



# An evaluation of inventory systems via an evidence theory for deteriorating items under uncertain conditions and advanced payment

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## KEYWORDS

Evidence theory;  
 Inventory;  
 Advance payment;  
 Deterioration;  
 Genetic algorithm;  
 Interval order relations.

**Abstract.** The inventory model for deteriorating items, which is developed by the Evidential Reasoning Algorithm (ERA) and imprecise inventory costs, is one of the most important factors in complex systems that plays a vital role in payment. The ERA is able to strengthen the precision of the model and give the perfect interval-valued utility. In this model, during lead-time and reorder level two different cases can occur in which the mathematical model turns into an imposed nonlinear mixed integer problem with an interval objective for each case. The placement of an order, which has been overlooked by many researchers until now, is normally connected with Advance Payment (AP) in business. Specifying the optimal profit and the optimal number of cycles in the finite time horizon and lot-sizing in each cycle, are the goals of this paper. In order to solve this model, the Real-Coded Genetic Algorithm (RCGA) with ranking selection is applied. Using the model, some numerical examples are represented and a sensitivity analysis with a variation in different inventory parameters.

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## 1. Introduction

Some of inventory management system studies discuss deterioration items, either deteriorated or outdated. For example, deterioration is typically seen during the normal storage period of commonly used goods such as fruit, vegetables, meat, foodstuffs, perfumes, alcohol, gasoline, radioactive substances, photographic films,

and electronic components, etc. Four meta-categories of inventoried goods can be categorized [1]:

- Abolished indicates items that lose their value through time because of swift changes in technology or the introduction of a new product by a rival;
- Deterioration mentions the damage, spoilage, dryness, evaporation, etc. of the products;
- Amelioration refers to an increment of value, utility or quantity to items by time;
- No abolished/deterioration/amelioration.

Wu et al. [2] developed and calculated previous studies about inventory models with a trapezoidal-type demand rate by:

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1. Discussing two inventory systems starting with and without deficiency;
2. Containing the purchasing cost into the total cost;
3. Developing the deterioration rate to any time-varying rate;
4. Considering the time value of money;
5. Maximizing the net present value of total profit.

Teng et al. [3] considered the deterioration rate of a product gently increases as the expiration date approaches. Also, a new inventory model with two warehouses for goods that have a fraction of imperfect quality items and those that are deteriorating in nature, was extended by Jaggi et al. [4]. Widyadana and Irohara [5] developed an inventory routing problem with time windows for deteriorating items. They used PSO (Particle Swarm Optimization) to solve the problem in a reasonable period with near optimal solutions. Their results showed that the deteriorating rate in the inventory has greater effects than the deteriorating rate in the vehicle, which is a significant contribution.

In competitive markets, wholesalers require Advance Payment (AP). The retailer should endure AP, and then apply a price discount at the time of final payment. The important point is that the interest on the amount of money paid as AP should be covered by the retailer. Thus, the entire profit of the inventory systems has been affected significantly by the amount of AP. The inventory model with AP was studied by Gupta et al. [1], in which they supposed interval-value for inventory costs. The seller's profit under three payment terms: AP, cash payment, and credit payment was derived by Li et al. [6]. Also, they performed sensitivity analyses to examine the impacts of financial related parameters on the seller's decisions and profits, and provided several managerial insights. Feng et al. [7] set their objectives on maximizing total profit through determining three decision variables (i.e., unit price, cycle time, and ending-inventory level) and then presented numerical examples to illustrate their theoretical results and managerial insights.

Yang et al. [8] considered the effect of defective items and inspection error, which are derived from non-cooperative and cooperative situations, in their mathematical model to optimize the equilibrium production and replenishment strategies for the supplier and retailer. Maiti et al. [9] studied an inventory model using stochastic lead-time by considering AP in their model. Taleizadeh [10] developed his model with a real case study of a gasoline station. In which, he considered AP in the inventory model with partial back ordering and with an evaporating item. Nodoust et al. [11] developed a new method based on the Evidential Reasoning (ER) approach. In their study, they applied various types of possible uncertainties that may occur in specifying

the inflation rate in the inventory decision making. Tiwari, et al. [12] proposed approach explicitly models the interdependence among price of a product, demand for the product and integration among the retailer and the supplier under four different policies: non-integrated, integrated, supplier-led Stackelberg policy and retailer-led Stackelberg policy. Using a differential calculus method, they found the unique maximum total relevant profit. Zhang et al. [13] studied Economic Order Quantity (EOQ) under AP. Sustainable economic production quantity using a partial backordering model, which is a general and more realistic model that can be used in many real cases, has been shown by Taleizadeh et al. [14]. Tiwari et al. [15] determined the retailer's optimal replenishment policies, in which the present worth of total optimal profit per unit time was maximized and the numerical example helped to validate their proposed inventory model.

Pourmohammad zia and Taleizadeh [16] investigated an EOQ model with back ordering and under hybrid AP and delayed payment. Maiti et al. [17] considered AP in inventory decisions. They considered a constant value for cost parameters but it is farfetched in real life. Special cases of static and state-of-the-world dependent walk-in market pricing strategies were studied by Elhafsi and Hamouda [18], in which they investigated the case of a spot market where the price is exogenously set.

In order to solve the problem with non-constant parameters, stochastic, fuzzy, and fuzzy-stochastic and interval valued methods may be used. In this paper, imprecise parameters with the evidential theory are introduced. This type of impreciseness is solved using the 6-step Dempster-Shafer (D-S) theory. In this method several Dempster-Shafer's (DMs) views are considered, where they could have various decisions. The 6-step D-S theory gives us the specific interval value for the imprecise parameter (interval utility for holding and shortage costs).

The theory was first developed by Arthur P. Dempster [19] and Shafer [20]. This is a mathematical theory of evidence that makes an evidence combination from different sources and acquires a degree of belief (represented by a belief function) that also considers all accessible evidence.

Moreover, the term D-S theory connects to the original idea behind the theory by Dempster and Shafer. However, this term usually is used in the more extensive sense of the same general method, as adapted to special types of condition. In particular, various rules for combining evidence have been proposed by many authors. They often have a glance to contrasts in the evidence, to handle it better [21].

The 6-step D-S theory gives one special interval utility. The inventory model with the interval value parameters has been solved by a strong computerized

heuristic search and optimization approaches such as, Genetic Algorithm (GA), which is based on the system of natural selection (inspired by the evolution principle “Survival of the Fittest”) (2009).

Park et al.[22] proposed a GA for the inventory routing problem with lost sales. Hiassat et al. [23] addressed a location-inventory-routing model for perishable products. They developed a GA approach to solving the problem efficiently. Azadeh et al. [24] presented a model of the inventory routing problem with transshipment in the presence of a single perishable product. They proposed a GA based approach to solve the problem. Taleizadeh et al. [25] optimized constrained inventory control systems with stochastic replenishments and fuzzy demand by the hybrid method of fuzzy simulation and the GA. Saracoglu, Topaloglu et al. [26] formulated an approach for a multi-product multi-period inventory model. A GA solution approach is proposed to solve the problem. O'Neill and Sanni [27] presented a framework derived from optimisation results for the cycle time and price. They showed how these results can be applied for particular deterioration functions and demand functions. Souri et al. [28] presented a framework to identify consumer behavior towards green products- goods rather than services- by measuring the gap between the expectations and perceptions of consumers. They made an importance-performance analysis based on their results.

In this study, due to interval value parameters, ER gave an interval valued objective function. To solve this kind of problem using the GA approach, one of the most important things when selecting/reproducing an operation and also in finding the best chromosome in each iteration, is ordering the relations of interval numbers, which is vital. Ishibuchi and Tanaka [29], Chanes and Kuchta [30] are among the very few to have specified the order relations of interval-valued numbers. Two different approaches (deterministic and fuzzy) were proposed by Sengupta and Pal [31] to compare any two interval numbers regarding the decision maker's perspective, although these approaches in some cases failed to figure out the order relations between two interval numbers. Recently, Mahato and Bhunia [32] proposed modified definitions of order relations regarding optimistic and pessimistic decision maker's points of view for maximization and minimization of problems.

The application of the evidence theory in the inventory models, particularly imprecise ones, is not seen in previous studies and literature. In this paper, an inventory model has been extended by incorporating the insight of the evidence theory in imprecise parameters (holding and shortage costs). Interval utilities of these costs are given by evidence reasoning. This inventory model, without application of the evidence theory, was solved in the customary way. But, to

solve the model with this feature in this paper, a Real-Coded Genetic Algorithm (RCGA) has been applied. This paper also extended this inventory model, whereas the latest research did not consider that the items could deteriorate by the passing of time in this slight inventory model.

It is assumed that a certain percentage of complete purchasing cost per cycle is to be paid as AP. Two cases have been studied by Gupta et al. [1], one of which was without shortage and the other by permitting partially backlogged shortages. For each case, a mixed integer constrained optimization problem with an interval objective was formulated by the authors helped by the use of interval arithmetic. To solve these problems, a RCGA with ranking selection was extended, using whole arithmetical crossover and mutation, considering the order relations of interval numbers with respect to a pessimistic decision maker's point of view. Finally, the model was elucidated using numerical examples and sensitivity analyses, with a variation of different inventory parameters on the optimal profit.

## 2. Assumptions and notations

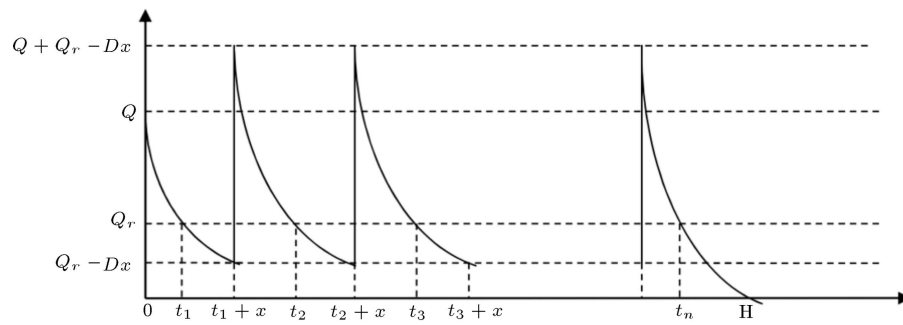
The following assumptions and notations are used for this model:

### *Assumptions:*

1. Holding and shortage costs (after using ER) stand in the known intervals;
2. AP is considered;
3. The demand rate is uniform. However, this rate varies during the stock-out periods;
4. Lead-time is constant;
5. The inventory planning horizon is finite and is sufficiently larger than the lead-time;
6. Items deteriorate throughout the course of time;
7. A single order will be placed at the beginning of each cycle and the whole lot is delivered in one batch;
8. The size of replenishment is finite;
9. This inventory model considers one item;
10. Shortages (if any) are allowed and partially backlogged;
11. A certain percentage will be accessible as discounts due to AP.

### *Notations:*

$P$ -size	Population size
$P$ -cros	Probability of crossover
$P$ -mute	Probability of mutation



**Figure 1.** The inventory status for Case (1).

$m\text{-gen}$	Maximum number of generations
$Q$	Order quantity
$Q_r$	Reorder level
$Q_s$	Shortage level
$q(t)$	Inventory level at time $t$
$D$	Uniform demand rate
$\theta$	The constant deterioration rate, where $0 < \theta < 1$
$\lambda$	The deduction by which the rate of demand decreases during the stock-out periods ( $0 < \lambda < 1$ )
$P > C_p$	Unit selling price of the product
$AP$	Advance Payment for purchasing quantities
$I_c$	Percentage of discount on unit purchase cost
$I_d$	The percentage of AP (Advance Payment) with respect to whole purchase cost per cycle
$I_b$	The prevailing interest rate
$C_o$	Ordering cost per order
$C_p$	Unit purchase cost
$C_h$	Holding cost per unit item per unit time
$C_s$	Shortage cost per unit item per unit time
$x$	Lead-time
$H$	Time horizon
$n$	Number of replenishments to be made during the time horizon
$T$	Cycle length
$Z$	Total profit in the time horizon

### 3. Mathematical formulation

Two cases are calculated by the amount of demand during lead time ( $\frac{D(e^{\theta x} - 1)}{\theta}$ ). The comparison of the  $Q_r$  (the inventory level at which the order takes place) and

the demand during lead time determine these cases.

$$\text{Case (1): } Q_r \geq \frac{D(e^{\theta x} - 1)}{\theta}, \quad (1)$$

$$\text{Case (2): } Q_r < \frac{D(e^{\theta x} - 1)}{\theta}. \quad (2)$$

#### Case (1)

There is no shortage in this case (Figure 1). Total holding cost is computed according to the following equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D \quad 0 \leq t \leq t_1 + x. \quad (3)$$

For the first period:

$$\ln \left( \frac{D + \theta I(t_1 + x)}{D + \theta I(t)} \right) = \theta(t - (t_1 + x)), \quad (4)$$

$$\begin{aligned} t_1 + x &= \frac{-1}{\theta} \ln \left( \frac{D + \theta Q_r - Dx\theta}{D + \theta Q} \right) \rightarrow t \\ &= \frac{1}{\theta} \ln \left( \frac{D + \theta Q}{D + \theta I(t)} \right). \end{aligned} \quad (5)$$

For the second period up to the penultimate period:

$$\ln \left( \frac{D + \theta I(t_1 + x)}{D + \theta I(t)} \right) = \theta(t - (t_1 + x)), \quad (6)$$

$$\begin{aligned} t_1 + x &= \frac{-1}{\theta} \ln \left( \frac{D + \theta Q_r - Dx\theta}{D + \theta Q + \theta Q_r - Dx\theta} \right) \rightarrow t \\ &= \frac{1}{\theta} \ln \left( \frac{D + \theta Q + \theta Q_r - Dx\theta}{D + \theta I(t)} \right). \end{aligned} \quad (7)$$

For the last period:

$$\ln \left( \frac{D + \theta I(t_1 + x)}{D + \theta I(t)} \right) = \theta(t - (t_1 + x)), \quad (8)$$

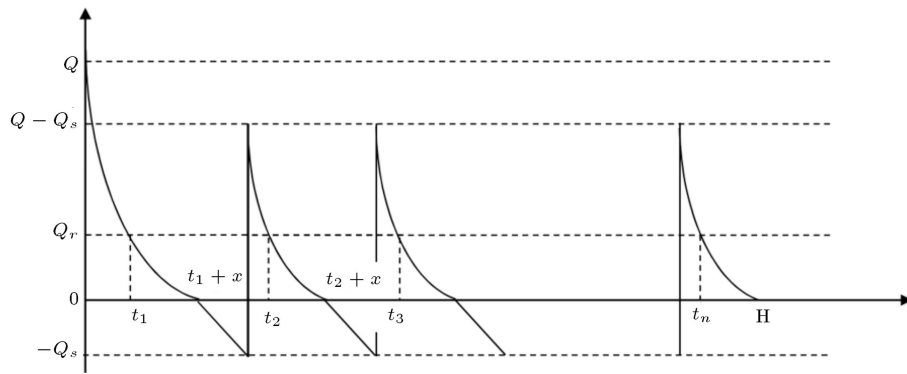


Figure 2. The inventory status for Case 2.

$$t_1 + x = \frac{-1}{\theta} \ln \left( \frac{D}{D + \theta Q + \theta Q_r - Dx\theta} \right) \rightarrow t$$

$$= \frac{1}{\theta} \ln \left( \frac{D + \theta Q + \theta Q_r - Dx\theta}{D + \theta I(t)} \right), \quad (9)$$

$$H_1(x) = C_h \left[ \int_{Q_r - Dx}^Q \frac{1}{\theta} \ln \left( \frac{D + \theta Q}{D + \theta q} \right) dq \right.$$

$$+ (n-2) \int_{Q_r - Dx}^{Q + Q_r - Dx} \frac{1}{\theta} \ln \left( \frac{D + \theta Q + \theta Q_r - Dx\theta}{D + \theta q} \right) dq$$

$$+ \int_0^{Q + Q_r - Dx} \frac{1}{\theta} \ln \left( \frac{D + \theta Q + \theta Q_r - Dx\theta}{D + \theta q} \right) dq \Big], \quad (10)$$

where:

$$Q = \int_0^T D e^{\theta t} dt = \int_0^{\frac{H}{n}} D e^{\theta t} dt = \frac{D \left( e^{\theta \frac{H}{n}} - 1 \right)}{\theta}, \quad (11)$$

and:

$$A_p = I_d (1 - I_c) Q C_p. \quad (12)$$

The total profit is equal to total sales revenue minus total purchase cost minus total interest rate on the loan from the bank minus total ordering cost minus total holding cost:

$$Z_1 = nPQ - nQC_p(1 - I_c) - (n-1)$$

$$A_p x I_b - nC_o - H_1(x)$$

$$= nPQ - nQC_p(1 - I_c) - (n-1)I_d Q C_p(1 - I_c)$$

$$x I_b - nC_o - H_1(x). \quad (13)$$

Due to the interval-value of  $C_h$  and  $C_s$  that are obtained from the Evidential Reasoning Algorithm

(ERA) (explained in the next section), the objective function is shown by the following:

$$Z \in [Z_l, Z_R],$$

$$Z_{1L} = nPQ - nQC_p(1 - I_c) - (n-1)I_d$$

$$QC_p(1 - I_c)x I_b - nC_o - H_{1R}(x), \quad (14)$$

and:

$$Z_{1R} = nPQ - nQC_p(1 - I_c) - (n-1)I_d$$

$$QC_p(1 - I_c)x I_b - nC_o - H_{1L}(x). \quad (15)$$

Therefore:

$$\max Z_1(Q_r, n) \quad (16)$$

$$\text{Subject to } Q_r \geq \frac{D(e^{\theta x} - 1)}{\theta} \text{ and } n \text{ is an integer.}$$

This is a nonlinear maximization problem with interval objective and variables.

### Case (2)

There will be a shortage in this case (Figure 2). Total holding cost is computed according to the following equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D \quad 0 \leq t \leq t_1 + y. \quad (17)$$

For the first period:

$$\ln \left( \frac{D + \theta I(t_1 + y)}{D + \theta I(t)} \right) = \theta(t - (t_1 + y)), \quad (18)$$

$$t_1 + y = \frac{-1}{\theta} \ln \left( \frac{D}{D + \theta Q} \right) \rightarrow t = \frac{1}{\theta}$$

$$\ln \left( \frac{D + \theta Q}{D + \theta I(t)} \right). \quad (19)$$

For the second period up to the last one:

$$\ln \left( \frac{D + \theta I(t_1 + y)}{D + \theta I(t)} \right) = \theta(t - (t_1 + y)), \quad (20)$$

$$t_1 + y = \frac{-1}{\theta} \ln \left( \frac{D}{D + \theta Q - \theta Q_s} \right) \rightarrow t$$

$$= \frac{1}{\theta} \ln \left( \frac{D + \theta Q - \theta Q_s}{D + \theta I(t)} \right), \quad (21)$$

$$H_2(x) = C_h \left[ \int_0^Q \frac{1}{\theta} \ln \left( \frac{D + \theta Q}{D + \theta q} \right) dq \right. \\ \left. + (n-1) \int_0^{Q-Q_s} \frac{1}{\theta} \ln \left( \frac{D + \theta Q - \theta Q_s}{D + \theta q} \right) dq \right]. \quad (22)$$

$Q$  is calculated according to the following equations:

$$Q_s = \lambda D t \rightarrow t = \frac{Q_s}{\lambda D}$$

$$Q'_s = \frac{D(e^{\theta t} - 1)}{\theta}$$

$$\rightarrow \text{extera } Q_s = D \left( \frac{(e^{\theta \frac{Q_s}{\lambda D}} - 1)}{\theta} - \frac{Q_s}{D} \right)$$

$$\text{extera } Q_s = D \left( \frac{(e^{\theta t} - 1)}{\theta} - \lambda t \right), \quad (23)$$

$$Q = \int_0^T D e^{\theta t} dt - \frac{(n-1)D}{n} \left( \frac{(e^{\theta \frac{Q_s}{\lambda D}} - 1)}{\theta} - \frac{Q_s}{D} \right)$$

$$= \int_0^{\frac{H}{n}} D e^{\theta t} dt - \frac{(n-1)D}{n} \left( \frac{(e^{\theta \frac{Q_s}{\lambda D}} - 1)}{\theta} - \frac{Q_s}{D} \right)$$

$$= \frac{D(e^{\theta \frac{H}{n}} - 1)}{\theta} - \frac{(n-1)D}{n}$$

$$\left( \frac{(e^{\theta \frac{Q_s}{\lambda D}} - 1)}{\theta} - \frac{Q_s}{D} \right), \quad (24)$$

and:

$$A_p = I_d(1 - I_c)QC_p. \quad (25)$$

Total shortage cost can be calculated by the following:

$$S_c(x) = (n-1)C_s \int_0^{Q_s} \frac{qdq}{\lambda D} = (n-1) \frac{C_s}{2\lambda D} Q_s^2. \quad (26)$$

The total profit is equal to total sales revenue minus

total purchase cost minus total interest on bank loan minus total ordering cost minus total holding cost minus total shortage cost:

$$Z_2 = nPQ - nQC_p(1 - I_c) - (n-1)A_p x I_b$$

$$- nC_o - H_2(x) - \frac{C_s(n-1)Q_s^2}{2\lambda D}$$

$$= nPQ - nQC_p(1 - I_c) - (n-1)$$

$$I_d QC_p(1 - I_c)x I_b - nC_o - H_2(x)$$

$$- \frac{C_s(n-1)Q_s^2}{2\lambda D}. \quad (27)$$

Due to the interval-value of  $C_h$  and  $C_s$  that is obtained from the ERA (explained in the next section), the objective function is shown by the following:

$$Z \in [Z_l, Z_R],$$

$$Z_{2L} = nPQ - nQC_p(1 - I_c) - (n-1)I_d QC_p$$

$$(1 - I_c)x I_b - nC_o - H_{2R}(x)$$

$$- \frac{C_{sR}(n-1)Q_s^2}{2\lambda D}, \quad (28)$$

and:

$$Z_{2R} = nPQ - nQC_p(1 - I_c) - (n-1)$$

$$I_d QC_p(1 - I_c)x I_b - nC_o - H_{2L}(x)$$

$$- \frac{C_{sL}(n-1)Q_s^2}{2\lambda D}. \quad (29)$$

Therefore:

$$\max Z_2(Q_s, n)$$

$$\text{Subject to } Q_s > 0 \text{ and } n \text{ is an integer.} \quad (30)$$

As is obvious, this is a nonlinear maximization problem with interval objective and variables [33]. Finally, the optimal profit will be attained by the following equation, i.e.:

$$\max Z = \max (Z_1, Z_2). \quad (31)$$

#### 4. The 6 steps of (D-S)

An ERA is used for the imprecise value parameters ( $C_h$  and  $C_s$ ). The steps of the algorithm are described below:

**Step 1.** Introduction of a multiple-attribute decision problem:

- (i) Define a set of attributes:

$$E = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_i, \dots, \varepsilon_L\}.$$

(ii) Assess the comparative weights of the attributes:

$$0 \leq \omega_i \leq 1, \sum_{i=1}^L \omega_i = 1.$$

(iii) Define  $N$  distinctive assessment degrees  $H_n$ :

$$H = \{H_1, H_2, \dots, H_n \dots H_N\}.$$

(iv) Determine a grade of belief  $\beta_n$  for each attribute  $\varepsilon_i$  and assessment degree  $H_n$ .

**Step 2.** Probability assignments for each attribute.  $m_{n,i}$  is a probability representing the degree to which the  $i$ th attribute supports an assumption that the main attribute is evaluated to the  $n$ th evaluation assessment degree  $H_n$ . The probability unassigned to each attribute is indicated by  $m_{H,i}$ .

$$m_{n,i} = \omega_i \beta_{n,i}, \quad (32)$$

$$m_{H,i} = 1 - \sum_{n=1}^N m_{n,i} = 1 - \omega_i \sum_{n=1}^N \beta_{n,i} (m_{H,i} = \bar{m}_{H,i} + \tilde{m}_{H,i}), \quad (33)$$

$$\bar{m}_{H,i} = 1 - \omega_i, \quad (34)$$

$$\tilde{m}_{H,i} = \omega_i (1 - \sum_{n=1}^N \beta_{n,i}). \quad (35)$$

**Step 3.** Incorporate probability assignments for the main attribute. The probability masses assigned to the various assessment grades, as well as the probability mass left at unassigned levels, are assumed [34] as follows:

$$m_{n,I(L)} (n = 1, \dots, N),$$

$$\bar{m}_{H,I(L)}, \tilde{m}_{H,I(L)}, m_{H,I(L)},$$

$$I(1) = 1 : m_{n,I(1)} = m_{n,1} (n = 1, \dots, N),$$

$$\bar{m}_{H,I(1)} = \bar{m}_{H,1}, \tilde{m}_{H,I(1)} = \tilde{m}_{H,1}, m_{H,I(1)} = m_{H,1}.$$

The following recursive ER algorithm produces the incorporating probability;

$$\{H_n\} : m_{n,I(i+1)} = K_{I(i+1)} \left[ m_{n,I(i)} \cdot m_{n,i+1} + m_{H,I(i)} \cdot m_{n,i+1} + m_{n,I(i)} \cdot m_{H,i+1} \right] \\ n = 1, \dots, N, \quad (36)$$

$$\{H\} : m_{H,I(i)} = \tilde{m}_{H,I(i)} + \bar{m}_{H,I(i)}, \quad (37)$$

$$\tilde{m}_{H,I(i+1)} = K_{I(i+1)} \left[ \tilde{m}_{H,I(i)} \cdot \tilde{m}_{H,i+1} + \bar{m}_{H,I(i)} \cdot \tilde{m}_{H,i+1} + \tilde{m}_{H,I(i)} \cdot \bar{m}_{H,i+1} \right], \quad (38)$$

$$\bar{m}_{H,I(i+1)} = K_{I(i+1)} [\bar{m}_{H,I(i)} \cdot \bar{m}_{H,i+1}], \quad (39)$$

$$K_{I(i+1)} = \left[ 1 - \sum_{t=1}^N \sum_{\substack{j=1 \\ j \neq t}}^N m_{t,I(i)} \cdot m_{j,i+1} \right]^{-1}, \quad (40)$$

$$i = \{1, 2, \dots, L-1\},$$

$$\sum_{n=1}^N m_{n,I(i+1)} + m_{H,I(i+1)} = 1. \quad (41)$$

**Step 4.** Calculation of the incorporated grades of belief for the main attribute.  $\beta_n$  indicates the incorporated grade of belief that the main attribute is evaluated as the grade  $H_n$ .

$$\{H_n\} : \beta_n = \frac{m_{n,I(L)}}{1 - \bar{m}_{H,I(L)}} \quad n = 1, 2, \dots, N, \quad (42)$$

$$\{H\} : \beta_H = \frac{\tilde{m}_{H,I(L)}}{1 - \bar{m}_{H,I(L)}}. \quad (43)$$

**Step 5.** Computation of the expected utility for a complete evaluation.

$$u = \sum_{n=1}^N \beta_n u(H_n). \quad (44)$$

**Step 6.** Computation of the utility interval of incomplete evaluations.

$$u_{\max} = \sum_{n=1}^{N-1} \beta_n u(H_n) + (\beta_N + \beta_H) u(H_N),$$

$$u_{\min} = (\beta_1 + \beta_H) u(H_1) + \sum_{n=2}^N \beta_n u(H_n), \quad (45)$$

$$u_{avg} = \frac{u_{\max} + u_{\min}}{2}. \quad (46)$$

## 5. Finite interval arithmetic

The interval numbers can be introduced according to the following:

$$A = \begin{cases} [a_L, a_R] = \{x : a_L \leq x \leq a_R, x \in R\} \\ < a_C, a_W > = \\ \{x : a_C - a_W \leq x \leq a_C + a_W, x \in R\} \end{cases}$$

The left and right limits are shown as  $a_L$  and  $a_R$ , and the center and radius of the intervals are shown as  $a_C = (a_L + a_R)/2$  and  $a_W = (a_R - a_L)/2$ ,  $R$  is the set of real numbers.

**Definition 1.** Four arithmetic operations are explained for two closed intervals  $A = [a_L, a_R]$  and  $B = [b_L, b_R]$ .

For  $*$   $\in (+, -, \cdot, /)$

$$A * B = \{a * b : a \in A \text{ and } b \in B\}.$$

In the case of division, it is assumed that  $0 \notin B$ .

Based on [35], all two intervals include one of the following three types:

- Type-I: Both the intervals are disjoint;
- Type-II: Intervals are partially overlapping;
- Type-III: One interval is contained in the other.

In this case, an order relation cannot be executed. Thus, Mahato and Bhunia [32] extended the order relations for maximization of the problems of the optimistic and pessimistic decision maker's point of view [36]. The following definitions are as below.

### 5.1. Pessimistic decision-making

According to the principle "Less uncertainty is better than more uncertainty" or "More uncertainty is worse than less uncertainty", the order relations are performed.

**Definition 2.** The order relation  $>_{p \max}$  between two intervals:

$$A = [a_L, a_R] = < a_C, a_W >$$

and:

$$B = [b_L, b_R] = < b_C, b_W >$$

is introduced for maximization problems:

- For Type-I and Type-II intervals:  $A >_{p \max} B$  and  $a_C > b_C$ ;
- For Type-III intervals:  $A >_{p \max} B$  and  $a_C \geq b_C$  and  $a_W < b_W$ ;
- Decision maker could not make a decision in Type-III with  $a_C \geq b_C$  and  $a_W < b_W$  conditions, in this case the optimistic decision maker would be considered.

### 5.2. Optimistic decision-making

In this case, the lowest cost/time for minimization

problems and the highest profit for maximization problems, ignoring uncertainty, are chosen by the decision maker.

**Definition 3.** The order relation  $\geq_{o \max}$  between two intervals  $A$  and  $B$  is introduced for maximization problems:

$$a_R \geq b_R \rightarrow A \geq_{o \max} B$$

$$A \geq_{o \max} B \text{ and } A \neq B \rightarrow A >_{o \max} B.$$

This order relation  $\geq_{o \max}$  is not symmetric but transitive.

## 6. Solution procedure

Solving these problems by different gradient based methods is difficult. Hence, the developed advanced GA was selected for solving the mixed integer maximization problems (10) and (18) with an interval objective.

### GA algorithm:

1. Choose an initial population

### If crossover condition is satisfied:

1. Select parent
2. Select crossover parameters
3. Perform crossover

### If the mutation condition is satisfied:

1. Select chromosomes
2. Select mutation points
3. Perform mutation
4. Evaluate fitness of offspring
5. Till adequate offspring created
6. Choose new population

### Till stopping standard is true:

1. Present estimate population

GA relies on some parameters like population size ( $P$ -size).  $P$ -size should not be too large or too small, because storing large amounts of data may cause some computer difficulty and crossovers cannot be best implemented at small  $P$ -size.

$Vj = [Vj1, Vj2]$  represented the chromosome in GA, where the components  $Vj1$  and  $Vj2$  denote, respectively, the decision variables,  $Qr/Qs$  and  $n$  of the problem. To initialize the chromosomes, a random value can be selected from the discrete set of values within the bounds.

The next step is checking how good the chromosomes are. Hence, the interval value of the objective



**Table 1.** Assessment grades.

$H_n$	1	2	3	4	5	6	7	8	9	10
$C_s$	2	4	6	8	10	12	14	16	18	20
$C_h$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

**Table 2.** Assigned belief grades and weights.

	$DM_1$	$DM_2$	$DM_3$	$DM_4$
$W_i$	0.3	0.25	0.2	0.25
$C_s$	6/40%, 8/10%, 10/30%	6/30%, 8/10%, 10/60%	6/40%, 8/10%, 10/40%	6/30%, 8/40%, 10/30%
$C_h$	0.5/60%, 0.7/30%, 1/0%	0.5/70%, 0.7/20%, 1/0%	0.5/80%, 0.7/10%, 1/0%	0.5/55%, 0.7/40%, 1/5%

**Table 3.** Calculated assigned and unassigned probability masses for  $C_s$ .

Evaluation grade											
$H_1, H_2, \dots, H_{13}$	Weight	Belief				Probability mass					
	$W_i$	$\beta_{3,i}$	$\beta_{7,i}$	$\beta_{11,i}$	$\beta_H$	$m_{3,i}$	$m_{7,i}$	$m_{11,i}$	$m_{H,i}$	$\tilde{m}_{H,i}$	$\tilde{m}_{H,i}$
$C_s$	–	0.357025	0.164458	0.406174	0.072341	–	–	–	–	–	–
$DM_1$	0.3	0.4	0.1	0.3	0.2	0.12	0.03	0.09	0.76	0.7	0.06
$DM_2$	0.25	0.3	0.1	0.6	0	0.075	0.025	0.15	0.75	0.75	0
$DM_3$	0.2	0.4	0.1	0.4	0.1	0.08	0.02	0.08	0.82	0.8	0.02
$DM_4$	0.25	0.3	0.4	0.3	0	0.075	0.1	0.075	0.75	0.75	0

function is calculated as the fitness value corresponding to the chromosome  $V_j$ . Then, the rank of the interval value (fitness value) of the chromosomes has been performed from the point of view of the pessimistic decision maker (Definition 3) for the selection process under the principle “survival of the fittest”.

Crossover and mutation operate on the resulting chromosomes.  $P$ -cros \*  $P$ -size number of chromosomes are selected as parents for the crossover operation and generate offspring by recombining the features of both parents.

The first component of two offspring will be created by:

$$\begin{aligned} V'_{k1} &= \lambda V_{k1} + (1 - \lambda) V_{i1}, \\ V'_{i1} &= \lambda V_{i1} + (1 - \lambda) V_{k1}, \quad \lambda \in [0, 1], \end{aligned} \quad (47)$$

and for the second component:

$$\begin{aligned} V'_{k2} &= V_{k2} - g \quad \text{and} \\ V'_{i2} &= V_{i2} + g \\ \text{if } V_{k2} > V_{i2}, \quad g &\in [0, |V_{k2} - V_{i2}|], \quad \text{and integer.} \end{aligned} \quad (48)$$

Mutation is executed on a single chromosome. It is usually fulfilled with low probability. Mutation changes one or all the genes of a randomly selected chromosome a little to shift the population gradually towards a slightly better course.

## 7. Numerical example

The model is illustrated by the following ten examples, all of whose values are real. These examples had been performed by the 6-step D-S algorithm. Four experts points of view on holding and shortage costs were considered. Seven distinctive evaluation grades  $H_n$  were defined for an interest rate, as shown in Table 1. Table 2 shows the experts' ideas about the holding and shortage costs. These assessments are incomplete obviously (this means that they had no idea of some percentages). The belief degrees and probable masses that are calculated in the second steps in the D-S algorithm are shown in Tables 3 and 4.

Aggregated probability masses, aggregated belief degrees for holding and shortage costs were computed and, at the end, the utility intervals are shown in Table 5. This algorithm gives one specific interval value for holding and shortage costs.

To continue solving this procedure, the proposed GA algorithm is used. For each example, 10 independent runs have been performed and, in each generation, the best value of total profit will be selected according to Definition 2 of the interval order relations. So, the best-found values  $Z$ ,  $Q$ ,  $Q_r$ ,  $Q_s$ ,  $n$ , and  $Ap$  have been obtained and displayed in Tables 6 and 7.

Also, to obtain the best performance of GA, the designing of experiments by the response surface methodology have been used to decide about the GA parameters.

**Table 4.** Calculated assigned and unassigned probability masses for  $C_h$ .

Evaluation grade											
$H_1, H_2, \dots, H_{13}$	Weight	Belief				Probability mass					
	$W_i$	$\beta_{3,i}$	$\beta_{7,i}$	$\beta_{11,i}$	$\beta_H$	$m_{3,i}$	$m_{7,i}$	$m_{11,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$
$C_h$	—	0.699196	0.232619	0.024069	0.044113	—	—	—	—	—	—
$DM_1$	0.3	0.6	0.3	0	0.1	0.18	0.09	0	0.73	0.7	0.03
$DM_2$	0.25	0.7	0.2	0	0.1	0.175	0.05	0	0.775	0.75	0.025
$DM_3$	0.2	0.8	0.1	0.1	0	0.16	0.02	0.02	0.8	0.8	0
$DM_4$	0.25	0.55	0.4	0.05	0	0.1375	0.1	0.0125	0.75	0.75	0

**Table 5.** Utility intervals for holding cost and shortage cost.

$U_{\max}$	8.242981481	$U_{\max}$	0.519250594
$U_{\min}$	7.953616495	$U_{\min}$	0.536896154

Accordingly, the following values of GA parameters are used:

For Case (1):  $Q_r \geq \frac{D(e^{\theta x} - 1)}{\theta}$

$P$ -size = 140,  $P$ -cros = 0.4,  $P$ -mute = 0.4,  $m$ -gen = 123.

For Case (2) :  $Q_r < \frac{D(e^{\theta x} - 1)}{\theta}$

$P\text{-size} = 140$ ,  $P\text{-cros} = 0.3$ ,  $P\text{-mute} = 0.3$ ,  $m\text{-gen} = 120$ .

## 8. Sensitivity analysis

Parameters have various effects on this model; these effects may be so significant or negligible and impress

**Table 7.** Solution of numerical examples.

Examples	$Q$	$Q_r$	$n$	$A_p$	$Z_1$
1	220.048	191.223	41	739.3628	[85221, 85265]
2	176.817	189.155	51	594.1053	[84717, 84894]
3	220.048	190.863	41	739.3628	[85221, 85265]
4	180.36	188.654	50	606.0112	[84716, 84896]
5	291.261	193.739	31	948.6374	[85552, 85842]
6	220.048	191.391	41	739.3628	[85221, 85265]
7	282.131	193.872	32	947.9593	[85555, 85837]
8	291.261	193.647	31	978.6374	[85552, 85842]
9	180.36	187.844	50	606.0112	[84716, 84896]
10	220.048	189.605	41	739.3628	[85089, 85418]

the optimal solution. Some of them increase and some of them decrease the optimal solution. The effects of every parameter are studied completely in this section and show how they influence the model. Table 8 shows these solutions completely in the numerical case, and it is described particularly in the following paragraphs.

**Table 6.** Values of parameters for numerical example.

[illegible]

**Table 8.** Sensitivity analysis.

	<b>2</b>	<b>1.5</b>	<b>1.1</b>	<b>1</b>	<b>0.9</b>	<b>0.5</b>	<b>0.1</b>		<b>2</b>	<b>1.5</b>	<b>1.1</b>	<b>1</b>	<b>0.9</b>	<b>0.5</b>	<b>0.1</b>
$C_p$	<b>24</b>	<b>18</b>	<b>15.4</b>	<b>14</b>	<b>10.8</b>	<b>6</b>	<b>1.2</b>	$C_o$	<b>70</b>	<b>52.5</b>	<b>38.5</b>	<b>35</b>	<b>31.5</b>	<b>17.5</b>	<b>3.5</b>
	$Z_1 = [53472.6004, 54032.6004]$	$Z_1 = [96603.4861, 97075.9861]$	$Z_1 = [13113.5800, 13148.2300]$	$Z_1 = [85221, 85265]$	$Z_1 = [14841.1946, 14871.4346]$	$Z_1 = [18293.8240, 18311.3240]$	$Z_1 = [21760.1352, 21763.4952]$		$Z_1 = [13849.4646, 13877.4646]$	$Z_1 = [13906.8968, 13938.3968]$	$Z_1 = [13943.2915, 13978.2915]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [14027.4558, 13993.8558]$	$Z_1 = [14104.1987, 14068.19879]$	$Z_1 = [14189.1125, 14228.6420]$
$P$	<b>46</b>	<b>34.5</b>	<b>25.3</b>	<b>23</b>	<b>20.7</b>	<b>11.5</b>	<b>2.3</b>	$I_d$	<b>1</b>	<b>0.75</b>	<b>0.55</b>	<b>0.5</b>	<b>0.45</b>	<b>0.25</b>	<b>0.05</b>
	$Z_1 = [36964.3279, 36999.3279]$	$Z_1 = [25464.2785, 25499.2785]$	$Z_1 = [16276.3563, 16309.9563]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [11676.3589, 11709.9589]$	$Z_1 = [24763.5768, 25099.5768]$	—		$Z_1 = [13747.2658, 13803.2658]$	$Z_1 = [13860.3486, 13907.5986]$	$Z_1 = [13951.4751, 13988.4351]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [14001.2238, 14031.4638]$	$Z_1 = [14093.8237, 14111.3237]$	$Z_1 = [14200.1347, 14203.4947]$
$I_c$	<b>0.6</b>	<b>0.45</b>	<b>0.33</b>	<b>0.3</b>	<b>0.27</b>	<b>0.15</b>	<b>0.03</b>	$x$	<b>1</b>	<b>0.75</b>	<b>0.55</b>	<b>0.5</b>	<b>0.45</b>	<b>0.25</b>	<b>0.05</b>
	$Z_1 = [17676.7971, 17694.7971]$	$Z_1 = [15829.6336, 15856.0336]$	$Z_1 = [14346.9951, 14379.1551]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [13606.9086, 13639.7586]$	$Z_1 = [12123.0680, 12163.8680]$	$Z_1 = [10646.9875, 10690.6375]$		$Z_1 = [13744.0882, 13807.0882]$	$Z_1 = [13860.4956, 13907.7456]$	$Z_1 = [13953.6168, 13988.2668]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [14000.1745, 14028.5245]$	$Z_1 = [14093.5825, 14111.0825]$	$Z_1 = [14199.7597, 14203.1197]$
$\lambda$	<b>1.6</b>	<b>1.2</b>	<b>0.88</b>	<b>0.8</b>	<b>0.72</b>	<b>0.4</b>	<b>0.08</b>	$D$	<b>200</b>	<b>150</b>	<b>110</b>	<b>100</b>	<b>90</b>	<b>50</b>	<b>10</b>
	$Z_2 = [26159.6199, 26330.6151]$	$Z_2 = [20968.4341, 21253.5334]$	$Z_1 = [13976.3592, 14009.9592]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [13976.3558, 14009.9558]$	$Z_1 = [13976.3306, 14009.9306]$	$Z_1 = [13976.8944, 14008.3944]$		$Z_1 = [28138.9148, 28208.9148]$	$Z_1 = [21051.6511, 21104.1511]$	$Z_1 = [15391.5272, 15428.4872]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [12565.1711, 12593.5211]$	$Z_1 = [69245.3103, 69385.3103]$	$Z_2 = Z_1 = [1325, 1325]$

**Table 8.** Sensitivity analysis (continued).

	<b>2</b>	<b>1.5</b>	<b>1.1</b>	<b>1</b>	<b>0.9</b>	<b>0.5</b>	<b>0.1</b>		<b>2</b>	<b>1.5</b>	<b>1.1</b>	<b>1</b>	<b>0.9</b>	<b>0.5</b>	<b>0.1</b>
<b>H</b>	<b>20</b>	<b>15</b>	<b>11</b>	<b>10</b>	<b>9</b>	<b>5</b>	<b>1</b>	<b>C<sub>h</sub></b>	<b>0.4</b>	<b>0.3</b>	<b>0.22</b>	<b>0.2</b>	<b>0.18</b>	<b>0.1</b>	<b>0.02</b>
	$Z_1 = [27890.5293, 27966.1293]$	$Z_1 = [20933.9640, 20987.9640]$	$Z_1 = [15361.0767, 15395.7267]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [12587.7064, 12616.0564]$	$Z_1 = [7024.2119, 7034.7066]$	$Z_1 = [1369.4491, 1371.5491]$		$Z_1 = [13802.8749, 13838.8749]$	$Z_1 = [13880.9944, 13915.9944]$	$Z_1 = [13951.8969, 13983.3969]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [14001.8999, 14033.3999]$	$Z_1 = [14121.1329, 14149.1329]$	$Z_1 = [14324.5999, 14345.5999]$
<b>C<sub>s</sub></b>	<b>16</b>	<b>12</b>	<b>8.8</b>	<b>8</b>	<b>7.2</b>	<b>4</b>	<b>0.8</b>								
	$Z_1 = [13976.3392, 14009.9392]$	$Z_1 = [13976.3127, 14009.9127]$	$Z_1 = [13976.3458, 14009.9458]$	$Z_1 = [13976.3261, 14009.9261]$	$Z_1 = [13976.8960, 14008.3960]$	$Z_1 = [13976.3569, 14009.9569]$	$Z_1 = [13976.8988, 14008.3988]$								

The first example is selected for this section; the effect of parameters upon the optimal solution will be studied by varying them in various percentages. A sensitivity analysis was undertaken by increasing the parameters by 10%, 50%, 100% and decreasing the parameters by 10%, 50%, 90%, taking each one at a time, and keeping the remaining parameters at their original values. The measures of system sensitivity considered here are profit function ( $Z$ ). Results of the sensitivity analysis are exhibited in Table 8.

The results show as the parameters  $D$ ,  $H$ ,  $P$  and  $I_c$  increase the profit function ( $Z$ ) increases and when these parameters decrease, the profit function ( $Z$ ) decreases. Conversely, when  $C_h$ ,  $C_p$ ,  $C_o$ ,  $I_d$  and  $x$  increase, the profit function ( $Z$ ) decreases and when these parameters decrease, the profit function ( $Z$ ) increases.

When parameters  $\lambda$  and  $C_s$  increase and parameter  $C_s$  decrease, there is no change in profit function value, because in the numerical example, the pessimistic decision maker chooses  $Z_1$  (between  $Z_1$  and  $Z_2$ ) as the best profit (Definition 2). In this case, there are no shortages in the model and parameter  $\lambda$  does not have any role in the profit function. But, when parameter  $\lambda$  increases, a different situation occurs. The pessimistic decision maker chooses  $Z_2$  for his optimal solution and in this case the profit function ( $Z$ ) increases.

## 9. Managerial implication

In the competitive world we face, many uncertain conditions exist and having a good concept about important decisions plays a crucial role in developing a prosperous business. This factor can be helpful for a manager (or any other decision maker) to be able to select the best way to manage their stock, in order to achieve maximum profit even in a shortage of time. The proposed model can help decision makers effectively in this way and since we have a pessimistic point of view it is very similar to real world situations. For instance, an organization could improve its current supply chain process by applying this model. As most industries (either food or other industries), have deteriorated items, it is a useful way to keep customers satisfied at the right time.

## 10. Conclusion

This paper has developed an inventory model based on deterioration, advance payment and also interpolation of the Dempster-Shafer theory. As the main problem for a decision-maker is to find the best and suitable membership functions, probable distributions and specific interval values, the algorithm aggregates different decisions and give us the interval utility for

incomplete assessments. The authors model solved these interval utility costs by the proposed developed Genetic Algorithm (GA) and found the best profit rate. The demand rate has been considered uniformly and it is assumed that it decreases by a certain fraction at the time of stock-out. In times of shortage, the inaccessibility of goods may causes some customers to seek elsewhere. In this case, we use a pessimistic decision-making point of view for selecting a better profit function in our model and for use in the real world, for selecting better chromosomes in each generation using an advanced GA.

The case of ameliorating items, multi-storage facilities, and time-dependent demand are important suggestions for furthering this research.

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