



Experimental evaluation of shape factor of axis-symmetric sunken structures

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KEYWORDS

Thermal simulation;
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Abstract. This paper investigates the dependency of shape factor of the axis-symmetric fully sunken structures (viz. cubical, square prismatic, pyramidal, and cylindrical) in terms of their depth and orientation. Experimental evaluations of the shape factor in reduced-scale models were carried out at laboratory using thermal simulation method in different conditions. The method was used to determine shape factor, which can be used to determine heat loss from ground to structure or structure to ground fully sunken with different orientations. Maximum and minimum values of the shape factor in set-I and set-II conditions were calculated as 90.18 and 9.93, respectively. In set-II condition, the value varied from 16.49 to 35.28. At $D/L = 2$, the shape factor in set-VI condition (17.26%) was compared to that in set-VII condition. Similarly, the shape factor in set-IX (33.47%) was compared to that in set-VIII condition. This comparison helps design a better building structure of fully buried nature to ensure higher thermal comfort.

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1. Introduction

Evaluating heat transfer between the ground and an earth-coupled structure is essential for a rational thermal design of the latter to ensure better human comfort. The solution to the corresponding three-dimensional Fourier equation of thermal conduction with relevant boundary conditions is in general very difficult to find, if at all possible; the analytical solutions of highly symmetrical structures can be obtained only for structures with high symmetry like a sphere or an infinite cylinder with horizontal axis [1,2]. Evaluation of the heat transfer through building structure to the

surrounding ground requires three-dimensional heat conduction relation with the appropriate boundary conditions to compute heat transfer between structure and earth surface. Numerical methods for few axis-symmetric (like cylinder and sphere) sunken structures are available, but a long computational time is required [3,4]. Effects of different earth-surface treatments of the surrounding surface for heating and cooling of earth-integrated building structures were reported by Deshmukh et al. [5] and Martinopoulos [6]. The theoretical basis of dynamic simulation (arbitrary time variation) of heat transfer between ground and structure using electrical simulation for fully buried structures of periodic nature was presented by Sodha [7]. These suggested methods are needed to adjust reduced-scale models for simulation purposes, which can further scale up in case of a larger realistic situation [7]. Mishra et al. [8] validated the basis of the experimental simulation of heat transfer between

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ground and structure, proposed by Sodha [9,10]. Geometrical optimization of heat storage unit using shape factor of structure was investigated by Solé et al. [11]. Experimental evaluation of heat transfer between fully or partially scaled buried structures and ground was not feasible due to the high costs; moreover, significant variations in size and shape of the structure would make a specific experiment somewhat relevant to the real problem. The application of wall-to-excavation shape factor concept to the preliminary design of deep cement mixing walls for identifying the cause of excavation failure was studied by Waichita et al. [12]. Effect of dam geometry and settlement of rock fill dams based on shape factor using finite element analysis was analyzed by Sukkarak et al. [13]. Dependence of berming, the dependence on the slope of berming, was reported by Sodha and Mishra [14]. Computational Fluid Dynamics (CFDs) simulation and analysis of the temperature distribution within the green house as well as solar heat gain and heat loss using shape factor were carried out by Tong et al. [15]. Shape factors of three different designs and their optimization using Response Surface Methodology (RSM) and validation with P-test for the steady state heat transfer between swimming pool water and surrounding ground were investigated by Somwanshi et al. [16]. Many different researchers employed shape factor of device/equipment to analyze the heat transfer from surface to ambient or from ambient to surface [17–22]. Evaluation of heat transfer from the fully sunken structure to the surrounding ground, or from surrounding ground to sunken structure, is a complex task.

Although fully sunken building structures are widely popular to consider in designing bunkers and partially sunken for high-rise buildings, one has to adopt approximation of doubtful merits for the evaluation of heat transfer from fully sunken structures to the surrounding ground or surrounding grounds to sunken structures. In brief, numerical simulation is difficult to perform and highly time-consuming in realistic cases. This drawback justifies the possibility of conducting thermal simulation experiments in order to evaluate heat transfer from fully sunken structures to the ground on a reduced-scale structure in a small time duration. The shape factor of fully buried structures with symmetrical axes was determined using thermal simulation method in this paper. Dependence of the shape factor corresponding to depth and orientation was investigated in this paper.

2. Experimental setup

A reduced-scale model involving dry fine silica sand in a wooden box with $1 \times 1 \times 1$ m dimensions (Figure 1a) was treated as a semi-infinite medium. A hollow sphere made of the copper material having 98% purity (chosen

due to large thermal conductivity) was energized by the incandescent lamp used for evaluating the thermal conductivity of silica sand. A constant DC power source of 24 V was considered.

The standard symmetrical structures made of a copper sheet of thickness 1.5 mm were modeled, i.e., hollow copper sphere, to determine the thermal conductivity of silica sand.

An incandescent lamp was placed inside a box in all the cases; then, normally after 8 – 10 hours, the steady-state difference in the temperature of the surface of the structures and the simulated earth surface was noted (by a temperature indicator called DTI-039T) with respect to eight values of the power consumption by the lamp (product of the current and potential differences).

The actual photograph and schematic arrangement of the experimental setup are illustrated in Figures 1a and 1b, respectively. Shape factor is a parameter used for predicting the thermal behavior of axis-symmetric sunken structures with different depths and orientations. Figures 2 and 3 represent a photograph of a cubical structure characterized by 10 cm and 5 cm sides of square prism with a 10 cm long axis. Figures 4a and 4b represent the photograph of triangular and square pyramids of 5 cm base side and 10 cm long axis. A photograph of the cylindrical structure characterized by a 5-cm base diameter and a



Figure 1a. Actual photograph of the experimental setup.

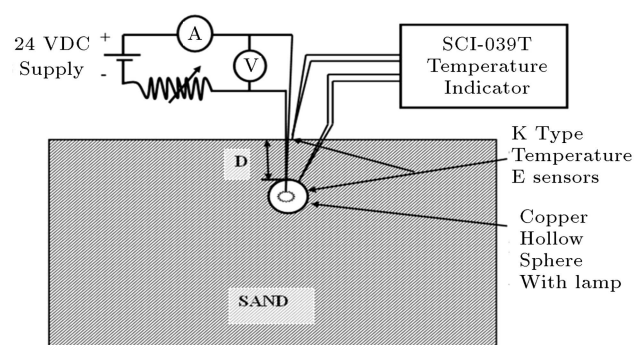


Figure 1b. Schematic diagram for determining thermal conductivity of sand.

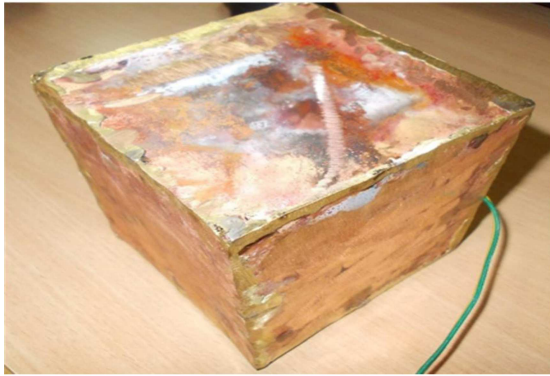


Figure 2. Cubical structure.



Figure 3. Square prismatic structure.

10 cm axis length is shown in Figure 5. The rate of heat transfer through a sphere deep in earth surface and the shape factor of the hollow sphere at a radius of 0.015 were used as the characteristics required for performing simulation [13,14]. Heat transfer through the structure can be evaluated using Eq. (1) as follows:

$$\dot{Q} = F' r K (D/r) (T_s - T_g), \quad (1)$$

where:

$$F' = 4\pi \left[\frac{1}{2} + e^{-\beta} \{1 + \cosh \beta\} + e^{-3\beta} \{1 + \cosh 3\beta\} \frac{\sinh \beta}{\sinh 3\beta} + e^{-5\beta} \{1 + \cosh 5\beta\} \frac{\sinh \beta}{\sinh 5\beta} + \dots \right]. \quad (2)$$

Rearranging Eq. (1), we can get:

$$K = \dot{Q} / [r F' (D/r) (T_s - T_g)]. \quad (3)$$

A different set of experiments was carried out to determine the shape factor of structures with different orientations.

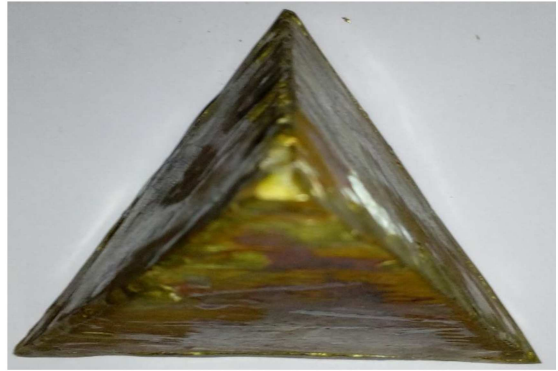


Figure 4a. The triangular pyramid structure.



Figure 4b. The square pyramid structure.



Figure 5. The cylindrical model.

3. Observation, result, and discussion

The thermal conductivity K of sand is in general dependent on the temperature; however, within the range of parameters of interest, the variation was less than the accuracy of determination of K . In any case, the experimental simulation does not allow for the temperature-dependent thermal conductivity. The thermal conductivity of simulating media (sand) was experimentally evaluated using thermal simulation method and tabulated in Table 1.

Based on six continuous observations and Eq. (2), the variation of the shape factor of a hollow copper cube with 10 cm long side with response to different

Table 1. Determination of thermal conductivity of sand.

Series no.	D/r	F	V (volts)	I (amp.)	T_s ($^{\circ}\text{C}$)	T_g ($^{\circ}\text{C}$)	K (W/mK)
01	1	16.86	12.01	0.32	62	40	0.69
02	2	15.31	12.01	0.34	64	36	0.64
03	3	14.46	12.03	0.35	65	32	0.59
04	4	13.94	12.06	0.37	68	30	0.56
05	5	13.69	12.02	0.37	66	29	0.59
06	6	13.60	12.61	0.42	69	28	0.63

depths and orientations of the structure was calculated as $0.62 \pm \frac{0.07}{0.06}$ W/mK by using different sets of the condition in Figure 6.

Three different sets of conditions for the cubical structure were used to determine the shape factor. The maximum value of shape factor was recorded as 90.18 at the D/L ratio of 2 for set-I cubical structure placed on the horizontal surface. The minimum value of F' was calculated as 9.93 for the set-II cubical structure placed in HP and about one of the corners and its edges were equally inclined to HP at the D/L ratio of 10. F'' varied from 35.28 to 16.49, while the experimentation for set III was equally inclined to HP and end surfaces were perpendicular to the HP when the cubical structure was placed on its edge and lateral surfaces.

Shape factor of the cubical structure for set-I, set-II, and set-III conditions was determined through Eqs. (4), (6), and (8), respectively:

$$F = 0.1971(D/L)^2 - 5.0624(D/L) + 48.927, \quad (4)$$

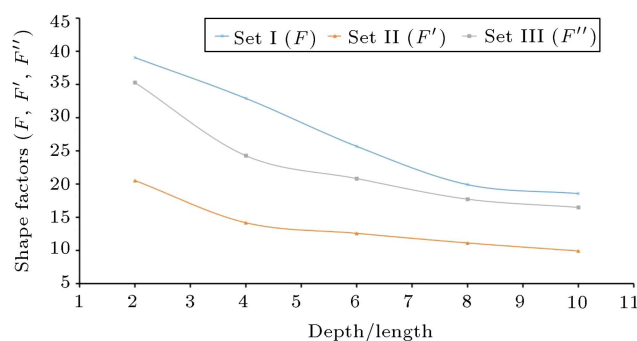
$$R^2 = 0.99, \quad (5)$$

$$F' = 0.1866(D/L)^2 - 3.4513(D/L) + 26.168, \quad (6)$$

$$R^2 = 0.9959, \quad (7)$$

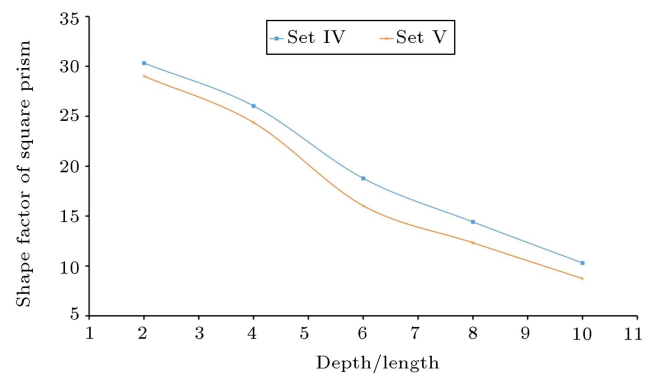
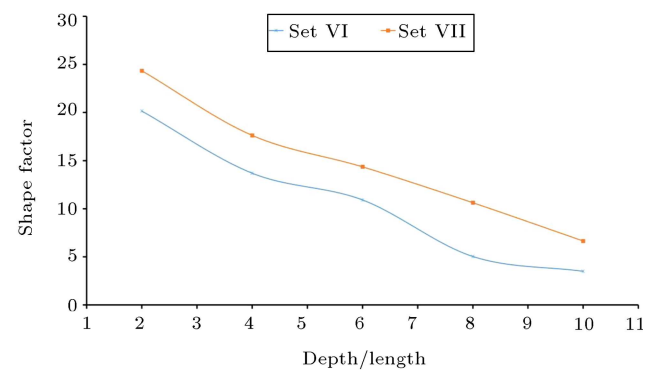
$$F'' = 0.3552(D/L)^2 - 6.469(D/L) + 46.11, \quad (8)$$

$$R^2 = 0.981. \quad (9)$$

**Figure 6.** Variation of shape factor with respect to different depths for sets I, II and III of cubical structure.

The shape factor of the square prism of base side 5 cm and 10 cm long axis of set-IV and set-V square prismatic structures placed about its lateral surface and edge of lateral surface and axis kept parallel to the horizontal surface, respectively. Experimental result of the evaluation of shape factor is shown in Figure 7.

Experimental results of the fully sunken structure at different D/L ratios and orientations are shown in Figure 8. The maximum value of shape factor for set-IV condition was measured as 30.32 at the D/L ratio of 2, and the minimum value of shape factor at the ratio of 10 in the set-V condition was calculated as 8.73. The shape factor at the D/L ratio of 10 for set IV was found 15.32% higher than that for set V.

**Figure 7.** Variation of shape factor with the depth of square prism.**Figure 8.** Variation of shape factor with the depth of triangular (set VI) and square (set VII) pyramids.

The factor indicates the maximum comfort provided by the underground structure of the square prismatic structure in the fully buried case with this relation and orientation. Eqs. (10) and (12) were employed for theoretical evaluation of shape factor for the IV and V sets of conditions, respectively:

$$F_{IV} = 0.058(D/L)^2 - 3.2779(D/L) + 37.086, \quad (10)$$

$$R^2 = 0.9924, \quad (11)$$

$$F_V = 0.1204(D/L)^2 - 4.073(D/L) + 37.24, \quad (12)$$

$$R^2 = 0.9867. \quad (13)$$

Thermal simulation result of shape factor for the triangular and square pyramids is shown in Figure 8 as set-VI and set-VII conditions with their base being placed on the horizontal surface and an axis perpendicular to it.

The maximum value of the shape factor in the set-VI condition was recorded at 24.35. Among triangular and square pyramid structures, best comfort can be achieved in case of a square pyramid (set VII), which was 3.49 at $D/L = 10$. At the $D/L = 2$, the shape factor in the set-VI condition was 17.26%, which remained the same throughout the thermal simulation experimentation. Maximum deviation of 52.68% was observed at $D/L = 2$ in the set-VI condition, compared to the set VII. Eqs. (13) and (15) are the empirical relations obtained from the experimental result, which can be utilized for the numerical evaluation of shape factor of the structure in set-VI and set-VII conditions, respectively.

$$F_{VI} = 0.1204(D/L)^2 - 3.5433(D/L) + 26.618, \quad (14)$$

$$R^2 = 0.9839, \quad (15)$$

$$F_{VII} = 0.0896(D/L)^2 - 3.1957(D/L) + 29.954, \quad (16)$$

$$R^2 = 0.9912. \quad (17)$$

Experimental result of the cylindrical structure is shown in Figure 9 for two different sets of conditions, namely sets VIII and IX.

Set VIII indicated that the cylindrical structure kept about its end surface on the horizontal plane and set IX resembled the condition placed on its lateral surface on the horizontal plane at different D/L ratios. At a ratio of $D/L = 2$, the shape factor of set IX was 33.47% higher than that in the set-VIII condition. The set-IX condition provided higher shape factor value throughout the experimentation than the set-VIII condition. It was indicated that the set-VIII

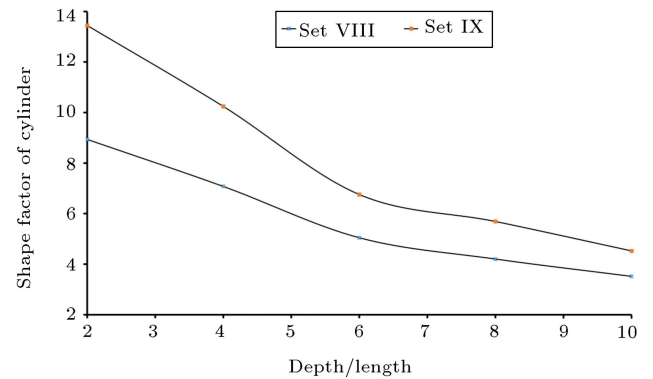


Figure 9. Variation of shape factor with the depth of cylinder placed on its end surface (set VIII) and on its lateral surface (set IX).

condition was optimal for creating thermal comfort for the fully buried cylindrical structure. The minimum value of 3.52 was recorded for it at a ratio of 10. For set-VIII and set-IX conditions, Eqs. (18) and (20) were employed to evaluate the shape factor of the structure, respectively:

$$F_{VIII} = 0.0633(D/L)^2 - 1.4454(D/L) + 11.644, \quad (18)$$

$$R^2 = 0.9953, \quad (19)$$

$$F_{IX} = 0.116(D/L)^2 - 2.5123(D/L) + 18.099, \quad (20)$$

$$R^2 = 0.9926. \quad (21)$$

4. Conclusion

The shape factor of the fully buried structure with symmetrical axes was found to be dependent on D/L ratio. It is an essential input parameter for the evaluation of thermal performance of fully sunken structure. After determining the shape factor of the fully buried structure, heat transfer from ground to structure or structure to ground was easily evaluated. This approach would help design a fully buried building structure to ensure better thermal comfort.

Nomenclature

D	Depth of the model below earth surface, m
F	Shape factor of cubical structure placed about its lateral surface on horizontal plane, set-I condition (dimensionless)
F'	Shape factor of cubical structure placed about its edge and lateral surfaces equally inclined to horizontal surface in the set-II condition (dimensionless)

F''	Shape factor of cubical placed on horizontal surface about its corner such that all edges are equally inclined to it in the set-III condition (dimensionless)
F_{IV}	Shape factor of square prismatic structure placed about its lateral surface in the set-IV condition (dimensionless)
F_V	Shape factor of square prismatic structure placed on horizontal plane about its one of the edge of the lateral surface in the set-V condition (dimensionless)
F_{VI}	Shape factor triangular pyramid structure placed about its base on horizontal plane in the set-VI condition (dimensionless)
F_{VII}	Shape factor square pyramid structure placed about its base on horizontal plane in the set-VII condition (dimensionless)
F_{VIII}	Shape factor of cylindrical structure placed about its end surface on horizontal plane in the set-VIII condition (dimensionless)
F_{IX}	Shape factor of cylindrical structure placed about its lateral surface on horizontal plane in the set-IX condition (dimensionless)
I	Current on heating bulb or element, A
K	Thermal conductivity of ground or sand, W/m°C
L	Characteristic length of object, m
\dot{Q}	Rate of heat loss from building to ground, W
r	Radius of the sphere, m
T_s	Temperature of surface of building or model, °C
T_g	Temperature of the earth surface, °C
V	Voltage across the heating element or bulb, V
β	$= \cosh^{-1}(D/R)$

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Biography

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