A mathematical model for an integrated airline fleet assignment and crew scheduling problem solved by vibration damping optimization

Alireza Rashidi Komijan[1*], Reza Tavakkoli-Moghaddam[2], Seyed-Ali Dalil[3]

[1]*Department of Industrial Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran*
[2]*School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*
[3]*Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran*

Abstract

Fleet assignment and crew scheduling are the most complex airline optimization problems. In this research, an optimized crew pairing set considered as an input, and a crew is chosen for assigning to each certain crew pairing. This paper presents a novel model to integrate fleet assignment and crew scheduling problems. In this model, closed routes for crews and fleet are considered simultaneously. Also, the model considers two consecutive flight legs and some characteristics such as time lag, Minimum permitted time lag and Maximum Economic time. Also, a vibration damping optimization (VDO) algorithm is introduced to find good solution for this problem in a reasonable time. Experimental design based on the Taguchi method is taken into account. To compare the proposed VDO algorithm performance, four designed test problems are solved by proposed VDO and compared with optimal solution and Particle Swarm optimization (PSO) algorithm. Then, 10 generated test problems in large scale are solved using VDO and PSO. The results show that in four designed test problems, VDO and PSO solutions have 1.62% and 2.95% gaps in average with optimal solution. Moreover, based on 10 generated test problems, in average VDO give 6.71% better solution in less time compare to PSO.

Keywords: Airline fleet assignment; Crew scheduling; Integrated mathematical model; VDO algorithm; Taguchi experimental design.

1. Introduction

Airlines seek for different methodologies to solve complex operational and technical airline problems. They consider different operations research methods to tackle these

*. *Corresponding author. Tel.: +98 21 88347422;*
Postal Address: Islamic Azad University, Firoozkooh, Iran.
E-mail addresses: rashidi@azad.ac.ir (A. Rashidi Komijan);
tavakoli@ut.ac.ir (R. Tavakkoli-Moghaddam);
ali.dalil7@gmail.com (S. A. Dalil)

problems. [1] The main critical problem is airline planning which includes four sub-problems as shown in Figure 1.

<Insert Figure 1. here>

Flight scheduling is the initial phase of airline planning. The output of this phase is a timetable that includes list of arrival and departure times of flight legs. Airline's decisions for designing flight legs depend on some factors, such as potential transportation demand, available aircraft seats, available human resource, rules and regulations, and the strategic marketing decisions of other airlines [1].

In fleet assignment problem, suitable fleet types are assigned to flight legs. The goal of this section is to assign each fleet type to a designed flight leg based on available aircraft of that type with a specific sequence while optimizing some objective functions under different operational and technical limitations [2]. It should be noted that this section only concerns different fleet type and it does not consider any specific aircraft.

In aircraft routing problem, schedule and route of each aircraft is determined in details considering maintenance requirements. The main objective of this section is to minimize aircraft costs with some considerations such as flight coverage, balance load of aircraft and maintenance requirement [3].

Crew scheduling is about assigning crew to designed flight legs. It can be considered for both cockpit and cabin crew according to the scheduling requirements. Aircraft routing and crew scheduling problems are usually considered after fleet assignment [4]. In the airline industry, crew cost is the second hugest cost after fuel cost [1]. Crew scheduling includes two sub-problems: crew pairing and crew rostering problems [2]. In crew pairing, a certain sequence of flight legs generates candidate pairings. In crew rostering, each crew is assigned to certain pairings according to the rules and regulations. Different approaches have been applied for solving above-mentioned sub-problems as column generation, benders decomposition, branch-and-price, heuristic and meta-heuristic algorithms [2,4]. In this paper, flight scheduling is given and an integrated model is presented for crew scheduling and fleet assignment problems. Contributions of this paper are proposing a novel mathematical model for integrated fleet assignment and crew scheduling problems, considering closed routes for crews and fleets at the same time and applying a vibration damping optimization algorithm as the solution approach. Solving the model leads to schedule of crew and routing of aircrafts. In other words, it is determined which pairings should be selected and assigned to each crew. Also, solving the model clears that which flights should be done by each aircraft. The objective function of the model is total cost minimization. Total cost includes fleet operational costs, crew costs, deadhead flight costs and aircraft-change costs. To have optimized crew pairings, Ahmadbeygi et al. [5] model is chosen and its generated pairings are considered as an input.

Among the many studies that integrate sub-problems of airline planning, the proposed model is the first model that has been solved by novel vibration damping optimization (VDO) algorithm. Moreover, Taguchi experimental method is taken in to account to tune the algorithm parameters. Also, closed routes are considered simultaneously for crews and fleet. Moreover, the model considers two consecutive flight legs and some characteristics such as time lag, Minimum permitted time lag and Maximum Economic time. In order to show the efficiency of VDO algorithm, some numerical examples have been developed and were solved using GAMS, VDO and PSO. The results show that in four designed test problems (small size problems), VDO and PSO solutions have 1.62% and 2.95% gaps in average with optimal solution. Moreover, based on 10 generated test problems (large scale problems), in average VDO give 6.71% better solution in less time compare to PSO.

2. Literature review

Several airline planning and operation problems with different horizon have been addressed in the literature. The fleet assignment problem is usually formulated as linear programming, mixed-integer programming or multi-commodity flow under operational and technical constraints [1,2].

Different types of airline fleet assignment models presented by Sherali et al. [6] considering technical and operational constraints. Also, they presented different integrated fleet assignment problems with other section of airline planning processes and surveyed different solution methods.

Sandhu and Klabjan [7] formulated integrated fleet assignment and crew scheduling problems. They presented column generation, Lagrangian relaxation and Benders decomposition methods as different solution approaches.

Papadakos [8] considers fleet assignment, aircraft maintenance routing and crew scheduling problems in a single model and solved it using Benders decomposition and column generation methods. Gao et al. [9] presented an integrated mathematical model for airline fleet assignment and crew planning problems. They proposed their model to find optimal fleet assignment results and cause robustness in crew planning and real-time operations. Ahmadbeygi et al. [5] presented a model to generate pairings based on rules and regulations that can be solved by exact methods such as Branch & Bound.

Weide et al. [10] solved the original aircraft routing and crew scheduling problems iteratively. They applied an iterative approach for model robustness. Also, they generated solutions that add costs as penalties and cause robustness to stochastic variability in operation. Pilla et al. [11] considered fleet assignment problem as a stochastic programming model and applied a multivariate adaptive regression based on the cutting planes method to solve it. Hu et al. [12] presented a model based on an approximate reduced time-band network for aircraft routing problem considering transit passengers under disruption.

An MIP mathematical model presented by Cacchiani and Salazar-Gonzalez [13] which integrates fleet assignment, aircraft routing and crew rostering problems and applied a heuristic method for Canary Islands airlines. They proposed binary variables for potential aircraft and crew route. For obtaining near-optimal solutions, they developed a column generation method. Salazar-Gonzalez [2] extended the model of Cacchiani and Salazar-Gonzalez [13] by considering a crew rostering problem and adding operational constraints, such as classical capacity constraints and infeasible-path inequalities. They made an analogy between crew rostering problem and multi-depot vehicle routing problem with the possibility of swapping drivers. They considered vehicles as aircraft and the drivers as crews.

Diaz-Ramirez et al. [14] applied a heuristic method for aircraft routing and crew scheduling problems considering one specific fleet, a single crew and maintenance base. The crew scheduling problem was solved by using a heuristic method to obtain an efficient initial feasible solution and applying a labeling algorithm and column generation technique to solve the pricing problem. Finally, the model was formulated and compared with a traditional method. Cheng et al. [15] introduced the concept of flight operation risk assessment system (FORAS) for an airline and the correlation between a risk factor and its sub components with fuzzy inference system. They also developed algorithms to identify critical risk factors based on sensitivity of the risk factor and heuristic search.

Kasirzadeh et al. [4] presented a survey for airline crew scheduling problems and solution methods. In addition, they formulated the personalized crew scheduling problem for cockpit crews based on a classic set covering problem and solved it by a column generation method.

a non-linear model formulated by Gurkan et al. [16] to integrate airline flight scheduling, fleet assignment and aircraft routing problems. They also considered cruise speed control, fuel consumption and CO_2 emission and solved it using two heuristic methods in large scale numerical cases. Dong et al. [17] formulated two models for flight scheduling and fleet assignment integration considering the itinerary price elasticity.

Safak et al. [18] introduced a robust schedule design considering cruise speed control and carbon emission. They applied a two-stage algorithm to decompose the problem and solve them sequentially.

Khaksar and Sheikholeslami [19] presented a method for airline delay prediction by machine learning algorithms based on data mining, random forest Bayesian classification, K-means clustering and hybrid approach and calculated delay occurrence and magnitude in both IRAN and USA networks.

A robust mathematical model presented by Jamili [20] which integrates flight scheduling, fleet assignment and maintenance routing problems and applied simulated annealing (SA) and particle swarm optimization (PSO) algorithms to solve his model.

Safaei & Jardine [21] addressed maintenance routing problem that ensures enough time for maintenance operations. The presented model minimizes maintenance misalignment using an interactive method between maintenance planning decisions and aircraft routing.

Chen et al. [22] formulated a mathematical programming model for aircraft leasing decisions which considered the aircraft requirements, budget and debt-ratio limit. Objective function of the model is cost minimization. Kenan et al. [23] developed an integrated and stochastic model for flight scheduling, fleet assignment and maintenance routing problems and solved it using column generation. Eltoukhy et al. [24] presented an operational maintenance routing model with a solution algorithm for finding suitable routing based on maintenance requirements. They also considered flying time, number of takeoff/landing and maintenance workforce as constraints. They applied the model in Egypt Air. Huu et al. [25] presented a multi-objective model to determine aircraft departure routes and developed and applied their model in Rotterdam and Amsterdam airports.

Ben Ahmed et al. [26] integrated aircraft routing and crew pairing considering maintenance restriction all in a robust mathematical model that generate elastic result to unpredictable disruption that causes delays and flight cancelation. An integrated and stochastic model presented by Kenan et al. [27] for flight scheduling and fleet assignment problems. They considered flight demand by stochastic parameter.

Based on the above literature review, there is no any research with the new variables and constraints for the proposed integrated model where for each fleet, the model is able to count the number of flight legs. Moreover, there is no any study applying the vibration damping optimization algorithm (VDO) algorithm for the integrated airline mathematical model after parameter tuning based on the Taguchi method. In this paper, the sequence of flight legs for a certain aircraft can be achieved and compared with the sequence of flight legs for a certain crew considering time lag and time limitation constraints. The aircraft routes are determined based on a set of assigned flight legs sequence. Meanwhile the VDO algorithm is used to find good solutions for the mathematical model and VDO performance compared with optimal results and PSO algorithm. Also, this model is able to count the number of flight legs. Furthermore, the sequence of flight legs for a certain aircraft can be achieved and compared with the sequence of flight legs for a certain crew that is assigned to a specific pairing.

3. Mathematical model

This section presents the integrated airline fleet assignment and crew scheduling problem for different pairings based on integer linear programming (ILP) formulation. A certain pairing is a sequence of flight legs in different days that begins and ends at the base. Figure 2 shows four different generated pairings, which are closed sequences of flights. After generating anonymous unassigned pairings based on time restrictions and cost limitation. It should be noted that these pairings are generated based on the crew pairings integer programming model proposed by Ahmadbeygi et al. [5]. It is time to assign each flight leg to the right aircraft type of each fleet family type based on the aircraft capacity and flying time limitation. After that, based on these generated pairings from the crew pairing model, each crew should be assigned to a certain pairing. It should be noted that constraints are applied for the cockpit-crews (e.g., pilots, co-pilots, flight engineers) and cabin-crews (e.g., flight attendants and flight guards) are not considered in this research. This approach can be applied to the planning according to this fact that each pairing takes place at last in three days. So from now on, cockpit-crews have been named crews in this research. Furthermore, each flight leg should be assigned only to one aircraft and one crew. It worth to note that the time lag between two consecutive flight legs are one of the most subject that should be considered when two consecutive flight legs are assigned to a specific aircraft. In the other words, it should be applied when there is no aircraft-change between two consecutive flight legs. This subject is worthy when the time lag should stand between two parameters, namely the minimum permitted time (i.e., least stopping time) that is needed between two consecutive flight legs and the maximum time (i.e., maximum economic time) that is logical for an aircraft to stay in a specific airport. According to the fact that, aircraft-change for each crew may result in delays for the next flight legs (connection time based on aircraft-change may increase the possibility of delays), airline companies consider costs for these subjects that may reduce these delays by minimizing the aircraft-change costs.

<Insert Figure 2. here>

Aircraft routes are the sequence of the flight legs with arrival and departure times. To illustrate the issue, Figure 3 shows possible flight legs among four airports. For example, in this figure, the *A-G-H* flights sequence is a closed route for aircraft. Each flight leg has five characters, arrival time, departure time, origin, destination and flight number. Time away from base (TAFB) is about the total time for a crew that starts with the departure time of the first flight leg until the arrival time of the last flight leg considering the sign in and sign out times.

<Insert Figure 3. here>

Model assumptions are as follows:

- In the beginning of planning, several crews and aircraft are known.
- Each aircraft can fly for the certain time period.
- Each flight leg should assign only to one crew and one aircraft.
- Aircraft and crew routes are closed. In the other words, each crew and aircraft start and end in the same base.
- There is no aircraft maintenance during the days of planning.
- There is no flight leg, maintenance and inspection for aircraft between 11:00pm and 7:00am of the next day, and the daily rest time for crews can be considered in this time period.

Minimizing the total cost (i.e., fleet operational costs, crew costs, deadhead flight costs and aircraft-change costs) is considered as objective function of this ILP model. It should be noted that this model considers aircraft types based on the fleet family type. Below, sets, indices, parameters and variables are introduced:

Sets and indices:

A	Set of aircraft fleets types that are available for assigning to flight legs
a	Aircraft index ($a=A_1, A_2, \ldots$)
P	Set of possible pairings
p	Pairing index ($p=1, 2, \ldots$)
F	Set of all flight legs
$FP_{f,f'}^{p}$	Set of consecutive flight legs f, f' of pairing p
θ_1	Set of flight legs with origin same as base
θ_2	Set of flight legs with destination same as base
ρ	Set of consecutive flight legs that destination of the first flight leg is the origin of the second flight
f, f'	Index of designed flight leg ($f, f'= A,B, \ldots$)
N	Set of flight counter
n	Index of flight counter (n=1, …, N)
C	Set of available crews for the designed schedule
C_a	Set of crews that are able to fly with aircraft a
c	Crews index (c=C1, C2,…)

Parameters:

FT_f	Flying time of flight leg f per minute
$CrewC_{p,c}$	Crew cost for each crew c for assigning to pairing p
$FleetC_{f,a}$	Fleet operational cost for aircraft a for assigning to the flight leg f
$ChangeC_f$	Aircraft-change cost for each flight leg f
DH_f	Deadhead flight cost for each flight leg f
$Lag_{f,f'}$	Time lag between two consecutive flight legs f and f' per minute
φ_a	Maximum time restriction for each for aircraft a per minute
LL	Minimum permitted time lag for two consecutive flight legs per minute
UL	Maximum economic time lag for two consecutive flight legs per minute
M	Large positive number

Decision variables:

$X_{f,a,n}$	1 if flight leg f assign to the aircraft a in the n-th flight leg; 0, otherwise
$Y_{p,c}$	1 if crew c assign to the pairing p; 0, otherwise
$Z_{f,c}$	1 if flight leg f in leading of crew c has aircraft-change; 0, otherwise

The mathematical model for the integrated fleet assignment and crew scheduling problem is proposed below.

$$\text{Min} \quad Z = \sum_{c\in C}\sum_{p\in P} CrewC_{p,c}.Y_{p,c} + \sum_{f\in F}\sum_{a\in A}\sum_{n\in N} FleetC_{f,a}.X_{f,a,n} + \sum_{f\in F}\sum_{c\in C} ChangeC_f.Z_{f,c} + \sum_{f\in F}\left(\left(\sum_{a\in A}\sum_{n\in N} X_{f,a,n}\right)-1\right).DH_f \tag{1}$$

s.t.

$$\sum_{n\in N}\sum_{a\in A} X_{f,a,n} = 1 \qquad \forall f \in F \tag{2}$$

$$\sum_{c\in C} Y_{p,c} = 1 \qquad \forall p \in P \tag{3}$$

$$\sum_{n\in N} X_{f',a,n} + 1 - \sum_{n\in N} X_{f,a,n} + Y_{p,c} \le Z_{f,c} + 2 \qquad \forall (f',f) \in FP^{p}_{f',f}, \forall a \in A, \forall c \in C_a, \forall p \in P \tag{4}$$

$$\begin{aligned} &LL - (2 - \sum_{n\in N} X_{f,a,n} - \sum_{n\in N} X_{f',a,n}).M \le lag_{f,f'} \\ &\le UL + (2 - \sum_{n\in N} X_{f,a,n} - \sum_{n\in N} X_{f',a,n}).M \qquad \forall (f,f') \in \rho, \forall a \in A \end{aligned} \tag{5}$$

$$X_{f',a,n} \le \sum_{f:(f,f')\in\rho} X_{f,a,n-1} \qquad \forall f' \in F - \theta_1, \forall n > 1, n \in N, \forall a \in A \tag{6}$$

$$\sum_{f\in\theta_1} X_{f,a,1} \le 1 \qquad \forall a \in A \tag{7}$$

$$\sum_{a\in A} X_{f,a,1} = 0 \qquad \forall f \in F - \theta_1 \tag{8}$$

$$\sum_{a\in A} X_{f,a,n} = 0 \qquad \forall f \in F - \theta_2, n = N \tag{9}$$

$$X_{f',a,n} \le \sum_{f:(f,f')\in\rho} X_{f,a,n-1} \qquad \forall f' \in \theta_1, \forall n > 2, n \in N, \forall a \in A \tag{10}$$

$$\sum_{f\in F}\sum_{a\in A} X_{f,a,n} \le 1 \qquad \forall n \in N \tag{11}$$

$$\mathrm{Tic}_f . \mathrm{X}_{f,a,n} \le Cap_a \qquad \forall f \in F, \forall a \in A, \forall n \in N \tag{12}$$

$$\sum_{n\in N}\sum_{f\in F} FT_f . X_{f,a,n} \le \varphi_a \qquad \forall a \in A \tag{13}$$

$$\sum_{a\in A}\sum_{n\in N} X_{f,a,n} \le \sum_{p\in P_f}\sum_{c\in C} Y_{p,c} \qquad \forall f \in F \tag{14}$$

$$\sum_{p\in P} Y_{p,c} \le 1 \qquad \forall c \in C \tag{15}$$

$$X_{f,a,n}, \mathrm{Y}_{p,c}, \mathrm{Z}_{f,c} \in \{0,1\} \tag{16}$$

The objective function (1) minimizes the total costs (i.e., fleet operational costs, crew costs, deadhead flight costs and aircraft-change costs). The first term of this objective function considers the related costs for the pairing *p* that is assigned to the specific crew *c*. It should be noted that each pairing has different number of flight legs, in which the related costs for each pairing are different from the others. The second term is about fleet operational costs (e.g., fuel costs, preparation costs, stopping costs, landing costs and taking off costs). The third term presents aircraft-change costs based on flight legs. Finally, the last term considers deadhead flight legs costs for the flight legs that appears more than once in different selected pairings (i.e., if flight leg *A* exists in two different selected pairing P_1={*A*,*E*} and P_2={*A*,*G*,*H*}, it means that flight leg *A* are covered twice and one of them should take place as a deadhead flight leg and one of the assigned crews should be considered as a passenger on that flight leg).

Equation (2) states that each flight leg should be assigned to one of aircraft in one of flight counters. Equation (3) indicates that some of the generated pairings are chosen to cover all of the flight legs. In the other words, only some of generated pairings are chosen according to the fact that all flight legs should be covered by aircraft and crews. Equation (4) ensures that if two consecutive flight legs of certain pairing which assigned to the specific crew with same aircraft took place, no aircraft-change would happen; otherwise, aircraft-change would happen for these consecutive flight legs which assigned to different aircraft. Equation (5) guarantees that a time lag between two consecutive flight legs is greater than the minimum permitted time, and these times are smaller than the maximum economic time. These two parameters are defined by the airline companies. The minimum permitted time defines the least time that are needed for an aircraft to be ready for the next flight. The maximum economic time defines the longest time that should take place according to the economical restriction of the companies.

Equation (6) guarantees that if a flight leg with different origin from base was chosen as the second flight, there would be a flight leg or flight legs that would take place before the mentioned flight leg. Equation (7) represents flight legs with origins same as base can be chosen as the first flight of aircraft. Equation (8) states flight legs with origins different from base should not be chosen as the first flight of aircraft. In fact, all aircraft should start from the base. Equation (9) states that flight legs with different destination from the base should not be chosen as the last flight of aircraft.

Equation (10) indicates that if flight legs with origins same as base are chosen as the third or next, there will be a flight leg or flight legs that take place before this mentioned flight leg. In fact, as mentioned before, the first flight leg should be chosen based on the θ_1 set, the subject that considered in Equation (7), the second flight leg should be chosen based on the θ_2 set, so the third flight leg and next can take place from the base again. This situation is possible based on the ρ set.

Equation (11) illustrates that aircraft transportation takes place when it is assigned for a flight leg. For more explanation, it means, in each flight counter of an aircraft only one flight leg happens. Equation (12) presents that sold tickets of each flight leg should be harmonized with the aircraft capacity. In the other words, sold tickets must be equal or less than the aircraft capacity that is assigned to a specific flight leg. Equation (13) indicates that total flying time for each aircraft should not violate the maximum time restriction that proposed by companies based on rules and regulations. Equation (14) guarantees that a specific flight leg in any flight positioning (flight 1, flight 2, etc.) is chosen to assign to aircraft and the pairing (P_f) that include that flight leg is selected.

Equation (15) presents that in any scheduling period each crew only assign to one pairing. Adding this equation is worthy when a planner wants to select crews more fairly. Moreover, based on assigned pairings and designed flight legs, TAFB is achievable for each assigned crew. Equation (16) presents types of the decision variables.

4. Proposed VDO algorithm

A vibration damping optimization (VDO) algorithm is based on the vibration damping process [28]. To apply the VDO algorithm for the integrated airline fleet assignment and crew scheduling problem, several effective choices must be made.

4.1. Solution representation

A solution representation is divided in to two parts. The first part presents a random sequence of aircraft that is assigned to flight legs sequence. It should be noted that {*f=1,2, …*} index is replaced with {*f=A, B, C, …*} index because large numbers of flight legs. The second part represents the random sequence of flight legs that based on them each flight leg is assigned to available aircraft. For clarifying the proposed algorithm, the following example is considered. Assume that five aircraft are available and nine flight legs are designed for assignment. Tables 1 and 2 show θ_1, θ_2 and ρ sets, respectively. Now the sequence of flight legs and aircraft is presented in Tables 3a and 3b.

<Insert Table 1. here>

<Insert Table 2. here>

<Insert Table 3. here>

Then based on the flight legs and aircraft sequence, flight legs are assigned to aircraft. For the given example, aircraft 5 is first chosen, and the flight legs with the value of 1, from θ_2, row of Table 1 are candidate for assignment. According to the proposed θ_2 set, it is obvious that flight legs 4, 8 and 9 have the value of 1. So for aircraft 5, the flight leg that is in the fourth position (i.e., flight leg 3) of the flight leg sequence is first chosen for assignment. Now, the sequence of flight legs is changed into Table 4a. According to the changed sequence, it is time to check the destination (θ_2) for the last flight leg that is assigned to aircraft 5. In the observation, the destination of the last flight leg is not same as base, so another flight should be chosen for aircraft 5. Based on ρ set, the flight legs with the origin as same as the destination of flight leg 7 are candidate to be chosen, based on the proposed example, flight legs 8 and 9 are candidate. It should be noted that these two flight legs can occur consecutively. So after the above discussion, flight leg 8 is assigned to aircraft 5 as well. This flight will be deleted from the proposed flight legs sequence and the new flight legs sequence is changed into Table 4b. Other necessary assignment can be followed as other part of Table 4. Based on the decoding, the assignment of flight legs to the different flight counters of the assigned aircraft is achievable. (i.e., for aircraft 5, the sequence of flight legs is flight 3-flight 7-flight 8, so flight 7 occurs as the second flight of aircraft 5). For assignment of pairings to crews, the proposed VDO algorithm assigns the chosen pairings to the available crews randomly. In the other words, $Y_{p,c}$ decision variable matrices are generated randomly. Based on the Equation (4), the $Z_{f,c}$ decision variable matrices are achievable.

<Insert Table 4. here>

4.2. Neighborhood structure

A neighborhood structure is considered for generating or developing a neighboring solution by adding some changes from the old solution to the new solution. Different neighborhood structures have been applied to airline planning and operation problems. These neighborhood structures should generate only feasible solutions and eliminate all infeasible solutions. In this paper, there are two types of operators which generate neighborhoods. These operators are swap and reversion, and applied to the flight legs and aircraft sequences. The swap is about changing the value of two positions, and the reversion operator is about arranging the positions from right to left for two candidate positions. Tables 5 and 6 show the process of swap and reversion operators.

<Insert Table 5. here>

<Insert Table 6. here>

4.3. Steps of the proposed VDO algorithm

Each of iteration of the proposed VDO algorithm adds some random changes in the current solution which generate a new solution in the neighborhood. Generation mechanisms define the neighborhood structure. As discussed in Section 4.2, the swap and reversion operators are chosen for the neighborhood structure. Once a new solution is generated, the corresponding changes in the cost function should be calculated to decide whether the new generated solution is acceptable or not. The steps of the proposed VDO algorithm are illustrated in Figure 4. The stopping criterion for the proposed VDO algorithm is reaching to the supposed maximum number of iterations. Moreover, to achieve the suitable results, penalties are added to the Equations (5), (12) and (13) to avoid violation in the algorithm.

<Insert Figure 4. here>

5. Parameter tuning

One of the main concerns of creating each meta-heuristic algorithm is to set suitable parameters to achieve acceptable performance for proposed algorithm. In this section, different parameter choices for the proposed VDO algorithm are studied. To tune the algorithm parameters, different methods of design of experimental (DOE) are introduced. One of these methods is the full factorial design used in different studies to design trials [28]. This method tests all possible combinations of factors, meanwhile, this method is not logical to use for a large number of factors, because of the unaccepted cost and run time.

For this condition, the Taguchi experimental method is one of the DOE methods that use the orthogonal arrays to find the suitable factors with a few numbers of experiments. Based on the Taguchi method, factors are divided into two parts, namely controllable and noise factors. According to this method, the inner orthogonal replaced by the controllable factors and the outer orthogonal array replaced by the noise factors [28,29]. Based on the behavior of the noise factors, the Taguchi method finds the best level for controllable factors while minimizing the corresponding effect of noise factors. Applying this method can cause robustness for the factors [29]. The Taguchi method transforms the repetitive data and cause variation. This repetitive data transforming is the signal to noise (S/N) ratio, which illustrates the variation in the response variable. The "signal" and the "noise" are about the suitable and unsuitable values, respectively.

The Taguchi method for parameter tuning is applied to gain suitable robustness for the proposed VDO algorithm. Based on parameters of the VDO algorithm, control factors of the algorithm are: A_0, σ, γ, L, *Maxit* and *Npop*. Three different levels for factors are illustrated in Table 7. The levels are called low, medium and high as denoted by *L*, *M* and *H*, respectively.

To achieve the appropriate orthogonal array, in the proposed algorithm, there are six factors with three levels. Based on the standard orthogonal arrays table, 27 different experiments should be considered. These 27 different trials are presented in Table 8, which control factors are shown in rows and three levels are shown in columns.

To gain results of the experiments, the proposed VDO algorithm is implemented in the MATLAB program on windows 7 a PC with 1 GB RAM memory and 2.0 GHz. When the parameters are set as A_0=30, L=8, γ=0.1, σ=0.6 with the maximum iteration of 1500 and the population number of 200, better robustness of the algorithm is gained as shown in Figure 5. After achieving the results based on different trials, the results of each trail are compared with the S/N ratio; this comparison is shown in Figure 6.

In order to adjust factors and compare trials, the relative percentage deviations (RPD) is used for the objective function value (OFV). The RPD is computed by:

$$RPD = \frac{Obj_{\text{alg}} - Obj_{\min}}{Obj_{\min}} \times 100 \tag{17}$$

Where Obj_{alg} is about the OFV for each trail in a specific problem and $Obj_{\min}$ is the best OFV for the same problem. First, it should convert the OFV to RDPs, and then, the RPD is calculated for each level. Figure 5 shows the RPD plot. Based on the RPD and S/N ratio plots, the best levels of factors are as follows: A(H), B(H), C(H), D(H), E(L), F(L).

<Insert Table 7. here>

<Insert Table 8. here>

<Insert Figure 5. here>

<Insert Figure 6. here>

6. Computational results

In this section, different test problems are proposed to evaluate the efficiency and effectiveness of the proposed VDO algorithm with the aim of finding good quality fleet and crew assignments. According to Section 5, the Taguchi experimental method is used to find the best factor of parameters and their levels. Moreover, after ten runs, the best OFV is selected for each test problem. In this section, Table 9 illustrates flight legs with their characteristics in Section 6.1. In this section, the proposed VDO algorithm is compared with the optimal method that is coded with the GAMS program. Table 10 compares the VDO and optimal results based on 4 designed test problems. In Section 6.2, based on Table 11, flight legs and pairings are generated randomly to create test problems. Table 12 shows comparison between the proposed VDO and PSO performances according to 10 generated test problems.

6.1. Comparison between the VDO, PSO algorithm and optimal method

In this section, four designed test problems are considered by using an airline crew pairing model presented by Ahmadbeygi et al. [5] to generate the candidate pairings. Table 9 illustrates proposed flights and their characteristics.

<Insert Table 9. here>

According to Table 10, pairings are generated for each problem and number of available aircraft and crews are given. Then, according to Section 2, each problem is formulated. Moreover, the second column of this table signifies the designed flight legs and the third column represents the generated pairings based on the airline crew pairing model.

The fourth, fifth and sixth columns signify the available aircraft, allowed number of flights and available crews, respectively. It should be noted that all four test problems are run for 15 times and the mean of each test problem is shown in the table. Other columns signify the best results, mean of results and run time for the model based on the VDO and PSO algorithms and optimal method. The last two columns represent gaps between different results obtained by the VDO and PSO algorithms and optimal method embedded in GAMS. A review of the results in Table 10 shows that the VDO, PSO algorithms and GAMS software with CPLEX solver are able to find suitable results for designed test Problems 1 to 4. Moreover, according to 4 designed test problems, the gaps and run times of two proposed methods versus optimal method are logical and VDO and PSO solutions have 1.62% and 2.95% gaps in average with optimal solutions respectively.

<Insert Table 10. here>

6.2. Proposed VDO algorithm versus PSO algorithm for large-scale test problems

To test the VDO algorithm and compare it with PSO algorithm, first 10 test problems are generated in different sizes based on Table 11 with the parameters generated randomly.

<Insert Table 11. here>

Then, the VDO and PSO algorithms are applied for solving these problems based on the generated parameters and proposed mathematical model. Table 12 shows the results of these problems. As it expected, more flights and generated pairings can cause more costs. All 10 test problems are run for 15 times, and the mean of each test problem is shown in the table. A review of the results in Table 12 shows that the VDO algorithm is able to solve large-scale problems. Moreover, the run times of the VDO algorithm are suitable for the different sizes of the problems and these times are smaller than six seconds and in average VDO gives 6.71% better solution in less time compare to PSO. It should be noted that GAMS couldn't find any result for these 10 generated problems. Based on results VDO performance is better than PSO, but if some delays or technical disruptions happen during the planning, VDO parameters should be tuned again based on Taguchi experimental method.

<Insert Table 12. here>

7. Conclusions and future research

This paper has presented a mathematical model for an integrated airline fleet assignment and crew scheduling problem. Generated pairings have been considered as the input data of this problem based on the integer programming model of the airline crew pairing. In the proposed mode, closed routes for crews and fleet are considered simultaneously. Also, the model considers two consecutive flight legs and some characteristics such as time lag, Minimum permitted time lag and Maximum Economic time. Moreover, to solve this model, a novel vibration damping optimization (VDO) algorithm has been applied and compared with PSO and optimal solutions. The objective function of this model has minimized the total cost (i.e., fleet and crew costs). Then, to find the best factor of parameters of the VDO algorithm, the Taguchi design method has been used and the robustness of this algorithms have been improved by tuning the VDO parameters. Furthermore, the results of the proposed VDO and PSO with optimal solutions have been compared and VDO and PSO solutions have 1.62% and 2.95% gaps respectively in average with optimal solutions. In order to examine the performance of the VDO algorithm, large-scale problems have been generated and VDO performance has been compared with PSO based on generated test problems. The computational results illustrate, applying the VDO algorithm has been suitable to optimize the given problems and also in average VDO gives 6.71% better solution in less time compare to PSO. It is suggested to add flight scheduling as a new part of the integrated mathematical model or apply maintenance issues to the aircraft routing problem. It is also worth to consider other constraints for fleets and crews, such as different weight of aircraft and different type of crews. Also, to solve the given problem, developing a hybrid algorithm is suggested, such as VDO-PSO and VDO-GA.

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Figure captions

Figure. 1. Sub problems of airline planning problem

Figure. 2. Examples of candidate pairings to assign to crews

Figure. 3. Typical sample of aircraft possible routes based on flight legs

Figure. 4. Steps of the proposed VDO algorithms

Figure. 5. RPD plot for factors at the proposed levels

Figure. 6. S/N plot for factors at the proposed levels

Table captions

Table 1. Θ_1 and Θ_2 sets for the given example

Table 2. ρ set for the given example

Table 3. Solution representation for the sequence of flight legs and aircraft

Table 4. Decoding process of solution representation for the given example

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Table 12. The Proposed VDO algorithm versus PSO algorithm for different test problems

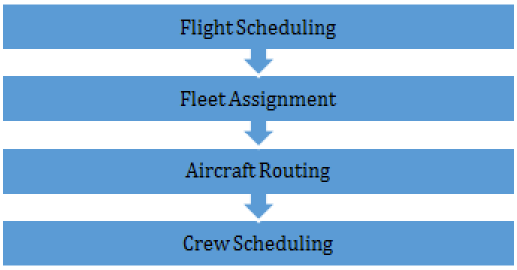

Figure. 1. Sub-problems of airline planning problem

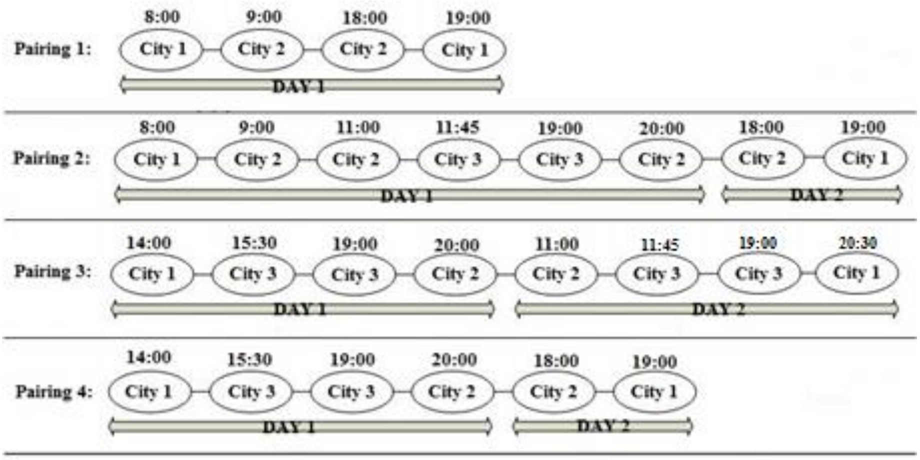

Figure. 2. Examples of candidate pairings to assign to crews

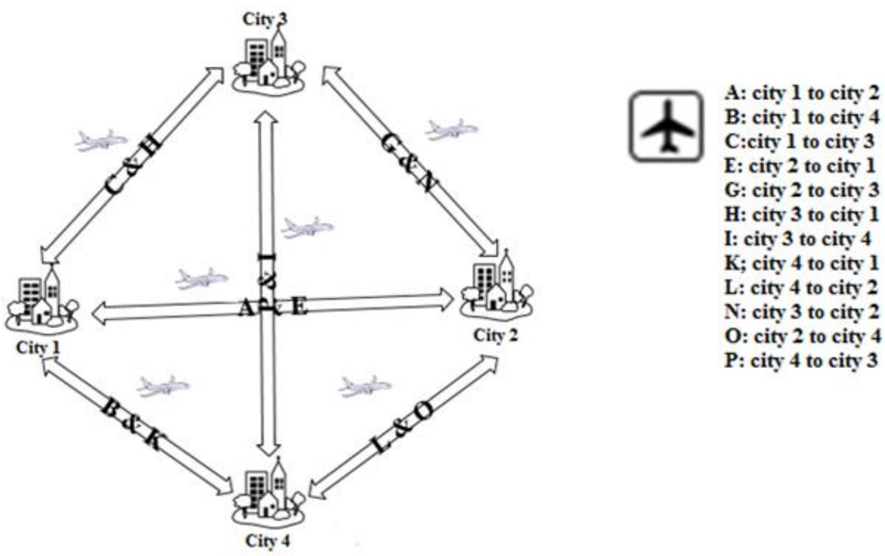

Figure. 3. Typical sample of aircraft possible routes based on flight legs

Step 1: Generate an initial flight legs and aircraft sequence (initial feasible solution).

Step 2: Initialize and tune the parameters of algorithm, which are: Maximum number of iteration (*Maxit*), Number of population (*Npop*), initial amplitude (A_0), maximum iteration at each amplitude (L), damping coefficient (γ), the standard deviation (σ). Finally, parameter t is set in one ($t=1$).

Step 3: Calculate the objective value Obj_0 for the generated solution.

Step 4: Setting the inner loop. The inner loop works from $l=1$ to $l=L$.

Step 5: Generate neighborhood structure. In this paper, two swap and reversion operator are being used.

Step 6: Accept the new solution

Set $diff=Obj-Obj_0$, Now if $diff<0$, the new solution is accepted as the result, otherwise

if $diff>0$, generate a random number r between (0,1);

if $r<1-\exp\left(\frac{-A_t^2}{2\sigma^2}\right)$, then accept the new solution; else reject it and accept the previous solution.

if $l>L$, then apply $t+1\rightarrow t$ and go to the step 7; else $l+1\rightarrow l$ and go back to step 5.

Step 7: Adjust the amplitude. In this step, $A_t=A_0\exp\left(\frac{-\gamma t}{2}\right)$ is used to decrease amplitude at each iteration in the outer loop, if the maximum iteration achieved go to Step 8. Else, go back to Step 4.

Step 8: Stop the criteria. The proposed algorithm will be stopped after reaching to the proposed maximum number of iterations.

Figure. 4. Steps of the proposed VDO algorithm

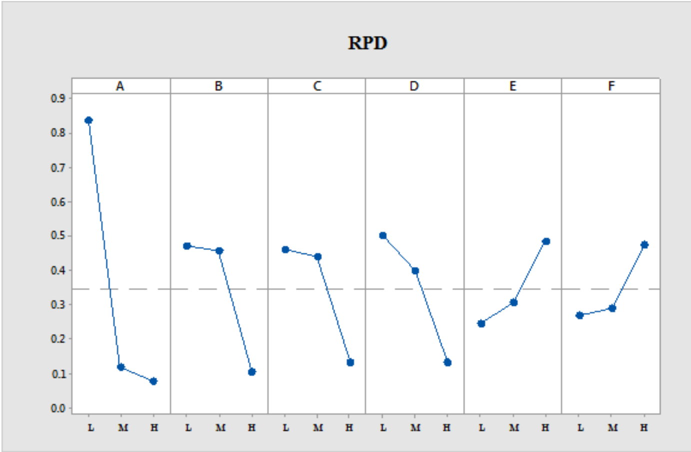

Figure. 5. RPD plot for factors at the proposed levels

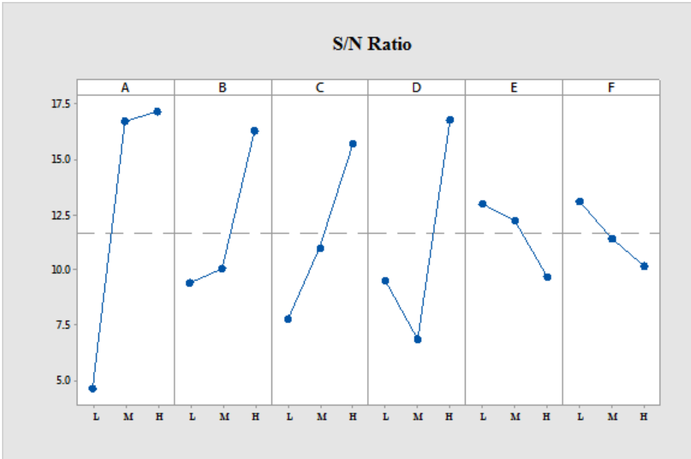

Figure. 6. S/N plot for factors at the proposed levels

Table 1. Θ_1 and Θ_2 sets for the given example

Flight Legs	1	2	3	4	5	6	7	8	9
Θ_1	1	1	1	0	0	0	0	0	0
Θ_2	0	0	0	1	0	1	0	1	0

Table 2. ρ set for the given example

ρ	1	2	3	4	5	6	7	8	9
1	0	0	0	1	1	0	0	0	0
2	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	1	1	0	0
4	1	1	1	0	0	0	0	0	0
5	0	0	0	0	0	1	1	0	0
6	1	1	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	1
8	1	1	1	0	0	0	0	0	0
9	0	0	0	1	1	0	0	0	0

Table 3. Solution representation for the sequence of flight legs and aircraft

(a)

Aircraft sequence	5	4	1	2	3

(b)

Flights	1	2	3	4	5	6	7	8	9
Flight sequence	8	7	6	3	9	5	4	2	1

Table 4. Decoding process of solution representation for the given example

(a)

Flight	1	2	3	4	5	6	7	8
Flight sequence	8	7	6	9	5	4	2	1

(b)

Flight	1	2	3	4	5	6
Flight sequence	6	9	5	4	2	1

(c)

Flight	1	2	3	4	5
Flight sequence	6	9	5	4	1

(d)

Flight	1	2	3	4
Flight sequence	6	5	4	1

Table 5. Sequences with swap operator

Current sequence for aircraft

Aircraft sequence	5	4	1	2	3

Current sequence for flight legs

Flight	1	2	3	4	5	6	7	8	9
Flight sequence	8	7	6	3	9	5	4	2	1

New sequence for aircraft

Aircraft sequence	5	2	1	4	3

New sequence for flight legs

Flight	1	2	3	4	5	6	7	8	9
Flight sequence	8	7	2	3	9	5	4	6	1

Table 6. Sequences with the reversion operator

Current sequence for aircraft

Aircraft sequence	5	4	1	2	3

Current sequence for flight legs

Flight	1	2	3	4	5	6	7	8	9
Flight sequence	8	7	6	3	9	5	4	2	1

New sequence for aircraft

Aircraft sequence	5	3	2	1	4

New sequence for flight legs

Flight	1	2	3	4	5	6	7	8	9
Flight sequence	8	7	2	4	5	9	3	6	1

Table 7. VDO algorithm factors and levels

Factors	Symbols	Levels		
		Low	Medium	High
Maxit	A	A(L)=500	A(M)=1000	A(H)=1500
Npop	B	B(L)=50	B(M)=100	B(H)=200
A_0	C	C(L)=10	C(M)=20	C(H)=30
L	D	D(L)=4	D(M)=6	D(H)=8
γ	E	E(L)=0.1	E(M)=0.2	E(H)=0.3
σ	F	F(L)=0.6	F(M)=0.7	F(H)=0.8

Table 8. Orthogonal array L27 based on the proposed levels

Trails	Different levels for factors					
	A	B	C	D	E	F
1	A(L)	B(L)	C(L)	D(L)	E(L)	F(L)
2	A(L)	B(L)	C(L)	D(L)	E(M)	F(M)
3	A(L)	B(L)	C(L)	D(L)	E(H)	F(H)
4	A(L)	B(M)	C(M)	D(M)	E(L)	F(L)
5	A(L)	B(M)	C(M)	D(M)	E(M)	F(M)
6	A(L)	B(M)	C(M)	D(M)	E(H)	F(H)
7	A(L)	B(H)	C(H)	D(H)	E(L)	F(L)
8	A(L)	B(H)	C(H)	D(H)	E(M)	F(M)
9	A(L)	B(H)	C(H)	D(H)	E(H)	F(H)
10	A(M)	B(L)	C(M)	D(H)	E(L)	F(M)
11	A(M)	B(L)	C(M)	D(H)	E(M)	F(H)
12	A(M)	B(L)	C(M)	D(H)	E(H)	F(L)
13	A(M)	B(M)	C(H)	D(L)	E(L)	F(M)
14	A(M)	B(M)	C(H)	D(L)	E(M)	F(H)
15	A(M)	B(M)	C(H)	D(L)	E(H)	F(L)
16	A(M)	B(H)	C(L)	D(M)	E(L)	F(M)
17	A(M)	B(H)	C(L)	D(M)	E(M)	F(H)
18	A(M)	B(H)	C(L)	D(M)	E(H)	F(L)
19	A(H)	B(L)	C(H)	D(M)	E(L)	F(H)
20	A(H)	B(L)	C(H)	D(M)	E(M)	F(L)
21	A(H)	B(L)	C(H)	D(M)	E(H)	F(M)
22	A(H)	B(M)	C(L)	D(H)	E(L)	F(H)
23	A(H)	B(M)	C(L)	D(H)	E(M)	F(L)
24	A(H)	B(M)	C(L)	D(H)	E(H)	F(M)
25	A(H)	B(H)	C(M)	D(L)	E(L)	F(H)
26	A(H)	B(H)	C(M)	D(L)	E(M)	F(L)
27	A(H)	B(H)	C(M)	D(L)	E(H)	F(M)

Table 9. Proposed designed flight legs and their characteristics

Flights	Origin	Destination	Departure time	Arrival time	Flying time
A	City 1	City 2	08:00	09:00	60
B	City 1	City 4	07:00	08:20	80
C	City 1	City 3	14:00	15:30	90
E	City 2	City 1	18:00	19:00	60
G	City 2	City 3	11:00	11:45	45
H	City 3	City 1	19:00	20:30	90
I	City 3	City 4	18:00	19:05	65
K	City 4	City 1	21:00	22:20	80
L	City 4	City 2	12:00	12:40	40
N	City 3	City 2	19:00	20:00	60
O	City 2	City 4	14:00	14:40	40
P	City 4	City 3	10:00	11:05	65

Table 10. Comparison between the VDO, PSO and optimal algorithms

						Proposed VDO			Proposed PSO			GAMS		Algorithm Gaps	
Problem number	Designed flight legs	generated pairings	A	N	C	Best OFV	Mean of solutions	Mean of run-times(S)	Best OFV	Mean of solutions	Mean of run-times(S)	Best OFV	Run-time(s)	VDO GAP (%)	PSO GAP (%)
1	A, C, E, G, H, N	P1={A,E} P2={A,G,H} P3={C,H} P4={C,N,E} P5={A,G,N,E} P6={C,N,G,H}	3	3	4	259800	261156.5	0.5835	261200	270148.25	0.7613	258700	1.91	0.42	0.95
2	A, B, C, E, G, H, I, K, L	P1={A,E} P2={A,G,H} P3={A,G,I,K} P4={B,K} P5={B,L,G,H} P6={C,H} P7={C,I,K} P8={C,I,L,E}	5	3	6	360800	360912.25	0.6246	362600	371132.5	0.8424	360400	1.838	0.11	0.61
3	A, B, C, E, G, H, I, K, L	P1={A,E} P2={A,G,H} P3={A,G,I,K} P4={B,K} P5={B,L,G,H} P6={C,H} P7={C,I,K} P8={C,I,L,E}	5	5	6	242900	243255.35	0.6323	246800	248422.5	0.9113	234900	1.796	3.4	4.82
4	A, B, C, E, G, H, I, K, L, N, O, P	P1={A,E} P2={A,G,H} P3={A,G,I,K} P4={B,K} P5={B,L,G,H} P6={C,H} P7={C,I,K} P8={C,I,L,E} P9={A,O,K} P10={A,O,P,H} P11={B,L,E} P12={B,P,H} P13={B,P,N,E} P14={C,N,E} P15={C,N,O,K}	5	5	6	340600	340815.75	0.7542	351200	354225.5	1.474	332100	6.687	2.55	5.43

Table 11. Generating parameters for large-scale problems

Parameters	Random generation
FT_f	Random number based on uniform distribution function (45, 90)
$Lag_{f,f'}$	Based on generated sequence 0 or 1
θ_1	Based on origin of generated flights 0 or 1
θ_2	Based on destination of generated flights 0 or 1
C_a	Take 0 or 1 value randomly
ρ	Based on generated sequence 0 or 1
Tic_f	Random number based on uniform distribution function (90, 100)
Cap_a	Generating random number based on capacity of selected aircraft
$ChangeC_f$	Random number based on uniform distribution function (1500, 2500)
φ	Random number based on uniform distribution function (500, 700)
$FP^{p}_{f,f'}$	Generating random number based on consecutive flights of each pairings
P_f	Take 0 or 1 value based on flights in each pairings
$CrewC_{p,c}$	Random number based on uniform distribution function (400, 800)
$FleetC_{f,a}$	Random number based on uniform distribution function (4500, 5500)
DH	Random number based on uniform distribution function (80, 100)

Table 12. The Proposed VDO algorithm versus PSO algorithm for different test problems

Problem	Number of flight legs	Number of generated pairings	Best OFV	Mean OFV	Mean of run time (Sec.)	BEST OFV	Mean OFV	Mean of run time (Sec.)
			PSO			VDO		
1	50	45	787370	793258.7	2.695	642750	653815.2	1.838
2	60	55	831200	853516.5	2.828	761300	764625.8	1.946
3	70	65	943580	987368.3	3.337	898850	901228.6	2.012
4	90	75	1184840	1221134.6	3.752	1156950	1161124.4	2.324
5	100	80	1202160	1214248.3	3.982	1198200	1200168.2	2.836
6	120	100	1752200	1786402.2	4.442	1542600	1587772.5	3.218
7	130	110	1812600	1868204.8	4.982	1649500	1652566.4	3.826
8	140	120	1864260	1912268.5	5.112	1776400	1778504.8	4.322
9	150	140	1988480	1994426.9	5.331	1928250	1930050.2	4.856
10	200	150	2682200	2699860.3	5.973	2571000	2575608.6	5.768

Alireza Rashidi-Komijan is associate professor of industrial engineering. He received his Ph.D. from Islamic Azad University in 2009. His major is operations research and focuses on mathematical modeling, specifically air transportation models. His interest field is large scale scheduling problems.

Reza Tavakkoli-Moghaddam is Professor of Industrial Engineering in University of Tehran, Iran. He received his M.Sc. degree in Industrial Engineering from University of Melbourne, and his Ph.D. degree from Swinburne University of Technology. His research interests include facility layout planning and location design, cellular manufacturing systems, sequencing and scheduling and applications of meta-heuristics in combinatorial optimization problems. He is the author of more than 400 journal papers and 150 papers in conference proceedings.

Seyed-Ali Dalil graduated his B.Sc. degree in Industrial Engineering/System Analysis from Qazvin Azad University, Iran and received his M.Sc. degree in Industrial Engineering from South Branch Islamic Azad University, Iran. He is currently a Product expert at SHATEL Telecommunication Company, Iran. His research interests include Airline Planning and Scheduling, Airline fleet assignment, Aircraft Routing and Airline Crew Scheduling problems, and also strategic product planning, project planning and controlling and using meta-heuristics for integrated airline planning problems.