Synchronized Timetables for Bus Rapid Transit Networks in Small and Large Cities

ShervinAtaeian[a], MaghsudSolimanpur[b], S.M. Mahdi Amiripour[c],Ravi Shankar[d]*

[a]MSc Graduate Student, Industrial Engineering Department, Urmia University, Urmia, Iran
Tel: (+98)9104051784, Email: st_sh.ataieyan@urmia.ac.ir
[b]Professor, Industrial Engineering Department, Urmia University, Urmia, Iran
Tel: (+98)9144400536, Email: m.solimanpur@urmia.ac.ir
[c]Assistant Professor,Toos Institute of Higher Education,Mashhad, Iran
Tel: (+98)9127246416, Email: amiripour@toos.ac.ir
[d]Professor, Department of Management Studies, Indian Institute of Technology Delhi, New Delhi, India, Tel: (+91)9811033937, Email: ravi1@dms.iitd.ac.in

Abstract

The quality of public transportation service has major effects on people's quality of life. During frequency and timetable setting, as an important step of the public transportation planning process, there is an important and complicated issue of synchronization which can directly influence the utility and attractiveness of the system; thus,it should be taken into account during the whole planning process, especially during the frequency timetable setting step. In this paper, a mixed-integer nonlinear programming model is proposed that aims at setting timetables on a bus transit network with the maximum synchronization and the minimum number of fleet size. The proposed model is shown to be applicable for both small and large-scale transit networks by employing it for setting timetables on two samples of both sizes. A simple problem is solvedby GAMS Software where the obtained timetable is substantially reasonable. Moreover, the proposed model is used to set timetables through the genetic algorithm on Tehran BRT networks as a real-life instance; then the NSGA-II is used to obtain the Pareto optimal solutions of the problem for five different scenarios.The overall results show that the proposed model is efficient for setting timetables on transit networks of different sizes.

Keywords: public transportation; bus line timetable setting; mathematical modeling; mixed-integer programming; genetic algorithm

1. Introduction

Public transportation is one of the fundamental parts of every city. The quality of the public transportation system in a city directly affects the quality of life. A reliable public transportation system can attract more passengers, while a poor public transit service can reduce the number of users[1]. Nowadays, a significant number of people, especially those with low income rely on public transportation infrastructures for their daily trips. Among different modes of public transportation, buses are more common due to their flexibility, compatibility, and required investment. Therefore, the studies for improving urban bus system are significant, and with growing technology and emergence of new modes of transportation and expansion of cities, the need for further studies in this area gains more importance.

Urban bus planning process is very complicated and requires many interdependent decisions to make. According to [2-8], the process can be divided into four basic steps; (1) network route design, (2) frequency and timetable setting, (3) vehicle scheduling, and (4) crew assignment. Each step is a complicated process and consists of several operations; therefore, they are usually performed in sequence, and the output of one step is the input for the next one. The focus of this paper is on the second step of urban bus planning process, i.e., frequency and timetablesetting.

Frequency setting deals with calculatingan efficient service frequency for each line based on the time of day and the day of week. In timetable setting, the scheduler prepares a list of departure times according to corresponding headways of each line and estimates the arrival time of each departure to each stop on that line. The scheduler should also consider available resources such as available fleet size. Timetables provide information about departure and arrival times of services on each line for passengers. Having this information, passengers can make better decisions onarriving servicesat the stations and adapt themselves to the bus departure times. In another word, by having a timetable, passengers know when they should arrive at a stop to minimize their waiting time. Urban bus timetable setting problem has received much attention because of its importance in the literature.

The quality of urban bus system highly depends on the service timekeeping and arrival times of vehicles. However, arrival times are easily affected by the number of passengers using the system, and the bus schedule is closely related to the dynamic motion of buses with passengers [9,10]. To offer an efficient service, planners face the responsibility of considering the synchronization of the schedules of different lines in a transit network. A proper synchronization can be achieved by maximizing the number of simultaneous arrivals of different departures of different lines at transfer points. Petersen et al. [11] mentioned that the average waiting time for a transfer is about 9.75 minutes on weekends in the Greater Copenhagen area. Synchronization decreases the waiting times of passengers transferring between lines of a network; thus, can lead to a higher level of service and encourages more people to use public transportation. Therefore, synchronization is a common objective which has gained much attention and has been studied in other modes of transportation too, e.g., urban railway networks [12,13].

Ceder et al. [14] proposed an optimization model with the objective of maximizing the number of simultaneous arrivals. The problem was formulated as a mixed-integer programming problem, and a heuristic algorithm was developed for solving it. They described simultaneous arrivals as the arrival of two departures from different lines to a transfer point at the same time. Eranki [15] extended the model developed by Ceder et al. [14] by adding a time window to the simultaneity definition and proposed a model for timetable setting with the same objective. The new model was also formulated as a mixed-integer linear programming problem and was solved by a heuristic algorithm. The model proposed by Ceder et al. [14] has received much attention and has been extended further by other researchers [16-18]. So many studieshave been devoted to transfer time minimization [19-24]. Gao et al. [25] proposed a bi-level programming technique to deal with frequency setting problem. The upper-level problem considers minimizing an objective that consists of in-vehicle and waiting time and frequency setting cost. The lower-level problem considers the path alternatives for passengers. A heuristic algorithm based on sensitivity analysis is developed to solve the model and set the optimal frequencies.

Bookbinder and Désilets [26] proposed a transfer optimization model in which travel times are considered stochastic and follow a truncated exponential distribution. The objective is to minimize total passenger inconveniences. Also, a heuristic algorithm is developed by Ting and Schonfeld [27] considering slack times for optimizing connecting lines. Knoppers and Muller [28] studied the chances of optimized transfers in order to minimize passenger transfer times. They considered probabilities and limitations of synchronized transfers and showed that when crossing lines have high frequencies, optimal synchronization is hard to achieve. The synchronization strategy proposed in theirstudy enables the vehicle handling the second part of the trip to delay its departure if the transferring passengers are supposed to arrive soon; therefore, sometimes transfer times are increased. Yu et al. [29] proposed an optimization model for bus transit network aiming for transfer minimization and passenger flow per unit length maximization. A coarse-grain parallel ant colony algorithm (CPACA) was used for solving the problem. The results show that a bus transit network with optimal transfers and less waiting times is an accessible goal. Ant colony algorithm has been used by other researchers as well [30,31]. Heuristic approaches have been used by many researchers for transfer optimization and synchronization among othersteps of the urban transportation process [32]. Fleurent et al. [33] used the idea of weighted transfers to describe the concepts used in Hastus commercial software for producing synchronized timetables. Furthermore, while concentrating on minimizing the transfer waiting times, Castelli et al. [34] proposed a mathematical model for scheduling problems. As one of the features of the proposed model, the number of dispatches does not depend on the frequency bounds but depends on the service quality and its costs. Service quality is evaluated by the sum of the transfer waiting times of all the passengers whose boarding and alighting times should be available. However, the authors approve that despite the limiting suppositions, their model cannot be used for optimizing a large scale network because of its numerous variables; therefore, a Lagrangian heuristic method is proposed, too. Rapp and Gehner [35] proposed a heuristic method for optimizing the transfer delays. The survey describes four coordinated processes for a practical transit planning. The operational tool for optimizing the transfer delays changes the departure times from terminal automatically and iteratively. The results indicate that the optimized timetable efficiently minimizes total transfer delay times and in comparison to manual scheduling causes no additional cost. Chakroborty et al. [22,23] focused on the application of Genetic algorithms on waiting time determination in transit networks in order to minimize the total passenger waiting times. The proposed mixed-integer nonlinear model is formulated and a hybrid genetic algorithm is selected for implementation purposes. The genetic display of a thorough timetable consists of a set of binary numbers that represent the headways and the stop times between successive vehicles on all lines; however, it is mentioned that the proposed display might not be suitable for large transit networks.

Genetic Algorithmhas also been applied to bus scheduling problem. Deb and Chakroborty [36] formulated an optimization problem for a bus transit system whose objective is to minimize the total passenger waiting times (with or without transfers) while satisfying the resource and service constraints. The study proves that the genetic algorithm is ideal for such problems. Chakroborty et al. [37] combined transfer synchronization with vehicle scheduling as a mixed-integer nonlinear problem. The problem, which aims at determining the fleet size and setting timetables with the objective of minimizing the passenger waiting times, is solved using the genetic algorithm. It should be noted that the problem is limited to one transfer point which is the intersection of multiple lines and it seems that the genetic display is not suitable for large problems. Ngamchai and Lovell [38] presented a new transfer optimization model considering the bus transit route designing and solved it using the genetic algorithm. The bus routes are designed in two phases. The route improvement algorithm employs the genetic operators, and another heuristic approach is used for headway synchronization in order to improve the efficiency of the system. The route improvement algorithm reduces the transfer times and the total cost. Moreover, headway synchronization methods are used for service frequency setting. Cevallos and Zhao [20] proposed an approach based on the genetic algorithm at the network level for bus transfer time optimization. The proposed algorithm attempts to find the best possible solution for the transfer time optimization problem through timetables movements. The authors aim to minimize the transfer waiting times of an existing timetable under strict evenly spaced headway constraints. The results indicate that the proposed algorithm efficiently reduces the waiting times.Beside these studies, genetic algorithms have been widely used by many researchers for different aspects of transit planning and optimization [39-47]. Table 1 presents a summary of the aforementioned research papers.

As previously stated, the mathematical model developed by Cederet. al. [14] has been used as a precedent by many researchers. In this paper, we develop a model to increase the simultaneous arrivals of buses in the network as first developed by Cederet. al. [14]. Moreover, we employ a different definition of simultaneous arrivals which is more practical, effective and realistic. The objective function of maximizing the simultaneous arrivals, and considering a time window as an allowable deadline for simultaneity are the concepts which have been borrowed from previous studies [14,15]. The timetable setting problem is formulated as a mathematical programming model whose implementation and solution methods on both small and large-scale networks are discussed and exemplified separately.

The remainder of this paper is organized as follows. Section 2 consists of the definition of the timetable setting problem with optimal synchronization. Section 3 presents the procedure of formulating the problem as a mathematical programming model. Section 4.1 presents a brief discussion on the scale of the problem. Converting the proposed bi-objective problem into a single objective one is described in Section 4.2. Numerical examples of both small and large scale networks are introduced and solved in Section 4.3 and 4.4 respectively, and in Section 5 the results are discussed and a conclusion is presented.

2. Problem Definition

Looking at the timetable setting problem from different angles, one can consider different objectives for the problem. According to Ceder et al. [14] a very important objective is to minimize the transfer waiting times that can be achieved by maximizing the number of simultaneous arrivals. Since in a large public transit network, all of the origins and destinations are not directly connected, several transfer points are needed to cover most of the area. However, an extra waiting time is imposed on the passengers that intend to travel between different lines of a network via transfer points. A perfect timetable is set only when the waiting time in transfer points are minimized by the simultaneous arrival of vehicles at transfer points.Figure 1 illustrates the importance of transfer points and simultaneous arrivals at transfer points. As shown in Figure 1, a passenger that intends to travel from zone A to zone B has to transfer from line 2 to line 1, since there is no direct link between these two zones. The transfer should be made at stop 12 that is thetransfer stop of two lines. If the arrival of the bus on which the passenger is riding and the bus the passenger needs to catch to continue the trip are not much apart, the transfer waiting time will be short, and the trip would remain attractive for the passenger.However, the arrival time of the buses on each line is an important issue. For instance, in this case, when the transferring passenger arrives at the stop 12 at 8:00, there is a short time needed to get off the first bus, walk to the next stop point and get on the immediate nextbus on line 1 which arrives at stop 12 at 8:05. Therefore, the passenger has a short time to move between the buses. The time interval between the buses arriving at the transfer point from different lines could be adjusted according to the time of day and system frequency. During peak hours, since the service has a high frequency the time intervals need to be very short, preventing possible delays, and during non-peak hours since the service frequency is lower, a small increase in the waiting time would not affect the trip attractiveness much.Furthermore, the interval also depends highly on the overall characteristics of the intersecting lines. For example, if one of the lines is insuburban areas, it is important to keep in mind that the service frequency would generally be low on that line even during the peak hours and the interval should not be too short as it is for intersecting Central Business District (CBD)lines.In real life the timetable scheduler cannot expect passengers to plan their trips such that they minimize their transfer time. The only thing they can do is to adapt themselves to the departure times at their origin. Thus it is the scheduler's task to synchronize the arrivals at transfer points in order to reduce the transfer time. Besides, the scheduler should plan the departures according to passenger demand trends in timetable setting process. Otherwise, passenger demand is not satisfied, and vehicles will be delayed by facing longer dwell times, and bunching phenomenon can occur [2].

A good timetable should also be cost-effective, that is in addition to satisfying passenger demand, it should also take into account the benefits of the transit company. Resource constraint is an important issue for all transit companies. However, they can attract more passengers by optimizing the current service. If the proposed synchronization objective is considered alone, the optimal solution will set the headways of all lines at their minimum to maximize the service level which will also increase the number of required vehicles. Therefore, another important objective is to minimize the required vehicles. The fleet size is an important factor and can greatly affect the company's costs. Headways, as the time between two successive departures, can be evenly or unevenly spaced. If the headways are evenly-spaced, the vehicles are departed in regular and equal intervals. In a period that headways are evenly-spaced the number of required vehicles on a line can be calculated by equation 1,

$$fleetsize = \frac{averageroundtriptime}{headway} \tag{1}$$

In this paper, by considering the objectives mentioned above, a bi-objective mixed-integer programming model is proposed whose mathematical formulation is presented in the next section.

3. Problem Formulation

The notations used in the mathematical model are presented in Table 2.

The objectives of the problem are formulated as Equations 2 and 3. The first objective maximizes the number of synchronized arrivals at transfer points. The second objective is defined in order to minimize the number of required vehicles for the system.

$$Max\ Z_1 = \sum_{m\in D_t^l} \sum_{n\in D_t^k} \sum_{q\in C^{lk}} \sum_{l\in L} \sum_{\substack{k\in L,\\ l\neq k}} \sum_{t\in T} \sum_{p\in T} y_{mnqtp}^{lk} \tag{2}$$

$$Min Z_2 = \sum_{l\in L} FS^l \tag{3}$$

$$x_t^l \le H\max_t^l \qquad l\in L, t\in T \tag{4}$$

$$x_t^l + (ND_t^l - 1)H_t^l \le U \qquad l \in L, t \in T \tag{5}$$

$$H\min_t^l \le H_t^l \le H\max_t^l \qquad l \in L, t \in T \tag{6}$$

$$A_{nit}^l = U(t-1) + x_t^l + (n-1)H_t^l + \sum_{k=1, k\in S^l}^{i-1} R_{k,k+1,t}^l + \sum_{k=1, k\in S^l}^{i-1} dw_{k,t}^l$$
$$l \in L, n \in D_t^l, i \in S^l, t \in T \tag{7}$$

$$y_{mnqtp}^{lk} = \begin{cases} 1 & \left| A_{mqt}^l - A_{nqp}^k \right| \le \delta^{lk} \quad l,k \in L, m \in D_t^l, n \in D_t^k, q \in C^{lk}, \\ 0 & therwise \qquad\qquad t, p \in T, l \ne k, l < k \end{cases} \tag{8}$$

$$FS^l = \max_{t\in T}\left({CT_t^l}/{H_t^l} \right) \qquad l \in L \tag{9}$$

$$y_{mnqtp}^{lk} \in \{0,1\} \qquad l,k \in L, m \in D_t^l, n \in D_t^k, q \in C^{lk}, t, p \in T, l \ne k, l < k \tag{10}$$

$$x_t^l, H_t^l \in N^+, l \in L, t \in T \tag{11}$$

The constraints of the model are formulated as Equations (4) to (11). Constraint (4) guarantees that the first departure would take place in a period shorter than the maximum allowable headway. Constraint (5) assures that the last departure of each period is set before the end of the corresponding period. Constraint (6) limits the headway of each period not to exceed the allowable range. Equation (7) calculates the arrival time of each departed vehicle at each stop. Constraint (8) defines the binary variable *y* introduced in Table 2. It compares all the arrival times which are calculated by Constraint (7) and when the interval between two arrivals from different lines at a transferpoint is less than the maximum allowable time window ($\delta^{lk}{}_{tp}$), *y* takes 1, otherwise it remains equal to 0. Equation (9) calculates the minimum number of required vehicles on each line which is equal to the maximum number of required vehicles in each period on the same line. Constraints (10) and (11) define the mathematical characteristics of the decision variables.

It is worth mentioning that in order to maintain the practicality of the problem, the following two constraints should also hold:

$$U \ge (ND_t^l - 1).H\min_t^l \qquad \forall l \in L, t \in T \tag{12}$$
$$U \le ND_t^l.H\max_t^l \qquad \forall l \in L, t \in T \tag{13}$$

According to Constraints (12) and (13) departures that are planned for a period do not violate the corresponding time interval. The presented mathematical formulation is the most comprehensive form which can be simplified for implementation purposes.

4. Solution Approaches

The proposed comprehensive model is widely applicable to networks with different sizes; therefore, the implementation approach is discussed in two sections for both small and large networks.

4.1 Scale of theProblem

Although binary and discrete variables increase the complexity of the problem, they are often used in mathematical modeling, yielding a mixed integer programming problem. As described in the previous section, the principal decision variables of the proposed model, i.e. simultaneity and headway variables are binary and discrete variables, respectively. Moreover, the nonlinearity of mathematical equations and constraints is, in some cases, unavoidable. For example, constraint 8, defining the simultaneity phenomena, couldn't be linearized efficiently, i.e. without increasing the number of constraints and variables, thus, leading to a more complicated problem. Furthermore, dimensions of the problem play an important role in complicating the problem and increasing the computation time. The complexity of the proposed mathematical model which consists of binary and discrete variables, nonlinear constraints and several dimensions could be usually judged by the number of integer variables of the problem. Variable $y^{lk}{}_{mnqtp}$ demonstrates the simultaneous arrivals of the vehicle m in period t on line l and vehicle n in period p on line k at transfer stop q. Accordingly, there is a binary variable for the combination of each two trips on every two lines which intersect at stop q. Assuming that L is the total number of lines on a network which at some point intersect with other lines, D is the average number of departures being planned on each line in a given period, C is the total number of transfer stops and T is the number of time periods, then the number of integer variables $y^{lk}{}_{mnqtp}$ would be $T^2 \times C \times L^2 \times D^2$.In a more realistic approach if C is substituted by the average transfer stops between each two lines, and the number of variables would be equal to $T^2 \times L^2 \times D^2$which would be a large number in real networks. Moreover, the number of the variables associated with the fleet size, FS^l, assuming $H = \left(Hmax_t^l - Hmin_t^l\right) + 1$ as the number of possible values for headways on each line in each period, would be equal to$T \times L \times H$. Consequently, the feasibility of efficiently implementing the proposed model on real-world transit networks with different sizes is quite controversial.

Additionally, regarding computational complexity theory, the basic model developed by Ceder et al.[14] which aims at maximizing the number of simultaneous arrivals is proved to be an NP-hard problem [48] whose complexity provides enough incentive for the authors to consider heuristic algorithms as a solution approach. Hence, the attempted problem in this paper is classified in the NP-hard class of problems.

4.2 Converting the Problem to a Single Objective Problem

In order to solve the problem with single objective solution approaches which are proposed in this paper, it needs to be converted into a single-objective problem; hence, the linear weighted-sum method is employed for this purpose. Equation 14 shows the objective function of the problem converted by the linear weighted-sum method.

$$MaxZ = (C_1 \sum_{m \in D_t^l} \sum_{n \in D_t^k} \sum_{q \in C^{lk}} \sum_{l \in L} \sum_{\substack{k \in L, \\ l \neq k}} \sum_{t \in T} \sum_{p \in T} y_{mnqtp}^{lk} - C_2 \sum_{l \in L} FS^l) \tag{14}$$

Equations 15,16 and 17 show the additional constraints added to the model by using this method.

$$C_1 + C_2 = 1 \tag{15}$$

$$C_1 \geq 0 \tag{16}$$

$$C_2 \geq 0 \tag{17}$$

In order to convert the bi-objective mathematical problem into a single objective one, the relative importance of the objectives should be assigned to their corresponding function as their weights. The weight values are very important and delicate; sincethey directly affect the final solution of the problem. Hence, much study is required for determining the weight values. There arealso some popular methods for this purpose, such as brainstorming and the Delphi methods;however, the ultimate decision should be made by the management team members who are known as the most experienced decision makers in this field. In this study, considering the monetary value of each simultaneous arrival, which savespassengers' time and the costof each vehicle, which can provide more simultaneous arrivals,the weights are assigned as C_1=0.23, C_2=0.77 only to provide an illustrative example of using methods designed for single-objective models.

Finally, in order to scale the objectives in the same unit and magnitude, the objective functions are normalized according to Equation 18:

$$\tilde{F}_i = \frac{F_i(x) - F_{i,\min}}{F_{i,\max} - F_{i,\min}} \tag{18}$$

where $F_{\mathrm{i,min}}$ and $F_{\mathrm{i,max}}$ are the minimum and maximum values of the objective function *i* respectively, and the normalized objective function $\tilde{F}_i \in [0,1]$, i=1,2,…,m has the same design space as before [49]. For a more perceivable depiction, two simplifications are made as Equations 19 and 20, and using Equation 18, the normalized objective function is presented in Equation 21.

$$\sum\nolimits_{m \in D_t^l} \sum\nolimits_{n \in D_t^k} \sum\nolimits_{q \in C^{lk}} \sum\nolimits_{l \in L} \sum\nolimits_{\substack{k \in L, \\ l \neq k}} \sum\nolimits_{t \in T} \sum\nolimits_{p \in T} y_{mnqtp}^{lk} = S \tag{19}$$

$$\sum_{l \in L} FS^l = FS \tag{20}$$

$$Z = C_1\left(\frac{S - S_{\min}}{S_{\max} - S_{\min}}\right) - C_2\left(\frac{FS - FS_{\min}}{FS_{\max} - FS_{\min}}\right) \tag{21}$$

The resulting Equation 21 will be used as the objective function of the model in cases of using single objective solution methods.

4.3 Small Cities

In case of small cities which have small bus networks that consist of only a few lines, a simplified version of the proposed model can be employed for timetable setting; however, the model needs to be solved for each period of a day. Thus, the variables and parameters lose the index t, and the simplified model could be solved by classic nonlinear mathematical methods efficiently.

In this section, a small numerical example is presented and solved using General Algebraic Modeling System (GAMS). Figure 2 illustrates a transit network with 4 lines and 6 stops. The planning horizon is 60 minutes (the morning peak hour) and the travel times between successive stops are mentioned in Figure 2. As illustrated in Figure 3, stops 1, 3, 5 and 6 are the transfer stops of at least two line intersects.

Table 3 presents the input data of the problem. Input data includes minimum and maximum allowable headways for each line, average dwell time of the vehicles in each stop, and average round-trip time on each line. Note that the dwell times of buses at the stops of the same line are assumed equal; hence the parameter loses the stop index*i*.

The maximumallowable time window for a simultaneous arrival (δ^{lk}) at each transfer point in the network is set equal to 2 minutes.The proposed model is used to set timetables on the example network illustrated in Figure 2 using GAMS. The problem is coded in GAMS 23.0.2 and run using DICOPT solver which is based on the OA/ER/AP algorithm, and the results are shown in Table 4.

Table 4 demonstrates the scheduled departure times of vehicles on each line during the planning horizon. The last column shows the number of required vehicles for each line. Figure 3 illustrates the occurrence of 11 of 23 consequent simultaneous arrivals at each transfer point in short intervals. Each line and its buses are recognized by a different geometric shape around the bus schematic and the time above is the arrival time of the bus at the transfer point. The same procedure can easily illustrate the remaining simultaneous arrivals.

As mentioned earlier, the best timetable would be the one that efficiently maintains the synchronization of different lines of a transit network requiring a minimum number of vehicles. Presented results show that this objective can be achieved and the model can,by this definition, provide optimal timetables under different conditions, therefore, can be used for setting timetables on a smallnew network as well as improving the timetables of an existing one.

4.4 Large Cities

The proposed MINLP formulation is difficult to be implemented on large transit networksusing classic methods due to the nonlinear constraints, discrete search space, and the large size of the problem. Hence, there is a strong motivation to consider metaheuristic algorithms for solving the proposed model efficiently.

In case of transit scheduling problems, among the most popular metaheuristic algorithms, genetic algorithms have shown efficiency in several cases [39-47]. Therefore, the proposed model along with the genetic algorithms is employed to set timetables on a real-world transit network with the minimum number of required vehicles. The process of employing genetic algorithm for solving the attempted problem is described as follows.

4.4.1Metaheuristic Approaches

Genetic algorithm (GA), which is based on Darwin's theory of evolution, is an optimization algorithm developed by Holland [50]. Given the nature of the genetic algorithm which can handle only problems with one objective function, the problem is converted into a single-objective problem using the linear weighted-sum method and normalized as Equation 21.

Solving the problem with the proposed mathematical model using genetic algorithm needs proper preparations and arrangements,which are a part of the GA procedure that is illustrated as a flowchart in Figure 4(presented by Amiripour et. al. [51]). As can be seen in the figure, an initial population of chromosomes is required first. Finding the proper configuration for the chromosomes is an essential primary stage in the genetic algorithms implementation. It not only affects the coding process but also provides a simple illustration of the problem for a better understanding. As shown in Figure 5, the configuration of the chromosomes is illustrated as a matrix.

Each row of this matrix contains the main variables of each transit line's timetable; the elements in each 1x2 block specify the time of the first departure and the headway of the corresponding period, respectively. The pattern is repeated for each line, and a chromosome is formed. As can be seen in Figure 5, after generating the initial population which consists of 200 chromosomes, each chromosome needs to be evaluated by a fitness function. In this case,similar to the small sample network problem,the fitness function which is the weighted linear combination of the introduced objective functions (Eq. 2 and 3)is normalized as Equation 21. All of the constraints are taken into account during the process of generating the chromosomes; hence, no penalty function is needed.After the evaluation process, the algorithm starts an iterative loop which will be repeated for a specified number of times until there are no significant improvements in the fitness value of the best chromosome.

Moreover, thenon-dominated sorting genetic algorithm for multi-objective optimization (NSGA-II) first proposed by Debet al.[52,53], is proved to be an efficient solution approach. A similar procedure to what was described is employed in the NSGA-II for a bi-objective problem; however instead of a fitness value for each chromosome, there is a fitness vector which consists of different fitness values of different fitness functions.

The following section presents a real-world case as an illustration of the applicability of the proposed model and solution approaches in large scale networks.

4.4.2A Case study – BRT Network of Tehran

In this section, the proposed model is applied to a large real-world network. Tehran BRT network, as a large scale network, consists of 9 lines (142 km), 8 of which have common stops between two or three lines, and according to the 2017statistics, it serves an average of 2.25 million travelers on a daily basis with 900 buses. Figure 6 illustrates a plain scheme of the network. Red lines are the urban bus lines, and blue lines are the BRT lines. The proposed model and solution methods are employed to set coordinated timetables on the network.

Collecting the required data is the first step of solving the problem, and since the vehicles are equipped with Automatic Fare collection system (AFC), and Automatic Vehicle Location (AVL) system which are two main sources of data in a bus system, this can be done conveniently.As an example, the AVL system records the time of opening and closing the bus doors at each stop. The required data includes the average travel time between two consecutive stops ($R^{l}_{i,i+1,t}$)which can be calculated by subtracting the time at which the doors are opened at stop $i+1$ from the time at which the doors are last closed at stop i. Similarly, the average round-trip time of a line(CT^{l}_{t})can be calculated by subtracting the time at which the bus doors are closed in the last stop from the time at which the bus doors are opened in the first stop on the corresponding line. Moreover, the average dwell time at each stop (dw^{l}_{it})is the time between the opening and closing of the bus doors at the same stop.For this purpose, thedata for the first two weeks of January 2016 is collected, refined, and sorted according to the required input parameters of the model. The maximum allowable time window for a simultaneous arrival ($\delta^{lk}_{\mathrm{tp}}$) at each transfer point in the network is set equal to 1 minute. The planning horizon is divided into 17 hours, and after tailoring the collected data in this format, the problem is coded in MATLAB 2017using the genetic algorithm. Then, the program is run on a PC with Pentium® Dual-Core CPU E5300 @ 2.60GHzand 4.00GB RAM, and the results are presented in Tables5 and 6.

Tables 4 and 5 demonstrate the obtained first departure times, headways, and the fleet size which are found by genetic algorithm for each line for all periods during a day. By this information, a comprehensive timetable could be built which is supposed to minimize the waiting times for transferring passengers and synchronize the lines while decreasing the number of the required vehicles. The improvements and convergence trend of the genetic algorithm is illustrated in Figure 7. As can be seen in the illustration, after about 8000 iterations, the algorithm converges to the presented solutionwith 10628 simultaneous arrivals and 257 vehicles.

As explained before, NSGA-II is a powerful tool to obtain a more general solution for a bi-objective problem, in case the importance weights are unknown or a financial analysis on the possible solutions is necessary for the final decision to be made. In this case, five scenarios are considered for the current problem of Tehran BRT network (δ=1,...,5 minutes). For each scenario, all the Pareto optimal solutions are obtained and the Pareto fronts are illustrated as Figure 8. The parameter δ acts as a controlling parameter which directly affects simultaneous arrivals and its value should be selected according to the capacity of the system and the level of service. As δ decreases, simultaneity gets rare and hard to happen. Therefore, δ=5 is the least strict scenario which offers the minimum level of service, assumed here, and δ=1 is the strictest scenario which offers the maximum level of service. In case of the strict scenarios, the waiting time for the transferring passengers reduces but this improvement would not be possible without the decrease in headways which demands a higher number of vehicles and consequently an increase in system costs. This proves that δ value is a sensitive, controversial, and impactful parameter. It is worth noting that as δ decreases, according to the increasing diagram gradient in Figure 8, in order to obtain a higher degree of synchronization, a bigger increase in the fleet size would be needed. A Pareto optimal front as Figure 8 can help the decision makers analyze the relationship between the objective values and weights closely and choose the best objective weight values according to the company's circumstances.

5. Conclusions

Scheduling (frequency determination and timetable setting) is one of the most important and complicated steps of the public transportation planning process. Synchronization is an important issue that should be considered during this step. Synchronization can have major effects on the public transit network attractiveness.

In this paper, by generalizing the primary concept of simultaneous arrivals and synchronization proposed by Ceder et al. [14], the frequency determination and timetable setting problem is formulated as a mixed-integer nonlinear programming model aiming for optimal synchronization of the timetables in a public transit network. The applicability of the proposed mathematical model is proved by its employment to set timetables on networks with different scales. In case of small networks, the problem is converted into a single objective problem, the formulation is simplified, and the model is implemented on a sample network with 4 lines and 6 stops as a case study. The problem is coded in GAMS and a thoroughly reasonable timetable is obtained.

Since the classic methods are not capable of solving large-scale mathematical programming problems, two solution methodsare proposedfor these cases. First, the genetic algorithm is employed to use the single objective problem in order toset timetables on the BRT network of Tehran which consists of 8 lines. Secondly, the NSGA-II is employed to extract the Pareto optimal solutions and fronts in five different scenarios.The overall results show that the proposed model can be easily used for synchronizing both small and large bus transit networks taking into account the vehicle costs by minimizing the required fleet size.

The problem discussed here, is a highly complex problem which is under theinfluenceofmany correlative factors. It could be more realistic, if the stochastic nature of some parameters such as average dwell time at stops, average travel time between two consecutive stops, and average round-trip time of lines were incorporated into the model as future studies. Furthermore, there are parameters regarding the level of service such as maximum headways, and maximum allowable time window for synchronized arrivals which need discussion and decisions of the experts.

References

[1]Liu, T., Ceder, A., Ma, J., and Guan, W. "Synchronizing public transport transfers by using intervehicle communication scheme: Case study", *Transportation Research Record: Journal of the Transportation Research Board*, **2417**(1), pp. 78-91(2014).

[2] Ceder, A. "Public Transit Planning and Operation: Theory, Modeling and Practice",In *Elsevier, Butterworth-Heinemann*, Oxford, UK(2007).

[3] Desaulniers, G. and Hickman, M. D. "Public transit", *Handbooks in Operations Research and Management Science,* **14**, pp. 69-127(2007).

[4] Guihaire, V. and Hao, J.K. "Transit network design and scheduling: A global review", *Transportation Research Part A: Policy and Practice*, **42**(10), pp. 1251-1273 (2008).

[5] Wu, Y., Yang, H., Tang, J., and Yu, Y. "Multi-objective re-synchronizing of bus timetable: Model, complexity and solution", *Transportation Research Part C: Emerging Technologies*, **67**, pp. 149-168 (2016).

[6] Ibarra-Rojas, O. J., Delgado, F., Giesen, R., and Muñoz, J. C. "Planning, operation, and control of bus transport systems: A literature review", *Transportation Research Part B: Methodological*, **77**, pp. 38-75 (2015).

[7] Wu, Y., Tang, J., Yu, Y., and Pan, Z. "A stochastic optimization model for transit network timetable design to mitigate the randomness of traveling time by adding slack time", *Transportation Research Part C: Emerging Technologies*, **52**, pp. 15-31 (2015).

[8] Ibarra-Rojas, O. J., López-Irarragorri, F., and Rios-Solis, Y. A. "Multiperiod bus timetabling", *Transportation Science*, **50**(3), pp. 805-822(2015).

[9] Nagatani, T."Nonlinear-map model for bus schedule in capacity-controlled transportation", *Applied Mathematical Modelling,* **37**(4), pp. 1823-1835 (2013).

[10] Gershenson, C. and Pineda, L. A. "Why does public transport not arrive on time? The pervasiveness of equal headway instability", *PloS one,* **4**(10), p. e7292 (2009).

[11] Petersen, H. L., Larsen, A., Madsen, O. B., Petersen, B. and Ropke, S. "The simultaneous vehicle scheduling and passenger service problem", *Transportation Science,* **47**(4), pp. 603-616 (2012).

[12] Kang, L. and Zhu, X. "A simulated annealing algorithm for first train transfer problem in urban railway networks", *Applied Mathematical Modelling,* **40**(1), pp. 419-435 (2016).

[13] Guo, X., Wu, J., Sun, H., Liu, R. and Gao, Z. "Timetable coordination of first trains in urban railway network: A case study of Beijing", *Applied Mathematical Modelling,* **40**(17), pp. 8048-8066 (2016).

[14] Ceder, A., Golany, B. and Tal, O. "Creating bus timetables with maximal synchronization", *Transportation Research Part A: Policy and Practice*, **35**(10), pp. 913-928 (2001).

[15] Eranki, A. "A model to create bus timetables to attain maximum synchronization considering waiting times at transfer stops", Master Thesis, University of South Florida(2004).

[16] Currie, G. and Bromley, L. "Developing measures of public transport schedule coordination quality",*28th Australasian Transport Research Forum*(2005).

[17] Quak, C.B. "Bus line planning", Master thesis, TU Delft (2003).

[18] Ibarra-Rojas, O.J. and Rios-Solis, Y.A. "Integrating synchronization bus timetabling and single-depot single-type vehicle scheduling", *ORP3 Meeting,* Cádiz, Spain (2011).

[19] Shafahi, Y. and Khani, A. "A practical model for transfer optimization in a transit network: Model formulations and solutions", *Transportation Research Part A: Policy and Practice,* **44**(6), pp. 377-389 (2010).

[20] Cevallos, F. and Zhao, F. "Minimizing transfer times in public transit network with genetic algorithm", *Transportation Research Record: Journal of the Transportation Research Board,* **1971**(1), pp. 74-79 (2006).

[21] Jansen, L.N., Pedersen, M.B. and Nielsen, O.A. "Minimizing passenger transfer times in public transport timetables",*Proceedings of the Seventh Conference of the Hong Kong Society for Transportation Studies: Transportation in the Information Age*, Hong Kong, China, pp.229-239 (2002).

[22] Chakroborty, P., Deb, K. and Subrahmanyam, P.S. "Optimal scheduling of urban transit systems using genetic algorithms",*Journal of Transportation Engineering,* **121**(6), pp. 544-553 (1995).

[23] Chakroborty, P., Deb, K. and Porwal, H. "A genetic algorithm based procedure for optimal transit systems scheduling",*Proceedings of the Fifth International Conference on Computers in Urban Planning and Urban Management*, Mumbai, India, pp.330-341 (1997).

[24] Daduna, J.R. and Voß, S. "Practical experiences in schedule synchronization", In *Computer-Aided Transit Scheduling, Lecture Notes in Economics and Mathematical Systems,* Springer, Berlin, Heidelberg, **430**, pp.39-55(1995).

[25] Gao, Z., Sun, H. and Shan, L.L. "A continuous equilibrium network design model and algorithm for transit systems", *Transportation Research Part B: Methodological,* **38**(3), pp. 235-250 (2004).

[26] Bookbinder, J. and Désilets, A. "Transfer optimization in a transit network",*Transportation Science*, **26**(2), pp. 106-118 (1992).

[27] Ting, C.J. andSchonfeld, P. "Schedule coordination in a multiple hub transit network", *Journal of urban planning and development*, **131**(2), pp. 112-124 (2005).

[28] Knoppers, P., and Muller, T. "Optimized transfer opportunities in public transport", *Transportation Science*, **29**(1), pp. 101-105(1995).

[29] Yu, B., Yang, Z., Cheng, C. and Liu, C. "Optimizing bus transit network with parallel ant colony algorithm", *Proceedings of the Eastern Asia Society for Transportation Studies*, **5**(1), pp.374-389 (2005).

[30] Poorzahedy, H., and Abulghasemi, F. "Application of ant system to network design problem",*Transportation*, **32**(3), pp.251-273 (2005).

[31] Yang, Z.Z., Yu, B., and Cheng, C.T. "A parallel ant colony algorithm for bus network optimization",*Computer-Aided Civil and Infrastructure Engineering*, **22**(1), pp.44-55(2007).

[32] Peng, K., and Shen, Y. "A variable iterated greedy algorithm based on grey relational analysis for crew scheduling", *Scientia Iranica. Transaction E, Industrial Engineering*, **25**(2), pp. 831-840 (2018).

[33] Fleurent, C., Lessard, R. and Séguin, L. "August. Transit timetable synchronization: Evaluation and optimization",*Proceedings of the 9th international conference on computer-aided scheduling of public transport*,San Diego, US, pp.9-11(2004).

[34] Castelli, L., Pesenti, R. and Ukovich, W. "Scheduling multimodal transportation systems", *European Journal of Operational Research*, **155**(3), pp.603-615(2004).

[35] Rapp, M.H. and Gehner, C.D. "*Transfer optimization in an interactive graphic system for transit planning"*, No. Intrm Rpt., pp. 27-33. (1967).

[36] Deb, K., and Chakroborty, P. "Time scheduling of transit systems with transfer considerations using genetic algorithms",*Evolutionary Computation*, **6**(1), pp.1-24(1998).

[37] Chakroborty, P., Deb, K., and Sharma, R.K. "Optimal fleet size distribution and scheduling of urban transit systems using genetic algorithms",*Transportation Planning and Technology*,**24**(3), pp.209-226(2001).

[38] Ngamchai, S. and Lovell, D.J. "Optimal time transfer in bus transit route network design using a genetic algorithm", *Journal of Transportation Engineering*, **129**(5), pp.510-521(2003).

[39] Pattnaik, S.B., Mohan, S., and Tom, V.M. "Urban bus transit route network design using genetic algorithm",*Journal of Transportation Engineering*, **124**(4), pp. 368-375(1998).

[40] Chien, S., Yang, Z. and Hou, E. "Genetic algorithm approach for transit route planning and design", *Journal of Transportation Engineering*, **127**(3), pp.200-207 (2001).

[41] Bielli, M., Caramia, M., and Carotenuto, P. "Genetic algorithms in bus network optimization",*Transportation Research Part C*, **10**(1), pp. 19-34(2002).

[42] Chakroborty, P., and Dwivedi, T. "Optimal route network design for transit systems using genetic algorithms",*Engineering Optimization*, **34**(1), pp. 83-100(2002).

[43] Tom, V.M., and Mohan, S. "Transit route network design using frequency coded genetic algorithm", *Journal of Transportation Engineering*, **129**(2), pp. 186-195(2003).

[44] Kuan, S.N., Ong, H.L. and Ng, K.M. "Solving the feeder bus network design problem by genetic algorithms and ant colony optimization", *Advances in Engineering Software*, **37**(6), pp.351-359(2006).

[45] Agrawal, J. and Mathew, T.V. "Transit route network design using parallel genetic algorithm", *Journal of Computing in Civil Engineering*, **18**(3), pp.248-256(2004).

[46] Fan, W., and Machemehl, R. "Optimal transit route network design problem with variable transit demand: genetic algorithm approach",*Journal of Transportation Engineering*, **132**(1), pp.40-51(2006).

[47] Zhao, F., and Zeng, X. "Simulated annealing–genetic algorithm for transit network optimization",*Journal of Computing in Civil Engineering*,**20**(1), pp. 57-68 (2006).

[48]Ibarra-Rojas, O. J., Rios, Y. "Synchronization of bus timetabling",*Transportation Research Part B: Methodological*, **46**(5), pp. 599-614 (2012).

[49] Abraham, A., and Jain, L. "Evolutionary multiobjective optimization", In*Evolutionary Multiobjective Optimization*,Abraham, A., Jain, L., and Goldberg, R., (Eds.), pp. 1-6, Springer, London, UK(2005).

[50] Holland, J.H. "Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence",*MIT press, Cambridge, MA.* (1992).

[51]Amiripour, S. M., Ceder, A. A., andMohaymany, A. S. "Designing large-scale bus network with seasonal variations of demand", *Transportation Research Part C: Emerging Technologies*, **48**, pp. 322-338 (2014).

[52] Deb, K., Pratap, A., Agarwal, S., andMeyarivan, T. A. M. T. "A fast and elitist multiobjective genetic algorithm: NSGA-II", *IEEE Transactions on Evolutionary Computation*, **6**(2), pp. 182-197 (2002).

[53] Habibnejad-Ledari, H., Rabbani, M., and Ghorbani-Kutenaie, N. "Solving a multi-objective model toward home care staff planning considering cross-training and staff's preferences by NSGA-II and NRGA",*Scientia Iranica*, in press (2018).

Shervin Ataeian received her B.S. in Industrial Engineering from the University of Tabriz in 2014 and her M.S. degree in the same field from Urmia University in 2016. Her research interests include public transportation planning, bus networks modeling and scheduling, and applications of operations research in transportation systems. She is currently expanding her studies in the field of transportation engineering.

Maghsud Solimanpur is the Professor of Industrial Engineering in the Department of Industrial Engineering, Urmia University, Urmia, Iran. He has received his BSc and MSc degrees from Sharif University of Technology in 1995 and 1997, respectively. He has been awarded a PhD degree from India Institute of Technology (IIT) Delhi in 2003. He has been awarded as the outstanding PhD student by the Ministry of Science, Research and Technology of Iran in 2003. His research areas of interest include Production and Operations Management, Operations Research, Cellular Manufacturing Systems, Facilities Planning and Layout, etc.

S.M. Mahdi Amiripour is an assistant professor at Toos Institute of Higher Education in Mashhad, Iran. He has his Ph.D. in Transportation Planning from Iran University of Science and Technology. His research interests include public transportation planning, designing and analysis of bus networks, bus networks modeling and scheduling, and public transport performance evaluation.

Ravi Shankar is the Professor of Operations and Supply Chain Management in the Department of Management Studies, Indian Institute of Technology (IIT) Delhi, India. Besides having a highly productive academic career at IIT Delhi, he has undertaken teaching in different highly prestigious international academic centers worldwide. He has supervised 50 defended/completed PhDs and over 100 Masters-level thesis for the M.Tech. and MBA degree programs. His research areas of interest include Operations and Supply Chain Management, Decision Sciences, Business Analytics, Project Management, Sustainable Freight Transportation, etc.

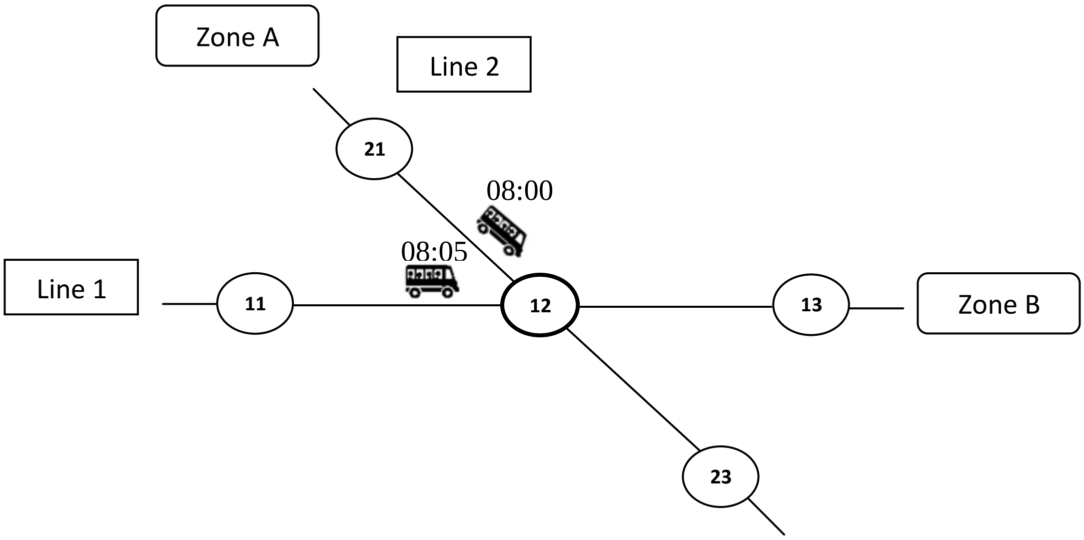


Figure 1

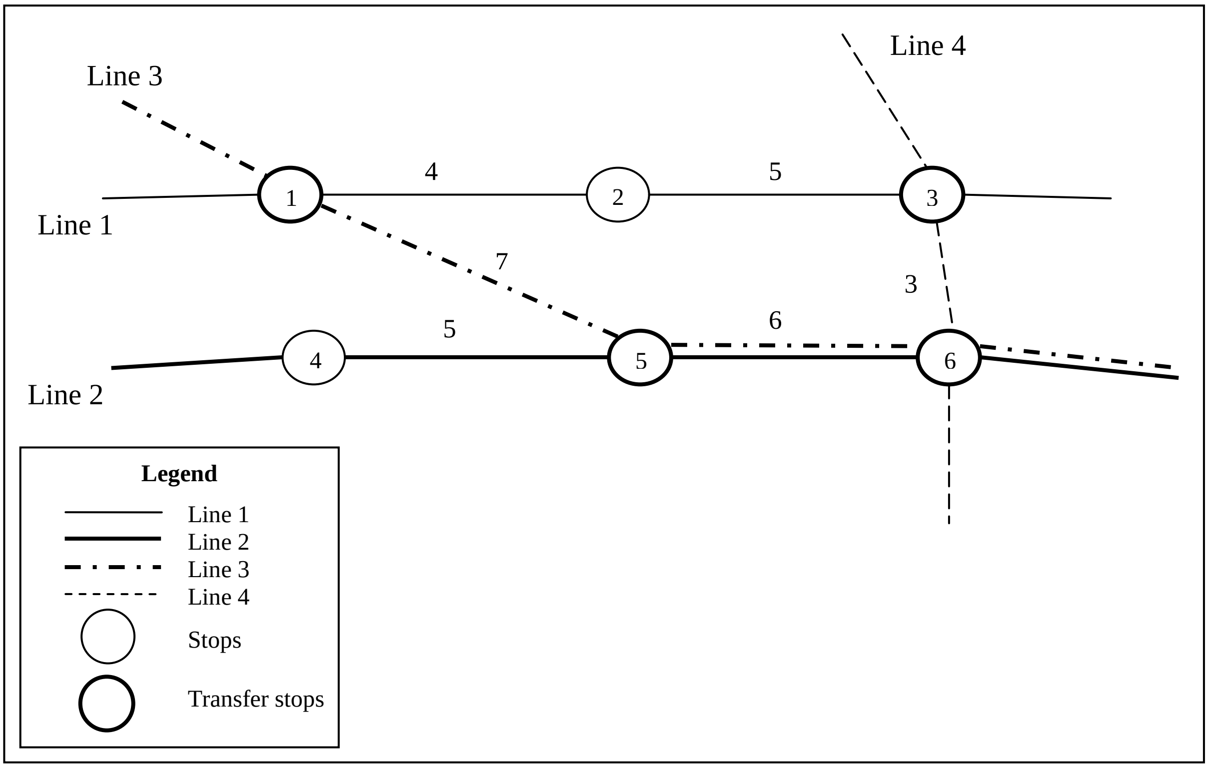

Line 3
Line 4
Line 1
Line 2
1
2
3
4
5
6
4
5
7
3
5
6
Legend
Line 1
Line 2
Line 3
Line 4
Stops
Transfer stops

Figure 2

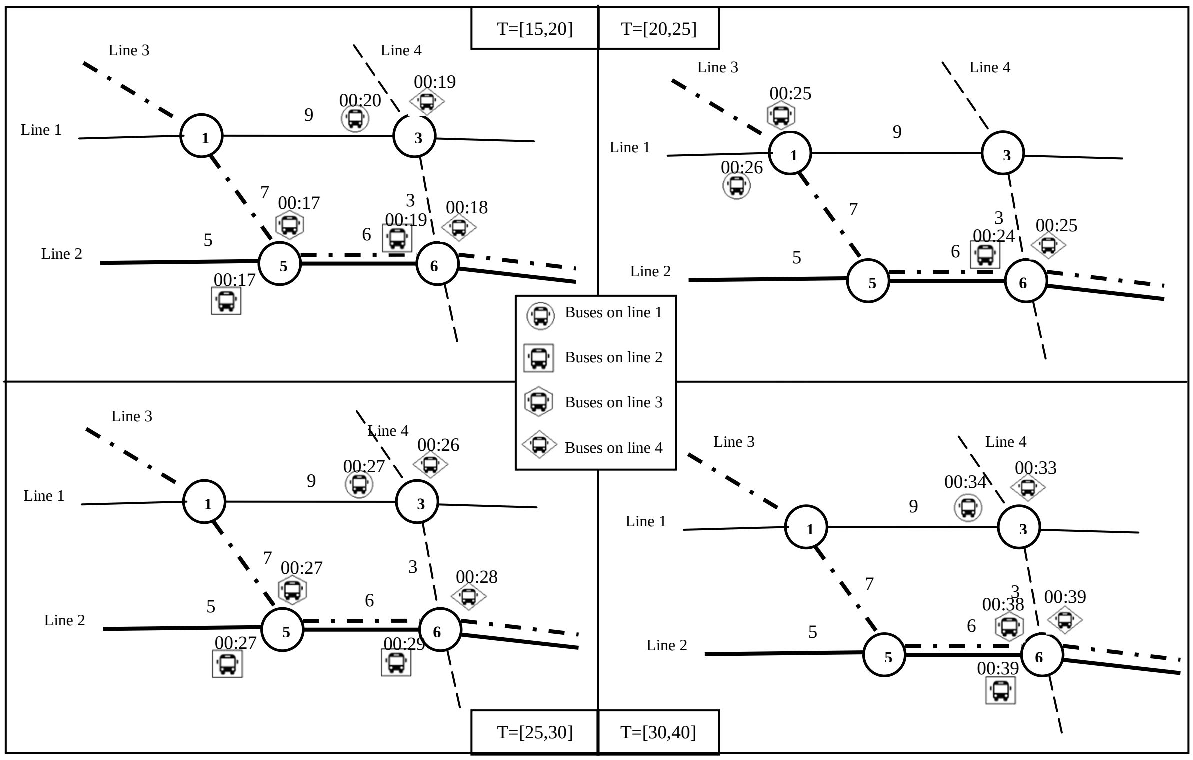
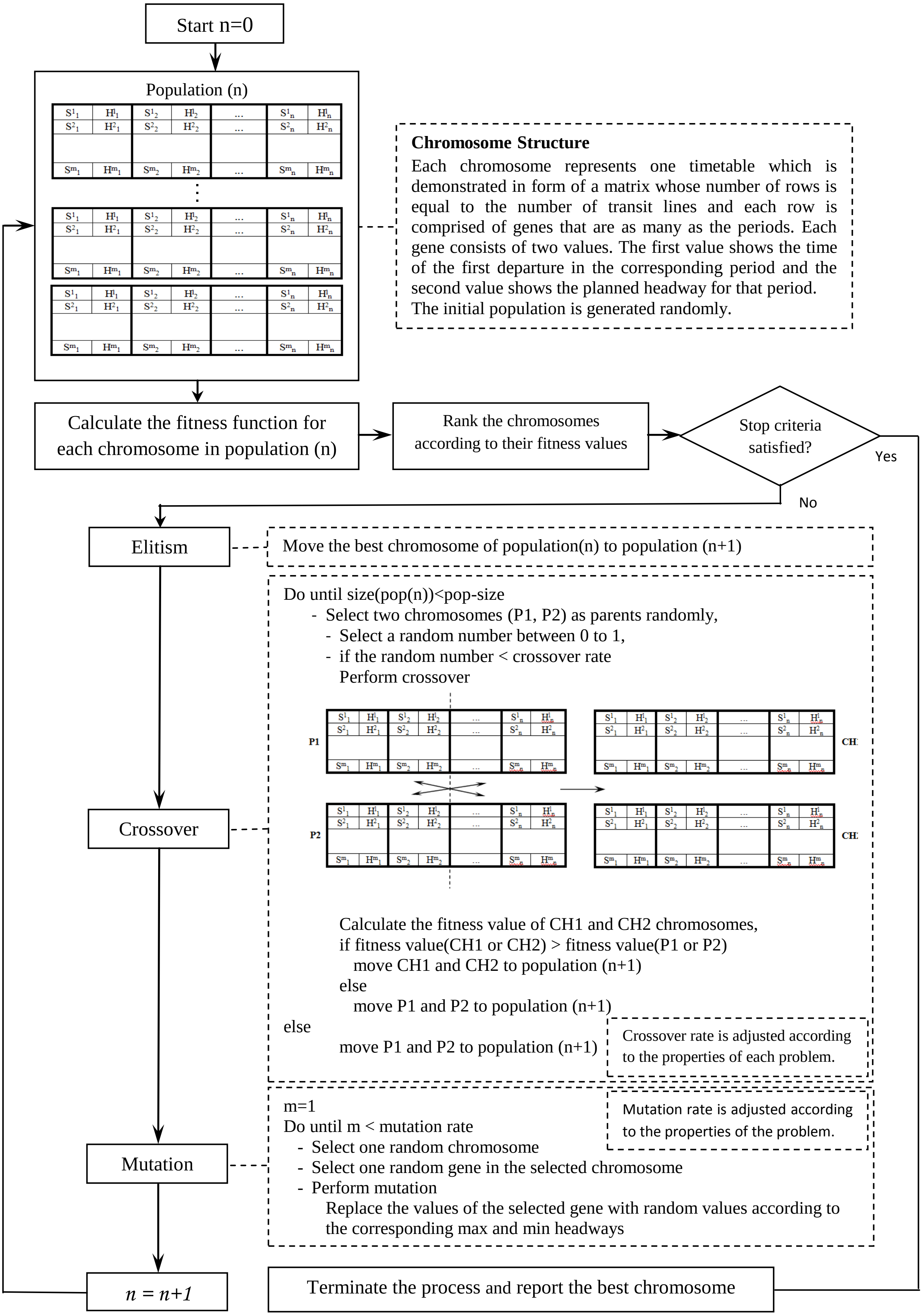

T=[15,20]
T=[20,25]
T=[25,30]
T=[30,40]
Line 1
Line 2
Line 3
Line 4
Buses on line 1
Buses on line 2
Buses on line 3
Buses on line 4

Figure 3



Start n=0
Population (n)
Chromosome Structure
Each chromosome represents one timetable which is demonstrated in form of a matrix whose number of rows is equal to the number of transit lines and each row is comprised of genes that are as many as the periods. Each gene consists of two values. The first value shows the time of the first departure in the corresponding period and the second value shows the planned headway for that period.
The initial population is generated randomly.
Calculate the fitness function for each chromosome in population (n)
Rank the chromosomes according to their fitness values
Stop criteria satisfied?
Yes
No
Elitism
Move the best chromosome of population(n) to population (n+1)
Do until size(pop(n))<pop-size
- Select two chromosomes (P1, P2) as parents randomly,
- Select a random number between 0 to 1,
- if the random number < crossover rate
Perform crossover
P1
P2
Crossover
Calculate the fitness value of CH1 and CH2 chromosomes,
if fitness value(CH1 or CH2) > fitness value(P1 or P2)
move CH1 and CH2 to population (n+1)
else
move P1 and P2 to population (n+1)
else
move P1 and P2 to population (n+1)
Crossover rate is adjusted according to the properties of each problem.
m=1
Do until m < mutation rate
- Select one random chromosome
- Select one random gene in the selected chromosome
- Perform mutation
Replace the values of the selected gene with random values according to the corresponding max and min headways
Mutation rate is adjusted according to the properties of the problem.
Mutation
n = n+1
Terminate the process and report the best chromosome

Figure 4

	Period 1		Period 2			Period n	
Line 1	$S^1{}_1$	$H^l{}_1$	$S^1{}_2$	$H^l{}_2$	...	$S^1{}_n$	$H^l{}_n$
Line 2	$S^2{}_1$	$H^2{}_1$	$S^2{}_2$	$H^2{}_2$	...	$S^2{}_n$	$H^2{}_n$
Line m	$S^m{}_1$	$H^m{}_1$	$S^m{}_2$	$H^m{}_2$	...	$S^m{}_n$	$H^m{}_n$

Figure 5

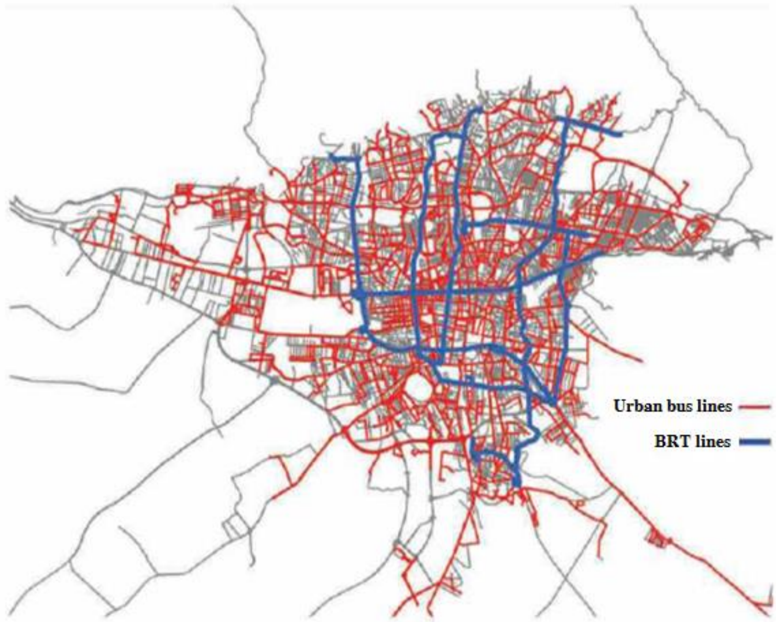


Figure 6

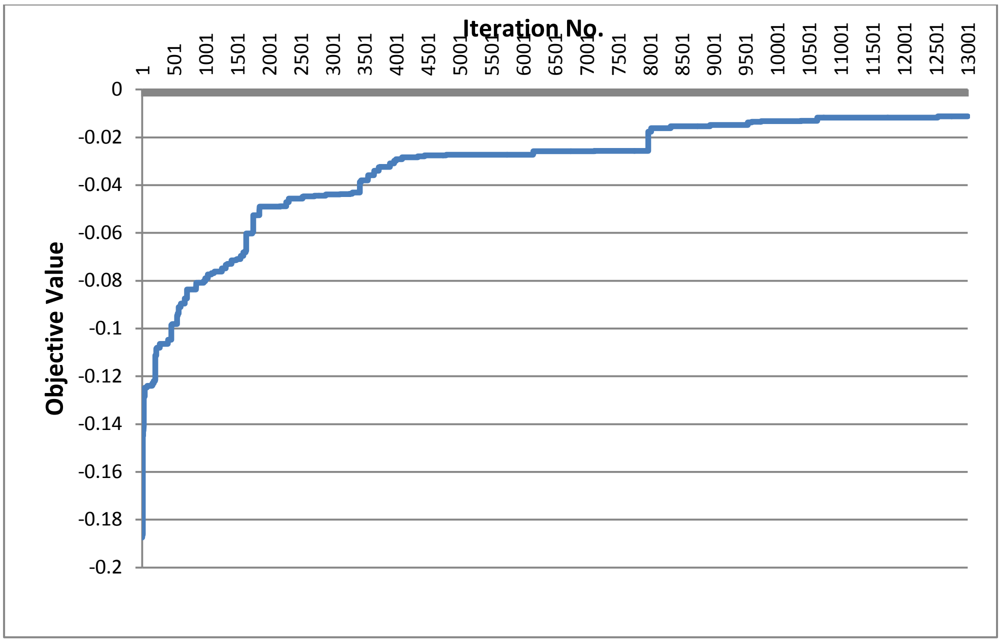
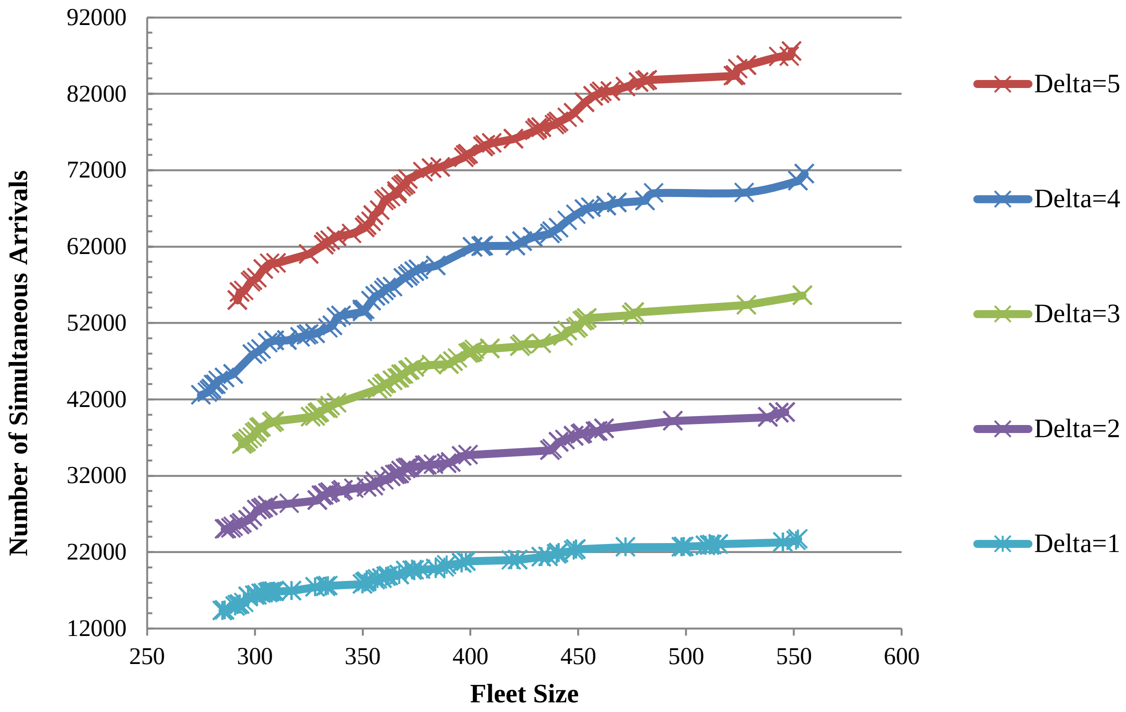

Iteration No.
1
501
1001
1501
2001
2501
3001
3501
4001
4501
5001
5501
6001
6501
7001
7501
8001
8501
9001
9501
10001
10501
11001
11501
12001
12501
13001
Objective Value
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2

Figure 7



92000
82000
72000
62000
52000
42000
32000
22000
12000
250
300
350
400
450
500
550
600
Number of Simultaneous Arrivals
Fleet Size
Delta=5
Delta=4
Delta=3
Delta=2
Delta=1

Figure 8

Table 1

Ref. No.	Year	Author(s)	Objective	Solution Approach
[14]	2001	Ceder et. al.	Number of simultaneous arrivals	Heuristic approach
[15]	2004	Eranki	Number of simultaneous arrivals, passengers waiting time	Heuristic approach
[17]	2003	Quak	Drive time, number of line runs, detour time, overloading, fleet size, waiting time	Heuristic approach
[20]	2006	Cevallos and Zhao	Transfer waiting time	Evolutionary
[21]	2002	Jansen et. al.	passenger transfer times	Neighborhood search
[22,23,36]	1995/1997/1998	Chakroborty et. al.	Passengers total waiting time	Evolutionary
[24]	1995	Daduna	Transfer time	Neighborhood search
[25]	2004	Gao et. al.	Deterrence, cost, system performance	Heuristic approach
[26]	1992	Bookbinder and Désilets	Waiting time	-
[29]	2005	Yu et al.	Number of transfers, maximum passenger flow per unit length	-
[33]	2004	Fleurent et al.	Trip synchronization, historical runs, fleet size, deadhead, layover	Heuristic approach
[34]	2004	Castelli et al.	Passenger transfer waiting time	Heuristic approach
[35]	1976	Rapp and Gehner	Transfer optimization	Heuristic approach
[37]	2001	Chakroborty et. al.	Transfer coordination (waiting time), fleet size	Evolutionary
[38]	2003	Ngamchai and Lovell	Fleet size, in-vehicle and waiting time	Evolutionary

Table 2

Symbol	Definition
Sets	
T	Set of time periods (indexed by t and p)
L	Set of lines (indexed by l and k)
S	Set of stops (indexed by i)
S^l	Set of stops on line l ($S^l \subset S$)
C^{lk}	Set of transfer points on lines l and k ($C^{lk} \subset S$) (indexed by q)
D_t	Set of departures in period t (indexed by m and n)
D^l_t	Set of departures on line l in period $t (D^l \subset D)$
Parameters	
U	Length of the time periods (usually 1 hour)
CT^l_t	Average round-trip time of line l in period t
$Hmin^l_t$	Minimum headway on line l in period t
$Hmax^l_t$	Maximum headway on line l in period t
dw^l_{it}	Average dwell time at stop i on line l in period t
δ^{lk}_{tp}	Maximum allowable time window for a synchronized arrival from line l in period t and line k in period p at a transfer point
$R^l_{i,i+1,t}$	Average travel time between two consecutive stops i and i+1 on line l in period t
Decision Variables	
x^l_t	The interval between the beginning of period t and the first departure on line l in that period
y^{lk}_{mnqtp}	A binary variable, which yields the value 1 if the m-th departure of period t on line l meets the n-th departure of period p on line k at transfer point q within a time window δ^{lk}, otherwise it yields the value 0
FS^l	Average number of required fleet size on line l
H^l_t	Planned headway on line l
A^l_{nit}	Arrival time of departure n to stop i on line l in period t
ND^l_t	The number of departures on line l in period t

Table 3

Line no.	**CT**[l]	**dw**[l]	**Hmin**[l]	**Hmax**[l]
1	33	3	2	7
2	27	1	1	6
3	51	5	3	10
4	15	3	2	7

Table 4

Line no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**H**	**FS**
1	5	12	19	26	33	40	47	54	-	-	-	7	5
2	6	11	16	21	26	31	36	41	46	51	56	5	6
3	5	15	25	35	45	55	-	-	-	-	-	10	6
4	5	12	19	26	33	40	47	54	-	-	-	7	3
Total	→												**20**

Table 5

Time Per. / Line no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	2	3	1	1	3	1	1	1	2	2	3	3	3	2	1	2
2	1	3	1	4	2	3	6	1	1	4	3	1	1	4	4	6	2
3	1	3	2	4	3	1	1	3	1	2	5	1	4	4	2	4	3
4	3	4	5	2	2	3	6	2	6	2	3	3	3	2	3	4	5
5	2	3	1	3	4	3	1	5	2	1	4	1	3	4	1	4	3
7	2	1	3	4	6	7	2	5	3	5	2	1	7	3	6	3	7
8	6	7	7	2	7	7	10	1	9	2	8	7	9	3	4	11	15
10	5	1	5	2	6	1	6	3	1	3	6	2	2	1	6	4	2

Table 6

Time per. / Line no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**Fleet Size**
1	2	2	2	3	2	2	2	2	2	3	2	3	3	3	2	2	4	57
2	4	3	3	3	3	4	4	6	3	4	3	4	4	5	4	5	4	45
3	3	3	3	3	4	3	3	3	5	3	4	3	3	4	4	3	5	32
4	5	4	5	6	4	5	6	5	6	6	5	5	4	5	5	8	7	36
5	3	3	3	3	5	5	4	5	6	4	6	4	4	4	5	6	9	34
7	5	4	4	4	4	4	6	5	4	6	4	6	6	7	5	6	7	30
8	10	6	7	6	10	9	10	8	10	10	8	9	9	7	8	11	11	6
10	6	5	4	5	6	4	4	5	6	5	6	5	5	5	6	4	5	17
Sum																		257