Applying a change-point control chart based on likelihood ratio to supply chain network monitoring

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Abstract: A supply chain network system is regarded as a serial-parallel multistage process; and the application of a change-point control chart based on likelihood ratio is explored to monitor this system. Firstly, state-space modeling is used to characterize complexities of the supply chain network system. Secondly, a change-point control chart based on likelihood ratio is used to trigger potential tardy orders in the system. Thirdly, a case study is illustrated to indicate that the change-point control chart can effectively signal mean shift in completion time of one order in one stage, and can accurately estimate the change point and the out-of-control stage in term of the performance indexes. In detail, when the mean shift is relatively small, the change-point control chart can effectively identify it, and more accurately detect the change point and the out-of-control stage comparing with the traditional Shewhart control chart. We also investigate the effect of misspecified parameters of state space equations on performance of the change-point control chart. The results show that the performance of the change-point control chart can still maintain relatively stable. In general, the change-point control can effectively monitor the supply chain network system, and the monitoring effect is relatively stable.

Keywords: supply chain network system, a serial-parallel multistage process, order completion time, state-space modeling, a change-point chart based on likelihood ratio

1 Introduction

A supply chain network system is full of uncertainties and variations, for example, demand uncertainty, lead time variability, supplier fault, which accordingly cause
degradation of system performance [1,2]. If these variations and uncertainties amass up to a point, the supply chain network system may fail. A collapse of supply chain network system will significantly affect quality performance of the system [3]. Heydari [4] contended that the control of lead time variability is a motivation for persuading retailers to take part in cooperation programs. Especially in MTO production mode, variability in response time is more significant. It would cause response time to go out of control. The response time is the time interval from the time one customer placing an order to the time products/service delivered to the customer. In the customized production environment, that is the order completion time. Due to many uncertainties in the order completion process, the response time must be monitored in real time in order to achieve a predetermined level of customer service.

Statistical process control (SPC) is a good technique for monitoring a supply chain network system. As one of the tools, control charts can effectively detect assignable causes in the process to ensure the stability of a system [5,6]. Wang [7] utilized the individual and moving range control chart on the base of finite state machine theory to monitor order completion of each echelon (stage) in a supply chain network. An exponentially weighted moving average (EWMA) control chart is adopted by Chen and Shaw to detect variations in temperature of the cold chain [8]. Wang et al. [9] proposed a control chart using Kalman filtering to issue potential assignable events in a supply chain network. Faraz et al. [10] utilized a multivariate control chart to monitor delivery chain. Lu et al. [11] applied non-parametric generalized EWMA control chart in green supply chain management to upgrade constantly customer satisfaction of suppliers’ stability. Zhong et al. [12] used the control chart based on the maximum likelihood ratio to monitor supply chain systems, and implemented corresponding maintenance behaviors according to alarm signals.

In this study, a supply chain network system can be abstracted as a series or series-parallel multi-stage process (see Figure 1) from the perspective of process. Therefore, when using SPC to monitor this system, we must consider the correlation between stages [13]. Zolfaghari and Amiri [14] used a discriminant analysis control
chart to monitor a two-stage system with correlated variable-attribute quality characteristics. Further, Amiri and Zolfaghami [15] extended this method to monitor a multi-stage system with clustering, and indicated its effectiveness in estimating the change point. A change-point control chart can identify abnormal variation in the process and timely track process changes by measuring the difference between observation points [16]. Furthermore, the change-point control chart can estimate specific change points and corresponding stages. The change-point control chart has been widely used in multistage processes (e.g. [17]). In addition, variation propagation is another important feature of the multi-stage process. Pirhooshyaran and Niaki [18] proposed a double-max multivariate MEWMA control chart to jointly monitor the parameters of a multivariate multi-stage auto-correlated process with prior knowledge of variation propagation. Bazdar et al. [19] put forward a within-stage fault diagnosis approach on the base of variation propagation modeling. Also, with respect to dynamic networks, Zou and Li [20] presented a network state space model (NSSM) to describe evolution of a dynamic network and integrated the NSSM with SPC for change detection.

[Insert Figure 1 Here]

In our study, we regard a supply chain network system as a serial-parallel multistage process, and explore the application of the change-point chart based on likelihood ratio to monitor the supply chain network. In detail, our monitoring object is the status (the time spent in various tasks or completion time) of each order at different stages in the supply chain network system. State-space model is used to characterize complexities of this system. Then, a change-point control chart based on likelihood ratio is used to trigger potential tardy orders in the system. Finally, a case study and a sensitivity analysis are provided to demonstrate effectiveness of the change-point control chart.

2 Modeling for a supply chain network system
A supply chain network system is a cross-organizational, cross-sectional composite system. Manufacturers, as the core business, may face multiple suppliers and distributors. The case that multiple suppliers provide parts or raw materials to one core
business can be abstracted as many-to-one relationship; and the case that one core business sales its products to multiple distributors or retailers can be abstracted as one-to-many relationship.

State-space modeling (see Equation 1) is used to quantitatively describe the system state of a supply chain network by transition equation, and the relationship between measurement and the system state by measure equation.

\[
x_i = A_k x_{i-1} + U_{i-1} + \omega_k \quad k = 1,2,\ldots,N
\]
\[
y_i = C_k x_i + \nu_k
\]  

(1)

In fact, state space equations contain two types of quality information, state variables \(x_i\) and measurement values \(y_i\). These two types of quality information propagate through all stages. And we assume that the quality characteristics in this paper follow normal distribution. \(A_k\) means how much quality information in stage \(k-1\) is transferred to that in stage \(k\), \(C_k\) represents the relationship between quality measurement \(y_k\) and state vector \(x_k\). Both \(A_k\) and \(C_k\) can be acquired by engineering knowledge and supply chain network information. \(U_i\) denotes a process fault or an out-of-control condition, e.g., a tardy order. If the supply chain network system is in-control, then \(U_i = 0\). In this paper, we assume that only step shifts of order completion time in one stage occur in the system, hence, \(U_i\) is constant. \(\omega_i\) denotes process noise, for example, background disturbance and un-modeled errors. \(v_i\) denotes measurement error.

For a specific order, the order completion time is the sum of the time spent in various tasks. Let \(x_{i,k}\) be the state variable, which denotes the time that order \(i(i=1,2,\ldots)\) finishes all the tasks in stage \(k(k=1,2,\ldots,N)\), including the waiting time for available resource before the start of processing task, the time spent in value-added processing task. We also assume that the time when the first task of the order starts is 0, i.e. \(x_{i,0} = 0\). When order \(i(i=1,2,\ldots)\) completes in stage \(k\), the time is measured, denoted by \(y_{i,k}\).

As shown in Figure 1, a supply chain network can be decomposed to three scenarios. Scenario 1, serial processes (i.e. from stage \(k+1\) to stage \(k+3\)), both upstream and downstream stages have only one stream, and have the same propagation pattern. Scenario 2, the outputs of stage \(k-1\) is separated at stage \(k\), that is, the downstream stage has more than one workstation with the same input source. Scenario
3, the outputs of stage $k$ are merged at stage $k+1$, i.e., the input has multiple sources, while the output has only one stream.

According to [21], scenario 1 (from stage $k+1$ to stage $k+3$) has the same propagation pattern. Therefore, Equation 1 can be directly used to this case:

$$
x_{i,k+2} = A_{i,k}x_{i,k+1} + U_{i,k+2} + \omega_{k+2} \quad i = 1,2,\ldots, k = 1,2,\ldots,N.
$$

Hence, when the supply chain network system is in-control, observation $y_n$ can be obtained

$$
y_{i,n} = \sum_{j=1}^{n} C_{n,j} \phi_{n,j} \omega_j + u_n,
$$

where $\phi_{n,j} = A_{n-1}A_{n-2}\ldots A_j$. Therefore, $E(y_{i,n}) = 0$, $\text{Var}(y_{i,n}) = \sum_{j=1}^{n} C_{n,j}^2 \sigma_{\omega_j}^2 + \sigma_{u_n}^2$.

Assume a mean shift occurs at stage $k+1$, if $n < k$, observation $y_n$ will not be affected, if $n \geq k$, then observation $y_n$ becomes

$$
y_{i,n} = \sum_{j=1}^{n} C_{n,j} \phi_{n,j} \omega_j + u_n + C_n \phi_{n,1} U_{1,k+1}.
$$

Therefore, $E(y_{i,n}) = C_n \phi_{n,1} U_{1,k+1}$, $\text{Var}(y_{i,n}) = \sum_{j=1}^{n} C_{n,j}^2 \sigma_{\omega_j}^2 + \sigma_{u_n}^2$.

In this case, once mean shifts occur in upstream stage, only means of the downstream stages are affected while their variances remain unchanged.

For scenario 2 (from stage $k-1$ to stage $k$ in Figure 1), we have one stream at stage $k-1$ while two streams at stage $k$. Two dummy variables of quality characteristics, $\bar{x}_{i,k-1}$ and $\bar{x}_{i,k-2}$, are used to model this case, as shown in Equation 5 [21].

$$
\begin{align*}
(x_{i,k-1}, x_{i,k-2}) &= A_{i,k-1}x_{i,k-1} + U_{i,k} + \omega_k = \left( A_{i,k-1}\bar{x}_{i,k-1,1} + U_{i,k-1} + \omega_{k,1} \right), \\
(y_{i,k-1}, y_{i,k-2}) &= C_{i,k-1}x_{i,k-1} + U_{i,k} + \omega_k = \left( C_{i,k-1}\bar{x}_{i,k-1,2} + U_{i,k-2} + \omega_{k,2} \right).
\end{align*}
$$

Where $\bar{x}_{i,k-1}$ and $\bar{x}_{i,k-1,2}$ represent the virtual quality characteristics in stage $k-1$ transformed to the two workstations in stage $k$, respectively. And we assume that the two workstations at stage $k$ are identical to one another. Hence, $\bar{x}_{i,k-1}$ and $\bar{x}_{i,k-1,2}$ follow the same distribution, and $x_{i,k-1}$ is the combination of them; $\omega_{k,1}$ and $\omega_{k,2}$ follow the same distribution $N(0, \sigma_{\omega_k}^2)$. Therefore, if a mean shift that occurs in stage $k-1$ will affect both workstations in stage $k$, and the effect is the same. Thus, the
propagation pattern of mean shifts under this case can be modeled as the scenario 1, i.e. mean shifts but variance remains unchanged.

For scenario 3 (from stage \( k \) to stage \( k+1 \)), two workstations are in stage \( k \). So two dummy variables, \( \tilde{x}_{i,k+1} \) and \( \bar{x}_{i,k+1} \), are also used to model this case, as shown in Equation 6,

\[
x_{i,k+1} = \begin{pmatrix} \tilde{x}_{i,k+1} \\ \bar{x}_{i,k+1} \end{pmatrix} = \begin{pmatrix} A_k x_{i,k} + U_{i,k+1} + \omega_{k+1} \\ A_k x_{i,k} + U_{i,k+2} + \omega_{k+2} \end{pmatrix}
\]

\[
y_{i,k+1} = C_k x_{i,k+1} + \nu_{k+1}.
\]

(6)

where \( \tilde{x}_{i,k+1} \) and \( \bar{x}_{i,k+1} \) denote the transformed quality characteristics from \( x_{i,k} \) and \( x_{i,k} \), respectively. \( x_{i,k+1} \) is a combination of \( \tilde{x}_{i,k+1} \) and \( \bar{x}_{i,k+1} \). If no mean shifts occur in the upstream stages, \( \tilde{x}_{i,k+1} \) and \( \bar{x}_{i,k+1} \) have the same distribution, thus the mixture distribution of \( x_{i,k+1} \) is the same as the distribution of \( \tilde{x}_{i,k+1} \) and \( \bar{x}_{i,k+1} \).

We suppose that the proportions of input to stage \( k+1 \) from workstation 1 and workstation 2 of stage \( k \) are \( p_t \) and \( 1-p_t \), respectively. Combining Equations 5-6 (i.e. from stage \( k-1 \) to stage \( k+1 \) in Figure 1), the state space model of stage \( k+1 \) becomes,

\[
x_{i,k+1} = \begin{pmatrix} \tilde{x}_{i,k+1} \\ \bar{x}_{i,k+1} \end{pmatrix} = \begin{pmatrix} A_k A_{k-1} \tilde{x}_{i,k-1} + A_k U_{i,k-1} + A_k \omega_{k-1} + \omega_{k+1} \\ A_k A_{k-1} \bar{x}_{i,k-1} + A_k U_{i,k-2} + A_k \omega_{k-2} + \omega_{k+2} \end{pmatrix}
\]

\[
y_{i,k+1} = \begin{pmatrix} \tilde{y}_{i,k+1} \\ \bar{y}_{i,k+1} \end{pmatrix} = \begin{pmatrix} C_k A_k A_{k-1} \tilde{x}_{i,k-1} + C_k A_k \omega_{k-1} + C_k \omega_{k+1} + \nu_{k+1} + C_{k+1} A_{k+1} U_{i,k-1} \\ C_k A_k A_{k-1} \bar{x}_{i,k-1} + C_k A_k \omega_{k-2} + C_k \omega_{k+2} + \nu_{k+2} + C_{k+1} A_{k+1} U_{i,k-2} \end{pmatrix}.
\]

(7)

On the base of mixture of normal distribution, we have the mean (denotes \( \mu_{i,k+1,0} \)) and variance (denotes \( \sigma_{i,k+1,0}^2 \)) of the observation \( y_{i,k+1} \) of order \( i \) when the system is in-control:

\[
\mu_{i,k+1,0} = 0
\]

\[
\sigma_{i,k+1,0}^2 = \sum_{j=1}^{k} C_k^2 \phi_{i,k-j}^2 \sigma_{0,j}^2 + \sigma_v^2,
\]

(8)

the mean (denotes \( \mu_{i,k+1} \)) and variance (denotes \( \sigma_{i,k+1}^2 \)) of the observation \( y_{i,k+1} \) of order \( i \) when mean shifts of \( U_{i,k,1} \) and \( U_{i,k,2} \) occur:

\[
\mu_{i,k+1} = p_s C_{k+1} A_{k+1} U_{i,k}
\]

\[
\sigma_{i,k+1}^2 = \sum_{j=1}^{k} C_k^2 \phi_{i,k-j}^2 \sigma_{0,j}^2 + \sigma_v^2 + p_s (1-p_s) (C_{k+1} A_{k+1} U_{i,k})^2.
\]

(9)

From Equation 9, we can know that the mean shift in the upstream stage impacts both means and variances of the downstream stages in scenario 3. Hence, a control chart, which is designed to monitor the supply chain network system, should be able to
detect and distinguish mean shift and variance change, and identify variation source in upstream stages or current stage.

3 A change-point control chart based on likelihood ratio
A change-point control chart based on likelihood ratio is adapted from Sullivan and Woodall [22]. The sample size is \( m \), and suppose that a change point \( \tau \) occurs in the \((m+1)\)th sample point, hence, the first \( m_1 \) samples follows normal distribution \( N(\mu_1, \sigma_1^2) \), and the remaining sample \( m_2 (m_2 = m - m_1) \) follows normal distribution \( N(\mu_2, \sigma_2^2) \). The log-likelihood function for \( i \)th \((i = 1, 2, \ldots)\) sample can be expressed as

\[
\frac{-1}{2} \log(2\pi\sigma^2) - \frac{1}{2} \frac{(y_i - \mu)^2}{\sigma^2}.
\] (10)

When the system is in-control, the log-likelihood function for population sample can be expressed as

\[
l_0 = \frac{m}{2} \log(2\pi) - \frac{m}{2} \log \sigma^2 - \frac{m}{2},
\] (11)

where \( \bar{y} = \frac{\sum_{i=1}^{m_1} y_i}{m_1} \) and \( \sigma^2 = \frac{\sum_{i=1}^{m_1} (y_i - \bar{y})^2}{m_1} \) denote mean and variance of the maximum likelihood estimate for \( m \) samples, respectively.

If a change point occurs, the log-likelihood function for the first \( m_1 \) samples can be expressed as

\[
\frac{-m_1}{2} \log(2\pi\sigma^2) - \frac{m_1}{2} \log \sigma_1^2 - \frac{m_1}{2} \frac{m_1}{2} \frac{(\bar{y}_1 - \mu)^2}{2\sigma_1^2},
\] (12)

where \( \bar{y}_1 = \frac{\sum_{i=1}^{m_1} y_i}{m_1} \) and \( \sigma_1^2 = \frac{\sum_{i=1}^{m_1} (y_i - \bar{y}_1)^2}{m_1} \) denote mean and variance of the maximum likelihood estimate for the first \( m_1 \) samples, respectively. Hence, the maximum log-likelihood function for the first \( m_1 \) samples is expressed as

\[
l_1 = \frac{-m_1}{2} \log(2\pi) - \frac{m_1}{2} \log \sigma_1^2 - \frac{m_1}{2},
\] (13)

Similarly, the maximum log-likelihood function for the remaining \( m_2 \) samples can be expressed as

\[
l_2 = \frac{-m_2}{2} \log(2\pi) - \frac{m_2}{2} \log \sigma_2^2 - \frac{m_2}{2},
\] (14)
where $\bar{y}_2 = \frac{\sum_{i=m_1+1}^{m} y_i}{m_2}$ and $\sigma^2_m = \frac{\sum_{i=m_1+1}^{m} (y_i - \bar{y}_2)^2}{m_2}$ denote mean and variance of the maximum likelihood estimate for the $m_2$ samples, respectively.

Based on the above mentioned likelihood ratio test statistic, Sullivan and Woodall [22] proposed the following test statistic

$$r_l[m_1, m_2] = -2 \left[ l_0 - (l_1 + l_2) \right] = m \log \left( \hat{\sigma}^2 \left( \hat{\sigma}^2_{\hat{\sigma}_{\hat{\sigma}}^2} \right)^{-m_1/m_2} \left( \hat{\sigma}^2_{\hat{\sigma}}^2 \right)^{-m_2/m_2} \right).$$

(15)

When the test statistic is larger than a pre-specified critical value, we can obtain an estimator of $\tau$, i.e.,

$$\hat{\tau} = \arg \max_{1 \leq m \leq m_1} \left( \text{lrt}[m_1, m_2] \right).$$

(16)

In order to explore the out-of-control state is caused by mean shift or variance change or both, an estimator of process variance for $m$ samples can be rewritten as

$$\hat{\sigma}^2 = \frac{m_1 \hat{\sigma}^2 + m_2 \hat{\sigma}^2}{m} + \frac{m_1 m_2}{m^2} (\bar{y}_1 - \bar{y}_2)^2.$$  

(17)

Hence, the test statistic can be given

$$\text{lrt}[m_1, m_2] = M_{in} + V_{in},$$

$$V_{in} = m \log \left( \frac{m}{m} r^{-m_1/m} + \frac{m_1}{m} r^{-2m_1/m} \right) = m \log \left( \frac{1 + c(r^2 - 1)}{r^{2c}} \right),$$

$$M_{in} = m \log \left( 1 + \frac{m_1 m_2}{m(m_1 \hat{\sigma}^2 + m_2 \hat{\sigma}^2)} (\bar{y}_1 - \bar{y}_2)^2 \right) = m \log \left( 1 + \frac{c(1 - c)}{1 + c(r^2 - 1)} d^2 \right),$$

(18)

where $c = \frac{m_1}{m}$, $d = \frac{\bar{y}_1 - \bar{y}_2}{\hat{\sigma}_b}$, $\tau = \frac{\hat{\sigma}^2}{\hat{\sigma}_b}$. $M_{in}$ and $V_{in}$ represent mean shift and variance change, respectively. For Equation 18, $M_{in} \geq 0$, we achieve the minimum when the mean of first $m_1$ samples equals to that of $m_2$ samples; $V_{in} \geq 0$, we achieve the minimum when the variance of first $m_1$ samples equals to that of $m_2$ samples.

The test statistic $\text{lrt}[m_1, m_2]$ follows asymptotically Gamma distribution $\chi^2(2)$ [22]. However, expectations of $\text{lrt}[m_1, m_2]$ are different for different $m_1$. Hence, the test statistic is modified to make expectations of $\text{lrt}[m_1, m_2]$ same for different $m_1$.

$$N_{\text{lrt}}[m_1, m_2] = \frac{\text{lrt}[m_1, m_2]}{E(\text{lrt}[m_1, m_2])}.$$  

(19)

Then control limits ($UCL$) and expectation $E(\text{lrt}[m_1, m_2])$ can be obtained by simulation:

$$UCL \approx \frac{1}{1.7} F^{-1}(1-\alpha)^{1/2},$$

(20)
\[ E(lrt[m_1, m_2]) = 2\left(\frac{m_1 + m_2}{(m_1 - 1)(m_2 - 1)} + 1\right), \]

where \( k^* = -4.76 + 3.18\log(m) \), \( \alpha \) denotes probability of Type I error, \( F(\cdot) \) is cumulative distribution function of \( \chi^2(2) \).

4 Case study

Take the production of computer server as an example. A server fulfillment supply chain network is complicated, including customer, server fulfillment production, packing, shipping and so on (as shown in Figure 2 [9]). It is a typical series-parallel process. From the perspective of end customers, good service means on-time receipt of server ordered. However, on-time delivery is not always met due to internal and external variations (e.g. machine downtime, the quality of integrated circuits from the supplier).

For simplicity, we focus on key elements of order fulfillment stages to verify the performance of the change-point control chart in detection of order tardiness. The average time that an order spent in each stage is shown in Table 1 when the system reaches a steady state. And the standard deviation of the measurement error is 6 hours.

We consider only the case of probability of Type I error when \( \alpha = 0.05 \) and fix the number of observations to \( m = 100 \). Without loss of generality, we suppose \( (C_1, A_1) = (1, 1) \) and \( \sigma_w^2 = \sigma_t^2 = 1 \), \( p_{B1} = p_{B2} = 0.5 \) according to engineering knowledge.

We first use the traditional Shewhart control chart to detect tardy orders at the end of each stage. Hence, \( y_i \) is the variable for monitoring the system states \( x \). Given probability of Type I error \( \alpha = 0.05 \), the control limit of the Shewhart control chart can be calculated by

\[ UCL_y(k) = E(y_k) + r\sqrt{\text{Var}(y_k)}, \]

where \( r \) denotes control limit parameters. Here the control limit for each stage can be calculated by simulation, as shown in Table 2. If the order completion time exceeds the control limit, the control chart signals a true alarm. Thus, we can conclude that a tardy order occurs in current stage.

We insert Figure 2 and Table 1 here.
Now, we use the change-point control chart based on likelihood ratio introduced in Section 3. The control limit for each stage can be calculated according to Equation 20. The results are given in Table 3.

[Insert Table 3 Here]

We choose the following performance indexes to judge the performance of the change-point control chart [23], the power of detection (the detection performance of the control chart when the order has been delayed, denotes \( pd \) for the change-point control chart), accuracy of estimation of the change point \( \tau \) (\( \mu_{\tau} \) and \( \sigma_{\tau} \)) and initial out-of-control stage (\( P(\hat{\tau} = \xi) \)), and judgment of mean shift or variance change (\( M_{si} \) and \( V_{si} \)). We also compare the two methods according to the power of detection (the power of detection for Shewhart control chart denotes \( pd_{s} \)).

As we known, performance of detecting change points in a multistage process depends on parameters \( A_{i} \) and \( C_{i} \); the initial out-of-control stage, \( \xi \); the magnitude of mean shifts in completion time in one stage, \( \delta \) (unexpected order delays); and the change point, \( \tau \). Hence, we consider the following situation, as shown in Table 4, where B1 and B2 denote the two workstations in parallel stage. Table 5 gives the performance obtained from 10,000 replications by MATLAB.

[Insert Table 4 Here]

[Insert Table 5 Here]

Apparently, the change-point control chart is superior to the Shewhart control chart in detecting mean shift in term of power of detection. For the change-point control chart, estimation of the process change point is more accurate, and the accuracy is gradually improved if mean shift increases in the case of mean shifts at 20, 30 and 40 (as shown in Column 4, Table 5). We also considered other magnitude of mean shifts, and we find that the power of detection equals to 1, the estimation of change point and initial out-of-control stage are close to the true value, especially for \( \delta \geq 45 \). Furthermore, according to the mean and standard deviation of the change point estimates (as shown in Columns 5-6, Table 5), \( \hat{\tau} \) performs uniformly better for any magnitude of mean shift. The probabilities \( P(\hat{\tau} = \xi) \) also indicate that estimation of out-of-control stage is fairly accurate, especially for large mean shifts. In addition, as shown in the last two columns of Table 5, values of \( \hat{M}_{si} \) are much larger than values of \( \hat{V}_{si} \), which indicate that the system is out-of-control because of mean shifts in completion time. In a word, the change-point control chart can identify the change.
point and the out-of-control stage, and distinguish the source of variation (mean shift or variance change).

5 Sensitivity analysis
Parameters $A_k, C_k$ in state-space equations usually are estimated by sample data or derived from engineering knowledge. In practice, we may obtain inaccurate estimation of these parameters, especially for many stages. Hence, we check the effectiveness and robustness of the change-point control chart under misspecification of values of $A_k, C_k$.

We consider only $\alpha = 0.05$, $m = 100$ and suppose that the true values of $A_k$ and $C_k$ are 1.0. Let $A_k$ and $C_k$ denote the incorrect values for the stage $k'$. We consider two situations, (1) $k' = 2$, (2) $k' = 1,3,5$. We also assume the change point in the serial stage $\xi = 3, \tau = 40$ and in the parallel stage $\xi = 2, \tau = 30$ (B1). For each situation, the following values of $(A_k, C_k)$ are considered (see Table 6). Table 7 gave the power of detection, accuracy of estimation of the change point $\tau$ ($\mu_i$ and $\sigma_i$), and judgment of mean or variance change ($M_i$ and $V_i$), which is obtained from 10000 replications by MATLAB. The values in the fourth column of Table 7 are obtained when the values of $A_k$ and $C_k$ are prespecified correctly.

For the case of the serial sub-process, we can see that for situation (1) of underestimate, the change-point control chart still performs better except for very small mean shift $\delta$, e.g. $\delta = 20$. In this case, the power of detection changes slightly, ranging from 0.89 to 0.95. Similarly, for the situation (2), when many parameters are estimated incorrectly, the change-point control chart also performs better except for very small mean shift $\delta$ in term of power of detection.

For the case of the parallel sub-process, as shown in Table 8, the change-point control chart also performs better, even in the case of very small mean shift $\delta$, the power of detection has remained largely unchanged.

The results show that the change-point control chart can still maintain a relatively stable monitoring effect when values of parameters $A_k, C_k$ are misspecified. $(A_{k'}, C_{k'})$ is a scalar greater than zero, and the system is one-dimensional. So there are
few changes in monitoring effect. In addition, the change-point control chart does not require any assumptions about process parameters. Moreover, the change-point control chart is applicable to the process with unknown process parameters and small number of in-control samples [23]. The sensitivity analysis verified this advantage. However, if \((A_k, C_k)\) is high-dimensional matrix, the results may not be the case. Of course, it is crucial to estimate \(A_k, C_k\) accurately when using a model-based method.

6 Conclusion and suggestion for future research

In this study, a supply chain network system can be viewed as a serial-parallel multistage process, and then state-space technique is used to model this system. On this basis, we explore the application of the change-point control chart based on likelihood ratio under step shift in mean of completion time in one stage. The case study is utilized to indicate that the change-point control chart can effectively signal process mean shifts, and effectively estimate the change point and the out-of-control stage. Finally, the sensitivity analysis illustrates that there is no significant changes in performance of the change-point control chart with misspecified parameters \(A_k\) and \(C_k\). In a word, the change-point control can effectively monitor the supply chain network system, and the monitoring effect is relatively stable.

However, the case that only one change point occurs in the supply chain network system is discussed. But in practice, several change points usually occur in the supply chain network, and how to design control charts to monitoring this case deserves further research. Furthermore, performance of the change-point control chart should be evaluated under different shifts such as drifts and outliers as well. This also may deserve further research.

Acknowledge

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References


Figure

Figure 1 A supply chain network system is viewed as a serial-parallel system

Figure 2 Six stage serial-parallel process of simplified production

Table

Table 1 The processing time at each stage (Hour)
Table 2 The control limit of the traditional Shewhart control chart for each stage (Hour)
Table 3 The control limit of the change-point control chart for each stage (Hour)
Table 4 Combination of the values of \((\delta, \tau, \xi)\)
Table 5 Performance comparison
Table 6 Combination of the values of \((C_{i^*}, A_{i^*})\)
Table 7 Performance with misspecified parameters \(A_i\) and \(C_i\) \((\tau = 40, \xi = 3)\)
Table 8 Performance with misspecified parameters \(A_i\) and \(C_i\) \((\tau = 30, \xi = 2)\)
Figure 1 A supply chain network system is viewed as a serial-parallel system.

Figure 2 Six stage serial-parallel process of simplified production.
Table 1 The processing time at each stage (Hour)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B1/B2</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>Total</th>
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<td>48.33</td>
<td>56.67</td>
<td>12.67</td>
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<tr>
<td>Std</td>
<td>2.39</td>
<td>9.35</td>
<td>15.82</td>
<td>10.27</td>
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<td>8.16</td>
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Table 2 The control limit of the traditional Shewhart control chart for each stage (Hour)

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<th>A</th>
<th>B1/B2</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>UCL</th>
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</thead>
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<tr>
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<td>76.77</td>
<td>81.49</td>
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Table 3 The control limit of the change-point control chart for each stage (Hour)

<table>
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<th>A</th>
<th>B1/B2</th>
<th>C</th>
<th>D</th>
<th>E</th>
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Table 4 Combination of the values of \( \delta, r, \xi \)

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<th>( (r, \xi) )</th>
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<td>20</td>
<td>( (30, 2 \text{ (B1) }) (20, 2 \text{ (B2) }) (40, 3) )</td>
</tr>
<tr>
<td>30</td>
<td>( (30, 2 \text{ (B1) }) (20, 2 \text{ (B2) }) (40, 3) )</td>
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</table>

Table 5 Performance comparison

<table>
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<tr>
<th>( \tau )</th>
<th>( \xi )</th>
<th>( \delta )</th>
<th>( pd, p' )</th>
<th>( pd, p' )</th>
<th>( \mu_i, \sigma_i )</th>
<th>( P(\hat{\xi} = \xi) )</th>
<th>( \tilde{M}_{st} )</th>
<th>( \tilde{V}_{st} )</th>
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<td>0.95</td>
<td>13.89</td>
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<td>31.89</td>
<td>0.48</td>
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<td>( \tau = 20 )</td>
<td>( \xi = 2 \text{(B2)} )</td>
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</tr>
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Table 6 Combination of the values of \((C_i, A_k)\)

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</tr>
<tr>
<td>b</td>
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<tr>
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</table>
Table 8 Performance with misspecified parameters \( A_t \) and \( C_t \). \( (\tau = 30, \xi = 2) \)

<table>
<thead>
<tr>
<th>( k^* )</th>
<th>( \delta )</th>
<th>( pd )</th>
<th>( \mu_i )</th>
<th>( \sigma_i )</th>
<th>( \bar{V}_{br} )</th>
<th>( \bar{M}_{br} )</th>
<th>( \bar{M}_{br} )</th>
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</tr>
<tr>
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</tbody>
</table>

\( \hat{\tau}_\mu \) and \( \hat{\tau}_\sigma \):
A brief technical biography of each author

Jianlan Zhong is currently an associated professor in the College of management & College of Tourism, Fujian Agriculture and Forestry University, P R China. Her main area of interest is in quality control under supply chain context.

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