Numerical investigation of the effect of viscosity on bubble dynamics in a narrow channel

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Abstract

In this paper, dynamic behavior of a vapor bubble inside a narrow channel filled with a viscous liquid has been studied numerically. The boundary integral equation method (BIEM) and the procedure of viscous correction of viscous potential flow (VCVPF) have been employed for obtaining the vapor bubble profiles during its pulsations inside the narrow channel filled with a viscous liquid. In the present paper a new method has been proposed for considering the effects of viscosity in a viscous liquid flow in the framework of the Green's integral formula together with the modified form of unsteady Bernoulli equation. The reported experimental and numerical results of the problem under investigation have been used for verification of the results of the present work. Numerical results show that, by increasing the viscosity of liquid around the vapor bubble, the bubble lifetime increases. Numerical results also indicate that for Reynolds numbers with the order of \(O(10^3)\), the viscosity effects are extremely reduced. Furthermore, the dynamic behavior of the bubble in water and oil is investigated at different Reynolds numbers and at different so-called dimensionless channel radii.

Keywords

Bubble dynamics, boundary element method, viscous pressure correction, narrow channel, viscous potential flow
1. Introduction

Understanding the bubble dynamics behavior near the boundary with axial symmetry can be important in industry, medical science, and different natural phenomena. For example, in the oil industry, to improve oil production in reservoirs that are damaged, cavitation bubbles are created near the walls by ultrasound waves [1-3]. In the petrochemical industry, at different stages such as oil processing and oil transport, the creation of Taylor bubbles due to two-phase flow in pipes causes slug flow [4,5]. Air guns fire produces low-frequency acoustic waves that lead to underwater level structures which are important in geology and exploration of oil and gas. At this application, the outflow of high-pressure gas from the main chamber toward the fluid creates a bubble with axial symmetry between the shuttle and main cylindrical chamber [6,7]. In marine engineering, injection and expansion of compressed gas bubbles in water jet cause net thrust augmentation [8-10]. In medical science, using Embolism technique due to therapeutic reasons, gas bubbles create inside the vein by high-frequency sound waves [11,12].

To our knowledge, the bubble dynamic behavior by BEM (Boundary Element Method) near the boundary with the axial symmetry like a narrow channel compared with the rigid boundary or free boundary has rarely been studied in the literature [13-18]. In most of the studies related to bubble dynamics inside the narrow channel based on BEM, viscosity effects are ignored. In these researches, the dynamics and bubble shape over a period of time, as well as the life of bubble, are studied along with the geometric parameters of tube and their results are presented. In these studies, to validate the results, those fluids are chosen that like water, have low viscosity[19-22]. However, it is not possible to generalize the results of this study to viscous fluids without more investigations.

The concept of applying the viscosity as an additional pressure term in boundary layer analysis has been first proposed by Moore [23]. In order to estimate drag force in a rising bubble, he attempted to implement the contribution of viscosity in boundary layer as an additional pressure. Nevertheless, he could not obtain a uniform relationship for the additional pressure. He concluded that to simulate the non-rectilinear motion observed for bubbles larger than about 2mm, a general 3D rather than an axi-symmetric model is required. Kang et al. [24], successfully estimated the drag coefficient on a spherical bubble accurately by calculating the normal stress and integrating it over the bubble surface. In their work, VPC (Viscous Pressure Correction) was estimated with a uniform relationship for the flow with high Reynolds numbers for the first time.
Their study showed that without considering the boundary layer and using the irrotational flow solution, VPC could be estimated accurately. In other words, they concluded that VPC is almost independent of vorticity. The present study also shows that the drag coefficient up to $O(R^{-1})$ depends only on the $O(1)$ vorticity distribution in the fluid.

Joseph [25] obtained the drag value with direct estimation based on VCVPF in several hydrodynamic examples. According to their theory, in problems where vorticity layers exist, VPC can be added to the estimated irrotational pressure. They suggested that the vorticity effects that are created by UISS (unbalanced irrotational shear stress) can be applied in problem-solving. In their study, for problems with axial symmetry, VPC was considered as series of surface harmonics. However, they could not propose a uniform relationship for all constant coefficients of the harmonic terms. In particular cases, the constant coefficient of one of these terms was determined by viscosity loss definition for Viscous Potential Flow (VPF) and VCVPF while other terms were ignored.

Klaseboer et al. [26] investigated the development and deformation of a rising bubble in water at high-Reynolds numbers. To simulate the non-rectilinear motion observed for bubbles larger than about 2mm, a general 3D rather than an axi-symmetric model is required. In this study, the probability of occurring vortex at the downstream flow was not investigated. For numerical simulation, they used BEM and inserted the VPC obtained by Joseph and Wang [27] in the unsteady Bernoulli equation. Joseph and Wang [27] investigated the viscous potential flow over a free surface. In their work, the dissipation method was used to study the viscosity effects. An explicit relation was found between the pressure correction and irrotational shear stress at the interface. They reported that the drag value obtained by utilizing the dissipation method is equal to $12\pi\mu aU$ which is the same as that calculated by using the pressure correction method.

Recently, several researchers have studied VCVPF method to investigate the bubble dynamic behavior near the boundary [28-30]. In all of these studies, VPC was assumed as a function of normal stress with a constant coefficient at each time step. The coefficient was estimated so that, the mechanical energy equation for Navier-Stokes equation was satisfied. However, no appropriate estimation was found in certain moments of the bubble life. Since according to the mentioned assumption, i.e., in some points on the bubble surface where the normal stress has a
large numerical value, the VPC value will be significant. However, this is not true, and according to the definition, VPC is directly related to UISS value [27].

In this study, the bubble dynamics inside a thin rigid channel is studied numerically by developing a computer code based on the BEM and implementing the VCVPF method for fluids with different viscosities. Also, a new technique is proposed to estimate the VPC parameter at the gas-fluid interface. Accordingly, the value of this parameter at each surface element is predicted so that the power of the traction integral is the same for VPF and VCVPF. In this method, in order to determine VPC, it is not necessary to consider additional assumption such as consistency with the normal stress. In the present work, unlike the previous researches, the assumption of linear relation between VPC and viscous normal stress are not considered. Also, the effect of parameters of UISS, VPC, and viscous normal stress on the bubble lifetime are investigated. In addition, the expression of equations in the discretization form for viscosity parameters on the bubble surface is another aspect of distinguishing the present work from the other researches. The numerical results obtained for the growth and collapse of bubble inside the water in the rigid tube are validated with the experimental one published by Ni et al. [14]. To the best of our knowledge, there is a lack of experimental results in literature for bubble dynamics in viscous fluids inside the narrow tube. In this regard, the present numerical results for the viscous fluid are compared with the theoretical results reported by Minsier et al. [31] based on solving two-dimensional Navier-Stokes equations near the rigid boundary.

2. Model description

Figure 1 shows a schematic drawing of a bubble with axial symmetry at the center of a thin rigid Channel. The bubble boundary and the wall of channel are divided to elements considering the axial symmetry. In order to prevent the implementation of the effects of other kinds of boundaries such as the free surface in bubble behavior, the channel length is assumed very larger than its diameter. The fluid around the bubble is considered as a Newtonian incompressible fluid. In addition, due to the small size and short life of the bubble, buoyancy force is ignored [32,33]. Furthermore, the primary shape of the bubble is considered spherical that its radius is determined by the Rayleigh-Plesset equation [34].

Integral representation of the solution for the Laplace equation at all points inside and at the boundary of the flow field is obtained from Green’s second identity[35].
\( c(p) \phi(p) = \int_{\Gamma} \left( \frac{\partial \phi(q)}{\partial n_q} G(p,q) - \phi(q) \frac{\partial G(p,q)}{\partial n_q} \right) ds \) \hspace{1cm} (1)

where \( p \) and \( q \) are the flow field point and the source point, respectively. In the equation, \( c(p) \) is the solid angle, \( \phi \) is the velocity potential, \( G \) is the Green function for the Laplace equation and \( \Gamma \) is the boundary of fluid domain. The Green's integral formula which in its classical form governs the potential fluid flow problems has been employed as the principal governing equation for the numerical simulation of the hydrodynamic behavior of the vapor bubble inside the viscous liquid filled narrow channel.

2.1. Boundary conditions

The fluid inside the bubble is composed of non-condensable gas and saturated vapor at the reference temperature. The gas behavior inside the bubble is ideal and adiabatic[36]. The normal stress balance at the bubble surface is expressed by the Young-Laplace equation as follows [26]:

\[ p_c + p_{ig} \left( \frac{R_i}{R} \right)^{3\gamma} - \sigma k + 2\mu \frac{\partial^2 \phi}{\partial n^2} = p_i, \] \hspace{1cm} (2)

where \( p_c \) and \( p_{ig} \) are the saturated pressure and the initial gas pressure inside the bubble, respectively. In above equation, \( R_i \) is the initial bubble radius, \( \sigma \) is the interface surface tension, \( k \) is the local mean bubble surface curvature, \( \mu \) is the dynamic viscosity, \( p_i \) is the pressure at the bubble surface and \( \gamma \) is the ratio of specific heat.

For the gas-bubble interface, to create a uniform boundary and to increase estimation accuracy, \( M \) number cubic spline elements with constant physical functions are applied. For the channel wall, \( N \) number linear elements with constant physical functions are considered. The cubic splines are used to discretize the bubble surface. The spline on the first and last nodes of bubble is clamped. In this way, due to the bubble symmetry, the spline on these nodes is tangent to the direction normal to the axis of symmetry:

\[ \frac{\partial r}{\partial l_j} = +1, \quad \frac{\partial z}{\partial l_j} = 0 \quad @ \quad j = 1, M \] \hspace{1cm} (3)
Discretization of the internal wall of the rigid cylinder is continued up to the physical infinity where the pulsation of the vapour bubble have a negligible effect on the fluid flow. At the center of each element, at each time scale, the velocity potential at collocation points is estimated by the Lagrangian form of modified Bernoulli’s equation. To represent the distribution of velocity potential on the bubble, cubic spline functions are used.

2.2. VCVPF method

The Navier-Stokes equation for an uncompressible viscose fluid with constant properties can be written as [37]:

$$\rho \frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{U}$$

(4)

where \( \rho \) is the fluid density, \( \mathbf{U} \) is the velocity vector, \( \mathbf{g} \) is the body force and \( p \) is the pressure.

The equation of mass conservation for an incompressible fluid is expressed as follows [37]:

$$\nabla \cdot \mathbf{U} = 0$$

(5)

The velocity vector in the flow field can be divided to irrotational velocity vector and rotational velocity vector, by employing Helmholtz decomposition [38].

$$\mathbf{U} = \mathbf{u} + \mathbf{v} = \nabla \phi + \mathbf{v}$$

(6)

By inserting Eq. (6) in Eq. (4), unsteady Bernoulli equation will result:

$$\nabla \left( \rho \frac{\partial \phi}{\partial t} + \frac{\rho}{2} \left| \nabla \phi \right|^2 + p + \rho g z \right) + \rho \frac{\partial \nabla \phi}{\partial t} + \rho \nabla \cdot \left[ \mathbf{v} \otimes \nabla \phi + \nabla \phi \otimes \mathbf{v} + \mathbf{v} \otimes \mathbf{v} \right] - \mu \nabla^2 \mathbf{v} = 0$$

(7)

in which,:

$$\nabla p_{vc} = \rho \frac{\partial \nabla \phi}{\partial t} + \rho \nabla \cdot \left[ \mathbf{v} \otimes \nabla \phi + \nabla \phi \otimes \mathbf{v} + \mathbf{v} \otimes \mathbf{v} \right] - \mu \nabla^2 \mathbf{v}$$

(8)

In some of the certain hydrodynamic problems such as Poiseuille flow or Hadamard–Rybczynski solution for the flow around a liquid sphere, based on Helmholtz decomposition, it is possible to calculate VPC, accurately[34]. However, for most the hydrodynamic problems such as the growth and collapse of bubble inside the channel, no accurate VPC estimation according
to Eq. (8) is presented. Therefore, in VCVPF technique, in order to generalize the potential flow method to viscous flows, a value for VPC should be estimated.

Replacing Eq. (8) in Eq. (7) and integrating, the modified Bernoulli equation will be obtained.

$$\rho \frac{\partial \phi}{\partial t} + \frac{\rho}{2} |\nabla \phi|^2 + p + p_{vc} + \rho g z = p_{\infty}$$

(9)

Dynamic condition at the bubble surface is estimated from the following equation that is obtained from rearranging Eqs. (3) and (9) based on Lagrangian form.

$$\rho \frac{D \phi}{Dt} = \frac{1}{2} \rho |\nabla \phi|^2 + \rho g z + \sigma k + p_{\infty} - p_c - p_{is} \left( \frac{R_L}{R} \right)^{3y} - 2\mu \frac{\partial^2 \phi}{\partial n^2} - p_{vc}$$

(10)

where $2\mu \frac{\partial^2 \phi}{\partial n^2}$ and $p_{vc}$ terms indicate the viscous normal stress and $p_{vc}$, respectively. These are the terms by which viscosity enters the potential estimations.

Stress at the bubble surface employing Helmholtz decomposition is defined as:

$$\tau = -pI + \mu (\nabla U + \nabla U^T) = -pI + \mu (\nabla \nu + \nabla \nu^T) + 2\mu \nabla \otimes \nabla \phi$$

(11)

Viscous rotational shear stress $\mu (\nabla \nu + \nabla \nu^T)$ at the bubble surface due to the low viscosity of gas inside the bubble is almost zero. On the other hand, the effects of viscous irrotational stress $\tau_i = 2\mu \nabla \otimes \nabla \phi$ within the solution domain is balanced internally, but in some boundaries such as bubble surface, it may be unbalanced and lead to power production [39]. The theory by Joseph [25] suggest that, in order to hold zero shear stress boundary conditions at the bubble boundary, a pressure correction should be considered to compensate non-zero irrotational shear stress. This parameter must be calculated so that the estimated power of the traction integral for VPF and VCVPF to be equal at the bubble surface. For this purpose, the following statement is obtained from the mechanical energy equation for Navier-Stokes equations [27]:

$$\int_A u.n(-p_{vc})dA = \int_A u.t \tau_i dA$$

(12)

$\tau_i$ and $p_{vc}$ values at the collocation points are estimated on the bubble surface as follows:

$$\sum_{j=1}^M u_j.n_j(-p_{vc,j})2\pi l_jR_j \sin \theta_j = \sum_{j=1}^M u_j.t_{ij} \tau_{ij} 2\pi l_jR_j \sin \theta_j$$

(13)
where \( M \) is the number of elements on the bubble surface and \( l_j \) is the length of the \( j \)th element.

In the above equation, \( R_j \) and \( \omega_j \) indicate radius and angle relative to \( z \)-coordinate in element \( j \), respectively.

In this study, it is assumed that the power of traction integral in VPF method for each bubble level element is equal to the power of traction integral in VCVPF method for the same element. In other words, viscosity difference resulted from uncompensated irrotational shear stress in each element is equal to the viscosity loss in that element. Therefore, simplifying Eq. (13), yields to the following expression:

\[
P_{vc,j} = -\frac{u_j \cdot t_j \overline{r}_{r,j}}{u_j \cdot n_j}
\]

(14)

2.3. Nondimensionalizing

Maximum bubble radius, \( R_m \), pressure difference driving the bubble collapse, \( \Delta p = p_{\infty} - p_c \) and fluid density are selected as the main nondimensionalizing parameters for the governing equations and numerical results.

\[
\begin{bmatrix}
\bar{R} \\
\bar{Z} \\
\bar{r}
\end{bmatrix} = \begin{bmatrix}
R \\
Z \\
r_m
\end{bmatrix},
\begin{bmatrix}
\bar{\tau} \\
\bar{p}_{vc}
\end{bmatrix} = \begin{bmatrix}
\bar{\tau} \\
\bar{p}_{vc}
\end{bmatrix},
\begin{bmatrix}
\bar{\psi} \\
\bar{\eta}
\end{bmatrix} = \begin{bmatrix}
\psi \\
\eta
\end{bmatrix} \left( \frac{\rho}{\Delta p} \right)^{0.5}
\]

(15)

\[
\lambda = \frac{R_{\text{cyl}}}{R_m}, \quad \bar{t} = \frac{t}{R_m} \left( \frac{\Delta p}{\rho} \right)^{0.5}, \quad \bar{\mu} = \mu \left( \frac{1}{\rho^{0.5} \Delta p^{0.5} R_m} \right), \quad \bar{\phi} = \phi \left( \frac{\rho}{\Delta p} \right)^{0.5}
\]

(16)

where \( \lambda \) is the dimensionless channel radius, \( \psi \) is the radial velocity and \( \eta \) is the tangential velocity. The nondimensional dynamic condition on the bubble surface is described as follows:

\[
\frac{D \bar{\phi}}{Dt} = 1 - \varepsilon \left( \frac{R_j}{R} \right)^{3y} + \frac{1}{2} \rho \nabla^2 \bar{\phi}^2 + \frac{2K}{We} \left( 1 - \frac{2}{Re} \frac{\partial \bar{\psi}}{\partial n} - \bar{p}_{vc} \right)
\]

(17)
where $\varepsilon = p_{g_0} / \Delta p$ is the strength parameter. In Eq. (17), Weber Number and Reynolds Number are defined as $We = \Delta p R_m / \sigma$ and $Re = R_m \sqrt{\Delta p \rho / \mu}$, respectively. The nondimensional form of Eq. (13) can be formulated as:

$$
\sum_{j=1}^{m} (\vec{\varphi}_j \vec{p}_j) 2\pi \vec{R}_j \sin \theta_j = \sum_{j=1}^{m} (\vec{\varphi}_j \vec{\tau}_j + \vec{\eta}_j \vec{\tau}_j) 2\pi \vec{R}_j \sin \theta_j
$$

(18)

2.4. Numerical implementation

In this study, in order to implement BEM on three-dimensional axisymmetric geometry, cylindrical coordinate system (r, z) is used.

2.4.1. Discretization

A second-order function is used to express the vertical velocity in each node (J) of the bubble surface based on two adjacent nodes (j-1 and j + 1).

$$
\phi_j = aS^2 + bS + c
$$

(19)

where a, b and c are constants of the second order equation. S is the distance of the desired node (j-1 or j + 1) from the node j. Then, by applying equation (19) to node j + 1 or j-1, the magnitude of S will be equal to the magnitude of $l_{j+1}$ or $l_{j-1}$, respectively.

The potential changes to the bubble surface profile (S) is obtained by [40]:

$$
\left( \frac{\partial \phi}{\partial S} \right)_j = \frac{\phi_{j+1}l_{j+1}^2 - \phi_{j-1}l_{j-1}^2 + \phi_j (l_{j+1}^2 - l_j^2)}{l_j l_{j+1}(l_{j+1} + l_{j-1})}
$$

(20)

Solving the linear equation set in Eq. (1), the vertical velocity at collocation points can be calculated. The final position of elements after a small time scale, $\Delta t$, is obtained from the following equations:

$$
\begin{align*}
\dot{r}_j^{t+\Delta t} & = \dot{r}_j + u_j \Delta t + O(\Delta t)^2 \\
\dot{z}_j^{t+\Delta t} & = \dot{z}_j + v_j \Delta t + O(\Delta t)^2
\end{align*}
$$

(21)

In order to calculate the velocity potential at the new time scale, the discretization form of the modified Bernoulli’s equation is used:
\( \phi_j^{t+\Delta t} = \phi_j + \frac{\Delta t}{\rho} \left\{ \frac{1}{2} \rho |\nabla \phi|^2 + \rho g z + \sigma k + p_\infty - p_\psi \left( \frac{R}{R} \right)^{3\gamma} - 2\mu \frac{\partial^2 \phi}{\partial n^2} - p_{vc} \right\} \)  

(22)

For more details about numerical implementations based on BEM, please refer to reference [40].

Nondimensional UISS and VPC at the bubble surface based on vertical and tangential velocity are calculated by:

\[
\overline{r}_{s_j} = 2\mu \left[ \frac{1}{2} \frac{\partial \overline{\eta}_j}{\partial n_j} - \frac{\overline{\eta}_j}{2\overline{r}_j} + \frac{1}{2\overline{r}_j} \frac{\partial \overline{\psi}_j}{\partial S_j} \right]
\]

(23)

\[
\overline{p}_{vc,j} = -\frac{\overline{\psi}_j \overline{r}_{s_j} + \overline{\eta}_j \overline{r}_{s_j}}{\overline{\psi}_j}
\]

(24)

Variation of tangential velocity in a vertical direction over the bubble surface can be estimated from the following equation:

\[
\frac{\partial \overline{\eta}_j}{\partial n_j} = \frac{\overline{\eta}_j^{i+1} - \overline{\eta}_j^i}{\Delta L}
\]

(25)

where \( \overline{\eta}_j^{i+1} \) shows the nondimensional tangential velocity that is considered at the \( \Delta L \) distance from collocation point \( j \) over the radial direction in fluid flow. For calculating the vertical velocity distribution over the bubble surface, cubic spline functions are employed. Therefore, changes in vertical velocity relative to the vector tangent on the bubble surface can be estimated as follows [40]:

\[
\frac{\partial \overline{\psi}_j}{\partial S_j} = \frac{\overline{\psi}_j^{i+1} \overline{T}_j^{i+1} - \overline{\psi}_j^{i+1} \overline{T}_j^{i+1} + \overline{\psi}_j (\overline{T}_j^{i+1} - \overline{T}_j^i)}{\overline{T}_j^{i+1} (\overline{T}_j^{i+1} + \overline{T}_j^{i+1})}
\]

(26)

3. Numerical model verification

3.1. Independence of results from the number of nodes

In order to investigate the independence of the numerical results from the number of nodes on the bubble surface, variations of \( V / V_{\text{max}} \) against the dimensionless time at \( \lambda = 1.15 \) is plotted. By referring to Fig 2 it is observed that for number of nodes greater than 18, the variations of the
\[ V/V_{\text{max}} \] against the dimensionless time the is very small (eventually about 2\%). Therefore, the optimum number of nodes on the bubble surface which results in the independence of the numerical results from the number of nodes is assumed to be \( M = 18 \).

### 3.2. Comparison with the empirical results

Ni et al. [14] conducted a series of experiments on bubble production by electric spark inside a narrow tube in water. Growth and collapse of the bubble were recorded by a high-velocity camera with the frame rate of 20000 frames/sec. The input parameters used in validation are presented in Table 1.

In Figure 3, the empirical results are compared with our numerical results. The figure shows that a good consistency exists between the empirical and numerical results. Figures 3(a) shows the spark bubble formation moment. In the empirical picture, at this stage, the bubble can be observed as a luminous domain. Figure 3(b) shows the moment where the bubble is at its maximum volume. The bubble cross-section at the expansion stage becomes elliptical since the cylindrical walls function as a barrier against the bubble growth. At the end of expansion stage, the bubble velocity at the two ends of bubble along the channel axis is almost zero.

The high static pressure in the fluid around the two ends of bubble located along the channel axis directs the bubble toward the contraction stage and then counter-jet emerges. According to Figures 3(c) and 3(d) with penetration of the counter jet, at the end of contraction stage, the bubble takes the shape of sand hour glass.

The oscillation period in numerical results is slightly larger than the experiment results. Also, a small deviation in time scale between the empirical photos and numerical model is observed. Error in measuring time and maximum bubble radius in experiments can be one of the most important reasons for the small difference between experimental and numerical results. In these experiments and according to the frame rate, the time between two recorded images is 0.05 ms. In other words, the measurement accuracy of time scale in empirical experiments is 0.05 ms. On the other hand, spatial resolution limitation of the camera causes errors for maximum bubble diameter estimation. The limitation in the number of pixels used in the photo structure makes the image resolution to be dependent on the size of the pixels. Therefore, measuring the maximum
diameter of the bubble is affected by the relative resolution of the image, which can be accompanied by an error.

3.3. Comparison with the numerical results

In order to validate the VCVPF method proposed in this study, the numerical results published by Minsier et al. [31] are used. In which, they applied the volume of fluid method for bubble growth and collapse near the rigid wall inside the viscous fluid. In their numerical model, the dimensionless channel radius, \( \lambda = 0.6 \), oil viscosity, \( \mu_{oil} = 0.05 \text{[kg / (m.s)]} \), and the maximum bubble radius, \( R_m = 1 \text{mm} \), were assumed. Figure 4 shows the comparison of numerical results obtained from the VCVPF method and volume of fluid model [31] for bubble dynamic in oil. Figure 4(a), reveals a slight difference between bubbles in the two models, at the maximum volume. Also, according to Figures 4(b) to 4(d), occurring phenomena such as bubble flattening near the wall, the formation of counter-jet at the same time, rapid jet penetration into the bubble, and almost similar time period confirm the validity of our numerical results during the contraction stage.

4. Result and discussion

Table 2 summarizes the main input parameters used in the numerical model.

4.1. The effect of viscosity and Reynolds number on bubble dynamics

In this section, the effect of fluid viscosity on the bubble dynamic, inside the narrow channel is investigated by using the proposed numerical model. Bubble shapes for two viscosity values of \( \mu_{oil} = 0.05 \text{[kg / (m.s)]} \) and \( \mu_{water} = 0.001 \text{[kg / (m.s)]} \) at \( \lambda = 1.15 \) are compared. The results in two maximum bubble radius of \( R_m = 1 \text{mm} \) and \( R_m = 10 \text{mm} \) are studied.

In Figure 5, the dynamic behavior of bubble for \( R_m = 1 \text{mm} \) at several dimensionless time is indicated. The required time for bubble growth until the maximum radius for oil \( (t = 193.0 \mu \text{s}) \) is slightly larger than water \( (t = 192.8 \mu \text{s}) \). Therefore, it can be concluded that with an increase in the viscosity, bubble growth becomes slower. The bubble volume for oil at the end of growth stage is slightly less than that for water. So that, the bubble volume ratio, which is the ratio of bubble volume to its maximum volume, for oil and water is 83.1 and 83.7, respectively. Comparison of bubble shapes at the end of contraction stage indicates this fact that, the bubble
volume further decreases with increasing viscosity. The bubble volume ratio at the end of the contraction stage for oil and water is 12.5 and 31.3, respectively. This is justified because; In both of growth and contraction stages, with an increase in viscosity, most part of the driving force is consumed to overcome viscous forces. For this reason, the bubble growth is less, at the end of growth stage. Also, at the end of contraction stage, the bubble is flattened like a sand-hour glass and its final volume becomes less.

In order to investigate the effect of Reynolds number, the results in $R_m = 10 mm$ are presented in Figure 6. As the figure indicates, at the growth stage like the contraction stage, no significant difference can be observed between the volume and shape of the bubble for oil and water. So that, the ratio of the bubble volume to its maximum volume at the end of the growth stage for oil and water is 83.65 and 83.70, respectively. Also, the value of this parameter at the end of the contraction stage is 30.5 and 31.5, respectively. This result can be explained in this way: An increase in bubble maximum radius results in a decrease of dimensionless viscosity (Eq. 15) and consequently, according to Eq. (23), UISS is decreased. On the other hand, it can be seen that Reynolds number increases as the bubble maximum radius increases. According to Eq. (15), normal stress has a reverse relationship with Reynolds number. Therefore, with an increase in the Reynolds number and maximum bubble radius, the effect of viscosity on bubble life and shape decreases. The comparison of Figures 5 and 6 shows that when the Reynolds number is in order of $O(10^3)$, the viscosity effects are greatly reduced which can be ignored.

4.2. The effect of $\lambda$ on bubble dynamics

The time history curves of bubble volume ratio to its maximum volume ($V_{\text{max}}/V$) in $\lambda = 0.95$, $\lambda = 1.15$ and $\lambda = 1.35$ is drawn in Figure 7. Initial time and final time are related to the begining and end of contraction stage. At all dimensionless channel radius, $V_{\text{max}}/V$ value for the bubble in oil is less than that in water. At the end of the contraction stage, by assumption of $\lambda = 0.95$, magnitude of $V_{\text{max}}/V$ for oil is approximately 53 percent less than magnitude of $V_{\text{max}}/V$ for water. While by assumption of $\lambda = 1.15$, this percentage is 47%. It means that during the contraction stage, the rate of bubble volume decrease for oil is always less than that in water for all $\lambda$ values. It is observed that higher viscosity brings about severe decrease in $V_{\text{max}}/V$ at the minimum dimensionless radius parameter ($\lambda = 0.95$). This trend is justified considering the point.
that the decrease in \( \lambda \) causes an increase in bubble shape deviation from the initial spherical shape. Therefore, the bubble elongates in the direction of channel axis. This increases UISS over the bubble surface that further decreases the bubble volume at the contraction stage.

Figure 8 depicts the maximum z-coordinate absolute value versus non-dimensional time. According to the symmetry, it is obvious that r-coordinate values for these points are almost zero. Also, Figure 8 reveals that with decreasing \( \lambda \), the bubble elongates and larger values of maximum z-coordinate occur at the end of the growth stage. Also, in similar \( \lambda \), with an increased viscosity, maximum z-coordinate values decrease. In other words, viscosity leads to flatten more and smaller bubbles. As an example, the Maximum z-coordinate of the bubble boundary for oil is approximately 13% less than that for water. Of course, with the increase in the amount of \( \lambda \) this difference will be reduced.

### 4.3. The pressure field

In order to fully understand the effect of viscosity on hydrodynamic of the bubble, path lines and pressure contours at different times are indicated in Figure 9 for two viscosities. As can be seen, the bubble behavior in both viscosities is almost similar. At first, the bubble rapidly growth due to the high pressure inside the bubble and subsequently with increasing in bubble volume, the pressure inside of it decreases. Therefore, approaching the end of the growth stage, the pressure inside bubble will be almost equal to the surrounding fluid pressure, but through motion inertia, the bubble still grows. The direction of path lines in growth stage - Figures 9 (a1) and (a2) and Figures 9 (b1) and (b2)- confirm that the bubble at this stage behaves as a source actuator of fluid. The path lines in Figures 9 (a3) and (b3) indicate the fluid tendency to create two counter jets inside the bubble. The increase of pressure around the bubble near symmetry axis leads to the rapid penetration of jets into the bubble. In the contraction stage, the direction of path lines is toward the inside of the bubble, which confirms the fact that bubble behaves as sink actuator of fluid (Figures 9 (a4) and (b4)). Also, it can be observed that, with respect to the bubble lifetime, with increasing viscosity, the bubble reacts slowly. The bubble lifetime in the oil is \( t = 4.570 \), while the corresponding value in the water is \( t = 4.213 \). According to Figure 9 (b3), it is found that at the contraction stage in the oil fluid the high-pressure zone surrounds most of the bubble surface while in water, the high-pressure zone occurs at around the channel axis. Therefore, at the end of contraction stage, in the oil fluid, those parts of the bubble surface that are closer to
the channel wall, take a shrinking mode toward the bubble due to the pressure distribution and UISS effects. Referring to Figure 9 (a4) and (b4), in the oil fluid, a further decrease in the bubble volume causes the pressure inside it to be higher than water. The pressure inside the bubble at the end of contraction stage in the oil is 115 kPa and in the water is 33 kPa.

4.4. Analyzing Viscosity component

In Figure 10, the behavior of UISS, VPC, and viscous normal stress versus to the dimensionless time are indicated in the third collocation point on the bubble surface. At the beginning of the growth stage, viscous normal stress experienced severe changes. After a rapid increase, the descending mode of this parameter continues until the end of the growth stage, so that at maximum bubble value, this parameter is almost zero. With the formation of the counter-jet and its penetration into the bubble, viscous normal stress increases slightly. On the other hand, at the beginning of the bubble life, due to the completely spherical shape, UISS does not exist at the bubble surface and the zero value of this parameter at the zero moment confirms this reality. The value of this parameter increases at the beginning of the bubble lifetime and then decreases until to formation of the counter-jet ($t_{jet}$). After $t_{jet}$, with a change in the bubble shape and severe deviation from the initial spherical shape, UISS value increases significantly. VPC value at the beginning of the growth stage is almost zero, too. During the growth stage, bubble shape deviation from spherical form increases gradually which leads to an increasing VPC. After the formation of the counter-jet, VPC increases slightly like UISS.

The velocity of the axis center point on the bubble surface is plotted versus the non-dimensional time in Figure 11. In this figure, the negative and positive values refer to the growth and contraction stages, respectively. It is interesting noted that, at the growth stage, increasing viscosity does not change much in velocity. But at the contraction stage, for a specific value of $\lambda$, the difference in velocity is increased, over time. At the expansion stage, the magnitudes of viscosity parameters are not significant. Therefore, by increasing of viscosity, there is no significant difference in the velocity of the interaction point of the axis of symmetry with the bubble surface. While at the bubble contraction, especially at the latest stages of the bubble contraction which the viscosity terms increases significantly, a portion of the bubble driving force is used to overcome the viscosity forces. This fact causes a significant reduction in the velocity of the interaction point of the axis of symmetry with the bubble surface. Under the
conditions of Re = 200 and \( \lambda = 1.15 \), ratio of the rate of velocity reduction on the interaction point of the axis of symmetry with the bubble surface in oil to the rate of velocity reduction in the corresponding point in water is approximately 37%. This reduction is equal to 22% in the case of \( \lambda = 1.35 \).

Changes in the absolute value of viscous normal stress on the bubble surface with respect to time are shown in Figure 12. Due to the existence of symmetry, changes on half of the collocation points are investigated. As can be seen from this figure, at the growth stage \((T < 2.4)\), viscous normal stress has maximum value in all collocation points. Extreme changes in the velocity of the early bubble growth stage result in a rapid increase in viscous normal stress. Then, with a decrease in velocity variations, this parameter shows descending behavior. So that, in the contraction stage, even at near-wall points where changes in velocity are small, the value of this parameter approaches to zero. At a specified time, the normal stress value in points near the axis of symmetry is greater than that in the near-wall points.

The variation of UISS in collocation points on the bubble surface as a function of time is illustrated in Figure 13. With the passage of time and approaching to the end of the growth stage, the value of UISS increases in all collocation points. At this time, the maximum value of UISS occurs at intermediate collocation points. It is worth to note that, at the end of the contraction stage a significant increase in the UISS value is observed in most collocation points. This can be justified by considering severe changes of tangential and normal velocities near the bubble surface. The maximum UISS value at the contraction stage is 6.70E-04 which occurs at the end of bubble lifetime on the node 3

By investigating Figures 12 and 13 the following results can be obtained:

The viscous normal stress reaches a maximum value in the growth stage only while, the UISS experiences the maximum value at the end of the contraction stage as well as the end of growth stage. Also the maximum value of viscous normal stress occurs in the nearest collocation point to the symmetry axis. Whereas, the UISS at both of the stages is maximized at the intermediate collocation points. The shape of the bubble in both of the expansion and contraction stages is influenced by the presence of the cylindrical wall and by the pressure at the far-field. It is observed that the shape of intermediate nodes on the bubble surface is further changed over the bubble lifetime. This is due to the effects of far-field flow (near the cylindrical symmetry axis)
and tube wall. In other words, extreme changes in the shape and physical parameters such as the pressure and velocity fields on the middle nodes of the bubble surface are the reason for higher values of UIS in these nodes than its values on the other nodes of the bubble surface. It is observed that the viscous normal stress in all of the collocation points, after experiencing the maximum amount at the early stage of growth, either has a decreasing trend or negligible values. While, UIS amount except in the early life of the bubble, is particularly significant at the middle collocation points of bubble surface. According to this point that VPC is considered for non-zero irrotational shear stress compensation, a direct relationship can be imagined between the numerical value of VPC and UIS. It can be concluded that behaviors of the viscous normal stress and VPC on the bubble surface during its evolution are not similar. In studies that have been done so far, in order to apply viscosity effects on the bubble dynamics, VPC is considered linearly as a function of normal stress. While the results of the present research which have been presented in Figures 12 and 13 show that the overall behaviors of these two parameters during evolution of the vapour bubble are not the same. Therefore, it should be noted that considering the proportion between these two parameters over the bubble lifetime can increase the numerical error of the calculations. Eventually, considering consistency between VPC and normal stress [25-27] in certain cases is not appropriate necessarily.

5. Conclusion

A numerical model based on BEM is used to simulate the dynamics of a single bubble in a rigid narrow channel filled with a viscous liquid. The effects of viscosity are evaluated by applying UIS and VPC parameters by using VCVPF method.

The numerical results of the present study have been verified by comparing them with the reported experimental results of the bubble profiles and its life-time when the bubble is inside a narrow channel filled with a viscous liquid [14]. The numerical results of this study have also been verified by comparing them with the reported numerical results of the bubble profiles and life-time when the bubble is in the vicinity of a flat rigid boundary [31]. These comparisons show good agreement between the numerical results of this study and the reported experimental and numerical results.

It is found that by increasing the viscosity of liquid around the vapor bubble,
(a) the lifetime of bubble increases. As an example, under the conditions of Re = 200 and \( \lambda = 1.15 \), the bubble lifetime for oil is \( t = 460.4 \mu s \) while for water is \( t = 428.0 \mu s \).

(b) at the end of the contraction stage, the velocity of the bubble boundary decreases. As an example, under the conditions of Re = 200 and \( \lambda = 1.35 \), the ratio of the rate of decrease of velocity of the interaction point of the axis of symmetry with the bubble surface in oil to the rate of decrease of velocity of the corresponding point in water is approximately 37%.

(c) the bubble contracts slowly. For example, under the conditions of Re = 200 and \( \lambda = 1.35 \), the bubble lifetime in oil is 7.8% higher than the bubble lifetime in water.

(d) the final bubble volume decreases. As an example, at the end of the contraction stage, by assumption of \( \lambda = 0.95 \), magnitude of \( V/V_{\text{max}} \) for oil is approximately 53 percent less than magnitude of \( V/V_{\text{max}} \) for water.

(e) and the sand hour glass profile of the bubble at the end of its contraction stage is flattened.

In addition, it is concluded that for Reynolds numbers with the order of \( O(10^3) \), the viscosity effects are extremely reduced which can be neglected. Moreover, the results indicate that viscous normal stress maximizes at all points of the bubble surface at the growth stage. Whereas, the magnitudes of UISS, except in the beginning of the bubble lifetime, are significant at the points of the bubble surface which are relatively near to the internal wall of the narrow channel. Finally, it is found that the variations of the viscous normal stress and VPC with respect to time on the surface of the bubble are not similar.

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**Mohammad-Taghi Shervani-Tabar** received the PhD degree in Mechanical Engineering from University of Wollongong in 1995. He is currently a Professor in University of Tabriz. His main interests are in the area of cavitation and bubble dynamics, boundary element method, and
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