Analytical Model of Recirculation Influence on Premixed Combustion of Lycopodium Dust Particle: Studying Thermal Resistance and Random Distribution of particles

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Abstract

In the present study, a comprehensive analytical method is developed to model the flame propagation through organic particles with air as a two-phase mixture, considering random distribution and particles thermal resistance. For this purpose, the structure of flame contains a preheat-vaporization zone, a reaction zone where vaporization and convection rates of particles are negligible and a post flame zone where diffusive terms are negligible in comparison of other terms zone. In order to enhance the combustion efficiency, the exhausted heat from the post flame zone is recirculated back to the preheat zone. Since this stream consists of high temperature gaseous mixture, it can enhance the temperature of the initial two-phase mixture entering the combustion chamber. A reasonable agreement between the results of the analytical approach and the experimental findings was obtained. Apart from the randomness distribution of particles and heat recirculation phenomena, the effect of thermal resistance on the combustion properties such as burning velocity and flame temperature is studied through non-zero Biot numbers in this model. Additionally, the variation of several parameters including equivalence ratio, particle diameter and Lewis number are studied in this research.

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1. Introduction

Dust cloud explosion had been a serious threat to mankind and their properties for the last 150 years. There is a high risk of explosion when combustible dust is suspended in the air in particular circumstances. This phenomenon is common and can cause high costs in wide-ranging industries. Therefore, a precise understanding of the origin, prevention and mitigation of dust explosions is vital. Several materials trigger this event such as coal, peat and metals [1-3]. Recent developments in technology and industries have led to a growing need for applicable energy. Biomass has been identified as a renewable energy resource and a good candidate as an alternative fuel [4-6]. In this mathematical work, lycopodium is considered as the volatile fuel especially due to its appreciable flammability and dispensability. Lycopodium has been known as a reference particle in organic dust combustion researches since being mono-size is a fundamental supposition in analytical models. Moreover, the amount of unfavorable residues produced by lycopodium during combustion is unimportant. A lycopodium particle generally contains about 50% fat oil, 24% sporopollenin, and 2% sucrose. In combustion processes, the particle misses some part of its mass during flame propagation since lycopodium particle is included volatile material. [7, 8]. Within the last few years, many studies have been done regarding combustion of organic dust particles. Bidabadi et al. [9] proposed a novel mathematical model to combustion of micro organic dust particles. In their study, the effect of random combustion, Lewis number and size of particles on the combustion properties of biofuel particles such as burning velocity, flame temperature and effective equivalence ratio were investigated. Mostafavi et al. [10] studied the experimental and mathematical modeling for
lycopodium dust particles. Extensive researches have been conducted by Proust [11]. That research was focused on the initiation and propagation of flame in dust clouds to provide a more in-depth understanding of the incidence of thermal radiation and turbulence in dust explosion. Continuing on this particular subject, the laminar flame speeds and flame temperatures were measured for several types of combustible dust-air mixtures [12]. Following this research with the more emphasis on the lycopodium as a combustible dust particle, Han et al. [13] empirically analyzed the flame propagation mechanism and dust particles behavior in a vertical duct. Some parametric studies were conducted by Daou [14] to realize the influence of heat loss, preferential diffusion and reversibility of reaction. Until 2009, there was no research addressing the recirculation effect in dust particles. Bidabadi et al. [15] analyzed this importance in a comprehensive manner. They developed a model to consider this effect in dust particles. Besides, Bidabadi and Rahbari [16] studied the influence of temperature alteration between particle and gas in a combustible mixture of homogeneous distribution of volatile fuel particles. In another study, Rockwell and Rangwala [17] scrutinized a premixed dust-air flame in existence of a homogeneous gas-phase reaction front. Moghadasi et al. [18] studied combustion of organic particles in a counterflow arrangement considering porosity and thermal radiation effects. Soltaninejad et al. [19] analyzed micro-organic dust combustion cogitating particles’ thermal resistance. In their model, it was presumed that gaseous fuel mixture was formed as a result of particle pyrolysis. Reviewing the existing literature indicates that few attempts have been made to numerically determine the non-uniform distribution of combustible dust-air mixture. This study reflects the combustion of micro-organic particles considering the random distribution of particles as well as particles’ thermal resistance. The particles have been dispersed in the preheat zone arbitrarily, presuming that the fuel particles volatilize initially to yield a gaseous fuel. The
flame structure is separated into three zones as exhibited in Fig. 1. The first region is a preheat-vaporization zone where the chemical reaction rate is neglected. The second one is a reaction zone where the volatile fuel particles are combusted in the gaseous phase. In this zone, the vaporization rate and convection terms are ignorable based on the order of magnitude. The third is a post flame zone where chemical reaction and vaporizations rates are supposed to be smaller than other terms. The exhaust heat recirculation from the post flame zone to the preheat zone is modeled in this research (leading to enhancement in the combustion performance of the process). The role of the particles’ thermal resistance is integrated into the governing equations via the dimensionless Biot number \((Bi = hr/\lambda)\). The Biot number states the easiness of heat flow through the interface of the powder deposit and its surroundings, relating to the easiness of heat conduction through the powder. The features of flame propagation and the structure of combustion zone with the impact of non-unity Lewis number and heat recirculation are explored mathematically in order to elucidate the flame propagation mechanisms in the organic dust clouds. Evaluating the accuracy of the present model will be made by comparing the burning velocity obtained from this model with the empirical and analytical data [11, 20, 21].

Fig. 1

2. Theoretical Analysis

In this study, it is assumed that the fuel particles volatilize first to shape an identified gaseous compound. The vaporization kinetic is formulated as [21]:

\[ \]
\[ \omega_v = \exp(-Bi)A_n 4\pi r^2 (T - T_u)^n \]  

(1)

where \( \omega_v \) is the rate of vaporization of fuel particles. The constant quantities \( A \) and \( n \) are considered to be known. One-step overall reaction is used to model the combustion process as:

\[ v_F[F] + v_{O_2}[O_2] \rightarrow v_P[P] \]  

(2)

where \( F, O_2, P \) signify the fuel, oxygen, product and \( v_F, v_{O_2}, v_P \) are their stoichiometric coefficients. The characteristic of Zeldovich number is obtained by:

\[ Ze = \frac{E_a(T_f - T_u)}{RT_f^2} \]  

(3)

where \( E_a, T_f, T_u \) and \( R \) are the activation energy, flame temperature, fresh mixture temperature and universal gas constant, respectively.

- Mass conservation equation:

\[ \rho v = \text{const} \]  

(4)

- Energy conservation equation:

\[ \rho v C \frac{dT}{dx} = \lambda_u \frac{d^2T}{dx^2} + \omega_v \frac{\rho_u}{\rho} Q - \left( \omega_v \frac{\rho_u}{\rho} Q_v + \text{sign}(x) \frac{\rho_u}{\rho} Q_L \right) \]  

(5)

where:

\[ \omega_F = v_F W_F C_F k_r \]  

(6)
\[ k_r = B_r \exp \left( -\frac{E_a}{RT_r} \right) \]  \hspace{1cm} (7)

where \( Q_v \) is the amount of heat required for vaporizing the solid particles and \( Q_L \) is symbolizing the amount of heat exchanged between the gas mixture and its surrounding estimated by:

\[ Q_L = k_2 (T - T_w) \]  \hspace{1cm} (8)

In this equation, \( T \), \( T_w \) and \( k_2 \) denote the mixture temperature, wall temperature and the ratio of convective heat transfer coefficient per chamber width. The total heat capacity of mixture can be outlined as below [21]:

\[ C = C_p + \left( \frac{4}{3} \pi R^3 \right) C_s \rho \frac{n_s}{\rho} \]  \hspace{1cm} (9)

In which \( C_p \) and \( C_s \) are the heat capacity of gaseous phase in the mixture and solid particles, respectively. The dimensionless parameters are implemented in the governing equations as:

\[ \theta = \frac{T - T_w}{T_f - T_w} , \quad y_F = \frac{Y_f}{Y_{FC}} , \quad y_s = \frac{Y_s}{Y_{FC}} \]  \hspace{1cm} (10)

\[ \xi = \frac{\rho v}{\rho_a v_a} , \quad z = \frac{\rho_a v_a C}{\lambda_a} \chi , \quad D = \frac{T_f - T_w}{T_f - T_w} \]

In which \( v_a \) is the burning velocity and \( Y_{FC} \) is defined as:

\[ Y_{FC} = \frac{C}{Q} (T_f - T_a) \]  \hspace{1cm} (11)

Introducing the aforementioned parameters into Eqs, (4) and (5) results in:
\[ \xi \frac{dy_x}{dz} = -Dy^3_{y^2} \theta'' \quad (12) \]

\[ \xi \frac{dy_y}{dz} = \frac{1}{Le} \frac{d^2 y_y}{dz^2} - \omega D \frac{\rho_y}{\rho} + Dy^3_{y^2} \theta'' \quad (13) \]

\[ \xi \frac{d \theta}{dz} = \frac{d^2 \theta}{dz^2} + \omega \frac{\rho_u}{\rho} - qy^3_{y^2} \theta'' - k \theta \text{sign}(z) \quad (14) \]

In which:

\[ \omega = \frac{\lambda_u \omega_F}{D \rho_u y^2_u} \quad , \quad \gamma = 4.836 \left( \frac{\sqrt{n_y}}{\sqrt{\rho_u \rho_y y_{FC}}} \right) A \lambda_u (T_f - T_u) \frac{v_u^2 CD}{\nu^2} \]

\[ q = \frac{Q_y}{Q} \quad , \quad k = \frac{h HPe^{-2}}{\lambda_u} \quad , \quad Le = \frac{\lambda_u}{\rho_u CD_u} \]

The heat recirculation coefficient is shown by parameter \( k \) in the above equation. In order to calculate the combustion parameters, the non-dimensional differential equations should be solved for all zones. Bear in mind that the quantity \( m \) can be considered as unity for solving these equations.

**2.1 preheat zone \( \{ x| -\infty < x < 0^- \} \)**

To solve equations, it is necessary to apply the boundary conditions. The required boundary conditions for solving conservation equations in this zone are as below:
The Zeldovich number is considered large enough in this zone, therefore it is possible to disregard the chemical reaction between the fuel and oxidizer in this zone. The amount of heat released by combustion is much larger than the heat needed for vaporization of solid particles ($q = 0$). The following correlations can be achieved by implementing the above assumptions and using the required boundary condition:

$$\theta^0 = \exp(k_1 z) \quad , \quad (k_1 = 0.5 + 0.5(1 - 4k)^{0.5}) \quad \quad \quad (17)$$

$$y_s = \left(\alpha^3 - a\exp(nk_1 z)\right)^3 \quad , \quad a = \frac{\gamma D}{3nk_i} \quad \quad \quad (18)$$

$$-\frac{dy_F}{dz}\Big|_{z = 0^-} = Le\left(3\exp(-Bi)\alpha^2 - 3\exp(-2Bi)\alpha^3 + \exp(-3Bi)\alpha^4 - y_{ef}\right) \quad \quad \quad (19)$$

In these equations, $\theta^0$ represent the value of $\theta$ calculated neglecting heat of vaporization of particles.

**2.2 Post flame zone** $\{x|0^+ < x < +\infty\}$
Solving the governing equations in this zone provides the appropriate boundary conditions for the second zone (reaction zone). As mentioned previously, the diffusive term can be omitted in this zone. Considering this assumption leads to the below correlations:

\[
\frac{d\theta^0}{dz} = -k\theta^0 \rightarrow \frac{d\theta^0}{dz} \bigg|_{z=0^+} = -k
\]

\[
\frac{dy_F}{dz} = 0 \rightarrow \frac{dy_F}{dz} \bigg|_{z=0^+} = 0
\]

\[
\frac{dy_s}{dz} = 0 \rightarrow y_s \big|_{z=0^+} = \text{const}
\]

**2.3 Reaction Zone \( \{x|0^- < x < 0^+\} \)**

In this zone, the convective and the vaporization terms can be ignored. The thickness of this zone is so thin. Therefore, an expansion procedure is applied to solve this zone. To achieve this objective, the following parameters are led into the differential equations:

\[
z = \varepsilon \eta \quad , \quad y_F = \varepsilon (b + y) \quad , \quad \theta^0 = 1 - \varepsilon t
\]

where \( b = \frac{y_F}{\varepsilon} \) and \( \varepsilon = \frac{1}{Ze} \). Conferring to the supposition of large Zeldovich number, \( \varepsilon \) would be too small. Introducing these parameters into Eq. (21) associated with the stated assumptions yields to:

\[
\frac{d^2t}{d\eta^2} = \varepsilon \omega \frac{\rho_v}{\rho}
\]
\[ \frac{d^2 y}{d\eta^2} = \varepsilon \omega D L e \frac{\rho_u}{\rho} \]

Introducing \( \omega \) from Eq. (15) into the above equation results in:

\[ \frac{d^2 t}{d\eta^2} = \Lambda (b + y) \exp(-t) \quad (23) \]

where:

\[ \Lambda = \frac{\lambda_u v_f B \varepsilon^2}{D \rho_a v_a^2 C} \exp\left(-\frac{E_a}{RT_f}\right) \quad (24) \]

By combining Eqs. (23) and (24), one can obtain:

\[ \frac{d^2 (y - L e D t)}{d\eta^2} = 0 \quad (25) \]

The required boundary conditions are as below:

\[ \begin{bmatrix} \frac{dt}{d\eta} \end{bmatrix}_{\eta \to \infty} = -\begin{bmatrix} \frac{d\theta^0}{dz} \end{bmatrix}_{z \to 0^+} = k \]

\[ \begin{bmatrix} \frac{dy}{d\eta} \end{bmatrix}_{\eta \to \infty} = -\begin{bmatrix} \frac{dy_f}{dz} \end{bmatrix}_{z \to 0^+} = 0 \quad (26) \]

\[ \begin{bmatrix} \frac{dt}{d\eta} \end{bmatrix}_{\eta \to -\infty} = -\begin{bmatrix} \frac{d\theta^0}{dz} \end{bmatrix}_{z \to 0^-} = k_1 \]

Solving Eq. (25) leads to the following expression:
\[ y = LeDt \]  

(27)

Finally, by using this expression and integrating the above equations, we would have:

\[ 2\alpha \Lambda (b + LeD) = k_i^2 + k^2 \]  

(28)

**2.4 Flame temperature and burning velocity**

The analytical expression for flame speed (burning velocity) is given by:

\[ v_u^2 = \frac{2(b + LeD)\lambda_a \nu_F B \varepsilon^2}{D \rho_u C \left( k_i^2 + k^2 \right)} \exp \left( -\frac{E_a}{RT_f} \right) \]  

(29)

Using the matching condition in the border of the reaction zone can provide the flame temperature as:

\[ \left[ \frac{d\theta^0}{d\xi} + \frac{1}{Le} \frac{dy_F}{d\xi} \right]_{\xi \to 0^+} = \left[ \frac{d\theta^0}{d\xi} + \frac{1}{Le} \frac{dy_F}{d\xi} \right]_{\xi \to 0^+} \]  

(30)

Hence:

\[ 3\exp(-Bi)a\alpha^3 - 3\exp(-2Bi)a^2\alpha^3 + \exp(-3Bi)a^3 + k + k_i = 0 \]  

(31)

Although the above expression obtains the flame temperature, its value must be limited by the adiabatic temperature (considering that the oxidizer is consumed at the end of the process).

The following expression calculates the adiabatic temperature:

\[ C(T_b - T_u) = \frac{v_F W_F Q}{v_{O_2} W_{O_2} Y_{O_2,u}} \]  

(32)

Until now, the whole solution procedure was based on neglecting the parameter \( q \).

Considering this parameter alters the burning velocity in the following formula:
\[ v_v = v_u \exp(-0.5qZe) \]  

(33)

3. Random model

Since the dispersion of the particles is not uniform in the flame front, the combustion properties including flame temperature, flame speed and the amount of released energy are subject to change in this region. The released energy in the reaction zone, controlled by the source term in the energy equation, is a function of net available fuel in gaseous phase that varies in the flame front because of randomness behavior of the fuel. The amount of this energy is dependent on the following parameters:

- **Size**

It has been found that the fuel particles with different sizes and shapes directly affect the combustion performance of the process. Here, it is assumed that the fuel particles are mono-sized [20].

- **Number density**

This parameter is explained as the total number of particles per unit volume by:

\[ n_s = \frac{N_s}{V} \]  

(34)

The equivalence ratio is calculated by:

\[ \phi_u = \left( \frac{Y_{Fu}}{Y_{air}} \right) f \left( \frac{Y_{Fu}}{Y_{air}} \right)_{st} = \frac{Y_{Fu}}{1 - Y_{Fu}} \left( \frac{v_F W_F}{v_{air} W_{air}} \right)_{st}^{-1} \]  

(35)

At the beginning of the preheat zone, parameters \( Y_{Fu} \) and \( Y_s \) are equal. Therefore, the equivalence ratio is given by:
\[ \phi_u = \frac{Y_s}{1 - Y_s} \left( \frac{v_p W_F}{v_{air} W_{air}} \right)_{st}^{-1} \]  

(36)

where \( Y_s \) is obtained from Eq (37):

\[ Y_s = \left( \frac{4}{3} \pi r^3 \right) \frac{n_s \rho_s}{\rho} \]  

(37)

It is assumed that the fuel particles volatilize to form methane [21]. Hence:

\[ \left( \frac{v_p W_F}{v_{air} W_{air}} \right)_{st}^{-1} = 17.12 \]  

(38)

Hence:

\[ n_s = \left( \frac{4}{3} \pi r^3 \right)^{-1} \frac{\rho \phi_u}{\rho_s (17.12 + \phi_u)} \]  

(39)

In an ideal case, not only the dispersion of these particles in the flame front is uniform, but also the condition of these particles is the same in the preheat zone. However, none of these hypotheses can be applied in the real case. Since the temperature changes from ambient to flame temperature in the preheat zone, particles are subjected to various temperatures determined by the gradient of temperature. Consequently, the amount of volatilized fuel from each particle would differ remarkably. Fig. 2 shows schematic of random distribution of particles (in preheat-vaporization zone).
In order to model the random distribution of the particles in the flame front, the source term in the energy equation needs to be amended. For this purpose, several groups of particles are assumed in which each group contains random amount of particles with a random temperature between flame temperature and ambient in the preheat zone. The total available fuel is calculated as the sum of volatilized fuel from each group as shown in Eqs. (40) and (41) [22]:

$$n_{s,i} = \sum_{j=1}^{g} n_{i,j}$$  \hspace{1cm} (40)

$$\omega_{r,i} = \sum_{j=1}^{g} \omega_{x,j} = \sum_{j=1}^{g} 4\pi r^2 A_n r T_j^n \hspace{1cm}, \hspace{0.5cm} T_u < T_j < T_f$$  \hspace{1cm} (41)

The reaction rate is obtained from the following equation:

$$\omega_f = \sum_{j=1}^{g} \omega_f = v_j k_r \sum_{j=1}^{g} \omega_{x,j}$$  \hspace{1cm} (42)

Additionally, the source or generation term is formulated as:

$$E_{gen} = \sum_{j=1}^{g} \frac{\rho_u}{\rho} Q \omega_f$$  \hspace{1cm} (43)

Where parameter $g$ signifies the number of particle groups. The number density in all groups varies from zero to $n_{st}$. In uniform distribution of particles, all particles have identical behaviors. Therefore, there is one group for all particles with the number density of $n_{st}$. Nevertheless, the condition of various groups of particles would vary in random consideration. The aim of this analysis is to estimate the effect of the Lewis number, heat recirculation and random distribution of particles along the wall temperature variation on the combustion behavior and flame structure of the organic dust particles.
4. Results and discussion

In this work, the physical geometry of the domain is an infinite open tube with the width of H. Another aspect of this research lies on addressing the influence of the particles thermal resistance in the form of Biot number primarily, it is presumed that the organic dust particles will be volatilized to form methane as the gaseous fuel. Methane is then combusted in the reaction zone.

Fundamentally, vaporization rate and the amount of released gaseous fuel is contingent on the temperature of particles in the preheat zone. Furthermore, the temperature distribution was analytically extracted versus the length in the preheat zone. It implies that the particle distribution strongly affects the vaporization rate. This importance is taken into account by dividing the particles into different groups to model the randomness dispersal of these particles. The heat capacity and adiabatic flame temperature are calculated as a function of equivalence ratio. Finally, analytical expression is derived for burning velocity and flame temperature.

Assessing the accuracy of the model is made by comparing the burning velocity obtained from the present model with the experimental findings of Proust [11] and Han et al. [20] as well as analytical data by Seshadri et al. [21]. As displayed in Fig. 3, both random and Seshadri non-random models have a practical compatibility with experimental data. Nonetheless, the random model has a better estimation. The achieved results from this research reveal that the random model leads to a better prediction of empirical burning velocity in comparison with the obtained data from uniform condition on the basis of Seshadri’s model [11, 20, 21]. Likewise, it is clear that the wall temperature and thermal resistance has a significant impact on the calculated burning velocity obtained from the presented mathematical approach. When
the wall temperature is considered to be constant and it becomes equal to the ambient temperature, Data in Seshadri theoretical work would be obtained.

Fig 3.
However, the wall temperature would increase due to the heat transfer to the wall from the reaction products.

Fig .4
As shown in Fig. 4, an increase in Biot number decreases the burning velocity and flame temperature, while an increase in thermal resistance of particles in varied temperature layers and pyrolysis rates inside the particle being observed. The main reason for this behavior can be explained by the reduction in the amount of produced gaseous fuel. This will result in less reaction rate and consequently lower flame speed and temperature. Pursuant to Fig. 4, as the equivalence ratio grows, the flame temperature and burning velocity are increased. In fact, pyrolysis resistance decreases as the mass particle density reduces.

Fig .5
Fig. 5 exhibits the flame temperature versus equivalence ratio for both uniform and random models. As seen, the maximum flame temperature cannot exceed the adiabatic flame temperature which is why there is a threshold for the equivalence ratio at each radius of the particle. As observed in this figure, the flame temperature is increased with the rise of the equivalence ratio. High equivalence ratio leads to more amounts of reaction heat, and as a result higher flame temperature. Another observation in this figure lies on the role of particle radius. A reduction in particle radius enhances the flame temperature because the two-phase
mixture approaches to the purely gaseous combustible mixture. It is revealed that there is a substantial difference between the uniform and random data at the fixed equivalence ratio. Although the random model has far greater flame temperature for smallest particle size than the uniform model, this gap gradually decreases for larger particles and at some stages, the flame temperature achieved from the uniform model outweigh the one by the random model. This is because of the less energy needed for vaporizing the smaller particle and higher number density for smaller particles at the constant equivalence ratio. Thus, it is more likely for particles to initiate the vaporization process in the preheat zone. However, in grander particle radii, this probability would decrease significantly.

Fig 6.

Fig. 6 clarifies the change in the burning velocity as a function of the equivalence ratio for various radii of the particles. Apparently, the enhancement in the equivalence ratio is associated with the growth in the flame speed. Additionally, the burning velocity declines as the particle radius rises. The same reasoning for the flame temperature can be applied for burning velocity to justify the trend of this figure. As mentioned earlier, the random model demonstrates lower amounts of flame speed for bigger particles and greater ones for smaller particles, comparing to the non-random mode.

According to Figs. 5 and 6, there is a reduction in the burning velocity and flame temperature after certain quantities of the equivalence ratio for various radii of particles. This is a consequence of following events. First, the preheat and flame temperature increase because of the augmentation in the equivalence ratio and released heat of reaction. Gradually, the quantity of vaporized fuel becomes equivalent to the stoichiometric value. At this situation, flame temperature and the effective equivalence ratio are identical to unit and adiabatic
temperature, respectively. On the other hand, with an increase in the equivalence ratio, the effective equivalence ratio would stay persistent. This implies that the heat of reaction doesn’t vary while more reactants are participating in the reaction process. Therefore, the flame speed and temperature start reducing after the specific equivalence ration (effective equivalence ratio becomes equal to unit).

Fig. 7
Heat recirculation is one of the dominating parameters on the combustion performance of biofuels. Fig. 7 attempts to demonstrate this importance on the burning velocity for \( r_u = 50 \mu m \). As shown, there is a straight relation between these two parameters. The flame temperature and flame speed go up with the heat recirculation coefficient. The reason is that recirculating the heat from post flame- to preheat zone enhances the temperature of reactants. Therefore, reactants require less energy for combustion process and the excess energy from the exothermic reaction further augments the flame temperature and as a consequence flame speed.

Fig. 8
Fig. 8 illustrates the variation of flame speed with equivalence ratio for different values of \( D \), \( D = \left( T_f - T_w \right) / \left( T_f - T_u \right) \). In figure 8, the flame temperature and the burning velocity rise, by the increase in the wall temperature or decrease in parameter \( D \). The wall temperature is proportional to the amount of heat recirculation. This can be done via two different mechanisms: (a) the energy can be conducted through the wall and then absorbed by the mixture in preheat zone via the convective heat transfer (b) the geometry of the combustion
chamber is designed in a way that the exhausted flow in post flame zone enters a tube and goes back to the preheat zone in the opposite direction of the fluid flow. In both methods, the temperature of wall, as well as convective heat transfer coefficient, play a vital role in the amount of recirculation.

Fig. 9

Fig. 9 indicates the flame temperature as a function of equivalence ratio for various Lewis numbers. As observed, there is a reduction in the flame temperature associating with an increase in the Lewis number due to a rise in the thermal diffusivity per mass diffusivity. This denotes that a lower amount of the heat is transferred to the combustion flow. Consequently, the flame temperature decreases with a growth in the Lewis number.

5. Conclusion

This research investigates the flame structure of organic dust particle considering the random distribution and the particles thermal resistance. The accuracy of the model is checked against corresponding experimental and analytical data. It is observed that the random model better predicts the experimental findings published in the literature on contrary to the uniform mode. After validation of the model, the impact of significant parameters including Biot number, heat recirculation, wall temperature and Lewis number on the combustion properties are conducted in this study. It is concluded that when the Biot number increases, different thermal layers with various pyrolysis rates will be formed inside the particles and consequently, less amount of gaseous fuel will be produced during pyrolysis procedure. Mainly, the amount of the heat recirculation approaches the maximum by taking the wall temperature. Therefore, the amount of the vaporization in the preheat zone, the flame temperature and the burning velocity of the
particles increase. It is monitored that the equivalence ratio approaches the stoichiometric condition at lower quantity by enhancement in the wall temperatures. Afterwards, the combustion properties will be the same as the adiabatic condition. It is also found out that a rise in parameter k leads to a growth in both flame speed and flame temperature. The rise in the Lewis number is accompanied by the reduction in the flame temperature. The flame temperature and flame speed augment by the equivalence ratio because of the reduction in the pyrolysis resistance. Consequently, smaller particles possess higher amount of the produced fuel mixture which has a tendency to increase the flame temperature and burning velocity dramatically.

**Nomenclature**

- $A$: parameter characterizing rate of vaporization of fuel particles
- $a$: defined in Eq. (18)
- $B$: pre-exponential factor of the reaction
- $Bi$: Biot number
- $C$: heat capacity of the mixture
- $C_p$: heat capacity of the gas
- $C_s$: heat capacity of a fuel particle
- $D$: dimensionless flame temperature
- $E_a$: activation energy of the reaction
- $H$: chamber width
- $h$: convective heat transfer coefficient between
- $k$: heat recirculation coefficient defined in Eq. (15)
\( k_r \)  
rate constant of the gas-phase reaction

\( k_i \)  
defined in Eq. (17)

\( L \)  
thickness of the thermal mixing layer

\( Le \)  
Lewis number

\( Pe \)  
Peclet number

\( Q \)  
heat release per unit mass of the fuel burned

\( q \)  
parameter defined in equation (15)

\( Q_v \)  
heat associated with vaporizing unit mass of fuel

\( R \)  
gas constant

\( r \)  
radius of fuel particle

\( v_u \)  
burning velocity calculated neglecting heat of vaporization of fuel particles

\( v_v \)  
burning velocity calculated including heat of vaporization of fuel particles

\( Y_F \)  
mass fraction of the gaseous fuel

\( Y_{FC} \)  
parameter defined in equation (11)

\( y_F \)  
dimensionless mass fraction of the gaseous fuel

\( Y_s \)  
mass fraction of the particles

\( y_s \)  
dimensionless mass fraction of the particles

\( n \)  
temperature exponent characterizing rate of vaporization of fuel particles

\( n_u \)  
number density of particles in the inlet stream

\( n_s \)  
local number density of particles (number of particles per unit volume)

\( W_F \)  
molecular weight of gaseous fuel

\( Ze \)  
Zeldovich number

\( z \)  
scaled independent variable defined in Eq. (10)

**Greek Letters**

\( \alpha \)  
defined in Eq. (16)

\( \phi_u \)  
equivalence ratio based on fuel available in the particles in the ambient reactant stream
ω  defined in Eq. (15)
ω_v  rate of vaporization of fuel particles
ω_F  rate of chemical kinetic for fuel particles
γ  parameter defined in equation (15)
λ  thermal conductivity
ξ  defined in Eq. (10)
Λ  defined in Eq. (24)
ν  stoichiometric coefficient
ρ  density of the mixture
ρ_s  density of a fuel particle
ε  expansion parameter (1/Zε)
η  independent variable defined in Eq. (21)
θ  dimensionless temperature of gas
θ^0  value of θ calculated neglecting heat of vaporization of particles

Subscripts
b  adiabatic conditions after completion of chemical reactions
F  gaseous fuel
f  conditions in the flame
s  solid fuel
u  conditions in the inlet stream
References


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**Caption of Figures:**

Fig. 1 Schematic view of the flame propagation mechanism and heat recirculation.

Fig. 2 schematic view of random distribution of particles in preheat-vaporization zone.
Fig 3. Variation of burning velocity as a function of mass concentration for the present model, experimental data [11, 20] and analytical model [21].

Fig. 4 Variation of (a) flame temperature (a) and burning velocity (b) as a function of equivalence ratio for different Biot numbers.

Fig. 5 Variation of flame temperature versus equivalence ratio for both uniform and random models with respect to different particle radii at Bi=0.2, Le=1.

Fig 6. Variation of burning velocity versus equivalence ratio for both uniform and random models with respect to different particle radii at Bi=0.2, Le=1.

Fig. 7 Variation of burning velocity versus equivalence ratio for different heat recirculation coefficients (k) at Bi=0.2, ru = 50 μm.

Fig. 8 Variation of the burning velocity versus equivalence ratio for different values of D at Bi=0.2, r=20 μm.

Fig. 9 Variation of the flame temperature versus equivalence ratio for different Lewis numbers.
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