Computational optimization of a UFAD system using large eddy simulation

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Abstract

In the present study, the Large Eddy Simulation (LES) turbulence closure is implemented, for the first time, to the best of our knowledge, to investigate the air conditioning system in a large space. The results of LES simulations are validated against experimental measurements and the model is used to study the effect of different design variables, including the Air Changes per Hour (ACH), supply temperature, and return air vent height, on design objectives, such as local and global thermal comfort indexes and the energy saving parameter, via a systematic multi-objective optimization approach. The sensitivity analysis shows that the global and local thermal comfort indexes are most sensitive to the air supply temperature while the energy saving is sensitive to ACH and the supply temperature to the same extent. In addition, the return air vent height affects the energy saving more than the other objectives. Finally, with the best design proposed by the multi-objective optimization, an energy saving of 22.9% is achievable while keeping the thermal comfort indexes within the allowable range.

Keywords: Under-Floor Air Distribution (UFAD); Thermal comfort; Energy saving; Multi-objective optimization; Large Eddy Simulation (LES)

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1. Introduction

Nowadays, by the growth of the population and industries around the world, the energy consumption crisis compels researchers to seek new ideas in order to manage the growing energy consumption rate for miscellaneous engineering systems, especially buildings. One of the ways to achieve this goal is using modern ventilation systems. For heating and cooling of spaces, air distribution strategies in conditioning systems have great effects not only on the thermal comfort of the indoor environment but also on energy costs. They also have direct impact on the space organization, interior design, floor height planning, and construction cost [1].

Air distribution systems are divided into three general categories: Mixing Ventilation (MV) [2], Stratum Ventilation (SV), and Displacement Ventilation (DV). Ventilation through Under-Floor Air Distribution (UFAD) is a newly developed DV system that has an impressive effect on the improvement of thermal comfort conditions. A UFAD system can effectively reduce the building life-cycle costs, energy usage, and floor-to-floor height in new constructions, and improve the thermal comfort, ventilation efficiency as well as the indoor air quality, productivity, and health. However, this system has several drawbacks, including its new and unfamiliar technology, lack of information and design guidelines, perceived higher costs, limited applicability to retrofit construction, and problems with applicable standards and codes [3]. To overcome these issues, studying different aspects of UFAD systems has become the main focus of many researchers and engineers, using experiments, computational fluid dynamics (CFD), or zonal models.

A summary of studies performed with different methods on UFAD systems is presented in Table 1. These studies have helped engineers to answer many questions regarding the use of UFAD systems. Lin et al. [4, 5] compared the performance of MV and DV systems. In another study, Lin et al. [6] showed that the supply should be located near the center of the room rather than on
one side of the room. Chung et al. [7] and Alajmi and El-Amer [8] indicated that the UFAD system is especially effective for high ceiling height buildings. According to the previous studies, the vent position has great effects on the performance of UFAD systems. In addition, except in special cases, some of the room air has to be returned to the ventilation system. However, in most of the previous studies, the return and exhaust air vents are combined as a single unit. Xu et al. [9] manifested that UFAD systems with the separate return and exhaust air vents can lead to a considerable energy consumption reduction. Fan et al. [10] studied the impact of return vent height in a room served by a UFAD system. Their results showed that by reducing the height of the return vent from the ceiling to floor height, energy saving and CO₂ removal increased. However, Lin and Tsai [11] pointed out that the vertical temperature profile is unaffected by the position of the return air vent. Peng et al. [12] compared air carrying energy radiant air-conditioning system (ACERS) and split air conditioner and indicated that ACERS can create thermal comfort with small vertical temperature gradients. Cao and Deng [13] investigated the effect of supplied air temperature on Indoor Air Quality (IAQ) and energy consumption. They concluded that higher supplied air temperatures lead to weaker turbulent diffusion and higher levels of indoor CO₂ concentration. More examples of studies on UFAD systems are given in Table 1. In spite of these developments, much more studies are required to reveal different aspects of this new system. Moreover, most of the CFD studies used a two-equation $k-\varepsilon$ or $k-\omega$ model for the turbulence closure. Stamou and Katsiris [14] compared different two-equation turbulence closures in a model office room. Their results showed that the computations with the SST $k-\omega$ model had the best agreement with measurements.

More elaborate turbulence closures, e.g. Large Eddy Simulation (LES), have not been assessed for UFAD systems yet. Here, we used the LES approach for a UFAD system for the first time, to
the best of our knowledge. We validated the model with two sets of experimental databases, namely with the new experimental measurements reported in this manuscript and with another one from the literature. In addition, the knowledge of the mutual effects of different design variables, including the input flow rate, supply temperature, and return air vent height, requires a multi-objective optimization study which has not been performed for a UFAD system yet, to the best of our knowledge. To fill this gap, in this study, a UFAD system is designed following a systematic approach starting with the design of experiments (DOE) technique. Then, the impacts of design variables on objective parameters, esp. the energy saving, are analyzed, and after that through multi-objective optimizations [15-17], the best design is proposed. The rest of the paper is organized as follows: the mathematical formulation is introduced in section 2, the objective parameters, including thermal comfort indexes and the energy saving parameter, are given in section 3, and the model is validated against experimental data in section 4. In section 5, the results and physical discussions are presented. Finally, the important conclusions of the study are given in section 6.

2. Mathematical model

In this study, the Large Eddy Simulation (LES) framework is used to account for the flow turbulence closure. In this framework, the governing equations are spatially filtered equations. Here, the governing equations include the mass (continuity), momentum, and energy equations. For a low-Mach-number thermally-driven flow, these equations can be written as [31], using the Cartesian tensor notation and summation convention for the repeated indices,

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0
\] (1)
where \( \rho \) and \( u_j \) are the density and \( j^{th} \) component of the velocity vector, respectively. All flow properties are filtered quantities. The momentum equation in \( i^{th} \) coordinate direction is

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i
\]  

(2)

where \( \bar{p} \) is the modified pressure [32], \( g_i \), \( i^{th} \) component of the gravitational acceleration vector, and \( \tau_{ij} \) the total stress tensor which is modelled by a gradient diffusion assumption here:

\[
\tau_{ij} = -2(\mu + \mu_t) \left[ S_{ij} - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]
\]  

(3)

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]  

(4)

and \( \mu \) is the molecular dynamic viscosity. In an LES framework, the turbulent viscosity, \( \mu_t \), is modeled by a Sub Grid-Scale (SGS) turbulence closure. Here, Deardorff’s model [33] is used as:

\[
\mu_t = \rho C_v \Delta \sqrt{k_{sgs}} \quad ; \quad k_{sgs} = \frac{1}{2} \left( (u-u)^2 + (v-v)^2 + (w-w)^2 \right)
\]  

(5)

where \( \Delta \) is the local filter width and \( \hat{u} \) is a weighted average of \( u \) over the adjacent cells (representing a test-filtered field at the length scale of \( 2\Delta \)). Here, the implicit filter (i.e. computational grid) is used and the filter width is considered as \( \Delta = (\Delta x \Delta y \Delta z)^{1/3} \), where \( \Delta x \), \( \Delta y \), and \( \Delta z \) are grid spacings in three spatial directions. The model constant is \( C_v = 0.1 \) [32]. This constant value of \( C_v \) does not result in the correct limiting value of the turbulent viscosity near the walls. To account for this deficiency, Van Driest’s damping function [34] is applied to compute \( \mu_t \) near wall boundaries.
The energy equation is as follows:

\[
\frac{\partial}{\partial t} (\rho h_s) + \frac{\partial}{\partial x_j} (\rho h_s u_j) = \frac{Dp}{Dt} + \frac{\partial}{\partial x_j} \left[ \left( k + \frac{\mu C_p}{Pr} \right) \frac{\partial T}{\partial x_j} \right]
\]

(6)

where \( h_s \), \( T \), and \( p \) are sensible enthalpy, temperature, and (background) pressure, respectively, and \( C_p \) and \( k \) are the specific heat capacity at constant pressure and thermal conductivity. Here, the standard wall functions [31] for the velocity and temperature is used at the wall boundaries.

To close the governing equations, the ideal gas relation

\[
p = \frac{\rho RT}{W}
\]

and the NIST-JANAF tables [35] (the sensible enthalpy versus temperature) is added to the set of equations mentioned above. Note that \( R \) and \( W \) are the universal gas constant and air molecular weight, respectively.

3. Objective parameters

In this section, the objective parameters quantifying the thermal comfort is introduced. These parameters are calculated from the flow fields, i.e. the CFD solution of the governing equations discussed in section 2.

The first objective is Fanger’s Predicted Mean Vote (PMV) index [36]. According to ISO7730 standard, for the thermal comfort, the average PMV over the occupant zone should be between -0.5 and +0.5 [37]. Sometimes, the Predicted Percentage of Dissatisfied (PPD) index [37], which is a function of PMV, is used instead. According to ISO7730 standard, the proper value of this index is between 0 and 15% [37].
As mentioned above, the global thermal comfort indexes, e.g. PMV or PPD, should be within the allowable range [37]. However, this is a necessary but sufficient condition to achieve thermal comfort and it is probable that some occupants feel a sense of discomfort in some portions of their bodies. This is called local thermal discomfort [38] which can be quantified by several parameters. One of the main causes of the local thermal discomfort is the environment Temperature Gradient in Vertical Direction (TGVD). This parameter is defined as the air temperature difference between the positions of an occupant head and ankle. According to ISO7730 standard, TGVD should not exceed 3 °C. The use of UFAD system can cause the air draft near the occupants’ bodies and local thermal discomfort. ASHRAE advocates that, to avoid the local thermal discomfort, the mean air velocity within the occupant zone, $V_{\text{MEAN}}$, should be less than 0.8 (m/s) [38].

Cheng et al. [39] found that the use of UFAD systems decreases the cooling coil load, compared to MV systems. The percentage of the cooling load energy reduction to the total space cooling load, which is called energy saving, is calculated by

$$E_r = \frac{\dot{m}_e C_p (T_e - T_{set})}{Q_{\text{space}} \times 100\%}$$  \hspace{1cm} (8)

where $Q_{\text{space}}$ is the space cooling load and is calculated by

$$Q_{\text{space}} = \dot{m}_e C_p (T_e - T_{set}) + \dot{m}_r C_p (T_r - T_{set})$$  \hspace{1cm} (9)

and $T_e$ is the exhaust air temperature, $T_r$ the return air temperature, $T_{set}$ the set-point air temperature which is considered as the average air temperature in the range of 0.8 m to 1.2 m from floor for UFAD systems, $\dot{m}_e$ the exhaust air flow rate, and $\dot{m}_r$ the return air flow rate.

4. Numerical method
The governing equations (of section 2) are solved using a finite difference discretization. Here, a standard 2\textsuperscript{nd} order central differencing scheme is applied to the diffusion terms and a 2\textsuperscript{nd} order fully conservative central differencing scheme, with both momentum and kinetic energy conservative properties, in conjunction with the Superbee flux limiter \cite{40} (to preserve the monotonicity) is incorporated for the convection terms. An explicit predictor-corrector method \cite{31} is used to decouple the governing equations, which results in the second-order temporal accuracy. The Courant-Friedrichs-Lewy (CFL) condition, CFL<1, is incorporated for the variable time step approach used in the computations. The numerical solutions are obtained using Fire Dynamics Simulator (FDS) (version 6.5.1) \cite{41} open-source software.

For the validation of the mathematical model and numerical method, a room with a displacement system experimentally studied by Loomans \cite{42} is considered in this section. This room with dimensions of 5.16 m (length), 3.65 m (width), and 2.5 m (height) is shown in Figure 1. The total air change rate and the air supply temperature are 0.047 m\textsuperscript{3}/s and 19.8 °C, respectively. The inflow boundary condition with a uniform velocity and temperature is assumed at the supply grille. The Neumann (zero-gradient) boundary condition is used for the outflow at the exhaust and no-slip boundary at the other surfaces. Heats emitted by each fluorescent lamp, computer, and reading light are set to 64.4, 98.4 and 27.9 W, respectively. A hollow block is used as the occupant with the surface equal to 1.6 m\textsuperscript{2} and 104.7 W heat output. The uniform heat flux boundary conditions are applied at the surface of all of these objects generating heat. The zero heat flux is applied at the walls of the desk and constant temperature boundary conditions at the room boundaries which are presented in Table 2.
For this problem, a block-structured grid with 1.23 million computational cells is generated. The grid spacing within each block is uniform but the blocks are devised such that the grid near the objects, heat sources, and supply air diffuser is finer, as can be seen in Figure 2.

The results of present large eddy simulation are shown in Figure 3. The forced flow from the inlet diffuser grille under the desk and the natural convection over the occupant and computer are clearly observable in the computed velocity field (Figure 3(a)). The stratified temperature field generated by the DV system is also observed in Figure 3(b). The filtered fields obtained by an LES are instantaneous quantities oscillating in time. To obtain mean temperature or velocity fields, a time averaging procedure is adopted. For the present case, the simulations are performed for 400 s to reach the statistically stationary condition. Then, the time-averaged statistics are computed by continuing the simulation between 400 s to 600 s and collecting the results with the frequency of 0.1 s, i.e. 2000 samples of the fields. To ensure the proper choice of the start (stationary state) and duration of the run-time averaging procedure, two different choices are tested and the results of the mean velocity and temperature profiles are compared in Figure 4. This comparison ensures that the stationary state and run-time averaging duration are chosen large enough for the mean statistics to be independent of these numerical parameters. This check is done for all simulations performed in this study. In Figure 5, the temperature profiles of the present simulation are compared with the experimental data [42]. The agreement between the simulation and measurement validates the accuracy of our modeling approach.

5. Results and discussion

5.1. Main case study: experimental vs. LES results
The main case study of this work is an amphitheater, located in Central Amphitheatre of Amirkabir University of Technology, with 146 occupants. This amphitheater is shown in Figure 6 and is of 15 m length, 12 m width, and 4 m height (Figure 6b). To perform preliminary tests, before constructing the main UFAD ventilation system, a floor stand air conditioner is placed at the south-east corner of the amphitheater. Two exhaust vents are on the bottom corners of the north wall. The volumetric flow rate and the air supply temperature for the floor stand air conditioner are 0.47 $m^3/s$ and 19°C, respectively. The heats emitted by each occupant and fluorescent lamp are 75 W and 45 W, respectively. The measured mean temperature of different walls is presented Table 3. The wall temperature measurements have been carried out by an Infrared Thermometer device with the accuracy of ±0.2%.

For the present amphitheater, the air temperature measurements were made at a distance of 1.5 m from the floor and in 6 different sections. For measuring the air temperature and relative humidity of the environment, Testo 605-H1 device with the accuracy of ±3% has been used. The results of the measurements are reported in Figure 7.

The case is simulated using the LES-CFD model introduced in section 2 and validated in section 4. The computational domain for this simulation is shown in Figure 6b for which a block-structured computational grid with 23 blocks and 3.41 million cells are constructed (see Figure 8). The blocks are arranged to have finer grids in areas with higher gradients like near the inlet, exhaust vents, occupied zone, and lamps. The simulation is performed for 400 s to reach the statistically stationary state and, then, the time-averaged quantities are computed during additional 200 s (from 400 s to 600 s). The computational time for each simulation of the amphitheater is about 36 hours on the workstation system at High Performance Computing
The results of the simulation are compared with the experimental measurements in Figure 7. The overall agreement is satisfactory, but there are deviations between the experimental data and LES in some regions. The main cause of deviations can be the assumption of constant wall temperature for the large walls of the amphitheater and the measurement uncertainties. Note also that the heat generated by the floor stand air conditioner has been neglected in this single simulation. This could be the reason of slight underprediction of the temperature profile near the air conditioner, at section x=1.2 m within 12.0<y<14.0 m.

5.2. Design and optimization of a UFAD system

In this section, a UAFD system is designed and optimized for the amphitheater introduced in the previous section using a computational optimization approach based on the LES model developed in the previous sections. A sample design of the UFAD ventilation system for the amphitheater is shown in Figure 9 as the base case. As it is observed in this figure, 146 direct inlet diffusers (0.2×0.2 m²) located at the floor elevation under the occupant seats, a return air vent (4×0.3 m²) located on the north wall, and 8 exhaust vents (0.4×0.4 m²) located at the ceiling elevation in two rows are placed in the computational domain. Note that the purpose of considering a return air vent is to save energy according to Eq. (8).

Other important design parameters of the UFAD system are the air change per hour, ACH, supply temperature, T_{sup}, and return air vent height, h_{vent}. These parameters are selected using a computational multi-objective optimization in the next section. In addition, this approach makes it possible to study the effect of the design parameters on the desired objectives systematically.
The boundary conditions for the simulations are as follows: the inflow boundary condition with the uniform temperature of $T_{\text{sup}}$ and the uniform velocity of $\text{ACH} \times V_{\text{space}} / (3600 \times 146 \times A_{\text{diffuser}})$, where $V_{\text{space}}$ and $A_{\text{diffuser}}$ are the room volume and diffuser area, respectively, is applied for each inlet diffuser. At the return vent, an outflow uniform velocity with the mass flow rate equal to 80% of the inlet mass flow rate and a Neumann boundary condition for the temperature are assumed. The Neumann outflow boundary condition is also applied at the exhaust vent and no-slip boundary at the other surfaces. For occupant and lamp surfaces, a constant heat flux boundary condition (according to the data given in section 5.1); for amphitheater walls, constant temperature boundary conditions (according to Table 3); and for seats and pilasters, zero heat flux boundary conditions are implemented.

The considered objectives are thermal comfort indexes, PMV and PPD, local thermal comfort parameters, TGVD and $V_{\text{MEAN}}$, and energy saving, $E_r$. The occupied zone used for the calculation of PMV and PPD indexes and $V_{\text{MEAN}}$ is shown in Figure 10. It includes 9 boxes, each of 10.2 m length (from x=0.8 m to x=11.1 m), 1.2 m width and 1.5 m height from the floor, surrounding the occupants.

5.2.1. The Design Of Experiments (DOE)

Air change per hour, ACH, supply temperature, $T_{\text{sup}}$, and return air vent height, $h_{\text{vent}}$, are the three design variables considered here. The range of each parameter variation is chosen as: ACH varies between 5 to 35, $T_{\text{sup}}$ 15 to 25°C, and $h_{\text{vent}}$ 0.6 to 4 m (the elevations of the north wall). At the first step, i.e. DOE, 20 design points (simulation cases) are selected by the Optimal Space
Filling (OSF) algorithm [43] such that the design points cover the whole range of design parameters effectively. These points, generated by OSF, are reported in Table 4.

For each of these 20 cases, a computational grid about 3.41 million cells (like the one described in section 5.1) is constructed and the numerical simulation with the LES model is carried out. Then, the objective parameters (defined in section 3) are computed for each of these design points to be used at the next step.

Several measures can be used to show the quality of the grid used for an LES simulation [44-46]. One of the most reliable assessments is the viscosity ratio, $\mu_\ell / \mu$, which is a measure of the modelled turbulence and the SGS diffusion introduced by LES with respect to the resolved turbulence. The value of viscosity ratio below 20, which is associated with the LES index of quality greater than 0.8 [44], is regarded as the appropriate value for LES and indicative of an acceptable mesh quality [44]. The viscosity ratio profiles at 12 different cross-sections of the amphitheater are reported in Figure 11. As it can be seen, the viscosity ratio is below 20 in all regions of the flow which advocates the high quality of the grid for the present simulations.

5.2.2. The response surfaces

In this step, a response surface is fitted for each objective parameter versus the (three) design parameters using the data obtained in the previous section for the 20 design points. Here, the Non-Parametric Regression (NPR) method [47] is adopted for the response surface generation. Samples of the generated response surfaces are shown in Figure 12. Note that the response surfaces are functions of three variables and only to show them in 3D physical space, the value of one parameter, say $h_{\text{vent}}$, is taken to be constant in this figure.
Before analyzing the responses, the quality of them is checked through the goodness of fit diagram in Figure 13. This diagram illustrates the exact values of the objectives obtained from the numerical simulation of the design points versus the predicted values of objectives from the response surfaces at these points. In this diagram, the larger deviations of points from the line $x = y$ reflect the higher levels of error of response surfaces. As it can be deduced from Figure 13, the responses are accurate at least at design points.

Valuable information can be extracted from response surfaces. The sensitivity of objective parameters to design variables is a piece of this information. The sensitivity parameter, $r$, shows the degree of dependence of an objective on a design variable. The larger the magnitude of $r$, the stronger the dependence of the objective on a design variable. The positive and negative values of $r$ represent the direct and inverse proportionalities, respectively. The local sensitivity of an objective parameter, $y$, to a design variable, $x$, can be defined by

$$r = \pm \frac{\max(y) - \min(y)}{\max(y) + \min(y)} \frac{\max(x) - \min(x)}{\max(x) + \min(x)}$$

(10)

where $\max()$ and $\min()$ functions represent the maximum and minimum values of $y = f(x)$ assuming all other design variables, except $x$, to be constant. The local sensitivity chart is reported in Figure 14. As it is realized from these results, $T_{\text{sup}}$ is the most effective parameter influencing both global thermal comfort, PMV, and local thermal comfort, TGVD and $V_{\text{MEAN}}$, condition while $T_{\text{sup}}$ and ACH are both effective parameters determining the energy saving.

On the average sense, increasing $T_{\text{sup}}$ raises the value of PMV significantly and decreases the other objectives, i.e. TGVD, $V_{\text{MEAN}}$ and $E_r$. On the other hand, when ACH grows PMV and
TGVD reduce while $V_{\text{MEAN}}$ and $E_r$ rise on average since the local sensitivity of $V_{\text{MEAN}}$ and $E_r$ upon ACH is negative according to Figure 14. Moreover, increasing the height of the return vent, $h_{\text{vent}}$, increases the value of PMV and reduces the other objectives, especially the energy saving.

The response curves which are the projections of a response surface onto a 2D space can be used to examine the details of the change of each objective parameter with the variation of the design variables. Figure 15 to Figure 18 show the response curves for different objectives as functions of design variables. According to Figure 15a, for $ \text{ACH}<10$, regardless of the $T_{\text{sup}}$ value, PMV magnitude, $|\text{PMV}|$, is below 0.5. For $ \text{ACH}>10$ and $T_{\text{sup}} < 19^\circ \text{C}$, PMV magnitude rises sharply by the increase in ACH, and this rise is more intense for smaller $T_{\text{sup}}$ values. This is due to the sense of coldness arises by the increase of the air flow rate at low supply air temperatures. Based on Figure 15b, most conditions satisfy the criterion of $|\text{PMV}| < 0.5$. Only a limited range of $\text{ACH}>26.6$ violates this criterion by a small amount.

Figure 16a shows that TGVD drops with the increase in ACH regardless of the value of $T_{\text{sup}}$. Only a range with $19^\circ \text{C} < T_{\text{sup}} < 24^\circ \text{C}$ and $15<\text{ACH}<30$ is in the allowable range ($\text{TGVD}<3^\circ \text{C}$) for this height. Based on Figure 16b, for $\text{ACH}>10$, the variation of the temperature gradient in the vertical direction with the change in $h_{\text{vent}}$ is insignificant. This result is consistent with the finding by Lin and Tsai [11]. However, for low ACH ($\text{ACH}<10$), TGVD increases with the reduction of the vent height. This is due to the rise of the room air temperature in upper layers as a consequence of the escape of fresh cold air from the return vent when $h_{\text{vent}}$ reduces (short circuit effect).
According to Figure 17, $V_{mean}$ increases as ACH grows but is in allowable range (<0.8 m/s) in all operating conditions. The change of $V_{mean}$ with $h_{ven}$ is minor at a specific ACH value.

Figure 18 illustrates the variation of the energy saving parameter. The maximum energy saving at all $T_{sup}$ values occurs at about ACH=18 (Figure 18a). At this local optimum ACH value, $E_r$ also increases with the growth of $T_{sup}$. However, at large ACHs, the variation of $E_r$ with the change in $T_{sup}$ significantly reduces, i.e. the air supply temperature effect on energy saving wears off at high ACH values. This response also indicates that at about ACH~18, $T_{sup} = 15^\circ{}C$, and $h_{ven} = 2.3 m$, an energy saving as large as 25% is achievable. However, this condition results in thermal discomfort as can be seen in Figure 15 (PMV~ -1.3) and for the best design a multi-objective optimization has to be carried out (the next section). For ACH>10, the lower return vent heights bring about a higher amount of energy saving (Figure 18b) which is in agreement with the result reported by Fan et al. [10]. As it is deduced from Figure 18b, the effect of ACH on $E_r$ is more pronounced for smaller $h_{ven}$ values.

The next valuable information is finding the cases (designs) with the maximum or minimum value of a specific objective. This is accomplished by finding the maximum or minimum of a response surface (single-objective optimization). The important maximums and minimums are reported in Table 5. According to the first and second rows of the table, a design with the PMV near zero (PPD~5%) and design with the PMV as large as -1.4 (PPD~46%) are possible in the range of this study. Moreover, it can be seen that the response surfaces include designs with a wide range of TGVD ranging from 2.18 to 7.13 $^\circ{}C$ and $E_r$ ranging from 5.33 to 25.21%.

5.2.3. The multi-objective optimizations
To find optimal designs, two sets of goals corresponding to two separate multi-objective optimizations are considered in this section. Based on one point of view, to achieve the best global and local thermal comfort condition, the magnitude of the PMV index, TGVD, and $V_{\text{MEAN}}$ are minimized simultaneously (Multi-Objective Optimizations (MOO) No. 1). In the second point of view, only the energy saving, $E_r$, is maximized while the value of the global and local thermal comfort parameters satisfy the criteria imposed by the ISO7730 standard, i.e. $|\text{PMV}| < 0.5$, $\text{TGVD} < 3^\circ \text{C}$, and $V_{\text{MEAN}} < 0.8 \text{ m/s}$ (MOO No. 2). These two multi-objective optimizations are summarized in Table 6. Here, the multi-objective optimizations are performed by the Non-dominated Sorting Genetic Algorithm II (NSGA II) [48] based on the response surfaces generated in the previous section. The optimal designs, the 3 best design candidates of each optimization, are reported in Table 7.

According to Table 7, for MOO No.1, a very small PMV index of about 0.002 (corresponding to PPD of 5.00%) is attainable while both $V_{\text{MEAN}}$ and $V_{\text{MAX}}$ in the occupant zone are below 0.8 m/s and TGVD is about $3^\circ \text{C}$ (the first row of Table 7). For this design, $ACH = 15.96$, $T_{\text{sup}} = 20.8^\circ \text{C}$, and $h_{\text{vent}} = 3.07 \text{ m}$. This case corresponds to the best global and local thermal comfort condition. However, the energy saving for this case is $E_r = 17.42\%$. To achieve higher energy saving while the thermal comfort complies with the ISO7730 standard, the best candidates of MOO NO. 2 can be considered. The best candidate, $ACH = 20.07$, $T_{\text{sup}} = 20.19^\circ \text{C}$, and $h_{\text{vent}} = 1.16 \text{ m}$, has nearly the same $T_{\text{sup}}$ as the previously mentioned optimal condition but a larger ACH and smaller $h_{\text{vent}}$. With this new optimal design, the energy saving is as large as 22.9% while keeping the local and global thermal comfort measures in the allowable range. This case is proposed as the best design in this study. It should be noted that in terms of the mean air
velocity within the occupant zone, $V_{MEAN}$, all proposed optimal cases in Table 7 also meet the more stringent standards like ISO EN7730 with the allowable $V_{MEAN}$ within the range 0.1 - 0.3 m/s.

To further validate our optimization results and the quality of response surfaces used for the optimization, in addition to the goodness of fit diagram presented in Figure 13, the best design obtained from MOO No. 2 is simulated using the LES approach (section 2). The results, the contours of velocity and temperature, for this simulation are illustrated in Figure 19 and the objectives are directly computed by the post-processing of the results of the simulations. In Table 8, the values obtained from the simulations are compared with the values predicted by the response surfaces. The low values of the relative errors of the response surfaces for this verification point indicate the accuracy and reliability of the generated responses.

6. Conclusion

In this work, a CFD model based on the LES closure was introduced and validated against experimental data. Then, this model was used to study the effect of various design parameters, including the Air Changes per Hour (ACH), supply air temperature ($T_{sup}$), and return air vent height ($h_{vent}$), on the performance objectives, namely the global (PMV or PPD) and local (TGVD and $V_{MEAN}$) thermal comfort measures and energy saving parameter ($E_r$), of an amphitheater equipped with a UFAD system. A systematic multi-objective optimization approach, comprising a design of experiments, response surface generation and analyses, and optimization steps, is adopted for this study. The main findings of this work can be summarized as follows:
- Based on the sensitivity analysis, the global and local thermal comfort indexes are most sensitive to $T_{sup}$ while the energy saving is sensitive to ACH and $T_{sup}$ to the same extent. $h_{vent}$ influences $E_r$ more than the other objectives.

- For ACH>10 and $T_{sup} < 19^\circ C$, PMV magnitude rises sharply with the growth of ACH.

- For low ACH (ACH<10), TGVD increases with decreasing $h_{vent}$, which is due to the short circuit effect, i.e. the rise of room temperature in upper layers as the result of the escape of fresh cold air from the low-height return vent. However, for the larger values of ACH (ACH>10), TGVD is insensitive to $h_{vent}$, which is in agreement with the findings of previous studies.

- $V_{mean} < 0.8 \, m/s$ is a less stringent criterion which is satisfied by almost all design points in the range of our study.

- For ACH>10, lower $h_{vent}$ results in a higher energy saving. At a low $h_{vent}$, the effect of ACH on $E_r$ is more pronounced.

- Based on the multi-objective optimization, the design with ACH~20, $T_{sup} \sim 20.2^\circ C$, and $h_{vent} / h_{space} \sim 0.3$, where $h_{space}$ is the height of the space, brings about the optimal condition with the global and local thermal comfort indexes within the allowable range and $E_r$ as large as 22.9%.

**Acknowledgement**

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Figure 18 Response curves: the variation of $E_r$ versus (a) $\text{ACH}$ for different values of the supply temperature and at the return air vent height of 2.3 m, (b) the return air vent height for different values of $\text{ACH}$ and at the supply temperature 20°C.
Figure 19 The instantaneous temperature and velocity contours on the central XY plane, for the best design: $ACH = 20.7$, $T_{sup} = 20.19$, $h_{vent} = 1.16$ (MOO No. 2).

### Tables

Table 1 A summary of recent studies performed on UFAD systems during the last decade.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Design parameters</th>
<th>Objective parameters</th>
<th>Method</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shan et al. (2019)[18]</td>
<td>Supply air temperature and velocity, radiant temperature</td>
<td>Thermal comfort (PMV)</td>
<td>Experiment /CFD (RNG $k-\varepsilon$)</td>
<td>The velocity values are below the draft sensation limitations. The distribution of PMV shows the fan coil unit is capable of providing thermally comfortable condition.</td>
</tr>
<tr>
<td>Ahmed and Gao (2017)[19]</td>
<td>Exhaust vent height</td>
<td>Energy saving, thermal comfort, draught risk, IAQ</td>
<td>CFD (RNG $k-\varepsilon$)</td>
<td>For a case with the 1.6 m combined exhaust height, the optimal performance was achieved (energy savings up to 22.56% and inhaled air quality improvement).</td>
</tr>
<tr>
<td>Fan et al. (2017)[10]</td>
<td>Return vent height</td>
<td>Thermal comfort, IAQ, contaminant removal, energy saving</td>
<td>CFD (RNG $k-\varepsilon$)</td>
<td>Energy saving was increased by reducing the height of the return vent from the ceiling to floor height. A descending return vent resulted in a lower CO₂ concentration but a larger mean age of air.</td>
</tr>
<tr>
<td>Zhang et al. (2016)[20]</td>
<td>Supply air temperature, airflow rate, number and type of diffusers</td>
<td>Temperature distribution</td>
<td>Experiment</td>
<td>The large-area grille diffusers, as an alternative choice, can create stratification comparable to the more conventional swirl diffusers.</td>
</tr>
</tbody>
</table>
UFAD system is capable of creating smaller vertical air temperature gradients than Over Head Air Distribution (OHAD) system. The optimum performance obtained at 18 °C supply air temperature, 0.8 m/s supply air velocity, and proper number and distribution of supply diffusers for a large theatre.

The Predicted Mean Vote (PMV) was misleading, whereas the Air Distribution Performance Index (ADPI) was more indicative of the relative level of occupants' thermal comfort.

When the supply air flow rate increases for a given diffuser, the vertical temperature gradient becomes milder and the indoor air stratification decreases. The vertical temperature profile seems unaffected by the change in the position of return air vent.

The thermal stratification establishment is the key factor for efficient operation of UFAD system.

SV, DV, and MV systems were compared. SV could provide satisfactory thermal comfort level to rooms of temperature up to 27 °C.

UFAD systems offers significant energy savings with respect to traditional OHAD systems, esp. for high ceiling height buildings.

UFAD systems, compared to MV systems, consume less energy (about 30% for high-ceiling-type buildings).

UFAD systems with separate return and exhaust air vents can lead to energy consumption reduction.

The thermal comfort is improved by the introduction of the partial partition. The presence of a gap above the partition walls improves the air distribution by reducing the re-circulation. The UFAD system may not be effective in situations where ground level contaminants are the main source of pollutants.
al. (2008) | exhaust location | temperature profiles | momentum flux) and buoyancy forces determine the position of the interface and level of stratification. The formation of a stratified region near the ceiling acts as an insulation reducing the cooling load.

Stamou and Katsiris (2006) | Turbulence models (SST $k-\omega$, standard $k-\varepsilon$, RNG $k-\varepsilon$ and laminar model) | Air flow velocity and temperature distributions | CFD
Computations with the SST $k-\omega$ model showed the best agreement with the measurements.

Lin et al. (2005) | Air supply and exhaust location | Thermal comfort, IAQ | CFD (RNG $k-\varepsilon$)
The supply should be located near the center rather than on one side of the room. The exhaust location was found to have a minor effect on the thermal comfort.

Lin and Linden (2005) | Heat load, ventilation rate, momentum flux | Flow pattern | Experiment and theory
A new UFAD theoretical model was developed and validated with the experiment.

Wan and Chao (2005) | Return air vent position and supply temperature | Airflow and temperature distributions | Experiment and CFD (standard $k-\varepsilon$)
When the thermal length scale, which is defined based on the jet momentum and buoyancy fluxes [30], of the supply jets is large, the temperature stratification became significant. Air exiting at the floor level produces larger stratification than that exiting at the ceiling level.

Lin et al. (2005) | System type (MV and DV) | Thermal comfort, IAQ | CFD (RNG $k-\varepsilon$)
Through a proper design, DV can maintain a thermally comfortable environment with a better IAQ, especially for breathing zone.

Table 2 Wall boundary temperatures of the validation case [42].

<table>
<thead>
<tr>
<th>Wall</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor</td>
<td>22.2</td>
</tr>
<tr>
<td>Ceiling</td>
<td>22.3</td>
</tr>
<tr>
<td>North</td>
<td>22.7</td>
</tr>
<tr>
<td>South</td>
<td>22.6</td>
</tr>
<tr>
<td>East</td>
<td>22.8</td>
</tr>
<tr>
<td>West</td>
<td>23.2</td>
</tr>
</tbody>
</table>

Table 3 The wall temperatures of the amphitheater.

<table>
<thead>
<tr>
<th>Wall</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor</td>
<td>23.9</td>
</tr>
<tr>
<td>Ceiling</td>
<td>25.1</td>
</tr>
<tr>
<td>North</td>
<td>24.6</td>
</tr>
<tr>
<td>South</td>
<td>24.1</td>
</tr>
<tr>
<td>East</td>
<td>24.7</td>
</tr>
<tr>
<td>West</td>
<td>24.3</td>
</tr>
</tbody>
</table>

Table 4 The 20 design points chosen by the DOE process.
### Table 5: Important minimums/maxima of response surfaces.

<table>
<thead>
<tr>
<th>Minimum/maximum</th>
<th>Design parameters</th>
<th>Objective values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACH (-)</td>
<td>PMV (-)</td>
</tr>
<tr>
<td>Minimum</td>
<td>22.18</td>
<td>0.0005</td>
</tr>
<tr>
<td>Maximum</td>
<td>22.98</td>
<td>-1.4134</td>
</tr>
<tr>
<td>Minimum TGVĐ</td>
<td>25.73</td>
<td>-0.263</td>
</tr>
<tr>
<td>Maximum TGVĐ</td>
<td>6.44</td>
<td>0.2076</td>
</tr>
<tr>
<td>Minimum VMEAN</td>
<td>10.64</td>
<td>0.396</td>
</tr>
<tr>
<td>Maximum VMEAN</td>
<td>23.93</td>
<td>0.3626</td>
</tr>
<tr>
<td>Minimum E_r</td>
<td>5</td>
<td>0.629</td>
</tr>
<tr>
<td>Maximum E_r</td>
<td>17.87</td>
<td>1.267</td>
</tr>
</tbody>
</table>

### Table 6: The definition of different Multi-Objective Optimizations (MOO) performed in this study.

<table>
<thead>
<tr>
<th>MOO Goals</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No. 1</strong></td>
<td>Min.</td>
</tr>
<tr>
<td><strong>No. 2</strong></td>
<td>Max. E_r</td>
</tr>
</tbody>
</table>

### Table 7: The best candidates of different Multi-Objective Optimizations (MOO).

<table>
<thead>
<tr>
<th>MOO</th>
<th>Design parameters</th>
<th>Objective values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACH (-)</td>
<td>Tsup (°C)</td>
</tr>
<tr>
<td><strong>No.1</strong></td>
<td>15.96</td>
<td>20.8</td>
</tr>
<tr>
<td></td>
<td>17.22</td>
<td>22.02</td>
</tr>
<tr>
<td></td>
<td>19.49</td>
<td>21.89</td>
</tr>
<tr>
<td><strong>No.2</strong></td>
<td>20.07</td>
<td>20.19</td>
</tr>
<tr>
<td></td>
<td>19.94</td>
<td>20.33</td>
</tr>
<tr>
<td></td>
<td>20.07</td>
<td>20.33</td>
</tr>
</tbody>
</table>
Table 8: Objectives values obtained from the original response surfaces and from new simulations for the verification point.

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>Objective values</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACH (–)</td>
<td>T_{sup} (°C)</td>
</tr>
<tr>
<td>Simulation</td>
<td>20.07</td>
</tr>
<tr>
<td>Response</td>
<td>20.07</td>
</tr>
<tr>
<td>Error (%)</td>
<td></td>
</tr>
</tbody>
</table>

Biographies

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