An Evaluation of Inventory System via Evidence Theory for Deteriorating Items under Uncertain Condition and Advanced Payment

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Abstract

The inventory model for deteriorating items, which is developed by The Evidential Reasoning Algorithm (ERA) and the imprecise inventory costs, is one of the most important factors in complex systems which plays a vital role in Payment. The ERA is able to strengthen the precision of the model and give the perfect interval-valued utility. In this model, during lead-time and reorder level two different cases can be happened which the mathematical model turns into an imposed nonlinear mixed integer problem with interval objective for each case. Placement of an order, which is overlooked by many researchers until now, is normally connected with the advance payment (AP) in business. Specifying the optimal profit and the optimal number of cycles in the finite time horizon and lot-sizing in each cycle, are our goals so. In order to solve this model, we apply the real-coded genetic algorithm (RCGA) with ranking selection. By the model, we represent some numerical examples and a sensitivity analysis with the variation of different inventory parameters.

Keywords: Evidence theory, Inventory, Advance payment, Deterioration, Genetic algorithm, Interval order relations.

1. INTRODUCTION

Some of the inventory management system's studies discuss deterioration items, deteriorate either deteriorated or outdated. For example, deterioration is typically seen during the normal storage period of commonly used goods such as fruits, vegetables, meat, foodstuffs, perfumes, alcohol, gasoline, radioactive substances, photographic films, electronic components, etc. Four meta-categories of inventoried goods can be categorized [1]:

(a) Abolished indicates items that lose their value through time because of swift changes in technology or the introduction of a new product by a rival.
(b) Deterioration mentions the damage, spoilage, dryness, evaporation, etc. of the products.
(c) Amelioration refers to an increment of value, utility or quantity to items by the time.
(d) No abolished /deterioration/amelioration.

Wu et al. [2] developed and calculated previous studies about inventory models with trapezoidal-type demand rate by 1. Discussing two inventory systems starting with and without deficiency, 2. Containing the purchasing cost into the total cost, 3. Developing the deterioration rate to any time-varying rate, 4. Considering the time value of money, 5. Maximizing the net present value of total profit. Teng et al. [3] considered the deterioration rate of a product gently increases as the expiration date approaches. Also a new inventory model with two warehouses: for goods that have a fraction of imperfect quality items and those that are deteriorating in nature, was extended by Jaggi et al. [4]. Gede Agus Widyadana and Takashi Irohara [5] developed an inventory routing problem with time windows for deteriorating items. They used PSO (Particle Swarm Optimization) to solve the problem in a reasonable period with near optimal solutions. Their results showed that the deteriorating rate in inventory has bigger effects than deteriorating rate in the vehicle which has a significant contribution.

In competitive markets, wholesaler requires advance payment. The retailer should endure AP(Advanced Payment), and then he/she applies price discount at the time of finalpayment. The important point is that the interest on the amount of money paid as AP should be devoted by the retailer. Thus the whole profit of the inventory systems has been affected significantly by the amount of AP. The inventory model with advance payment was studied by Gupta et al. [1], they supposed interval-valued for inventory costs. The seller's profit under three payment terms: advance payment, cash payment, and credit payment was derived by Li et al. [6]. Also they performed sensitivity analyses to examine the impacts of financial related parameters on the seller's decisions and profits, and then they provided several managerial insights. Feng, L., et al. [7] set their objectives on maximizing the total profit through determining three decision variables (i.e., unit price, cycle time, and ending-inventory level) and then presented numerical examples to illustrate their theoretical results and managerial insights.

Chih-Te Yang et al. [8] considered the effect of defective items and inspection errors , which are derived from non-cooperative and cooperative situations, in their mathematical model to optimize the equilibrium production and replenishment strategies for the supplier and retailer. Maiti et al [9] studied inventory model with stochastic lead-time by considering AP in their model. Taleizadeh [10] developed his model with a real case study of a gasoline station. In which, he considered AP in inventory model with partial back ordering and with evaporating item. Nodoust et al.[11] developed a new method based on the evidential reasoning (ER) approach. In their study, They applied various types of possible uncertainties that may happen in the specifying of the inflation rate in the inventory decision making. Tiwari, et al [12] proposed approach explicitly models the interdependence among price of a product, demand for the product and the integration among the retailer and the supplier under four different policies: non-integrated, integrated, supplier-led Stackelberg policy and retailer-led Stackelberg policy. By using differential calculus method, they found the unique maximum total relevant profit. Zhang, Tsao, and Chen [13] studied EOQ under advance payment. The sustainable economic production quantity with partial backordering model, which is a general and more realistic model that can be used in many real cases, has been showed by Taleizadeh et al. [14]. Tiwari, Sunil et al. [15] determined the retailer's optimal replenishment policies in which the present worth of total optimal profit per unit time was maximized and the numerical example helped to validate their proposed inventory model.

Pourmohammad zia and Taleizadeh [16] investigated an EOQ model with back ordering and under hybrid advance payment and delayed payment. Maiti, Bhunia, and Maiti [17] considered AP in inventory decisions. They considered constant value for cost parameters but it is farfetched in real life. The special cases of static and state-of-the-world dependent walk-in market pricing strategies was studied by Elhafsi and Hamouda [18], in which they investigated the case of a spot market where the price is exogenously set.

In order to solve the problem with non-constant parameters, stochastic, fuzzy, and fuzzy-stochastic and interval valued methods may be used. In this paper, we introduce the imprecise parameters with the Evidential Theory. We solve this type of impreciseness by 6-step Dempster-Shafer (D-S) theory. In this method several DMs views are considered, where they could have various decisions, 6-step Dempster-Shafer theory gives us the specific interval value for the imprecise parameter (interval utility for holding cost and shortage cost).

The theory was first developed by Arthur P. Dempster [19] and Glenn Shafer [20]. This is a mathematical theory of evidence. Which makes an evidence combination from different sources and acquires a degree of belief (represented by a belief function) that consider all the accessible evidence as well.

Moreover, the term Dempster–Shafer theory connects to the original idea of the theory by Dempster and Shafer. However, this term usually is used in the more extensive sense of the same general method, as adapted to special types of conditions. In particular, various rules for combining evidence has been proposed by many authors. They often have a glance to contrasts in evidence, to handle it better (Kari, S., and Ferson, S., [21].

6-step Dempster-Shafer (D-S) theory gives one special interval utility. The inventory model with the interval value parameters has been solved by a strong computerized heuristic search and optimization approaches such as, genetic algorithm (GA), which is based on the systems of natural selection (inspired by the evolution principle “Survival of the fittest”) (2009).

Park, Yoo, and Park [22] proposed a genetic algorithm for the inventory routing problem with lost sales. Hiassat, Diabat, and Rahwan [23] addressed a location-inventory-routing model for perishable products. They developed a Genetic Algorithm approach to solving the problem efficiently. Azadeh et al. [24] presented a model of inventory routing problem with transshipment in the presence of a single perishable product. They proposed a genetic algorithm based approach to solve the problem. Taleizadeh et al [25] optimized constrained inventory control systems with stochastic replenishments and fuzzy demand by the hybrid method of fuzzy simulation and genetic algorithm. Saracoglu, Topaloglu, and Keskinturk [26] formulated an approach for multi-product multi-period inventory model. A genetic algorithm (GA) solution approach is proposed to solve the problem. B. O'Neill, S. Sanni [27] presented a framework which derive from optimisation results for the cycle time and price. They showed how these results can be applied for particular deterioration functions and demand functions. Mohammad Ehsan Souri et al. [28] presented a framework to identify consumer's behavior towards green products- goods rather than services- by measuring the gaps between expectations and perceptions of consumers. They made importance-performance analysis based on their results.

In this study, due to interval value parameters, evidential reasoning gave an interval valued objective function. To solve this kind of problem by GA approach, one of the most important things to select/ reproduce an operation and also to find the best chromosome in each iteration is ordering relations of interval numbers which is vital. Ishibuchi and Tanaka [29], Chanes and Kuchta [30] are among the very few to have specified the order relations of interval-valued numbers. Two different approaches (deterministic and fuzzy) were proposed by Sengupta and Pal [31] to compare any two interval numbers regarding the decision maker's perspective. Although, approaches in some cases fail to figure out the order relations between two interval numbers. Recently, Mahato and Bhunia [32] proposed modified definitions of order relations regarding optimistic and pessimistic decision maker's point of view for maximization and minimization of problems.

The application of the evidence theory in the inventory models particularly imprecise ones is not seen in the previous studies and literature. In this paper, we have extended an inventory model by incorporating the insight of evidence theory in imprecise parameters (holding cost and shortage cost). Interval utilities of these costs are given by evidence reasoning. This inventory model without the application of the evidence theory was solved by the customary way but to solve the model with this feature in this paper, we apply a real-coded genetic algorithm (RCGA). Besides this paper extended this inventory model whereas the deteriorating items, that the last research didn't consider that the items could deteriorate by pass the time in this slightly inventory model.

It is assumed that a certain percentage of completely purchasing cost per cycle is to be paid as AP. Two cases have been studied by [1], which one of them was without shortage, and the other one was by permitting partially backlogged shortages. For each case, we have formulated mixed integer constrained optimization problem with interval objective by helping of interval arithmetic. To solve these problems, a real-coded genetic algorithm with the ranking selection was extended, whole arithmetical crossover and mutation considering the order relations of interval numbers with respect to pessimistic decision maker's point of view. Finally, we have elucidated the model with numerical examples and sensitivity analyses with the variation of different inventory parameters on the optimal profit.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used for this model.

Assumptions

1. Holding and shortage costs (after using evidential reasoning) stand in the known intervals.
2. Advance payment is considered.
3. The demand rate is uniform. However, this rate varies during the stock-out periods.
4. Lead-time is constant.
5. The inventory planning horizon is finite and is sufficiently larger than the lead-time.
6. Items deteriorate throughout course of time
7. A single order will be placed at the beginning of each cycle and the whole lot is delivered in one batch.
8. The size of replenishment is finite.
9. This inventory model considers one item.
10. Shortages (if any) are allowed and partially backlogged.
11. A certain percentage will be accessible as discounts due to advance payment.

Notations

p-size	population size.
P-cros	probability of crossover.
P-mute	probability of mutation.
m-gen	Maximum number of generation.
Q	Order quantity.
Q_r	Reorder level.
Q_s	Shortage level.
$q(t)$	Inventory level at time t.
D	Uniform demand rate.
θ	The constant deterioration rate, where $0<\theta<1$
λ $(0<\lambda<1)$	The deduction by which the rate of demand decreases during the stock-out periods.
$P>Cp$	Unit selling price of the product.
Ap	Advance payment for purchasing quantities.
I_c	Percentage of discount on unit purchase cost.
I_d	The percentage of A_p (advance payment) with respect to whole purchase cost per cycle.

I_b	The prevailing interest rate.
C_o	Ordering cost per order.
C_p	Unit purchase cost.
C_h	Holding cost per unit item per unit time.
C_s	Shortage cost per unit item per unit time.
x	Lead-time.
H	Time horizon.
n	Number of replenishment to be made during the time horizon.
T	Cycle length.
Z	Total profit in the time horizon

3. MATHEMATICAL FORMULATION

Two cases calculated by the amount of demand during lead time ($\frac{D\left(e^{\theta x}-1\right)}{\theta}$). The comparison of the Q_r (the inventory level that the order take place) and demand during lead time determine these cases.

$$case(1): Q_r \geq \frac{D\left(e^{\theta x}-1\right)}{\theta} \qquad (1)$$

$$case(2): Q_r < \frac{D\left(e^{\theta x}-1\right)}{\theta} \qquad (2)$$

Case (1):
There is no shortage in this case. Total holding cost computed according to the following equation:

$$\frac{dI(t)}{dt}+\theta I(t)=-D \qquad 0 \leq t \leq t_1+x \qquad (3)$$

For the first period:

$$\ln\left(\frac{D+\theta I(t_1+x)}{D+\theta I(t)}\right)=\theta\left(t-(t_1+x)\right) \qquad (4)$$

$$t_1+x=\frac{-1}{\theta}\ln\left(\frac{D+\theta Q_r-Dx\theta}{D+\theta Q}\right) \rightarrow t=\frac{1}{\theta}\ln\left(\frac{D+\theta Q}{D+\theta I(t)}\right) \qquad (5)$$

For the second period up to penultimate period:

$$\ln\left(\frac{D+\theta I(t_1+x)}{D+\theta I(t)}\right)=\theta\left(t-(t_1+x)\right) \qquad (6)$$

$$t_1+x=\frac{-1}{\theta}\ln\left(\frac{D+\theta Q_r-Dx\theta}{D+\theta Q+\theta Q_r-Dx\theta}\right) \rightarrow t=\frac{1}{\theta}\ln\left(\frac{D+\theta Q+\theta Q_r-Dx\theta}{D+\theta I(t)}\right) \qquad (7)$$

For the last period:

$$\ln\left(\frac{D+\theta I(t_1+x)}{D+\theta I(t)}\right)=\theta\left(t-(t_1+x)\right) \qquad (8)$$

$$t_1 + x = \frac{-1}{\theta}\ln\left(\frac{D}{D+\theta Q+\theta Q_r - Dx\theta}\right) \rightarrow t = \frac{1}{\theta}\ln\left(\frac{D+\theta Q+\theta Q_r - Dx\theta}{D+\theta I(t)}\right) \qquad (9)$$

$$H_1(x) = C_h\left[\int_{Q_r-Dx}^{Q}\frac{1}{\theta}\ln\left(\frac{D+\theta Q}{D+\theta q}\right)dq + (n-2)\int_{Q_r-Dx}^{Q+Q_r-Dx}\frac{1}{\theta}\ln\left(\frac{D+\theta Q+\theta Q_r-Dx\theta}{D+\theta q}\right)dq + \int_{0}^{Q+Q_r-Dx}\frac{1}{\theta}\ln\left(\frac{D+\theta Q+\theta Q_r-Dx\theta}{D+\theta q}\right)dq\right] \qquad (10)$$

Where

$$Q = \int_0^T De^{\theta t}dt = \int_0^{\frac{H}{n}} De^{\theta t}dt = \frac{D\left(e^{\theta\frac{H}{n}}-1\right)}{\theta} \qquad (11)$$

And

$$A_p = I_d\left(1-I_c\right)QC_p \qquad (12)$$

The total profit = total sales revenue – total purchase cost – total interest rate on loan from bank – total ordering cost – total holding cost;

$$Z_1 = nPQ - nQC_p\left(1-I_c\right) - (n-1)A_p xI_b - nC_o - H_1(x) = nPQ - nQC_p\left(1-I_c\right) - (n-1)I_d QC_p\left(1-I_c\right)xI_b - nC_o - H_1(x) \qquad (13)$$

Due to the interval-valued of C_h and C_s that obtained from Evidential Reasoning Algorithm (explained in the next section), the objective function is shown by following: $Z \in \left[Z_l, Z_R\right]$

$$Z_{1L} = nPQ - nQC_p\left(1-I_c\right) - (n-1)I_d QC_p\left(1-I_c\right)xI_b - nC_o - H_{1R}(x) \qquad (14)$$

And

$$Z_{1R} = nPQ - nQC_p\left(1-I_c\right) - (n-1)I_d QC_p\left(1-I_c\right)xI_b - nC_o - H_{1L}(x) \qquad (15)$$

Therefore:

Maximize $Z_1(Q_r, n)$ *(16)*

Subject to $Q_r \geq \frac{D\left(e^{\theta x}-1\right)}{\theta}$ and n is an integer.

This is a nonlinear maximization problem with interval objective and variables.

(Please insert Figure 1)

Case (2):
There will be a shortage in this case. Total holding cost computed according to the following equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D \qquad 0 \leq t \leq t_1 + y \qquad (17)$$

For the first period:

$$\ln\left(\frac{D+\theta I\left(t_1+y\right)}{D+\theta I(t)}\right) = \theta\left(t-\left(t_1+y\right)\right) \qquad (18)$$

$$t_1+y=\frac{-1}{\theta}\ln\left(\frac{D}{D+\theta Q}\right)\rightarrow t=\frac{1}{\theta}\ln\left(\frac{D+\theta Q}{D+\theta I(t)}\right) \tag{19}$$

For the second period up to last one:

$$\ln\left(\frac{D+\theta I(t_1+y)}{D+\theta I(t)}\right)=\theta\left(t-(t_1+y)\right) \tag{20}$$

$$t_1+y=\frac{-1}{\theta}\ln\left(\frac{D}{D+\theta Q-\theta Q_s}\right)\rightarrow t=\frac{1}{\theta}\ln\left(\frac{D+\theta Q-\theta Q_s}{D+\theta I(t)}\right) \tag{21}$$

$$H_2(x)=C_h\left[\int_0^Q\frac{1}{\theta}\ln\left(\frac{D+\theta Q}{D+\theta q}\right)dq+(n-1)\int_0^{Q-Q_s}\frac{1}{\theta}\ln\left(\frac{D+\theta Q-\theta Q_s}{D+\theta q}\right)dq\right] \tag{22}$$

Q is calculating according to following equations:

$$\begin{cases} Q_s=\lambda Dt\rightarrow t=\dfrac{Q_s}{\lambda D} \\ Q_s^{'}=\dfrac{D\left(e^{\theta t}-1\right)}{\theta} \\ extera\,Q_s=D\left(\dfrac{\left(e^{\theta t}-1\right)}{\theta}-\lambda t\right)\end{cases}\rightarrow extera\,Q_s=D\left(\frac{\left(e^{\theta\frac{Q_s}{\lambda D}}-1\right)}{\theta}-\frac{Q_s}{D}\right) \tag{23}$$

$$Q=\int_0^T De^{\theta t}dt-\frac{(n-1)D}{n}\left(\frac{\left(e^{\theta\frac{Q_s}{\lambda D}}-1\right)}{\theta}-\frac{Q_s}{D}\right)=\int_0^{\frac{H}{n}}De^{\theta t}dt-\frac{(n-1)D}{n}\left(\frac{\left(e^{\theta\frac{Q_s}{\lambda D}}-1\right)}{\theta}-\frac{Q_s}{D}\right)=\frac{D\left(e^{\theta\frac{H}{n}}-1\right)}{\theta}-\frac{(n-1)D}{n}\left(\frac{\left(e^{\theta\frac{Q_s}{\lambda D}}-1\right)}{\theta}-\frac{Q_s}{D}\right) \tag{24}$$

And

$$A_p=I_d\left(1-I_c\right)QC_p \tag{25}$$

Total shortage cost can be calculated by following:

$$S_c(x)=(n-1)C_s\int_0^{Q_s}\frac{qdq}{\lambda D}=(n-1)\frac{C_s}{2\lambda D}Q_s^2 \tag{26}$$

The total profit = total sales revenue – total purchase cost – total interest on loan from bank – total ordering cost – total holding cost – total shortage cost;

$$Z_2=nPQ-nQC_p(1-I_c)-(n-1)A_pxI_b-nC_o-H_2(x)-\frac{C_s(n-1)Q_s^2}{2\lambda D}=nPQ-nQC_p(1-I_c)-(n-1)I_dQC_p(1-I_c)xI_b-nC_o-H_2(x)-\frac{C_s(n-1)Q_s^2}{2\lambda D} \tag{27}$$

Due to the interval-valued of C_h and C_s that obtained from Evidential Reasoning Algorithm (explained in the next section), the objective function is shown by following: $Z\in\left[Z_l,Z_R\right]$

$$Z_{2L} = nPQ - nQC_p(1-I_c) - (n-1)I_dQC_p(1-I_c)xI_b - nC_o - H_{2R}(x) - \frac{C_{sR}(n-1)Q_s^2}{2\lambda D} \tag{28}$$

And

$$Z_{2R} = nPQ - nQC_p(1-I_c) - (n-1)I_dQC_p(1-I_c)xI_b - nC_o - H_{2L}(x) - \frac{C_{sL}(n-1)Q_s^2}{2\lambda D} \tag{29}$$

Therefore:

Maximize $Z_2(Q_s, n)$

Subject to $Q_s>0$ and n is an integer.

(30)

As is it obvious, this is a nonlinear maximization problem with interval objective and variables [33].

Finally, the optimal profit will be attained by following equation:

i.e., Maximize Z=Maximize (Z_1, Z_2) *(31)*

(Please insert Figure 2)

4. THE 6 STEPS OF DEMPSTER-SHAFER

Evidential Reasoning Algorithm is used for the imprecise value parameters (C_h and C_s). The steps of the algorithm are described below.

Step 1. Introduction of a multiple-attribute decision problem.

i) Define a set of attributes; $E = \{\varepsilon_1, \varepsilon_2, \ldots \varepsilon_i, \ldots \varepsilon_L\}$

ii) Assessment the comparative weights of the attributes; $0 \le \omega_i \le 1, \sum_{i=1}^{L} \omega_i = 1$

iii) Define N distinctive assessment degrees H_n; $H = \{H_1, H_2, \ldots H_n \ldots H_N\}$

iv) A grade of belief β_n is determined for each attribute ε_i and assessment degree H_n;

Step 2: probability assignments for each attribute.

$m_{n,i}$ is a probability representing the degree that the ith attribute supports an assumption that the main attribute is evaluated to the nth evaluation assessment degree H_n.

$m_{H,i}$ is the probability unassigned to each attribute.

$$m_{n,i} = \omega_i \beta_{n,i} \tag{32}$$

$$m_{H,i} = 1 - \sum_{n=1}^{N} m_{n,i} = 1 - \omega_i \sum_{n=1}^{N} \beta_{n,i}, (m_{H,i} = \bar{m}_{H,i} + \tilde{m}_{H,i}) \tag{33}$$

$$\bar{m}_{H,i} = 1 - \omega_i \tag{34}$$

$$\tilde{m}_{H,i} = \omega_i (1 - \sum_{n=1}^{N} \beta_{n,i}) \tag{35}$$

Step 3: Incorporate probability assignments for the main attribute.

The probability masses assigned to the various assessment grades as well as the probability mass left unassigned levels are assumed [34] as follows:

$$m_{n,I(L)} \left(n = 1, \ldots, N \right) \ , \ \bar{m}_{H,I(L)} \ , \ \tilde{m}_{H,I(L)} \ , \ m_{H,I(L)}$$

$$I\left(1\right) = 1 : m_{n.I(1)} = m_{n,1} \left(n = 1, \ldots .N \right) \quad , \quad \bar{m}_{H,I(1)} = \bar{m}_{H,1} \quad , \quad \tilde{m}_{H,I(1)} = \tilde{m}_{H,1} \quad , \quad m_{H,I(1)} = m_{H,1}$$

The following recursive ER algorithm produces the incorporating probability;

$$\left\{ H_n \right\} : m_{n,I(i+1)} = K_{I(i+1)} \left[m_{n,I(i)} . m_{n,i+1} + m_{H,I(i)} . m_{n,i+1} + m_{n,I(i)} . m_{H,i+1} \right] \qquad n = 1, \ldots, N \tag{36}$$

$$\left\{ H \right\} : m_{H,I(i)} = \tilde{m}_{H,I(i)} + \bar{m}_{H,I(i)} \tag{37}$$

$$\tilde{m}_{H,I(i+1)} = K_{I(i+1)} \left[\tilde{m}_{H,I(i)} . \tilde{m}_{H,i+1} + \bar{m}_{H,I(i)} . \tilde{m}_{H,i+1} + \tilde{m}_{H,I(i)} . \bar{m}_{H,i+1} \right] \tag{38}$$

$$\bar{m}_{H,I(i+1)} = K_{I(i+1)} \left[\bar{m}_{H,I(i)} . \bar{m}_{H,i+1} \right] \tag{39}$$

$$K_{I(i+1)} = \left[1 - \sum_{t=1}^{N} \sum_{\substack{j=1 \\ j \neq t}}^{N} m_{t,I(i)} . m_{j,i+1} \right]^{-1} \tag{40}$$

$$i = \left\{ 1, 2, \ldots, L-1 \right\} \qquad \sum_{n=1}^{N} m_{n,I(i+1)} + m_{H,I(i+1)} = 1 \tag{41}$$

Step 4: Calculation of the incorporated grades of belief for the main attribute.

β_n indicates the incorporated grade of the belief that the main attribute evaluated as the grade H_n.

$$\left\{ H_n \right\} : \beta_n = \frac{m_{n,I(L)}}{1 - \bar{m}_{H,I(L)}} \qquad n = 1, 2, \ldots ., N \tag{42}$$

$$\left\{ H \right\} : \beta_H = \frac{\tilde{m}_{H,I(L)}}{1 - \bar{m}_{H,I(L)}} \tag{43}$$

Step 5: Computation of the expected utility for a complete evaluation.

$$u = \sum_{n=1}^{N} \beta_n u\left(H_n \right). \tag{44}$$

Step 6: Computation of the utility interval of incomplete evaluations.

$$u_{max} = \sum_{n=1}^{N-1} \beta_n u\left(H_n \right) + \left(\beta_N + \beta_H \right) u\left(H_N \right) \quad , \quad u_{min} = \left(\beta_1 + \beta_H \right) u\left(H_1 \right) + \sum_{n=2}^{N} \beta_n u\left(H_n \right) \tag{45}$$

$$u_{avg} = \frac{u_{max} + u_{min}}{2} \quad (46)$$

5. FINITE INTERVAL ARITHMETIC

The interval numbers can be introduced according to following:

$$A = \begin{cases} [a_L, a_R] = \{x : a_L \le x \le a_R, x \in R\} \\ < a_C, a_W >= \{x : a_C - a_W \le x \le a_C + a_W, x \in R\} \end{cases}$$

The left and right limits showed as a_L and a_R and, the center and radius of the intervals showed as $a_C=(a_L+a_R)/2$ and $a_W=(a_R-a_L)/2$: R is the set of real numbers.

Definition 1.

Four arithmetic operations are explained for two closed intervals A= [a_l, a_R] and B= [b_l, b_R]

$$For\ * \in (+, -, ., /)$$
$$A * B = \{a * b : a \in A\, and\, b \in B\}$$

In the case of division, it is assumed that $0 \notin B$.

Based on [35], all two intervals include one of the following three types:

Type-I: Both the intervals are disjoint.
Type-II: Intervals are partially overlapping.
Type-III: One interval is contained in the other.

In this case order relation cannot be executed, therefore Mahato and Bhunia [32] extended the order relations for maximization problems of optimistic and pessimistic decision maker's point of view [36], The following definitions are as below:

5.1. Pessimistic decision-making

According to the principle "Less uncertainty is better than more uncertainty" or "More uncertainty is worse than less uncertainty", the order relations is performed.

Definition 2. The order relation $>_{pmax}$ between two intervals

$A = [a_L, a_R] =< a_C, a_W >\quad and \quad B = [b_L, b_R] =< b_C, b_W >$ is introduced for maximization problems.

$$for\, type - I\, and\, type - II\, intervals \quad : \quad A >_{pmax} B\ and\ a_C > b_C$$

$$for\, type - III\, intervals : A >_{pmax} B\, and\ a_C \ge b_C\ and\ \ a_W < b_W$$

$for\, type - III\, intervals : a_C \ge b_C\ and\ \ a_W < b_W$ in this case the last optimistic may be consider.

5.2. Optimistic decision-making

In this case, the lowest cost/time for minimization problems and the highest profit for maximization problems ignoring the uncertainty are chosen by the decision maker.

Definition3. The order relation $\ge_{o\,max}$ between two intervals A and B is introduced for maximization problems.

$$a_R \geq b_R \quad \rightarrow \quad A \geq_{omax} B$$

$$A \geq_{omax} B \quad and \quad A \neq B \quad \rightarrow \quad A >_{omax} B$$

This order relation $\geq_{omax}$ is not symmetric but transitive.

6. SOLUTION PROCEDURE

Solving these problems by different gradient based methods is difficult hence the developed advanced genetic algorithm was selected for solving the mixed integer maximization problems (10) and (18) with interval objective.

GA algorithm:

1. Choose an initial population

If crossover condition is satisfied:

2. Select parent
3. Select crossover parameters
4. Perform crossover

If the mutation condition is satisfied:

5. Select chromosomes
6. Select mutation points
7. Perform mutation
8. Evaluate fitness of offspring
9. Till adequate offspring created
10. Choose new population

Till stopping standard is true

11. Present estimate population

GA is relied on some parameters like Population size (p_size), p-size should not be so large or too small, because storing large data may cause some difficulties in our computer and crossovers cannot do the best implementation in small p-size.

Vj=[Vj1, Vj2] represented the chromosome in GA, where the components Vj1 and Vj2 denote respectively the decision variables, Qr/Qs and n of the problem. To initialize the chromosomes, a random value can be selected from the discrete set of values within the bounds.

The next step is checking how good the chromosomes are. Hence, the interval value of the objective function is calculated as the fitness value corresponding to the chromosome Vj. Then the rank of interval value (fitness value) of chromosomes has been performed from the point of view of the pessimistic decision maker (Definition 3) for selecting process under the principle “survival of the fittest”.

Crossover and mutation operate on resulting chromosomes. P-cros*p-size number of chromosomes were selected as parents for crossover operation and generate offspring by recombining the features of both parents.

The first component of two offspring will be created by

$$V_{k1}^{'} = \lambda V_{k1} + (1-\lambda) V_{i1} \quad , \quad V_{i1}^{'} = \lambda V_{i1} + (1-\lambda) V_{k1} \quad , \quad \lambda \epsilon [0,1] \tag{47}$$

And for the second component

$$V_{k2}^{'} = V_{k2} - g \quad and \quad V_{i2}^{'} = V_{i2} + g \quad if \ V_{k2} > V_{i2} \quad , \quad g \epsilon \left[0, |V_{k2} - V_{i2}|\right] and\ integer \tag{48}$$

Mutation is executed to a single chromosome. It is usually fulfilled with low probability. Mutation changes single or all the genes of a randomly selected chromosome a little to shift the population gradually into a little better course

7. NUMERICAL EXAMPLE

The model is illustrated by the following ten examples which all of their values are real. These examples had been performed by the 6-step Dempster-Shafer algorithm. We considered four experts' points of view on holding and shortage cost. We defined seven distinctive evaluation grades H_n for an interest rate as shown in table 1. Table 2 shows the experts' ideas about the holding ~~cost~~ and shortage cost. These assessments are incomplete obviously (this means that they had no idea in some percentage). The belief degrees and probable masses that are calculated in second step~~s~~ in Dempster-Shafer algorithm are shown in Table 3 and 4.

(Please insert Table 1)

(Please insert Table 2)

(Please insert Table 3)

(Please insert Table 4)

Aggregated probability masses, aggregated belief degrees for holding cost and shortage cost were computed and at the end, the utility intervals are shown in Table 5. This algorithm gives one specific interval value for holding cost and shortage cost.

(Please insert Table 5)

To continue for solving in this procedure, the proposed GA algorithm is used, for each example, 10 independent runs have been performed, in each generation, the best value of total profit will be selected according to Definition 2 of interval order relations. So, the best-found values Z, Q, Qr, Qs, n, and Ap have been obtained and displayed in Table 6 and Table 7.

Also to have the best performance of GA, we have used designing of experiments by response surface methodology to decide about the GA parameters.

According to this, the following values of GA parameters are used:

For $case-1: Q_r \geq \dfrac{D\left(e^{\theta x}-1\right)}{\theta}$

P-size=140,P-cros=0.4,P-mute=0.4,m-gen=123.

For $case-2: Q_r < \dfrac{D\left(e^{\theta x}-1\right)}{\theta}$

P-size=140,P-cros=0.3,P-mute=0.3,m-gen=120.

(Please insert Table 6)

(Please insert Table 7)

8. SENSITIVITY ANALYSIS

Parameters have various effects on this model; these effects may be so significant or negligible and impress the optimal solution. Some of them increase and some of them decrease the optimal solution. The effects of every parameter are studied completely in this section and show how they influence the model. Table 8 shows these solutions completely in the numerical case and it is described particularly in the following paragraphs.

Example 1 is selected for this section; the effect of parameters upon optimal solution will be studied by varying them in various percentages. a sensitivity analysis was undertaken by increasing the parameters by 10, 50, 100% and decreasing the parameters to 10, 50, 90% by taking each one at a time, and keeping the remaining parameters at their original values. The measures of system sensitivity considered here are profit function (Z). Results of the sensitivity analysis are exhibited in Table 8.

The results show as the parameters D, H, P and I_c increase, the profit function (Z) increases and when these parameters decrease, the profit function (Z) decreases. Conversely, when C_h, C_p, C_o, I_d and x increase, the profit function (Z) decreases and when these parameters decrease, the profit function (Z) increases.

When parameters λ and C_s increase and parameter C_s decrease, there is no change in profit function value, because in numerical example pessimistic decision maker choose Z_1 (between Z_1 and Z_2) as the best profit (Definition 2). And in this case, we do not have any shortages in the model and parameter λ does not have any role in profit function. But, when parameter λ increase, the other situation happened. The pessimistic decision maker chooses Z_2 for his optimal solution and in this case profit function (Z) increases.

(Please insert Table 8)

9. MANEGERIAL IMPLICATION

In competitive world to which we are facing, many uncertain conditions, having a good concept about important decisions, plays a crucial role in prospering a business. This factor can be helpful for a manager (or any other decision maker) to be able to select the best way to manage their stocks as well, in order to achieve maximum profit even in shortage of time. Our proposed model can help decision makers effectively in this way and since we have a pessimistic point of view it is very similar in the real world. For instance, an organization could improve its current supply chain processes by applying this model. As most of them (either food or other industries), have deteriorated items and they want to keep and satisfy their customers at the right time too, it is a useful way.

10. CONCLUSION

This paper has developed an inventory model based on Deterioration, advance payment and also interpolation of Dempster-Shafer theory. As the main problem for a decision-maker, finding the best and suitable membership functions, probable distributions and specific interval values, the algorithm aggregates different decisions and give us the interval utility for incomplete assessments. We solved our model with these interval utility costs by the proposed developed GA algorithm and found the best profit rate. The demand rate has been considered uniformly and assumed decreases by a certain fraction at the time of stock-out. In shortage times, inaccessibility of goods make causes some customers to refer to others. In this case, we use pessimistic decision-making point of view for selecting better profit function in ~~this~~ our model and as a result in real world, for selecting better chromosomes in each generation in advanced GA algorithm.

In the end, the case of ameliorating items, multi-storage facilities, and time-dependent demand are the most important suggestions that can be made to further this research.

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Figures & Tables

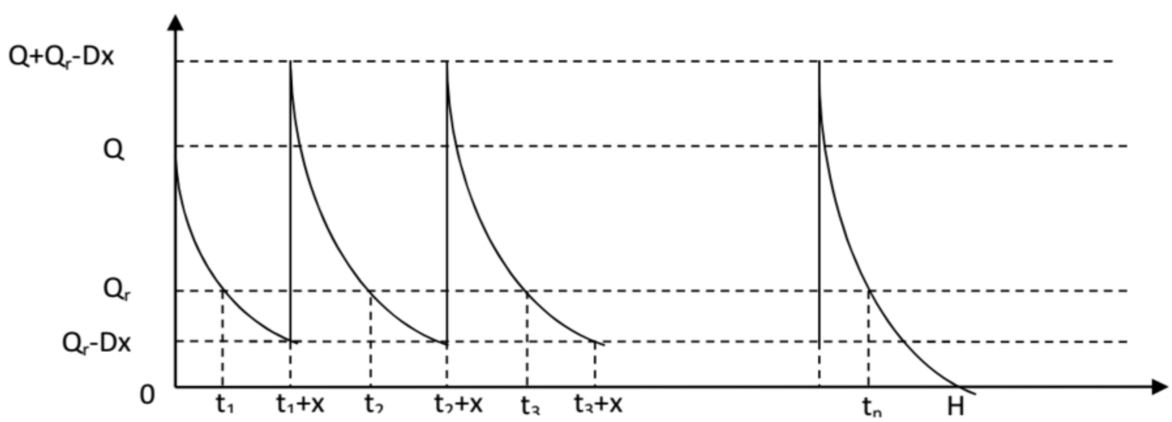

Figure 1.

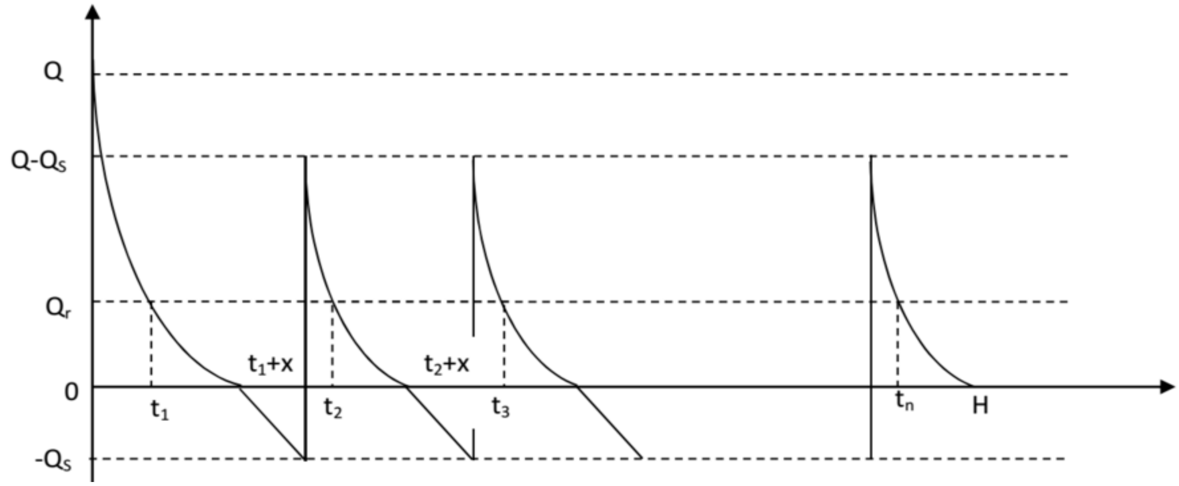

Figure 2.

Hn	1	2	3	4	5	6	7	8	9	10
Cs	2	4	6	8	10	12	14	16	18	20
Ch	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

Table 1.

	DM1	DM2	DM3	DM4
Wi	0.3	0.25	0.2	0.25
Cs	6/40% 8/10% 10/30%	6/30% 8/10% 10/60%	6/40% 8/10% 10/40%	6/30% 8/40% 10/30%
Ch	0.5/60% 0.7/30% 1/0%	0.5/70% 0.7/20% 1/0%	0.5/80% 0.7/10% 110%	0.5/55% 0.7/40% 1/5%

Table 2.

Evaluation Grade											
$H_1, H_2, ., H_{13}$	weight	Belief				Probability Mass					
	W_i	$\beta_{3,i}$	$\beta_{7,i}$	$\beta_{11,i}$	β_H	$m_{3,i}$	$m_{7,i}$	$m_{11,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$
Cs		0.357025	0.164458	0.406174	0.072341						
DM_1	0.3	0.4	0.1	0.3	0.2	0.12	0.03	0.09	0.76	0.7	0.06
DM_2	0.25	0.3	0.1	0.6	0	0.075	0.025	0.15	0.75	0.75	0
DM_3	0.2	0.4	0.1	0.4	0.1	0.08	0.02	0.08	0.82	0.8	0.02
DM_4	0.25	0.3	0.4	0.3	0	0.075	0.1	0.075	0.75	0.75	0

Table 3.

Evaluation Grade											
$H_1,H_2,.,H_{13}$	weight	Belief				Probability Mass					
	W_i	$\beta_{3,i}$	$\beta_{7,i}$	$\beta_{11,i}$	β_H	$m_{3,i}$	$m_{7,i}$	$m_{11,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$
C_h		0.699196	0.232619	0.024069	0.044113						
DM_1	0.3	0.6	0.3	0	0.1	0.18	0.09	0	0.73	0.7	0.03
DM_2	0.25	0.7	0.2	0	0.1	0.175	0.05	0	0.775	0.75	0.025
DM_3	0.2	0.8	0.1	0.1	0	0.16	0.02	0.02	0.8	0.8	0
DM_4	0.25	0.55	0.4	0.05	0	0.1375	0.1	0.0125	0.75	0.75	0

Table 4.

Umax	8.242981481	Umax	0.519250594
Umin	7.953616495	Umin	0.536896154

Table 5.

Parameters	Values for examples									
	1	2	3	4	5	6	7	8	9	10
C_p	14	14	14	14	14	14	14	14	14	14
C_o	23	23	23	23	23	23	23	23	23	23
P	21	21	21	21	21	21	21	21	21	21
I_d	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
I_b	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
I_c	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.5	0.5
x	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
λ	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
D	450	450	450	450	450	450	450	450	450	450
H	20	20	20	20	20	20	20	20	20	20
C_h	[0.52, 0.54]	[0.7,0.8]	[0.52,0.54]	[0.7,0.8]	[0.3,0.4]	[0.52,0.54]	[0.3,0.4]	[0.3,0.4]	[0.7,0.8]	[0.52,0.54]
C_s	[7.954,8.243]	[7.954,8.243]	[8.9,9.2]	[8.9,9.2]	[7.954,8.243]	[6.1,7.4]	[6.1,7.4]	[8.9,9.2]	[6.1,7.4]	[7.5,8.5]
θ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table 6.

Examples	Q	Qr	n	Ap	Z1
1	220.048	191.223	41	739.3628	[85221,85265]
2	176.817	189.155	51	594.1053	[84717,84894]
3	220.048	190.863	41	739.3628	[85221,85265]
4	180.36	188.654	50	606.0112	[84716,84896]
5	291.261	193.739	31	948.6374	[85552,85842]
6	220.048	191.391	41	739.3628	[85221,85265]
7	282.131	193.872	32	947.9593	[85555,85837]
8	291.261	193.647	31	978.6374	[85552,85842]
9	180.36	187.844	50	606.0112	[84716,84896]
10	220.048	189.605	41	739.3628	[85089,85418]

Table 7.

	2	1.5	1.1	1	0.9	0.5	0.1
C_p	24	18	15.4	14	10.8	6	1.2
	Z1=[53472.6004, 54032.6004]	Z1=[96603.4861, 97075.9861]	Z1=[13113.5800, 13148.2300]	Z1=[85221, 85265]	Z1=[14841.1946, 14871.4346]	Z1=[18293.8240, 18311.3240]	Z1=[21760.1352, 21763.4952]
C_o	70	52.5	38.5	35	31.5	17.5	3.5
	Z1=[13849.4646, 13877.4646]	Z1=[13906.8968, 13938.3968]	Z1=[13943.2915, 13978.2915]	Z1=[13976.3261, 14009.9261]	Z1=[14027.4558, 13993.8558]	Z1=[14104.1987, 14068.19879]	Z1=[14189.1125, 14228.6420]
P	46	34.5	25.3	23	20.7	11.5	2.3
	Z1=[36964.3279,36999.3279]	Z1=[25464.2785,25499.2785]	Z1=[16276.3563,16309.9563]	Z1=[13976.3261, 14009.9261]	Z1=[11676.3589,11709.9589]	Z1=[24763.5768, 25099.5768]	-
I_d	1	0.75	0.55	0.5	0.45	0.25	0.05
	Z1=[13747.2658,13803.2658]	Z1=[13860.3486,13907.5986]	Z1=[13951.4751, 13988.4351]	Z1=[13976.3261, 14009.9261]	Z1=[14001.2238, 14031.4638]	Z1=[14093.8237,14111.3237]	Z1=[14200.1347, 14203.4947]
I_c	0.6	0.45	0.33	0.3	0.27	0.15	0.03
	Z1=[17676.7971,17694.7971]	Z1=[15829.6336,15856.0336]	Z1=[14346.9951,14379.1551]	Z1=[13976.3261, 14009.9261]	Z1=[13606.9086,13639.7586]	Z1=[12123.0680,12163.8680]	Z1=[10646.9875, 10690.6375]
x	1	0.75	0.55	0.5	0.45	0.25	0.05

C_s		C_h		H		D		λ		
16	Z1=[13802.8749,13838.8749]	0.4	Z1=[27890.5293,27966.1293]	20	Z1=[28138.9148,28208.9148]	200	Z2=[26159.6199,26330.6151]	1.6	Z1=[13744.0882,13807.0882]	2
12	Z1=[13880.9944,13915.9944]	0.3	Z1=[20933.9640,20987.9640]	15	Z1=[21051.6511,21104.1511]	150	Z2=[20968.4341,21253.5334]	1.2	Z1=[13860.4956,13907.7456]	1.5
8.8	Z1=[13951.8969,13983.3969]	0.22	Z1=[15361.0767,15395.7267]	11	Z1=[15391.5272,15428.4872]	110	Z1=[13976.3592,14009.9592]	0.88	Z1=[13953.6168,13988.2668]	1.1
8	Z1=[13976.3261, 14009.9261]	0.2	Z1=[13976.3261, 14009.9261]	10	Z1=[13976.3261, 14009.9261]	100	Z1=[13976.3261, 14009.9261]	0.8	Z1=[13976.3261, 14009.9261]	1
7.2	Z1=[14001.8999,14033.3999]	0.18	Z1=[12587.7064,12616.0564]	9	Z1=[12565.1711,12593.5211]	90	Z1=[13976.3558,14009.9558]	0.72	Z1=[14000.1745,14028.5245]	0.9
4	Z1=[14121.1329,14149.1329]	0.1	Z1=[7024.2119,7034.7066]	5	Z1=[69245.3103,69385.3103]	50	Z1=[13976.3306,14009.9306]	0.4	Z1=[14093.5825,14111.0825]	0.5
0.8	Z1=[14324.5999,14345.5999]	0.02	Z1=[1369.4491,1371.5491]	1	Z2=Z1=[1325,1325]	10	Z1=[13976.8944,14008.3944]	0.08	Z1=[14199.7597,14203.1197]	0.1

	2	1.5	1.1	1	0.9	0.5	0.1
	Z1=[13976.3392,14009.9392]	Z1=[13976.3127,14009.9127]	Z1=[13976.3458,14009.9458]	Z1=[13976.3261, 14009.9261]	Z1=[13976.8960,14008.3960]	Z1=[13976.3569,14009.9569]	Z1=[13976.8988,14008.3988]

Table 8.

Biographies

Motahareh Soleimani Amiri graduated in MSc in Industrial Engineering from Kharazmi University in 2014. Also, she received her BSc degree in Industrial Engineering from Al-Zahra University in 2011. She is the project manager in the electrical industry. Her research interests include Evidence theory, Inventory control, and Genetic algorithm.

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