Integrated supply chain decisions with credit-linked demand- A Stackelberg approach

Chong Zhang¹, Mingchuan You¹, Guanghua Han²*

¹School of Management, Nanjing University of Posts and Telecommunications, 66 Xin Mofan Road, Nanjing, PR.China, 210003
²School of International and Public Affairs, Shanghai Jiao Tong University, 1954 Huashan Road, PR. China, 200030

Abstract. Market demand is likely to be affected by the seller’s credit in many industrial practices, this study aims to investigate the beneficial performance of the supply chain considering the credit-linked demand. Without loss of generality, we conduct the study under three different decision circumstances, i.e., supplier dominated, retailer dominated and centralized supply chain. In this paper, we develop a demand model which is a function of the trade credit period. Moreover, the supplier determines the optimal trade credit period and the retailer determines the optimal replenishment cycle time. Under such conditions, we first present optimal solutions for the centralized decision and the decentralized decision in a Supplier Stackelberg model and a Retailer Stackelberg model. Then, a set of theorems are developed to determine the optimal results. Finally, a numerical example and sensitivity analysis are provided to illustrate the efficiency of the proposed models and optimal solutions. The finding reveals under the condition of the trade credit, the supplier dominated supply chain has a better performance than that the retailer dominated one. However, when the supplier doesn’t provide the trade credit period, the result is the opposite.

KEYWORDS EOQ model · Supply chain coordination · Stackelberg model · Trade credit · Credit-linked demand

1. Introduction

In economic order quantity (EOQ) model, retailers are often assumed to pay immediately after receiving the goods. However, suppliers usually offer the retailers a trade credit period to enlarge market demand and occupy lot size in many industrial practices. Trade credit has been an important source of external financing. According to the Financial Times, trade credit has accounted for 90% of world merchandise trade in 2007 with the total amount of $14 trillion [1]. Then, as of June 2016, the application of trade credit was 1.3 times that of bank loans on the total balance sheet of non-financial business in the United States [2]. And in Fortune, Quick [3] reported that some large enterprises, such as Procter and Gamble, Unilever, Merck, Mondelez International,
have implemented a policy that retailers could use of trade credits of 120 days to delay the payments with suppliers.

When a supplier offers trade credit to allow its retailers to postpone payments, the retailer can sell the products and deposit income from the sale in the bank during the trade credit period. However, if the retailers fail to pay up during the trade credit period, then a higher interest is charged by the supplier. On the contrary, during the period, the retailer can earn interest while suppliers will lose the interest they should have earned. Currently, more and more evidence shows that trade credit is an important short-term financial strategy. For example, Wal-Mart Store Inc and Carrefour Gome (a large European home appliance chain retailer) often delay payments to their suppliers due to their market powers in supply chain. In this way, Wal-Mart and Carrefour Gome can make reasonable use of sales revenue to obtain additional interest. Therefore, the trade credit period is of economic significance to the retailers to delay the replenishment settlement until the last moment of the trade credit period.

Since the supplier and retailer make their decisions subsequently over time, the mathematical models with Stackelberg game process are formulated under different supply chain structures. For a supply chain with the background of Stackelberg game characteristics, each member in the supply chain aims to maximize their own profit and can make their own decisions independently and rationally. For example, Toyota and General Motors are giant manufacturers and leaders in the market, who will impact the decisions of other members in the supply chain. In addition, there are some gigantic retailers play key roles and own remarkable power in their market, such as Tesco, Gome and WalMart, who act like the channel leaders. As a matter of fact, both of supplier-dominated and retailer-dominated supply chains aim to maximize their own profit and the total profit of supply chain.

We focus on the following research questions:

1. Under the decentralized and centralized decision instances, what are the optimal trade credit period and replenishment cycle time, and how do they compare?
2. How the optimal decision variables affect the expected profits of all supply chain members in three supply chain structures?
3. Under the decentralized decision, whether the Supplier Stackelberg model or Retailer Stackelberg model is more beneficial to the supply chain?

To address the above three questions, this paper considers both the decentralized decision and centralized decision simultaneously under the condition that the demand is related to the trade credit period. Both the trade credit period offered by the supplier and the retailer’s replenishment cycle time are considered to be decision variables in our model. Furthermore, to fully understand the impacts of different structures on the supply chain performance, the following factors are taken into account: (1) suppliers usually allow retailers to delay payment, while the retailers do not provide trade credit period to customers, (2) the trade credit period offered by the supplier is positively correlated with the annual demand rate, (3) the profit of the supplier and the retailer are considered, (4) the respective effect of trade credit period and retailer’s replenishment cycle time on optimal decisions under different decision-making structures are investigated respectively, (5) Supply chain members’ decisions under three different structures: (i) Supplier Stackelberg (SS), (ii) Retailer Stackelberg (RS) and (iii) centralized decision are explored. Then, sufficient conditions are derived, which is helpful to search for the optimal solutions for both the supplier and retailer under different structures. In addition, some theorems are developed to presents the retailer and supplier’s decision concerns. Finally, we have the equilibrium solutions
and numerical examples to gain some management inspiration.

The remainder of the paper is organized as follows. Section 2 conducts the literature review. In section 3, notations and assumptions are presented. Section 4 formulates the supplier’s and retailer’s model. In section 5, we present the centralized and decentralized inventory models in Stackelberg games. Section 6 provides a numerical example and sensitivity analysis to investigate the effects of the trade credit period and retailer’s replenishment cycle time on SC performance. Conclusions and future research are provided in Section 7.

2. Literature review

Over the past five decades, many researchers had paid attention to the trade credit period. In 1967, Beranek [4] pointed out the importance of trade credits when determining order lot size. Then Goyal [5] became the first man to study the economic order quantity (EOQ) model under trade credit conditions. And after that, many scholars extended and studied the trade credit strategies. For example, Jamal et al. [6] extended Goyal [5]’s model with the consideration of shortages. Moreover, Dye [7] considered that demand is a function of stock, and made an assumption of trade credit in the model. Huang [8] developed an EOQ model under the condition of trade credits and assumed that when the order quantity was smaller than the predetermined quantity, the supplier provides partial allowable deferred payment to retailer. Huang and Hsu [9] studied the retailer’s inventory policy when suppliers offered partial trade credit to its retailers. Gupta and Wang [10] proposed a stochastic inventory model under the assumption of trade credit. Jaggi and Kausar [11] determined the optimal solutions for retailers, with the consideration of the credit-related demand function, and proposed the optimal replenishment strategy for vulnerable items. Under the condition that demand is related to credit period, Jaggi et al. [12] examined the optimal replenishment cycle time and credit period via EOQ model. Teng et al. [13] assumed that the demand rate is an incremental function of time under the trade credits assumption. Kumar Sett et al. [14] studied a two-warehouse inventory model in which the goods deteriorate as demand increases and time changes. Jaggi et al. [15] developed a new inventory model for incomplete quality items to optimize total profit and orders under delayed payment, because it was observed that in the real environment, there might be some defects in an order batch. Lou and Wang [16] established an EOQ model, in which they did not account for the retailer’s interests and supplier’s increased capital opportunity cost through the provision of trade credit period.

Then, Wang et al. [17] extended Lou and Wang [16]’s model to consider the loss of capital opportunity during the trade credit period. Teng et al. [18] extended the retailer’s EOQ model under conditionally deferred payment, and proposed a new and simple arithmetic geometry method for solving optimal solutions. Kumar and Triphthi [19] considered inflation and exponential demand rate for deteriorating items in their model. Jaggi et al. [20] developed an inventory model for minimizing retailers’ cost in different scenarios, and solved the optimal the replenishment cycle length and storage period simultaneously. Assuming the vendor offering credit period to the retailer, Khanna et al. [21–22] set up an integrated vendor-retailer inventory model from the aspect of substandard products. And then, allowing delay payment, they developed an inventory model for retailers dealing with deteriorating substandard products, in which the order quantity and stock shortage are optimized by maximizing the expected total profit. In 2016, with the trade credit policy of price-dependent demand and the assumption of allowing shortage and complete backlog, Khanna et al. [23] set up an inventory management system model for deteriorating substandard products.
Recently, Peura et al. [24] examined if suppliers can obtain the benefits of trade credit through a horizontal channel. They found that a financially stronger firm with trade credit can exclude its weaker competitor from the market and trade credit can coordinate supply chain and achieve the function of balancing contract if customers attach importance to trade credit. Chen and Zhang [25] studied the capital flow constraints and trade credit in lot-sizing problems. Their results indicated that the incorporating the capital flow constraints and trade credit into lot-sizing problems can affect the optimal solutions. Jaggi et al. [26] developed a retailer inventory model by considering the imperfect quality and deterioration with trade credit, they also examined the optimal trade credit period and the length of a replenishment cycle. In the same year, Jaggi et al. [27] formulated an inventory model for retailers dealing with deteriorating substandard products with a certain credit period. In their paper, the demand was assumed to increase exponentially, and the backlog rate was inversely proportional to the waiting time for subsequent replenishment. In addition, they also jointly optimized the shortcomings and cycle length. Kouvelis and Zhao [28] examined the effect of credit rating on the optimal decisions of a supply chain under the situation that both the supplier and the retailer are capital constrained. Feng et al. [29] developed a joint economic lot-size model to examine the integrated production-inventory policy for a product in deterioration and credit period. The results showed that suppliers could benefit from working with good ratings retailers, while retailers prefer to cooperate with suppliers outside of their credit rating loopholes. However, all these articles mentioned above only considered the retailer’s or supplier’s profit or cost issues.

The articles above are all about the trade credit period from the perspective of the trade credit providers. However, only a few articles considered the trade credit period from the game perspective. In practice, some researchers have developed game-theoretic models to solve the interactive optimization problems about suppliers and retailers. For example, Zhou and Zhou [30] considered an EOQ model on the basis of the provision of unconditional and conditional trade credit under a supplier-Stackelberg model. In addition, Chern et al. [31], extended Lou and Wang [16]’s model, obtained the optimal equilibrium solutions by proposing a vendor-Stackelberg model. Hoseininia et al. [32] studied inventory management in a multi-channel distribution system, in which the Stackelberg and Nash game were applied to solve the optimal solutions. They found under the condition that the whole prices are equal to production cost, more inventory are carried by the manufacturer in the simultaneous game. Then, Chern et al. [33] established vendor-buyer supply chain models and derived the optimal solutions under non-cooperative Nash equilibrium. Tsao et al. [34] developed a model to optimize the retail shelf-space and introduced a retailer Stackelberg game in the supply chain. Recently, Wu et al. [35] developed a supply chain model, in which demand was associated with default risk under a supplier-Stackelberg model, and proposed two inventory models: (i) centralized decision, (ii) decentralized decision. Chua et al. [36] proposed a make-to-order supply chain to analyze a production planning problem by taking the Stackelberg game into consideration. Based on an assumption that the market demand depends on the displayed stock, Jaggi et al. [37] obtained the optimal solutions under three decision circumstances, i.e., centralized, supplier dominated Stackelberg and Nash equilibrium solution. Nazari et al. [38] studied the optimal ordering and pricing policies using game theory in the closed-loop supply chain. Two types of game strategies, including Nash and Stackelberg game in a decentralized decision are considered in the paper. More related articles can be found in Pal et al. [39], and Wang et al.[40] and their references. However, these articles did not compare the results in centralized decision with that in Stackelberg game under decentralized decision in detail.

Based on the trade credit policy, some of the above articles only considered the supplier profit models or retailer profit models. And some of them considered decentralized decision without considering centralized
decision or only developed one supply chain member Stackelberg model structure. For example, Chern et al. [31] only considered the supplier Stackelberg model without mentioning the retailer Stackelberg model. However, till now, the model that considers both centralized decision with trade credit and two Stackelberg structures in decentralized decision has not been considered.

Overall, this paper makes three contributions. Firstly, this study comprehensively consider and compare both of the supplier’s and retailer’s profit under different decision circumstances, i.e., supplier dominated, retailer dominated and centralized decision, while most of the previous literature only focused on the profit of individual supply chain member. Secondly, it studies a supplier-retailer replenishment model under both centralized and decentralized decisions, while other scholars only studied decentralized decision without considering centralized decision which is incomplete and not specific. Thirdly, we use MATLAB software for example analysis and get some new conclusions that have not been obtained before. For example, we find that under the condition of a credit period, supplier-Stackelberg model is more beneficial to the supply chain than retailer-Stackelberg model. While when there is no credit period, the result is opposite.

A summary of the related literature and its contrast to our research are given in Table 1, considering the following factors: (i) the retailer’s profit or cost, (ii) the supplier’s profit or cost, (iii) Supplier Stackelberg model, (iv) Retailer Stackelberg model, (v) trade credit, and (vi) centralized decision.

Table 1. Summary of related literature

3. Notation and assumptions

We list the notations and assumptions used in this paper in the following (Table 2).

Table 2. Notations

Assumptions:

Following the industrial practices and our motivation to develop this study, five assumptions are made in the model:

(1) Replenishment rate is instantaneous.
(2) In the context of increasingly fierce market competition, we assume that shortages are not allowed.
(3) Time horizon is infinite.
(4) Trade credit has become an important financial resource and is considered by retailers as a tool for price reduction. So according to Jaggi et al. [44], trade credit period offered by sellers to retailers has a positive impact on the demand for goods. Hence, for simplicity, we assume that the relationship between demand rate $D$ and the supplier’s trade credit period $M$ is given by $D = ke^{\alpha M}$, where $k$ and $\alpha$ are positive constants.

(5) As stated in Abad and Jaggi [45], the supplier’s opportunity cost of capital is an incremental linear function about $M$. $I_s = a + bM$, $a > 0$, $b > 0$. That is to say, the credit period offered by the supplier enhances the supplier’s risk. The values of $a$ and $b$ can be determined by the relationship between interest rate and loan term as well as specific factors. In addition, we assume the supplier follows a lot-for-lot strategy. Thus, the supplier does not need to pay the holding cost related to the order quantity $Q$. 

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4. Model formulation

In this section, we formulate the profit models for the retailer and supplier under different situations, i.e., \( M \leq T \) and \( M \geq T \). In addition, since the interest earned, interest payable and the inventory depletion are different under different circumstances, both charts are depicted in order to formulate the profit functions.

4.1 The retailer’s problem

The retailer wants to obtain its optimal cycle time \( T \) to maximize the total profit. The retailer decides its optimal cycle time under the following two conditions: (1) \( M \leq T \) or (2) \( M \geq T \).

**Case 1. \( M \leq T \)**

Seen from Figure (1), we know that the retailer’s interest charged per year is \( \frac{V I_{1} D (T - M)^{2}}{2T} \), and the interest earned is \( \frac{P I_{2} D M^{2}}{2T} \). Figure 2(a) illustrates the inventory level during the interval \([M, T]\). Hence, the retailer’s profit can be expressed as follows:

\[
TP_{r}^{(1)}(T) = (P-V)D - A_{t} - \frac{hDT}{T} - \frac{PI_{2} DM^{2}}{2T} - \frac{V I_{1} D (T - M)^{2}}{2T} \tag{1}
\]

The first term in Equation 1 is the retailer’s sales revenue. The second and third term represent the retailer’s ordering cost per cycle and holding cost, respectively. The fourth term in Equation 1 represents the retailer’s interest earned, and the last term represents the interest payable during the interval \([M, T]\).

**Case 2. \( M \geq T \)**

In this case, the account is settled at \( T = M \) and the retailer does not need to pay any interest charge. And Figure 2(b) illustrates the inventory level during the interval \([T, M]\). Seen from Figure 3, we know that the retail’s interest earned is \( \frac{PI_{2} D (2M - T)}{2} \). Hence, the retailer’s profit can be expressed as follows:

\[
TP_{r}^{(2)}(T) = (P-V)D - A_{t} - \frac{hDT}{T} + \frac{PI_{2} D (2M - T)}{2} \tag{2}
\]

Similar to the interpretation of Equation 1, the first three terms in Equation 2 represent the retailer’s sales revenue, ordering cost and holding cost, respectively. Then, since the retailer has no interest payable during the interval \([T, M]\), the last term in Equation 2 means the interest earned.

4.2 The supplier’s problem
According to assumption 5, the supplier does not need to pay any holding cost. Hence, the supplier’s annual profit total profit is

\[ TP_s(M) = (V - C)D - \frac{A}{T} - (a + bM)DVM \] (3)

The first term in Equation 3 represents the supplier’s sales revenue, and the second term represents the supplier’s ordering cost. According to assumption (5), we can know the supplier’s opportunity cost is \( I_s = a + bM \). Hence, the last term in Equation 3 means the supplier’s total opportunity cost.

5. Equilibrium decisions under different structures

In this section, three models are considered, namely the Supplier Stackelberg model \((SS)\), the Retailer Stackelberg model \((RS)\) and the centralized supply chain model \((c)\).

5.1 Supplier Stackelberg model

5.1.1 the retailer’s decision

Under the assumptions of \(SS\), the retailer’s reaction function is derived firstly when choosing this strategy. Then, the supplier decides on \(M\) to maximize its profit by taking the retailer’s reaction into consideration. This section refers to Chern et al. (2013).

Case 1. \(M \leq T\)

To maximize the retailer’s profit, we calculate the first and second derivatives of Equation 1 with respect to \(T\), and obtain the optimal replenishment cycle time is

\[ T^* = \sqrt{\frac{2A_r - D M (P_r \frac{4}{V})}{h D + \frac{V_r I}{D}}} \] (4)

Proof See Appendix

Case 2. \(M \geq T\)

By calculation, the optimal replenishment cycle time is

\[ T^* = \sqrt{\frac{2A_r}{hD + PLcD}} \] (5)

Proof See Appendix

Theorem 1. Let \( \Delta = 2A_r - DM^2 (PLc + h) \), for any given \(M\), we obtain:

(a) If \( \Delta > 0 \), then the optimal replenishment period is \( T^* > M \) as shown in Equation 4.

(b) If \( \Delta < 0 \), then the optimal replenishment period is \( T^* < M \) as shown in Equation 5.

(c) If \( \Delta = 0 \), then the optimal replenishment period is \( T^* = M \).

Proof See Appendix

The economic interpretation of Theorem 1 is similar to the results in Chern et al. [31].

5.1.2 the supplier’s decision

Case 1. \(M \leq T\)
After substituting Equation 4 into Equation 3, the supplier’s profit becomes

\[ TP_{s}^{(1)} = ke^{\alpha M} [V - C - (a + bM)VM] - A_s \sqrt{\frac{(h+VI_c)ke^{\alpha M}}{2A_r - ke^{\alpha M} M^2 (PI_c - VI_c)}} \]  

By calculation, the optimal trade credit period is

\[ M^* = 0 \]  

**Proof**  
See Appendix

It can be seen that when the supplier does not offer the trade credit, its profit can reach maximum. In such case, the demand rate is \( D^* = k \), and the supplier’s opportunity cost is \( I_s = \alpha \). Therefore, the profit of the supplier and the retailer become

\[ TP_{s}^{(2)} = ke^{\alpha M} [V - C - (a + bM)VM] - A_s \sqrt{\frac{(h + PI_c)ke^{\alpha M}}{2A_r}} \]

**Case 2.** \( M \geq T \)

After substituting Equation 5 into Equation 3, we find that the supplier’s profit becomes

\[ TP_{s}^{(2)} = ke^{\alpha M} [V - C - (a + bM)VM] - A_s \sqrt{\frac{(h + PI_c)ke^{\alpha M}}{2A_r}} \]

Similar to case 1, we can also derive that \( TP_{s}^{(2)} \) is a decreasing function with respect to \( M \) when \( M \geq T \). Hence, the maximization of \( TP_{s}^{(2)} \) over the interval \([T, +\infty)\) is \( TP_{s}(M = T) \). Thus, the optimal trade credit is

\[ M^* = \sqrt{\frac{2A_r}{(h + PI_c)ke^{\alpha M}}} \]

**Proof.**  
See Appendix

That means when the trade credit offered by the supplier is equal to the retailer’s replenishment cycle time, the supplier’s profit can reach maximum. And the demand rate \( D^* = ke^{\alpha \sqrt{\frac{2A_r}{(h + PI_c)ke^{\alpha M}}}} \), so the profit of the supplier and the retailer becomes

\[ TP_{s}^{(2)} = ke^{\alpha \sqrt{\frac{2A_r}{(h + PI_c)ke^{\alpha M}}}} [V - C - \left(\sqrt{\frac{2A_r}{(h + PI_c)ke^{\alpha M}}} + \sqrt{\frac{2bA_r}{(h + PI_c)ke^{\alpha M}}}\right) \]  

\[ TP_{s}^{(2)} = ke^{\alpha \sqrt{\frac{2A_r}{(h + PI_c)ke^{\alpha M}}}} [V - C - \left(2A_r \sqrt{\frac{h + PI_c}{ke^{\alpha M}}} + \frac{2bA_r}{(h + PI_c)ke^{\alpha M}}\right)] \]  

\[ -A_s \frac{(h + PI_c)ke^{\alpha \sqrt{\frac{2A_r}{(h + PI_c)ke^{\alpha M}}}}}{2A_r} \]
Corollary 1. If $PI_e > VI_e$

(1) The higher the value of $A_r$, the higher the value of $A_1$, $T^*$, $TP^{rs*}$ and the lower the value of $TP^{rs*}$.

(2) A higher value of $k$, $\alpha$, $M$ and $h$ causes a lower value of $A_1$ and $T^*$.

From Theorem 1 and Corollary 1, the following conclusions can be obtained. First, the optimal solutions when $A_1 > 0$, as expressed in case 1, is $T^* > M^*$, and vice versa. Specifically, if $A_1 = 0$, the optimal replenishment cycle time degenerates into the trade credit period. The second insight is that if $PI_e > VI_e$, then the value of $A_1$ increases with a higher value of retailer’s ordering cost and a lower value of demand rate. It implies that under SS model, the lower the demand rate and the higher the retailer’s ordering cost, the lower the replenishment cycle time.

Additionally, some managerial insights can be obtained as below. In the case of $M < T$, we can find the supplier’s profit will reach maximum only when the supplier does not offer the trade credit period. In the case of $M \geq T$, if the trade credit period offered is equal to the retailer’s replenishment cycle time, the supplier can get the optimal profit. It implies that from the supplier’s perspective, it’s better for the supplier not to provide the trade credit period or the trade credit period offered is less than the replenishment cycle time.

5.2 Retailer Stackelberg Model

In the RS model, the supplier’s reaction function is deduced. Then, the retailer considers the supplier’s response to determine $T$ by maximizing its own profit.

5.2.1 Supplier’s Decision

To maximize the supplier’s profit, by taking the first and second derivatives of Equation 3 with respect to $M$, we obtain

$$M^* = \frac{-(\alpha aV + 2bV) + \sqrt{(\alpha aV + 2bV)^2 - 4abV(aV - \alpha V + \alpha C)}}{2abV}$$

(14)

Proof. See Appendix

5.2.2 Retailer’s Decision

Case 1. $M \leq T$

By substituting Equation 13 into Equation 1, the retailer’s profit is

$$TP_r^*(T) = (P - V)ke^{\alpha M} - \frac{A_r}{T} - \frac{ke^{\alpha M}T(h + VI_e)}{2} + \frac{ke^{\alpha M}M^* (PI_e - VI_e)}{2T} + VI_eke^{\alpha M}M^*$$

(15)

After calculation, the optimal trade credit period is

$$T^* = \sqrt{\frac{2A_r - (PI_e - VI_e)ke^{\alpha M}M^*}{(h + VI_e)ke^{\alpha M}}}$$

(16)

Proof. See Appendix

Hence, the optimal profit of the supplier and retailer becomes
\[ TP_r^{*} = (P-V)ke^{\alpha M^*} - \frac{A_1\sqrt{(h+VI_c)ke^{\alpha M^*}}}{\sqrt{2A_r - ke^{\alpha M^*}M^*(Pl_c-VI_c)}} - ke^{\alpha M^*}(Pl_c-VI_c) \left( Pl_c-VI_c \right) \]
\[ + \frac{ke^{\alpha M^*}M^*(Pl_c-VI_c)}{2\sqrt{2A_r - ke^{\alpha M^*}M^*(Pl_c-VI_c)}} + VI_c ke^{\alpha M^*} M^* \]
\[ = ke^{\alpha M^*} [V-C-(a+bM^*)VM^*] - \frac{A_1\sqrt{(h+VI_c)}}{\sqrt{2A_r - ke^{\alpha M^*}M^*(Pl_c-VI_c)}} \] (17)

**Case 2.** \( M \geq T \)

After substituting Equation 14 into Equation 2, we find

\[ TP_r^{(2)} = (P-V)ke^{\alpha M^*} - \frac{A_1}{T} - \frac{hTke^{\alpha M^*}}{2} + Pl_c ke^{\alpha M^*} \frac{2M^*-T}{2} \]
\[ = \frac{2A_r}{(h + Pl_c)ke^{\alpha M^*}} - \frac{2A_r}{(h + Pl_c)ke^{\alpha M^*}} + \frac{2A_r}{(h + Pl_c)ke^{\alpha M^*}} \] (19)

After calculation, we can get

\[ T^* = \frac{2A_r}{(h + Pl_c)ke^{\alpha M^*}} \] (20)

**Proof.** See Appendix

So the profit of the supplier and the retailer becomes

\[ TP_r^{*} = (P-V)ke^{\alpha M^*} - \frac{A_1\sqrt{(h+VI_c)ke^{\alpha M^*}}}{2} - \frac{hke^{\alpha M^*}}{2} \left( 2M^*-T \right) \]
\[ + \frac{Pl_c ke^{\alpha M^*}}{2} \left( 2M^*-\frac{2A_r}{(h + Pl_c)ke^{\alpha M^*}} \right) \] (21)

\[ TP_s^{*} = ke^{\alpha M^*} [V-C-(a+bM^*)VM^*] - A_1 \sqrt{\frac{(h + Pl_c)ke^{\alpha M^*}}{2A_r}} \] (22)

**Theorem 2.** Let \( \Delta_2 = 2A_r - ke^{\alpha M^*}M^*(Pl_c+h) \), for any given \( T \), we obtain:

(a) If \( \Delta_2 > 0 \), then the optimal replenishment period is \( T^* > M^* \) as shown in Equation 16.

(b) If \( \Delta_2 < 0 \), then the optimal replenishment period is \( T^* < M^* \) as shown in Equation 20.

(c) If \( \Delta_2 = 0 \), then the optimal replenishment period is \( T^* = M^* \).

**Proof.** See Appendix

The economic interpretation of Theorem 2 is as follows. The benefit the retailer received from the trade credit period is \( DM^2.Pl_c/2 \), so the true ordering cost is \( (A_r - DM^2.Pl_c/2) \), and the inventory cost is \( DM^2.h/2 \).

Hence, if the ordering cost is higher than the inventory cost \( (A_r > 0) \), then we can know that the optimal replenishment period \( T^* \) is higher than the trade credit period \( M^* \) as shown in Equation 16, and vice versa. If the condition \( 2A_r - DM^2(Pl_c-VI_c) > 0 \) is false, then we can get

\[ \Delta_2 = 2A_r - DM^2(Pl_c+h) < DM^2(Pl_c-VI_c) - DM^2(Pl_c+h) = -DM^2(h+VI_c) < 0. \]
Therefore, if \( 2A_r - DM^2(PI_e - VI_e) < 0 \), then we know the optimal replenishment period is given in Equation 20. Since the wholesale price \( V \) is lower than retailer’s selling price \( P \), we can assume \( PI_e > VI_e \).

Then the following results can be obtained.

**Corollary 2.** If \( PI_e > VI_e \), then we have

1. The higher the value of \( A_r \), the higher the value of \( T^* \), \( TP_{rs}^{*,*} \) and the lower the value of \( TP_r^{*,*} \).
2. A higher value of \( k \), \( \alpha \), \( M \) and \( h \) causes a lower value of \( A_r \) and \( T^* \).

From Theorem 2 and Corollary 2, we can also get the similar conclusions to the previous subsection.

In addition, we can get managerial insights as below. The first point is that the trade credit period has nothing to do with the retailer’s replenishment cycle time under \( RS \) model. It is determined by the production cost, wholesale price, demand rate and the opportunity cost. It implies that under \( RS \), the supplier will not change the size of the trade credit period no matter how long the replenishment cycle time offered by the retailer is. The second is that if the holding cost is reduced and the trade credit period offered is increased, then the replenishment cycle time and the demand rate will be greater. In this way, the profit of the supplier and retailer will increase and the whole supply chain will be optimized.

### 5.3. Centralized decision

In the centralized decision, the supplier and the retailer cooperate as one decision maker. That is to say, they decide the replenishment plan together to maximize their whole expected profits.

**Case 1.** \( M \leq T \)

Combing Equation 1 and Equation 3, the total profit of the whole supply chain becomes

\[
TP_{sc} = k\alpha M (P - C) - \frac{A_r + A_p}{T} - \frac{k\alpha M T(h + VI_e)}{2} \\
+ \frac{k\alpha M^2 (PI_e - VI_e)}{2T} + k\alpha M V M[I_e - (a + bM)]
\]

(23)

After calculation, the optimal solution that maximize the centralized supply chain profit is achieved at

\[
T^* = \sqrt{\frac{2(A_r + A_p) - k\alpha M^2 (PI_e - VI_e)}{k\alpha M (h + VI_e)}}
\]

(24)

\[
M^* = \frac{-2(PI_e - VI_e) + (I_e - a)\alpha VT - 2bVT}{2\alpha(PI_e - VI_e - 2bVT)} \\
+ \sqrt{\frac{4[(PI_e - VI_e) + (I_e - a)\alpha VT - 2bVT]^2 - 8\alpha(PI_e - VI_e - 2bVT)[2TV(I_e - a) + \alpha T(2P - 2C - hT - VI_e T)]}{2\alpha(PI_e - VI_e - 2bVT)}}
\]

(25)

**Proof.** See Appendix

And the optimal profit for the centralized supply chain becomes to be

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\[ TP_{sc}^* = ke^{\alpha M} (P - C) - (A_y + A_x) \sqrt{\frac{ke^{\alpha M} (h + V_{I_e})}{2(A_y + A_x) - ke^{\alpha M} M^2 (PI_e - V_{I_e})}} \]

\[ - \frac{ke^{\alpha M} (h + V_{I_e})}{2} \sqrt{\frac{2(A_y + A_x) - ke^{\alpha M} M^2 (PI_e - V_{I_e})}{ke^{\alpha M} (h + V_{I_e})}} \]

\[ + \frac{ke^{\alpha M} M^2 (PI_e - V_{I_e})}{2} \sqrt{\frac{ke^{\alpha M} (h + V_{I_e})}{2(A_y + A_x) - ke^{\alpha M} M^2 (PI_e - V_{I_e})}} \]

\[ + ke^{\alpha M} VM^*[I_e - (a + bM^*)] \]  

(26)

In order to prove that \( TP_{sc} \) is concave in \( T \) and \( M \), it is sufficient to show that \( \frac{\partial TP_{sc}}{\partial T} < 0 \), \( \frac{\partial^2 TP_{sc}}{\partial M^2} < 0 \) and \( |H| \geq 0 \), where \( H \) is the Hessian matrix of \( TP_{sc} \). We set

\[ W_1 = \frac{ke^{\alpha M} M^2 (PI_e - V_{I_e}) - 2(A_y + A_x)}{T^3} \]

\[ \times ke^{\alpha M} \{\alpha^2 [P - C - \frac{T(h + V_{I_e})}{2}] + \frac{(PI_e - V_{I_e})(\alpha^2 M^2 + 4\alpha M + 2)}{2T} \]

\[ + (I_e - a - bM)(\alpha^2 VM + 2\alpha V) - 2abVM - 2bV \} \]

\[ W_2 = -\{ke^{\alpha M} \left[\frac{\alpha(h + V_{I_e})}{2} + \frac{(\alpha M^2 + 2M)(PI_e - V_{I_e})}{2T^2}\right]\}^2 \]

Theorem 3. If \( W_1 + W_2 \geq 0 \), then \( TP_{sc} \) is a concave function in \( M \) and \( T \).

Proof. See Appendix

Case 2. \( M \geq T \)

Combing Equation 2 and Equation 3, the profit of the whole supply chain becomes to be

\[ TP_{sc} = ke^{\alpha M} (P - C) - \frac{A_y + A_x}{T} - \frac{ke^{\alpha M} Th}{2} + \frac{ke^{\alpha M} PI_e (2M - T)}{2} - ke^{\alpha M} VM (a + bM) \]  

(27)

By calculation, the optimal solutions that maximize the centralized supply chain profit are achieved at

\[ T^* = \sqrt{\frac{2(A_y + A_x)}{ke^{\alpha M} (h + PI_e)}} \]  

(28)

\[ M^* = \frac{2a PI_e - 2aaV - 4bV}{4abV} \]

\[ + \sqrt{4(\alpha PI_e - aV - 2bV)^2 - 16abV[PI_e \alpha - 2PI_e + 2aV - (2P - 2C - Th)]} \]  

(29)

Proof. See Appendix

The optimal profit for the centralized supply chain are formulated as

\[ TP_{sc}^* = ke^{\alpha M^*} (P - C) - (A_y + A_x) \sqrt{\frac{ke^{\alpha M^*} (h + PI_e)}{2(A_y + A_x) - ke^{\alpha M^*} M^2 (PI_e - V_{I_e})}} \]

\[ - \frac{ke^{\alpha M^*} (h + V_{I_e})}{2} \sqrt{\frac{2(A_y + A_x) - ke^{\alpha M^*} M^2 (PI_e - V_{I_e})}{ke^{\alpha M^*} (h + V_{I_e})}} \]

\[ + \frac{ke^{\alpha M^*} M^2 (PI_e - V_{I_e})}{2} \sqrt{\frac{ke^{\alpha M^*} (h + V_{I_e})}{2(A_y + A_x) - ke^{\alpha M^*} M^2 (PI_e - V_{I_e})}} \]

\[ + ke^{\alpha M^*} PI_e (M^* - \sqrt{\frac{2ke^{\alpha M^*} (h + PI_e)}{2ke^{\alpha M^*} (h + PI_e)}}) - ke^{\alpha M^*} VM^* (a + bM^*) \]  

(30)
In order to prove that $TP_{sc}$ is concave in $T$ and $M$, it is sufficient to show that $\frac{\partial^2 TP_{sc}}{\partial T^2} < 0$, $\frac{\partial^2 TP_{sc}}{\partial M^2} < 0$ and $|H| \geq 0$, where $H$ is the Hessian matrix of $TP_{sc}$. We set

$$W_3 = -\frac{2(A_r + A_s)}{T^3} \times \alpha e^{\alpha M} [\alpha (P - C - \frac{Th}{2}) + \frac{\alpha P_l (2M - T)}{2} - \alpha VM (a + bM) + 2P M - 2V (a + bM)] - 2ke^{\alpha M} b V$$

$$W_4 = -\frac{\alpha e^{\alpha M} (h + P I_r)}{2}$$

**Theorem 4.** If $W_3 + W_4 \geq 0$, then $TP_{sc}$ is a concave function in $M$ and $T$.

**Proof.** See Appendix

The optimal decisions under different decision circumstances have been derived in Sections 5.1-5.3. In section 6, we will describe in detail the difference of the optimal solutions under three supply chain models, and study the effects of changes in some parameters on the supply chain performance.

### 6. Numerical example and analysis

In this section, some numerical examples are provided to illustrate the above theoretical results and gain some insights into managerial areas.

**Example 1.** Let $A_s = \$50/\text{order}$, $A_r = \$100/\text{order}$, $P = \$3.0/\text{unit}$, $V = \$2.50/\text{unit}$, $C = \$1/\text{unit}$, $h = \$1/\text{unit/year}$, $l_c = 0.05/\text{year}$, $I_c = 0.05/\text{year}$, $\alpha = 2$, $k = 1000$, $I_s = 0.6 + 0.5M$.

In the SS model, for the case of $M \leq T$, the optimal solution that maximizes Equation 6 is $M^* = 0$. Then we can know from Equation 6 that the optimal supplier’s profit is $TP^*_S = 1381.4$. Then, in the case of $M \geq T$, the optimal solution maximizing Equation 10 are $M^* = 0.3068$, and then we can get the optimal supplier’s profit is $TP^*_S = 1936.0087$. By comparing those two cases, we can get $M^{rs*} = 0.3068$. So we can get $M = 2A_s - DM^2 (P l_c - V l_c) > 0$, then the optimal replenishment cycle time derived from Theorem 1 is $T^{rs*} = 0.3068$, and the optimal profit of supplier and retailer is $TP^{rs*}_S = 1538.5$, $TP^{rs*}_R = 356.7499$ respectively. Hence, the optimal profit of the supply chain under SS is $TP^{rs*}_S + TP^{rs*}_R = 1895.2499$.

In the RS model, the optimal trade credit that maximizes the retailer’s profit is $M^* = 0.2454$. We can obtain $2A_s - DM^2 (P l_c - V l_c) = 198.08 > 0$, so we can know from Theorem 2 that the optimal replenishment cycle time is $T^r > M^*$ as shown in Equation 16. Hence, the optimal solution maximizing Equation 15 are $M^{rs*} = 0.2454$ and $T^{rs*} = 0.3279$, and the optimal profit of supplier and retailer is $TP^{rs*}_S = 1573.6$, $TP^{rs*}_R = 226.5316$ respectively. So the optimal total profit of the supply chain under RS is $TP^{rs*}_S + TP^{rs*}_R = 1800.1316$.

By comparing the optimal decision solutions in the two decentralized cases, we find that $M^{rs*} > M^{rs*}$ and $T^{rs*} < T^{rs*}$. Namely, compared with RS model, the supplier under SS model should offer a longer credit
period to improve his/her benefits. Similarly, compared with SS model, the retailer under RS should increase the replenishment period to obtain the optimal solution. Additionally, the supply chain can earn more profit in SS model than in RS model.

In the Centralized decision, for the case of $M \leq T$, the value of $(W_1 + W_2)$ is less than zero, so $TP_{sc}$ is not concave in $M$ and $T$, which means the optimal solutions do not exist. In the case of $M \geq T$, the optimal solutions maximizing Equation 27 are $M^* = 0.3667$, $T^* = 0.3540$ and $W_1 + W_2 > 0$, so the optimal profit for the centralized decision is $TP_{sc} = 1657.52$. Hence, the optimal decision solutions are $M^* = 0.3667$, $T^* = 0.3540$, $TP_{sc} = 1657.52$.

By comparing the results in the centralized decision and decentralized decision, we can find that $TP_{sc}^c > TP_{sc}^a > TP_{sc}^r$. That is to say, the supplier and retailer should make joint decisions to improve supply chain performance.

In addition, we use the value of $M$ under centralized decision to draw an image of market demand changing with the parameters $\alpha$ and $k$ (Figure 4). In Figure 4, we find that both the parameters $\alpha$ and $k$ have positive effects on the market demand. And the demand is more sensitive to the changes in the value of $\alpha$, which means in the same range of change, the larger the value of $\alpha$ is, the greater the market demand is.

**Figure 4.** The influence of $\alpha$ and $k$ on the market demand

### 6.1 Sensitivity analysis

**Example 2.** To gain the management insights, we use the same data as in Example 1 to study the effects of changes in some parameters on the optimal solution. And according to Ho [46], Chern et al. [31] and other related references, the optimal solutions for different parameters of $A_s(30,34,38,42,46,50)$, $A_r(90,94,98,102,106,110)$, $P(3,3.4,3.8,4.2,4.6,5)$, $C(0,0.4,0.8,1.2,1.6,2)$, $I_c(0,0.04,0.08,0.12,0.16,0.2)$, $I_r(0,0.04,0.08,0.12,0.16,0.2)$ as shown in Tables 3-6.

**Table 3** The impact of $A_s$ and $A_r$ on the optimal solutions under different structures

The influence of $A_s$ and $A_r$ on $M$, $T$, $TP_r$ and $TP_c$ can be shown in Table 3. Hence, the following can be deduced from it:

(i) The credit period and replenishment cycle time keep constant as $A_s$ increases in both SS model and RS model. However, under centralized decision, the credit period decreases and the replenishment period increases with the increase of $A_s$. Additionally, under both SS model and RS model, when the supplier’s ordering cost increases, the retailer’s profit remains unchanged while the supplier’s profit decreases. Thus it can be seen that under decentralized decision, the value of $A_s$ has no effect on the credit period, replenishment cycle time and the retailer’s profit, while the supplier benefits from the smaller value of $A_s$. And under centralized decision, a smaller value of $A_s$ can result in a longer credit period and a shorter replenishment
cycle time, and then the total profit of supply chain increases.

(ii) Under $SS$ model, both the credit period and replenishment cycle time increase as $A_r$ increases. Hence, the retailer’s profit decreases due to a larger value of $A_r$, and the supplier’s profit also decreases because the supplier has to pay more interest during a longer credit period. Under $RS$ model, when the retailer’s ordering cost increases, the credit period keeps constant while the replenishment cycle time increases, which will inevitably lead to a reduction in the retailer’s profit. However, since the credit period remains unchanged and the replenishment cycle time increases, the supplier’s ordering cost per cycle decreases, and the supplier’s profit increases eventually. Under centralized decision, the credit period decreases while the replenishment cycle time increases as $A_r$ increases. This shows that the smaller the value of $A_r$, the longer the credit period, and the shorter the replenishment period. Hence, the supply chain benefits from a lower value of $A_r$.

Table 4. The impact of $P$ and $C$ on the optimal solutions under different structures

The influence of $P$ and $C$ on optimal solutions can be shown in Table 4. Hence, the following can be deduced from Table 3:

(i) Under $SS$ model, both the credit period and replenishment cycle time decrease as $P$ increases. Additionally, the retailer’s profit increases greatly while the supplier’s profit decreases slightly with the increase of $P$. Under $RS$ model, the credit period remains unchanged and the replenishment cycle time decreases as $P$ increases. Therefore, the retailer’s profit increases due to a larger value of $P$ and shorter replenishment period. However, the supplier’s profit decreases because a shorter replenishment cycle time can add the supplier’s ordering cost per cycle. Under centralized decision, the credit period increases while the replenishment cycle time decreases with the increase of $P$. This implies that the retailer’s profit increases greatly and the supplier’s profit decreases slightly, and the total profit of supply chain increases eventually.

(ii) Under $SS$ model, when the value of $C$ is less than a certain value, the credit period, replenishment cycle time and the retailer’s profit keep constant while the supplier’s profit decreases as $C$ increases. While when the value of $C$ is more than a certain value, the supplier will not choose to provide the credit period to retailer. And under this circumstance, the replenishment cycle time and retailer’s profit remain unchanged and the supplier’s profit decreases as $C$ increases. Under $RS$ model, a higher value of $C$ can cause a higher value of the replenishment cycle time while a lower value of the credit period. And both the retailer’s and supplier’s profit decrease as $C$ increases. Under centralized decision, the credit period decreases while the replenishment cycle time increases as $C$ increases. Then, the profit of the supply chain will decreases due to a larger production cost and a shorter credit period. Hence, both the supplier and retailer benefit from a lower production cost in any supply chain power structurers.

Table 5. The impact of $V$ and $h$ on the optimal solutions under different structures

The influence of $V$ and $h$ on $M$, $T$, $TP_r$, and $TP_s$ is shown in Table 5. Hence, the following can be deduced from it:

(i) In $SS$ model, both the credit period and replenishment cycle time keep constant with the increase of $V$. Therefore, the retailer's profit will be inevitably decreased because of the unchanged credit period and the
increase of wholesale price. On the contrary, the supplier’s profit will be increased. In \( RS \) model, the credit period increases while the replenishment period decreases with the increase of wholesale price. Although the credit period has increased, for retailers, it cannot compensate for the loss caused by the rise in wholesale prices. Hence, the retailer’s profit will be decreased while the supplier’s profit has been increased as \( V \) increases. In centralized decision model, the credit period decreases while the replenishment cycle time increases as \( V \) increases. Then, the total profit of supply chain will be decreased, due to a higher wholesale price and a shorter credit period. Hence, in centralized decision model, the supply chain benefits from a lower wholesale price.

(ii) Under \( SS \) model, two decision variables, i.e., trade credit period and replenishment cycle time, decrease as \( h \) increases. Then, both the profit of the supplier and retailer will decrease due to the decrease of credit period and the increase of holding cost. This is understandable, since the increase in profit due to the reduction of credit period cannot compensate for the loss of profit due to the increase in holding cost. Under \( RS \) model, the credit period remains unchanged while the replenishment cycle time decreases as \( h \) increases. Then, since the supplier’s profit from the reduction of replenishment cycle time cannot compensate for the loss caused by the increase of holding cost, the profit of both the supplier and retailer decreases with the increase of the holding cost. Under centralized decision, both the credit period and replenishment period decrease as \( h \) increases, which will lead to a reduction in the total profit of the supply chain. Therefore, either decentralized decision or centralized decision, both of supplier and retailer will benefit from a lower holding cost.

Table 6. The impact of \( I_c \) and \( I_e \) on the optimal solutions under different structures

The influence of \( I_c \) and \( I_e \) on optimal solutions can be shown in Table 6. Hence, the following can be deduced from Table 6:

(i) The credit period, replenishment cycle time keep constant under as \( I_c \) increases under \( SS \) model. Hence, the retailer’s profit also keeps constant, and the supplier’s profit increases with the increase of \( I_c \). Under \( RS \) model, the value of \( I_c \) has no effect on the credit period, while leads to a shorter replenishment cycle time. And then, as \( I_c \) increases, the interest payable and the supplier’s ordering cost per cycle increase. Hence, the profit of both the retailer and supplier is falling. Under centralized decision, both the credit period and replenishment period decrease, while will lead to an increase in interest payable. Therefore, a lower interest rate is conducive to the supply chain.

(ii) Under \( SS \) model, two decision variables, trade credit period and replenishment cycle time, decrease as \( I_e \) increases. And the retailer’s profit increases, while the supplier’s profit decreases with the increase of \( I_e \). This is a good understanding, when \( I_e \) increases, the retailer can make use of credit period to earn more interest, while the supplier has no advantage. Hence, the supplier will provide a shorter credit period than before to reduce the interest payable during the credit period. Under \( RS \) model, as \( I_e \) increases, the credit period keeps constant and the replenishment cycle time decreases, which leads to an increase in the retailer’s profit and a decrease in the supplier’s profit. Under centralized decision, the credit period increases and the replenishment period decreases with the increase of \( I_e \). This will result in an increase in the profit of both the supplier and retailer. Hence, whether it is decentralized or centralized decision, both the supplier and retailer benefit from a
higher interest rate.

6.2 The influence of parameters on the total profit of the supply chain

The influence of parameters on optimal solutions has been investigated, while the influence of parameters on total profit of supply chain remains unclear. Hence, we conduct a numerical analysis to examine the influence based on the data in Example 1. Figures 5-8 depict the influence of parameters, \( A_s, A_r, P, C, V, h, I_s \) and \( I_r \), on the total profit of the whole supply chain.

**Figure 5.** The influence of parameter \( A_s \) and \( A_r \) on the profit of the supply chain

From the view of comprehensive Figure 5(a) and Figure 4(b), the total profit of the supply chain is consistent with the same change of both \( A_s \) and \( A_r \). On the one hand, the total profit decreases as \( A_r \) increases under both the decentralized and centralized decision. On the other hand, the total profit under centralized decision is significantly larger than that under decentralized decision, and the profit of the supply chain under \( SS \) is better than that under \( RS \). This implies that the total profit of the supply chain benefits from the lower ordering cost of both the supplier and retailer.

**Figure 6.** The influence of parameter \( P \) and \( C \) on the profit of the supply chain

From Figure 6(a), on the one hand, we find that \( TP_{sc}^{*} > TP_{sc}^{ars} > TP_{sc}^{rs} \) as \( P \) increases. On the other hand, the total profit of the supply chain increases under each decision circumstance. Figure 6(b) shows that the total profit under centralized decision is slightly larger than that under both \( SS \) and \( RS \) model. When the value of \( C \) is between two certain values, we find the total profit under \( SS \) model is more than that under \( RS \) model. Besides, when the value of \( C \) is more than a certain value, the supplier does not provide credit period to the retailer, and the result is opposite. Hence, the supply chain benefits from a higher retail price and a lower production cost.

**Figure 7.** The influence of parameter \( V \) and \( h \) on the profit of the supply chain

As can be seen from Figure 7(a), we find that \( TP_{sc}^{*} > TP_{sc}^{ars} > TP_{sc}^{rs} \) as \( V \) increases. And the total profit of supply chain decreases with the increase of wholesale price under both \( SS \) model and centralized decision. Besides, under \( RS \) model, the total profit increases at first and then decreases with the increase of wholesale price. And when \( V \approx 2.3 \), the total profit of the supply chain is maximized. Figure 7(b) shows that under centralized decision and \( RS \) model, the total profit decreases as \( h \) increases. And under \( SS \) model, the total profit increases first and then decreases with the increase of the holding cost. And when \( h \approx 0.2 \), the total profit will reach maximum. Hence, the supply chain benefits from a lower wholesale price and holding cost.

**Figure 8.** The influence of parameter \( I_s \) and \( I_r \) on the profit of the supply chain

The total profit of the supply chain under centralized decision is found to be higher than that under both \( SS \) and \( RS \) model from Figure 8(a) and 8(b). Additionally, from Figure 8(a), the profit under centralized decision
and RS model decreases while increases under SS model as I increases. Figure 8(b) shows that the total profit increases under both the centralized decision and RS model, while it is basically unchanged under SS model. This implies that the supply chain benefits from a lower value of interest payable and a higher value of interest earned.

Overall, we can conclude from Tables 3-6 and Figures that:

(i) The parameters P and I have a positive effect on the total profit of supply chain, while the remaining parameters have a negative effect on it. It shows that in reality, firms should raise price and interest earned, or reduce related costs and interest payable to improve their own profit.

(ii) By comparing the optimal results under Supplier Stackelberg and Retailer Stackelberg, we can find that when the value of the credit period is greater than zero, the profit of the supply chain under Supplier Stackelberg is greater than that under Retailer Stackelberg. That means under the condition of trade credit period, Supplier Stackelberg model is better for the whole supply chain. However, when the supplier does not offer the trade credit period, Retailer Stackelberg model is better for the whole supply chain;

(iii) After comparing the optimal values in the decentralized decision and centralized decision, we can find the total profit of the centralized supply chain is always higher than that of two decentralized cases. This indicates that under the condition of trade credit, the centralized decision of supplier and retailer is more conducive to the benefit of the supply chain.

7. Conclusions

In this paper, we establish a supplier-retailer replenishment model in which the credit period has a positive impact on the demand rate to determine the optimal solutions under different supply chain structures. In addition, the profit function of the supplier and retailer under different conditions are established according to the relationship between the trade credit period and the replenishment cycle time. And then we obtain the optimal retailer’s replenishment cycle time according to the theorems. Finally, the optimal solutions can be received by using the method of profit maximization. Moreover, this paper simultaneously considers the supplier’s profit and the retailer’s profit. This is also the main feature of this study. Then, the optimal strategies for the retailer and the supplier are derived in different supply chain structures. We find a higher value of the supplier’s ordering cost and supplier’s production cost can cause a lower value of the profit of the supply chain under both the decentralized decision and centralized decision. While a higher value of the retailer’s retail price increases the profit of the supply chain under each power structure. Besides, the whole supply chain will benefit from a lower wholesale price and holding cost. We also find the higher the value of the interest rate, the higher the profit of the supply chain under the centralized decision while the lower the profit under decentralized decision.

In addition, there are some managerial implications. On the one hand, the supplier and the retailer should adopt different strategies under different conditions, which can not only improve their own profit, but also optimize the management of supply chain. For example, when the supplier does not offer trade credit period to the retailer, the retailer can get the optimal profit solution under the Retailer Stackelberg model. On the other hand, from the aspect of the date, we can see that the profit of the supply chain under the centralized decision is far greater than that under the decentralized decision, so the supplier and the retailer should make joint decision to optimize the supply chain.
In future research, we will investigate the two-level trade credit policy in order to understand the effect of the trade credit period offered by the retailer on channel strategies. Then we will extend the model with the consideration of other demand functions, such as price-credit demand, etc. Finally, the permissible delay may result in default risk to suppliers which is an issue deserved to study.

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Appendix
Proof of 5.1.1
By calculation, we have

$$\frac{\partial T^{(1)}_P}{\partial T} = \frac{A_y}{T^2} \frac{D(h+VI)}{2} - \frac{DM^2 (PI_e - VI_e)}{2T^2}$$

$$\frac{\partial^2 T^{(1)}_P}{\partial T^2} = -\frac{2A_y}{T^3} + \frac{DM^2 (PI_e - VI_e)}{T^3}$$

If $$DM^2 (PI_e - VI_e) - 2A_y < 0$$, then $$\frac{\partial^2 TP}{\partial T^2} < 0$$, $$TP^{(1)}$$ is concave in $$T$$, we can obtain

$$T^* = \sqrt{\frac{2A_y - DM^2 (PI_e - VI_e)}{hD + VI_e D}}$$

Taking the first and second derivatives of Equation 2, we have

$$\frac{\partial T^{(2)}_P}{\partial T} = \frac{A_y}{T^2} hD - \frac{PI_e D}{2}$$

$$\frac{\partial^2 T^{(2)}_P}{\partial T^2} = -\frac{2A_y}{T^3} < 0$$
Since the retailer’s ordering cost $A_r > 0$ and the replenishment cycle time $T > 0$, $TP_r^{(2)}$ is concave in $T$, we can get

$$T^* = \sqrt{\frac{2A_r}{hD + PI_eD}}$$

**Proof of Theorem 1**

If $2A_r - DM^2(PI_e + h) > 0$, then we can know that $\frac{\partial^2 TP_r^{(1)}(T | M)}{\partial T^2} < 0$ for $M \leq T$, which implies the retailer’s profit is strictly decreasing in $T$. In addition, for $M \leq T$, we can get $\frac{\partial TP_r^{(1)}(M | M)}{\partial T} = 2A_r - DM^2(PI_e + h)/2M^2 > 0$, and $\frac{\partial TP_r^{(1)}(\infty | M)}{\partial T} = -D(h + VI_e)/2 < 0$. Consequently, there exists a unique $T^* > M$ such that $TP_r^{(1)}(T^*) \geq TP_r^{(1)}(T)$ for all $M \leq T$.

Similarly, for $M \geq T$, we have $\frac{\partial^2 TP_r^{(2)}(T | M)}{\partial T^2} < 0$, $\frac{\partial TP_r^{(2)}(0 | M)}{\partial T} = 2A_r - DM^2(h + PI_e)/2M^2 > 0$. Therefore, we have $TP_r^{(2)}(M) \geq TP_r^{(2)}(T)$ for all $M \geq T$.

Then we have $TP_r^{(2)}(T^*) \geq TP_r^{(2)}(M) \geq TP_r^{(2)}(T)$ for all $M \geq T$.

Likewise, if $2A_r - DM^2(PI_e + h) < 0$, then we can know that $\frac{\partial TP_r^{(1)}(M | M)}{\partial T} = 2A_r - DM^2(PI_e + h)/2M^2 < 0$ and $\frac{\partial TP_r^{(1)}(\infty | M)}{\partial T} = -D(h + VI_e)/2 < 0$. Hence, we can know that $\frac{\partial TP_r^{(1)}(T | M)}{\partial T} < 0$ and $TP_r^{(1)}(M) \geq TP_r^{(1)}(T)$ for all $M \leq T$.

Similarly, for $M \geq T$, if $2A_r - DM^2(PI_e + h) < 0$, then we can know that $\frac{\partial TP_r^{(2)}(0 | M)}{\partial T} = 2A_r - DM^2(h + PI_e)/2M^2 < 0$. Consequently, we obtain that there is a unique $T^* < M$ such that $TP_r^{(2)}(T^*) \geq TP_r^{(2)}(T)$ for all $M \geq T$.

So we can get $TP_r^{(2)}(T^*) \geq TP_r^{(2)}(M) \geq TP_r^{(2)}(T)$ for all $M \leq T$.

This completes the proof of Theorem 1.

**Proof of 5.1.2**

By calculating the first-order derivatives of Equation 6 with respect to $M$, we have

$$\frac{\partial TP_r^{(1)}}{\partial M} = ke^{\alpha M} [\alpha V - \alpha C - \alpha VM(a + bM) - aV - 2bVM]$$

$$- A_r \sqrt{ke^{\alpha M}(h + VI_e)\{\alpha[2A_r - M^2ke^{\alpha M}(PI_e - VI_e)] + ke^{\alpha M}M(PI_e - VI_e)(\alpha M + 2)\}/2[2A_r - ke^{\alpha M}M^2(PI_e - VI_e)]^{1/2}}$$

In the case of $M \leq T$, we can know that $2A_r - DM^2(PI_e - VI_e) > 0$. And from the assumption (5), we can know that $a > 0, b > 0$. And according to the above assumption $PI_e > VI_e$, we can derive that $TP_r^{(1)}$ is a decreasing function of $M$ when $M \leq T$. Hence, the maximum of $TP_r$ over the interval $[0, T]$ is $TP_r^{(1)}(M = 0)$. So the optimal trade credit is $M^* = 0$.

Taking the first derivative on Equation 10, we have
\[
\frac{\partial T P_r^{(2)}}{\partial M} = k e^{a M} \left[\alpha V - \alpha C - \alpha VM (a + b M) - aV - 2bVM\right] - \alpha A \sqrt{\frac{k e^{a M} (h + P I)}{8 A_r}}
\]

According to the assumption (4) and (5), we can know that \( a > 0, b > 0, D = k e^{a M} > 0 \), so we can derive that \( T P_r^{(2)} \) is a decreasing function of \( M \) when \( M \geq T \). Hence, the maximization of \( T P_r^{(2)} \) over the interval \([T, +\infty]\) is \( T P_r^{(2)}(M = T) \). So the optimal trade credit is \( M^* = T = \sqrt{\frac{2 A_r}{(h + P I) k e^{a M}}} \).

**Proof of 5.2.1**

By calculation, we have
\[
\frac{\partial T P_r}{\partial M} = k e^{a M} \left[\alpha V - \alpha C - \alpha VM (a + b M) - aV - 2bVM\right]
\]
\[
\frac{\partial^2 T P_r}{\partial M^2} = k e^{a M} \left[\alpha^2 V - \alpha^2 C - \alpha^2 VM (a + b M) - 2aV - 4abVM - 2b\right]
\]

We can know from the assumption (5) and the above assumption \( P I_e > V I_e \) that \( \frac{\partial^2 T P_r}{\partial M^2} < 0 \), which means \( T P_r \) is concave in \( M \), we can get
\[
M^* = \frac{-(\alpha a V + 2bV) + \sqrt{(\alpha a V + 2bV)^2 - 4\alpha bV (aV - \alpha V + \alpha C)}}{2\alpha bV}
\]

**Proof of 5.2.2**

By calculation of the first-order and second-order derivations of \( T P_r^{(1)} \) by \( T \), we have
\[
\frac{\partial T P_r^{(1)}}{\partial T} = \frac{A_r}{T^2} - \frac{k e^{a M^*} (h + V I_e)}{2} - \frac{(P I_e - V I_e) k e^{a M^*} M^*}{2T^2}
\]
\[
\frac{\partial^2 T P_r^{(1)}}{\partial T^2} = -\frac{2A_r}{T^3} + \frac{(P I_e - V I_e) k e^{a M^*} M^*}{T^3}
\]

If \( (P I_e - V I_e) k e^{a M^*} M^* - 2A_r < 0 \), then \( T P_r^{(1)} \) is concave in \( T \), we obtain
\[
T^* = \sqrt{\frac{2A_r - (P I_e - V I_e) k e^{a M^*} M^*}{(h + V I_e) k e^{a M^*}}}
\]

Taking the first and second derivatives of Equation 19, we have
\[
\frac{\partial T P_r^{(2)}}{\partial T} = \frac{A_r}{T^2} - \frac{(h + P I_e) k e^{a M^*}}{2}
\]
\[
\frac{\partial^2 T P_r^{(2)}}{\partial T^2} = -\frac{2A_r}{T^3} < 0
\]

Since \( \frac{\partial^2 T P_r^{(2)}}{\partial T^2} < 0 \), \( T P_r^{(2)} \) is concave in \( T \), we can get
\[
T^* = \sqrt{\frac{2A_r}{(h + P I_e) k e^{a M^*}}}
\]
Proof of Theorem 2

Similar to the Proof of Theorem 1, assuming \( A_2 = 2A_r - ke^{\alpha M} M^2 (P_{1r} + h) \), for any given \( T \), we obtain:

If \( 2A_r - ke^{\alpha M} M^2 (P_{1r} + h) > 0 \), then we can know that \( \partial^2 T_{p_{1r}}^{(1)} T / \partial T^2 < 0 \) for \( M \leq T \). In addition, we can get that \( \partial T_{p_{1r}}^{(1)} (M) / \partial T > 0 \) and \( \partial T_{p_{1r}}^{(1)} (\infty) / \partial T < 0 \). Hence, there exists a unique \( T^* > M^* \) such that \( T_{p_{1r}}^{(1)} (T^*) \geq T_{p_{1r}}^{(1)} (T) \) for all \( M \leq T \). Similarly, for \( M \geq T \), we can know that \( T_{p_{1r}}^{(1)} \) is an increasing function for all \( M \geq T \). Therefore, we have \( T_{p_{1r}}^{(1)} (M) \geq T_{p_{1r}}^{(1)} (T) \) and \( T_{p_{1r}}^{(1)} (T^*) \geq T_{p_{1r}}^{(1)} (M) \geq T_{p_{1r}}^{(1)} (T) \) for all \( M \geq T \).

Likewise, if \( 2A_r - DM^2 (P_{1r} + h) < 0 \), we can get \( T_{p_{1r}}^{(2)} (T^*) \geq T_{p_{1r}}^{(2)} (M) \geq T_{p_{1r}}^{(2)} (T) \) for all \( M \leq T \).

This completes the proof of Theorem 2.

Proof of Theorem 3

By calculation the first and second orders of \( T_{p_{sc}} \) by \( T \), we obtain

\[
\frac{\partial T_{p_{sc}}}{\partial T} = \frac{A_x + A_y - ke^{\alpha M} (h + VI_e)}{T^2} - \frac{ke^{\alpha M} M^2 (P_{1e} - VI_e)}{2T^2}
\]

\[
\frac{\partial^2 T_{p_{sc}}}{\partial T^2} = \frac{ke^{\alpha M} M^2 (P_{1e} - VI_e) - 2(A_x + A_y)}{T^3}
\]

\[
\frac{\partial T_{p_{sc}}}{\partial M} = \alpha ke^{\alpha M} \{ P - C - T(h + VI_e) \}
\]

\[
+ \alpha ke^{\alpha M} \left\{ \frac{(P_{1e} - VI_e)(\alpha M^2 + 2M)}{2T} + (I_e - a - bM)(aVM + V) - bVM \right\}
\]

\[
\frac{\partial^2 T_{p_{sc}}}{\partial M^2} = ke^{\alpha M} \left\{ \alpha^2 \left[ P - C + \frac{T(h + VI_e)}{2} \right] + \frac{(P_{1e} - VI_e)(\alpha M^2 + 4\alpha M + 2)}{2T} \right. \\
+ (I_e - a - bM)(\alpha^2 VM + 2aV) - 2abVM - 2bV \}
\]

\[
\frac{\partial^2 T_{p_{sc}}}{\partial T \partial M} = \frac{\partial^2 T_{p_{sc}}}{\partial T^2} = -ke^{\alpha M} \left\{ \frac{\alpha(h + VI_e)}{2} + \frac{(\alpha M^2 + 2M)(P_{1e} - VI_e)}{2T^2} \right\}
\]

The optimal solutions are achieved at

\[
T^* = \sqrt{\frac{2(A_x + A_y) - ke^{\alpha M} M^2 (P_{1e} - VI_e)}{ke^{\alpha M} (h + VI_e)}}
\]

\[
M^* = \frac{-2[(P_{1e} - VI_e) + (I_e - a)\alpha VT - 2bVT]}{2\alpha(P_{1e} - VI_e - 2bVT)}
\]

\[
+ \sqrt{\frac{4[(P_{1e} - VI_e) + (I_e - a)\alpha VT - 2bVT]^2 - 8\alpha(P_{1e} - VI_e - 2bVT)(2T (I_e - a) + \alpha T (2P - 2C - hT - VI_T))}{2\alpha(P_{1e} - VI_e - 2bVT)}}
\]

Then we can know that the Hessian matrix is
\[ |H| = \begin{vmatrix} \frac{\partial^2 TP_{sc}}{\partial M^2} & \frac{\partial^2 TP_{sc}}{\partial M \partial T} \\ \frac{\partial^2 TP_{sc}}{\partial T \partial M} & \frac{\partial^2 TP_{sc}}{\partial T^2} \end{vmatrix} \]

\[ = ke^{\alpha M} \left\{ \alpha^2 \left[ P - C - \frac{T(h + VI_e)}{2} \right] + \frac{(PI_e - VI_e)(\alpha^2 M^2 + 4\alpha M + 2)}{2T} \right. \]

\[ + (I_e - a - bM)(\alpha^2 VM + 2\alpha V) - 2\alpha b VM - 2bV \]

\[ \times \frac{ke^{\alpha M} M^2 (PI_e - VI_e)}{T^3} - \frac{ke^{\alpha M} [\alpha(h + VI_e)]}{2} + \frac{(\alpha M^2 + 2M)(PI_e - VI_e)}{2T^2} \right\}^2 \]

\[ = W_1 + W_2 \]

Since the optimal retailer’s replenishment cycle time \( T^* > 0 \), \( 2(A_e + A_r) - ke^{\alpha M} M^2 (PI_e - VI_e) > 0 \) and \( \frac{\partial^2 TP_{sc}}{\partial T^2} < 0 \).

If \( W_1 + W_2 \geq 0 \), then we can know that \( \frac{\partial^2 TP_{sc}}{\partial M^2} < 0 \) and \( |H| \geq 0 \).

This completes the proof of theorem 3.

**Proof of Theorem 4**

By calculating the first and second derivations of \( TP_{sc} \) by \( T \), we obtain

\[ \frac{\partial TP_{sc}}{\partial T} = \frac{A_r + A_e}{T^2} - \frac{ke^{\alpha M} (h + PI_e)}{2} \]

\[ \frac{\partial^2 TP_{sc}}{\partial T^2} = -\frac{2(A_r + A_e)}{T^3} \]

\[ \frac{\partial TP_{sc}}{\partial M} = \alpha ke^{\alpha M} \left[ P - C - \frac{Th}{2} - VM(a + bM) + \frac{PI_e(2M - T)}{2} \right] + ke^{\alpha M} [PI_e - V(a + 2bM)] \]

\[ \frac{\partial^2 TP_{sc}}{\partial M^2} = \alpha ke^{\alpha M} \left[ \alpha(P - C - \frac{Th}{2}) + \frac{\alpha PI_e(2M - T)}{2} - \alpha VM(a + bM) + 2PI_e - 2V(a + 2bM) \right] - 2ke^{\alpha M} bV \]

\[ \frac{\partial^2 TP_{sc}}{\partial T \partial M} = \frac{\partial^2 TP_{sc}}{\partial M \partial T} = -\frac{ke^{\alpha M} (h + PI_e)}{2} \]

We can know that \( TP_{sc} \) is concave in \( T \), so the optimal retailer’s replenishment cycle time is

\[ T^* = \sqrt{\frac{2(A_r + A_e)}{ke^{\alpha M} (h + PI_e)}} \]

And we also can get

\[ M^* = \frac{2\alpha PI_e - 2\alpha aV - 4bV}{4\alpha bV} \]

\[ + \sqrt{\frac{4(\alpha PI_e - a\alpha V - 2bV)^2 - 16abV[PI_e(\alpha - 2PI_e) + 2aV - \alpha(2P - 2C - Th)]}{4abV}} \]
Then we can know that the Hessian matrix is

\[ |H| = \begin{vmatrix} \frac{\partial^2 TP}{\partial M^2} & \frac{\partial^2 TP}{\partial M \partial T} \\ \frac{\partial^2 TP}{\partial T \partial M} & \frac{\partial^2 TP}{\partial T^2} \end{vmatrix} \]

\[ = -\frac{2(A_i + A_j)}{T^3} \times \left[ ake^{\alpha M} \left[ a(P - C - \frac{Th}{2}) + \frac{\alpha P I_e (2M - T)}{2} - aVM(a + bM) + 2PI_e - 2V(a + 2bM) \right] - 2ke^{\alpha M} bV \right] \]

\[ - \left[ \frac{a ke^{\alpha M} (h + PI_e)}{2} \right] = W_3 + W_4 \]

Since we know that \( \frac{\partial^2 TP}{\partial T^2} < 0 \), if \( W_3 + W_4 \geq 0 \), then we can get \( \frac{\partial^2 TP}{\partial M^2} < 0 \) and \( |H| \geq 0 \).

This completes the proof of theorem 4.

**Biography**

**Chong Zhang** received his PhD degree in Systems Engineering from the School of Economics and Management in Southeast University, China in 2012. During his doctoral studies, he joined as a PhD Research Scholar at Worcester Polytechnic Institute for one year. Currently, he is an associate professor of Management Science and Engineering at the School of Management, Nanjing University of Posts and Telecommunications. His mainly research interests include supply chain logistics management, consumer behavior and supply chain finance.

**Mingchuan You** received a bachelor's degree in Information Management and Information Systems from the School of Management in Xuzhou Institute of Technology, China in 2013. Currently, she is pursuing a master's degree in Management Science and Engineering at the School of Management, Nanjing University of Posts and Telecommunications. Her main research interests include supply chain management, supply chain finance and risk preference.

**Guanghua Han** had his Ph.D. degree in Management from Shanghai Jiao Tong University. After receiving Ph.D degree, he joined National University of Singapore as a Research Fellow. At present, he is an associate professor at School of International and Public Affairs, Shanghai Jiao Tong University. Dr. Han has more than 10-years research experience on operations management and has published more than 20 papers in the research field.
Table 1. Summary of related literature

Table 2. Notations

Figure 1. Total amount of interest earned and interest payable when $M \leq T$

Figure 2. Retailer’s Inventory level when $M \leq T$ and $M \geq T$

Figure 3. Total amount of interest earned when $M \geq T$

Figure 4. The influence of $\alpha$ and $k$ on the market demand

Table 3. The impact of $A_s$ and $A_r$ on the optimal solutions under different structures

Table 4. The impact of $P$ and $C$ on the optimal solutions under different structures

Table 5. The impact of $V$ and $h$ on the optimal solutions under different structures

Table 6. The impact of $I_c$ and $I_e$ on the optimal solutions under different structures

Figure 5. The influence of parameter $A_s$ and $A_r$ on the profit of the supply chain

Figure 6. The influence of parameter $P$ and $C$ on the profit of the supply chain

Figure 7. The influence of parameter $V$ and $h$ on the profit of the supply chain

Figure 8. The influence of parameter $I_c$ and $I_e$ on the profit of the supply chain

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Table 2

$M$               the retailer’s trade credit period offered by supplier in years, decision variable

$T$               the retailer’s replenishment cycle time in years, decision variable

27
The retailer’s order quantity in units
annual demand rate for the item
supplier’s unit production cost
the wholesale price charged by the supplier
the retail price of products charged by the retailer
interest that can be earned per unit time
interest payable per unit time
supplier’s capital opportunity cost
the unit holding cost per year excluding interest charge
the retailer’s ordering cost
the supplier’s ordering cost
the \( i \)'s annual total profit, \( i = s, r, sc \) and \( j = ss, rs, c \)
Supplier Stackelberg model
Retailer Stackelberg model
Centralized supply chain structure
The optimal solution
the supplier
the retailer
the supply chain

```
Revenue

Interest earned

Interest payable

0 M T
Time
```
Figure 1

![Figure 1](image1.png)

Figure 2

![Figure 2](image2.png)

Figure 3

![Figure 3](image3.png)
### Figure 4

#### Table 3

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Figure 5
Figure 6

Figure 7
Figure 8