal and Non-Normal Processes

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Abstract

Statistical process control techniques are commonly used to monitor process performance. Control charting technique is the most sophisticated tool of SPC and is categorized as memory-less and memory-type control charts. Shewhart-type control charts have low efficiency in detecting the small changes in the process parameters and named as memory-less control charts, and memory-type control charts (for example cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts) are very sensitive to small persistent shifts. In connection with enhancing the performance of CUSUM and EWMA charts, an efficient variant of memory-type charts for the location parameter is developed based on mixing the double exponentially weighted moving average (DEWMA) chart and CUSUM chart by performing exponential smoothing twice. Performance of the proposed efficient variant is compared with existing counterparts under normal and non-normal (heavy tails and skewed) environments. The study also provides an industrial application related to the monitoring of weights of quarters made by mint machine placed into service at U.S. Mint. From theoretical and numerical studies, it is revealed that proposed variant of memory-type charts outperforms the counterparts in detecting shifts of small and moderate magnitude.

Keywords: Average run length; Control charts; CUSUM; Double EWMA; Location parameter; Memory-type charts.

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1. Introduction

Control charts, one of the key tools of statistical process control (SPC), are categorized into two main types named as memory-less and memory control charts. Shewhart charts are named as memory-less control charts are quite efficient in detecting the larger shifts in the process location or dispersion. However, their efficiencies are under consideration in detecting the small and moderate shifts in the process parameters. On the other hand, the memory control charts such as cumulative sum (CUSUM) control charts introduced by Page [1] and exponentially weighted moving average (EWMA) control charts suggested by Roberts [2] are more effective in detecting the small process shift because they make use of the current as well as past sample information.

The most significant and commonly used measure to assess the performance of control charts is the average run length (ARL) which is simply mean of the random variable run length (RL). The RL is the number of samples after that first out-of-control signal revealed. Some researchers have discouraged the only use of ARL due to the skewed behavior of its RL distribution (cf. Nasir [3], Zaman et al. [4] and Abid et al. [5-7]). Therefore, in order to further explain the run length distribution, it is better to report different important characteristics of the run length such as the standard deviation ($SDRL$) and some percentile points. When the process is in-control, the ARL is indicated by ARL_0 and is expected to be large and if the process is out-of-control, the ARL is represented by ARL_1 and is anticipated as small as possible.

To enhance the performance of CUSUM and EWMA charts, several amendments have been made (cf. Lucas [8], Lucas and Saccucci [9], Abid et al. [10], Raza et al. [11], Abujiya et al. [12], Riaz et al. [13], Mehmood et al. [14]). Shamma and Shamma [15] proposed a control chart for evaluating the smaller and moderate shifts in the process mean, using the method of double exponentially weighted moving average (DEWMA) by performing exponential smoothing twice. Riaz et al. [16] improved the performance of CUSUM scheme in detecting the small to large shifts by utilizing the concept of runs rules scheme. Abbas et al. [17] proposed the implementation of different runs rules for EWMA schemes. Abbas et al. [18] improved the design structure of EWMA and CUSUM control chart in such a way that EWMA statistic is served as the input for the CUSUM structure and named as mixed EWMA-CUSUM (MEC) chart. Zaman et al. [19] proposed a reverse version of the said MEC chart in such a way that the CUSUM statistic will use the input for the EWMA structure and hereafter, named as mixed CUSUM-EWMA (MCE) chart. On the further developments of the control charts using different approaches the interested reader can see the work of Nazir et al. [20], Ahmad et al. [21], Riaz and Ali [22], Abujiya et al. [23], Abbasi et al. [24], Ajadi and Riaz [25] and Hussain et al. [26].

With quality becoming more and more vital in today's industry and quality standards becoming higher and higher, a natural question comes to mind is: Is there a method to make the existing EWMA and CUSUM charts more sensitive to very small shifts in a process location parameter? In this study, following Abbas et al. [18] and Zaman et al. [19], we explore such a possibility by combining features of the structures of CUSUM and DEWMA charts and proposed an efficient chart by mixing the structures of CUSUM and DEWMA charts for the location parameter of the process. The control charts, designed under the assumption of normality, do not perform well under the violation of this assumption. Non-normal processes are more common in practice; hence it is indispensable to develop the structure of the control charts under non-normality. So, the performance of the proposed control chart is also under consideration in case of non-normal environments. The rest of the paper is organized as follows: In Section 2, we give the basic design structures of the CUSUM and DEWMA control charts and the proposed efficient variant scheme. Section 3, consists of design structure and derivation of the control limits of the proposed chart. Performance comparisons of the proposed scheme with its counterparts are presented in Section 4. An industrial application of the proposed chart is given in Section 5. At the end, Section 6 ends with conclusions.

2. Description of CUSUM, DEWMA and the proposed charts

Quality characteristic of interest, say X, is an independent sequence of observations $\{X_t\}$ ($t = 1, 2, 3, \dots\dots,$) following the normal distribution with mean $\mu_0 + \delta\sigma_0$ and variance σ_0^2, i.e. $X_t \sim N(\mu_0 + \delta\sigma_0, \sigma_0^2)$, where μ_0 and σ_0^2 are the mean and variance of the process, respectively. The value of $\delta = 0$, shows that the process is in-control, if not, the process mean has shifted and objective of the process monitoring is to detect the mean shift $\mu_0 + \delta\sigma_0$ as early as possible following its occurrence. Without loss of generality, we assume that $\mu_0 = 0$ and $\sigma_0 = 1$. Thus, we assume the phase II application of control charts with the in-control values of the parameters assumed to be known. The following subsection contains details about the memory-type control charts.

2.1. Cumulative sum (CUSUM) chart

Page [1] introduced the CUSUM chart by utilizing the method of accumulating the positive and negative deviations from μ_0 into two statistics C_i^+ and C_i^-, respectively. These two statistics are defined as:

$$\left.\begin{aligned} C_t^+ &= \max[0, (X_t - \mu_0) - K + C_{t-1}^+] \\ C_t^- &= \max[0, -(X_t - \mu_0) - K + C_{t-1}^-] \end{aligned}\right\} \tag{1}$$

where t is the sample number, μ_0 is the target value and K is the reference or slack value which is commonly selected equal to half of the shift (in standard deviation unit) to be detected. The starting values of C_0^+ and C_0^- are generally chosen equal to zero or the process location μ_0, that is $C_0^+ = C_0^- = \mu_0$, although it may be specified otherwise for a fast initial response (cf. Lucas and Crosier [9]). The statistics C_i^+ and C_i^- are plotted against the decision interval or control limit H and the chart signals if either one of the statistics (C_i^+ or C_i^-) exceeds the decision interval H. The K and H are two parameters of the CUSUM chart and these are defined as:

$$K = k \times \sigma_0, \quad H = h \times \sigma_0 \tag{2}$$

Here k and h are the constants which are selected to fulfill a pre-defined ARL_0 or according to the desired design conditions.

2.2. Double exponentially weighted moving average (DEWMA) chart

Shamma and Shamma [15] proposed the DEWMA chart by performing exponential smoothing twice. The main disadvantage associated with EWMA statistic is that it always offers strictly decreasing weights to historical data, but, this will not happen in case of DEWMA statistic (cf. Zhang and Chen [27). The DEWMA statistic Z_t is written as:

$$\left.\begin{aligned} Y_t &= \lambda_1 X_t + (1 - \lambda_2) Y_{t-1} \\ Z_t &= \lambda_3 Y_t + (1 - \lambda_4) Z_{t-1} \end{aligned}\right\} \tag{3}$$

where $\lambda_1 + \lambda_2 = 1$, and $\lambda_3 + \lambda_4 = 1$, and λ_1 and $\lambda_3 \in (0,1]$ are the smoothing parameters of the DEWMA chart. Also, shown by Zhang and Chen [27], the DEWMA statistic in Equation (3) may be expressed as:

$$Z_t = \lambda_1\lambda_3 \sum_{j=1}^{t}\left\{\lambda_2^{t-j} \sum_{k=0}^{t-j}\left(\frac{\lambda_4}{\lambda_2}\right)^k\right\} X_t + \lambda_3 \sum_{j=0}^{t-1} \lambda_2^{t-j}\lambda_4^j Y_0 + \lambda_4^t Z_0\,,\ t \geq 1 \tag{4}$$

If $\lambda_1 = \lambda_3$, then

$$Z_t = \lambda_1^2 \sum_{j=1}^{t}(t-j+1)\lambda_2^{t-j} X_t + t\lambda_1\lambda_2^t Y_0 + \lambda_2^t Z_0 \tag{5}$$

Equation (4) can be rewritten as following if $\lambda_1 \neq \lambda_3$:

$$Z_t = \lambda_1\lambda_3 \sum_{j=1}^{t} \frac{1-(\lambda_4/\lambda_2)^{t-j+1}}{1-(\lambda_4/\lambda_2)} \lambda_2^{t-j} X_t + \lambda_2\lambda_3 \frac{\lambda_2^t - \lambda_4^t}{\lambda_2 - \lambda_4} Y_0 + \lambda_4^t Z_0 \tag{6}$$

The starting values of Y_t and Z_t are generally taken equal to the target values, i.e. $Y_0 = Z_0 = \mu_0$. The chart, DEWMA, signals if the statistic Z_t falls beyond the following limits:

$$\left.\begin{aligned} LCL_t &= \mu_0 - L\sigma_{Z_t} \\ CL &= \mu_0 \\ UCL_t &= \mu_0 + L\sigma_{Z_t} \end{aligned}\right\} \tag{7}$$

where if $\lambda_1 = \lambda_3$, then

$$\sigma_{Z_t} = \sqrt{\sigma_0^2 \lambda_1^4 \frac{1+\lambda_2^2-(t^2+2t+1)\lambda_2^{2t}+(2t^2+2t-1)\lambda_2^{2t+2}-t^2\lambda_2^{2t+4}}{(1-\lambda_2^2)^3}} \tag{8}$$

and if $\lambda_1 \neq \lambda_3$, then

$$\sigma_{Z_t} = \sqrt{\sigma_0^2 \frac{\lambda_1^2\lambda_3^2}{(\lambda_4-\lambda_2)^2}\left\{\frac{\lambda_4^2(1-\lambda_4^{2t})}{1-\lambda_4^2} + \frac{\lambda_2^2(1-\lambda_2^{2t})}{1-\lambda_2^2} - 2\frac{\lambda_2\lambda_4(1-(\lambda_2\lambda_4)^t)}{1-\lambda_2\lambda_4}\right\}} \tag{9}$$

The constant L in Equation (7), is the control limit coefficient and can carefully be chosen to satisfy the pre-specified ARL_0 or according to the design conditions, that, together with λ_1 and λ_3, control the performance of DEWMA chart.

2.3. Proposed DEWMA-CUSUM chart

To improve the design structure of EWMA and CUSUM control charts, Abbas et al. [18] and Zaman et al. [19], suggested the mixed versions of EWMA and CUSUM charts. The proposed chart is based on mixing the features of DEWMA and CUSUM charts by using the concept of double exponential smoothing which makes the proposed chart sensitive to very small shifts in the process location parameter. The proposed mixed DEWMA-CUSUM chart, hereafter, is named as efficient variant (EV) chart. The charting statistics (EV_t^+ and EV_t^-) for this proposed EV chart are given as:

$$\left.\begin{aligned} EV_t^+ &= max[0,(Z_t-\mu_0)-P_t+EV_{t-1}^+] \\ EV_t^- &= max[0,-(Z_t-\mu_0)-P_t+EV_{t-1}^-] \end{aligned}\right\} \quad (10)$$

where Z_t is defined as in Equation (3) and P_t is the reference value. The initial values for the statistics EV_0^+ and EV_0^+ are generally put equal to zero or the target value, μ_0, i.e., $EV_0^+ = EV_0^- = \mu_0$, although the initial values may be specified according to the desired design conditions. The statistics (given in Equation (10)) are plotted alongside the control limit Q_t and if either one of these statistics (EV_t^+ or EV_t^-) goes outside the control limit Q_t, then the process is considered to be an out-of-control, otherwise, in-control. . The standardized versions of P_t and Q_t are given below:

if $\lambda_1 = \lambda_3$, then

$$P_t = p \times \sqrt{\sigma_0^2 \lambda_1^4 \frac{1+\lambda_2^2-(t^2+2t+1)\lambda_2^{2t}+(2t^2+2t-1)\lambda_2^{2t+2}-t^2\lambda_2^{2t+4}}{(1-\lambda_2^2)^3}} \quad (11)$$

and if $\lambda_1 \neq \lambda_3$, then

$$P_t = p \times \sqrt{\sigma_0^2 \frac{\lambda_1^2\lambda_3^2}{(\lambda_4-\lambda_2)^2}\left\{\frac{\lambda_4^2(1-\lambda_4^{2t})}{1-\lambda_4^2}+\frac{\lambda_2^2(1-\lambda_2^{2t})}{1-\lambda_2^2}-2\frac{\lambda_2\lambda_4(1-(\lambda_2\lambda_4)^t)}{1-\lambda_2\lambda_4}\right\}} \quad (12)$$

Unlike the usual CUSUM chart, the EV chart has time-varying reference values P_t that are due to the variance of DEWMA statistic in Equation (8) and Equation (9) and are functions of λ_1 and λ_3. The threshold control limit Q_t is as if $\lambda_1 = \lambda_3$:

$$Q_t = q \times \sqrt{\sigma_0^2 \lambda_1^4 \frac{1+\lambda_2^2-(t^2+2t+1)\lambda_2^{2t}+(2t^2+2t-1)\lambda_2^{2t+2}-t^2\lambda_2^{2t+4}}{(1-\lambda_2^2)^3}} \tag{13}$$

and when $\lambda_1 \neq \lambda_3$,

$$Q_t = q \times \sqrt{\sigma_0^2 \frac{\lambda_1^2\lambda_3^2}{(\lambda_4-\lambda_2)^2}\left\{\frac{\lambda_4^2(1-\lambda_4^{2t})}{1-\lambda_4^2} + \frac{\lambda_2^2(1-\lambda_2^{2t})}{1-\lambda_2^2} - 2\frac{\lambda_2\lambda_4(1-(\lambda_2\lambda_4)^t)}{1-\lambda_2\lambda_4}\right\}} \tag{14}$$

where p and q are constants similar to k and h in Equation (2), respectively and can carefully be chosen to satisfy the pre-specified ARL_0 or according to the design conditions. The flow chart of the proposed chart is provided in Figure 1.

2.4. Derivation of the limits of the proposed scheme

The construction of the Phase II control limits in Equation (13) and Equation (14) of the proposed EV chart depends on the choice of the smoothing parameters, λ_1 and λ_3, the reference value, P_t and the decision interval, Q_t. These parameters need to be chosen with care as these parameters control the performance of the proposed scheme. The q in Equation (13) and Equation (14) is determined to obtain the desired ARL_0 by setting $p = 0.5$, as an optimal constant to detect a shift of size $\delta = 1$, with different choices of λ_1 and λ_3, taking inspiration from Lucas [8], Shamma and Shamma [15], Abbas et al. [18] and Zaman et al. [19]. The values of q to satisfy, $ARL_0 = 168, 200, 370$ and 500, are evaluated when $\lambda_1 = \lambda_3$ and are given in Table 1 and when $\lambda_1 \neq \lambda_3$, values of q to satisfy $ARL_0 = 168$ are provided in Table 2 with their in-control $SDRL$ values. The in-control $SDRL$ are also reported in Table 3, when $\lambda_1 = \lambda_3$. Numerically these values are hard to find, and hence are determined by the use of Monte Carlo simulation.

For a fixed in-control ARL (ARL_0), the values of q increase as the smoothing parameters ($\lambda_1$and λ_3) tend to zero, and when $\lambda_1 = \lambda_3$ approaches to one, this phenomenon is opposite (cf. Table 1). However, when a too small value of $\lambda_1 = \lambda_3$, is used, the in-control $SDRL$ is often very large (cf. Table 3) and on the contrary, when the value of λ_1 and λ_3 has tendency to one, the in-control variability in the run length is small (cf. Table 2).

3. Performance of the charts

To judge the performance of the proposed EV chart, the *ARL* is used as a performance measure. Monte Carlo simulation is conducted to find ARL_0 and ARL_1 of the process. The simulation details are: we have generated 10^5 random observations from the distributions given in section 2. The control limits of the EV chart are established using the expressions given in Equation (13) and Equation (14) and the values of design parameters given in Tables 1-3. Then, we noted the number of sample points at which the plotting statistics (EV_t^+ or EV_t^-) breach the control limits. At the end, we repeated this procedure 10^5 times to get the distribution of the run lengths. The structure of proposed scheme can easily be implemented in any statistical software. In this study, R language is utilized for the implementation and to evaluate the properties of the charts.

ARL_1 and $SDRL$ of EV chart for $\lambda_1 = \lambda_3$ with fixed $ARL_0 = 168, 200, 370,\ 500$, are given in Tables 4-7 and when $\lambda_1 \neq \lambda_3$, the ARL_1 of the proposed chart with fixed $ARL_0 = 168$ are provided in Table 8. The following observations can be made from Tables 4-8:

i. The detection ability of the proposed chart for small shifts is higher for small values of smoothing parameters as compared to the large choices of λ_1 and λ_3. To detect the shift of size $\delta = 0.25$, the average run length is much lower for

$\lambda_1 = \lambda_3 = 0.01$ as compared to any other choices of the smoothing parameter (cf. Table 4).

ii. The performance of the EV chart is substantial with smaller values of $\lambda_1 = \lambda_3$.

iii. With the moderate value of $\lambda_1 = \lambda_3 = 0.10$, the shift of size $\delta = 0.50$ can be identified with smaller variability in the run length distribution.

iv. When $\lambda_1 \neq \lambda_3$, the efficient choices of $\lambda_1$and λ_3 for detecting $\delta = 0.50$ quickly is to use $\lambda_1 = 0.05$ and $\lambda_3 = 0.01$ along with the choice $\lambda_1 = 0.1$ and $\lambda_3 = 0.01$.

v. The proposed chart works efficiently in finding undesirable process level with $\lambda_1 = \lambda_3$ as compared to chart with $\lambda_1 \neq \lambda_3$.

vi. The recommendation is to use the $0 < \lambda_1 = \lambda_3 < 0.25$, for quick detection of shifts of magnitude i.e. $\delta = 0.50$ (cf. Tables 4-7) and in case of $\lambda_1 \neq \lambda_3$, better to select $0.05 \leq \lambda_1 \leq 0.1$ and $\lambda_3 = 0.01$.

4. Comparisons with other mixed charts under normal environment

Since the goal is to provide an efficient chart from the existing mixed charts e.g. mixed EWMA-CUSUM (MEC) chart and mixed CUSUM-EWMA (MCE) chart. We compare the performance of the EV chart, only, with that of MEC and MCE charts because papers of MEC and MCE charts provide detailed comparisons with some other charts. For valid comparisons, we let the EV chart, MEC chart and MCE chart have the same in-control average run length that is ARL_0 and then compare their respective out-of-control average run lengths that is ARL_1. For the said purpose, we simulate the ARLs of the MEC and MCE charts. Some representative results are provided in Table 9.

It can be seen from Table 4 and Table 9 that EV chart is slightly proficient than the MEC chart but outperforms MCE chart in detecting small to moderate changes in the process location parameter when the smoothing parameters of EV chart are equal i.e. $\lambda_1 = \lambda_3$. The performance of the proposal is more obvious and substantial with larger values of $\lambda_1 = \lambda_3$.

Comparing the results of Table 8 of EV chart having $\lambda_1 \neq \lambda_3$ with Table 9 of the MEC and MCE charts, it can be observed the EV chart is even more sensitive to small shift. The above discussion is made having $ARL_0 \cong 168$, but this is generally true when other in-control ARLs are considered.

As we discussed in section 2, the only use of ARL is criticized by many researchers due to its skewed behavior. So for a better understanding of the RL distribution of EV, MEC and MCE charts, some other measures such as standard deviation of run length ($SDRL$) and different percentiles (P_i, i^{th} percentile) along with smallest and largest run lengths of the in-control process are reported in Table 10 and as these measures help in studying the short run and long run behavior of the RL distribution. For instance, the 5% percentiles of the RL distribution of the EV, MEC and MCE charts are on average about 17, 4, 18, and 8 observations (cf. Table 10).

To get more insight into the out-of-control RL distribution, Figure 2 presents the run length distribution curves of all the charts considering the value of smoothing parameter equal to 0.10 with $\delta = 0.25$ under a normal environment. The curves give the cumulative probability of detecting an out-of-control situation. A higher curve shows the superiority of a chart in terms of its quick detection of shifts in the process parameter.

It can be observed from Figure 2 that EV chart with has higher probabilities for small run lengths to detect the shift than that of other memory charts. For detecting a shift of magnitude $\delta = 0.25$ at a run length equal to 30, the practitioner has to wait less with the mixed EV chart as compared to the MEC and MCE charts.

Overall, we see that the smaller values of smoothing parameters of the proposed EV chart offer better performance in spotting smaller changes in the process location parameter.

4.1. Evaluation under non-normal environments

Design and implementation of the proposed EV chart, discussed in the preceding subsection is based on the assumptions that: process measurements are independent and identically distributed, both the in-control and out-of-control distributions are normal, and the process parameters of the in-control distribution are known. But there are many practical situations where these assumptions can be invalid. Coming section discusses the effect, on the performance of the proposed EV chart, of the case when process measurements collected at different time periods are from non-normal environments. For the sake of comparisons, the counterpart charts MEC and MCE are also considered.

4.1.1. Limits based on normality

In this sub-section, the impact of non-normal observations on EV, MEC and MCE charts with control limits based on normality is evaluated. Consider the following scenario: When process measurements are from a non-normal distribution i.e. t distribution with 4 degrees of freedom (t_4) having heavy tails and being flatter than that of normal distribution. Looking at the results given in Table 11, it can be observed that the proposed EV chart and MEC chart are insensitive to change in the environment (i.e. t_4) in Phase II data, keeping the in-control properties nearly same as are in normal environment, whereas the MCE chart is effected by the change in distribution. The charts EV and MEC are observed to be robust under symmetric non-normal distribution. The standard deviation of RL of EV is smaller as compared to the SDRL of MEC and MCE charts under in-control and out-of-control conditions.

4.1.2. Limits based on non-normality

The distributions of many quality characteristics (capacitance, insulation resistance and surface finish, roundness, mold dimensions and customer waiting times, impurity levels in semiconductor process chemicals, in nuclear reactions, the interval between beta particle emissions) of different processes follow non-normal distribution. Hence, the performance of the proposed EV and its competitors MEC and MCE is evaluated under different non-normal environments when the control limits are set from same environment unlike the previous paragraph. For this purpose, *t*, Laplace and Logistic from symmetric family of distributions and Gamma and Lognormal from skewed family of distributions are considered. Without loss of generality, parameters of the distributions are set to have mean zero and variance one for valid comparisons. The results for symmetric distributions are given in Table 12 and Table 13 contains ARL for skewed distributions.

It can be observed from Table 12 that under *t* distribution the performance of MEC chart is similar to EV chart for shifts of small magnitude and EV chart outperforms the MEC chart when the value of $\delta \geq 1.5$. For small shifts in the process location, MCE is not good but its detection ability is higher as compared to other charts for large shifts as when $\delta \geq 1.5$, ARL for EV, MEC and MCE charts, respectively, are 12.29, 12.52 and 7.14. Similar kind of behavior is observed for other distributions such as Logistic and Laplace (cf. Table 12).

On the other hand, in case of Lognormal and Gamma distributions the EV chart performs more efficiently as compared to the MCE chart when the value of δ is relatively small that is $\delta \leq 0.75$ and the performance of the EV is also relatively better than the MEC chart when the value of δ is quite large as $\delta \geq 1.5$. So, in general, we can say that the EV chart outperforms the

MCE chart in detecting shifts of small magnitude and outperforms the MEC chart in detecting the shifts of large magnitude.

5. An industrial application

This section demonstrates how to construct the proposed EV chart. The data set is supplied by Zhang and Chen [27] and Triola [28], taking the first 16 samples each of size 5 giving 80 observations to apply the proposed chart. The operation concerns the monitoring of the weights of quarters made by a mint machine that was placed into service at U.S. Mint. The run chart, histogram and probability plot are provided, respectively, in Figures 3-5 for the behavior of the weights of quarters. The run chart depicts that there is more variation in the samples (50^{th} to 80^{th}) as compared to other samples. Anderson-Darling test is applied for the identification of the distribution of the weights of quarters. The test indicates that data do not follow normal distribution as p-value is less than 0.005. For comparisons, MEC and MCE charts are also constructed. The smoothing parameter is set equal to 0.10 for all said charts and control limits are calculated to guarantee that $ARL_0 \cong 168$. The graphical displays of all three charts are presented in Figures 6-8.

The proposed chart indicates that a signal at 50^{th} sample number whereas MEC identifies out-of-control point on 52^{nd} sample number and MCE chart observes no point out of the control limits. The proposed EV and MEC charts are proficient to detect decrease in the process location parameter (cf. Figures 6-7) while the MCE chart shows incapability to spot such change in the location parameter for the same data set (cf. Figure 8).

6. Conclusions

Control charts are widely used in monitoring the process parameters. Memory-less control charts (e.g. Shewhart-type charts) have low efficiency in detecting the small changes in the process parameters and memory-type control charts (e.g. CUSUM and EWMA charts) are very sensitive to small persistent shifts. Enhancing the performance of CUSUM and EWMA charts, an efficient variant of memory-type charts is developed based on mixing the double exponentially weighted moving average chart and CUSUM chart. Performance of the proposed efficient variant is compared with existing counterparts (i.e. mixed EWMA-CUSUM and mixed CUSUM-EWMA charts) under normal and non-normal environments. The proposed variant of memory-type charts outperforms the counterparts in detecting small and moderate persistent shifts. Some feature of this structure can be useful for monitoring the dispersion parameter of the process and may be the topic of next investigation.

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References

1. Page ES. "Continuous inspection schemes", *Biometrika*, **41**(1), pp.100–115 (1954).
2. Roberts SW. "Control chart tests based on geometric moving averages", *Technometrics*, **3**(1), pp. 239-250 (1959).
3. Abbas, N. "Homogeneously weighted moving average control chart with an application in substrate manufacturing process", *Computers and Industrial Engineering*, **120**(6), pp.460-470 (2018).
4. Zaman, B., Riaz, M., Lee, M. H. "On the performance of control charts for simultaneous monitoring of location and dispersion parameters", *Quality and Reliability Engineering International*, **33**(1), pp. 37-56 (2016).

5. Abid M, Nazir ZA, Riaz M, et al. “Use of ranked set sampling in nonparametric control charts”, *Journal of the Chinese Institute of Engineers*, **39**(5), pp. 1-19 (2016).
6. Abid M, Nazir ZA, Riaz M, et al. “Investigating the impact of ranked set sampling in nonparametric CUSUM control charts”, *Quality and Reliability Engineering International*, **33**(1), pp. 203-214 (2017).
7. Abid M, Nazir ZA, Riaz M, et al. “An efficient nonparametric EWMA Wilcoxon signed-rank chart for monitoring location”, *Quality and Reliability Engineering International*, **33**(3), pp. 665-679 (2017).
8. Lucas JM. “Combined Shewhart-CUSUM quality control schemes”, *Journal of Quality Technology*, **14**(2), pp. 51-59 (1982).
9. Lucas JM, Saccucci MS. “Exponentially weighted moving average control schemes: properties and enhancements”, *Technometrics*, **32**(1), pp. 1–12 (1990).
10. Abid M, Nazir ZA, Riaz M, et al. “In-control robustness comparison of different control charts”, *Transaction of the Institute of Measurement and Control*, **40**(13), pp. 3860-3871 (2018).
11. Raza, S. M. M., Riaz, M., Ali, S. “EWMA control chart for Poisson–exponential lifetime distribution under type I censoring”, *Quality and Reliability Engineering International*, **32**(3), pp. 995-1005 (2016).
12. Abujiya, M. R., Abbasi, S. A., Riaz, M. “A new EWMA control chart for monitoring Poisson observations. average control chart for monitoring the coefficient of variation”, *Computers and Industrial Engineering*, **78, pp.** 205-212 (2016).
13. Riaz, S., Riaz, M., Hussain, Z., et al. “Monitoring the performance of Bayesian EWMA control chart using loss functions”, *Computers and Industrial Engineering*, **112**, pp. 426-436 (2018).
14. Mehmood, R., Lee, M.H., Hussain, S., et al. “On efficient construction and evaluation of runs rules based control chart for known and unknown parameters under different distributions”, *Quality and Reliability Engineering International*, **35**(2), pp.582-599 (2019).
15. Shamma SE, Shamma AK. “Development and evaluation of control charts using double exponentially weighted moving averages”, *International Journal of Quality and Reliability Management*, **9**(6), pp. 18-25 (1992).
16. Riaz M, Abbas N, Does RJMM. “Improving the performance of CUSUM charts”, *Quality and Reliability Engineering International*, **27**(4), pp. 415 – 424 (2011).
17. Abbas N, Riaz M, Does RJMM. “Enhancing the performance of EWMA charts”, *Quality and Reliability Engineering International*, **27**(6), pp. 821 – 833 (2011).
18. Abbas N, Riaz M, Does RJMM. “Mixed exponentially weighted moving average-cumulative sum charts for process monitoring”, *Quality and Reliability Engineering International*, **29**(3), pp. 355 – 365 (2013).
19. Zaman B, Riaz M, Does RJMM. “Mixed Cumulative Sum-Exponentially Weighted Moving Average Control Charts: An Efficient Way of Monitoring Process Location”, *Quality and Reliability Engineering International*, **31**(8), pp. 1407-1421 (2014).
20. Nazir HZ, Riaz M, Does RJMM, et al. “Robust CUSUM control charting”, *Quality Engineering*, **25**(3), pp. 211-224 (2013).
21. Ahmad L, Aslam M, Jun CH. “Designing of X-bar control charts based on process capability index using repetitive sampling”, *Transactions of the Institute of Measurement and Control*, **36**(3), pp. 367-374 (2014).

22. Riaz M, Ali S. "On process monitoring using location control charts under different loss functions", *Transactions of the Institute of Measurement and Control*, **38**(9), pp. 1107-1119 (2015).
23. Abujiya MR, Lee MH, Riaz M. "New EWMA S2 control charts for monitoring process dispersion", *Scientia Iranica*, **24**(1), pp. 378-389 (2017).
24. Abbasi SA, Ahmad S, Riaz M. "On enhanced sensitivity of nonparametric EWMA control charts for process monitoring", *Scientia Iranica*, **24**(3), pp. 1603-1614 (2017).
25. Ajadi JO, Riaz M. "New memory-type control charts for monitoring process mean and dispersion", *Scientia Iranica*, **24**(6), pp. 3423-3438 (2017).
26. Hussain S, Song L, Ahmad S, et al. "On auxiliary information based improved EWMA median control charts", *Scientia Iranica*, **25**(2), pp. 954-982 (2018).
27. Zhang L, Chen G. "An extended EWMA mean chart", *Quality Technology and Quality Management*, **2**(1), pp. 39-52 (2005).
28. Triola MF. "Elementary Statistics U sing Excel", Pearson and Addison Wesley, Boston (2004).

Figure Captions

Figure 1: Flowchart of the proposed charts.

Figure 2: Run length curves of EV, MEC and MCE charts with $\delta = 0.25$ and $ARL_0 \cong 168$.

Figure 3: Run chart of weights of quarter.

Figure 4: Histogram of weights of quarter.

Figure 5: Probability plot of weights of quarter.

Figure 6: An industrial application of the EV chart.

Figure 7: An industrial application of the MEC chart.

Figure 8: An industrial application of the MCE chart.

Table Captions

Table 1: q values of the EV chart for given ARL_0, and $\lambda_1 = \lambda_3$ with $p = 0.5$.

Table 2: q values of the EV chart for $ARL_0 \cong 168$, and $\lambda_1 \neq \lambda_3$, with $p = 0.5$.

Table 3: $SDRL$ of the EV chart for given ARL_0, and $\lambda_1 = \lambda_3$ with $p = 0.5$.

Table 4: ARL_1 and $SDRL$ of EV chart when $\lambda_1 = \lambda_3$ with $ARL_0 \cong 168$.

Table 5: ARL_1 and $SDRL$ of EV chart when $\lambda_1 = \lambda_3$ with $ARL_0 \cong 200$.

Table 6: ARL_1 and $SDRL$ of EV chart when $\lambda_1 = \lambda_3$ with $ARL_0 \cong 370$.

Table 7: ARL_1 and $SDRL$ of EV chart when $\lambda_1 = \lambda_3$ with $ARL_0 \cong 500$.

Table 8: ARL_1 of EV chart when $\lambda_1 \neq \lambda_3$ with $ARL_0 \cong 168$.

Table 9: ARLs of MEC and MCE charts with $ARL_0 \cong 168$.

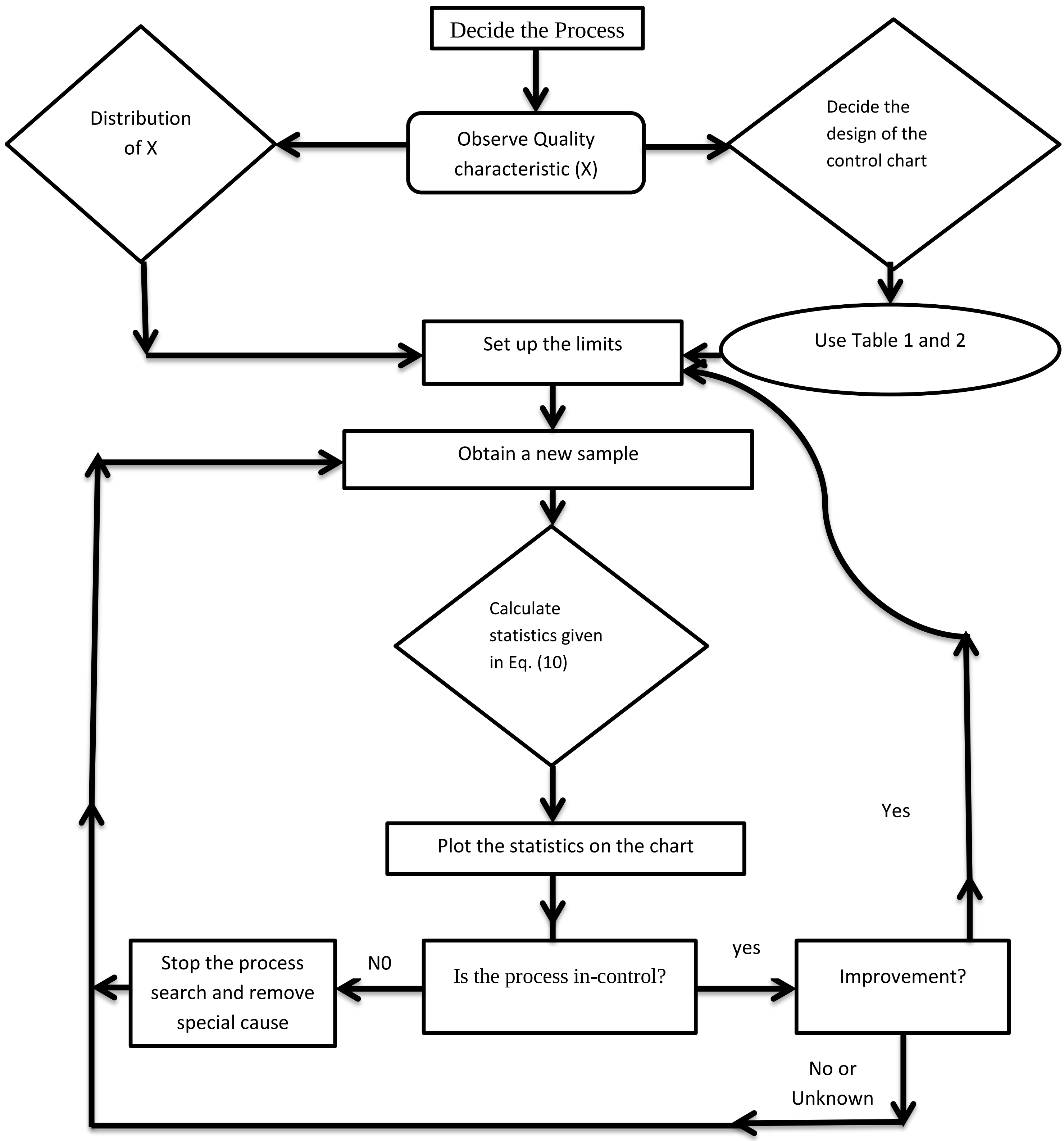
Table 10: Characteristics of in-control run length for EV, MEC and MCE charts with $ARL_0 \cong 168$.

Table 11: Characteristics of run length distribution of mixed charts under uncorrected limits with $ARL_0 \cong 168$.

Table 12: ARLs of mixed charts for symmetric distributions with $ARL_0 \cong 370$.

Table 13: ARLs of mixed charts for skewed distributions with $ARL_0 \cong 370$.

Figure 1



Decide the Process
Distribution of X
Observe Quality characteristic (X)
Decide the design of the control chart
Use Table 1 and 2
Set up the limits
Obtain a new sample
Calculate statistics given in Eq. (10)
Plot the statistics on the chart
Yes
Stop the process search and remove special cause
N0
Is the process in-control?
yes
Improvement?
No or Unknown

Figure 2

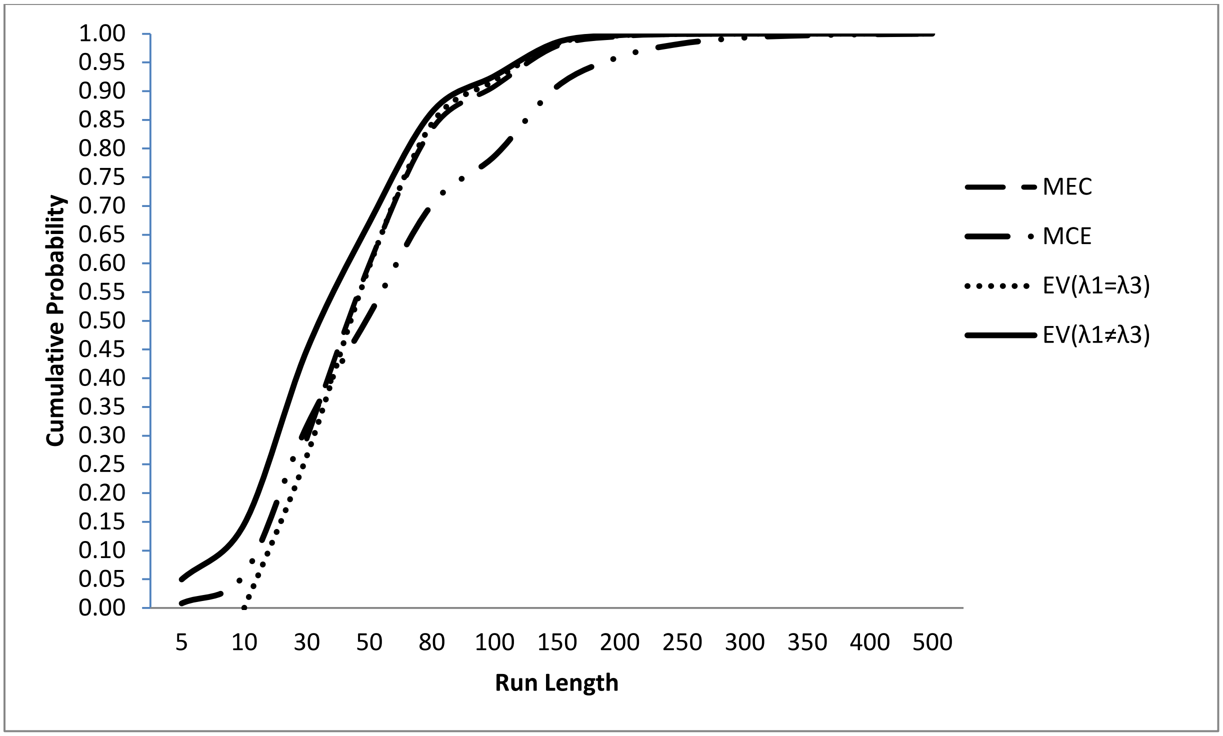


Figure 3

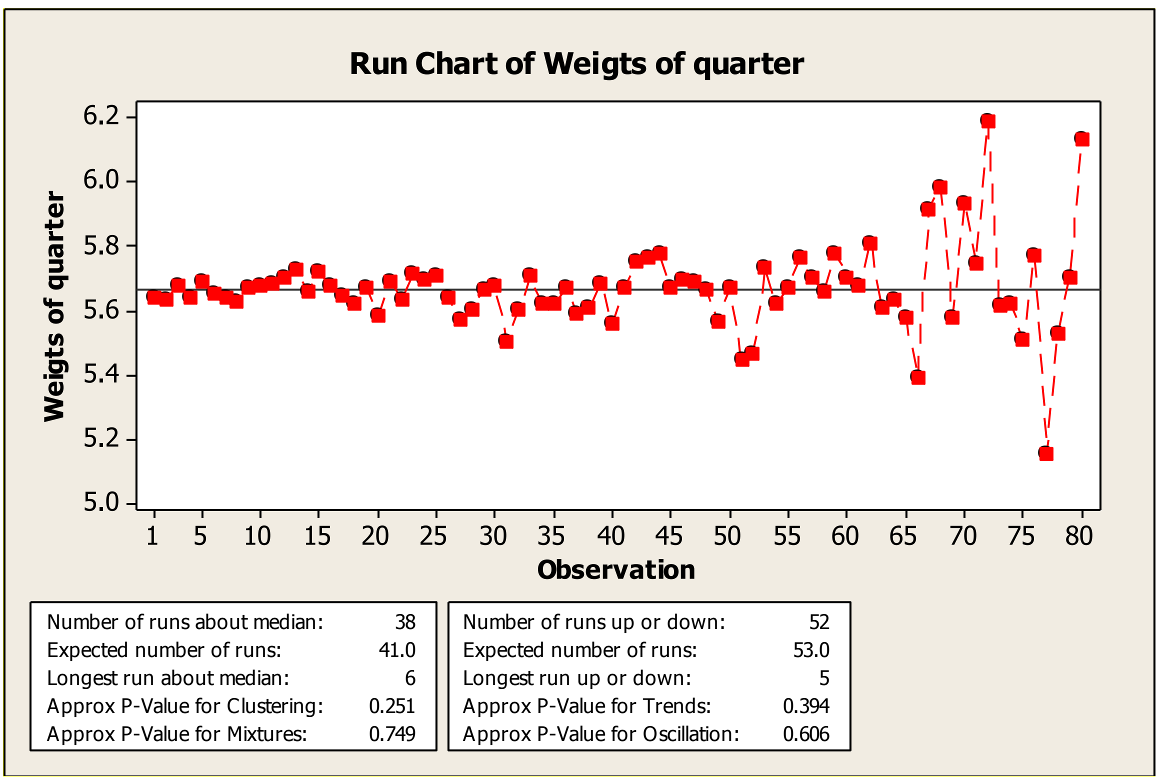


Figure 4

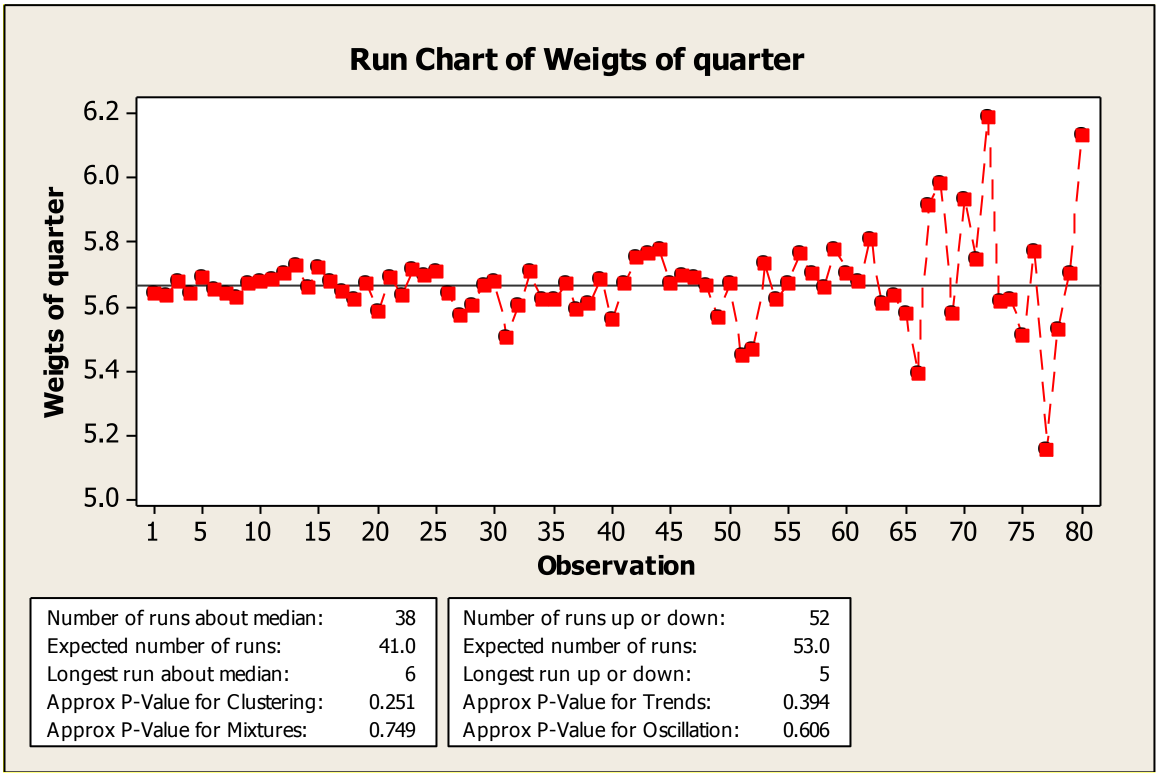


Figure 5

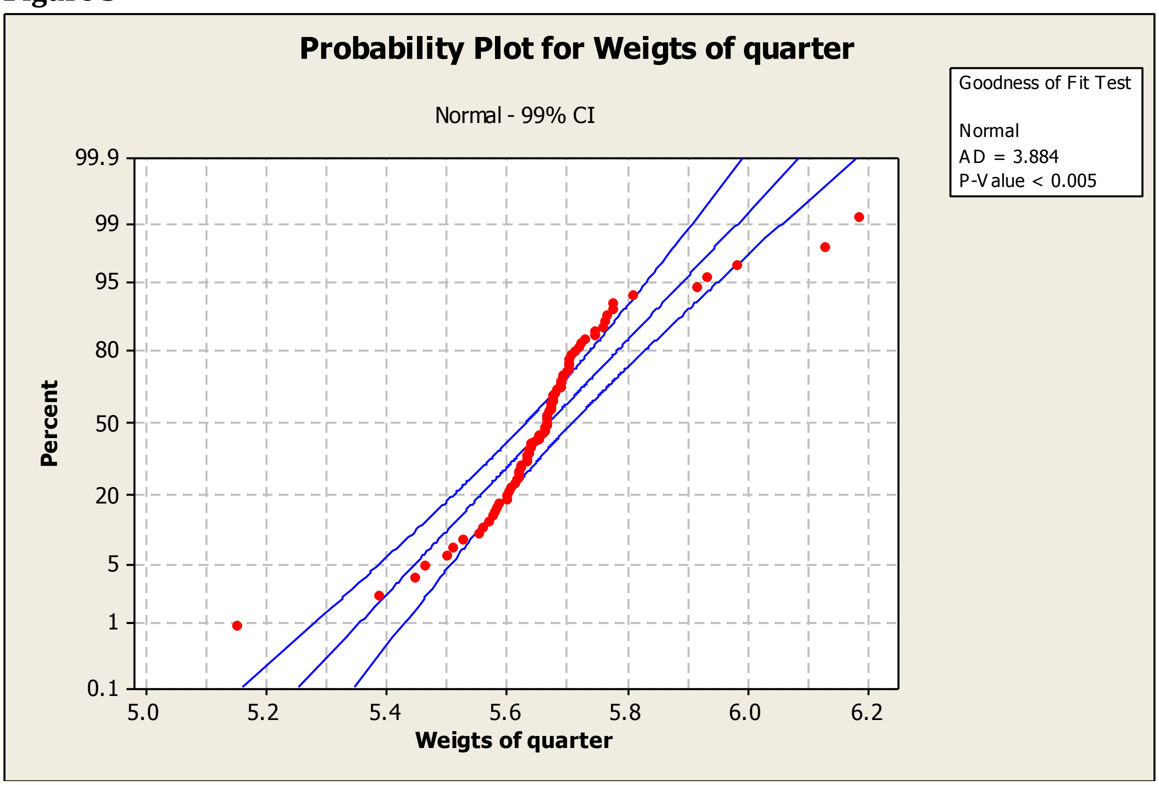


Figure 6

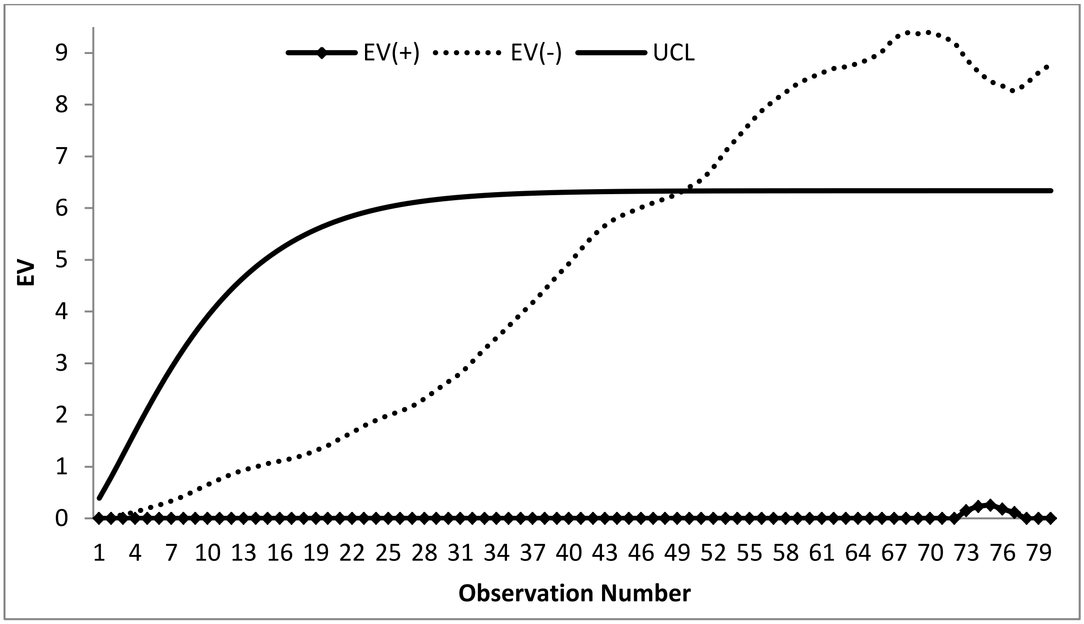


Figure 7

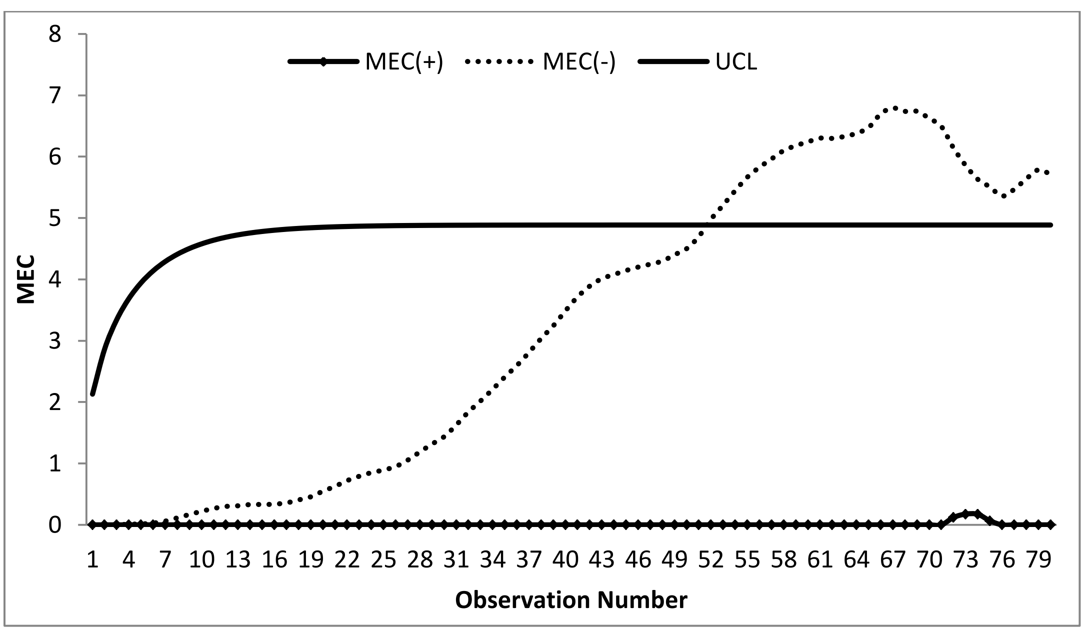
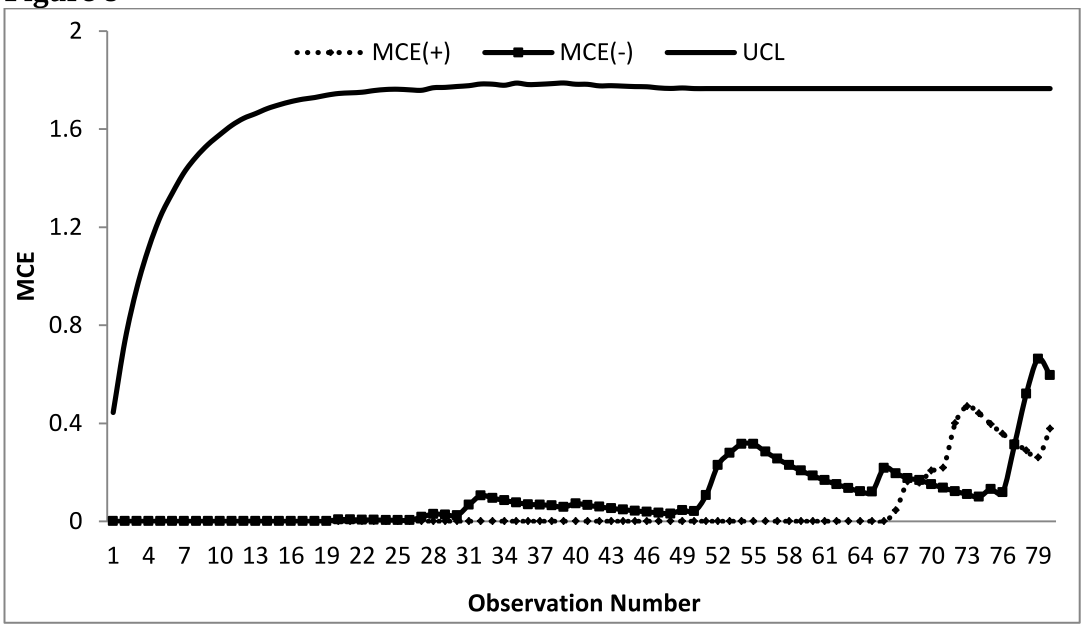


Figure 8



MCE(+)
MCE(-)
UCL
2
1.6
1.2
0.8
0.4
0
MCE
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79
Observation Number

Table 1

ARL_0	$\lambda_1 = \lambda_3$						
	0.01	0.05	0.1	0.25	0.5	0.75	1
168	116.04	51.34	39	23.91	13.1	7.4	4
200	124	58.04	43.99	26	14	7.8	4.18
370	165	84.6	60	32.8	17	9.24	4.78
500	193.6	100.4	68.84	36.74	18.6	9.95	5.08

Table 2

λ_1	0.01	0.01	0.01	0.05	0.05	0.05	0.1	0.1	0.1
λ_3	0.05	0.1	0.25	0.01	0.1	0.25	0.01	0.05	0.25
q	38.28	30.06	25.89	150.84	38.28	30.3	169.4	55.9	28
ARL	169.79	168.33	167.09	168.46	168.90	169.70	169.31	169.07	169.04
$SDRL$	181.85	161.34	154.45	187.88	146.48	142.79	174.04	145.27	145.53
λ_1	0.25	0.25	0.25	0.5	0.5	0.5	0.75	0.75	0.75
λ_3	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
q	193	61.26	39.8	209.7	66	40.68	218.6	68	41.5
ARL	168.61	168.35	169.66	169.31	167.88	169.10	168.55	168.17	168.88
$SDRL$	167.59	143.32	146.80	165.11	139.62	145.74	167.70	141.28	146.05

Table 3

ARL_0	$\lambda_1 = \lambda_3$						
	0.01	0.05	0.1	0.25	0.5	0.75	1
168	252.46	149.18	143.49	148.73	154.85	159.21	161.73
200	291.57	176.35	171.60	180.99	185.76	195.13	198.40
370	417.52	317.56	324.32	336.72	351.91	359.34	371.45
500	509.96	430.64	437.63	466.81	475.50	491.99	500.54

Table 4

δ	$\lambda_1 = 0.01$ $\lambda_3 = 0.01$		$\lambda_1 = 0.05$ $\lambda_3 = 0.05$		$\lambda_1 = 0.1$ $\lambda_3 = 0.1$		$\lambda_1 = 0.25$ $\lambda_3 = 0.25$		$\lambda_1 = 0.5$ $\lambda_3 = 0.5$		$\lambda_1 = 0.75$ $\lambda_3 = 0.75$		$\lambda_1 = 1$ $\lambda_3 = 1$	
	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$
0.05	136.1	198	151	129.5	149.5	125.3	154.3	133.8	156.1	143.1	159.2	150.9	162.1	155.9
0.1	86.4	114.9	112.5	91.38	114.9	91.72	120.5	102.1	125	111.2	131.4	121.5	140.3	138.6
0.2	43.75	50.19	63.83	44.34	65.86	44.65	68.23	50.67	71.92	59.35	78.74	70.17	92.75	86.43
0.25	32.98	34.9	50.27	32.39	52.01	32.34	53.44	37.42	55.65	43.88	61.11	52.59	74.97	69.93
0.5	13.35	11.7	23.61	12.13	25.52	10.84	24.15	11.37	22.77	13.03	23.13	15.83	26.74	21.43
0.75	7.81	5.44	14.76	6.73	17.04	5.93	15.77	5.49	13.74	6.05	12.75	6.9	13.34	9.02
1	5.61	3.21	10.53	4.41	12.8	4.02	12.03	3.42	9.97	3.53	8.73	3.87	8.36	4.74
1.5	3.76	1.51	6.69	2.27	8.6	2.29	8.52	1.84	6.75	1.71	5.51	1.77	4.75	2.01
2	3.02	0.9	5.01	1.37	6.5	1.49	6.74	1.23	5.29	1.07	4.14	1.05	3.35	1.18
2.5	2.59	0.65	4.08	0.93	5.24	1.06	5.62	0.93	4.43	0.77	3.39	0.72	2.61	0.78
3	2.33	0.51	3.53	0.69	4.48	0.81	4.89	0.74	3.88	0.62	2.94	0.58	2.19	0.58
4	2.07	0.25	2.95	0.44	3.54	0.57	3.94	0.54	3.15	0.41	2.3	0.46	1.71	0.49
5	2.01	0.07	2.54	0.5	3.06	0.31	3.29	0.46	2.8	0.41	2.02	0.16	1.31	0.46

Table 5

δ	$\lambda_1 = 0.01$ $\lambda_3 = 0.01$		$\lambda_1 = 0.05$ $\lambda_3 = 0.05$		$\lambda_1 = 0.1$ $\lambda_3 = 0.1$		$\lambda_1 = 0.25$ $\lambda_3 = 0.25$		$\lambda_1 = 0.5$ $\lambda_3 = 0.5$		$\lambda_1 = 0.75$ $\lambda_3 = 0.75$		$\lambda_1 = 1$ $\lambda_3 = 1$	
	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$
0.05	151.7	209.9	178.4	153.6	180.4	153.4	184.8	162.8	185	170.6	185.9	174.4	189.3	184.2
0.1	100.6	124.7	130.1	102.6	133.1	105.4	141.5	122.5	145.9	132.7	154.6	147.1	166.5	161.6
0.2	47.98	50.92	70.8	46.38	72.91	48.37	74.72	55.6	79.3	65.58	87.41	78.2	108	103.9
0.25	36.99	38.4	56.02	34.62	57.18	34.34	58.14	39.48	61.05	47.08	68.04	59.75	83.71	78.99
0.5	14.29	12.02	26.59	12.61	27.93	11.23	25.55	11.99	24.07	13.9	24.27	16.72	28.47	23.07
0.75	8.4	5.88	16.55	7.07	18.75	6.1	16.62	5.63	14.62	6.44	13.23	6.94	13.97	9.41
1	5.92	3.38	11.88	4.6	14.16	4.13	12.73	3.47	10.46	3.6	9.09	3.96	8.65	4.79
1.5	3.95	1.55	7.49	2.4	9.55	2.38	9.02	1.88	7.04	1.74	5.7	1.82	4.95	2.04
2	3.12	0.92	5.56	1.48	7.22	1.57	7.14	1.24	5.51	1.09	4.28	1.09	3.47	1.21
2.5	2.69	0.68	4.49	1.02	5.81	1.13	5.97	0.93	4.63	0.78	3.51	0.76	2.72	0.81
3	2.41	0.54	3.84	0.76	4.95	0.86	5.18	0.76	4.03	0.63	3.03	0.58	2.27	0.59
4	2.09	0.29	3.14	0.44	3.88	0.59	4.16	0.53	3.27	0.46	2.39	0.5	1.77	0.47
5	2.01	0.1	2.8	0.42	3.25	0.44	3.52	0.51	2.92	0.31	2.04	0.21	1.37	0.48

Table 6

δ	$\lambda_1 = 0.01$ $\lambda_3 = 0.01$		$\lambda_1 = 0.05$ $\lambda_3 = 0.05$		$\lambda_1 = 0.1$ $\lambda_3 = 0.1$		$\lambda_1 = 0.25$ $\lambda_3 = 0.25$		$\lambda_1 = 0.5$ $\lambda_3 = 0.5$		$\lambda_1 = 0.75$ $\lambda_3 = 0.75$		$\lambda_1 = 1$ $\lambda_3 = 1$	
	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$
0.05	270.7	293.6	296.5	245.4	300.6	258	305.9	276.4	323.3	304.9	332.7	321.2	348.7	346.1
0.1	157.1	152.4	192	144.2	196.4	156.6	208	177.5	228.9	208.6	252.6	240.9	283.5	276.3
0.2	73.07	63.57	95.2	55.09	94.57	59.02	97.22	70.77	107.9	89.3	126.7	116.2	164.1	160.1
0.25	53.93	45.04	74.81	38.36	73.35	40.56	73.08	48.73	78.7	61.37	92.52	81.4	123.2	117.5
0.5	20.2	15.05	36.64	13.49	35.14	12.4	30.86	13.35	28.64	15.39	28.74	18.86	35.28	28.92
0.75	11.29	7.13	23.94	7.77	23.86	6.53	19.88	6.31	16.91	6.89	15.65	7.98	16.34	10.66
1	7.74	4.17	17.4	5.32	18.27	4.38	15.03	3.81	12.11	3.96	10.46	4.3	9.9	5.23
1.5	4.89	1.92	11.03	2.97	12.53	2.58	10.62	2	8.1	1.91	6.49	1.94	5.52	2.18
2	3.78	1.09	8.02	1.86	9.53	1.74	8.43	1.32	6.3	1.17	4.83	1.16	3.86	1.26
2.5	3.18	0.74	6.28	1.29	7.65	1.29	7.04	0.99	5.24	0.83	3.94	0.8	3	0.85
3	2.83	0.6	5.27	0.97	6.47	1	6.13	0.79	4.59	0.66	3.38	0.6	2.48	0.63
4	2.35	0.48	4.07	0.62	4.97	0.68	4.91	0.59	3.73	0.52	2.73	0.48	1.96	0.4
5	2.08	0.27	3.38	0.5	4.12	0.48	4.14	0.42	3.14	0.35	2.2	0.4	1.61	0.49

Table 7

δ	$\lambda_1 = 0.01$ $\lambda_3 = 0.01$		$\lambda_1 = 0.05$ $\lambda_3 = 0.05$		$\lambda_1 = 0.1$ $\lambda_3 = 0.1$		$\lambda_1 = 0.25$ $\lambda_3 = 0.25$		$\lambda_1 = 0.5$ $\lambda_3 = 0.5$		$\lambda_1 = 0.75$ $\lambda_3 = 0.75$		$\lambda_1 = 1$ $\lambda_3 = 1$	
	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$	ARL_1	$SDRL$
0.05	349.7	335.3	378.4	312	384.2	330.4	405.2	369.5	430.3	410.7	444.3	430	461.3	455.8
0.1	195.1	165.9	228.6	169.3	233.9	185.4	258.3	223.5	288.1	268.6	315.1	307.2	374	376.3
0.2	89.08	67.7	109.3	61.59	106.8	65.55	112	82.12	125	104.3	150	136.2	199.2	188.4
0.25	66.05	48.95	85.54	41.27	82.09	43.82	81.69	52.68	89.02	69.04	105.2	91.68	145.8	137.3
0.5	24.64	16.64	41.97	13.89	38.87	13	33.57	14.23	31.09	16.37	31.37	20.26	38.91	31.43
0.75	13.59	8.03	28.16	8.12	26.47	6.75	21.72	6.7	18.22	7.23	16.6	8.28	17.35	11.26
1	9.16	4.74	20.61	5.53	20.38	4.51	16.31	3.97	13	4.15	11.13	4.47	10.53	5.57
1.5	5.64	2.16	13.14	3.18	14.1	2.65	11.46	2.07	8.64	1.99	6.9	2.03	5.86	2.27
2	4.24	1.24	9.52	2.06	10.77	1.81	9.1	1.36	6.7	1.21	5.13	1.21	4.08	1.3
2.5	3.51	0.8	7.47	1.45	8.66	1.35	7.65	1.02	5.56	0.86	4.14	0.83	3.14	0.88
3	3.11	0.6	6.16	1.08	7.31	1.05	6.63	0.82	4.86	0.68	3.56	0.64	2.61	0.66
4	2.6	0.51	4.68	0.69	5.58	0.72	5.31	0.59	3.96	0.47	2.88	0.44	2.03	0.39
5	2.22	0.41	3.89	0.5	4.58	0.56	4.47	0.52	3.32	0.47	2.35	0.48	1.72	0.45

Table 8

δ	$\lambda_1 = 0.01$			$\lambda_1 = 0.05$			$\lambda_1 = 0.10$			$\lambda_1 = 0.25$			$\lambda_1 = 0.50$			$\lambda_1 = 0.75$		
	$\lambda_3 = 0.05$	$\lambda_3 = 0.10$	$\lambda_3 = 0.25$	$\lambda_3 = 0.01$	$\lambda_3 = 0.10$	$\lambda_3 = 0.25$	$\lambda_3 = 0.01$	$\lambda_3 = 0.05$	$\lambda_3 = 0.25$	$\lambda_3 = 0.01$	$\lambda_3 = 0.05$	$\lambda_3 = 0.10$	$\lambda_3 = 0.01$	$\lambda_3 = 0.05$	$\lambda_3 = 0.25$	$\lambda_3 = 0.01$	$\lambda_3 = 0.05$	$\lambda_3 = 0.25$
0.05	143.7	147.1	147.9	144.9	151.2	151.3	146.5	148.7	150.4	145.6	146.9	148.5	149.8	153.9	153.5	148.2	152.2	155
0.1	103.6	108.9	110.5	100.8	113.9	114.5	105.9	114.6	117.6	107	114.3	117	108.1	118	118.7	106.8	117.4	120
0.2	57.24	61.29	63.53	53.71	65.64	66.36	56.62	65.1	66.47	58.05	66.27	67.3	58.34	67.63	67.98	57.97	67.54	68.87
0.25	44.15	48.97	51.39	40.65	52.19	52.82	43.69	51.78	52.9	45.1	52.75	53.34	44.87	53.58	53.72	44.62	53.56	54.29
0.5	19.37	23.49	26.6	16.7	26.26	27.55	17.9	24.07	25.76	18.55	24.3	24.8	18.67	24.96	24.91	18.52	25.05	25.15
0.75	11.84	15.34	18.69	9.75	17.59	19.46	10.67	15.39	18	10.9	15.27	16.16	10.89	15.55	15.93	10.77	15.6	16.03
1	8.2	11.11	14.45	6.86	13.16	15.35	7.39	10.96	14.02	7.68	10.98	12.02	7.64	11.08	11.69	7.51	11.07	11.67
1.5	5.27	7.25	10.24	4.47	8.75	11.09	4.78	7.01	10.1	5.03	7.12	8.12	4.93	7.05	7.71	4.8	6.96	7.61
2	4.04	5.44	8.03	3.5	6.58	8.79	3.72	5.24	8.02	3.87	5.33	6.16	3.82	5.25	5.83	3.7	5.12	5.68
2.5	3.38	4.43	6.65	2.98	5.29	7.29	3.16	4.28	6.71	3.3	4.36	5.04	3.22	4.25	4.73	3.1	4.11	4.56
3	2.98	3.79	5.7	2.65	4.51	6.31	2.81	3.67	5.79	2.92	3.75	4.31	2.87	3.67	4.08	2.75	3.53	3.91
4	2.45	3.09	4.5	2.23	3.55	5	2.33	3.04	4.61	2.44	3.09	3.44	2.39	3.04	3.27	2.3	2.93	3.15
5	2.11	2.7	3.79	2.04	3.06	4.18	2.07	2.68	3.9	2.11	2.75	3.02	2.09	2.69	2.92	2.06	2.52	2.8

Table 9

δ	MEC chart				MCE chart			
	$\lambda = 0.1$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 0.1$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$
0	168.04	168.07	169.88	171.04	168.30	169.39	168.36	170.28
0.25	52.64	54.18	59.78	68.15	67.79	70.42	72.90	73.45
0.5	24.86	22.41	22.55	24.13	25.61	24.94	25.61	25.43
0.75	17.02	14.02	12.86	12.61	13.52	12.77	12.64	12.43
1	13.33	10.48	8.96	8.27	9.34	8.39	7.78	7.54
1.5	9.74	7.33	5.79	5.00	5.87	5.05	4.42	4.09
2	7.91	5.82	4.43	3.74	4.41	3.73	3.14	2.79

Table 10

Characteristics	EV ($\lambda_1 = \lambda_3 = 0.1$)	EV ($\lambda_1 = 0.1, \lambda_3 = 0.01$)	MEC ($\lambda = 0.1$)	MCE ($\lambda = 0.1$)
Min	5	2	9	3
P_1	17	4	18	8
P_5	38	7	36	25
P_{10}	47	13	45	35
P_{25}	66	41	64	55
P_{50}	125	116	125	122
P_{75}	222	239	229	234
P_{90}	356	399	368	382
P_{95}	457	521	470	494
P_{99}	688	792	699	741
Max	1746	1879	2282	1796
ARL	167.67	169.24	170.60	170.40
SDRL	143.72	174.73	149.58	161.94

Table 11

Chart	Smoothing Parameter	Reference Value	Limit	δ	ARL	$SDRL$	Min	P_5	P_{25}	P_{50}	P_{75}	P_{95}	Max
EV	0.1	0.5	39	0	173.4	151.33	2	28	68	129	230	470	1542
				0.05	153.55	130.5	2	27	62	115	205	410	1655
				0.1	117.94	94.52	2	25	51	90	155	302	959
				0.2	66.45	44.93	3	19	35	54	85	156	455
				0.25	52.65	32.39	2	18	30	44	67	116	317
				0.5	25.33	10.5	2	12	18	24	31	45	91
				0.75	16.95	5.81	2	9	13	16	20	28	63
				1	12.78	3.92	2	7	10	12	15	20	38
				1.5	8.54	2.22	2	5	7	8	10	12	26
				2	6.45	1.43	2	4	6	6	7	9	15
				2.5	5.26	1.04	2	4	5	5	6	7	12
				3	4.49	0.8	2	3	4	4	5	6	11
				4	3.54	0.59	2	3	3	4	4	4	9
				5	3.05	0.32	2	3	3	3	3	4	7
MCE	0.1	0.5	5.96	0	154.49	149.85	1	12	47	109	214	448	1397
				0.05	145.05	137.97	1	13	46	103	199	422	1329
				0.1	131.27	127.7	1	12	42	92	178	382	1391
				0.2	93.43	85.78	1	10	32	67	129	271	829
				0.25	74.17	67.38	1	9	26	54	100	211	751
				0.5	27.18	20.48	1	7	13	21	35	67	226
				0.75	13.7	7.69	1	5	8	12	17	29	78
				1	9.09	4.04	1	4	6	8	11	17	37
				1.5	5.48	1.88	1	3	4	5	6	9	18
				2	3.99	1.17	1	2	3	4	5	6	11
				2.5	3.16	0.86	1	2	3	3	4	5	8
				3	2.6	0.7	1	2	2	3	3	4	6
				4	1.95	0.46	1	1	2	2	2	3	5
				5	1.47	0.52	1	1	1	1	2	2	4
MEC	0.1	0.5	21.3	0	176.3	157.46	3	26.95	65	128	239	490	1710
				0.05	156.94	136.55	5	25	59	115	210	430	1209
				0.1	119.03	98.59	3	23	49	90	157	315	931
				0.2	66.77	48.04	3	19	33	53	85	161	470
				0.25	52.63	34.81	5	17	28	43	66	123	335
				0.5	24.75	10.14	4	13	18	22	29	44	107
				0.75	17.06	5.18	3	11	14	16	20	27	55
				1	13.39	3.17	1	9	11	13	15	19	37
				1.5	9.78	1.76	3	7	9	10	11	13	24
				2	7.9	1.19	4	6	7	8	9	10	17
				2.5	6.72	0.88	3	6	6	7	7	8	14
				3	5.92	0.7	3	5	5	6	6	7	11
				4	4.85	0.52	2	4	5	5	5	6	9
				5	4.09	0.35	2	4	4	4	4	5	7

Table 12

δ	t_4			Logistic			Laplace		
	EV	MEC	MCE	EV	MEC	MCE	EV	MEC	MCE
0	369.47	371.42	370.31	369.64	367.95	371.58	368.62	367.68	369.8
0.25	72.32	71.12	140.78	73.04	71.97	116.48	73.11	72.36	130.12
0.5	34.37	32.06	39.54	34.91	32.16	34.67	34.94	32.22	37.31
0.75	23.4	21.88	18.02	23.69	22.01	16.85	23.7	22	17.63
1	17.95	17.14	11.63	18.17	17.29	11.09	18.17	17.32	11.47
1.5	12.29	12.52	7.14	12.47	12.61	6.83	12.47	12.71	7.04
2	9.3	10.22	5.21	9.44	10.29	4.97	9.45	10.26	5.13
3	6.32	7.66	3.44	6.44	7.71	3.27	6.44	7.72	3.38
4	4.87	6.28	2.56	4.95	6.32	2.4	4.96	6.31	2.5
5	4.05	5.35	2.04	4.11	5.39	1.96	4.1	5.4	2.02

Table 13

δ	Lognormal			Gamma		
	EV	MEC	MCE	EV	MEC	MCE
0	374.64	370.71	372.5	370.06	373.08	375.73
0.25	74.77	74.17	177.84	73.86	73.08	104.55
0.5	33.83	31.21	65	35.03	32.71	36.52
0.75	22.79	20.95	25	23.64	22.11	17.81
1	17.35	16.44	14.6	18.29	17.31	11.43
1.5	11.9	12.04	8.55	12.45	12.66	6.92
2	8.96	9.74	6.23	9.46	10.3	5.06
3	6.1	7.34	4.15	6.42	7.72	3.29
4	4.71	5.92	3.01	4.94	6.32	2.44
5	3.89	4.99	2.58	4.09	5.43	1.92

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