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Optimal production policy for a closed-loop supply chain with stochastic lead time and learning in production

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KEYWORDS
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Remanufacturing.

Abstract. This paper considers a closed-loop supply chain with one manufacturer and one retailer for trading a single product. On behalf of the manufacturer, the retailer collects the used items from the end customers for possible remanufacturing. The production of finished products (manufactured and remanufactured) is subject to learning. The lead time for the retailer is assumed to be stochastic. The manufacturer delivers the retailer's order quantity in a number of equal-sized shipments. The objective is to determine the optimal number of shipments and shipment size by minimizing the average expected total cost of the closed-loop supply chain. A solution method for the model is presented, and important results are obtained for numerical examples. According to the numerical study, an impressive cost reduction due to consideration of learning in production and remanufacturing is observed. To investigate the impact of key model parameters on the optimal results, a sensitivity analysis is also carried out. The proposed model is applicable to those business firms whose production process is executed by the human beings.

1. Introduction

Manufacturing industries all over the world are nowadays giving greater importance to remanufacturing of used items to protect the environment and the society from being over-polluted. Several companies take up the recycling process to add to their organization some value from recovery, too. In electronic industries, many products collected at the end of their useful life may have components with intrinsic economic value. In the steel manufacturing industry, production cost could be reduced by mingling the metals obtained from collected used products with the virgin raw materials. However, the collection of used products from the market and their reinsertion into the downstream flow of materials form closed-loop supply chains. Sometimes, the used products with good quality are returned for new ones to upgrade purposes with some new features or advanced technology; sometimes, these are left at the end of life in poor conditions. Therefore, there is a need for inspection, selection, or a sorting process in order to determine the quality of used items acceptable for recycling. There are plenty of ways in which used items are returned from end customers. The manufacturer can collect the used products through his own channel, and a third party can be appointed for this collection, or even the retailer can be in charge of collecting them on behalf of the manufacturer.

In competitive business environment, most of the production runs are conducted by machines to save time, energy, or achieve errorless production. However, there are also some products that cannot be designed only by a machine; a human mind is required. For

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example, consider manufacturing of some sophisticated garment products or leather goods where human involvement is very much needed. When a human being is included in a manufacturing system, the erroneous (learning-forgetting) nature is also included there. Learning nature can be seen in cases of repetitive jobs. When the number of repetition increases, workers become more confident and spend less time doing their jobs. The human characteristic 'learning' was well defined by Wright [1] with a curve known as learning curve. According to the theory, it can be stated that when the produced quantity doubles, a certain percentage of cost reduction takes place. In the literature, learning is mostly considered in production in the forward channel. What is the impact of worker learning in the production of finished products (manufactured and remanufactured) in a closed-loop supply chain? How is the situation handled if the delivery lead time from the manufacturer to the retailer is stochastic in a closed-loop scenario? In practice, the delivery lead time cannot be always known as constant; it may vary due to many reasons such as variable transportation time, production time, or loading/unloading time. To find the answers to the above questions, in this paper, stochastic delivery lead time is considered, and its effect jointly with worker learning in production on the performance of the associated closed-loop supply chain is investigated.

2. Literature review

In this section, the literature on two key topics viz. variable/stochastic lead time and learning in production is briefly reviewed. Liao and Shyu [2] and Ben-Daya and Raouf [3] proposed inventory models where lead time was assumed to be a decision variable. Ben-Daya and Hariga [4] discussed variable lead time due to production time, transferring time, setup time, etc. Ouyang et al. [5], Hoque and Goyal [6], and Mandal and Giri [7] controlled the lead time with some additional costs. Lin [8] considered variable lead time that can be reduced by investment. Taleizadeh et al. [9] assumed changeable lead time, which is linearly dependent on the lot size. Several other studies were also conducted by Taleizadeh and his coauthors [10-12] considering multi-product single/multi-constraint inventory situations, stochastic replenishment, and dynamic/fuzzy demands.

Yano [13] developed an inventory model with stochastic lead time in a two-level assembly system with the objective of minimizing the sum of the holding cost and tardiness cost. Ouyang et al. [14] extended Yano's [13] model allowing shortages in inventory. Sajadieh et al. [15] determined an optimal policy for an integrated vendor-buyer model with stochastic lead time. They allowed shortages and assumed batches of equal sizes. Hoque [16] extended the model of Sajadieh et al. [15], assuming unequal batch deliveries from the vendor to the buyer. He considered the lead time distribution as normal instead of exponential distribution adopted by Sajadieh et al. [15]. Maity et al. [17] developed a model with probabilistic lead time and shortages. They assumed a practical pricing decision that can be much effective in real life. Isotupa and Samanta [18] obtained a model with stochastic lead time, which follows an Erlang distribution. Disney et al. [19] ignored the normal distribution of lead time and presented an optimal condition with greater generalization. Roldan et al. [20] presented a survey on inventory-routing problems with stochastic lead time and demand.

Learning is an important human factor that has attracted the attention of many researchers throughout the last few decades. Wright [1] was the first to apply the learning curve to industrial problems. The workers, the employees, even the managers who do the same work repeatedly can automatically do their subsequent jobs with a comparatively better speed. Learning curve is a curve where time per repetition decreases as the number of repetitions increases. Yelle [21] provided a comprehensive survey of works on learning curve. Like learning, forgetting is also an important human factor. Someone who learns in the production period can forget some portion in the non-production period. Elmaghraby [22] and Jaber and Bonney [23] were among the first authors who observed the forgetting phenomenon along with learning. Jaber and Bonney [23] made a comparison of their model with the forgetting model developed by Elmaghraby [22] and Carlson and Rowe [24], and found that their proposed model was more efficient than others. Chiu et al. [25] studied learning and forgetting in the case of production and, also, setup. Khan et al. [26] extended Huang's [27] model considering learning in production and error in inspections. Burr and Pearne [28], Teng et al. [29], Van Hoof [30], and Grosse et al. [31] also studied the learning process. Teyarachakul et al. [32] assumed forgetting curve and compared larger and shorter non-production periods. For a shorter non-production period, the rate of forgetting is low and, also, the batch size reduces. They found optimal policy that implies whether the smaller batch size is more effective or the bigger one. Glock and Jaber [33] proposed a model for an imperfect production process in which items that were not good in quality were reproduced. In both production and reproduction processes, learning and forgetting both were involved. Lolli et al. [34] highlighted the learning and forgetting curve with imperfect production and rework for imperfect items like Glock and Jaber [33]. Later, Jaber and Givi [35] assumed that the production interruption and the forgetting occurred at the break time due to a change in the production and rework processes. Recently, Giri and Glock [36] studied learning and forgetting effects in production as well as inspection. They considered stochastic production return, and observed that more learning effect caused recovery of more used items.

In a Closed-Loop Supply Chain (CLSC), Clark and Scarf [37] first proposed multi-echelon inventory system. Guide and Wassenhove [38] developed a model to investigate profits that can be made by remanufacturing through reusable materials, which can help change many business policies made by different decision-makers in business management. Majumdar and Groenevelt [39] presented a two-period model in the context of recycling with a competing face. In their model, two vendors (one original and the other local) compete in the market for reusable items. It was shown that the local vendor gave motivations for reducing the remanufacturing cost. Koh et al. [40] assumed a model where a fixed portion of used items as remanufactured and the remanufactured items are exactly the same as the freshly manufactured products. Savaskan et al. [41] considered a model where the manufacturer determines how to take the used items from the consumer for remanufacturing. There were three possible choices: manufacturer himself, retailer, and third party. It was shown that the most fruitful way is to assemble the wastage by the retailer. Chung et al. [42] provided some effective policy for remanufacturing with the used items. Jaber and El Saadany [43] assumed that the demands for manufactured and remanufactured items are different. Hong et al. [44] proposed a model where the used items were collected by the manufacturer, the retailer, and the collector (third party) but two of them collected at the same time in all possible ways. De Giovanni and Zaccour [45] made a comparison of different ways of returning used products due to environmental consciousness. Naeem et al. [46] discussed the recycling process where demand and return rate were taken as constant and variable both. Giri and Sharma [47] assumed return rate based on product quality. They [48] developed a closed-loop supply chain model with stochastic demand and random return. Cobb [49] proposed a model with an uncertain return rate. Jena et al. [50] proposed a CLSC model with uncertain return rates where advertising plays a vital role in different cases such as advertising by manufacturer, advertising by retailer, centralized advertising, decentralized advertising, etc. Masoudipour et al. [51] also discussed a bi-objective model with a quality-based return rate in a closed-loop supply chain system.

Table 1 shows a comparison of the present model and the relevant existing models. It should be noticed that although significant works have been done considering variable/stochastic lead time and learning/forgetting in production, most of them are bound in EOQ/EPQ model with an integrated or non-integrated system where the manufacturer is not allowed to produce items using the refused ones. Recently, Angius et al. [52] analyzed stochastic lead time in a closed-loop supply chain with a Markovian approach. However, learning in production along with the stochastic lead time has not yet been considered in CLSC. To fill the research gap, this paper studies learning in the production process in CLSC with stochastic lead time. The rest of the paper is organized as follows. Section 3 presents problem description and modeling assumptions. In Section 4, notations are given, which are used throughout the paper. Section 5 deals with the formulation and development of the model. Section 6 exhibits numerical analysis which ends in sensitivity analysis of some important model parameters. The paper is concluded with some managerial insights and future research directions in Section 7.

3. Problem description

A closed-loop supply chain, which consists of a manufacturer and a retailer, is considered. The manufacturer produces a single product in lots and delivers each lot in n batches to the retailer. On behalf of the manufacturer, the retailer collects the used items from the end customers and supplies them to the man-

Table 1. Comparison between the existing models and our proposed model.

Authors	CLSC	Lead time	Learning in production	Inspection
Savaskan et al. [41]	Yes	No	No	No
Hoque and Goyal [6]	No	Controllable	No	No
Hoque [16]	No	Stochastic	No	No
Glock and Jaber [33]	No	No	Yes	No
Khan et al. [26]	No	No	Yes	Yes
Disney et al. [19]	No	Stochastic	No	No
Angius et al. [52]	Yes	Stochastic	No	No
Giri and Glock [36]	Yes	No	Yes	Yes
This paper	Yes	Stochastic	Yes	Yes

ufacturer for possible remanufacturing. However, the manufacturer bears the purchase cost and holding cost of returned items. Besides, the following assumptions are made:

(i) The return rate of used items is influenced by the investment in collection and product demands. Here, the investment implies the economic amount of effort, which is applied to the end users to add incentive in order to receive high return;

(ii) The collected used items are inspected. A fraction β ($0 < \beta \leq 1$) of inspected used items that qualifies for remanufacturing is accepted and returned to the manufacturer for remanufacturing;

(iii) The demand rate and the production rate of the finished product are both constant. The production rate is greater than the demand rate;

(iv) Delivery lead time is stochastic and independent of deliveries to the retailer;

(v) The retailer's inventory is continuously reviewed, and a batch is requested to deliver when the stock level drops to a certain level;

(vi) Shortages in the retailer's inventory are allowed and are completely backlogged;

(vii) A joint setup is made for manufacturing and remanufacturing. Remanufactured products are as good as manufactured products, i.e., it is not possible to distinguish between manufactured and remanufactured products [37];

(viii) One unit of raw material is required to produce one unit of the finished good;

(ix) Learning occurs during the production of the finished goods (manufactured and remanufactured). Learning leads to an increase in the rate of production over time and depends on the length of the production interval.

A configuration of the proposed model is depicted in Figure 1, which illustrates the flow of products throughout the closed-loop supply chain. The return rate of used items is a fraction of demand, which is influenced by the investment. A fraction of the returned items is found acceptable for remanufacturing. The rejected items are disposed with a disposal cost.

4. Notations

In this section, the notations used to develop the proposed model are described.

Manufacturer:

c_m	Production cost per unit time
A_m	Setup cost per set up
P	Production rate
h_m	Holding cost of the finished product per unit per unit time
h_{mu}	Holding cost of returned items per unit per unit time
c_{mr}	Unit purchase cost of raw material
c_{mu}	Unit cost for purchase and recovery of used items selected for remanufacturing
c_{md}	Unit disposal cost
β	Fraction of used items acceptable for remanufacturing, $0 < \beta \leq 1$
n	Number of shipments

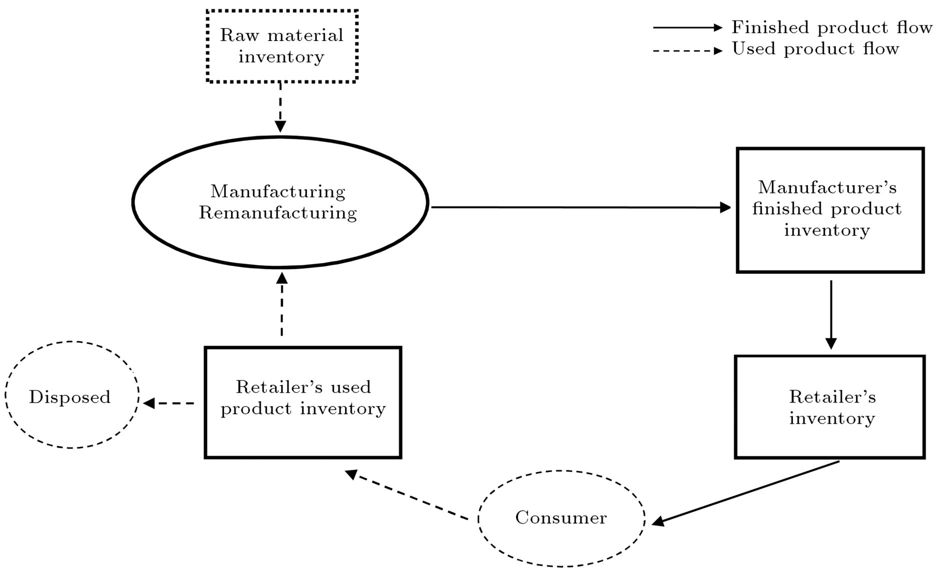

Figure 1. Product flow diagram.

b	Learning exponent, $(0 < b < 1)$
$T_1(=1/P)$	Manufacturer's production time to produce the first unit in case of learning
$T_{pi}(T_{di})$	Manufacturer's production (non-production) period in cycle i
TCm_i	Total cost per unit time for the ith cycle
R	Return rate of used items

Retailer

D	Demand rate
A_r	Ordering cost per order
h_r	Holding cost per unit per unit time
s	Shortage cost per unit per unit time
T_r	Length of an ordering cycle
TC_r	Total cost per unit time
Q	Batch size
L	Lead time as a random variable
σ	Standard deviation of lead time
$f_L(l)$	Probability density function of L
k	Reorder level

5. Model formulation and analysis

It is assumed that the manufacturer produces and delivers a lot of size nQ to the retailer in n equal-sized batches of size Q. Since the lead time is random, the retailer requests the manufacturer for a batch when the stock level drops to the reorder level k. Therefore, the next batch is expected to receive when the stock at the reorder point comes to zero level, i.e., after the mean lead time k/D [16]. The deviation of lead time may occur due to the variation of loading/unloading time. The used items collected by the retailer are inspected and, after inspection, only a fraction is accepted for remanufacturing. The manufacturer has to order that amount of fresh raw materials that cannot be met by the accepted used items. The manufacturer bears all the costs for holding, promotion for collection, and inspection of used items.

5.1. Retailer's total cost

When random variable L takes value l, the following three cases that may arise are considered:

- **Case (i):** *A batch Q arrives early for the retailer, i.e.,* $0 < l < k/D$. In this case, using Figure 2(a), the inventory holding area of the retailer for one batch can be calculated as given below:

  Inventory holding area

$$= \text{Area}(\square\text{OPNM} + \triangle\text{QRS} + \square\text{PTSQ})$$

$$= \frac{1}{2}(k-Dl+k)l+\frac{1}{2}(k-Dl+Q-k)\left(\frac{Q-Dl}{D}\right)$$

$$+\frac{(Q-Dl)k}{D} = \frac{1}{2}\left[\frac{Q^2}{D}+2Q\left(\frac{k}{D}-l\right)\right],$$

  where:

$$k=\frac{DQ}{P}.$$

  Thus, for the entire lot, Q_p, of the manufacturer, the expected inventory holding cost of the retailer is computed as follows:

$$h_r\int_0^{k/D}\frac{n}{2}\left[\frac{Q^2}{D}+2Q\left(\frac{k}{D}-l\right)\right]f_L(l)dl.$$

- **Case (ii):** *A batch Q arrives late for the retailerlimiting the lead time l in the range* $\frac{k}{D} \le l < \frac{k+Q}{D}$. In this case, shortages may occur. Based on Figure 2(b), the inventory shortage area for a single batch Q is obtained as $\frac{1}{2D}(Dl-k)^2$. Hence

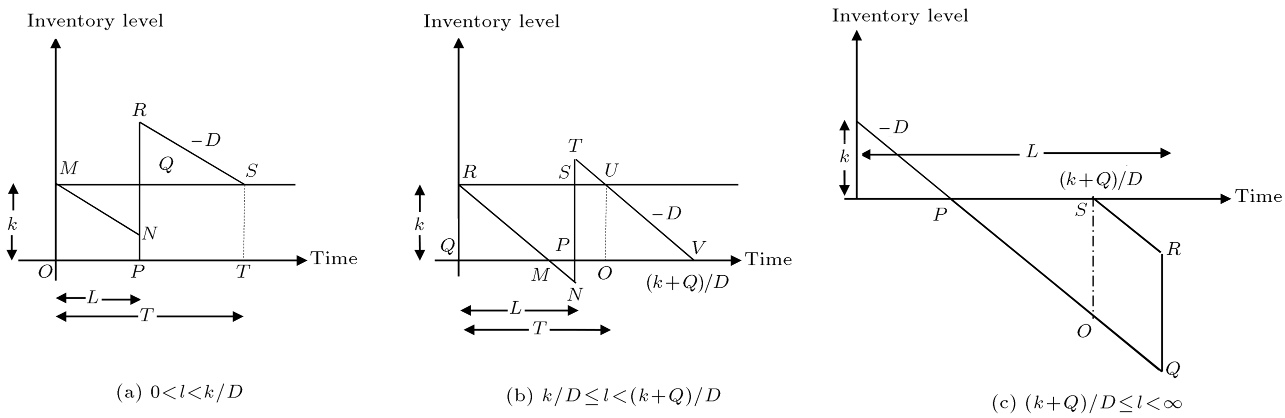

Figure 2. Retailer's inventory under stochastic lead time.

the retailer's expected shortage cost for all batches is as follows:

$$\frac{ns}{2}\int\limits_{k/D}^{\frac{k+Q}{D}}\left(\frac{Dl-k}{D}\right)(Dl-k)f_L(l)dl.$$

Similar to Case (i), the inventory holding area for one batch Q is obtained as follows:

$$\text{Area }(\triangle\text{QMR}+\triangle\text{STU}+\square\text{POUS})$$

$$=\left[\frac{k^2}{2D}+\frac{(Q-Dl)^2}{2D}+\frac{k(Q-Dl)}{D}\right]$$

$$=\frac{1}{2D}(Q-Dl+k)^2.$$

Therefore, the expected inventory holding cost of the retailer for all batches is as follows:

$$nh_r\int\limits_{k/D}^{\frac{k+Q}{D}}\frac{1}{2D}(Q-Dl+k)^2f_L(l)dl.$$

- **Case (iii):** *A batch Q arrives late for the retailer limiting the lead time l in the range $\frac{k+Q}{D}\leq l<\infty$.* In this case, there is no inventory, except only shortages (see Figure 2(c)). The shortage area for one batch Q is given by:

$$\text{Area }(\square\text{PQRS})=\frac{Q^2}{2D}+\text{Area }(\square\text{OQRS})$$

$$=\frac{Q^2}{2D}+Q\left(l-\frac{Q+k}{D}\right).$$

Therefore, the expected shortage cost for all batches is:

$$ns\int\limits_{\frac{k+Q}{D}}^{\infty}\left[Ql-\frac{Q^2}{2D}-\frac{Qk}{D}\right]f_L(l)dl.$$

Considering all the three cases, the retailer's expected inventory holding cost for all batches is:

$$nh_r\left[\frac{1}{2}\int\limits_{0}^{k/D}\left(\frac{Q^2}{D}+2Q\left(\frac{k}{D}-l\right)\right)f_L(l)dl\right.$$

$$\left.+\int\limits_{k/D}^{\frac{k+Q}{D}}\frac{1}{2D}(Q-Dl+k)^2f_L(l)dl\right],$$

and the expected shortage cost for all batches is:

$$ns\left[\int\limits_{k/D}^{\frac{k+Q}{D}}\frac{1}{2}\left(\frac{Dl-k}{D}\right)(Dl-k)f_L(l)dl\right.$$

$$\left.+\int_{\frac{k+Q}{D}}^{\infty}\left(Ql-\frac{Q^2}{2D}-\frac{Qk}{D}\right)f_L(l)dl\right].$$

Hence, the expected total cost of the retailer for one cycle is:

$$TC_r=A_r+\frac{nh_r}{2}\left[\int\limits_{0}^{k/D}\left(\frac{Q^2}{D}+2Q\left(\frac{k}{D}-l\right)\right)f_L(l)dl\right.$$

$$\left.+\int\limits_{k/D}^{\frac{k+Q}{D}}\frac{1}{D}(Q-Dl+k)^2f_L(l)dl\right]$$

$$+ns\left[\int\limits_{k/D}^{\frac{k+Q}{D}}\frac{1}{2}\left(\frac{Dl-k}{D}\right)(Dl-k)f_L(l)dl\right.$$

$$\left.+\int\limits_{\frac{k+Q}{D}}^{\infty}\left(Ql-\frac{Q^2}{2D}-\frac{Qk}{D}\right)f_L(l)dl\right]. \quad (1)$$

5.2. Manufacturer's cost

5.2.1. Change in holding cost

The manufacturer's change in holding cost due to random delivery lead time is first calculated. In Case (ii), i.e., when batch Q arrives late by a time $l-k/D$ limiting l to the range $\frac{k}{D}\leq l<\frac{k+Q}{D}$, the batch is kept with the manufacturer over the delay period and it creates an extra inventory of $nQ(l-k/D)$. Thus, in this case, the expected additional inventory holding cost of the manufacturer is:

$$h_m\int\limits_{k/D}^{\frac{k+Q}{D}}nQ(l-k/D)f_L(l)dl.$$

In Case (iii), i.e., when batch Q arrives late by a time $l-k/D$ limiting l to the range $\frac{k+Q}{D}\leq l<\infty$, the batch is kept with the manufacturer over the delay period and it creates an extra inventory of $nQ(l-k/D)$. Thus, the expected additional inventory holding cost of the manufacturer, in this case, is as follows:

$$h_m\int\limits_{\frac{k+Q}{D}}^{\infty}nQ(l-k/D)f_L(l)dl.$$

Therefore, the expected net change in the manufacturer's holding cost due to stochastic delivery lead time at the retailer is:

$$h_m\left[\int_{\frac{k}{D}}^{\infty} nQ(l-k/D)f_L(l)dl\right].$$

5.2.2. Learning in production

Since learning in production has impact on production time, in general, the ith production cycle is considered. The production time in the ith cycle [26] is given by:

$$Tp_i=\int_{(i-1)Q_p}^{iQ_p} T_1x^{-b}dx=\frac{T_1}{1-b}Q_p^{1-b}\left[i^{1-b}-(i-1)^{1-b}\right]. \tag{2}$$

Based on the above, the production quantity in the ith cycle can be written as a function of time, t, as follows:

$$Q(t)=\left[\frac{(1-b)t}{T_1}\frac{1}{i^{1-b}-(i-1)^{1-b}}\right]^{\frac{1}{1-b}}. \tag{3}$$

Then, the average inventory in the ith cycle during production is as follows:

$$ITp_i=\int_0^{Tp_i}Q(t)dt=\frac{T_1}{2-b}\left[i^{1-b}-(i-1)^{1-b}\right](nQ)^{(2-b)}. \tag{4}$$

Therefore, the time for the first dispatch after the start of production in the ith cycle is as follows:

$$T_{1i}=\int_{(i-1)nQ}^{Q+(i-1)nQ} T_1x^{-b}dx$$
$$=\frac{T_1}{1-b}Q^{1-b}\left[(1+(i-1)n)^{1-b}-((i-1)n)^{1-b}\right]. \tag{5}$$

Now, the manufacturer's inventory in the non-production period of the ith cycle is calculated based on Figure 3 [26] as follows:

$$ITd_i=\text{Area }(\square\text{ABFE}+\square\text{BIHF}-\square\text{ACGE})$$
$$=\frac{nT_1}{1-b}Q^{2-b}\left[(1+(i-1)n)^{1-b}-((i-1)n)^{1-b}\right]$$
$$+\frac{n(n-1)Q^2}{D}-\frac{T_1(i^{1-b}-(i-1)^{1-b})}{2-b}(nQ)^{2-b}. \tag{6}$$

The total inventory moved from the manufacturer to the retailer in a cycle is $\frac{n(n-1)Q^2}{2D}$. Thus, the manufacturer's average inventory in the ith cycle is:

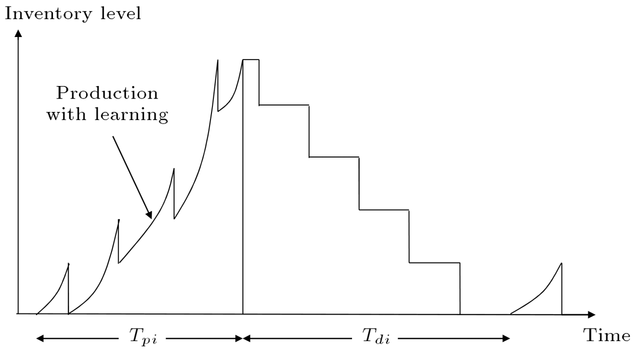

Figure 3a. Manufacturer's inventory level under learning.

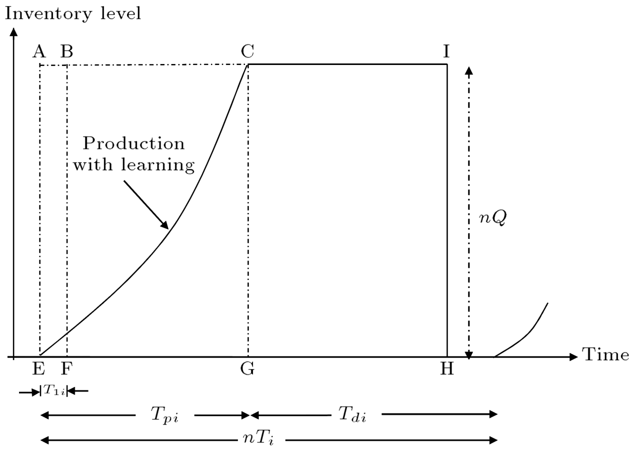

Figure 3b. Manufacturer's total inventory under learning.

$$ITp_i+ITd_i-\frac{n(n-1)Q^2}{2D}$$
$$=\frac{nT_1}{1-b}Q^{2-b}\left[(1+(i-1)n)^{1-b}-((i-1)n)^{1-b}\right]$$
$$+\frac{n(n-1)Q^2}{2D}-\frac{T_1(i^{1-b}-(i-1)^{1-b})}{(2-b)(1-b)}(nQ)^{2-b}. \tag{7}$$

Therefore, considering the result of Subsection 5.2.1, the manufacturer's expected holding cost for the ith cycle is:

$$h_m\left[\frac{nT_1Q^{2-b}((1+(i-1)n)^{1-b}-((i-1)n)^{1-b})}{1-b}\right.$$
$$+\frac{n(n-1)Q^2}{2D}-\frac{T_1(i^{1-b}-(i-1)^{1-b})}{(2-b)(1-b)}(nQ)^{2-b}$$
$$\left.+\int_{k/D}^{\infty}nQ(l-k/D)f_L(l)dl\right].$$

Hence, for the ith cycle, the manufacturer's expected total cost, which is the sum of the setup cost, production cost, and expected holding cost, is as follows:

$$TC_{mi} = A_m + c_m T p_i + h_m \left[\int_{k/D}^{\infty} nQ(l - r/D) f_L(l) dl + \frac{nT_1 Q^{2-b}}{1-b}((1+(i-1)n)^{1-b} - ((i-1)n)^{1-b}) + \frac{n(n-1)Q^2}{2D} - \frac{T_1(i^{1-b} - (i-1)^{1-b})}{(2-b)(1-b)}(nQ)^{2-b} \right]. \quad (8)$$

5.2.3. Cost for used items

It is assumed that the used items are collected by the retailer on behalf of the manufacturer at a rate, R, which is influenced by the collection investment and demand. The Collection Investment (CI) indicates the economic amount of effort (e.g., promotion, marketing, etc.), which is applied to the end users to create the necessary incentive to receive targeted return. It is assumed that $R = \sqrt{\frac{CI}{\gamma}} D$ where γ is a scaling parameter and $\sqrt{\frac{CI}{\gamma}} < 1$. When the collected items are used for remanufacturing, the inventory level of the collected items after time t is given by:

$$I(t) = n\beta R T r_i - Q(t) = \sqrt{\frac{CI}{\gamma}}(n\beta Q) - Q(t). \quad (9)$$

If the stock of the collected used items is exhausted after a time period T_2, then we have $I(T_2) = 0$. This gives:

$$Q(T_2) = \sqrt{\frac{CI}{\gamma}}(n\beta Q), \quad (10)$$

which further gives:

$$T_2 = \left[\sqrt{\frac{CI}{\gamma}}(n\beta Q) \right]^{1-b} \left[\frac{T_1}{1-b} [i^{1-b} - (i-1)^{1-b} \right]. \quad (11)$$

Using Figure 4, the inventory holding area for collected used items is calculated as follows:

Area $(\triangle ABC + EFC)$

$$= \frac{1}{2} R n T r_i (n T r_i) + \int_0^{T_2} I(t) dt$$

$$= \frac{1}{2D} \sqrt{\frac{CI}{\gamma}} n^2 Q^2 + \int_0^{T_2} I(t) dt.$$

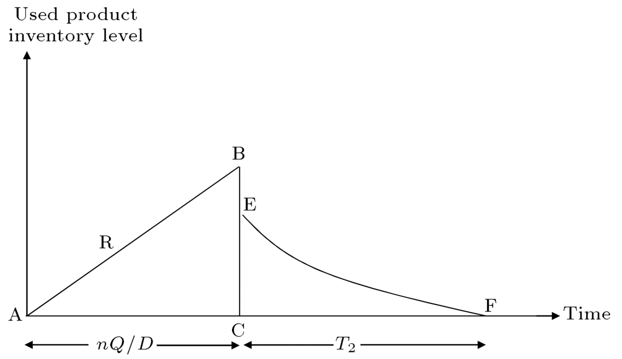

Figure 4. Used product inventory.

Therefore, the total cost for remanufacturing is:

$$TC_{mu} = h_{mu} \left[\frac{1}{2D} \alpha n^2 Q^2 + \int_0^{T_2} I(t) dt \right] + c_{mu} \alpha n Q,$$

$$\alpha = \sqrt{\frac{CI}{\gamma}}. \quad (12)$$

5.3. Total cost of the CLSC

By combining the retailer's cost and the manufacturer's cost as derived from Subsections 5.1 and 5.2, respectively, the average expected cost of the closed-loop supply chain for the ith cycle is given by:

$$TC_i(n,Q) = \frac{1}{nTr_i}[TC_r + TC_{mi} + c_{mr}(1-\alpha\beta)nQ + c_{md}(1-\beta)\alpha n Q + TC_{mu}] = \frac{1}{nTr_i} \left[A_r + \frac{nh_r}{2} \left(\int_0^{k/D} \left(\frac{Q^2}{D} + 2Q\left(\frac{k}{D} - l\right) \right) f_L(l) dl + \int_{k/D}^{\frac{k+Q}{D}} \frac{1}{D}(Q - Dl + k)^2 f_L(l) dl \right) + ns \left(\int_{k/D}^{\frac{k+Q}{D}} \frac{1}{2}\left(\frac{Dl-k}{D}\right)(Dl-k) f_L(l) dl + \int_{\frac{k+Q}{D}}^{\infty} \left(Ql - \frac{Q^2}{2D} - \frac{Qk}{D} \right) f_L(l) dl \right) + A_m \right.$$

$$+c_mTp_i+h_m\left(\int_{k/D}^{\infty} nQ(l-k/D)f_L(l)dl\right.$$

$$+\frac{nT_1Q^{2-b}}{1-b}((1+(i-1)n)^{1-b}-((i-1)n)^{1-b})$$

$$\left.+\frac{n(n-1)Q^2}{2D}-\frac{T_1(i^{1-b}-(i-1)^{1-b})}{(2-b)(1-b)}(nQ)^{2-b}\right)$$

$$+h_{mu}\left(\frac{1}{2D}\alpha n^2Q^2+\int_0^{T_2} I(t)dt\right)+c_{mu}\alpha nQ$$

$$\left.+c_{mr}(1-\alpha\beta)nQ+c_{md}(1-\beta)\alpha nQ\right]. \quad (13)$$

Our objective is to find the optimal values of n and Q that minimize the average expected total cost $TC_i(n,Q)$, $i=1,2,\cdots$. Since n is discrete and Q is continuous, it is not possible to apply the calculus method to optimize $TC_i(n,Q)$ with respect to n and Q jointly. First, it should be assumed that n is real, not just an integer. Then, the following results are presented:

Theorem 1. *The average expected total cost function $TC_i(n,Q)$ is convex in n for any given Q, $i=1,2,3,\cdots$.*

Proof: Differentiating Eq. (13) with respect to n, we have:

$$\frac{\partial TC_i}{\partial n}=-\frac{A_r+A_m+c_mT_{pi}}{n^2T_{ri}}+\frac{h_mQ^2}{2DT_{ri}}$$
$$+\frac{h_{mu}\alpha Q^2}{2DT_{ri}}+\frac{h_mT_1Q^{2-b}}{T_{ri}}$$
$$[(1+(i-1)n)^{-b}(i-1)$$
$$-(i-1)^{1-b}n^{-b}]$$
$$-\frac{h_mT_1(i^{1-b}-(i-1)^{1-b})}{(2-b)T_{ri}}n^{-b}Q^{2-b}$$
$$-\frac{h_{mu}}{n^2T_{ri}}\int_0^{T_2}\left(\frac{(1-b)Pt}{i^{1-b}-(i-1)^{1-b}}\right)^{\frac{1}{1-b}}dt. \quad (14)$$

Differentiating again with respect to n, we get:

$$\frac{\partial^2 TC_i}{\partial n^2}=\frac{2(A_r+A_m+c_mT_{pi})}{n^3T_{ri}}$$
$$+\frac{2h_{mu}}{n^3T_{ri}}\int_0^{T_2}\left(\frac{(1-b)Pt}{i^{1-b}-(i-1)^{1-b}}\right)^{\frac{1}{1-b}}dt$$
$$+\frac{bh_mT_1Q^{2-b}}{T_{ri}}\left[(i-1)^{1-b}n^{-b-1}\right.$$
$$\left.-(1+(i-1)n)^{-b-1}(i-1)^2\right]$$
$$+\frac{bh_mT_1(i^{1-b}-(i-1)^{1-b})}{2-b}n^{-b-1}Q^{2-b}$$
$$=\frac{2(A_r+A_m+c_mT_{pi})}{n^3T_{ri}}$$
$$+\frac{2h_{mu}[\alpha\beta(nQ)]^{2-b}}{n^3T_{ri}(2-b)}\left[\frac{i^{1-b}-(i-1)^{1-b}}{P}\right]$$
$$+\frac{bh_mT_1Q^{2-b}}{T_{ri}}\left[(i-1)^{1-b}n^{-b-1}\right.$$
$$\left.-(1+(i-1)n)^{-b-1}(i-1)^2\right]$$
$$+\frac{bh_mT_1(i^{1-b}-(i-1)^{1-b})}{2-b}n^{-b-1}Q^{2-b}, \quad (15)$$

since:

$$\frac{2h_{mu}}{n^3T_{ri}}\int_0^{T_2}\left(\frac{(1-b)Pt}{i^{1-b}-(i-1)^{1-b}}\right)^{\frac{1}{1-b}}dt$$
$$=\frac{2h_{mu}}{n^3T_{ri}}\left(\frac{(1-b)P}{i^{1-b}-(i-1)^{1-b}}\right)^{\frac{1}{1-b}}\frac{1-b}{2-b}$$
$$\left[[\alpha\beta(nQ)]^{1-b}\frac{T_1}{1-b}[i^{1-b}-(i-1)^{1-b}]\right]^{\frac{2-b}{1-b}}$$
$$=\frac{2h_{mu}[\alpha\beta(nQ)]^{2-b}}{n^3T_{ri}(2-b)}\left[\frac{i^{1-b}-(i-1)^{1-b}}{P}\right]. \quad (16)$$

Since $0<b<1$ and based on Eq. (15), the third term is positive as $((i-1)n)^{-b-1}>(1+(i-1)n)^{-b-1}$ and each of the second and fourth terms is positive as $i^{1-b}>(i-1)^{1-b}$ for all i. Therefore, $\frac{\partial^2 TC_i}{\partial n^2}>0$, implying that $TC_i(n,Q)$ is convex in n.

Theorem 2. *If $h_m>h_r/2$, then the average expected cost function $TC_i(n,Q)$ is convex with respect to Q for all $Q<Q_i^0$ where Q_i^0 is obtained by the equation shown in Box I.*

$$Q_i^0 = \frac{(b+1)c_m n^{-b}\frac{i^{1-b}-(i-1)^{1-b}}{(1-b)}}{h_m((1+(i-1)n)^{1-b}-((i-1)n)^{1-b})+\frac{(i^{1-b}-(i-1)^{1-b})}{2-b}\left[h_{mu}(\alpha\beta)^{2-b}n^{1-b}-h_m n^{-b}\right]},$$

$$i=1,2,3,\cdots.$$

Box I

Proof: Differentiating Eq. (13) twice with respect to Q, we get:

$$\frac{\partial^2 TC_i}{\partial Q^2} = \frac{2D}{nQ^3}(A_r+A_m)+\frac{\exp^{\frac{-Q^2}{2\sigma^2 D^2}}}{\sqrt{2\pi}\sigma}\left(\frac{h_m}{D}-\frac{h_r}{2D}\right)$$
$$+b(b+1)c_m DT_1 n^{-b}Q^{-b-2}\frac{i^{1-b}-(i-1)^{1-b}}{(1-b)}$$
$$-bh_m DT_1 Q^{-b-1}((1+(i-1)n)^{1-b}-((i-1)n)^{1-b})$$
$$+bh_m DT_1 Q^{-b-1}n^{-b}\frac{(i^{1-b}-(i-1)^{1-b})}{2-b}$$
$$+\int_{\frac{k}{D}}^{\frac{k+Q}{D}}\frac{(Dl-k)^2(h_r+s)}{Q^3}f_L(l)dl$$
$$-\frac{b}{2-b}h_{mu}DT_1(\alpha\beta)^{2-b}n^{1-b}Q^{-b-1}$$
$$(i^{1-b}-(i-1)^{1-b}). \quad (17)$$

The second term in the right-hand side of Eq. (17) is positive if $h_m > h_r/2$. The sixth term is positive as it is the sum of holding cost and shortage cost of retailer. The sum of the third, fourth, fifth, and seventh terms will be positive if:

$$(b+1)c_m n^{-b}Q^{-1}\frac{i^{1-b}-(i-1)^{1-b}}{(1-b)}$$
$$> h_m((1+(i-1)n)^{1-b}-((i-1)n)^{1-b})$$
$$+\frac{(i^{1-b}-(i-1)^{1-b})}{2-b}\left[h_{mu}(\alpha\beta)^{2-b}n^{1-b}-h_m n^{-b}\right],$$

i.e.: $Q < Q_i^0$, $i=1,2,3,\cdots$

Therefore, if $h_m > h_r/2$, then we see that $\frac{\partial^2 TC_i}{\partial Q^2} > 0$ for all $Q < Q_i^0$, $i=1,2,3,\cdots$.

Hence, the theorem is proved.

Solution algorithm: In the following, the step by step procedure is used to find the optimal values of n and Q for successive production cycles [26].

Step 1. Set $i=1$ and $n=1$;

Step 2. Find the optimal value of Q from (13) and, then, determine the corresponding value of TC_i. Set $n=n+1$;

Step 3. Repeat Step 2 until the value of TC_i for the nth batch is greater than that for the $(n-1)$th batch;

Step 4. Write $Q^* = Q$ and $TC_i^* = TC_i$ for $n^* = n-1$;

Step 5. Set $i=i+1$ and $n=1$;

Step 6. Repeat Steps 2 to 4 up to a certain value of i, say, $i=10$.

6. Numerical example

Now, it is time to demonstrate the proposed model numerically with the following parameter values: $c_m =$ \$100000 per year, $A_m =$ \$400 per set up, $h_m =$ \$4 per unit per year, $h_{mu} =$ \$3 per unit per year, $c_{mr} =$ \$200 per unit, $c_{mu} =$ \$50 per unit, $c_{md} =$ \$1 per unit, $b=0.32$, $R=\sqrt{\frac{CI}{\gamma}}D$ where $\sqrt{\frac{CI}{\gamma}}=0.65$, $\beta=0.8$, $P=3200$ units per year, $D=1000$ units per year, $A_r =$ \$100 per order, $h_r =$ \$5 per unit per year, and $s =$ \$8 per unit per year.

The probability density function of L is also assumed here:

$$f_L(l)=\frac{1}{\sqrt{2\Pi}\sigma}e^{-\frac{1}{2\sigma^2}(l-\frac{k}{D})^2},$$

where $\sigma = 0.12$.

For this data set, we check that the average expected cost function $TC_i(n,Q)$ is convex with respect to Q, see Figure 5.

For $i=1$, the optimal solution is obtained as $n^*=4$ and $Q^*=204.245$, and the corresponding average expected cost of the closed-loop supply chain is $TC =$ \$137195. Table 2 shows the optimal results of consecutive ten cycles.

One can see from Table 2 that there is an impressive cost reduction in the first few cycles. After that, the cost reduction slows down and the cost curve almost plateaus after 8 or 9 cycles. A similar observation is made in production time T_{pi}. This happens because of the human factor 'learning' in production. Initially, workers do their jobs faster than the previous

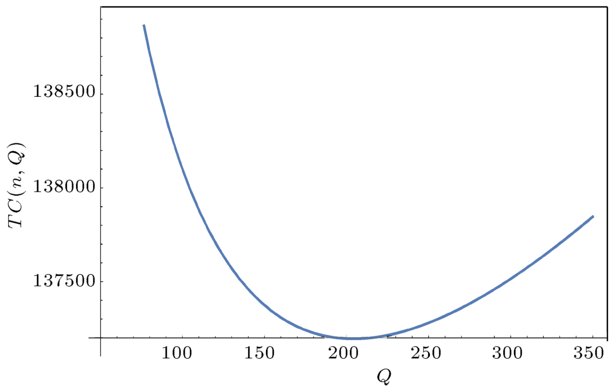

Figure 5. Convexity of $TC(n,Q)$ for the first cycle, given $n=4$.

Table 2. Optimal results of consecutive ten cycles.

i	n^*	Q^*	T^*_{pi}	TC^*_i
1	4	204.245	0.0439	137195
2	3	218.083	0.0228	134960
3	3	207.257	0.0186	134416
4	3	201.121	0.0163	134107
5	3	196.961	0.0148	133896
6	3	193.871	0.0137	133740
7	3	191.446	0.0129	133616
8	3	189.467	0.0123	133515
9	2	265.536	0.0112	133419
10	2	263.653	0.0108	133344

Table 3. Comparison of the results of the base model and the proposed model.

Model	n^*	Q^*	TC^*
Base model	4	115.1	162500
Proposed model	3	222.81#	134220.8#

average of the optimal results of the first 10 cycles.

cycle; however, after a few production cycles, their efficiency reaches a threshold level, and no significant improvement in their actions is observed. Table 3 presents a comparison of the optimal results of the proposed model and the base model, ignoring learning in production. As observed from Table 3, the proposed model with learning in production provides lower cost than the model without learning in production (base model).

Now, we take $\alpha = \sqrt{\frac{CI}{\gamma}}$ and discuss the effect of α on the average expected total cost. Table 4 shows the optimal results for different values of α. Here again, the average values of the results are calculated considering the first 10 cycles for each α.

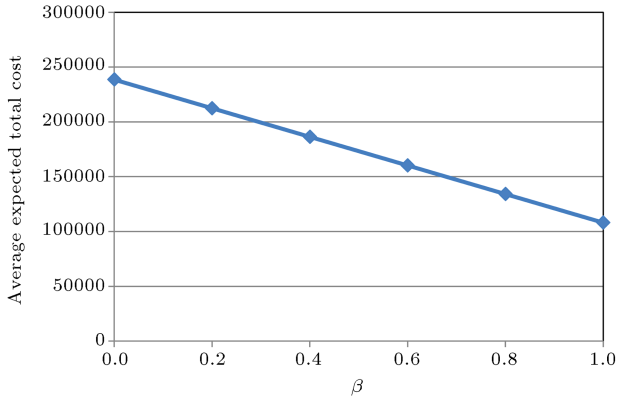

Figure 6. β versus average expected total cost.

Table 4. Optimal results for different values of α.

α	n^*	Average Q^*	Average TC^*
0.0	3	232.92	204917.4
0.25	3	224.13	177731.2
0.45	3	223.88	155966
0.65	3	222.81	134220.8
0.85	2	242.59	112394

Table 5. Sensitivity of the average expected cost with respect to P.

P	Average TC^*
3200	134220.8
4000	133571
5000	133067.4
6000	132730
7000	132488.8

When $\alpha = 0$, i.e., there is no returned item, the average expected cost is higher than that of the case of returned items. As α increases, the average expected cost decreases. This implies that it is beneficial for the manufacturer to collect the returned items as much as possible. Based on Figure 6, it is obvious that as β increases, the average expected total cost decreases. The average expected cost is maximum when $\beta = 0$, i.e., no used items are accepted for remanufacturing.

We now look into the ratio of P/D keeping P and D fixed alternatively. As the ratio P/D approaches 1, i.e., the demand rate is getting closer to the production rate, the average expected cost increases. Similarly, Figure 7 depicts that the average expected cost decreases steadily with the increasing value of P/D. Now, keeping D fixed, if $P = 4000$, 5000, 6000, and 7000 are considered, it is seen from Table 5 that the rate of change in the average expected cost is very small and gradually decreases. It ultimately becomes stable after a certain higher value of P.

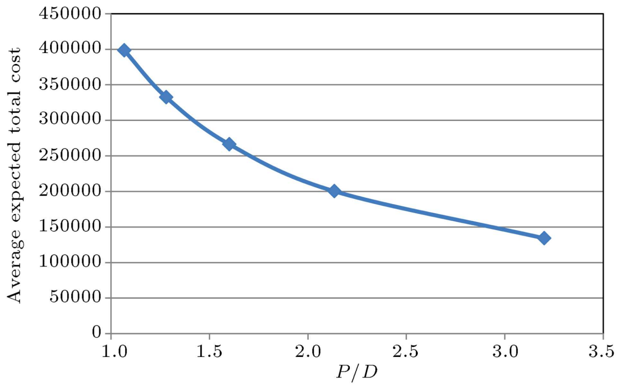

Figure 7. P/D versus average expected total cost.

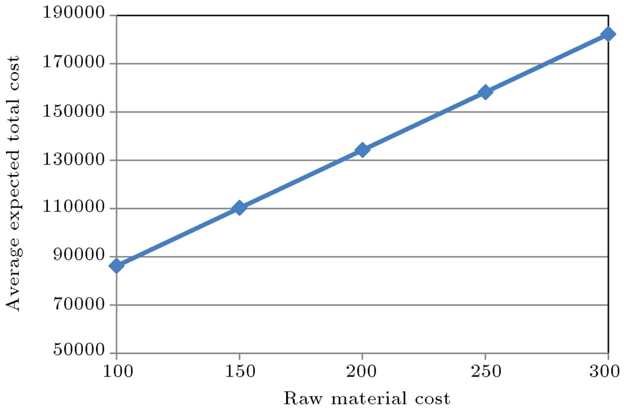

Figure 8. Raw material cost versus average expected total cost.

Figure 8 shows how the raw material cost influences the average expected total cost. We observe an adverse effect of raw material cost on the average expected total cost. It is obvious that if the manufacturer uses costly raw materials, then very high cost will ensue. The learning exponent, b, has an important role in optimal decisions. As b increases, the average production rate increases. This is mainly because of continuously gained experience of the workers due to the repetition of the same work. In practice, the power of gaining experience of the workers is limited to a certain level and varies from person to person as human beings have different capacities. Figure 9 shows that the rate of increase in the average production rate decreases, yet it is up to a certain value of b after which there is almost no change. As the production rate increases for increasing b, the production time decreases, resulting in a noticeable cost reduction (see Figure 10). Figure 10 exhibits cost reduction for increasing value of the learning exponent, b. Figure 11 illustrates the role of standard deviation of mean lead time in the proposed model, more specifically in the cost function. It is also observed that the change of the growth level of the average expected total cost with respect to the standard deviation σ is very low.

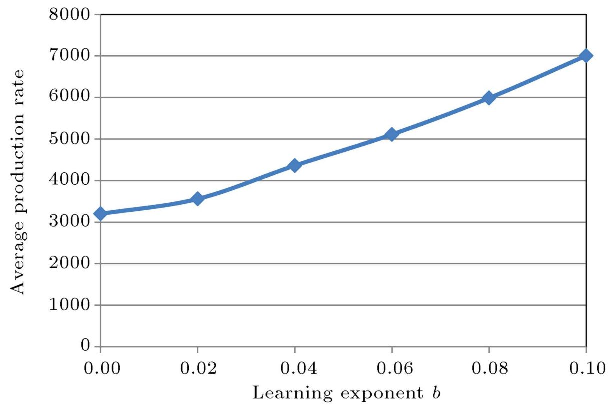

Figure 9. Learning exponent b versus average production rate.

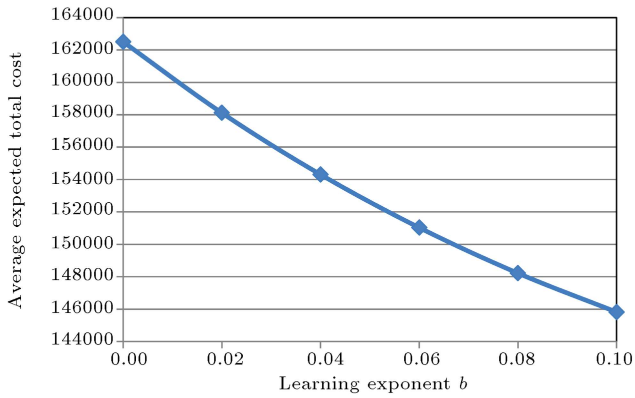

Figure 10. Learning exponent b versus average expected total cost.

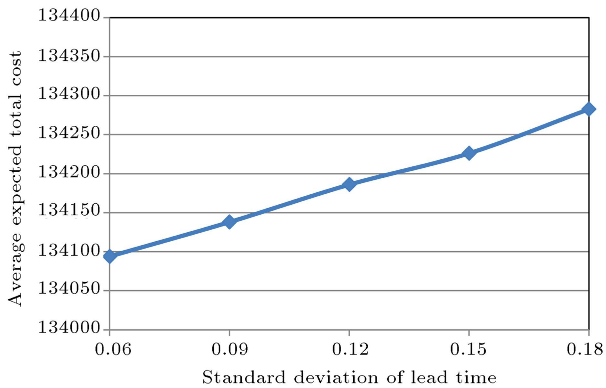

Figure 11. Standard deviation of lead time versus average expected total cost.

The proposed model can be applied to several industrial problems in practice. Some real applications of the key topics of this study observed with positive results throughout the years are given below:

- Primarily, the remanufacturing process was experienced (industrial level) by tank remanufacturing in World War I. After World War II in the UK, all the car manufacturers found the recycling process profitable. Therefore, Caterpillar and Xerox have developed ongoing revenue opportunities from the 2nd, 3rd, ... nth life products. At the expert remanufacturing workshop (CfSD, 2006), Xerox reported that the products could be remanufactured and useful up to the 7th life [53];

- In European Union, new regulations on end of life for automotive products (2000/EC), electrical and electronic equipment (2003/EU) are made to protect the environment from pollution due to waste [54];
- In 1990, Kodak initiated a single-use camera recovery program to recycle and reuse the items. In 2003, the return rate was more than 70% in the United States and 60% throughout the whole world [55];
- Xerox Europe responsible for 25% of Xeroxs worldwide business launched regulations for a waste-free company, which results in cost saving of Xerox of over US$76 million in 1999. Equipment recycling was a fundamental approach to achieving Waste-Free Product goals. Today, 90% of Xerox-designed equipment is recyclable, which detracts about 145 million pounds of waste from landfills in 1998 [55];
- Using data from the Census of Industrial Production (CIP) from the Department of Statistic, Ministry of Trade and Industry in Singapore for the years 1980 to 2007, it was observed that, in the transport equipment industry and the electronic industry, when the gained skill doubles, the unit labor input is reduced to 30% and 56% of the initial labor, respectively [56].

7. Conclusion

The paper analyzed a closed-loop supply chain with a manufacturer and a retailer where the retailer collects the used items on behalf of the manufacturer. The production of the finished goods (manufactured and remanufactured) is subject to learning, i.e., the production process is executed by workers who perform the same job again and again with increasing speed. The proposed model was formulated considering stochastic lead time.

Several managerial insights can be drawn from the findings of the proposed model:

1. As more returned items with a greater fraction of accepted used items reduce the expected total cost of the CLSC, managers should strive for higher return rates. They should determine a suitable offer price to attract more customers to sell their rejected items. In doing so, management not only improves the intrinsic economic value of used items, but also protects the environment from pollution due to possible improper disposal of used products by the end users;
2. As observed in the numerical study, a higher value of learning exponent leads to a significant cost reduction of the whole system. However, after a few production cycles, the cost curve plateaus. At that time, the management has to make a decision whether investment in providing supports, such as sophisticated equipment, technology, etc., for the workers would be a viable tool to improve further the performance of the workers or not.

In practice, the proposed model can be applied to the manufacturing systems where the production largely depends on workers/labors, e.g., garment factories, leather goods manufacturing industries, etc. [26]. There are many ingredients that can be used for future research. For example, the model can be extended by incorporating stochastic demand, which may be influenced by factors such as price, quality, availability, etc. Further, there can be errors in inspection. As human being is involved in inspection, there can be two error types: wrong acceptance of bad items and wrong rejection of good items. One may also consider forgetting in production along with learning; the workers can do a better job by repeated work; however, at the same time, they can lose the experience if there is a significant gap after learning.

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References

1. Wright, T.P. "Factors affecting the cost of airplanes", *J. Aeron. Sci.*, **3**(2), pp. 112-128 (1936).
2. Liao, C.J. and Shyu, C.H. "An analytical determination of lead time with normal demand", *Int. J. Oper. Prod. Manag.*, **11**(9), pp. 72-78 (1991).
3. Ben-Daya, M. and Raouf, A. "Inventory models involving lead time as a decision variable", *J. Oper. Res. Soc.*, **45**(5), pp. 579-582 (1994).
4. Ben-Daya, M. and Hariga, M. "Integrated single vendor single buyer model with stochastic demand and variable lead time", *Int. J. Prod. Econ.*, **92**(1), pp. 75-80 (2004).

5. Ouyang, L.Y., Wu, K.S., and Ho, C.H. "Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time", *Int. J. Prod. Econ.*, **92**(3), pp. 255-266 (2004).

6. Hoque, M.A. and Goyal, S.K. "A heuristic solution procedure for an integrated inventory system under controllable lead-time with equal or unequal sized batch shipments between a vendor and a buyer", *Int. J. Prod. Econ.*, **102**(2), pp. 217-225 (2006).

7. Mandal, P. and Giri, B.C. "A single-vendor multi-buyer integrated model with controllable lead time and quality improvement through reduction in defective items", *Int. J. Sys. Sci.: Oper. & Logis.*, **2**(1), pp. 1-14 (2015).

8. Lin, H.J. "Investing in lead-time variability reduction in a collaborative vendor-buyer supply chain model with stochastic lead time", *Comp. Oper. Res.*, **72**, pp. 43-49 (2016).

9. Taleizadeh, A.A., Niaki, S.T.A., Shafii, N., Meibodi, R.G., and Jabbarzade, A. "A particle swarm optimization approach for constraint joint single buyer single vendor inventory problem With changeable lead-time and (r, Q) policy in supply chain", *Int. J. Adv. Manuf. Tech.*, **51**(9-12), pp. 1209-1223 (2010).

10. Taleizadeh, A.A., Niaki, S.T.A., Aryanezhad, M.B., and Tafti, A.F. "A genetic algorithm to optimize multi-product multi-constraint inventory control systems with stochastic replenishments and discount", *Int. J. Adv. Manuf. Tech.*, **51**(1-4), pp. 311-323 (2010).

11. Taleizadeh, A.A., Niaki, S.T.A., and Seyedjavadi, S.M.H. "Multiproduct multichance-constraint stochastic inventory problem with dynamic demand and partial backordering: A harmony search algorithm", *J. Manuf. Sys.*, **31**(2), pp. 204-213 (2012).

12. Taleizadeh, A.A., Niaki, S.T.A., Aryanezhad, M.B., and Shafii, N. "A hybrid method of fuzzy simulation and genetic algorithm to optimize multi-product single-constraint inventory control systems with stochastic replenishments and fuzzy demand", *J. Inf. Sci.*, **220**(20), pp. 425-441 (2013).

13. Yano, C.A. "Stochastic lead times in two-level assembly systems", *IIE Transac.*, **19**(4), pp. 371-378 (1987).

14. Ouyang, L.Y., Yeh, N.C., and Wu, K.S. " Mixture inventory model with backorders and lost sales for variable lead time", *J. Oper. Res. Soc.*, **47**(6), pp. 829-832 (1996).

15. Sajadieh, M.S., Jokar, M.R.A., and Modarres, M. "Developing a coordinated vendorbuyer model in two-stage supply chains with stochastic lead-times", *Comp. Oper. Res.*, **36**(8), pp. 2484-2489 (2009).

16. Hoque, M. "A manufacturer buyer integrated inventory model with stochastic lead times for delivering equal-and/or unequal-sized batches of a lot", *Comp. Oper. Res.*, **40**(11), pp. 2740-2751 (2013).

17. Maity, A.K., Maity, K., Mondal, S.K., and Maiti, M. "A production recycling inventory model with learning effect", *Optimization and Eng.*, **10**(3), pp. 427-438 (2009).

18. Isotupa, K.S. and Samanta, S. "A continuous review (s, q) inventory system with priority customers and arbitrarily distributed lead times", *Math. Comp. Model.*, **57**(5), pp. 1259-1269 (2013).

19. Disney, S.M., Maltz, A., Wang, X., and Warburton, R.D. "Inventory management for stochastic lead times with order crossovers", *Euro. J. Oper. Res.*, **248**(2), pp. 473-486 (2016).

20. Roldan, R.F., Basagoiti, R., and Coelho, L.C. "A survey on the inventory-routing problem with stochastic lead times and demands", *J. Appl. Logic*, **24**(A), pp. 15-24 (2017).

21. Yelle, L.E. "The learning curve: Historical review and comprehensive survey", *Decision Sciences*, **10**(2), pp. 302-328 (1979).

22. Elmaghraby, S.E. 'Economic manufacturing quantities under conditions of learning and forgetting (EMQ/LaF)", *Prod. Plan. Cont.*, **1**(4), pp. 196-208 (1990).

23. Jaber, M.Y. and Bonney, M. "Production breaks and the learning curve: the forgetting phenomenon", *Appl. Math. Model.*, **20**(2), pp. 162-169 (1996).

24. Carlson, J.G. and Rowe, A.J. "How much does forgetting cost", *Indus. Eng.*, **8**(9), pp. 40-47 (1976).

25. Chiu, H.N., Chen, H.M., and Weng, L.C. "Deterministic time-varying demand lotsizing models with learning and forgetting in setups and production", *Prod. Oper. Manag.*, **12**(1), pp. 120-127 (2003).

26. Khan, M., Jaber, M.Y., and Ahmad, A.R. "An integrated supply chain model with errors in quality inspection and learning in production", *Omega*, **42**(1), pp. 16-24 (2014).

27. Huang, C.K. "An integrated vendor-buyer cooperative inventory model for items with imperfect quality", *Prod. Plan. Cont.*, **13**(4), pp. 355-361 (2002).

28. Burr, W. and Pearne, N. "Learning curve theory and innovation", *Circuit World*, **39**(4), pp. 169-173 (2013).

29. Teng, J.T., Lou, K.R., and Wang, L. "Optimal trade credit and lot size policies in economic production quantity models with learning curve production costs", *Int. J. Prod. Econ.*, **155**, pp. 318-323 (2014).

30. van Hoof, B. "Organizational learning in cleaner production among Mexican supply networks", *J. Cleaner Prod.*, **64**, pp. 115-124 (2014).

31. Grosse, E.H., Glock, C.H., and Muller, S. "Production economics and the learning curve: A meta-analysis", *Int. J. Prod. Econ.*, **170**, pp. 401-412 (2015).

32. Teyarachakul, S., Chand, S., and Ward, J. "Effect of learning and forgetting on batch sizes", *Prod. Oper. Manag.*, **20**(1), pp. 116-128 (2011).

33. Glock, C.H. and Jaber, M.Y. "A multi-stage production-inventory model with learning and forgetting effects, rework and scrap", *Comp. Indus. Eng.*, **64**(2), pp. 708-720 (2013).

34. Lolli, F., Messori, M., Gamberini, R., Rimini, B., and Balugani, E. "Modelling production cost with the effects of learning and forgetting", *IFAC-Papers Online*, **49**(12), pp. 503-508 (2016).

35. Jaber, M. and Givi, Z. "Imperfect production process with learning and forgetting effects", *Computational Manag. Sci.*, **12**(1), pp. 129-152 (2015).

36. Giri, B. and Glock, C.H. "A closed-loop supply chain with stochastic product returns and worker experience under learning and forgetting.", *Int. J. of Prod. Res.*, **55**(22), pp. 6760-6778 (2017).

37. Clark, A.J. and Scarf, H. "Optimal policies for a multi-echelon inventory problem", *Manag. Sci.*, **6**(4), pp. 475-490 (1960).

38. Guide, V.D.R. and Wassenhove, L.N. "Managing product returns for remanufacturing", *Prod. Oper. Manag.*, **10**(2), pp. 142-155 (2001).

39. Majumder, P. and Groenevelt, H. "Competition in remanufacturing", *Prod. Oper. Manag.*, **10**(2), pp. 125-141 (2001).

40. Koh, S.G., Hwang, H., Sohn, K.I., and Ko, C.S. "An optimal ordering and recovery policy for reusable items", *Comp. Indus. Eng.*, **43**(1), pp. 59-73 (2002).

41. Savaskan, R.C., Bhattacharya, S., and Van Wassenhove, L.N. "Closed-loop supply chain models with product remanufacturing", *Manag. Sci.*, **50**(2), pp. 239-252 (2004).

42. Chung, S.L., Wee, H.M., and Yang, P.C. "Optimal policy for a closed-loop supply chain inventory system with remanufacturing", *Math. Comp. Model.*, **48**(5), pp. 867-881 (2008).

43. Jaber, M.Y. and El Saadany, A.M. "The production, remanufacture and waste disposal model with lost sales", *Int. J. Prod. Econ.*, **120**(1), pp. 115-124 (2009).

44. Hong, X., Wang, Z., Wang, D., and Zhang, H. "Decision models of closed-loop supply chain with remanufacturing under hybrid dual-channel collection", *Int. J. Adv. Manuf. Tech.*, **68**(5-8), pp. 1851-1865 (2013).

45. De Giovanni, P. and Zaccour, G. "A two-period game of a closed-loop supply chain", *Euro. J. Oper. Res.*, **232**(1), pp. 22-40 (2014).

46. Naeem, M.A., Dias, D.J., Tibrewal, R., Chang, P.C., and Tiwari, M.K. "Production planning optimization for manufacturing and remanufacturing system in stochastic environment", *J. Intell. Manuf.*, **24**(4), pp. 717-728 (2013).

47. Giri, B. and Sharma, S. "Optimizing a closed-loop supply chain with manufacturing defects and quality dependent return rate", *J. Manuf. Sys.*, **35**, pp. 92-111 (2015).

48. Giri, B. and Sharma, S. "Optimal production policy for a closed-loop hybrid system with uncertain demand and return under supply disruption", *J. Cleaner Prod.*, **112**, pp. 2015-2028 (2016).

49. Cobb, B.R. "Inventory control for returnable transport items in a closed-loop supply chain", *Trans. Res. Part E: Logis. Trans. Rev.*, **86**, pp. 53-68 (2016).

50. Jena, S.K., Sarmah, S., and Sarin, S.C. "Joint-advertising for collection of returned products in a closed-loop supply chain under uncertain environment", *Comput. Ind. Eng.*, **113**, pp. 305-322 (2017).

51. Masoudipour, E., Amirian, H., and Sahraeian, R. "A novel closed-loop supply chain based on the quality of returned products", *J. Cleaner Prod.*, **151**, pp. 344-355(2017).

52. Angius, A., Colledani, M., Horvath, A., and Gershwin, S.B. "Analysis of the lead time distribution in closed loop manufacturing systems", *IFAC-Papers Online*, **49**(12), pp. 307-312 (2016).

53. Gray, C. and Charter, M. "Remanufacturing and product design", *Int. J. Prod. Dev.*, **6**(3/4), pp. 375-342 (2008).

54. Zeballos, L.J., Gomes, M.I., Povoa, A.P.B., and Novais, A.Q. "Addressing the uncertain quality and quantity of returns in closed-loop supply chains", *Comp. Chem. Eng.*, **47**, pp. 237-247 (2012).

55. Guide Jr., V.D.R., Jayaraman, V., and Linton, J.D. "Building contingency planning for closed-loop supply chains with product recovery", *J. Oper. Man.*, **21**, pp. 259-279 (2003).

56. Heng, T.M. "Learning curves & productivity in Singapore manufacturing industries", *Second Annual Conference of the Academic Network for Development in Asia (ANDA)*, Phnom Penh, Cambodia (2003).

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