Synchronized Timetables for Bus Rapid Transit Networks in Small and Large Cities

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Abstract

The quality of public transportation service has major effects on people’s quality of life. During frequency and timetable setting, as an important step of the public transportation planning process, there is an important and complicated issue of synchronization which can directly influence the utility and attractiveness of the system; thus, it should be taken into account during the whole planning process, especially during the frequency timetable setting step. In this paper, a mixed-integer nonlinear programming model is proposed that aims at setting timetables on a bus transit network with the maximum synchronization and the minimum number of fleet size. The proposed model is shown to be applicable for both small and large-scale transit networks by employing it for setting timetables on two samples of both sizes. A simple problem is solved by GAMS Software where the obtained timetable is substantially reasonable. Moreover, the proposed model is used to set timetables through the genetic algorithm on Tehran BRT networks as a real-life instance; then the NSGA-II is used to obtain the Pareto optimal solutions of the problem for five different scenarios. The overall results show that the proposed model is efficient for setting timetables on transit networks of different sizes.

Keywords: public transportation; bus line timetable setting; mathematical modeling; mixed-integer programming; genetic algorithm

1. Introduction

Public transportation is one of the fundamental parts of every city. The quality of the public transportation system in a city directly affects the quality of life. A reliable public transportation system can attract more passengers, while a poor public transit service can reduce the number of users\cite{1}. Nowadays, a significant number of people, especially those with low income rely on public transportation infrastructures for their daily trips. Among different modes of public transportation, buses are more common due to their flexibility, compatibility, and required investment. Therefore, the studies for improving urban bus system are significant, and with growing technology and emergence of new modes of transportation and expansion of cities, the need for further studies in this area gains more importance.
Urban bus planning process is very complicated and requires many interdependent decisions to make. According to [2-8], the process can be divided into four basic steps; (1) network route design, (2) frequency and timetable setting, (3) vehicle scheduling, and (4) crew assignment. Each step is a complicated process and consists of several operations; therefore, they are usually performed in sequence, and the output of one step is the input for the next one. The focus of this paper is on the second step of urban bus planning process, i.e., frequency and timetable setting.

Frequency setting deals with calculating an efficient service frequency for each line based on the time of day and the day of week. In timetable setting, the scheduler prepares a list of departure times according to corresponding headways of each line and estimates the arrival time of each departure to each stop on that line. The scheduler should also consider available resources such as available fleet size. Timetables provide information about departure and arrival times of services on each line for passengers. Having this information, passengers can make better decisions on arriving services at the stations and adapt themselves to the bus departure times. In another word, by having a timetable, passengers know when they should arrive at a stop to minimize their waiting time. Urban bus timetable setting problem has received much attention because of its importance in the literature.

The quality of urban bus system highly depends on the service timekeeping and arrival times of vehicles. However, arrival times are easily affected by the number of passengers using the system, and the bus schedule is closely related to the dynamic motion of buses with passengers [9,10]. To offer an efficient service, planners face the responsibility of considering the synchronization of the schedules of different lines in a transit network. A proper synchronization can be achieved by maximizing the number of simultaneous arrivals of different departures of different lines at transfer points. Petersen et al. [11] mentioned that the average waiting time for a transfer is about 9.75 minutes on weekends in the Greater Copenhagen area. Synchronization decreases the waiting times of passengers transferring between lines of a network; thus, can lead to a higher level of service and encourages more people to use public transportation. Therefore, synchronization is a common objective which has gained much attention and has been studied in other modes of transportation too, e.g., urban railway networks [12,13].

Ceder et al. [14] proposed an optimization model with the objective of maximizing the number of simultaneous arrivals. The problem was formulated as a mixed-integer programming problem, and a heuristic algorithm was developed for solving it. They described simultaneous arrivals as the arrival of two departures from different lines to a transfer point at the same time. Eranki [15] extended the model developed by Ceder et al. [14] by adding a time window to the simultaneity definition and proposed a model for timetable setting with the same objective. The new model was also formulated as a mixed-integer linear programming problem and was solved by a heuristic algorithm. The model proposed by Ceder et al. [14] has received much attention and has been extended further by other researchers [16-18]. So many studies have been devoted to transfer time minimization [19-24]. Gao et al. [25] proposed a bi-level programming technique to deal with frequency setting problem. The upper-level problem considers minimizing an objective that consists of in-vehicle and waiting time and frequency setting cost. The lower-level problem considers the path alternatives for passengers. A heuristic algorithm based on sensitivity analysis is developed to solve the model and set the optimal frequencies.

Bookbinder and Désilels [26] proposed a transfer optimization model in which travel times are considered stochastic and follow a truncated exponential distribution. The objective is to minimize total passenger inconveniences. Also, a heuristic algorithm is developed by Ting and Schonfeld [27] considering slack times for optimizing connecting lines. Knoppers and Muller [28] studied the
chances of optimized transfers in order to minimize passenger transfer times. They considered probabilities and limitations of synchronized transfers and showed that when crossing lines have high frequencies, optimal synchronization is hard to achieve. The synchronization strategy proposed in their study enables the vehicle handling the second part of the trip to delay its departure if the transferring passengers are supposed to arrive soon; therefore, sometimes transfer times are increased. Yu et al. [29] proposed an optimization model for bus transit network aiming for transfer minimization and passenger flow per unit length maximization. A coarse-grain parallel ant colony algorithm (CPACA) was used for solving the problem. The results show that a bus transit network with optimal transfers and less waiting times is an accessible goal. Ant colony algorithm has been used by other researchers as well [30,31]. Heuristic approaches have been used by many researchers for transfer optimization and synchronization among other steps of the urban transportation process [32]. Fleurent et al. [33] used the idea of weighted transfers to describe the concepts used in Hastus commercial software for producing synchronized timetables. Furthermore, while concentrating on minimizing the transfer waiting times, Castelli et al. [34] proposed a mathematical model for scheduling problems. As one of the features of the proposed model, the number of dispatches does not depend on the frequency bounds but depends on the service quality and its costs. Service quality is evaluated by the sum of the transfer waiting times of all the passengers whose boarding and alighting times should be available. However, the authors approve that despite the limiting suppositions, their model cannot be used for optimizing a large scale network because of its numerous variables; therefore, a Lagrangian heuristic method is proposed, too. Rapp and Gehner [35] proposed a heuristic method for optimizing the transfer delays. The survey describes four coordinated processes for a practical transit planning. The operational tool for optimizing the transfer delays changes the departure times from terminal automatically and iteratively. The results indicate that the optimized timetable efficiently minimizes total transfer delay times and in comparison to manual scheduling causes no additional cost. Chakroborty et al. [22,23] focused on the application of Genetic algorithms on waiting time determination in transit networks in order to minimize the total passenger waiting times. The proposed mixed-integer nonlinear model is formulated and a hybrid genetic algorithm is selected for implementation purposes. The genetic display of a thorough timetable consists of a set of binary numbers that represent the headways and the stop times between successive vehicles on all lines; however, it is mentioned that the proposed display might not be suitable for large transit networks.

Genetic Algorithm has also been applied to bus scheduling problem. Deb and Chakroborty [36] formulated an optimization problem for a bus transit system whose objective is to minimize the total passenger waiting times (with or without transfers) while satisfying the resource and service constraints. The study proves that the genetic algorithm is ideal for such problems. Chakroborty et al. [37] combined transfer synchronization with vehicle scheduling as a mixed-integer nonlinear problem. The problem, which aims at determining the fleet size and setting timetables with the objective of minimizing the passenger waiting times, is solved using the genetic algorithm. It should be noted that the problem is limited to one transfer point which is the intersection of multiple lines and it seems that the genetic display is not suitable for large problems. Ngamchaim and Lovell [38] presented a new transfer optimization model considering the bus transit route designing and solved it using the genetic algorithm. The bus routes are designed in two phases. The route improvement algorithm employs the genetic operators, and another heuristic approach is used for headway synchronization in order to improve the efficiency of the system. The route improvement algorithm reduces the transfer times and the total cost. Moreover, headway synchronization methods are used for service frequency setting. Cevallos and Zhao [20] proposed an approach based on the genetic algorithm at the network level for bus transfer time optimization. The proposed algorithm attempts to
find the best possible solution for the transfer time optimization problem through timetables movements. The authors aim to minimize the transfer waiting times of an existing timetable under strict evenly spaced headway constraints. The results indicate that the proposed algorithm efficiently reduces the waiting times. Beside these studies, genetic algorithms have been widely used by many researchers for different aspects of transit planning and optimization [39-47]. Table 1 presents a summary of the aforementioned research papers.

As previously stated, the mathematical model developed by Ceder et al. [14] has been used as a precedent by many researchers. In this paper, we develop a model to increase the simultaneous arrivals of buses in the network as first developed by Ceder et al. [14]. Moreover, we employ a different definition of simultaneous arrivals which is more practical, effective and realistic. The objective function of maximizing the simultaneous arrivals, and considering a time window as an allowable deadline for simultaneity are the concepts which have been borrowed from previous studies [14,15]. The timetable setting problem is formulated as a mathematical programming model whose implementation and solution methods on both small and large-scale networks are discussed and exemplified separately.

The remainder of this paper is organized as follows. Section 2 consists of the definition of the timetable setting problem with optimal synchronization. Section 3 presents the procedure of formulating the problem as a mathematical programming model. Section 4.1 presents a brief discussion on the scale of the problem. Converting the proposed bi-objective problem into a single objective one is described in Section 4.2. Numerical examples of both small and large scale networks are introduced and solved in Section 4.3 and 4.4 respectively, and in Section 5 the results are discussed and a conclusion is presented.

2. Problem Definition

Looking at the timetable setting problem from different angles, one can consider different objectives for the problem. According to Ceder et al. [14] a very important objective is to minimize the transfer waiting times that can be achieved by maximizing the number of simultaneous arrivals. Since in a large public transit network, all of the origins and destinations are not directly connected, several transfer points are needed to cover most of the area. However, an extra waiting time is imposed on the passengers that intend to travel between different lines of a network via transfer points. A perfect timetable is set only when the waiting time in transfer points are minimized by the simultaneous arrival of vehicles at transfer points. Figure 1 illustrates the importance of transfer points and simultaneous arrivals at transfer points. As shown in Figure 1, a passenger that intends to travel from zone A to zone B has to transfer from line 2 to line 1, since there is no direct link between these two zones. The transfer should be made at stop 12 that is the transfer stop of two lines. If the arrival of the bus on which the passenger is riding and the bus the passenger needs to catch to continue the trip are not much apart, the transfer waiting time will be short, and the trip would remain attractive for the passenger. However, the arrival time of the buses on each line is an important issue. For instance, in this case, when the transferring passenger arrives at the stop 12 at 8:00, there is a short time needed to get off the first bus, walk to the next stop point and get on the immediate next bus on line 1 which arrives at stop 12 at 8:05. Therefore, the passenger has a short time to move between the buses. The time interval between the buses arriving at the transfer point from different lines could be adjusted according to the time of day and system frequency. During peak hours, since the service has a high frequency the time intervals need to be very short, preventing possible delays, and during non-peak
hours since the service frequency is lower, a small increase in the waiting time would not affect the trip attractiveness much. Furthermore, the interval also depends highly on the overall characteristics of the intersecting lines. For example, if one of the lines is in suburban areas, it is important to keep in mind that the service frequency would generally be low on that line even during the peak hours and the interval should not be too short as it is for intersecting Central Business District (CBD) lines. In real life the timetable scheduler cannot expect passengers to plan their trips such that they minimize their transfer time. The only thing they can do is to adapt themselves to the departure times at their origin. Thus it is the scheduler’s task to synchronize the arrivals at transfer points in order to reduce the transfer time. Besides, the scheduler should plan the departures according to passenger demand trends in timetable setting process. Otherwise, passenger demand is not satisfied, and vehicles will be delayed by facing longer dwell times, and bunching phenomenon can occur [2].

A good timetable should also be cost-effective, that is in addition to satisfying passenger demand, it should also take into account the benefits of the transit company. Resource constraint is an important issue for all transit companies. However, they can attract more passengers by optimizing the current service. If the proposed synchronization objective is considered alone, the optimal solution will set the headways of all lines at their minimum to maximize the service level which will also increase the number of required vehicles. Therefore, another important objective is to minimize the required vehicles. The fleet size is an important factor and can greatly affect the company’s costs. Headways, as the time between two successive departures, can be evenly or unevenly spaced. If the headways are evenly-spaced, the vehicles are departed in regular and equal intervals. In a period that headways are evenly-spaced, the number of required vehicles on a line can be calculated by equation 1,

$$fleetsize = \frac{average\text{-}round\text{-}trip\text{time}}{headway}$$ (1)

In this paper, by considering the objectives mentioned above, a bi-objective mixed-integer programming model is proposed whose mathematical formulation is presented in the next section.

3. Problem Formulation

The notations used in the mathematical model are presented in Table 2.

The objectives of the problem are formulated as Equations 2 and 3. The first objective maximizes the number of synchronized arrivals at transfer points. The second objective is defined in order to minimize the number of required vehicles for the system.

$$Max \ Z_1 = \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{p=L}^{L} \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{t=1}^{T} y_{mnpklt}$$

(2)

$$Min Z_2 = \sum_{i=L}^{I} FS^i$$

(3)

$$x_i^l \leq H max_I^l \quad l \in L, t \in T$$

(4)
\[ x^l_t + (ND^l_t - 1)H^l_t \leq U \quad l \in L, t \in T \] (5)

\[ H \min^l_t \leq H^l_t \leq H \max^l_t \quad l \in L, t \in T \] (6)

\[ A^l_{nt} = U(t - 1) + x^l_t + (n - 1)H^l_t + \sum_{k=1, k \neq s}^{i-1} R^l_{k,k+1,t} + \sum_{k=1, k \neq s}^{i-1} dw^l_{k,t} \]

\[ l \in L, n \in D^l_t, i \in S', t \in T \] (7)

\[ y^l_{mnpq} = \begin{cases} 1 & |A^l_{mnp} - A^l_{nq}| \leq \delta^l_{kp} \quad l, k \in L, m \in D^l_t, n \in D^k_t, q \in C^l_t, \\ 0 & \text{otherwise} \quad t, p \in T, l \neq k, l < k \end{cases} \] (8)

\[ FS^l_t = \max_{z \in T} \left( \frac{CT^l_z}{H^l_t} \right) \quad l \in L \] (9)

\[ y^l_{mnpq} \in \{0, 1\} \quad l, k \in L, m \in D^l_t, n \in D^k_t, q \in C^l_t, t, p \in T, l \neq k, l < k \] (10)

\[ x^l_t, H^l_t \in \mathbb{N}^+, l \in L, t \in T \] (11)

The constraints of the model are formulated as Equations (4) to (11). Constraint (4) guarantees that the first departure would take place in a period shorter than the maximum allowable headway. Constraint (5) assures that the last departure of each period is set before the end of the corresponding period. Constraint (6) limits the headway of each period not to exceed the allowable range. Equation (7) calculates the arrival time of each departed vehicle at each stop. Constraint (8) defines the binary variable \( y \) introduced in Table 2. It compares all the arrival times which are calculated by Constraint (7) and when the interval between two arrivals from different lines at a transfer point is less than the maximum allowable time window \( (\delta^l_{kp}) \), \( y \) takes 1, otherwise it remains equal to 0. Equation (9) calculates the minimum number of required vehicles on each line which is equal to the maximum number of required vehicles in each period on the same line. Constraints (10) and (11) define the mathematical characteristics of the decision variables.

It is worth mentioning that in order to maintain the practicality of the problem, the following two constraints should also hold:

\[ U \geq (ND^l_t - 1)H \min^l_t \quad \forall l \in L, t \in T \] (12)

\[ U \leq ND^l_t.H \max^l_t \quad \forall l \in L, t \in T \] (13)

According to Constraints (12) and (13) departures that are planned for a period do not violate the corresponding time interval. The presented mathematical formulation is the most comprehensive form which can be simplified for implementation purposes.

4. Solution Approaches

The proposed comprehensive model is widely applicable to networks with different sizes; therefore, the implementation approach is discussed in two sections for both small and large networks.
4.1 Scale of the Problem

Although binary and discrete variables increase the complexity of the problem, they are often used in mathematical modeling, yielding a mixed integer programming problem. As described in the previous section, the principal decision variables of the proposed model, i.e. simultaneity and headway variables are binary and discrete variables, respectively. Moreover, the nonlinearity of mathematical equations and constraints is, in some cases, unavoidable. For example, constraint 8, defining the simultaneity phenomena, couldn’t be linearized efficiently, i.e. without increasing the number of constraints and variables, thus, leading to a more complicated problem. Furthermore, dimensions of the problem play an important role in complicating the problem and increasing the computation time. The complexity of the proposed mathematical model which consists of binary and discrete variables, nonlinear constraints and several dimensions could be usually judged by the number of integer variables of the problem. Variable \( y_{lk}^{npqp} \) demonstrates the simultaneous arrivals of the vehicle \( m \) in period \( t \) on line \( l \) and vehicle \( n \) in period \( p \) on line \( k \) at transfer stop \( q \). Accordingly, there is a binary variable for the combination of each two trips on every two lines which intersect at stop \( q \). Assuming that \( L \) is the total number of lines on a network which at some point intersect with other lines, \( D \) is the average number of departures being planned on each line in a given period, \( C \) is the total number of time periods, then the number of integer variables \( y_{lk}^{npqp} \) would be \( T^2 \times C \times L^2 \times D^2 \). In a more realistic approach if \( C \) is substituted by the average transfer stops between each two lines, and the number of variables would be equal to \( T^2 \times L^2 \times D^2 \) which would be a large number in real networks. Moreover, the number of the variables associated with the fleet size, \( FS \), assuming \( H = (H_{max} - H_{min}) + 1 \) as the number of possible values for headways on each line in each period, would be equal to \( T \times L \times H \). Consequently, the feasibility of efficiently implementing the proposed model on real-world transit networks with different sizes is quite controversial.

Additionally, regarding computational complexity theory, the basic model developed by Ceder et al.[14] which aims at maximizing the number of simultaneous arrivals is proved to be an NP-hard problem [48] whose complexity provides enough incentive for the authors to consider heuristic algorithms as a solution approach. Hence, the attempted problem in this paper is classified in the NP-hard class of problems.

4.2 Converting the Problem to a Single Objective Problem

In order to solve the problem with single objective solution approaches which are proposed in this paper, it needs to be converted into a single-objective problem; hence, the linear weighted-sum method is employed for this purpose. Equation 14 shows the objective function of the problem converted by the linear weighted-sum method.

\[
\text{Max} Z = (C_1 \sum_{m \in L_l} \sum_{n \in L_n} \sum_{t \in T} \sum_{k \in L} \sum_{p \in T} \sum_{q \in C} y_{lk}^{npqp} - C_2 \sum_{l \in L} FS_l)
\]  

(14)

Equations 15, 16 and 17 show the additional constraints added to the model by using this method.

\[
C_1 + C_2 = 1
\]  

(15)

\[
C_1 \geq 0
\]  

(16)

\[
C_2 \geq 0
\]  

(17)
In order to convert the bi-objective mathematical problem into a single objective one, the relative importance of the objectives should be assigned to their corresponding function as their weights. The weight values are very important and delicate; since they directly affect the final solution of the problem. Hence, much study is required for determining the weight values. There are also some popular methods for this purpose, such as brainstorming and the Delphi methods; however, the ultimate decision should be made by the management team members who are known as the most experienced decision makers in this field. In this study, considering the monetary value of each simultaneous arrival, which saves passengers’ time and the cost of each vehicle, which can provide more simultaneous arrivals, the weights are assigned as $C_1=0.23$, $C_2=0.77$ only to provide an illustrative example of using methods designed for single-objective models.

Finally, in order to scale the objectives in the same unit and magnitude, the objective functions are normalized according to Equation 18:

$$
\tilde{F}_i = \frac{F_i(x) - F_{i,\min}}{F_{i,\max} - F_{i,\min}}
$$

(18)

where $F_{i,\min}$ and $F_{i,\max}$ are the minimum and maximum values of the objective function $i$ respectively, and the normalized objective function $\tilde{F}_i \in [0,1]$, $i=1,2,\ldots,m$ has the same design space as before [49].

For a more perceivable depiction, two simplifications are made as Equations 19 and 20, and using Equation 18, the normalized objective function is presented in Equation 21:

$$
\sum_{m=1}^{m} \sum_{nc=1}^{nc} \sum_{q=1}^{q} \sum_{l=L}^{L} \sum_{t=T}^{T} \sum_{p=1}^{p} y_{lkt} \nu_{ijklmnp} = S
$$

(19)

$$
\sum_{l=L}^{L} F S_i = FS
$$

(20)

$$
Z = C_1 \left( \frac{S - S_{\min}}{S_{\max} - S_{\min}} \right) - C_2 \left( \frac{FS - FS_{\min}}{FS_{\max} - FS_{\min}} \right)
$$

(21)

The resulting Equation 21 will be used as the objective function of the model in cases of using single objective solution methods.

4.3 Small Cities

In case of small cities which have small bus networks that consist of only a few lines, a simplified version of the proposed model can be employed for timetable setting; however, the model needs to be solved for each period of a day. Thus, the variables and parameters lose the index $t$, and the simplified model could be solved by classic nonlinear mathematical methods efficiently.

In this section, a small numerical example is presented and solved using General Algebraic Modeling System (GAMS). Figure 2 illustrates a transit network with 4 lines and 6 stops. The planning horizon is 60 minutes (the morning peak hour) and the travel times between successive stops are mentioned in Figure 2. As illustrated in Figure 3, stops 1, 3, 5 and 6 are the transfer stops of at least two line intersects.
Table 3 presents the input data of the problem. Input data includes minimum and maximum allowable headways for each line, average dwell time of the vehicles in each stop, and average round-trip time on each line. Note that the dwell times of buses at the stops of the same line are assumed equal; hence the parameter loses the stop index.

The maximum allowable time window for a simultaneous arrival ($\delta^k$) at each transfer point in the network is set equal to 2 minutes. The proposed model is used to set timetables on the example network illustrated in Figure 2 using GAMS. The problem is coded in GAMS 23.0.2 and run using DICOPT solver which is based on the OA/ER/AP algorithm, and the results are shown in Table 4.

Table 4 demonstrates the scheduled departure times of vehicles on each line during the planning horizon. The last column shows the number of required vehicles for each line. Figure 3 illustrates the occurrence of 11 of 23 consequent simultaneous arrivals at each transfer point in short intervals. Each line and its buses are recognized by a different geometric shape around the bus schematic and the time above is the arrival time of the bus at the transfer point. The same procedure can easily illustrate the remaining simultaneous arrivals.

As mentioned earlier, the best timetable would be the one that efficiently maintains the synchronization of different lines of a transit network requiring a minimum number of vehicles. Presented results show that this objective can be achieved and the model can, by this definition, provide optimal timetables under different conditions, therefore, can be used for setting timetables on a small new network as well as improving the timetables of an existing one.

### 4.4 Large Cities

The proposed MINLP formulation is difficult to be implemented on large transit networks using classic methods due to the nonlinear constraints, discrete search space, and the large size of the problem. Hence, there is a strong motivation to consider metaheuristic algorithms for solving the proposed model efficiently.

In case of transit scheduling problems, among the most popular metaheuristic algorithms, genetic algorithms have shown efficiency in several cases [39-47]. Therefore, the proposed model along with the genetic algorithms is employed to set timetables on a real-world transit network with the minimum number of required vehicles. The process of employing genetic algorithm for solving the attempted problem is described as follows.

#### 4.4.1 Metaheuristic Approaches

Genetic algorithm (GA), which is based on Darwin's theory of evolution, is an optimization algorithm developed by Holland [50]. Given the nature of the genetic algorithm which can handle only problems with one objective function, the problem is converted into a single-objective problem using the linear weighted-sum method and normalized as Equation 21.

Solving the problem with the proposed mathematical model using genetic algorithm needs proper preparations and arrangements, which are a part of the GA procedure that is illustrated as a flowchart in Figure 4 (presented by Amiripour et. al. [51]). As can be seen in the figure, an initial population of chromosomes is required first. Finding the proper configuration for the chromosomes is an essential
primary stage in the genetic algorithms implementation. It not only affects the coding process but also provides a simple illustration of the problem for a better understanding. As shown in Figure 5, the configuration of the chromosomes is illustrated as a matrix.

Each row of this matrix contains the main variables of each transit line’s timetable; the elements in each 1x2 block specify the time of the first departure and the headway of the corresponding period, respectively. The pattern is repeated for each line, and a chromosome is formed. As can be seen in Figure 5, after generating the initial population which consists of 200 chromosomes, each chromosome needs to be evaluated by a fitness function. In this case, similar to the small sample network problem, the fitness function which is the weighted linear combination of the introduced objective functions (Eq. 2 and 3) is normalized as Equation 21. All of the constraints are taken into account during the process of generating the chromosomes; hence, no penalty function is needed. After the evaluation process, the algorithm starts an iterative loop which will be repeated for a specified number of times until there are no significant improvements in the fitness value of the best chromosome.

Moreover, the non-dominated sorting genetic algorithm for multi-objective optimization (NSGA-II) first proposed by Deb et al. [52,53], is proved to be an efficient solution approach. A similar procedure to what was described is employed in the NSGA-II for a bi-objective problem; however instead of a fitness value for each chromosome, there is a fitness vector which consists of different fitness values of different fitness functions.

The following section presents a real-world case as an illustration of the applicability of the proposed model and solution approaches in large scale networks.

4.4.2A Case study – BRT Network of Tehran

In this section, the proposed model is applied to a large real-world network. Tehran BRT network, as a large scale network, consists of 9 lines (142 km), 8 of which have common stops between two or three lines, and according to the 2017 statistics, it serves an average of 2.25 million travelers on a daily basis with 900 buses. Figure 6 illustrates a plain scheme of the network. Red lines are the urban bus lines, and blue lines are the BRT lines. The proposed model and solution methods are employed to set coordinated timetables on the network.

Collecting the required data is the first step of solving the problem, and since the vehicles are equipped with Automatic Fare collection system (AFC), and Automatic Vehicle Location (AVL) system which are two main sources of data in a bus system, this can be done conveniently. As an example, the AVL system records the time of opening and closing the bus doors at each stop. The required data includes the average travel time between two consecutive stops \((R_{i,i+1,t})\) which can be calculated by subtracting the time at which the doors are opened at stop \(i+1\) from the time at which the doors are last closed at stop \(i\). Similarly, the average round-trip time of a line \((CT_l)\) can be calculated by subtracting the time at which the bus doors are closed in the last stop from the time at which the bus doors are opened in the first stop on the corresponding line. Moreover, the average dwell time at each stop \((d_{it})\) is the time between the opening and closing of the bus doors at the same stop. For this purpose, the data for the first two weeks of January 2016 is collected, refined, and sorted according to the required input parameters of the model. The maximum allowable time window for a simultaneous arrival \((\delta^{g}_{ijkl})\) at each transfer point in the network is set equal to 1 minute. The planning horizon is divided into 17 hours, and after tailoring the collected data in this format, the problem is
coded in MATLAB 2017 using the genetic algorithm. Then, the program is run on a PC with Pentium® Dual-Core CPU E5300 @ 2.60GHz and 4.00GB RAM, and the results are presented in Tables 5 and 6.

Tables 4 and 5 demonstrate the obtained first departure times, headways, and the fleet size which are found by genetic algorithm for each line for all periods during a day. By this information, a comprehensive timetable could be built which is supposed to minimize the waiting times for transferring passengers and synchronize the lines while decreasing the number of the required vehicles. The improvements and convergence trend of the genetic algorithm is illustrated in Figure 7. As can be seen in the illustration, after about 8000 iterations, the algorithm converges to the presented solution with 10628 simultaneous arrivals and 257 vehicles.

As explained before, NSGA-II is a powerful tool to obtain a more general solution for a bi-objective problem, in case the importance weights are unknown or a financial analysis on the possible solutions is necessary for the final decision to be made. In this case, five scenarios are considered for the current problem of Tehran BRT network (δ=1,...,5 minutes). For each scenario, all the Pareto optimal solutions are obtained and the Pareto fronts are illustrated as Figure 8. The parameter δ acts as a controlling parameter which directly affects simultaneous arrivals and its value should be selected according to the capacity of the system and the level of service. As δ decreases, simultaneity gets rare and hard to happen. Therefore, δ=5 is the least strict scenario which offers the minimum level of service, assumed here, and δ=1 is the strictest scenario which offers the maximum level of service. In case of the strict scenarios, the waiting time for the transferring passengers reduces but this improvement would not be possible without the decrease in headways which demands a higher number of vehicles and consequently an increase in system costs. This proves that δ value is a sensitive, controversial, and impactful parameter. It is worth noting that as δ decreases, according to the increasing diagram gradient in Figure 8, in order to obtain a higher degree of synchronization, a bigger increase in the fleet size would be needed. A Pareto optimal front as Figure 8 can help the decision makers analyze the relationship between the objective values and weights closely and choose the best objective weight values according to the company’s circumstances.

5. Conclusions

Scheduling (frequency determination and timetable setting) is one of the most important and complicated steps of the public transportation planning process. Synchronization is an important issue that should be considered during this step. Synchronization can have major effects on the public transit network attractiveness.

In this paper, by generalizing the primary concept of simultaneous arrivals and synchronization proposed by Ceder et al. [14], the frequency determination and timetable setting problem is formulated as a mixed-integer nonlinear programming model aiming for optimal synchronization of the timetables in a public transit network. The applicability of the proposed mathematical model is proved by its employment to set timetables on networks with different scales. In case of small networks, the problem is converted into a single objective problem, the formulation is simplified, and the model is implemented on a sample network with 4 lines and 6 stops as a case study. The problem is coded in GAMS and a thoroughly reasonable timetable is obtained.

Since the classic methods are not capable of solving large-scale mathematical programming problems, two solution methods are proposed for these cases. First, the genetic algorithm is employed to use the
single objective problem in order to set timetables on the BRT network of Tehran which consists of 8 lines. Secondly, the NSGA-II is employed to extract the Pareto optimal solutions and fronts in five different scenarios. The overall results show that the proposed model can be easily used for synchronizing both small and large bus transit networks taking into account the vehicle costs by minimizing the required fleet size.

The problem discussed here, is a highly complex problem which is under the influence of many correlative factors. It could be more realistic, if the stochastic nature of some parameters such as average dwell time at stops, average travel time between two consecutive stops, and average round-trip time of lines were incorporated into the model as future studies. Furthermore, there are parameters regarding the level of service such as maximum headways, and maximum allowable time window for synchronized arrivals which need discussion and decisions of the experts.

**References**


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Ravi Shankar is the Professor of Operations and Supply Chain Management in the Department of Management Studies, Indian Institute of Technology (IIT) Delhi, India. Besides having a highly productive academic career at IIT Delhi, he has undertaken teaching in different highly prestigious international academic centers worldwide. He has supervised 50 defended/completed PhDs and over 100 Masters-level thesis for the M.Tech. and MBA degree programs. His research areas of interest include Operations and Supply Chain Management, Decision Sciences, Business Analytics, Project Management, Sustainable Freight Transportation, etc.
Figure 1. The simultaneous arrival of two departures

Figure 2. Example network

Figure 3. The first simultaneous arrivals at each transfer point

Figure 4. Genetic algorithm procedure flowchart [51]

Figure 5. The configuration of the chromosomes

Figure 6. Tehran bus network

Figure 7. The GA convergence diagram

Figure 8. The Pareto optimal fronts of the Tehran BRT problem

Table 1. Literature Summary

Table 2. Table of notations

Table 3. Required parameters (in minutes)

Table 4. Departure times of vehicles and the number of required vehicles for each line

Table 5. The best first departure times found by GA in each time period

Table 6. The best headways and fleet sizes found by GA
Figure 1
Figure 2
Figure 3
Population (n)

Chromosome Structure
Each chromosome represents one timetable which is demonstrated in form of a matrix whose number of rows is equal to the number of transit lines and each row is comprised of genes that are as many as the periods. Each gene consists of two values. The first value shows the time of the first departure in the corresponding period and the second value shows the planned headway for that period. The initial population is generated randomly.

Calculate the fitness function for each chromosome in population (n)

Rank the chromosomes according to their fitness values

Stop criteria satisfied?

Yes

No

Elitism
Move the best chromosome of population(n) to population (n+1)

Crossover
Do until size(pop(n))<pop-size
- Select two chromosomes (P1, P2) as parents randomly,
- Select a random number between 0 to 1,
- if the random number < crossover rate
  Perform crossover
- Calculate the fitness value of CH1 and CH2 chromosomes,
  if fitness value(CH1 or CH2) > fitness value(P1 or P2)
    move CH1 and CH2 to population (n+1)
  else
    move P1 and P2 to population (n+1)
else
  move P1 and P2 to population (n+1)

Mutation
Do until m < mutation rate
- Select one random chromosome
- Select one random gene in the selected chromosome
- Perform mutation
  Replace the values of the selected gene with random values according to the corresponding max and min headways

n = n+1

Terminate the process and report the best chromosome

n=0

Start

Figure 4
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<th>Period n</th>
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Figure 5
Figure 6
Figure 7
Figure 8
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<td>$CT_{lt}$</td>
<td>Average round-trip time of line $l$ in period $t$</td>
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<td>Minimum headway on line $l$ in period $t$</td>
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<td>Maximum headway on line $l$ in period $t$</td>
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<td>$dw_{li}^t$</td>
<td>Average dwell time at stop $i$ on line $l$ in period $t$</td>
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<td>$\delta_{lk}^{tp}$</td>
<td>Maximum allowable time window for a synchronized arrival from line $l$ in period $t$ and line $k$ in period $p$ at a transfer point</td>
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<tr>
<td>$R_{li+1,t}^l$</td>
<td>Average travel time between two consecutive stops $i$ and $i+1$ on line $l$ in period $t$</td>
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<td><strong>Decision Variables</strong></td>
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<td>$x^l_t$</td>
<td>The interval between the beginning of period $t$ and the first departure on line $l$ in that period</td>
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<tr>
<td>$y_{mnpq}^{lk}$</td>
<td>A binary variable, which yields the value 1 if the $m$-th departure of period $t$ on line $l$ meets the $n$-th departure of period $p$ on line $k$ at transfer point $q$ within a time window $\delta_{lk}^{tp}$, otherwise it yields the value 0</td>
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<td>$FS^l_t$</td>
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