Improved Multi-Ant-colony algorithm for solving Multi-Objective Vehicle Routing Problems

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Abstract: Classical vehicle routing problems (VRP) involves supply of goods/services from a central depot to geographically scattered customers. Besides the classical objective of minimizing the total travelled distance, the present work also considers simultaneous optimization of two additional objectives namely minimizing make span and minimizing distance imbalance. A mathematical model has been developed to deal with this multi-objective version of VRP (MO-VRPTW). A meta-heuristic based on multiple ant colony system for solving this MO-VRPTW has also been proposed. Firefly optimization algorithm (FA) has also been applied to avoid local optima. Two new migration operators named Migration-I and Migration-II and multi-pheromone matrices have been developed to further improve the solution quality. The proposed algorithm has been tested on a number of benchmark problems and its superiority over other state of art approaches and NSGA-II is demonstrated.

Keywords: Ant Colony System, Firefly optimization, Load balancing, Multi-objective optimization, Vehicle Routing Problem.

1 Introduction

Vehicle routing problem (VRP) is a well-known classical combinatorial optimization problem in transportation logistics and supply chain management. It is concerned with the supply of goods/services from a central distribution/collection center to finite number of geographically dispersed customers in an efficient and economical way. As the cost and time of transportation of goods have direct influence on business and industry, VRP has gained importance amongst the researchers to produce business enterprises economical solutions. Furthermore, global warming, traffic jams, road congestions and other issues such as depleting natural resources have motivated researchers to search for efficient routing strategies for supply chain activities.

Graphically, VRP can be formulated as an undirected graph $G(V,E)$, where the vertex set $N = \{0,1,2,3 \dots, n\}$ is the set of nodes and $E = \{(i,j)\ i,j \in V\ ,\ i \neq j\}$ is the set representing the link between nodes (i,j). Node 0, is considered as the central depot where a fleet of homogeneous vehicles (each with identical capacity Q) are available to serve customers $(N/\{0\})$ each having a fixed demand q_i. Each arc (i,j) is associated with a fixed symmetric travel cost d_{ij} and travel time t_{ij} satisfying triangle inequality i.e. $d_{ij} \leq d_{ik} + d_{kj}\ \ \forall\ i,j,k\ \in N$.

The objective of VRP is to find an optimal set of routes with optimal sequence of customers satisfying the following constraints:

- Each route must start from central depot and end there.
- Each of the customers must be fully serviced by a single vehicle in a single visit.
- Demand of any customer does not exceed the vehicle capacity.

Most of the real life models of VRP primarily aim at optimizing a single objective of VRP such as minimizing the total travelled distance or minimizing the required fleet size. However, supply chain management/logistic management and other transportation industries mostly encounter multi-objective scenarios of minimizing simultaneously the number of vehicles as well as total travelled distance. Multi-objective optimization problems (MOP) are mainly concerned with the optimization of more than one objective simultaneously under the same set of conditions. The introduction of additional objectives to be optimized simultaneously further enhances the utility of classically defined single objective optimization problem [1].

Most of the available literature on the subject deals with optimization of single objective functions such as minimizing the total travelled distance or minimizing the overall fleet size required. In fact, most of the companies/ enterprises try not only to minimize the primary objective of total travelled distance but also focus towards balancing the individual route lengths (i.e. make span), number of customers covered by a single vehicle and balancing of working hours of the drivers etc. Such types of problems result in multi-objective optimization problems with conflicting objectives.

In this paper, firstly a new mathematical model has been developed to tackle the MO-VRPTW considering the minimization of total travelled distance, length of longest route (make-span) and driver's load imbalance (in terms of distance travelled) simultaneously. Another goal of proposed work is to develop an algorithm which can solve the developed model efficiently and effectively.

In order to achieve this goal, an improved ant colony system based multi-objective optimization algorithm has been proposed to optimally solve proposed multi-objective vehicle routing problem with time windows. The proposed multiple ACS (named MACS) can enhance the basic ACS in the following three aspects:

- Multiple pheromone matrices have been introduced to fit MACS for solving different objective functions of proposed MO-VRPTW.
- Proposed ACS has been hybridized with firefly optimization algorithm (FA) to avoid being trapped into local optima.
- Furthermore, two new migration operators have been proposed to improve the obtained solutions.

Rest of the paper is organized as follows. Literature related to MOVRPs has been presented in Section 2. Mathematical formulation of the present multi-objective VRPTW is in Section 3 presented. Section 4 describes the details of the proposed algorithm. Experimentation details and results are presented in Section 5. Conclusions based on present study are finally drawn in Section 6.

2 Literature Review

Literature currently available on multi-objective VRP, has generally considered capacited vehicle routing problem (CVRP) without time windows constraint. [2] presented a heuristic algorithm to minimize the total travelled distance as well as to balance the route length of vehicles. Similarly, [3] proposed an evolutionary algorithm along with elitist diversification based meta heuristic to simultaneously reduce the total length of routes as well as optimizing route balancing. [4] worked towards balancing of load carried by each vehicle. Recently, [5] proposed a genetic algorithm based approach for bi-objective CVRP where they considered total travelled distance as well as distance balance of active vehicles of simultaneously. [6] designed two scatter search based algorithms and test them for the bi-objective VRPTW. They tackled a real life problem presented by a Spain based company with the aim of minimizing the total tour length as well as trying to balance time spent by each of the drivers on duty. [7] presented a hybrid meta-heuristic to solve MO-VRPTW. GA and Tabu search were employed to optimize number of vehicles and travelled distance respectively. [8] thoroughly discuss the potential benefits of using multi-objective vehicle routing problems (MO-VRPs). In 2011, [9] proposed an improved multi objective evolutionary algorithm for MO-VRPTW. They proposed similarity measure to improve the diversity and quality of the Pareto optimal front solution sets. Ten different types of mutation operators were defined and tested and the superiority of proposed improved evolutionary algorithm was shown on a standard bench mark set while considering bi-objective and tri-objective formulations. Recently [10] presented a tissue p-system based on discrete glowworm evolution mechanism and variable neighborhood evolution mechanism to solve bi-objective VRPTW involving minimization of primarily the fleet size and total travelled distance. They were able to find some new Pareto front optimal solution sets for small scale instances and provide comparable results for large benchmark problems. Additional information on recent methods dealing with multi-objective vehicle routing problems is available in [1], [11], [12], [13], [14].

Most of the literature considered above mainly emphasizes on minimization of travelled distance and/or fleet size required to complete the task. However, besides the minimization of transportation cost real life VRP has also to deal with some other additional objectives like customer satisfaction, work load balancing (in terms of time and distance) etc. For instance, Norwegian distribution company "Innovation AS" provides routing services for newspaper distribution in large parts of country. Its main goal is to find well balanced routes with a maximum of 20% imbalance [15], [16] solved a MO-VRP for designing a robust evacuation plan in case of earthquakes. They proposed a hierarchical multi-objective optimization model including the minimization of waiting time for the medical treatment of injured persons as well as minimizing the response time for supply of relief materials with least number of emergency vehicles. [17] define the work load imbalance as difference between the load carried out by maximum utilized and minimum utilized vehicles. Distance imbalance directly influences the time travelled by the particular vehicle. [18] claimed that in the waste collection problem the collection should be finished as soon as possible so that the remaining time can be utilized to categorize the collected waste material. Furthermore, the delivery of perishable food items, emergency services and disaster evacuation strategies require the shortest travel time. Most of the work in field of MO-VRP focused to managerial benefits, ignoring the customer satisfaction and driver's work distribution. However, such factors greatly influence the market share of the companies. For example, late delivery of items due to longer make-span makes customers unsatisfied and may cause loss of future sales. Additionally, the work-load imbalance may create differences among the lorry drivers which results to lower throughputs. Hence such real-life situations need consideration of multi-objective routing plans involving the total travelled distance, minimizing length of longest route (i.e. make-span) as well as balancing travelled route length of each vehicle.

Like VRP, MO-VRP is also a NP-Hard problem. Exact solution algorithms are possible for problems having small instances only. Moreover, in an MOP more than one objectives are to be combined, which can-not be separated or solved sequentially. As a result, exact methods are seldom available for such problems (Dong, W. et al. 2017). So, in recent years, meta-heuristics based approaches have been tried to solve MO-VRP. Evolutionary algorithms have the ability to approximate the whole Pareto front. As a result, a large number of studies have used evolutionary algorithms to deal with MO-VRPs [9], [13].

However, evolutionary algorithms suffer from the problem of being trapped in local optima and a large amount of computational effort is generally needed to escape from being trapped in local optima. Besides, EAs, [5] proposed a local search based optimization technique namely LSMO-VRPTW. They handled the multi objective problem sequentially by optimizing different objectives, one at a time. However, in such a case there may be chances of distortion of objectives which are already optimized. Some other recent literature in MOVRPs is tabulated as under in Table 1.

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Interested readers may also refer to [27] for detailed survey on MO-VRPs.

No doubt, MO-VRPTW is still a less explored topic in the field of MO-VRPs. Our MO-VRPTW aim towards minimizing total tour length (from economical perspective), minimizing make span (for early delivery) and route imbalance (from social perspective of fair distribution of work among drivers) also.

3 Mathematical formulation

Before formulating the mathematical model of multi-objective VRPTW we first briefly describe the classical VRPTW model.

VRPTW is an extension of VRP with additional constraints of time windows associated with each of the customer. In this paper we assume the case of hard time windows where customer must be serviced within specified time window. If a vehicle arrives earlier than the starting/opening time of the customer's window, then it has to wait till the opening of window. However, if a vehicle arrives after the closing time of the window, then this customer cannot be served at all by this vehicle.

The set of notations used in mathematical formulation are presented in Table 2.

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With these notations mathematical model of VRPTW gets formulated as follows:

$$\min f_1 = \sum_{k \in K} \sum_{i,j \in V} d_{ij} X_{ij}^k \quad (1)$$

subjected to:

$$\sum_{k \in K} \sum_{j \in V} X_{ij}^k = 1, \quad \forall\, i = N/\{0\} \quad (2)$$

$$\sum_{j=1}^{N} X_{0j}^k = 1 \quad \forall k \in K \quad (3)$$

$$\sum_{i=1}^{N} X_{i0}^k = 1 \quad \forall k \in K \quad (4)$$

$$\sum_{i=1}^{N} X_{ih}^k - \sum_{j=1}^{N} X_{hj}^k = 0 \quad \forall\, h = \frac{N}{\{0\}}, \quad \forall k \in K \quad (5)$$

$$\sum_{i=1}^{N} q_i \sum_{j=1}^{N} X_{ij}^k \le Q \quad \forall k \in K \quad (6)$$

$$T_i^k + s_i + t_{ij} + w_j \leq T_j^k \quad \forall \mathrm{i}, j = N/\{0\}, \quad \forall k \in K \tag{7}$$

$$e_i \leq T_i^k \leq l_i \quad \forall \mathrm{i}, = N/\{0\}, \quad \forall k \in K \tag{8}$$

$$X_{ij}^k \in \{0, 1\} \quad \forall k \in K, \ \forall i, j \in N \tag{9}$$

The objective function (1) seeks to minimize the total traveled distance. Constraint (2) specifies that every customer is visited by exactly one vehicle and splitting of deliveries is forbidden. Eqs. (3) and (4) ensures that all routes start and end at the central depot respectively. Constraint (5) ensures that the vehicle has to leave the customer location being serviced after visiting it. Inequality (6) restricts the violation of capacity constraints such that total serving on the route must be less than or at the most equal to the available capacity of the vehicle. Eqs. (7) and (8), preserve the time window constraints by checking that the sum of travelling time to the customer, service time and waiting time if any are less than the closing time of customer's time-window. Finally, the partial or split deliveries are prohibited by (9).

Literature on MO-VRPTW is mostly formulated as bi-objective VRPTW, working towards minimization of total fleet size as well as total travelled distance simultaneously. In this paper, along with these objective we have tried to solve the VRP with hard time windows minimizing at the same time make span as well as the distance imbalance. The latter two objectives gain insight from social perspective and guarantee a fair share scheduling of the work. These objectives can be mathematically formulated as:

$$f_2 = \min(\max_{k \in K}(makespan_k))$$

$$\text{where} \quad makespan_k = \sum_{i,j \in V} d_{ij} X_{ij}^k \tag{10}$$

$$f_3 = (\max_{k \in K}(makespan_k) - \min_{k \in K}(makespan_k)) / \max_{k \in K}(makespan_k) \tag{11}$$

whereas the objective function f_2 as given in (10) corresponds to the minimization of make-span i.e. length of longest route, f_3 as given in (11) defines the distance imbalance among the distances covered by vehicles to complete the delivery. Make-span can directly affect the cost of transportation. For example, as mentioned by [28], in real life scenario of school bus routing where due to large distances between the pickup points, certain route lengths far exceed average route. As a result, there are less chances of using the full capacity of the bus, in case when most of customers belong to last locations. It may also create late servicing of end customers. f_2 is mainly associated with timely delivery and satisfaction of customers [27].

A multi-objective optimization problem can be mathematically formulated as

$$\min f(x) = \{f_1(x), f_2(x), \ldots f_n(x)\} \tag{11a}$$

Here $f_i(x) (i = 1,2, \ldots n)$ are the objective functions to be optimized simultaneously under specified set of constraints. MOP tries to find a set of non-dominated solutions called Pareto sets (PS). The dominance among the solution sets is defined as follows:

Definition 1: A solution set $A = (a_1, a_2, a_3 \ldots a_n)$ dominates another solution set

$B = (b_1, b_2, b_3 \ldots b_n) \ iff \quad \forall i \quad b_i \leq a_i$ & $\exists i \in (1,2 \ldots n) \quad s.t. \quad b_i < a_i$ i.e. solution set A must be better than set B in at least one objective and not worse in others. If $b_i = a_i \forall i$ then solutions are indifferent i.e. incomparable. Furthermore a solution set Z is said to be Pareto optimal if there does not exist any other solution Z' such that Z' dominates Z.

Solution methods for dealing with multi-objective optimization problems are broadly categorized into two groups (i) a priori approach (ii) a posteriori approach [8]. In a priori approach, a preference index is associated with each of the objectives under consideration. Then all of the objectives are aggregated into a single objective function. However, the problem with this approach is that weights associated with different objective functions are very difficult to decide, particularly when there are tradeoffs between the objectives (as are generally there in real life applications). In a-posteriori approach all the possible Pareto fronts or a subset of it is produced. However, in multi-objective optimization, generally the objectives are conflicting in nature.

4 Proposed MACS for MOP

It has been observed that some of commonly used approaches such as GA, PSO, Tabu search have certain disadvantages. These don't use heuristic information for solution construction, they rather rely on their self-optimization mechanism for the same. However, ACS takes advantage of heuristic function during solution construction process [27]. Furthermore, as discussed in section-1 that EAs also suffer from huge computation efforts for being trapped out of local optima. However, ants in ant colony systems due to their cooperative behavior are better suited for combinatorial optimization problems such as VRP and bin packing problems.

4.1 Ant colony System ACS

Ant colony system gets inspired from the real ants. Ants in nature live in colonies. In their search for food to begin with search of paths for the food source in random manner i.e. exploring various possible alternatives. Real ants release a smelling agent called pheromone on the paths they follow. At the return trip from the source ant deposits the pheromone on the path, which guides other ants for food source. Greater the amount of pheromone on the path more likely is that this path will be followed by other ants. However, pheromone left by an ant gradually decays with passage of time. In addition to pheromone information, ants also use their insight to explore new paths. Keeping these facts in view some efficient ant colony optimization algorithms have been designed. For applying ACO on real life problems, the problem has to be modeled as connected graphs and the solution to problem is an ordered sequence of all the nodes. In the mathematical model developed on this base an ant probabilistically visits from current node i to next node j. The state transition from current node i to next node is influenced by exploration and exploitation, and mathematically defined by the following formula:

$$j = \begin{cases} argmax_{j\in J} \frac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{j\in J}\tau_{ij}^{\alpha}\eta_{ij}^{\beta}} & if\ \varphi \leq \varphi_o (exploitation) \\ J & otherwise\ \ (exploration) \end{cases} \quad (12)$$

Exploitation refers to the selection of edges already visited by most of previous ants whereas to explore new solution set ant choose next node to be visited by following pseudo transition probability rule:

$$p_{ij} = \begin{cases} \frac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{j\in J}\tau_{ij}^{\alpha}\eta_{ij}^{\beta}} & if\ j \in J \\ 0 & otherwise \end{cases} \quad (13)$$

Here p_{ij} is the probability of moving from current node i to next node j, τ_{ij} and η_{ij} are the pheromone concentration and heuristic value of the edge $i-j$ respectively, α and β parameters showing the relative importance of pheromone and heuristic respectively. J is the set of nodes currently available for visit from node i. Solution construction process in ACO involves pheromone concentration and heuristic information. The two factors are biased by different weights. For MOP, multiple pheromone and/or heuristics contribute for the construction of solution.

Heuristic Information ($\boldsymbol{\eta_{ij}}$): In general, the heuristic function can be calculated as the reciprocal of the objective function (for minimization problems). For MOP heuristic is the aggregation of different objectives to be optimized i.e. multiple objectives are aggregated into heuristic calculation. In our formulated MO-VRPTW model , 2nd and 3rd objectives (i.e. f_2 and f_3) correspond to minimization of make-span and route imbalance respectively. Since the said two objectives are known only after the completion of route so these two cannot be included in heuristic η_{ij} while constructing the route using ACO. Therefore we have assumed, heuristic function η_{ij} depends simply upon first objective f_1 and is given by

$$\eta_{ij} = \frac{1}{d_{ij}+w_{ij}} \qquad (14)$$

where w_{ij} is the waiting time at location j (if reached earlier than opening time a_j of location j otherwise it is 0).

Pheromone matrix $\boldsymbol{\tau_{ij}}$: As discussed earlier, the proposed approach uses Pareto-front optimal solutions, so we avoid combining multiple objectives in heuristic calculation, but we use multiple pheromone matrices corresponding to each objective. Since our formulated model includes three objectives, so here we consider three pheromone matrices (namely $\tau(1), \tau(2)$ and $\tau(3)$ corresponding to the proposed three objectives namely f_1, f_2 and $f_3$respectively. Initially, all the three matrices are initialized to equal values given by $\tau_{ij} = \frac{1}{d_{ij}}$ i.e. pheromone trial on each of the arc $i - j$ in all matrices is reciprocal of the arc length. Furthermore, to deal with different objectives, we divide the ant system into three different colonies $(col_u)\ u = 1,2,3)$. Each colony constructs an independent solution corresponding to its pheromone matrix. When all ants of corresponding colony construct their solutions, the best solution of that colony update pheromone locally according to the quality of the corresponding objective amongst those solutions. Pareto front solutions among three colonies are used to update pheromone globally. Local pheromone updation can be given by following equation:

$$\tau_{ij}^{new}(u) = \begin{cases} (1-\rho)\tau_{ij}^{old}(u) + \rho\Delta\tau_{ij} & \text{if arc}(i-j) \text{ followed by } u^{th} \text{ ant} \\ (1-\rho)\tau_{ij}^{old}(u) & \text{otherwise} \end{cases} \qquad (15)$$

Here $\Delta\tau_{ij}$ is the pheromone deposited on arc $(i - j)$ by the ants following that arc (if any). $\Delta\tau_{ij}$ has been defined for three colonies differently as here three different colonies have been used to optimize three different objectives accordingly. It can be given by following equation as follows:

$$\Delta\tau_{ij} = \begin{cases} \frac{\overline{f_1}}{k_1 * f_1} & \text{if } u = 1 \\ \frac{\overline{f_2}}{k_2 * f_2} & \text{if } u = 2 \\ \frac{\overline{f_3}}{k_3 * f_3} & \text{if } u = 3 \end{cases} \qquad (15a)$$

$\overline{f_u}$ is the average value of objective function of all the ants of u^{th}colony. After several simulations it was observed that $k_1 = 5, k_2 = 5$ and $k_3 = 5$ give most appropriate outcome.

Similarly, in each iteration after finding the best solution, the pheromone can be updated globally by the following equation

$$\tau_{ij}^{new} = \begin{cases} (1-\rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij} & \text{if arc}(i-j) \text{ followed in current best solution} \\ (1-\rho)\tau_{ij}^{old} & \text{otherwise} \end{cases} \quad (16)$$

$$\text{where } \Delta\tau_{ij} = 1/\sum f_u. \quad (16a)$$

Here the denominator is sum of all the objective functions for the current best solution.

ACS is well suitable for solving VRP and VRPTW in terms of computational complexity and convergence rate. However, the traditional ACS has the drawback of pre mature convergence and getting easily trapped into local optima. So, the researchers improved basic ACS to solve VRP and its variants by hybridizing ACS with GA, PSO, Local search, neighborhood search etc. [1].

On the other hand, the advantages of FA in various optimization fields have been analyzed in the survey presented by [29]. It has also been noticed that whereas on one hand ACS has relatively high convergence rate on the other hand FA provides a more diversified solution space due to its multi-modality nature [30]. Despite of such a high popularity of FA and multi-modality nature, it has been rarely applied in the field of routing and multi-objective routing as well.

So it was felt that an appropriate hybridization of these two approaches might prove more effective in solving multiobjective VRPs. Hence to take advantages of good features of above said approaches we, combine FA and ACS to solve MO-VRPTW.

4.2 Firefly algorithm (FA)

Firefly approach (FA) is a recent population based stochastic algorithm that uses a probabilistic approach for problem solutions. It is based on flashing behavior and the phenomenon of bioluminescent communication of fireflies. Fireflies flash the light primarily for mating. Interested user can refer to [30, [31] for more technical details of FA.

In the simplest form as suggested by [30] the attractiveness is proportional to the light seen by the observing fireflies; the variation of attractiveness with distance is approximated as

$$\mathcal{B} = \mathcal{B}_0 e^{-\gamma r_{ij}^2} \quad (17)$$

where $\mathcal{B}_0$ is the attractiveness at $r = 0$ and r_{ij} is Euclidean distance between two fireflies at positions i and j. Finally the movement(x_i) of firefly (i)towards the firefly $(j)$is determined by

$$x_i = x_i + \mathcal{B}e^{-\gamma r_{ij}^2} + \sigma \quad (18)$$

Here the second term is the attractiveness and third term σ is the randomness control parameter such that $\sigma \in [0, 1]$.

The ants in ACS follow the path with higher concentration of pheromone, causing the ACS to get trapped into local optima. So, FA is applied to search for the other promising regions which are less explored.

Firefly Encoding

To adapt FA for discrete optimization problems and to encode fireflies, here we adopted the representation scheme proposed by [32]. The fireflies are encoded as set of customers where the index represents customer visiting sequence number. "0" indicates the start of a new route and last "0" end of the overall tour. For example, for the tour (T1 and T2), the fireflies are represented as depicted in Figure 1.

Another key issue of defining the distance r_{ij} between two fireflies is defined as sum of the links connected by different customers in between two fireflies (shown by red links in Figure 2. For instance, the distance between firefly 1 and 2 is 3 as depicted in Figure 2.

---Insert Figure 1 Here-----------------------------------
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---Insert Figure 2 Here-----------------------------------
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Pseudo code for FA search

Step 1. Initialization.

The $'m'$ tours generated by ACS can be treated as population of $'m'$ fireflies.

Step 2. Light Intensity calculation.

Since VRPs are the minimization objective problems so intensity of flashing light of each firefly is set to be equal to inverse of the objective function value.

Step 3. Attractiveness and Movement

Sort $'m'$ fireflies according to their light intensity. The firefly with the least objective function value can be considered as the best firefly. The attractiveness $\mathcal{B}$ of other fireflies towards best firefly is determined as a light intensity function and random move (σ) and given by eq (18).

Step 4. Return 'm' explored tours to ACS algorithm.

The values of parameters can be taken as $m = 15$ and $\sigma = .4$ as suggested by [32].

4.3 Migration operators

For combinatorial optimization problems, it is a general practice to augment local search with basic meta heuristic algorithms to enhance the results obtained from the basic algorithms. However, for MOP Pareto dominance based local search (called PLS) is applied. In PLS, the new solutions are accepted on the basis of directions in which the improvement is expected. In this paper, PLS based on migration moves is applied. Migration refers to the removal of a customer from one route and reinsertion into another route provided the obtained solution is feasible and better one.

Here two migration operators namely Migration-I and Migration-II are proposed and brief details are as follows:

Migration-I: This operator is used to reduce the make-span of the largest route. A route with the largest make-span is selected and a customer nearest to the neighboring route is selected as seed. The seed is removed from its parent route and reinserted into the neighboring route. The new solution is accepted on the basis of quality of improved make-span and solution feasibility. Diagrammatically Migration-I is shown as below in Figure 3. The lastly served customer in route R1, can be migrated from that route R1 to R2 as first customer of route R2 (without any constraint violation) to shorten the make-span.

---Insert Figure 3 Here-----------------------------------
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Migration-II: This operator tries to minimize f_3. The route with length less than half of average route length (if any) is selected. The customers of this route are considered one by one and readjusted to other neighboring routes (if possible) resulting in minimization of distance imbalance. It has been depicted in Figure 4. Additionally, the operator also tries to reduce the number of routes. For instance, as route R4 in Figure 4, has only two customers which can be migrated to routes R1 and R3 (without any constraint violation) reducing the overall distance imbalance among vehicles.

---Insert Figure 4 Here-----------------------------------
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The general frame work of our proposed ant colony system algorithm is given by the MACS Algorithm.

$\mathbb{S}$ in line 7 represents the tour constructed using the nearest neighbor heuristic. $\mathbb{U}$ is the archive set of Pareto front solutions and initially it contains only single solution $\mathbb{S}$ as described in line 8. Lines 11-37 represent main loop iterations until a specified termination criterion is found. Loop involving line 13-29 undergoes repetition for all of the ants. Inner while loop (line 15-26) is repeated until all the customers are not served. Lines 19-23 decides the exploration of new possibilities or exploitation of the previous routes according to Eq 12. In line 30 local search is applied to improve the obtained results. Line 33 selects all the non-dominated solutions among current k ants. In line 34 the archive set $\mathbb{U}$ is updated by inserting all non-dominated solutions and discarding dominated solutions. Pheromone trials are updated in line 27 and 35 as described in section 3.2. Finally, all the Pareto front solutions are presented in line 38.

MACS Algorithm:

```
1. /* Initialization*/
2. Parameter Setting: Set values of No. of ants (m), Stopping criteria i.e. max number of
3. Initialize Pareto Front 𝕌 as empty.
4. Initialize Pheromone matrix τ0.
5.         /* Main Loop*/
6.         /* Construct Initial Solution*/
```

Apply Nearest Neighbor heuristic and construct initial feasible solution $\mathbb{S}$.
Update $\mathbb{U}$ as $\mathbb{U} = \mathbb{U} \cup \mathbb{S}$.
/* MACS*/
Set $itr = 1$
while $itr \leq N_{max}$ **do**
for each ant colony (col_u) $(u = 1{:}\,3)$ **do**
for $i = 1$ to m $(no.of\ ants)$ **do**
set pool of visited customers $\mathcal{S}_i = \emptyset$ and veh_used i.e. $k = 1$
while $\mathcal{S}_i \neq V$ **do**
for each of unvisited customer v **do**
calculate the probability of movement acc. to transition rule
Generate a random number φ
if $\varphi \leq \varphi_o$
exploration
otherwise
exploitation
end if
If vehicle gets full or any other constraint violation set $k = k + 1$
end for
end while
update pheromone locally as $\tau_{ij}^{new}(u)$
Apply FA search as described in section 3.2.
end for
find best solution $best\_sol(u)$ among all ants for each colony
Apply Migration-I and Migration-II operators on $best\_sol(u)$
end for
Among the u best solutions find the set $\mathbb{S}$ of non-dominated Pareto solutions.
Update $\mathbb{U}$ as $\mathbb{U} = \mathbb{U} \cup \mathbb{S}$ and remove all of dominated solutions (if any).
For each of the $\mathbb{S}$ apply update pheromone globally
Set $itr = itr + 1$
end while
Output Pareto Front $\mathbb{U}$

5 Experimentation and Discussion

Evaluation of performance of multi objective optimization algorithms is more complicated in comparison with that of single objective algorithms. The researchers have proposed a number of valuable criteria. However, till now no single criteria is available to evaluate the overall performance of MO- algorithms. Here we use two most commonly used metrics namely hyper volume (HV) and coverage metric (C-Metric) to evaluate the performance of proposed MACS [17], [34].

The said two criteria are defined as follows:

5.1 Performance Metrics

Hyper-volume (HV): As proposed by[35], HV indicates the area bounded by a reference point X_{ref}, which is dominated by at least one of the non-dominated solutions. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of non-dominated solutions. Therefore, larger the $HV(x)$ better will be the X. In general, HV determines the closeness of solution space towards the

Pareto fronts. Diagrammatically, HV for the 3-objective function is as shown in Figure 5. Since the volume of solution set Y is greater than that of solution set X, so the set Y will be better as compared to the set X as the former is closer to Pareto front. In our case, X_{ref} is the reference point which is equal to twice the objective function obtained from initial nearest neighbor heuristic as suggested by [13].

Coverage Metric (C-Metric): This metric proposed by [36], compares two non-dominated algorithms X and Y (say), by following metric:

$$C(X,Y) = \frac{|\{y \in Y \;\; |\exists x \in X: \;\; x \geq y\}|}{|Y|} \tag{19}$$

i.e. $C(X,Y)$ is an ordered pair representing the percentage of non-dominated solutions obtained by algorithm Y, which are dominated by at least one solution of algorithm X. Furthermore, $C(X,Y) + C(Y,X)$ need not to be equal to 1 as there may be some solutions which are non-comparable. Also greater the value of $C(X,Y)$ better will be X as compared to Y. From the given Figure 5a, below it is noticed that $C(X,Y) = 55\%$ and $C(Y,X) = 72\%$, hence Algorithm Y performs better than Algorithm X. Furthermore, HV of Algorithm Y (Figure 5b) is wider than Algorithm X (Figure 5c), it also verifies the better performance of Y over X.

5.2 Benchmarks and parameter setting

Since no standard benchmarks are available for comparing currently proposed MO-VRPTW so, the proposed MACS algorithm is tested on standard benchmark problems of Solomon's datasets available in [37]. The 56 problem sets are broadly categorized into 6 groups namely C-1, R-1, RC-1, C-2, R-2 and RC-2. In C type problems the customers are clustered into groups, whereas in R datasets the customers are randomly scattered. RC type is a collection of partially grouped as well as partially geographically scattered customers. Moreover, type-1 problems are associated with smaller time windows while the time span for type 2 problems is large. In type-2 problems the capacity of vehicles is also large as compared to that of type-1 sets. Hence type-2 problems use smaller fleet size as compared to type-1 problem sets. All the computations have been performed on a laptop with 2.0 GB Ram, Core-2 Duo processor having speed of 2.3 GHZ using Matlab R-2013. As proposed MACS uses a number of parameters which significantly effects the performance of algorithm so optimal values of these parameters should be selected appropriately. For our experimentation we choose these values from the recent literature on use of ACS for VRPTW [32], Used values are given as in .

---Insert Table 3 Here-----------------------------------
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---Insert Figure 5 Here----------------------------------
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5.3 Simulation results

To check the performance, the proposed MACS algorithm the results are compared with other approaches available in literature considering single objective, two objectives and then all the three objectives simultaneously.

5.3.1 Comparison among different versions of proposed MACS

Here first of all we have analyzed the effect of introducing FA search and migration operators in the proposed MACS. The performance comparison among different versions of proposed MACS (i.e. MACS with FA search only, MACS with migration operators only and MACS with FA search and migration operators both) on all the considered objectives has been presented in following Table 4. First column in this table presents the dataset taken and other three main columns namely TD, Make-span and Load Balancing represent the considered three objectives. From the TD column it has been found that FA search tries to optimize the total travelled distance as it is providing better results (for all the datasets taken) as compared to results obtained from migration operators only. Alternately, Migration operators perform well for load balancing among the drivers/routes as shown in last column. However, the performance of FA and Migration operators is incomparable in case of make-span as shown by bold entries in respective column. It might be due to the fact that Migration-I tries to reduce the make-span whereas Migration-II operator will contribute to larger make-span by eliminating the routes with very small number of visited customers. The underlined italic entries in make-span column four show best make-span among all the three versions. However, these best results are the cost of unfair load balancing.

---Insert Table 4 Here-----------------------------------
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5.3.2 Comparison of Proposed MACS with some of other available techniques

In this subsection we have analyzed the performance of our proposed MACS with some of available techniques available in literature.

Observations for f_1 alone:

The proposed MACS is firstly tested on the Solomon datasets considering only single objective of minimization of total travelled distance. For this the number of ant colonies i.e. $u = 1$, consequently single pheromone matrix and only one local pheromone updation ($\tau_{ij}^{new}(1)$) is required. A total of 100 simulation runs has been performed for all the 56 datasets and the average of them compared with other proposed approaches available in literature are reported in Table 5. Proposed MACS has been compared with standard ant colony system (ACO), ACO hybridized with tabu (ACO-Tabu) p-system based evolutionary algorithm (PDVA) (all three results are reported by [10]), MOEA [38], MOGA [7], HAFA [18] and [39] and best known solutions (BKS) reported in literature. For each approach Table 5, has three rows representing total travelled distance, number of vehicle used and computation time in seconds respectively. (-) in any cell indicated that the data is not available.

It was observed that our approach performs better as compared to ACO, ACO-Tabu and EDFA in terms of travelled distance (TD) as well as number of vehicles used to complete the tour. Also for C1 and C2 datasets it is comparable to other approaches with a maximum percentage error of 1% from PDVA. For R1 and R2 problems, it appears to be better than MOEA, MOGA and EDFA in term of TD (except for MOEA and MOGA in R2). Additionally, proposed approach require less computation time (for all data sets) and produce better results (in case of R2) as compared to PDVA, however there is an error of 2%, 1% and 8% in TD of R1, RC1 and RC2 with our approach in comparison to best known solutions (BKS). Hence the overall performance of proposed algorithm (for single objective problems) is found comparable to other existing algorithms designed specially for single objective VRPTWs.

---Insert Table 5 Here----------------------------------
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5.3.3 Observations for MO-VRPTW

Next proposed MACS has been tested on same Solomon benchmarks while considering all the three objectives simultaneously. Since NSGA-II (Non-dominated Sorting GA) is one of the successfully used evolutionary algorithm for MOPs [34], [40], [41] so here the NSGA-II is taken as comparative algorithm for the proposed approach. The approach used by [34] has been adopted to fit NSGA-II for MO-VRPTW and accordingly a population of 200 chromosomes, and 100000 generations has been used. The cross-over rate and probability of mutation are .6 and .3 respectively. Again a total of 2000 simulation runs has been performed for both the algorithms and the average values of the metrics defined in Section 5.1 has been presented in Table 6. Since true Pareto fronts are unknown for current problem so, Pareto fronts obtained using current approach has been taken as true Pareto fronts additionally, the value of each objective function found using nearest neighbor approach has been taken as reference point for calculating hyper-volume (HV).

Bold entries in Table 6, depict the better values among the two compared approaches. Last row in the Table 6 summarizes the overall results of 56 instances as (b/s/w), indicating that proposed MACS performs better, nearly equal to and worse than NSGA-II respectively.

---Insert Table 6 Here----------------------------------
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Considering HV metric, it has been found that HV values ranges from 0.32 to 1.03 and 0.37 to 1.68 for NSGA-II and MACS respectively confirming better Pareto fronts of proposed approach in contrast to that of NSGA-II. Furthermore, it has been also concluded that, HV obtained using MACS is better in 49 instances with a maximum increase of 178% for RC-107, and minimum increase of 6% for R-211 with an average increase of 57%. On the other hand, NSGA-II appears to better in only 5 cases with a maximum rise of 37% (for R-205). The superiority of proposed approach can also be supported with the results obtained while comparing C-metric value. It has been found that in 42 cases (out of 56), proposed MACS dominate the NSGA-II, while the later could dominate the former only in 6 cases. Moreover, both the approaches are non-comparable (i.e. no one dominated another) in 8 cases. On comparing computation time needed for the two approaches, our algorithm performed better in all the 56 instances with nearly one tenth of time need as compared to NSGA-II. Hence, proposed approach proved to be more stable. The two algorithms are further compared on the basis of total travelled distance and Make-span (the two most conflicting objectives). Resultant graphs of six problems (one from each category of Solomon Dataset) have been depicted in Figure 6. From all the graphs, the superiority of proposed MACS over better NSGA-II both in terms of solution quality and solution diversity has been confirmed.

---Insert Figure 6 Here-----------------------------------
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6 Conclusion and scope for future work

In this paper we have investigated multi objective vehicle routing problem with time windows focusing on simultaneously minimization of total travelled distance, make-span and route imbalance using ant colony system. The ant system is divided into multiple colonies to deal with each objective separately. Additionally, multiple pheromone matrices are used to update the pheromone according to obtained Pareto front solutions. FA has been hybridized to avoid being trapped into local optima. Two new local search operators namely Migration-I and Migration-II have been designed to further improve the solution quality. The algorithm has been validated by testing on 56- Solomon benchmark problems considering minimization of total travelled distance. Furthermore, MACS has been proved much better than widely used multi-objective NSGA-II while considering multiple objectives of VRPTW. Proposed MACS can also be easily adapted to many-objective problems, by just changing the number of colonies and pheromone matrices corresponding to each objective.

In future, proposed approach can be further extended to real life applicability of VRPTW such as VRPTW with split delivery, stochastic VRPTW and time dependent VRPTW etc. Future works also include experimentation on hybridization of proposed MACS with EAs to solve many-objective real life VRPTWs.

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Figure 5. (a) C-Metric (Algorithm X, Algo. Y) (b) Algorithm Y Hyper-volume (c) Algorithm X Hyper-volume

Figure 6. Make-span vs Total Travelled Distance

Table 7. Some other Related work

Author/s	Problem	Objectives	Approach	Metric	Data Set
[19]	DVRPTW with fuzziness	TD, Total Travel Time, Max Customer, Preference, NV	GA	----	Solomon
[20]	MOVRPTW	TD, NV	EA	Coverage	Solomon
[17]	MOVRPTW	TD, Make Span, Total Waiting Time, NV	LS	IGD, HV, Coverage metric	Real Life
[21]	MDVRP	Reachability Time, Profit, Satisfaction Level, NV	PSO	----	Real Life
[22]	MOVRP	TD, NV	EA	HV	Solomon
[23]	MOVRPTW	Customer Satisfaction, Energy Consumption NV	EA	----	Random
[24]	MOVRP	Total Travel Time, Total Reliability	FA	Spacing Metric	Real World
[25]	MOMDVRP TW	TD, NV, Makespan, Total Waiting Time and Total Delay Time	EA	HV	Real World
[26]	MOVRPTW	Total Travel Time, Servicing Level	Memetic Algorithm	----	Modified Solomon

Table 8. Mathematical notations used in MO-VRPTW

Symbol used	Description
X_{ij}^{k}	Binary variable (0/1), equals 1 if edge (i-j) is followed by vehicle k, otherwise 0.
d_{ij}	Distance between node i and j
K	maximum number of vehicles
Q	maximum capacity of each vehicle

q_i	demand of customer i
s_i	service time at customer i
w_i	waiting time at customer i
T_i^k	service start time at customer i by vehicle k
$[e_i, l_i]$	Earliest start time and latest start time respectively specified for customer i

Table 9. Decision Parameters

Parameter	**Description**	**Value**
N_{max}	Max no. of iterations	5000
m	Number of ants	10
φ_0	Exploration vs exploitation decision	.9
α	Importance of pheromone	1
β	Importance of heuristic	5
ρ	pheromone evaporation constant	.1

Table 10. Performance comparison among different versions of MACS

Data Type	**TD**			**Make-span**			**Load Balancing**		
	MACS with FA only	**MACS with Migration only**	**Proposed MACS**	**MACS with FA only**	**MACS with Migration only**	**Proposed MACS**	**MACS with FA only**	**ACS with Migration only**	**Proposed MACS**
R101	1847.63	1882.16	1650.799	**142.16**	143.93	132.49	27%	35%	56%
R102	1532.04	1583.94	1486.859	143.04	**141.26**	134.25	47%	57%	70%
R103	1391.27	1444.46	1292.675	**142.16**	143.93	132.49	37%	44%	57%
R104	1158.03	1193.25	1007.31	145.62	**141.78**	134.25	27%	43%	61%
R105	1465.41	1498.04	1377.11	140.37	**136.42**	132.49	29%	46%	58%
R106	1338.15	1375.26	1252.03	**143.04**	146.94	134.25	28%	40%	56%
R107	1209.82	1255.1	1104.655	140.37	**136.42**	132.49	47%	60%	72%
R108	1024.32	1072.17	960.8753	145.62	**136.42**	134.25	37%	48%	59%
R109	1257.8	1302.18	1194.734	**140.37**	141.78	132.49	57%	68%	76%
R110	1204.06	1256.2	1118.838	**145.62**	146.94	134.25	37%	48%	59%
R111	1201.97	1249.14	1096.726	**140.37**	145.62	132.49	47%	63%	79%
R112	1083.14	1152.9	982.1392	**145.62**	146.94	134.25	37%	49%	62%
R201	1426.85	1500.07	1252.371	481.6	**466.07**	427.87	72%	78%	81%
R202	1256.22	1322.62	1191.703	492.71	**458.26**	438.72	90%	91%	94%
R203	979.65	997.44	939.5029	**419.72**	458.26	358.4	72%	83%	90%
R204	907.06	907.06	825.5188	***419.72***	501.4	476.65	90%	91%	93%

R205	1083.16	1107.14	994.4272	***365.27***	384.28	384.28	74%	80%	87%
R206	998.14	1052.84	906.1416	365.27	**351.97**	335.37	89%	91%	93%
R207	953.18	1008.26	890.6078	***365.27***	453.27	453.27	91%	92%	93%
R208	848.01	943.16	726.8223	***365.27***	379.82	367.44	91%	91%	93%
R209	1024.11	1097.52	909.1629	330.16	**328.41**	316.34	87%	91%	91%
R210	1032.9	1097.52	939.3722	***310.16***	328.41	324.46	89%	89%	94%
R211	949.62	997.44	885.7109	481.6	**481.6**	469.24	87%	88%	91%
RC101	1803.89	1881.83	1696.949	***151.04***	173.85	162.7	72%	80%	86%
RC102	1701.2	1762.07	1554.747	**173.16**	173.85	162.7	72%	81%	85%
RC103	1345.12	1389.04	1261.671	173.16	**149.06**	142.73	66%	71%	82%
RC104	1261.95	1321.17	1135.479	**140.84**	152.24	134.45	90%	90%	93%
RC105	1802.65	1881.83	1629.436	***151.04***	170.81	167.05	74%	83%	89%
RC106	1508.23	1544.04	1424.733	**173.16**	173.85	167.76	88%	91%	91%
RC107	1295.26	1321.17	1230.477	***140.84***	152.24	150.63	46%	59%	73%
RC108	1198.46	1237.85	1139.821	**145.58**	170.81	136.29	90%	92%	93%
RC201	1510.02	1568.06	1406.94	**462.88**	481.4	447.66	88%	90%	93%
RC202	1446.37	1506.24	1365.645	557.16	**543.02**	526.87	74%	81%	89%
RC203	1123.85	1144.16	1049.624	510.96	**503.34**	498.26	60%	64%	76%
RC204	849.01	919.43	798.4632	343.14	**343.14**	321.72	69%	72%	84%
RC205	1391.43	1402.18	1297.647	436.02	**423.5**	418.43	75%	81%	90%
RC206	1208.27	1256.92	1146.317	**499.16**	510.12	476.61	70%	76%	82%
RC207	1203.18	1256.92	1061.144	436.02	**431.67**	419.83	55%	66%	79%
RC208	861.48	880.06	828.1411	343.14	**316.9**	309.42	79%	80%	86%
C101	857.16	861.28	828.9366	141.4	**129.75**	127.3	70%	74%	81%
C102	839.15	857.16	828.9366	141.4	**130.08**	127.3	68%	73%	74%
C103	863.25	863.25	828.0646	138.07	**132.92**	128.04	70%	73%	82%
C104	849.46	857.16	824.7765	141.4	**130.08**	128.04	50%	59%	69%
C105	901.12	909.26	828.9366	138.07	**135.6**	127.3	49%	59%	68%
C106	863.38	884.62	828.9366	138.07	**132.92**	127.3	38%	49%	59%
C107	863.38	909.26	828.9366	138.07	**135.6**	127.3	68%	72%	80%
C108	849.46	863.25	828.9366	141.4	**127.3**	127.3	39%	51%	70%
C109	863.38	884.62	828.9366	138.07	**129.75**	127.3	48%	58%	68%
C201	603.14	647.62	591.5563	262.76	**251.06**	238.21	87%	90%	92%
C202	603.14	647.62	591.5563	262.76	**251.06**	238.21	86%	91%	92%
C203	617.26	650.14	591.1732	273.14	**257.81**	238.21	88%	91%	93%
C204	642.32	642.32	590.5985	273.14	**249.01**	238.21	76%	82%	91%
C205	617.26	647.62	588.8757	273.14	**251.06**	235.53	88%	91%	90%
C206	642.32	650.14	588.4926	262.76	**257.81**	235.53	66%	70%	78%
C207	617.26	647.62	588.286	273.14	**251.06**	235.53	85%	89%	90%
C208	607.24	624.46	588.3235	273.14	**249.64**	235.53	83%	87%	92%

Table 11. Comparison with other approaches considering single objective f1

Data set	C1	C2	R1	R2	RC1	RC2
ACO [10]	881.44 10 112	641.25 3.5 89	1383.2 13.6 405	1098.22 4.1 395	1211.12 12.7 204	1209.44 5.6 188
ACO-Tabu [10]	841.92 10 210	612.75 3.3 142	1213.16 13.1 698	952.3 4.6 655	1415.62 12.7 317	1120.37 5.6 407
MOEA [38]	828.38 10 -	591.49 3 --	1236.17 13.89 -	912.81 4 -	1392.09 12.63 -	1162.4 5.13 -
MOGA [7]	828.48 10 -	590.6 3 -	1253.86 13.89 -	907.38 4.37 -	1370.84 12.75 -	1070.38 4.75 -
PDVA [10]	828.38 10 3800	591.49 3 4000	1228.6 12.92 3800	1033.53 3.45 3900	1362.09 12.75 3800	1068.26 3.75 7700
HAFA [33]	843.32 10 -	599.04 3 -	1244.48 13.18 -	976.2 3.18 -	1347.72 12.96 -	1163.98 3.5 -
EDFA [39]	907.11 10 -	666.23 3 -	1442.71 13.18 -	1243.18 3.18 -	1568.94 12.96 -	1490.36 3.5 -
Proposed MACS	828.38 10 500	599.04 3 2000	1250.61 13.9 3600	1024.41 3.5 3750	1374.82 13.1 3400	1158.97 4 3400
BKS	828.38 10 -	589.86 3 -	1209.89 12.92 -	951.19 3.45 -	1245.79 12.37 -	1119.35 4 -

Table 12. Comparison with NSGA-II on Average values of HV, C-metric and computation time on Solomon benchmarks considering all three objectives

Data Type	HV		C-Metric		Computation Time (s)	
	NSGA-II	MACS	C(NSGA-II, MACS)	C(MACS, NSGA-II)	NSGA-II	MACS
R101	0.4357	**0.6267**	0.3233	**0.3964**	11575	**2028**
R102	0.4227	**0.6093**	0.3901	**0.4216**	13229	**2219**
R103	0.4906	**0.8922**	0.3531	0.3528	14707	**2127**
R104	0.4853	**0.8698**	**0.4002**	0.3542	10348	**2563**
R105	0.3156	**0.5549**	**0.5216**	0.2709	14264	**2214**
R106	0.3769	**0.5483**	0.4137	**0.431**	14314	**2302**
R107	0.4227	**0.665**	0.3143	**0.5249**	15938	**1997**
R108	0.4373	**0.6034**	**0.5609**	0.5385	12649	**2553**
R109	0.4706	**1.0533**	0.2145	**0.6085**	11872	**2626**
R110	0.4518	**1.1976**	0.4722	**0.5102**	16439	**2321**

R111	**0.3905**	0.363	0.4308	**0.467**	9507	**1964**
R112	**0.532**	0.4904	0.4461	**0.4793**	13108	**1927**
R201	0.7001	**1.0962**	0.4678	**0.5106**	14482	**1900**
R202	0.6806	**1.045**	0.2534	**0.3891**	10990	**2119**
R203	0.6477	**0.904**	0.3502	**0.6215**	12077	**2310**
R204	0.5908	**1.025**	0.4583	0.4583	10463	**2171**
R205	**0.9021**	0.5645	0.4426	**0.5238**	12467	**2444**
R206	0.6386	**0.9455**	0.438	**0.4904**	14182	**2634**
R207	0.6102	**0.8349**	0.4398	**0.46**	11108	**2627**
R208	0.5344	**0.7046**	0.4701	**0.5113**	11562	**2537**
R209	0.5906	**1.1609**	**0.6218**	0.5365	13608	**2750**
R210	0.6004	**0.9324**	0.3204	**0.4128**	12427	**2600**
R211	0.5284	**0.5592**	0.338	**0.6581**	11352	**2154**
RC101	0.5407	**1.0042**	0.2932	0.2947	13134	**2156**
RC102	0.4642	**0.7967**	0.2541	**0.6548**	11729	**1947**
RC103	0.5495	**0.9821**	0.4205	**0.5488**	10821	**2090**
RC104	0.4882	**1.1523**	0.4273	**0.5679**	12587	**1888**
RC105	0.379	**0.7503**	0.4094	**0.5802**	10403	**1793**
RC106	0.456	0.4557	0.3984	0.3992	13788	**2068**
RC107	0.44	**1.2238**	0.4386	**0.5578**	10555	**2515**
RC108	**0.5186**	0.4816	0.4901	0.4943	9140	**2137**
RC201	0.8221	**0.9084**	0.4374	**0.6075**	14356	**2881**
RC202	0.7802	**1.6803**	0.4607	**0.4632**	14496	**2719**
RC203	0.8379	0.8388	**0.6255**	0.3578	12875	**2410**
RC204	0.6846	**1.4303**	0.449	**0.5672**	11332	**2291**
RC205	0.7205	**1.1266**	0.4916	0.4908	14697	**2752**
RC206	0.6793	**1.3835**	0.2729	**0.7041**	13076	**1876**
RC207	0.4609	**0.584**	0.3886	**0.6248**	12167	**2153**
RC208	0.6628	**0.8764**	0.2908	**0.6108**	12534	**1825**
C101	0.9692	**1.5189**	0.3871	**0.5389**	10256	**2282**
C102	1.039	**1.615**	0.3442	**0.6304**	4515	**929**
C103	**0.9255**	0.697	**0.316**	0.2938	4833	**922**
C104	0.5612	**0.7724**	0.3549	**0.4802**	7371	**982**
C105	0.6378	**0.9278**	0.3004	**0.6103**	8604	**1012**
C106	0.6004	**1.2699**	0.3794	**0.5349**	5693	**1127**
C107	0.6128	**0.933**	0.3052	**0.4586**	8679	**1050**
C108	0.5587	**1.2751**	0.2556	**0.4702**	8854	**1148**
C109	0.728	**1.009**	0.2578	0.2594	3225	**1138**
C201	0.605	**1.3785**	0.3557	**0.4027**	5706	**835**
C202	0.5892	**1.034**	0.4314	**0.4805**	4928	**1132**

C203	**0.5904**	0.5822	0.4372	**0.5621**	3920	**858**
C204	0.5591	**0.8942**	0.3627	**0.5427**	7077	**810**
C205	0.5994	**0.7247**	0.4619	**0.5093**	5668	**1098**
C206	0.6237	**0.8248**	0.5243	0.5226	8516	**883**
C207	0.559	**0.7935**	**0.5738**	0.4166	5488	**866**
C208	0.6702	**0.8567**	0.4644	**0.5003**	8058	**724**
b/s/w	**49**/2/5		**42**/8/6		**56**/0/0	

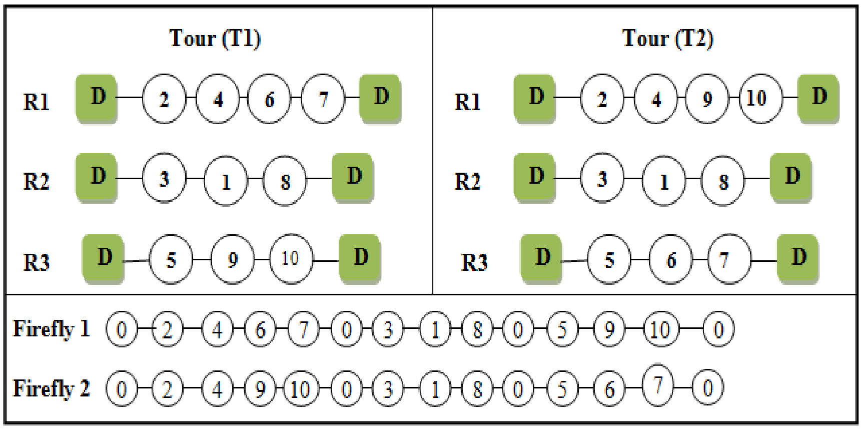

Figure 6. Firefly Encoding

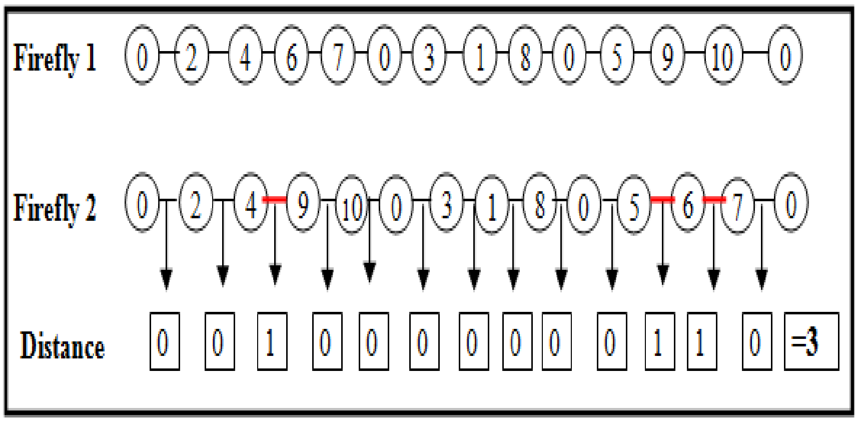

Figure 7 Distance b/w Firefly 1 and 2

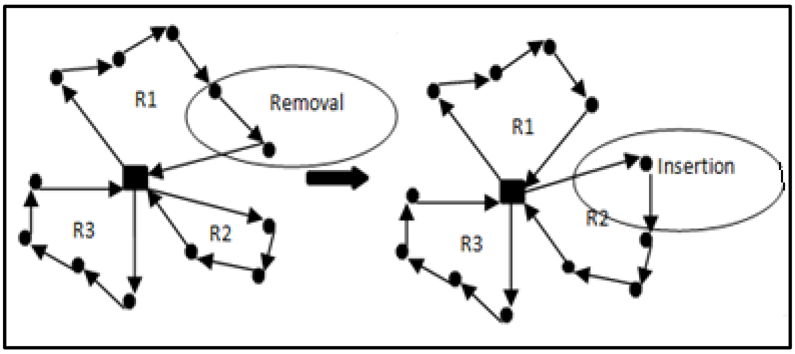

Figure 8 Migration-I operator

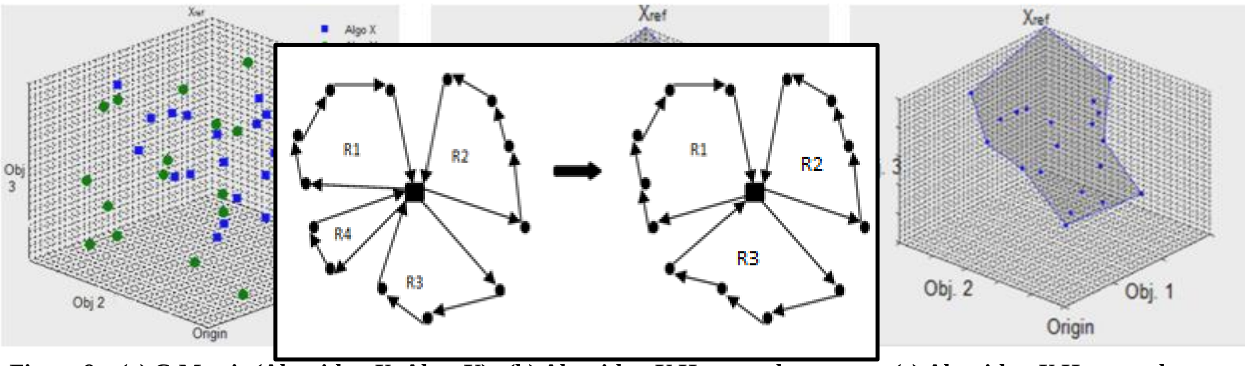

Figure 9.   (a) C-Metric (Algorithm X, Algo. Y)   (b) Algorithm Y Hyper-volume   (c) Algorithm X Hyper-volume

Figure 10 Migration-II operator

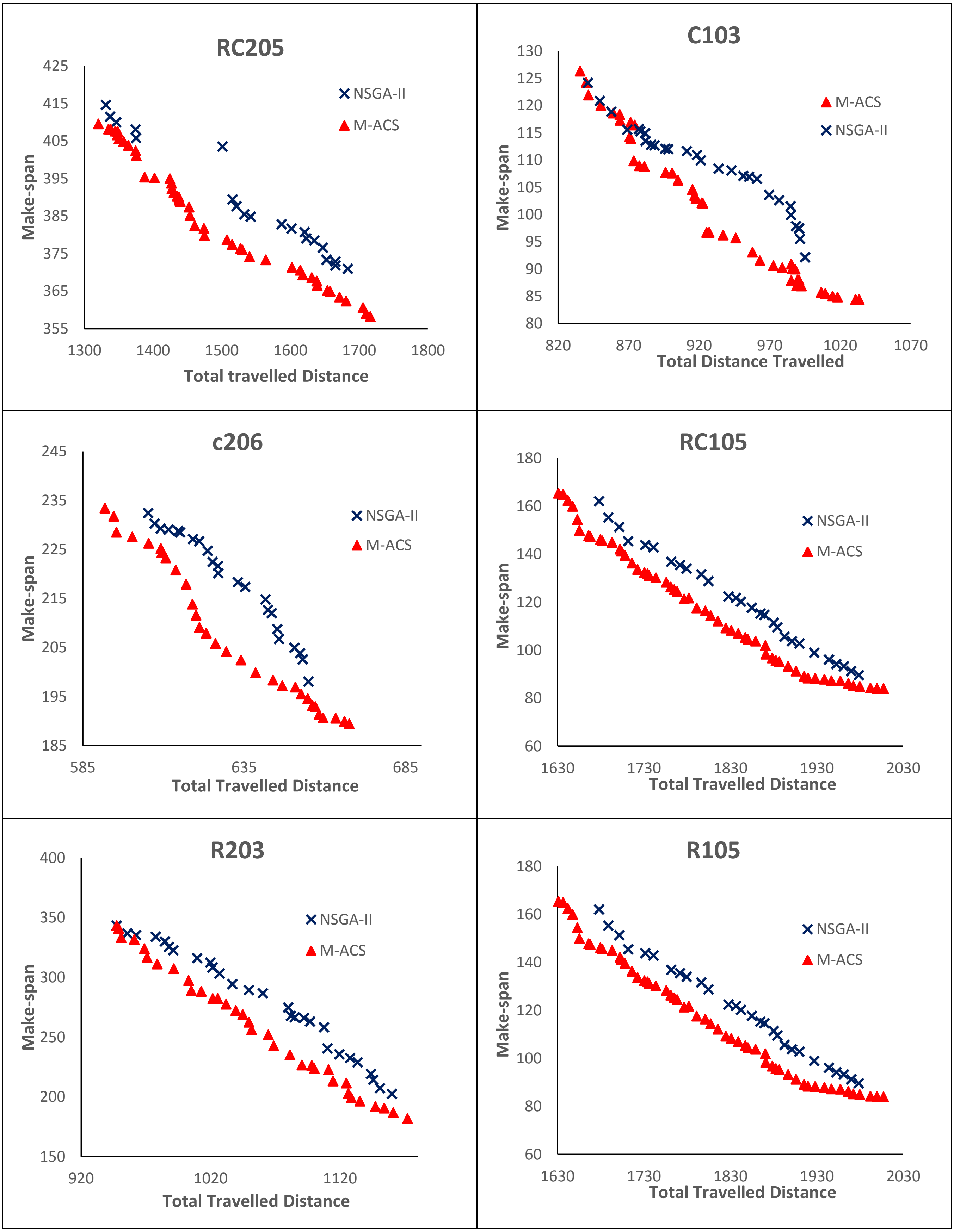

Figure 6. Make-span vs Total Travelled Distance

Short Biography

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Rajeev goel is working as assistant professor in Govt. College Naraingarh, Ambala in Computer Science Department. He has done his Graduation and Post-Graduation from Kurukshetra university Kurukshetra, India. Currently, he is pursuing his Ph.D. from Punjabi University Patiala India. His research interest includes meta heuristics and the operations research.

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Raman Maini received B.Tech (Computer Science & Engineering) from Beant College of Engineering, Gurdaspur, Punjab, India in 1999 and M.Tech(Computer Science & Engineering) from PAU, Ludhiana, India, in 2002. He got Merit certificate in his M.Tech thesis at PAU. He is currently working as a Professor in Computer Engineering at University College of Engineering, Punjabi University, Patiala, India. He is a life member of ISTE (Indian Society of Technical Education), India and IETE (Institution of Electronics & Telecommunication Engineers), India. His current area of research is Computer Vision (Specialty Noise Reduction in Medical Images, Edge Detection and Image Enhancement) and development of soft computing based algorithms.