



# Evaluation of vehicle braking parameters by multiple regression method

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## KEYWORDS

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 Friction coefficient;  
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 Multiple regression method.

**Abstract.** This study uses two pairs of OEM brake disc-pads. One of these discs belongs to a passenger car and the other one to a light commercial vehicle. The disc-pad pair of the passenger car is subject to the global brake effectiveness test by a full-scale inertia dynamometer according to SAE J2522 test standard; the other one is subject to the tests by a full-scale inertia dynamometer according to Fiat 7-H4020 and 7-H2000 standards. During these tests, 13 variables for the passenger car disc-pad pair and 11 variables for light commercial vehicle disc-pad pair were measured and recorded. The interrelation of the parameters was analyzed by the multiple regression method, and importance levels were determined. In this study, dependent variables in the multiple regression method including braking time, friction coefficient, disc final temperature, brake speed, and brake pressure were selected for each braking pair. In multiple regression analysis concerning the passenger car, for each unit of increase in deceleration and friction coefficient, braking time decreases with 7.3 and 60.9 units, respectively. Moreover, for each unit of increase in brake pressure and friction coefficient for the light commercial vehicle, braking time increases with 1.267 and 91.887 units, respectively.

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## 1. Introduction

A braking system is one of the most important safety systems of a vehicle [1]. It enables controlled dissipation of energy to slow down, stop, or control the speed of a vehicle [2-4]. The kinetic energy of the vehicle is converted into mechanical energy, while braking leads to heat dissipation and temperature rise of the disc and pads pairs [1]. Friction disc brakes are commonly used in vehicles as wheel brakes. With braking, the temperature of disc-pad interface and a change in speed have

effect on friction coefficient ( $\mu$ ).  $\mu$  drops with increasing temperature and sliding speed; therefore, brake fade (brake fading) and speed fade take place [2]. Brake fade is an occurrence that takes place in every vehicle with a friction brake system when  $\mu$  significantly drops based on the temperature [3,5,6]. High temperature values during braking cause brake fade, premature wear, brake fluid vaporization, bearing failure, thermal cracking, and thermally-excited vibrations [7]. Ideal brake pads must ensure uniform and stable friction in every working condition without developing brake fade at any temperature. Brake friction materials comprise more than 10 ingredients required to achieve the desirable braking performance including stable friction coefficients, low wear, and low noise generation in a wide range of braking conditions [8]. They are classified as organic, semi-metallic, metallic, synthetic,

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and ceramic [9]. In another study, they were classified into semi-metallic, low-steel low-metallic, no-steel low-metallic (also known as non-asbestos organic-NAO), and European metallic depending on ferrous and non-ferrous metal content [10]. It was shown that  $\mu$  value for braking ranged 0.30-0.35 for automobiles [6], 0.45 for automobiles, over 0.50 for sports cars, and around 0.35 for rail vehicles [11]. Another study detected that a couple of ventilated discs based on gray cast iron and semi-metallic pad had  $\mu$  values of 0.246 and 0.412 between the temperatures of 98°C and 632°C [3].

Braking performance can be defined as the ability of a vehicle to stop as soon as possible while maintaining its driving stability [12]. It is basically evaluated according to braking distance, braking torque, or braking efficiency (Carlos & Ferro, 2005). For this purpose, dynamometers are frequently used. Performance, durability, and noise tests are the most common tests for dynamometers. Most of the inertia dynamometer procedures (SAE, JASO, ISO, AK, FMVSS, and JIS) used by OEMs, pad suppliers, and component manufacturers are carried out with single-ended dynamometers [13].

Deterministic mathematical models that are devoted to analyzing braking parameters of vehicles are frequently used [14]. However, the values of deceleration and braking distance parameters are random values in practice. A number of studies have investigated the results of the evaluation of vehicle braking parameters, which are considered as random values with known possibility properties [15].

Tribological conditions of the braking components during operation have a dominant effect on brake fade [6]. Tribological analysis is one of the most important mechanical fields in the industry. The tribological properties of two contact surfaces of engines and machines generally depend on factors such as load, speed, temperature, sliding time, lubricant, and additive formulation [16]. Individual mechanisms of friction are dependent upon *temperature, normal load, and sliding velocity*; thus, it seems reasonable to assume that  $\mu$  is dependent upon these parameters [17]. In many studies [17-21],  $\mu$  is found to be dependent on *temperature*. In many studies [17,21-24],  $\mu$  is found to be dependent on *braking force and velocity*. In most of these studies,  $\mu$  shows a decreasing trend with increasing velocity, while it shows a mixed trend with the increasing load [25].

During the last decades, ever more sophisticated models have been developed. The techniques in the literature include static and dynamic models, neural networks, and state observers. The most commonly used model is the static model [26]. An *analytical formulation* considering only the friction dependence on *the speed* was proposed [27]. A very simple analytical formulation based on steady-state experimental

tests that correlate *pressure, speed, and temperature* dependences to *friction and wear* was assessed [28]. An *alternative formulation* was put forth that, in addition to *the sliding speed*, involved thermal effects due to an increase in the temperature of the friction materials [29].

In this study, two pairs of an original brake disc-pad are used. One of these discs belongs to a Passenger Car (PC), and the other one belongs to a Light Commercial Vehicle (LCV). The PC disc-pad pair is subject to the global brake effectiveness test by a full-scale inertia dynamometer according to SAE J2522 test standard; the other one is subject to the tests by the full-scale inertia dynamometer according to FIAT 7-H4020 and 7-H2000 standards. For the PC disc-pad pair tests, 276 braking tests from 21 different categories were performed including features such as green effectiveness, speed sensitivity, fade resistance, friction recovery, and friction stability; for the LCV disc-pad pair tests, a total of 130 braking tests were performed in 2 different categories including burnish and hot judder I-II procedures. During these tests, braking parameters such as the number of brakes performed, cycle time, brake speed, release speed, braking duration, deceleration, braking torque (min, avg, max), pad actuator pressure (min, avg, max), friction coefficient (min, avg, max), and initial and final temperatures (disc, inner and outer pads) were measured and recorded. By using all the data collected from these tests, braking parameters were evaluated according to a multiple regression method. This study is divided into four parts. The first section includes an introduction and a short literature abstract. The second section includes the selection of the discs, braking test unit, test standards, multiple regression model, and dependent and independent variables examined by the model. The third section includes the equations of dependent braking parameters estimated according to multiple regression method such as braking time, friction coefficient, disc final temperature, brake speed, and braking pressure. Conclusions are presented in the final section.

## 2. Materials and method





### 2.1. Selection of the discs

In this study, two pairs of front ventilated disc-pad pairs are used. Basic characteristics and material components of the disc-pad pairs are shown in Table 1.

### 2.2. Testing standard and brake test mechanism

SAE-recommended practice defines an inertia-dynamometer test procedure to assess the effective behavior of friction material with respect to pressure, temperature, and speed for motor vehicles fitting with hydraulic brake actuation [30,31]. The efficiency

**Table 1.** Properties of the disc and pad.

	PC		LCV	
	Disc	Pad	Disc	Pad
Thickness (mm)	22	12	21.805	17.663
Disc diameter (mm)	255	–	257	–
Mass (g)	5,004	292.2	5,328.73	335.43
Disc and pad materials	3.58C2.28Si0.572Mn 0.02P	PN529H-FF NAC	83.2Fe4.9Si0.4Mn2. 8P0.9S0.2Cr1.8Al2. 4Cu2.5Zn0.1Sn0.2Zr	11.6Fe4Si2.7P2S11Cr 31.3Ni6Sn7.3Zr0.3Bi 8.9W13.8Hf0.6Ti
				

of brake systems is measured by braking distance depending on vehicle speed or deceleration and running-up time. This efficiency can be determined by various experiments. Certain parameters are required for brake disc characteristics. SAE J2522 is a universal effectiveness test that is useful only when target friction levels in specific sections or a baseline material is available for comparison. The SAE J2522 has become the baseline for several test versions with cold temperature, wet effectiveness, parking brake evaluation, and ramp applications. It is a useful friction behavior evaluation regarding green effectiveness, speed sensitivity, fade resistance, friction recovery, and friction stability [30]. PC disc-pad pair is subject to the J2522 global brake effectiveness test of SAE with a full-scale inertia dynamometer (Table 2). In tests, such parameters as braking number, cycle time, brake speed, release speed, stop time, deceleration, braking torque, pad actuator pressure, coefficient of friction, rotor, and input/output temperature were saved for 276-braking. While the total test time of disc was 41,400 s (11.5 h), averagely, 1,770 seconds ( $\sim 0.5$  h) of this time was determined as the effective braking time. The test of the LCV disc-pad pair was carried out by a full-scale inertia dynamometer according to FIAT 7-H4020 and/or 7H2000 standard, as shown in Table 3. The braking tests were performed using an inertia dynamometer with a maximum resolution rate of 2400 rpm, a maximum moment of 166 kgm<sup>2</sup>, a maximum power rate of 140 kW, and a maximum torque rate of 5000 Nm. The temperature of the rubbing interface was measured using K-type thermocouple on the disc surface. The inertia dynamometers used in the tests are shown in Appendix – Figure A.1; their technical specifications and measurement parameters are provided in Tables A.1 and A.2.

### 2.3. Regression analysis

Detecting if there is a relation between two variables and, if so, determining the degree of this relation is a common problem in statistical analysis. Regression is the first technique that comes to mind when analyzing the relation between variables. Regression analysis is to explain the relations between a dependent variable and an independent variable (simple regression) or more than one independent variable (multiple regression) by a mathematical equation. In regression analysis, the relationship between independent variables  $X_i$  and dependent variables  $Y_i$  is expressed as a mathematical function. For example, if a linear relationship such as  $Y_i = +X_i + \varepsilon_i$  ( $i = 1, 2, 3, \dots, n$ ) is foreseen between  $Y$  and  $X$ , the first step is to predict the unknown parameters of the model. When the unknown parameters of the model are predicted, to predict the value of the dependent variable for different values of the independent variables is another objective of regression analysis. In the multiple linear regression model,  $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_p X_{ip} + \varepsilon_i$  ( $i = 1, 2, \dots, n$ ) can be written for  $p$  number of explanatory variables and  $n$  number of observations [33,34]. In order to make reliable parameter predictions for the regression model that will be obtained after the results of both simple and multiple linear regression analysis, some of the assumptions about the model must be ensured.

### Dependent and Independent Variables (DV and IV) in multiple regression method

In this study, braking time, friction coefficient, disc final temperature, brake speed, and brake pressure for each braking pair are examined with the multiple regression method. In the analysis of PC disc-pad pair, 13 variables are used as friction coefficient ( $\mu_{avg}$ ),

**Table 2.** SAE J2522 dynamometer global brake effectiveness [30].

Section	Number of stops/snubs	Braking release speed (km/h)	Control	Initial brake temperatures (°C)
Green effectiveness	30	80-30	30 bar	100
Burnish (or bedding)	64	80-30	various pressures	100
Characteristic check I	6	80-30	30 bar	100
Speed/pressure sensitivity I	8	40-<5	10, 20, ... 80 bar	100
Speed/pressure sensitivity II	8	80-40	10, 20, ... 80 bar	100
Speed/pressure sensitivity III	8	120-80	10, 20, ... 80 bar	100
Speed/pressure sensitivity IV	8	160-130	10, 20, ... 80 bar	100
Speed/pressure sensitivity V	8	200-170	10, 20, ... 80 bar	100
Characteristic check II	6	80-30	30 bar	100
Cold braking check	1	40-<5	30 bar	40
Motorway braking check I	1	100-5	0.6 g	50
Motorway braking check II	1	180-100	0.6 g	50
Characteristic check III	18	80-30	30 bar	100
1st fade (maximum 160 bar)	15	100-<5	0.4 g	100-500
Recovery	18	80-30	30 bar	100
Pressure sensitivity (100°C)	8	80-30	10, 20, ... 80 bar	100
Increasing temperature sensitivity (500°C)	9	80-30	30 bar	100, 150, ... 500
Pressure sensitivity (500°C)	8	80-30	10, 20,... 80 bar	500
Characteristic check IV	18	80-30	30 bar	100
2nd fade (maximum 160 bar)	15	100-<5	0.4 g	100-500
Characteristic check V	18	80-30	30 bar	100

**Table 3.** Test parameters of a light commercial vehicle disc-pad pair [32].

Section	Number of brakes	Initial speed (km/h)	Release speed (km/h)	Initial brake temperatures (°C)	Deceleration (m/s <sup>2</sup> )
Burnish	100	80	0	100	–
Hot Judder-I	15	152	0	100	1, 2, and 3
Hot Judder-II	15	152	0	300	1, 2, and 3

braking time ( $t$ ), braking torque ( $M_{avg}$ ), brake pressure ( $p_{avg}$ ), brake speed ( $\nu_{BS}$ ), brake release speeds ( $\nu_{BR}$ ), deceleration ( $a$ ), disc initial and final temperatures ( $T_{DI}$  and  $T_{DF}$ ), inpad initial and final temperatures ( $T_{IPI}$  and  $T_{IPF}$ ), outpad initial and final temperatures ( $T_{OPI}$  and  $T_{OPF}$ ); for *LCV* disc-pad pair, 11 variables are used as friction coefficients ( $\mu_{avg}$ ), braking time ( $t$ ), braking torque ( $M_{avg}$ ), brake pressure ( $p_{avg}$ ), brake speed ( $\nu_{BS}$ ), brake release speeds ( $\nu_{BR}$ ), deceleration ( $a$ ), inertia ( $I$ ), fluid absorption (FB), and disc initial and final temperatures ( $T_{DI}$  and  $T_{DF}$ ). Dependent

variables in the multiple regression method such as braking time, friction coefficient, disc final temperature, brake speed, and brake pressure are selected for each braking pair. The reasons for choosing dependent variables are given in Table 4.

### 3. Results and discussion

In this study, two pairs of OEM disc-pad were used. The PC disc-pad pair was subjected to the global brake effectiveness test by a full-scale inertia dynamometer

**Table 4.** Dependent variables and reasons for selection.

DV	Reasons
Braking time	It is a key indicator of braking performance.
Friction coefficient	Many studies in the literature are based on friction coefficient. The friction coefficient directly affects both the stopping distance and the temperature of the brake elements.
Disc final temperature	It is a factor that can directly affect braking performance.
Brake speed	It is one of the basic parameters affecting friction coefficient.
Brake pressure*	It has been considered as a braking parameter in some studies.

\*Braking force has been considered in some studies/papers.

**Table 5(a).** Summary of multiple regression analysis for PC.

DV	O	R square (p<0.05)	IV	Equation(s)	The highest effect
Braking time	276	0.917	10	$t = 29.16 + 0.59v_{BS} - 0.06v_{BR} - 7.307a + 0.057M_{avg} - 0.454p_{avg} - 60.868\mu + 0.012T_{DI} - 0.012T_{DF} - 0.182T_{OPI} + 0.181T_{OPF}$	$\mu(1), a(1)$
Friction coefficient	276	0.962	11	$\mu = 0.338 + 0.0004v_{BS} + 0.0003v_{BR} - 0.004t + 0.0005M_{avg} - 0.008p_{avg} + 0.0004T_{DI} - 0.0001T_{DF} - 0.0009T_{IPI} + 0.0004T_{IPF} + 0.001T_{OPI} + 0.0009T_{OPF}$	$p_{avg}(1), t(1)$
Disc final temperature	276	0.990	11	$T_{DF} = 149.972 + 1.549v_{BS} - 0.602v_{BR} - 4.105t + 0.260M_{avg} - 4.011p_{avg} - 514.2\mu + 0.935T_{DI} + 4.699T_{IPI} - 3.776T_{IPF} - 7.038T_{OPI} + 6.09T_{OPF}$	$\mu(1), T_{OPI}(1)$
Brake speed	276	0.973	11	$v_{BS} = -78.11 + 0.730v_{BR} + 2.199t + 35.01a - 0.264M_{avg} + 1.914p_{avg} + 225.76\mu - 0.255T_{DI} + 0.167T_{DF} - 1.724T_{IPI} + 1.275T_{IPF} + 0.473T_{OPI}$	$\mu(1), a(1)$
Brake pressure	276	0.997	9	$p_{avg} = 38.016 + 0.0517v_{BS} - 0.034v_{BR} - 0.452t + 0.062M_{avg} - 112.26\mu + 0.034T_{DI} - 0.0117T_{DF} + 0.102T_{OPI} - 0.081T_{OPF}$	$\mu(1), t(1)$

**Table 5(b).** Summary of multiple regression analysis for LCV.

DV	O	R square (p<0.05)	IV	Equation(s)	The highest effect
Braking time	130	0.964	8	$t = -32.688 + 0.395I + 0.107v_{BS} + 1.267p_{avg} - 0.148M_{avg} + 0.204a + 0.003FA + 91.887\mu - 0.012T_{DI}$	$\mu(1), p_{avg}(1)$
Friction coefficient	130	0.984	8	$\mu = 0.289 - 0.00099I + 0.0044t + 0.0012M_{avg} - 0.016p_{avg} - 0.0011a - 0.00001FA + 0.0005T_{DI} - 0.00045T_{DF}$	$p_{avg}(1)$
Disc final temperature	130	0.989	8	$T_{DF} = -119.027 + 2.051I + 1.651v_{BS} + 0.423M_{avg} - 5.219p_{avg} - 0.951a + 0.0195FA - 388.18\mu + 0.865T_{DI}$	$\mu(1), p_{avg}(1)$
Brake speed	130	0.977	7	$v_{BS} = 130.12 - 1.276I - 2.392v_{BR} + 0.977t + 0.190a - 0.0198FA - 0.266T_{DI} - 0.362T_{DF}$	$v_{BR}(1)$
Brake pressure	130	0.986	8	$p_{avg} = 16.186 - 0.0925I + 0.192t + 0.0656M_{avg} - 0.0543a + 0.001FA - 50.75\mu + 0.022T_{DI} - 0.019T_{DF}$	$\mu(1)$

according to SAE J2522 test standard; the other pair was subjected to the tests by a full-scale inertia dynamometer according to FIAT 7-H4020 and 7-H2000 standards. For the PC disc-pad pair tests, 276 braking tests from 21 different categories were performed; for the LCV tests, a total of 130 braking cases in 2 different categories consisting of burnishing and judder I-II

procedures were performed. During these tests, 13 variables for the PC disc-pad pair and 11 variables for the LCV disc-pad pair were measured and recorded. The interrelation of these parameters was analyzed with the multiple regression method, and effects of the explanatory variable(s) were determined. A summary of all regression analyses is given in Tables 5(a) and 5(b).

### 3.1. Evaluating braking time

The correlation coefficients between Independent Variables (IV) and braking time (1) of the PC disc-pad pair are provided in Table A.3. The multiple regression output, prepared using Excel, is given in Table A.4. The  $R$  square for the braking time regression is 0.917, or 91.7% ( $R$  square,  $R^2$ , varies from 0 to 1, and higher values indicate a better regression). One popular interpretation is that  $R^2$  is the percent explained variability [33]. This means that 91.7% of the change in the dependent variable is explained by the independent variables in the model. In simple words, the model is 91.7% good). Each predictor has a coefficient, its standard error, a  $t$ -ratio, and the corresponding  $P$ -value. Ten of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $\nu_{BS}$ ,  $\nu_{BR}$ ,  $a$ ,  $M_{avg}$ ,  $p_{avg}$ ,  $\mu_{avg}$ ,  $T_{DI}$ ,  $T_{DF}$ ,  $T_{OPI}$ , and  $T_{OPF}$ ) is a significant predictor of the braking time. The constant term (intercept) in the regression equation (1) is 29.16. For each unit of increase in  $\nu_{BS}$ ,  $M_{avg}$ ,  $T_{DI}$ , and  $T_{OPF}$ , the braking time is predicted to increase by 0.059, 0.057, 0.012, and 0.181 units, respectively. For each unit of increase in  $\nu_{BR}$ ,  $a$ ,  $p_{avg}$ ,  $\mu$ ,  $T_{DF}$ , and  $T_{OPI}$ , braking time decreases with 0.06, 7.307, 0.454, 60.868, 0.012, and 0.182 units, respectively. The other independent variables,  $T_{IPI}$  and  $T_{IPF}$ , are not statistically significant (Table A.4).

The estimated regression equation for braking time of the PC is:

$$t = 29.16 + 0.59\nu_{BS} - 0.06\nu_{BR} - 7.307a + 0.057M_{avg} - 0.454p_{avg} - 60.868\mu + 0.012T_{DI} - 0.012T_{DF} - 0.182T_{OPI} + 0.181T_{OPF}. \quad (1)$$

The correlation coefficients and the multiple regression output for the LCV disc-pad pair are given in Tables A.5 and A.6.  $R$  square equals 0.964, which is a very good fit. Then, 96.4% of the variations in braking time are explained by the independent variables. Eight of the coefficients in the regression statistics have  $P$ -values less than 5%. The constant term is -32.688. For each unit of increase in  $I$ ,  $\nu_{BS}$ ,  $p_{avg}$ ,  $a$ ,  $FA$ , and  $\mu_{avg}$  in Eq. (2), the braking time is predicted to increase by 0.395, 0.107, 1.267, 0.204, 0.003, and 91.887 units, respectively. For each unit of increase in  $M_{avg}$  and  $T_{DI}$ , braking time decreases with 0.148 and 0.024 units, respectively. The other independent variables,  $\nu_{BR}$  and  $T_{DF}$ , are not statistically significant (Table A.6).

The estimated regression equation for the braking time of the LCV is as follows:

$$t = -32.688 + 0.395I + 0.107\nu_{BS} + 1.267p_{avg} - 0.148M_{avg} + 0.204a + 0.003FA + 91.887\mu - 0.012T_{DI}. \quad (2)$$

### 3.2. Evaluating friction coefficient

The correlation coefficients and the multiple regression output for the PC disc-pad pair are given in Tables A.7 and A.8.  $R$  square equals 0.962, and 96.2% of the variations in  $\mu_{avg}$  are explained by the independent variables. Eleven of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $\nu_{BS}$ ,  $\nu_{BR}$ ,  $t$ ,  $M_{avg}$ ,  $p_{avg}$ ,  $T_{DI}$ ,  $T_{IPF}$ ,  $T_{IPI}$ ,  $T_{DF}$ ,  $T_{OPI}$ , and  $T_{OPF}$ ) is a significant predictor of the friction coefficient. The constant term is 0.338. For each unit of increase in  $\nu_{BS}$ ,  $M_{avg}$ ,  $T_{DI}$ ,  $T_{IPF}$ , and  $T_{OPI}$  in Eq. (3), the friction coefficient is predicted to increase by 0.0004, 0.0005, 0.0004, 0.0004, and 0.001 units, respectively. For each unit of increase in  $\nu_{BR}$ ,  $t$ ,  $p_{avg}$ ,  $T_{DF}$ ,  $T_{IPI}$ , and  $T_{OPF}$ , friction coefficient decreases with 0.0003, 0.004, 0.008, 0.0001, 0.0009, and 0.0009 units, respectively. The other independent variable,  $a$ , is not statistically significant (Table A.8).

The estimated regression equation for  $\mu_{avg}$  of the PC is as follows:

$$\begin{aligned} \mu = & 0.338 + 0.0004\nu_{BS} + 0.0003\nu_{BR} - 0.004t \\ & + 0.0005M_{avg} - 0.008p_{avg} + 0.0004T_{DI} \\ & - 0.0001T_{DF} - 0.0009T_{IPI} + 0.0004T_{IPF} \\ & + 0.001T_{OPI} + 0.0009T_{OPF}. \end{aligned} \quad (3)$$

The correlation coefficients and the multiple regression output for the LCV disc-pad pair are given in Tables A.9 and A.10.  $R$  square equals 0.984, and 98.4% of the variations in  $\mu_{avg}$  are explained by the independent variables. Eight of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $I$ ,  $t$ ,  $p_{avg}$ ,  $M_{avg}$ ,  $a$ ,  $FA$ ,  $T_{DI}$ , and  $T_{DF}$ ) is a significant predictor of the friction coefficient. The constant term is 0.289. For each unit of increase in  $t$ ,  $M_{avg}$ , and  $T_{DI}$  in Eq. (4), the friction coefficient is predicted to increase by 0.004, 0.0012, and 0.0005 units, respectively. For each unit of increase in  $I$ ,  $p_{avg}$ ,  $a$ ,  $FA$ , and  $T_{DF}$ , friction coefficient decreases with 0.00099, 0.016, 0.0011, 0.00001, and 0.00045 units, respectively. The other independent variables  $\nu_{BS}$  and  $\nu_{BR}$  are not statistically significant (Table A.10).

The estimated regression equation for  $\mu_{avg}$  of the LCV is as follows:

$$\begin{aligned} \mu = & 0.289 - 0.00099I + 0.0044t + 0.0012M_{avg} \\ & - 0.016p_{avg} - 0.0011a - 0.00001FA \\ & + 0.0005T_{DI} - 0.00045T_{DF}. \end{aligned} \quad (4)$$

### 3.3. Evaluating disc final temperature

The correlation coefficients and the multiple regression

output of the PC disc-pad pair are given in Tables A.11 and A.12.  $R$  square equals 0.990, and 99% of the variations in  $T_{DF}$  are explained by the independent variables. Eleven of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $\nu_{BS}$ ,  $\nu_{BR}$ ,  $t$ ,  $M_{avg}$ ,  $p_{avg}$ ,  $\mu_{avg}$ ,  $T_{DI}$ ,  $T_{IPF}$ ,  $T_{IPI}$ ,  $T_{OPI}$ , and  $T_{OPF}$ ) is a significant predictor of  $T_{DF}$ . The constant term is 149.972. For each unit of increase in  $\nu_{BS}$ ,  $M_{avg}$ ,  $T_{DI}$ ,  $T_{IPI}$ , and  $T_{OPF}$  in Eq. (5),  $T_{DF}$  is predicted to increase by 1.549, 0.260, 0.935, 4.699, and 6.090 units, respectively. For each unit of increase in  $\nu_{BR}$ ,  $t$ ,  $p_{avg}$ ,  $T_{DF}$ ,  $\mu_{avg}$ ,  $T_{IPF}$ , and  $T_{OPI}$ , the disc final temperature decreases with 0.602, 4.105, 4.011, 514.234, 3.776, and 7.038 units, respectively. The other independent variable,  $a$ , is not statistically significant (Table A.12).

$$\begin{aligned}
 T_{DF} = & 149.972 + 1.549\nu_{BS} - 0.602\nu_{BR} - 4.105t \\
 & + 0.260M_{avg} - 4.011p_{avg} - 514.2\mu \\
 & + 0.935T_{DI} + 4.699T_{IPI} - 3.776T_{IPF} \\
 & - 7.038T_{OPI} + 6.09T_{OPF}. \tag{5}
 \end{aligned}$$

The correlation coefficients and the multiple regression output for the LCV disc-pad pair are given in Tables A.13 and A.14.  $R$  square equals 0.989, and 98.9% of the variations in  $T_{DF}$  are explained by the independent variables. Eight of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $I$ ,  $\nu_{BS}$ ,  $a$ ,  $M_{avg}$ ,  $p_{avg}$ ,  $FA$ ,  $\mu_{avg}$ , and  $T_{DI}$ ) is a significant predictor of  $T_{DF}$ . The constant term is -119.027. For each unit of increase in  $I$ ,  $\nu_{BS}$ ,  $M_{avg}$ ,  $FA$ , and  $T_{DI}$  in Eq. (6),  $T_{DF}$  is predicted to increase by 2.051, 1.651, 0.423, 0.0195, and 0.865 units, respectively. For each unit of increase in  $p_{avg}$ ,  $a$ , and  $\mu_{avg}$ , the disc final temperature decreases with 5.219, 0.951, and 388.176 units, respectively. The other independent variables  $\nu_{BR}$  and  $t$  are not statistically significant (Table A.14).

$$\begin{aligned}
 T_{DF} = & -119.027 + 2.051I + 1.651\nu_{BS} \\
 & + 0.423M_{avg} - 5.219p_{avg} - 0.951a \\
 & + 0.0195FA - 388.18\mu + 0.865T_{DI}. \tag{6}
 \end{aligned}$$

**3.4. Evaluating brake speed**

The correlation coefficients and the multiple regression output for the PC disc-pad pair are given in Tables A.15 and A.16.  $R$  square equals 0.973, and 97.3% of the variations in  $\nu_{BS}$  are explained by the independent variables. Eleven of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $\nu_{BR}$ ,  $t$ ,  $a$ ,  $M_{avg}$ ,  $p_{avg}$ ,  $\mu_{avg}$ ,  $T_{DI}$ ,  $T_{DF}$ ,

$T_{IPI}$ ,  $T_{IPF}$ , and  $T_{OPI}$ ) is a significant predictor of  $\nu_{BS}$ . The constant term is -78.110. For each unit of increase in  $\nu_{BR}$ ,  $t$ ,  $a$ ,  $p_{avg}$ ,  $\mu_{avg}$ ,  $T_{DF}$ ,  $T_{IPF}$ , and  $T_{OPI}$  in Eq. (7),  $\nu_{BS}$  is predicted to increase by 0.730, 2.199, 35.013, 1.914, 225.763, 0.167, 1.275, and 0.473 units, respectively. For each unit of increase in  $M_{avg}$ ,  $T_{DI}$ , and  $T_{IPI}$ , the brake speed decreases with 0.264, 0.255, and 1.724 units, respectively. The other independent variable  $T_{OPF}$  is not statistically significant (Table A.16).

$$\begin{aligned}
 \nu_{BS} = & -78.11 + 0.730\nu_{BR} + 2.199t + 35.01a \\
 & - 0.264M_{avg} + 1.914p_{avg} + 225.76\mu \\
 & - 0.255T_{DI} + 0.167T_{DF} - 1.724T_{IPI} \\
 & + 1.275T_{IPF} + 0.473T_{OPI}. \tag{7}
 \end{aligned}$$

The correlation coefficients and the multiple regression output for the LCV disc-pad pair are given in Tables A.17 and A.18.  $R$  square equals 0.977, and 97.7% of the variations in  $\nu_{BS}$  are explained by the independent variables. Seven of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $I$ ,  $\nu_{BR}$ ,  $a$ ,  $t$ ,  $FA$ ,  $T_{DI}$ , and  $T_{DF}$ ) is a significant predictor of  $\nu_{BS}$ . The constant term is 130.119. For each unit of increase in  $t$  and  $a$  in Eq. (8),  $\nu_{BS}$  is predicted to increase by 0.977 and 0.190 units, respectively. For each unit of increase in  $I$ ,  $\nu_{BR}$ ,  $FA$ ,  $T_{DI}$ , and  $T_{DF}$ , brake speed decreases with 1.276, 2.392, 0.0198, 0.266, and 0.362 units, respectively. The other independent variables,  $p_{avg}$ ,  $M_{avg}$ , and  $\mu_{avg}$ , are not statistically significant (Table A.18).

$$\begin{aligned}
 \nu_{BS} = & 130.12 - 1.276I - 2.392\nu_{BR} + 0.977t \\
 & + 0.190a - 0.0198FA - 0.266T_{DI} \\
 & - 0.362T_{DF}. \tag{8}
 \end{aligned}$$

**3.5. Evaluating average braking pressure**

The correlation coefficients and the multiple regression output for the PC disc-pad pair are given in Tables A.19 and A.20.  $R$  square equals 0.997, and 99.7% of the variations in  $p_{avg}$  are explained by the independent variables. Nine of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $\nu_{BS}$ ,  $\nu_{BR}$ ,  $t$ ,  $M_{avg}$ ,  $\mu_{avg}$ ,  $T_{DI}$ ,  $T_{DF}$ ,  $T_{OPI}$ , and  $T_{OPF}$ ) is a significant predictor of  $p_{avg}$ . The constant term is 38.016. For each unit of increase in  $\nu_{BS}$ ,  $M_{avg}$ ,  $T_{DI}$ , and  $T_{OPI}$  in Eq. (9),  $p_{avg}$  is predicted to increase by 0.0517, 0.062, 0.034, and 0.102 units, respectively. For each unit of increase in  $\nu_{BR}$ ,  $t$ ,  $\mu_{avg}$ ,  $T_{DF}$ , and  $T_{OPF}$ , average braking pressure decreases with 0.034, 0.452, 112.26, 0.0117, and 0.0814 units, respectively. The other independent variables  $a$ ,  $T_{IPI}$ ,  $T_{IPF}$  are not statistically significant (Table A.20).

$$\begin{aligned}
p_{avg} = & 38.016 + 0.0517v_{BS} - 0.034v_{BR} - 0.452t \\
& + 0.062M_{avg} - 112.26\mu_{avg} + 0.034T_{DI} \\
& - 0.0117T_{DF} + 0.102T_{OPI} - 0.081T_{OPF}. \quad (9)
\end{aligned}$$

The correlation coefficients and the multiple regression output for the LCV disc-pad pair are given in Tables A.21 and A.22.  $R$  square equals 0.986, and 98.6% of the variations in  $p_{avg}$  are explained by the independent variables. Eight of the coefficients in the regression statistics have  $P$ -values less than 5%. Each of the variables ( $I$ ,  $a$ ,  $t$ ,  $M_{avg}$ ,  $FA$ ,  $\mu_{avg}$ ,  $T_{DI}$ , and  $T_{DF}$ ) is a significant predictor of  $p_{avg}$ . The constant term is 16.186. For each unit of increase in  $t$ ,  $M_{avg}$ ,  $FA$ , and  $T_{DI}$  in Eq. (10),  $p_{avg}$  is predicted to increase by 0.192, 0.0656, 0.001, and 0.0221 units, respectively. For each unit of increase in  $I$ ,  $a$ ,  $\mu_{avg}$ , and  $T_{DF}$ , the average braking pressure decreases with 0.0925, 0.0543, 50.750, and 0.0191 units, respectively. The other independent variables  $\nu_{BS}$  and  $\nu_{BR}$  are not statistically significant (Table A.22).

$$\begin{aligned}
p_{avg} = & 16.186 - 0.0925I + 0.192t + 0.0656M_{avg} \\
& - 0.0543a + 0.001FA - 50.75\mu + 0.022T_{DI} \\
& - 0.019T_{DF}. \quad (10)
\end{aligned}$$

### 3.6. Discussion

The friction and wear behaviors of brake's friction materials are mainly affected by factors such as material characteristics, *braking conditions*, surrounding conditions, surface conditions, and structural parameters [35]. Friction is highly dependent on brake linings' chemical composition, environmental conditions, and operating conditions. The braking pair's  $\mu$  model is to correlate the friction coefficient to system inputs, such as *brake pressure*, and to system states such as *brake temperature and disc speed* [26].

From the information in the literature, the following points were identified:

- In the analyses of both PC and LCV braking pairs, *disc final temperature* has been affected positively by brake speed and negatively by pressure and the  $\mu$  value. These results partially validate the study results of Verma [36]. The average temperature rise in the contact region due to sliding is directly proportional to the dissipated frictional power, given by the product of the friction coefficient, the applied load, and the sliding velocity and is inversely proportional to the thermal conductivities of the mating materials [36];
- An increase in  $\mu$  in the braking pair affects the braking time in the PC negatively and in LCV

positively. Generally, an increase in friction coefficient reduces braking time. This is probably due to a negative intercept coefficient in the LCV-braking time equation and the effects of all other independent variables;

- Luo and Yang [37] emphasized that the brake linings must be investigated at different *pressures, speeds, and temperatures*. In addition to the three parameters mentioned, motorway braking check I-II (0.6 g) and fade (maximum 160 bar) I-II (0.4 g) test procedures were carried out at different **decelerations** in this study;
- Ricciardi et al. [26] stated that the  $\mu$  value correlates with *braking pressure, brake temperature, and disc speed* at brake linings'  $\mu$  model. When the correlation tables (Table A.7) in the study are examined, brake pressure, disc initial and final temperatures, brake speed, and release speed coefficients of the PC brake pair are determined as 0.359, 0.460, 0.429, 0.0914, and  $-0.109$ , respectively. Coefficients of the same parameters for the LCV brake pair are determined as  $-0.484$ ,  $-0.187$ ,  $-0.153$ ,  $-0.421$ , and 0.356, respectively (Table A.9). The correlation stated by Ricciardi et al. [26] was provided positively by the PC and negatively by the LCV;
- Rhee [17] stated that  $\mu$  can be dependent upon *temperature, load, and sliding velocity*. In this study, three additional parameters including *braking time, deceleration, and braking torque* are added to the list of the other parameters. Further, in the studies [18–21],  $\mu$  was found to be dependent on *temperature*. In this study, effect levels of 6 additional parameters for the PC and 8 additional parameters for the LCV on the friction coefficient were determined;
- Heussaff et al. [21], El-Tayeb and Liew [22], Saffar et al. [23], and Liew and Nirmal [24] individually stated that  $\mu$  was found to be dependent on braking force and velocity. In this study, the effects of additional 9 parameters were investigated.

### 4. Conclusions

In this study, two pairs of OEM brake disc-pad were used. One of these pairs belongs to a Passenger Car (PC) and the other one to a Light Commercial Vehicle (LCV). The PC disc-pad pair was subject to the global brake effectiveness test according to SAE J2522 test standard; the other one was subject to the tests according to FIAT 7-H4020 and 7-H2000 standards. For the PC disc-pad pair, 276 braking tests from 21 different categories were performed; in addition, for the LCV, a total of 130 braking tests were performed in 2 different categories. The basic brake parameters in the tests were measured and recorded. The interrelation of the parameters was analyzed with a multiple regression



method, and effect levels were determined. In this study, dependent variables in the multiple regression method including braking time, friction coefficient, disc final temperature, brake speed, and brake pressure for both PC and LCV were selected. In this analysis of PC disc-pad pair, 13 variables such as friction coefficient, avg. braking torque, avg. brake pressure, brake speed, brake release speeds, deceleration, disc initial and final temperatures, inpad initial and final temperatures, outpad initial and final temperatures, and braking time were used; in the regression model of LCV disc-pad pair, 11 variables such as brake speed, brake release speeds, deceleration, inertia, avg. braking torque, avg. brake pressure, fluid absorption, disc initial and final temperatures, friction coefficient, and braking time are used. From these analyses, the conclusions below were obtained:

- The  $R$  square in this study ranges from 0.917 to 0.997. The interpretation is that about 91.7% to 99.7% of the variability in the dependent variables can be explained by variations in the explanatory/independent variables;
- In the literature, 1, 2, or 3 variable approaches are generally used to examine the braking parameters. In this study, the *predictor coefficients* of at least 9 to 11 independent variables in the regression statistics for PC braking pair have  $P$ -values less than 5%. Furthermore, the coefficients of 7 to 8 independent variables in the regression statistics for LCV braking pair have  $P$ -values less than 5%. The statistical significance of each individual independent variable has been determined;
- According to the estimated regression equations for PC, for each unit of increase in deceleration and friction coefficient, *braking time* decreases with 7.3 and 60.9 units, respectively. For each unit of increase in braking time and brake pressure, friction coefficient decreases with 0.004 and 0.008 units, respectively. For each unit of increase in *friction coefficient* and outpad initial temperature, *disc final temperature* decreases with 514.2 and 7.03 units, respectively. For each unit of increase in friction coefficient and deceleration, *brake speed* increases with 225.8 and 35 units, respectively. For each unit of increase in friction coefficient, *braking pressure* decreases with 112.3 units;
- According to the estimated regression equation for LCV, for each unit of increase in brake pressure and friction coefficient, *braking time* increases with 1.267 and 91.887 units, respectively. For each unit of increase in brake pressure, *friction coefficient* decreases with 0.016 units. For each unit of increase in friction coefficient and brake pressure, *disc final temperature* decreases with 388.2 and 5.219 units,

respectively. For each unit of increase in brake release speed, *brake speed* decreases with 2.4 units. For each unit of increase in friction coefficient, *braking pressure* decreases with 50.8 units.

### Note

Data of the passenger car brake disc-pad pair used in regression analysis are taken from the doctoral thesis by Demir (2009), and from the doctoral thesis by Öz (2012) for the light commercial vehicle.

### Nomenclature

$D$	Disc
$DV$	Dependent Variable
$DV$	Dummy Variable
$FMVSS$	Federal Motor Vehicle Safety Standards
$ISO$	International Organization for Standardization
$IV$	Independent Variable
$JASO$	Japan Automobile Standards Organization
$JIS$	Japanese Industrial Standard
$LCV$	Light Commercial Vehicle
$O$	Observation
$OEM$	Original Equipment Manufacturer
$PC$	Passenger Car
$SAE$	Society of Automotive Engineers

### Symbols

$\varepsilon$	Error term
$\nu$	Hz
$\alpha$	Linear function constant
$\beta$	Linear function elevation
$\mu$	Friction coefficient
$a$	Deceleration
FA	Fluid Absor
$I$	Inertia
$M$	Torque
$p$	Pressure
$t$	Braking time
$T$	Temperature
$X$	Independent variable
$Y$	Dependent Variable

### Subscript

$avg$	Average
$BR$	Brake Release speed
$BS$	Brake speed

<i>DF</i>	Disc Final temperature
<i>DI</i>	Disc Initial temperature
<i>IPF</i>	Inpad final temperature
<i>IPI</i>	Inpad initial temperature
<i>LCV</i>	Light Commercial Vehicle
<i>n</i>	Number (1, 2, 3, ...)
<i>OPI</i>	Outpad initial temperature
<i>OPF</i>	Outpad final temperature
<i>p</i>	Exploratory variable

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## Appendix

In this section, some technical information about dynamometers used in experiments, as well as regression analysis and correlation matrices were included.

**Table A.1.** Technical specifications and measurement parameters of the inertia dynamometer used in the automobile disc-pad pair test.

No	Definitions	Manufacturer	Model	Serial no	Full-scale standards	Units	Uncertainty (+%FS)	Uncertainty (+Unit)	Full-scale metric	Unit	
1	Air velocity	RM Young	27105R-2400	jenerator	AS17A	55	mph	1.44	0.79	88.55	kph
2	Maximum inertia	Link Engineering				68	Slug ft <sup>2</sup>	0.44	0.30	92.82	kgm <sup>2</sup>
3	Base inertia					8	Slug ft <sup>2</sup>			10.20	kgm <sup>2</sup>
4	Inertia intervals					28 at 2.2	Slug ft <sup>2</sup>			2.99	kgm <sup>2</sup>
5	Engine power	General motors				100	hp			74.57	kW
6	Pressure	Sensotec	TJE-0743-03	TJG-3000	478656	3,000	psi	0.31	9.33	206.70	bar
7	Angular velocity	Sick stegmann	DRS25-4F	400512	71002439	2,000	rpm	0.29	5.79	2,000.00	rpm
8	Temperature	Link engineering	1484	CAQ	TL17A	2,400	°F	0.54	13.07	1,315.56	°C
9	Torque	Siebe Lebow	2112-50K		332	4,167	ft*lbs	0.34	14.03	5,667.12	Nm
10	Corrosion	Mitutoyo	Çeşitli	Çeşitli		1	in	0.34	0.00	2.54	cm
11	Liquid volume	Balluff	BTL-5-A21-M0102-2-532	FD17		2.4409	in <sup>3</sup>	0.30%	0.01	40.00	cm <sup>3</sup>
12	Capacitive prob	Capacitec	4008-P115			0.100	in	0.24	0.0002	2,540.00	micron



(a) LINK full scale inertia dynamometer (Link Engineering Company, Detroit, USA)



(b) ESAM brand inertia dynamometer (Gebze/Kale Balata)

**Figure A.1.** Full-scale inertia dynamometers used in the tests.

**Table A.2.** Specifications of the inertia dynamometer used in the light commercial vehicle disc-pad pair test.

Technical specification parameters	Units	Value
Maximum engine revolutions	rpm	2,400
Maximum inertia moment	kgm <sup>2</sup>	166
Maximum braking torque	Nm	5,000
Max, speed for both directions (hardware limit)	rpm	2,400
Max, speed for both directions (software limit)	rpm	2,160
Ventilation maximum speed	rpm	2,600
Test equipment weight	DA.N	13,000
Maximum dynamometer load	kgmsq	166
Minimum revolutions stopping engine	rpm	±20
Hydraulic system power unit pressure limit	bar	160
Hydraulic system applicable pressure size	bar	150
Hydraulic system pressure alarm limit	bar	152
Maximum body movement	mm	1,900
Maximum and minimum torque weight	Nm	±4, 500
Torque meter mechanic accuracy	Nm	±2.5
Thermocouple temperature range	°C	0...1000
Thermocouple telemetry channel precision	°C	±3
Pressure transformation scale	bar	200
Pressure transformation precision	bar	±0, 1
Flowmeter flow range	l/min	0.004 - 4
Flowmeter precision	mm <sup>3</sup>	10
Ventilation pipe diameter	mm	255
Maximum air flow	m <sup>3</sup> /h	2,200
Engine power	kW	140
Height	mm	2,250
Flywheel	N°	4
Length	mm	6,950

**Table A.3.** The correlation coefficients between independent variables and braking time (for PC).

	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	$a$ (m/s <sup>2</sup> )	$M_{avg}$ (Nm)	$p_{avg}$ (bar)	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{DF}$ (°C)	$T_{IPI}$ (°C)	$T_{IPF}$ (°C)	$T_{OPI}$ (°C)	$T_{OPF}$ (°C)	$t$ (s)
$\nu_{BS}$ , k/h	1												
$\nu_{BR}$ , k/h	0.86474	1											
$a$ , m/s <sup>2</sup>	0.25746	0.02039	1										
$M_{avg}$ , Nm	0.27948	0.06384	0.99566	1									
$p_{avg}$ , bar	0.27637	0.09255	0.97251	0.97977	1								
$\mu_{avg}$	0.0914	-0.1094	0.548	0.52566	0.35945	1							
$T_{DI}$ , °C	0.06749	-0.2759	0.43353	0.39634	0.32349	0.46047	1						
$T_{DF}$ , °C	0.30583	-0.07	0.50268	0.47278	0.40824	0.42956	0.95515	1					
$T_{IPI}$ , °C	0.09596	-0.2459	0.43795	0.40235	0.33933	0.41708	0.9914	0.94929	1				
$T_{IPF}$ , °C	0.09633	-0.2553	0.43222	0.3959	0.32849	0.43057	0.99244	0.94989	0.99905	1			
$T_{OPI}$ , °C	0.09342	-0.2466	0.43325	0.39784	0.33751	0.40396	0.98776	0.9443	0.99941	0.99786	1		
$T_{OPF}$ , °C	0.10378	-0.2451	0.42734	0.39192	0.32988	0.40222	0.9896	0.95182	0.99916	0.99861	0.99923	1	
$t$ , s	-0.2314	-0.2764	-0.7113	-0.7091	-0.7198	-0.4268	-0.0146	-0.0628	-0.0242	-0.0075	-0.0221	-0.0018	1

**Table A.4.** Multiple regression model statistics of the factors that contributed to the stopping/braking time (for PC).

Regression statistics	
Multiple <i>R</i>	0.95796
<i>R</i> square	0.91769
Adjustable <i>R</i> square	0.91393
Standard error	0.73315
Observation	276

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>
Regression	12	1576.03	131.336	244.343	6E-135
Difference	263	141.365	0.53751		
Total	275	1717.4			

	Coefficients	Standard error	<i>t</i> Stat	<i>P</i> -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	29.1689	1.88393	15.483	7.1E-39	25.4593	32.8784	25.4593	32.8784
$\nu_{BS}$ , k/h	0.0596	0.00946	6.29804	1.3E-09	0.04096	0.07823	0.04096	0.07823
$\nu_{BR}$ , k/h	-0.06	0.00711	-8.4325	2.3E-15	-0.074	-0.046	-0.074	-0.046
<i>a</i> , m/s <sup>2</sup>	-7.3079	0.77638	-9.4128	2.5E-18	-8.8366	-5.7791	-8.8366	-5.7791
$M_{avg}$ , Nm	0.05709	0.00394	14.506	2E-35	0.04934	0.06483	0.04934	0.06483
$p_{avg}$ , bar	-0.4543	0.05509	-8.2472	7.8E-15	-0.5628	-0.3458	-0.5628	-0.3458
$\mu_{avg}$	-60.868	6.38996	-9.5256	1.1E-18	-73.45	-48.287	-73.45	-48.287
$T_{DI}$ , °C	0.01249	0.00576	2.16807	0.03105	0.00115	0.02383	0.00115	0.02383
$T_{DF}$ , °C	-0.012	0.00326	-3.698	0.00026	-0.0184	-0.0056	-0.0184	-0.0056
$T_{IPI}$ , °C	<b>-0.0081</b>	<b>0.0433</b>	<b>-0.1868</b>	<b>0.85194</b>	<b>-0.0933</b>	<b>0.07716</b>	<b>-0.0933</b>	<b>0.07716</b>
$T_{IPF}$ , °C	<b>0.00932</b>	<b>0.02481</b>	<b>0.37557</b>	<b>0.70754</b>	<b>-0.0395</b>	<b>0.05816</b>	<b>-0.0395</b>	<b>0.05816</b>
$T_{OPI}$ , °C	-0.1823	0.04214	-4.325	2.2E-05	-0.2652	-0.0993	-0.2652	-0.0993
$T_{OPF}$ , °C	0.1815	0.03403	5.33411	2.1E-07	0.1145	0.24849	0.1145	0.24849

**Table A.5.** The correlation coefficients between independent variables and braking time (for LCV).

	<i>I</i>	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	$p_{avg}$ (bar)	$M_{avg}$ (Nm)	<i>a</i> (m/s <sup>2</sup> )	<i>FA</i>	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{DF}$ (°C)	<i>t</i> (s)
I	1										
$\nu_{BS}$ , k/h	-0.53874	1									
$\nu_{BR}$ , k/h	0.515739	-0.82696	1								
$p_{avg}$ , bar	-0.71998	0.247353	-0.40849	1							
$M_{avg}$ , Nm	-0.12483	-0.26334	0.023628	0.563023	1						
<i>a</i> , m/s <sup>2</sup>	-0.81658	0.416691	-0.44359	0.68695	0.373259	1					
<i>FA</i>	0.23632	-0.52126	0.274012	0.395285	0.533453	-0.08364	1				
$\mu_{avg}$	0.607629	-0.4212	0.356912	-0.48446	0.423433	-0.3228	0.059694	1			
$T_{DI}$ , °C	-0.085	0.721645	-0.5822	-0.01083	-0.32088	-0.05859	-0.27456	-0.18781	1		
$T_{DF}$ , °C	0.063896	0.725292	-0.57266	-0.11098	-0.37223	-0.18011	-0.26583	-0.15384	0.951683	1	
<i>t</i> , s	0.321696	0.455721	-0.23309	-0.61766	-0.76367	-0.31432	-0.5021	-0.03842	0.554085	0.656019	1

**Table A.6.** Multiple regression model statistics of the factors that contributed to the stopping/braking time (for LCV).

Regression statistics								
Multiple <i>R</i>	0.98186							
<i>R</i> square	0.964049							
Adjustable <i>R</i> square	0.961028							
Standard error	1.583748							
Observation	130							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>			
Regression	10	8004.082	800.4082	319.1091	6.18E-81			
Difference	119	298.4827	2.508258					
Total	129	8302.564						

	Coefficients	Standard error	<i>t</i> stat	<i>P</i> -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	-32.6885	5.392565	-6.06177	1.63E-08	-43.3663	-22.0106	-43.3663	-22.0106
<i>I</i>	0.395474	0.04869	8.122278	4.82E-13	0.299063	0.491886	0.299063	0.491886
$\nu_{BS}$ , k/h	0.107291	0.028741	3.733061	0.000292	0.050381	0.164201	0.050381	0.164201
$\nu_{BR}$ , k/h	<b>-0.16296</b>	<b>0.278037</b>	<b>-0.58611</b>	<b>0.558909</b>	<b>-0.7135</b>	<b>0.387579</b>	<b>-0.7135</b>	<b>0.387579</b>
$p_{avg}$ , bar	1.267735	0.204418	6.201687	8.36E-09	0.862968	1.672503	0.862968	1.672503
$M_{avg}$ , Nm	-0.14812	0.010935	-13.5453	7.3E-26	-0.16977	-0.12646	-0.16977	-0.12646
<i>a</i> , m/s <sup>2</sup>	0.204403	0.020655	9.896078	3.28E-17	0.163504	0.245302	0.163504	0.245302
<i>FA</i>	0.003222	0.001272	2.532618	0.012623	0.000703	0.005741	0.000703	0.005741
$\mu_{avg}$	91.88736	10.11557	9.083757	2.76E-15	71.85753	111.9172	71.85753	111.9172
$T_{DI}$ , °C	-0.02416	0.012807	-1.88678	0.061627	-0.04952	0.001195	-0.04952	0.001195
$T_{DF}$ , °C	<b>0.020328</b>	<b>0.014111</b>	<b>1.44053</b>	<b>0.152343</b>	<b>-0.00761</b>	<b>0.04827</b>	<b>-0.00761</b>	<b>0.04827</b>

**Table A.7.** The correlation coefficients between independent variables and friction coefficient (for PC).

	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	<i>t</i> (s)	<i>a</i> (m/s <sup>2</sup> )	$M_{avg}$ (Nm)	$p_{avg}$ (bar)	$T_{DI}$ (°C)	$T_{DF}$ (°C)	$T_{IPI}$ (°C)	$T_{IPF}$ (°C)	$T_{OPI}$ (°C)	$T_{OPF}$ (°C)	$\mu_{avg}$
$\nu_{BS}$ , k/h	1												
$\nu_{BR}$ , k/h	0.8647	1											
<i>t</i> , s	-0.231	-0.276	1										
<i>a</i> , m/s <sup>2</sup>	0.2575	0.0204	-0.711	1									
$M_{avg}$ , Nm	0.2795	0.0638	-0.709	0.9957	1								
$p_{avg}$ , bar	0.2764	0.0926	-0.72	0.9725	0.9798	1							
$T_{DI}$ , °C	0.0675	-0.276	-0.015	0.4335	0.3963	0.3235	1						
$T_{DF}$ , °C	0.3058	-0.07	-0.063	0.5027	0.4728	0.4082	0.9552	1					
$T_{IPI}$ , °C	0.096	-0.246	-0.024	0.438	0.4023	0.3393	0.9914	0.9493	1				
$T_{IPF}$ , °C	0.0963	-0.255	-0.008	0.4322	0.3959	0.3285	0.9924	0.9499	0.9991	1			
$T_{OPI}$ , °C	0.0934	-0.247	-0.022	0.4333	0.3978	0.3375	0.9878	0.9443	0.9994	0.9979	1		
$T_{OPF}$ , °C	0.1038	-0.245	-0.002	0.4273	0.3919	0.3299	0.9896	0.9518	0.9992	0.9986	0.9992	1	
$\mu_{avg}$	0.0914	-0.109	-0.427	0.548	0.5257	0.3594	0.4605	0.4296	0.4171	0.4306	0.404	0.4022	1

**Table A.8.** Multiple regression model statistics of the factors that contributed to the friction coefficient (for PC).

		Regression statistics					ANOVA				
		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>					
		Multiple <i>R</i>	0.981								
		<i>R</i> square	0.9624								
		Adjustable <i>R</i> square	0.9607								
		Standard error	0.0061								
		Observation	276								
		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>					
Regression		12	0.2506	0.0209	561.14	1E-179					
Difference		263	0.0098	4E-05							
Total		275	0.2604								

	Coefficients	Standard error	<i>t</i> Stat	<i>P</i> -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	0.3381	0.0059	57.055	4E-150	0.3264	0.3497	0.3264	0.3497
$\nu_{BS}$ , k/h	0.0004	8E-05	5.2737	3E-07	0.0003	0.0006	0.0003	0.0006
$\nu_{BR}$ , k/h	-3E-04	6E-05	-4.204	4E-05	-4E-04	-1E-04	-4E-04	-1E-04
<i>t</i> , s	-0.004	0.0004	-9.526	1E-18	-0.005	-0.003	-0.005	-0.003
<b><i>a</i>, m/s<sup>2</sup></b>	<b>0.0085</b>	<b>0.0075</b>	<b>1.1378</b>	<b>0.2562</b>	<b>-0.006</b>	<b>0.0231</b>	<b>-0.006</b>	<b>0.0231</b>
<i>M<sub>avg</sub></i> , Nm	0.0005	3E-05	15.746	8E-40	0.0004	0.0006	0.0004	0.0006
<i>p<sub>avg</sub></i> , bar	-0.008	0.0002	-43.19	2E-121	-0.008	-0.007	-0.008	-0.007
<i>T<sub>DI</sub></i> , °C	0.0004	4E-05	8.2242	9E-15	0.0003	0.0004	0.0003	0.0004
<i>T<sub>DF</sub></i> , °C	-1E-04	3E-05	-3.863	0.0001	-2E-04	-5E-05	-2E-04	-5E-05
<i>T<sub>IPI</sub></i> , °C	-9E-04	0.0004	-2.604	0.0097	-0.002	-2E-04	-0.002	-2E-04
<i>T<sub>IPF</sub></i> , °C	0.0004	0.0002	1.9767	0.0491	2E-06	0.0008	2E-06	0.0008
<i>T<sub>OPI</sub></i> , °C	0.001	0.0004	2.8787	0.0043	0.0003	0.0017	0.0003	0.0017
<i>T<sub>OPF</sub></i> , °C	-9E-04	0.0003	-2.901	0.004	-0.001	-3E-04	-0.001	-3E-04

**Table A.9.** The correlation coefficients between independent variables and friction coefficient (for LCV).

	<i>I</i>	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	<i>t</i> (s)	<i>p<sub>avg</sub></i> (bar)	<i>M<sub>avg</sub></i> (Nm)	<i>a</i> (m/s <sup>2</sup> )	<i>FA</i>	<i>T<sub>DI</sub></i> (°C)	<i>T<sub>DF</sub></i> (°C)	$\mu_{avg}$
I	1										
$\nu_{BS}$ , k/h	-0.5387	1									
$\nu_{BR}$ , k/h	0.51574	-0.827	1								
<i>t</i> , s	0.3217	0.45572	-0.2331	1							
<i>p<sub>avg</sub></i> , bar	-0.72	0.24735	-0.4085	-0.6177	1						
<i>M<sub>avg</sub></i> , Nm	-0.1248	-0.2633	0.02363	-0.7637	0.56302	1					
<i>a</i> , m/s <sup>2</sup>	-0.8166	0.41669	-0.4436	-0.3143	0.68695	0.37326	1				
<i>FA</i>	0.23632	-0.5213	0.27401	-0.5021	0.39528	0.53345	-0.0836	1			
<i>T<sub>DI</sub></i> , °C	-0.085	0.72165	-0.5822	0.55409	-0.0108	-0.3209	-0.0586	-0.2746	1		
<i>T<sub>DF</sub></i> , °C	0.0639	0.72529	-0.5727	0.65602	-0.111	-0.3722	-0.1801	-0.2658	0.95168	1	
$\mu_{avg}$	0.60763	-0.4212	0.35691	-0.0384	-0.4845	0.42343	-0.3228	0.05969	-0.1878	-0.1538	1

**Table A.10.** Multiple regression model statistics of the factors that contributed to the friction coefficient (for LCV).

		Regression statistics							
		Multiple $R$	0.992189						
		$R$ square	0.984439						
		Adjustable $R$ square	0.983132						
		Standard error	0.011029						
		Observation	130						
		ANOVA							
		$df$	$SS$	$MS$	$F$	Significance $F$			
Regression		10	0.915772	0.091577	752.8396	1.5E-102			
Difference		119	0.014475	0.000122					
Total		129	0.930248						

	Coefficients	Standard error	$t$ stat	$P$ -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	0.289506	0.033785	8.569083	4.44E-14	0.222609	0.356404	0.222609	0.356404
$I$	-0.00099	0.000413	-2.40528	0.0177	-0.00181	-0.00018	-0.00181	-0.00018
$\nu_{BS}$ , k/h	<b>0.000147</b>	<b>0.000211</b>	<b>0.695873</b>	<b>0.487865</b>	<b>-0.00027</b>	<b>0.000565</b>	<b>-0.00027</b>	<b>0.000565</b>
$\nu_{BR}$ , k/h	<b>-0.00055</b>	<b>0.001938</b>	<b>-0.28231</b>	<b>0.778193</b>	<b>-0.00439</b>	<b>0.003291</b>	<b>-0.00439</b>	<b>0.003291</b>
$t$ , s	0.004456	0.000491	9.083757	2.76E-15	0.003485	0.005428	0.003485	0.005428
$p_{avg}$ , bar	-0.01619	0.000691	-23.4308	2.12E-46	-0.01756	-0.01483	-0.01756	-0.01483
$M_{avg}$ , Nm	0.001279	3.16E-05	40.41932	2.36E-71	0.001216	0.001341	0.001216	0.001341
$a$ , m/s <sup>2</sup>	-0.00118	0.000162	-7.28234	3.87E-11	-0.0015	-0.00086	-0.0015	-0.00086
$FA$	-1.2E-05	9.03E-06	-1.29634	0.197367	-3E-05	6.18E-06	-3E-05	6.18E-06
$T_{DI}$ , °C	0.000499	7.81E-05	6.395335	3.28E-09	0.000345	0.000654	0.000345	0.000654
$T_{DF}$ , °C	-0.00045	9E-05	-5.04478	1.65E-06	-0.00063	-0.00028	-0.00063	-0.00028

**Table A.11.** The correlation coefficients between independent variables and disc final temperature (For PC).

	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	$t$ (s)	$a$ (m/s <sup>2</sup> )	$M_{avg}$ (Nm)	$p_{avg}$ (bar)	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{IPI}$ (°C)	$T_{IPF}$ (°C)	$T_{OPI}$ (°C)	$T_{OPF}$ (°C)	$T_{DF}$ (°C)
$\nu_{BS}$ , k/h	1												
$\nu_{BR}$ , k/h	0.8647	1											
$t$ , s	-0.231	-0.276	1										
$a$ , m/s <sup>2</sup>	0.2575	0.0204	-0.711	1									
$M_{avg}$ , Nm	0.2795	0.0638	-0.709	0.9957	1								
$p_{avg}$ , bar	0.2764	0.0926	-0.72	0.9725	0.9798	1							
$\mu_{avg}$	0.0914	-0.109	-0.427	0.548	0.5257	0.3594	1						
$T_{DI}$ , °C	0.0675	-0.276	-0.015	0.4335	0.3963	0.3235	0.4605	1					
$T_{IPI}$ , °C	0.096	-0.246	-0.024	0.438	0.4023	0.3393	0.4171	0.9914	1				
$T_{IPF}$ , °C	0.0963	-0.255	-0.008	0.4322	0.3959	0.3285	0.4306	0.9924	0.9991	1			
$T_{OPI}$ , °C	0.0934	-0.247	-0.022	0.4333	0.3978	0.3375	0.404	0.9878	0.9994	0.9979	1		
$T_{OPF}$ , °C	0.1038	-0.245	-0.002	0.4273	0.3919	0.3299	0.4022	0.9896	0.9992	0.9986	0.9992	1	
$T_{DF}$ , °C	0.3058	-0.07	-0.063	0.5027	0.4728	0.4082	0.4296	0.9552	0.9493	0.9499	0.9443	0.9518	1



**Table A.12.** Multiple regression model statistics of the factors that contributed to disc final temperature (for PC).

		Regression statistics			
		Multiple <i>R</i>	0.99517		
		<i>R</i> square	0.99037		
		Adjustable <i>R</i> square	0.98993		
		Standard error	13.5391		
		Observation	276		

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>
Regression	12	4955552	412963	2252.84	3E-257
Difference	263	48209.8	183.307		
Total	275	5003762			

	Coefficients	Standard error	<i>t</i> stat	<i>P</i> -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	149.972	47.2031	3.17716	0.00166	57.0276	242.916	57.0276	242.916
$\nu_{BS}$ , k/h	1.54922	0.1613	9.60479	6.4E-19	1.23162	1.86682	1.23162	1.86682
$\nu_{BR}$ , k/h	-0.60234	0.14332	-4.20262	3.6E-05	-0.88455	-0.32013	-0.88455	-0.32013
<i>t</i> , s	-4.10558	1.11023	-3.69796	0.00026	-6.29165	-1.91952	-6.29165	-1.91952
<b><i>a</i>, m/s<sup>2</sup></b>	<b>7.14833</b>	<b>16.5715</b>	<b>0.43136</b>	<b>0.66656</b>	<b>-25.4814</b>	<b>39.7781</b>	<b>-25.4814</b>	<b>39.7781</b>
<i>M</i> <sub>avg</sub> , Nm	0.26013	0.09618	2.7048	0.00728	0.07076	0.4495	0.07076	0.4495
<i>p</i> <sub>avg</sub> , bar	-4.0108	1.11414	-3.5999	0.00038	-6.20457	-1.81703	-6.20457	-1.81703
$\mu_{avg}$	-514.234	133.13	-3.86264	0.00014	-776.37	-252.097	-776.37	-252.097
<i>T</i> <sub>DI</sub> , °C	0.93568	0.09051	10.3376	3E-21	0.75746	1.11391	0.75746	1.11391
<i>T</i> <sub>IPI</sub> , °C	4.69961	0.74522	6.30632	1.2E-09	3.23225	6.16698	3.23225	6.16698
<i>T</i> <sub>IPF</sub> , °C	-3.77634	0.39468	-9.56804	8.4E-19	-4.55348	-2.9992	-4.55348	-2.9992
<i>T</i> <sub>OPI</sub> , °C	-7.03853	0.67849	-10.3738	2.3E-21	-8.37449	-5.70257	-8.37449	-5.70257
<i>T</i> <sub>OPF</sub> , °C	6.09007	0.54454	11.1839	5.2E-24	5.01786	7.16227	5.01786	7.16227

**Table A.13.** The correlation coefficients between independent variables and disc final temperature (for LCV).

<i>I</i>	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	<i>t</i> (s)	<i>p</i> <sub>avg</sub> (bar)	<i>M</i> <sub>avg</sub> (Nm)	<i>a</i> (m/s <sup>2</sup> )	<i>FA</i>	$\mu_{avg}$	<i>T</i> <sub>DI</sub> (°C)	<i>T</i> <sub>DF</sub> (°C)	
I	1										
$\nu_{BS}$ , k/h	-0.53874	1									
$\nu_{BR}$ , k/h	0.51574	-0.82696	1								
<i>t</i> , s	0.3217	0.45572	-0.23309	1							
<i>p</i> <sub>avg</sub> , bar	-0.71998	0.24735	-0.40849	-0.61766	1						
<i>M</i> <sub>avg</sub> , Nm	-0.12483	-0.26334	0.02363	-0.76367	0.56302	1					
<i>a</i> , m/s <sup>2</sup>	-0.81658	0.41669	-0.44359	-0.31432	0.68695	0.37326	1				
<i>FA</i>	0.23632	-0.52126	0.27401	-0.5021	0.39528	0.53345	-0.08364	1			
$\mu_{avg}$	0.60763	-0.4212	0.35691	-0.03842	-0.48446	0.42343	-0.3228	0.05969	1		
<i>T</i> <sub>DI</sub> , °C	-0.085	0.72165	-0.5822	0.55409	-0.01083	-0.32088	-0.05859	-0.27456	-0.18781	1	
<i>T</i> <sub>DF</sub> , °C	0.0639	0.72529	-0.57266	0.65602	-0.11098	-0.37223	-0.18011	-0.26583	-0.15384	0.95168	1

**Table A.14.** Multiple regression model statistics of the factors that contributed to disc final temperature (for LCV).

		Regression statistics	
		Multiple $R$	0.994744
		$R$ square	0.989515
		Adjustable $R$ square	0.988634
		Standard error	10.1997
		Observation	130

ANOVA					
	$df$	$SS$	$MS$	$F$	Significance $F$
Regression	10	1168328	116832.8	1123.027	9.9E-113
Difference	119	12380.03	104.0339		
Total	129	1180708			

	Coefficients	Standard error	$t$ stat	$P$ -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	-119.027	38.20344	-3.11562	0.002301	-194.674	-43.3807	-194.674	-43.3807
$I$	2.0512	0.342761	5.984351	2.35E-08	1.3725	2.729901	1.3725	2.729901
$\nu_{BS}$ , k/h	1.651812	0.123874	13.33464	2.26E-25	1.40653	1.897095	1.40653	1.897095
$\nu_{BR}$ , k/h	<b>0.673467</b>	<b>1.792139</b>	<b>0.37579</b>	<b>0.707742</b>	<b>-2.87515</b>	<b>4.222081</b>	<b>-2.87515</b>	<b>4.222081</b>
$t$ , s	<b>0.843133</b>	<b>0.585294</b>	<b>1.44053</b>	<b>0.152343</b>	<b>-0.31581</b>	<b>2.002073</b>	<b>-0.31581</b>	<b>2.002073</b>
$p_{avg}$ , bar	-5.21942	1.436801	-3.63266	0.000415	-8.06443	-2.37441	-8.06443	-2.37441
$M_{avg}$ , Nm	0.423476	0.105351	4.019661	0.000103	0.21487	0.632081	0.21487	0.632081
$a$ , m/s <sup>2</sup>	-0.95167	0.156992	-6.06186	1.63E-08	-1.26253	-0.64081	-1.26253	-0.64081
$FA$	0.019543	0.008218	2.377933	0.019003	0.00327	0.035816	0.00327	0.035816
$\mu_{avg}$	-388.176	76.94604	-5.04478	1.65E-06	-540.537	-235.815	-540.537	-235.815
$T_{DI}$ , °C	0.865545	0.026665	32.45975	5.83E-61	0.812746	0.918345	0.812746	0.918345

**Table A.15.** The correlation coefficients between independent variables and brake speed (for PC).

	$\nu_{BR}$ (k/h)	$t$ (s)	$a$ (m/s <sup>2</sup> )	$M_{avg}$ (Nm)	$p_{avg}$ (bar)	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{DF}$ (°C)	$T_{IPI}$ (°C)	$T_{IPF}$ (°C)	$T_{OPI}$ (°C)	$T_{OPF}$ (°C)	$\nu_{BS}$ (k/h)
$\nu_{BR}$ , k/h	1												
$t$ , s	-0.276	1											
$a$ , m/s <sup>2</sup>	0.0204	-0.711	1										
$M_{avg}$ , Nm	0.0638	-0.709	0.9957	1									
$p_{avg}$ , bar	0.0926	-0.72	0.9725	0.9798	1								
$\mu_{avg}$	-0.109	-0.427	0.548	0.5257	0.3594	1							
$T_{DI}$ , °C	-0.276	-0.015	0.4335	0.3963	0.3235	0.4605	1						
$T_{DF}$ , °C	-0.07	-0.063	0.5027	0.4728	0.4082	0.4296	0.9552	1					
$T_{IPI}$ , °C	-0.246	-0.024	0.438	0.4023	0.3393	0.4171	0.9914	0.9493	1				
$T_{IPF}$ , °C	-0.255	-0.008	0.4322	0.3959	0.3285	0.4306	0.9924	0.9499	0.9991	1			
$T_{OPI}$ , °C	-0.247	-0.022	0.4333	0.3978	0.3375	0.404	0.9878	0.9443	0.9994	0.9979	1		
$T_{OPF}$ , °C	-0.245	-0.002	0.4273	0.3919	0.3299	0.4022	0.9896	0.9518	0.9992	0.9986	0.9992	1	
$\nu_{BS}$ , k/h	0.8647	-0.231	0.2575	0.2795	0.2764	0.0914	0.0675	0.3058	0.096	0.0963	0.0934	0.1038	1

**Table A.16.** Multiple regression model statistics of the factors that contributed to brake speed (for PC).

		Regression statistics				
		Multiple $R$	0.986615			
		$R$ square	0.97341			
		Adjustable $R$ square	0.972196			
		Standard error	4.453452			
		Observation	276			
		ANOVA				
		$df$	$SS$	$MS$	$F$	Significance $F$
Regression		12	190949.7	15912.47	802.3133	2.3E-199
Difference		263	5216.141	19.83324		
Total		275	196165.8			

	Coefficients	Standard error	$t$ stat	$P$ -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	-78.1101	15.07086	-5.18286	4.36E-07	-107.785	-48.4352	-107.785	-48.4352
$\nu_{BR}$ , k/h	0.730576	0.018504	39.48298	1.5E-112	0.694142	0.76701	0.694142	0.76701
$t$ , s	2.199016	0.349159	6.298038	1.26E-09	1.511513	2.886518	1.511513	2.886518
$a$ , m/s <sup>2</sup>	35.01316	5.007219	6.992536	2.23E-11	25.15382	44.87249	25.15382	44.87249
$M_{avg}$ , Nm	-0.26449	0.027616	-9.5774	7.83E-19	-0.31886	-0.21011	-0.31886	-0.21011
$p_{avg}$ , bar	1.914492	0.356352	5.372471	1.71E-07	1.212825	2.616158	1.212825	2.616158
$\mu_{avg}$	225.7637	42.8092	5.273718	2.79E-07	141.4713	310.0561	141.4713	310.0561
$T_{DI}$ , °C	-0.25524	0.031605	-8.07606	2.42E-14	-0.31747	-0.19301	-0.31747	-0.19301
$T_{DF}$ , °C	0.16762	0.017452	9.604791	6.43E-19	0.133257	0.201983	0.133257	0.201983
$T_{IPI}$ , °C	-1.72456	0.240553	-7.16914	7.63E-12	-2.19821	-1.2509	-2.19821	-1.2509
$T_{IPF}$ , °C	1.275339	0.128595	9.917485	6.69E-20	1.022132	1.528545	1.022132	1.528545
$T_{OPI}$ , °C	0.473436	0.263319	1.797955	0.073331	-0.04505	0.991917	-0.04505	0.991917
$T_{OPF}$ , °C	<b>0.121407</b>	<b>0.21745</b>	<b>0.558322</b>	<b>0.577099</b>	<b>-0.30676</b>	<b>0.549573</b>	<b>-0.30676</b>	<b>0.549573</b>

**Table A.17.** The correlation coefficients between independent variables and brake speed (for LCV).

	$I$	$\nu_{BR}$ (k/h)	$t$ (s)	$p_{avg}$ (bar)	$M_{avg}$ (Nm)	$a$ (m/s <sup>2</sup> )	$FA$	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{DF}$ (°C)	$\nu_{BS}$ (k/h)
$I$	1										
$\nu_{BR}$ , k/h	0.51574	1									
$t$ , s	0.3217	-0.2331	1								
$p_{avg}$ , bar	-0.72	-0.4085	-0.6177	1							
$M_{avg}$ , Nm	-0.1248	0.02363	-0.7637	0.56302	1						
$a$ , m/s <sup>2</sup>	-0.8166	-0.4436	-0.3143	0.68695	0.37326	1					
$FA$	0.23632	0.27401	-0.5021	0.39528	0.53345	-0.0836	1				
$\mu_{avg}$	0.60763	0.35691	-0.0384	-0.4845	0.42343	-0.3228	0.05969	1			
$T_{DI}$ , °C	-0.085	-0.5822	0.55409	-0.0108	-0.3209	-0.0586	-0.2746	-0.1878	1		
$T_{DF}$ , °C	0.0639	-0.5727	0.65602	-0.111	-0.3722	-0.1801	-0.2658	-0.1538	0.95168	1	
$\nu_{BS}$ , k/h	-0.5387	-0.827	0.45572	0.24735	-0.2633	0.41669	-0.5213	-0.4212	0.72165	0.72529	1

**Table A.18.** Multiple regression model statistics of the factors that contributed to the brake speed (for LCV).

		Regression statistics						
		Multiple <i>R</i>	0.988557					
		<i>R</i> square	0.977245					
		Adjustable <i>R</i> square	0.975333					
		Standard error	4.779327					
		Observation	130					
		ANOVA						
		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>		
Regression		10	116736.8	11673.68	511.0629	9.9E-93		
Difference		119	2718.194	22.84197				
Total		129	119455					
	Coefficients	Standard error	<i>t</i> stat	<i>P</i> -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	130.1191	14.29389	9.103124	2.48E-15	101.8157	158.4224	101.8157	158.4224
<i>I</i>	-1.27612	0.140974	-9.05215	3.28E-15	-1.55526	-0.99697	-1.55526	-0.99697
$\nu_{BR}$ , k/h	-2.39205	0.811133	-2.94902	0.003839	-3.99817	-0.78592	-3.99817	-0.78592
<i>t</i> , s	0.977068	0.261734	3.733061	0.000292	0.458809	1.495327	0.458809	1.495327
$p_{avg}$ , bar	<b>0.595826</b>	<b>0.707493</b>	<b>0.842166</b>	<b>0.401384</b>	<b>-0.80508</b>	<b>1.996732</b>	<b>-0.80508</b>	<b>1.996732</b>
$M_{avg}$ , Nm	<b>0.020714</b>	<b>0.052575</b>	<b>0.393986</b>	<b>0.694296</b>	<b>-0.08339</b>	<b>0.124818</b>	<b>-0.08339</b>	<b>0.124818</b>
<i>a</i> , m/s <sup>2</sup>	0.190521	0.082325	2.314246	0.022369	0.027509	0.353534	0.027509	0.353534
<i>FA</i>	-0.0198	0.003499	-5.65948	1.07E-07	-0.02673	-0.01287	-0.02673	-0.01287
$\mu_{avg}$	27.58663	39.64322	0.695873	0.487865	-50.9109	106.0842	-50.9109	106.0842
$T_{DI}$ , °C	-0.26623	0.030705	-8.67055	2.58E-14	-0.32703	-0.20543	-0.32703	-0.20543
$T_{DF}$ , °C	0.362677	0.027198	13.33464	2.26E-25	0.308822	0.416531	0.308822	0.416531

**Table A.19.** The correlation coefficients between independent variables and average braking pressure (for PC).

	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	<i>t</i> (s)	<i>a</i> (m/s <sup>2</sup> )	$M_{avg}$ (Nm)	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{DF}$ (°C)	$T_{IPI}$ (°C)	$T_{IPF}$ (°C)	$T_{OPI}$ (°C)	$T_{OPF}$ (°C)	$p_{avg}$ (bar)
$\nu_{BS}$ , k/h	1												
$\nu_{BR}$ , k/h	0.8647	1											
<i>t</i> , s	-0.2314	-0.2764	1										
<i>a</i> , m/s <sup>2</sup>	0.2575	0.0204	-0.7113	1									
$M_{avg}$ , Nm	0.2795	0.0638	-0.7091	0.9957	1								
$\mu_{avg}$	0.0914	-0.1094	-0.4268	0.548	0.5257	1							
$T_{DI}$ , °C	0.0675	-0.2759	-0.0146	0.4335	0.3963	0.4605	1						
$T_{DF}$ , °C	0.3058	-0.07	-0.0628	0.5027	0.4728	0.4296	0.9552	1					
$T_{IPI}$ , °C	0.096	-0.2459	-0.0242	0.438	0.4023	0.4171	0.9914	0.9493	1				
$T_{IPF}$ , °C	0.0963	-0.2553	-0.0075	0.4322	0.3959	0.4306	0.9924	0.9499	0.9991	1			
$T_{OPI}$ , °C	0.0934	-0.2466	-0.0221	0.4333	0.3978	0.404	0.9878	0.9443	0.9994	0.9979	1		
$T_{OPF}$ , °C	0.1038	-0.2451	-0.0018	0.4273	0.3919	0.4022	0.9896	0.9518	0.9992	0.9986	0.9992	1	
$p_{avg}$ , bar	0.2764	0.0926	-0.7198	0.9725	0.9798	0.3594	0.3235	0.4082	0.3393	0.3285	0.3375	0.3299	1

**Table A.20.** Multiple regression model statistics of the factors that contributed to average braking pressure (for PC).

Regression statistics	
Multiple <i>R</i>	0.9987
<i>R</i> square	0.9974
Adjustable <i>R</i> square	0.9973
Standard error	0.7315
Observation	276

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	Significance <i>F</i>
Regression	12	53554	4462.8	8339.7	0
Difference	263	140.74	0.5351		
Total	275	53694			

	Coefficients	Standard error	<i>t</i> stat	<i>P</i> -value	Low 95%	High 95%	Low 95.0%	High 95.0%
Intersection	38.016	1.1221	33.879	6E-98	35.806	40.225	35.806	40.225
$\nu_{BS}$ , k/h	0.0517	0.0096	5.3725	2E-07	0.0327	0.0706	0.0327	0.0706
$\nu_{BR}$ , k/h	-0.0347	0.0077	-4.4995	1E-05	-0.0499	-0.0195	-0.0499	-0.0195
<i>t</i> , s	-0.4523	0.0548	-8.2472	8E-15	-0.5603	-0.3443	-0.5603	-0.3443
<b><i>a</i>, m/s<sup>2</sup></b>	<b>1.3339</b>	<b>0.8919</b>	<b>1.4956</b>	<b>0.136</b>	<b>-0.4222</b>	<b>3.0901</b>	<b>-0.4222</b>	<b>3.0901</b>
$M_{avg}$ , Nm	0.0622	0.0036	17.222	5E-45	0.0551	0.0693	0.0551	0.0693
$\mu_{avg}$	-112.26	2.5994	-43.188	2E-121	-117.38	-107.14	-117.38	-107.14
$T_{DI}$ , °C	0.034	0.0054	6.2958	1E-09	0.0234	0.0447	0.0234	0.0447
$T_{DF}$ , °C	-0.0117	0.0033	-3.5999	0.0004	-0.0181	-0.0053	-0.0181	-0.0053
<b><math>T_{IPI}</math>, °C</b>	<b>-0.0395</b>	<b>0.0431</b>	<b>-0.9148</b>	<b>0.3611</b>	<b>-0.1244</b>	<b>0.0455</b>	<b>-0.1244</b>	<b>0.0455</b>
<b><math>T_{IPF}</math>, °C</b>	<b>-0.0121</b>	<b>0.0247</b>	<b>-0.49</b>	<b>0.6245</b>	<b>-0.0609</b>	<b>0.0366</b>	<b>-0.0609</b>	<b>0.0366</b>
$T_{OPI}$ , °C	0.1019	0.0431	2.3673	0.0186	0.0172	0.1867	0.0172	0.1867
$T_{OPF}$ , °C	-0.0814	0.0354	-2.3013	0.0222	-0.1511	-0.0118	-0.1511	-0.0118

**Table A.21.** The correlation coefficients between independent variables and average braking pressure (for LCV).

	<i>I</i>	$\nu_{BS}$ (k/h)	$\nu_{BR}$ (k/h)	<i>t</i> (s)	$M_{avg}$ (Nm)	<i>a</i> (m/s <sup>2</sup> )	<i>FA</i>	$\mu_{avg}$	$T_{DI}$ (°C)	$T_{DF}$ (°C)	$p_{avg}$ (bar)
<i>I</i>	1										
$\nu_{BS}$ , k/h	-0.5387	1									
$\nu_{BR}$ , k/h	0.51574	-0.827	1								
<i>t</i> , s	0.3217	0.45572	-0.2331	1							
$M_{avg}$ , Nm	-0.1248	-0.2633	0.02363	-0.7637	1						
<i>a</i> , m/s <sup>2</sup>	-0.8166	0.41669	-0.4436	-0.3143	0.37326	1					
<i>FA</i>	0.23632	-0.5213	0.27401	-0.5021	0.53345	-0.0836	1				
$\mu_{avg}$	0.60763	-0.4212	0.35691	-0.0384	0.42343	-0.3228	0.05969	1			
$T_{DI}$ , °C	-0.085	0.72165	-0.5822	0.55409	-0.3209	-0.0586	-0.2746	-0.1878	1		
$T_{DF}$ , °C	0.0639	0.72529	-0.5727	0.65602	-0.3722	-0.1801	-0.2658	-0.1538	0.95168	1	
$p_{avg}$ , bar	-0.72	0.24735	-0.4085	-0.6177	0.56302	0.68695	0.39528	-0.4845	-0.0108	-0.111	1

**Table A.22.** Multiple regression model statistics of the factors that contributed to average braking pressure (for LCV).

		Regression statistics							
		Multiple $R$	0.993268						
		$R$ square	0.986581						
		Adjustable $R$ square	0.985454						
		Standard error	0.61742						
		Observation	130						
		ANOVA							
		$df$	$SS$	$MS$	$F$	Significance $F$			
Regression		10	3335.259	333.5259	874.9183	2.3E-106			
Difference		119	45.36376	0.381208					
Total		129	3380.623						
	Coefficients	Standard error	$t$ stat	$P$ -value	Low 95%	High 95%	Low 95.0%	High 95.0%	
Intersection	16.18699	1.892729	8.5522	4.87E-14	12.4392	19.93478	12.4392	19.93478	
$I$	-0.09253	0.022093	-4.18834	5.42E-05	-0.13628	-0.04879	-0.13628	-0.04879	
$\nu_{BS}$ , k/h	<b>0.009944</b>	<b>0.011807</b>	<b>0.842166</b>	<b>0.401384</b>	<b>-0.01344</b>	<b>0.033323</b>	<b>-0.01344</b>	<b>0.033323</b>	
$\nu_{BR}$ , k/h	<b>-0.11271</b>	<b>0.108055</b>	<b>-1.04312</b>	<b>0.299009</b>	<b>-0.32667</b>	<b>0.101246</b>	<b>-0.32667</b>	<b>0.101246</b>	
$t$ , s	0.192672	0.031068	6.201687	8.36E-09	0.131155	0.254189	0.131155	0.254189	
$M_{avg}$ , Nm	0.065653	0.003158	20.79209	1.98E-41	0.0594	0.071905	0.0594	0.071905	
$a$ , m/s <sup>2</sup>	-0.05438	0.009662	-5.62858	1.23E-07	-0.07351	-0.03525	-0.07351	-0.03525	
$FA$	0.001019	0.000501	2.036694	0.0439	2.83E-05	0.002011	2.83E-05	0.002011	
$\mu_{avg}$	-50.7501	2.165956	-23.4308	2.12E-46	-55.0389	-46.4613	-55.0389	-46.4613	
$T_{DI}$ , °C	0.022117	0.004644	4.762815	5.44E-06	0.012922	0.031312	0.012922	0.031312	
$T_{DF}$ , °C	-0.01913	0.005265	-3.63266	0.000415	-0.02955	-0.0087	-0.02955	-0.0087	

## Biographies

**Abdullah Demir** was born in Trabzon, Turkey in 1973. He received an MS degree from Marmara University in 1997 and PhD degree from Kocaeli University in 2009. He is now an Assistant Professor at the Mechanical Engineering Department of Marmara University. His research interests include power-train, vehicle test technique, transportation, and car-parking management.

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