Phase II Monitoring of Generalized Linear Profiles Under Different Types of Changes

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Abstract

Various control charts have been proposed to monitor generalized linear profiles in Phase II. However, robustness of the proposed methods in detecting different types and especially different directions of changes is not well-studied in the literature. In real-world applications different kinds of changes such as drift and multiple change are likely to happen which can be isotonic (increasing) or antitonic (decreasing). This paper studies the robustness of Rao Score Test (RST) method, T^2, and multivariate exponential weighted moving average (MEWMA) in different types, drift and multiple, and directions of changes. Rao Score Test method also benefits from a change-point detection approach whose performance is studied as well. According to the results, generally RST method shows a better performance in detecting different types of changes. Moreover, the performance of the RST method is robust to direction of the change, while T^2 and MEWMA are not ARL-unbiased and show different performances under isotonic and antitonic changes. Therefore, to address this issue, we proposed a bias-reduced estimator to be used in T^2. Our results demonstrate that the proposed control chart outperforms T^2 and is less biased than T^2. Finally, a real-world problem is presented in which aforementioned methods are applied to real data.

Keywords: Change-point, Control Charts, MEWMA, RST, T^2

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1. Introduction

A comprehensive review of the relevant literature indicates that there has been much interest in the area of profile monitoring within the past decade. A profile is the functional relationship between a response variable and one or more predictor variables. Profile monitoring is the use of control charts to monitor this functional relationship. Profiles can be utilized in a wide variety of manufacturing and service areas to monitor product or process performance over time. For instance, Kang and Albin [1] studied the case of aspartame which is an artificial sweetener. They also considered mass flow controller in which using profile monitoring was more advantageous than a single measurement over time.

Studies in the field of profile monitoring can be categorized by the types of the profiles being monitored and types of the change-points that occur in the process. Type of a profile being monitored can be simple linear, nonlinear/multiple/polynomial regression, and nonparametric. For a review of the studies in this area one can refer to Noorossana et al. [2]. The most important types of changes are single step, drift, and multiple step. Most of the research efforts have focused on single step change, but in real applications sometimes it is not practical to model changes as a simple single step change. For instance, in chemical applications or real tear and wear examples in which change occurs gradually, it can be modeled better by a drift model. Niaki and Khedmati [3] studied the change time of a multivariate binomial process for step and drift types of change. Kazemzadeh et al. [4] considered change-point estimation of multivariate linear profiles under linear drift. Korkas and Fryzlewicz [5] proposed a Wild Binary Segmentation method (WBS) to detect number and location of multiple change-points in the second order structure of time series. For a review of change-point estimation methods one can refer to Atashgar [6] and Aminikhanghahi and Cook [7].

All of the aforementioned research works can be classified into Phase I control and Phase II monitoring. According to Woodall et al. [8], in the Phase I analysis practitioners use a set of historical process data to evaluate process stability over time. Once anomalous observations are omitted, in-control process is modeled and unknown parameters are estimated. In the Phase II analysis, online data is utilized to quickly identify shifts in the process.

There are some limiting assumptions in most of the aforementioned control schemes. For instance, they mostly assume that response variables are continuous, while in many real examples the response variable may be discrete. For example, response variable may be a Poisson random variable (the number of defects per item) or a binomial random variable. These limitations in earlier methods encouraged us to use a more general model, including a wide variety of distributions and link functions. Generalized Linear Models (GLMs) look appropriate for this purpose.

Qiu [9] for the first time introduced the GLM to the statistical process control. Yeh et al. [10] developed five methods based on Hotelling T^2 statistic and assuming that response variable is binary, and applying them to Phase I control, compared performance of the proposed methods. Paynabar et al. [11] presented Phase I risk-adjusted control charts which are based on likelihood ratio test approach extracted from change-point model. Soleymanian et al. [12] presented four control charts, including Hotelling T^2, MEMWA, likelihood ratio test and LRT/EWMA, to monitor a binary response variable in Phase II. Thereafter, the performances of the proposed methods were evaluated by average run length (ARL) measure. According to simulation results, all methods perform quite well. Moreover, the performances of the methods improve as the size of the samples increases. According to the results, MEWMA method performs better than others when the changes in the parameters of logistic regression are small or moderate in size, and LRT/EWMA performs better than MEWMA when the changes are large in size. Furthermore, LRT/EWMA control chart performs somewhat better than T^2 and LRT control charts for every size of change and every size of sample. Koosha and Amiri [13] showed that ignoring autocorrelation between observations in different levels of a binary profile may lead to misleading results in evaluating the performance of different monitoring methods. Afterward, they evaluated the performance of five T^2 control charts, proposed by Yeh et al. [10] under step and drift types of change assuming that autocorrelation exists. Their results demonstrated that T_I^2 is the best method as already shown by Yeh et al. [10]. Amiri et al. [14] assessed the performance of five methods proposed by Yeh et al. [10] in Phase I monitoring of Gamma profiles. Gamma parameter estimation process was another contribution of this research. According to their results T_I^2 performs better than other methods in detecting step change and drift. Nevertheless, T_R^2 performs better than other methods when the step change or drift occurs in both regression parameters simultaneously. Noorossana et al. [15] evaluated four methods in Phase II monitoring of polytomous profiles. Their results demonstrate that MEWMA method performs better in detecting small size changes. Moreover, they showed that MEWMA and χ^2 are susceptible to the size of samples, whereas EWMA/R and χ^2/EWMA are less affected by the size of samples and roughly give the same results for different sample sizes. Amiri et al. [16] modified three methods which include T^2, likelihood ratio test and F to make them applicable in generalized linear model regression profiles following Poisson distribution. Simulation results show that likelihood ratio test method shows the best performance in detecting changes and the F method performs better than others in detecting outliers. Qi et al. [17] presented a method to monitor generalized linear profiles via weighted likelihood ratio charts. Simulation results show that their proposed method shows a better performance compared to some existing methods. Sogandi and Amiri [18] proposed a maximum likelihood estimator to monitor GLM based regression profiles under step change and drift. Simulation results show that the proposed method performs better than others in detecting small and moderate changes. Khedmati and Niaki [19] developed an approach based on the U statistic for Phase II monitoring of generalized linear profiles and removed impact of between-profile autocorrelation of error terms.

One of the attractive and practical topics in the process monitoring area is the effective estimation of the time at which a change has occurred in the process. The change-point approach provides experts with a diagnostic tool which can help to identify the time of change in process. This tool simplifies the identification of the root causes. Maximum likelihood estimation, clustering analysis, and artificial neural networks are examples of the common change-point identification methods. Noorossana et al. [20] used maximum likelihood method to detect step changes. Perry et al. [21] utilized maximum likelihood estimation to estimate a change-point when a linear change occurs in the process. Noorossana and Shadman [22] estimated time of a monotonic change through a maximum likelihood estimator. Alaeddini et al. [23] presented a clustering approach to identify the change-point of the process. Atashgar and Noorossana [24] took advantage of artificial neural networks to identify the change-point in a process. Ahmadzadeh [25] used artificial neural network to estimate change-point for multivariate control charts.

Some researchers have studied change-point estimation in the profile monitoring content. In Phase II Zou et al. [26] utilized likelihood ratio method to identify the time of change when a step shift takes place in the mean of linear profile. Moreover, Paynabar et al. [27] developed a change-point estimation approach for monitoring multivariate profiles. They applied their presented method in a real-world problem related to a forging process. Simulation results show that their method performs better in estimating change-point in comparison with some other methods. Maleki et al. [28] proposed two maximum likelihood estimators to identify real time of step changes and drifts in Phase II monitoring of binary profiles where within-profile autocorrelation exists.

Maximum likelihood framework comprises three general tests: Wald, Likelihood Ratio, and Rao Score test. There are some differences between these three tests. In order to carry out a likelihood ratio test, one must estimate both models under the null and alternative hypotheses. Rao Score test requires estimation of the model subject to the null hypothesis while the Wald test needs to estimate the model based on the alternative hypothesis.

In previous researches less efforts have been devoted to study susceptibility of the proposed methods to type and especially direction, increasing or decreasing, of the change. Most of the proposed methods use iterative weighted least square as an estimator which is a biased estimator [29]. Therefore, methods that use IWLS as an estimator are expected to be susceptible to direction of the change, and they are not ARL-unbiased, while RST method which does not use an estimator is robust to this issue. This paper studies this problem and proposes a bias-reduced estimator to be used in T^2 which satisfactorily attempts to solve this problem.

2. Methodology

2.1 Rao Score Test Method

Suppose that profile samples are collected over time and the j^{th} random profile comprises of n observations. We have a set of observations $\{(\boldsymbol{x}_{ij},y_{ij}), i=1,2,\dots,n\}$, in which y_{ij} is the i^{th} response observation in the j^{th} profile and $\boldsymbol{x}^{ij}$ is a vector consisting of q predictor variables ($\boldsymbol{x}_{ij} =(x_{ij1},x_{ij2},\dots,x_{ijq})$). In this research, we assume that predictor variables in each profile are known and constant over time. Moreover, we assume that the relationship between the response variable and predictor variables can be adequately modeled by generalized linear models which means:

1) Response variables are from an exponential family with a canonical form.

2) Linear combination of predictor variables with the coefficient vector $\boldsymbol{\beta}_j$ is as follows:

$$\eta_{ij} = \boldsymbol{x}_{ij}^T \boldsymbol{\beta}_j \tag{1}$$

$$\boldsymbol{\beta}_j = (\beta_{j1}, \beta_{j2}, \dots, \beta_{jq}) \tag{2}$$

3) A monotone link function g exists that connects mean of the response variable μ_{ij} to the linear predictor:

$$g(\mu_{ij}) = \eta_{ij} = \boldsymbol{x}_{ij}^T \boldsymbol{\beta}_j \tag{3}$$

$$\mu_{ij} = E(y_{ij}) \tag{4}$$

Most of the times, as in this research, x_{ij1} is set to be 1 for all i and j, therefore β_{j1} is the intercept of the model. As mentioned earlier Rao Score test method is discussed in Shadman et al. [30] in details but the following steps give a summary of the method:

1) For $j = 1,2,3,\ldots,t$,we have:

$$\boldsymbol{w}_j = \boldsymbol{w}_{j-1} + \boldsymbol{X}^T \boldsymbol{y}_j \tag{5}$$

In Equation (5) t is current time of monitoring, $\boldsymbol{w}_0$ is a q dimensional **0** vector, $\boldsymbol{X}$ is an $n \times q$ regressor matrix and $\boldsymbol{y}_j$ is an n-variate response vector.

2) For $k=1,2,\ldots, t-1$,we have:

$$\boldsymbol{U}_0(k,t) = \boldsymbol{w}_t - \boldsymbol{w}_k - (t-k)\boldsymbol{X}^T \boldsymbol{\mu}_0 \tag{6}$$

In Equation (6) $\boldsymbol{\mu}_0$ is an n-dimensional in-control mean vector of response variable.

And:

$$R_{k,t} = \boldsymbol{U}_0^T(k,t) \times \left(\frac{1}{t-k}\right) \boldsymbol{J}_0^{-1} \times \boldsymbol{U}_0(k,t) \tag{7}$$

In Equation (7) $\boldsymbol{J}_0^{-1}$ is in-control variance-covariance matrix

3) The statistic of method equals to:

$$R_{\max,t} = \max_{\max(0,t-window) \le k<t}(R_{k,t}) \tag{8}$$

In Equation (8), *window* is a rather newly adapted notion which is used because as t becomes very large, it will be difficult to record all the past data and find the maximum. Choosing a *window* is a trade-off between recording less data and having a good performance for the control chart, and theoretically having an infinite *window* (recording all of the data) should show the best performance. However, it can be shown that in most of the control charts if the *window* is not too small, it will show a performance close to the method without using *window*.

If $R_{max,t} \ge h_t$ then control chart signals and t^{th} profile is out of control. h_t is control limit for the t^{th} profile and the method to determine this limit will be described later. The change-point is estimated by:

$$\hat{\tau} = \arg\max_{\max(0,t-window \le k<t)}(R_{k,t}) \tag{9}$$

In most research efforts the control limits are constant for every t but Margavio et al. [31] showed that this approach could result in variation of false alarm over time. Therefore, in this research the following conditional probability is used which generates a constant alarm rate for every t:

$$\Pr(R_{\max,t} > h_t \mid R_{\max,i} \leq h_i, 1 \leq i < t) = \alpha \quad (10)$$

Since Equation (10) is not easily tractable, simulation is used to determine the sequence of control limits [32]. Similar to Shadman et al. [30], it is assumed that after 100th profile the control limit converges to a constant control limit, and the control limit is set equal to control limit of the 100th profile. In order to estimate control limits using simulation, we generated an 80000*100 matrix in which ith row is an in-control simulation of the following vector:

$$\boldsymbol{V}_i = [R_{\max,1}, R_{\max,2}, \ldots, R_{\max,100}] \quad (11)$$

In order to set ARL at 200, 0.995th quantile of the first column in the matrix was calculated and determined as h_1. After that, for elements of the column 1 which are greater than h_1 the relevant row was removed from the matrix. A similar procedure was applied to second and other columns in an order to estimate all of the control limits.

2.2 Bias-Reduced T^2

Firth [29] proposed an approach called bias reduction of maximum likelihood estimates. In fitting process of the logistic model sometimes one or more parameter estimates diverge to ±infinity, this phenomenon is called separation which is ideally solved by Firth procedure [33]. This method is built on the generic iteration proposed in Kosmidis and Firth [34] to solve the bias-reducing adjusted score equations. According to Kosmidis and Firth [34] this estimator has two advantages: 1) It is a second order unbiased estimator and has smaller variance compared to MLE. 2) The resulted estimates and their standard errors are finite. A detailed description of this estimator is given in Heinze and Schemper [35]. As mentioned earlier, most of the methods that are proposed to monitor generalized linear profiles are biased due to using a biased estimator. Therefore, to deal with this problem we enhance the T^2 method proposed by Yeh et al. [10] using the estimator proposed by Firth [29] instead of the usual IWLS method.

3. Performance comparisons

This section is devoted to evaluate performances of the RST, T^2, MEWMA and propose using bias reduced method under different types and direction of changes. This comparison is done using Monte Carlo simulation and ARL measure is used to evaluate the methods. RST method also benefits from a change-point estimation approach whose performance is studied as well.

3.1 Hotelling T^2

Hotelling T^2 control chart was used by Kang and Albin [1] for monitoring simple linear profiles. Yeh et al. [10] developed five Hotelling T^2 control charts to monitor binary profiles in Phase I. T^2 statistic for the j^{th} profile is calculated as:

$$T_j^2 = (\boldsymbol{\beta}_j - \boldsymbol{\beta}_0)^T \boldsymbol{\Sigma}_0^{-1} (\boldsymbol{\beta}_j - \boldsymbol{\beta}_0) \quad (12)$$

The above statistic can be rewritten as:

$$T_j^2 = (\boldsymbol{\beta}_j - \boldsymbol{\beta}_0)^T \boldsymbol{X}^T \boldsymbol{W} \boldsymbol{X} (\boldsymbol{\beta}_j - \boldsymbol{\beta}_0) \tag{13}$$

where $\widehat{\boldsymbol{\beta}}_j$ is the estimated vector of parameters via iterative weighted least square algorithm, $\boldsymbol{\beta_0}$ is the in-control vector of parameters and $\boldsymbol{\Sigma_0}$ is in-control variance-covariance matrix. In Equation (13), $\boldsymbol{W}$ is a N×N diagonal matrix in which diagonal elements are calculated as follows:

$$w_{ii} = \frac{1}{Var(y_i)} \left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2 \tag{14}$$

As long as this statistic is less than upper control limit, it is assumed that the process is in-control but when T_j^2 > UCL it is assumed that the process is out of control. The upper control limit is estimated according to a given ARL.

3.2 MEWMA Method

MEWMA which was proposed by Zou et al. [26], is also applicable in monitoring generalized linear profiles. Soleymanian et al. [12] and Noorossana et al. [15] used this method for monitoring generalized linear profiles. MEWMA statistic for the j^{th} profile is calculated as:

$$Q_j = \boldsymbol{Z}_j^T \boldsymbol{\Sigma}_{Z_j}^{-1} \boldsymbol{Z}_j \tag{15}$$

In Equation (15), $\boldsymbol{Z_j}$ is a q-dimensional vector which equals to:

$$\boldsymbol{Z}_j = \lambda(\boldsymbol{\beta}_j - \boldsymbol{\beta}_0) + (1-\lambda)\boldsymbol{Z}_{j-1} \tag{16}$$

$\boldsymbol{\Sigma}_{\boldsymbol{Z_j}}$ is asymptotic variance-covariance matrix of $\boldsymbol{Z_j}$ which is calculated as following:

$$\boldsymbol{\Sigma}_{Z_j} = \frac{\lambda}{2-\lambda} \boldsymbol{J}_0^{-1} \tag{17}$$

$\widehat{\boldsymbol{\beta}}_j$ is maximum likelihood estimator of $\boldsymbol{\beta_j}$ which is calculated by iterative weighted least square algorithm, λ is weighting parameter and $\boldsymbol{Z_0} = \boldsymbol{0}$. This control chart gives a signal when $Q_j > L_{MEWMA}$ and L_{MEWMA} , the MEWMA upper control limit, is simulated to give a known ARL which is 200.

3.3 Binomial profiles

To simplify the problem suppose that there is only one predictor variable (q=2) and y_{ij} $\sim Binomial\ (m, \pi_{ij})$ in which m is the number of observations and π_{ij} is the probability of the success for the j^{th} profile and i^{th} observation. Link function is considered as:

$$g(\pi_{ij}) = \log(\frac{\pi_{ij}}{1-\pi_{ij}}) = \beta_0 + \beta_1 x_{ij} \tag{18}$$

Equation (18) can be simplified as:

$$\pi_{ij} = \frac{\exp(x_{ij}^T \beta)}{1+\exp(x_{ij}^T \beta)} = \frac{\exp(\eta_{ij})}{1+\exp(\eta_{ij})} \tag{19}$$

In-control parameters are assumed as $\boldsymbol{\beta_0} = (\beta_{00},\beta_{10})^T = (-2.8,1)^T$. Predictor variables are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1 and n=10. We set m=30 and simulate control charts for three different time windows: 20, 50,100.

3.3.1 Isotonic drift

Change-point is assumed to be τ=30 , this means that for j=1,2,…,30 the profiles are in-control and parameters are as $\boldsymbol{\beta}_0 = (-2.8,1)^T$ and after 30^{th} profile for j=31,32,… process undergoes a change which is modeled as:

$$\boldsymbol{\beta}_1 = (\beta_{00} + (j-30)\times\delta_1\times\sigma_1, \beta_{10} + (j-30)\times\delta_2\times\sigma_2) \tag{20}$$

σ_1 and σ_2 are the standard deviation of intercept and slope respectively which are calculated as:

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} = (\boldsymbol{X}^T\boldsymbol{WX})^{-1} = \begin{bmatrix} 0.2186 & -0.2936 \\ -0.2936 & 0.4771 \end{bmatrix} \tag{21}$$

Therefore, standard deviations are calculated as σ_1= 0.4676 and σ_2= 0.6907 and the performance of the method is evaluated by ARL and estimated change-point criterion for different δ_1 and δ_2. Results are given in Table 1 and Table 2. In this research three cases are simulated: 1) change in intercept 2) change in slope 3) change in intercept and slope, simultaneously. However, for the sake of brevity simulation results are presented only for case 1. In the following tables the numbers stand for ARL's and the numbers inside parentheses are standard deviations of run lengths. All of the results are determined based on 10000 iterations.

3.3.2 Isotonic multiple step change

In isotonic multiple step change case two step changes occur in the process at τ_1=30 and τ_2=35. The monitoring is done in Phase II and according to the literature our aim is to detect the change as soon as possible and estimate the change-point. Let δ_{ij} control the size of change in the i^{th} parameter and j^{th} step. The model of change is as:

$$\boldsymbol{\beta}_0 = (-2.8,1),\ j = 1,2,...,30 \tag{22}$$

$$\boldsymbol{\beta}_1 = (\beta_{00} + \delta_{11}\times\sigma_1, \beta_{10} + \delta_{21}\times\sigma_2),\ j = 31,32,...,35 \tag{23}$$

$$\boldsymbol{\beta}_2 = (\beta_{00} + \delta_{12} \times \sigma_1, \beta_{10} + \delta_{22} \times \sigma_2), j = 36, 37, \ldots \quad (24)$$

Standard deviations of intercept and slope are calculated as shown in the previous section and the number of simulation replications is set at 10000. Moreover, since the performance of Hotelling T^2 was very poor its results are not given for this case. Results are shown in Table 3 and Table 4. In Table 4 which is related to change-point the first change has been assumed as *the* change-point.

As discussed before, to simplify the problem we considered q=2 and n=10. In order to see if the RST method is still efficient for larger values of q and n we doubled both of the parameters (q=4 and n=20) and run the simulations. In this case, the link function, in-control and out of control parameters are defined by:

$$g(\pi_{ij}) = \log(\frac{\pi_{ij}}{1-\pi_{ij}}) = \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \beta_3 x_{ij}^3 \quad (25)$$

$$\beta_0 = (\beta_{00}, \beta_{10}, \beta_{20}, \beta_{30}) = (-2.8, 1, 2, 3)^T, j = 1, 2, \ldots, 30 \quad (26)$$

$$\beta_0 = (\beta_{00} - (j-30) \times \delta_1 \times \sigma_1, \beta_{10}, \beta_{20}, \beta_{30}) = (-2.8, 1, 2, 3)^T, j = 31, 32, \ldots \quad (27)$$

Table 5 demonstrates that RST still outperforms T^2.

3.4 Poisson Profiles

Similar to binomial example suppose that there is only one predictor variable (q=2) and $y_{ij} \sim Poisson\ (\mu_{ij})$ in which μ_{ij} is the mean for the j^{th} profile and i^{th} observation. Link function is considered as:

$$g(\mu_{ij}) = \log(\mu_{ij}) = \beta_0 + \beta_1 x_{ij} \quad (28)$$

The Equation (28) can be simplified as:

$$\mu_{ij} = \exp(x_{ij}^T \beta) = \exp(\eta_{ij}) \quad (29)$$

In-control parameters are assumed as $\boldsymbol{\beta_0} = (\beta_{00}, \beta_{10})^T = (3,2)^T$. Predictor variables are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1 and therefore n=10. Simulations are run for three different time windows: 20, 50,100.

3.4.1 Antitonic drift

Change-point is assumed to be τ=30 , that is for j=1,2,…,30 profiles are in-control and parameters are as $\boldsymbol{\beta_0} = (3,2)^T$ and after 30th profile for j=31,32,… process undergoes a change which is modeled as:

$$\boldsymbol{\beta}_1 = (\beta_{00} - (j-30)\times\delta_1\times\sigma_1, \beta_{10} - (j-30)\times\delta_2\times\sigma_2) \tag{30}$$

σ_1 and σ_2 are the standard deviation of intercept and slope, respectively, which are calculated as:

$$\Sigma_0 = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} = (\boldsymbol{X}^T\boldsymbol{W}\boldsymbol{X})^{-1} = \begin{bmatrix} 0.0117 & -0.0146 \\ -0.0146 & 0.0207 \end{bmatrix} \tag{31}$$

Therefore, σ_1= 0.1082 and σ_2= 0.1440 and the performance of the methods are evaluated by ARL and estimated change-point criterion for different δ_1 and δ_2. All of the simulation results are based on 10000 replications. Results are given in Table 6 and Table 7.

3.4.2 Antitonic multiple change

In antitonic multiple change two step changes occur in the process at τ_1=30 and τ_2=35. We let δ_{ij} control the size of change in the i^{th} parameter and j^{th} step. The model of the change is as:

$$\boldsymbol{\beta}_0 = (-2.8, 1),\ j = 1, 2, \ldots, 30 \tag{32}$$

$$\boldsymbol{\beta}_1 = (\beta_{00} - \delta_{11}\times\sigma_1, \beta_{10} - \delta_{21}\times\sigma_2),\ j = 31, 32, \ldots, 35 \tag{33}$$

$$\boldsymbol{\beta}_2 = (\beta_{00} - \delta_{12}\times\sigma_1, \beta_{10} - \delta_{22}\times\sigma_2),\ j = 36, 37, \ldots \tag{34}$$

Standard deviations of intercept and slope are calculated as shown in the subsection 3.4.1 and simulation results are based on 10000 replications. Results are given in Table 8 and Table 9 in which the first change has been assumed as *the* change-point.

3.5 A summary of simulation results

3.5.1 Isotonic drift

In isotonic drift case results agree with those results presented by Shadman et al. [30] for isotonic single step change. Rao Score test method outperforms MEWMA and T^2 in all cases and such better performance is more noticeable for small changes. When the changes are very small out of control ARL's related to MEWMA and T^2 methods are even larger than in-control ARL (200) which shows these control charts are not ARL-unbiased.

Results of MEWMA for different values of λ show that MEWMA with a small λ performs better in detecting small changes and MEWMA with a large λ performs better in detecting large changes. Comparing MEWMA with T^2 shows that MEWMA performs better in detecting small changes while T^2 performs better in detecting large changes.

Different time windows give close results for moderate and large changes but this similarity is caused because the change occurs early. For small changes the effect of time window is relatively discernible and increasing the time window improves performance of the control chart.

Nevertheless, time window of 50 gives satisfactory results. Moreover, the change-point estimator estimates the change-point close to the real change-point for moderate and large changes.

3.5.2 Isotonic multiple step change

In isotonic multiple step change superiority of Rao Score test method over MEWMA is evident. Since the T^2 method's performance was poor, it was not used in simulations. As mentioned earlier, since the research is done in Phase II, the aim of research is to detect the change as soon as possible and estimate the first change. This assumption agrees with Perry et al. [21]. In simulations a constant size for the first change and 10 different sizes for the second change are assumed. According to the results as the size of the second change increases, at first the performance of change-point estimation gets better. However, for larger sizes of the second change beyond a point, the performance of change-point estimation gets worse, and the change-point is estimated closer to the second step change. It is obvious that for large sizes of the second step change most of the times estimator estimates the second step as *the* change-point.

3.5.3 Antitonic drift

In antitonic drift Rao Score test method is not always superior to the other two methods. For small changes MEWMA with a small λ performs better than Rao Score test method. Moreover, for moderate size changes sometimes MEWMA method with a moderate λ performs better than Rao Score test method. For large changes T^2 performs better than Rao Score test method. Similar to isotonic case for large and moderate changes change-point estimation approach comes up with estimates close to the real change-point.

3.5.4 Antitonic multiple step change

In antitonic multiple step change Rao Score test method outperforms the other two methods most of the times. Rao Score test method is always superior to T^2 method but MEWMA with a small or moderate λ performs better than Rao Score Test method in detecting small changes. Similar to isotonic multiple example, increasing the size of the second step change cause change-point approach to estimate the second change as *the* change-point most of the times.

3.6 Bias-Reduced T^2

According to the results, since T^2 and MEWMA use a biased estimator, performance of these methods depend on the direction of the change, and they are not ARL-unbiased. Therefore, we used Firth bias-reduced estimator as the estimator of the T^2 and run the simulations based on following model:

$$\beta_1 = (\beta_{00} \pm \delta_1 \times \sigma_1, \beta_{10}) \tag{35}$$

The only difference is that here we used both increasing and decreasing changes to see the results. Our results show that the proposed method is significantly less biased than T^2. The results are given in Table 10 and Figure 1. According to the results, the proposed method shows less difference between isotonic change ARL and antitonic change ARL. Therefore, our proposed method is less biased than T^2.

4. A Real-Data Analysis

In this section, Rao Score Test method is used to monitor a real-world data example in Phase II. This example was used and monitored in Shadman et al. [36] and Shadman et al. [30] in both Phase I and Phase II, but this research focuses on Phase II and drift. An instrument named Dispergrader was used to get data. This instrument is used to assess dispersion of carbon black filler in rubber mix.

Evaluation of filler dispersion is very important in resin substances especially in the tire industry. The amount of dispersion affects the quality of the product; and therefore it is considered as a parameter in quality control systems. In the literature different methods are introduced to evaluate the dispersion. These methods are mostly subjective, time-consuming and costly. Dispergrader using a microscope provides an appropriate method for measuring the dispersion of fillers.

In resin production filler pellets which are mostly 1milimeter in diameter are broken down into aggregates which are mostly 1 micron in diameter to produce the desirable substance. In the process of production this breaking down does not occur in all particles and particles which are not broken down remain as agglomerates and can be considered as defects. These defects can highly affect mechanical features such as tensile strength, rupture, and fatigue.

In Dispergrader, beams are sent out to the surface of the sample in a way that beams' direction and surface meet at a 30-degree angle to each other. This equipment magnifies the image 100 times.

A sample curved rubber bar is put in Dispergrader and beams are sent out on its surface. The beams reflected from agglomerates larger than 3 microns in diameter make white spots in the image. Afterward, the number of white spots are recorded for each given spot size. Recording is in a way that sizes between 3 and 6 microns are recorded as 3, sizes between 6 and 9 microns are recorded as 6, and this approach is also used for larger spots. All the white spots which are 57 microns in diameter or greater are recorded as 57. The quality characteristic is defined as a profile in which agglomerate count is the response variable and the agglomerate diameter is the predictor variable.

The aim of Phase II is online monitoring of Dispergrader. To start Phase II a set of in-control historical data are needed to estimate parameters and control limits. For this purpose, in-control data used by Shadman et al. [30] is chosen. They assumed that response variable can be adequately modeled by negative binomial distribution. They used a log function and assumed that there is second order polynomial relationship between response variable and predictor variable:

$$Y \sim NegativeBinomial(\upsilon = 1; p = 1/(1+\mu/\upsilon)) \quad (36)$$

$$Log(\mu) = \beta_0 + \beta_1 X + \beta_2 X^2 \quad (37)$$

In this research, we used the same model and the approach of Ver Hoef and Boveng [37] to estimate parameters. Parameters were estimated as:

$\beta_0^{in-control} = 6.0781$
$\beta_1^{in-control} = -0.0104$
$\beta_2^{in-control} = -0.0030$

Three different scenarios are considered, in which parameters of the model undergo isotonic drift. Scenario 1 is as follows:

$$\begin{cases} \beta_0 = \beta_0^{in-control}, t = 1,2,...,16 \\ \beta_0 = \beta_0^{in-control} + 0.5 \times \sigma_0 \times (t-16), t = 17,18,... \end{cases} \tag{38}$$

$$\begin{cases} \beta_1 = \beta_1^{in-control}, t = 1,2,...,16 \\ \beta_1 = \beta_1^{in-control} + 0.5 \times \sigma_1 \times (t-16), t = 17,18,... \end{cases} \tag{39}$$

$$\begin{cases} \beta_2 = \beta_2^{in-control}, t = 1,2,...,16 \\ \beta_2 = \beta_2^{in-control} + 0.5 \times \sigma_2 \times (t-16), t = 17,18,... \end{cases} \tag{40}$$

The models for scenario 2 and 3 are similar to scenario 1, and the only difference is that for scenario 2 the parameters which determine the slope of change are $\delta_0=\delta_1=\delta_2=0.05$. The same parameters for scenario 3 are 0.005.

Finally, a control chart based on change-point approach and Rao Score test statistic is implemented to monitor a negative binomial profile as specified in the following steps:

1) A simulated control limit which is not constant over time is determined in a way that leads to a 200 in-control ARL.
2) After obtaining new observations, $R_{max,t}$ statistic is calculated and compared to the control limit. If it is below control limit, new observations will be used, otherwise control chart signals and the change-point is estimated.

Three methods, RST, T^2 and MEWMA, are compared under the three mentioned scenarios. The proposed method, bias reduced T^2, is not used in real-data analysis for two following reasons:

1) Phase I of this example is done using IWLS which is a biased estimator. Therefore, using another estimator (bias reduced estimator) in Phase II might be misleading.

2) Bias reduced GLM package in *R* (brglm *R*) currently is developed only for binomial response GLMs.

The time window of RST method is 100 and λ of MEWMA method is 0.2. The results and control charts are given in Figure 2 and Table 11 respectively.

5. Conclusions

The aim of this study is to evaluate the performance of RST method compared to MEWMA and T^2 in monitoring generalized linear profiles in presence of drift and multiple change which can be increasing or decreasing, and also attempts to solve the problem of being ARL-biased in control charts that use IWLS as estimator.

In this research, the performances of the mentioned methods are evaluated in detecting isotonic drift and multiple change in parameters of binomial profile and antitonic drift and multiple change in parameters of Poisson profile. For the case of increasing drift and multiple change the results are in agreement with Shadman et al. [30] and Rao Score test method outperforms MEWMA and T^2. Furthermore, the change-point estimator estimates the change-point closely to the real change-point in large and moderate drifts. For the case of multiple change in which the first step is constant and second step can have ten different sizes, increasing the size of the second step at first makes change-point estimation approach to estimate the change-point closely to the real change-point. However, beyond a point increasing the size of *the* second step makes the change-point estimation to estimate the second step as *the* change-point most of the times. In decreasing changes Rao Score test is not superior to the other two methods in all cases. In terms of detecting a small decreasing drift MEWMA with small λ value performs better than RST method; for a moderate change some times MEWMA with moderate λ values shows a better performance and for a large change T^2 outperforms RST method. In detecting a small decreasing multiple change MEWMA with a small or moderate λ outperforms RST method. The performance of the change-point estimator in estimating the change-point for case of antitonic drift and multiple change was similar to the case of isotonic change. Another important concern might be the effect of number of predictors and observations on performance of RST method. In order to study this issue, we doubled the numbers of the predictors and observations for binomial profile under drift. Simulation results show that RST method still works well compared to T^2.

We noticed that the performance of MEWMA and T^2 method differs for the case of isotonic and antitonic change, and they are not ARL-unbiased. This happens because MEWMA and T^2 methods does not have the characteristic of being unbiased which is a result of biased estimator that they use. Therefore, to deal with this problem we proposed using a bias reduced GLM approach as an estimator in T^2. This estimator was proposed by Firth [29], but it had never been used in profile monitoring. Our results show that the proposed method reduces the bias of the control chart satisfactorily.

A real world example of resin industry is presented to show the implementation of the methods in Phase II. At first through statistical analysis a negative binomial profile with a log function which relates mean of the response variable to a second order polynomial of predictor variable is chosen to model the data. Afterward, three methods are used to monitor negative binomial profiles in Phase II and under drift. According to the results in the scenario with large drift, RST and T^2 show similar performance and outperform MEWMA. In the scenario with moderate drift three methods perform closely. In the scenario with small drift, MEWMA outperforms RST and RST outperforms T^2. As mentioned earlier, bias reduced GLM package in *R* (brglm *R*) currently works only for binomial responses. Developing and testing the package for other families and using it in profile monitoring can be considered as a future study.

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Appendix A (Figures)

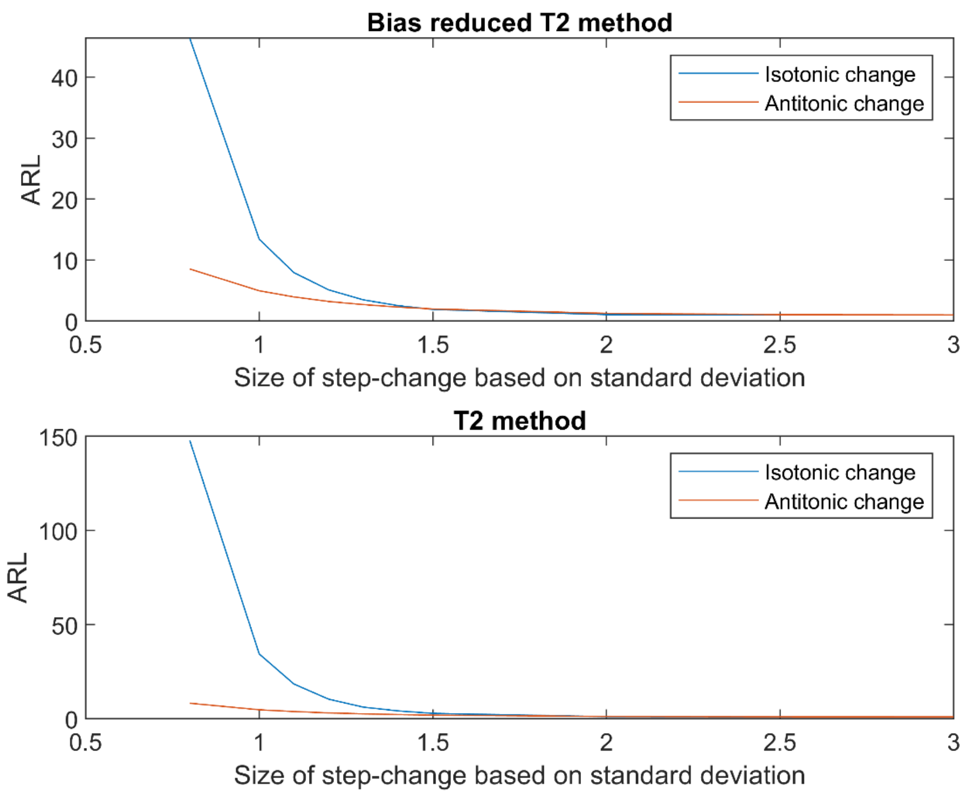

Figure 1) Comparison of the Bias reduced T^2 with T^2

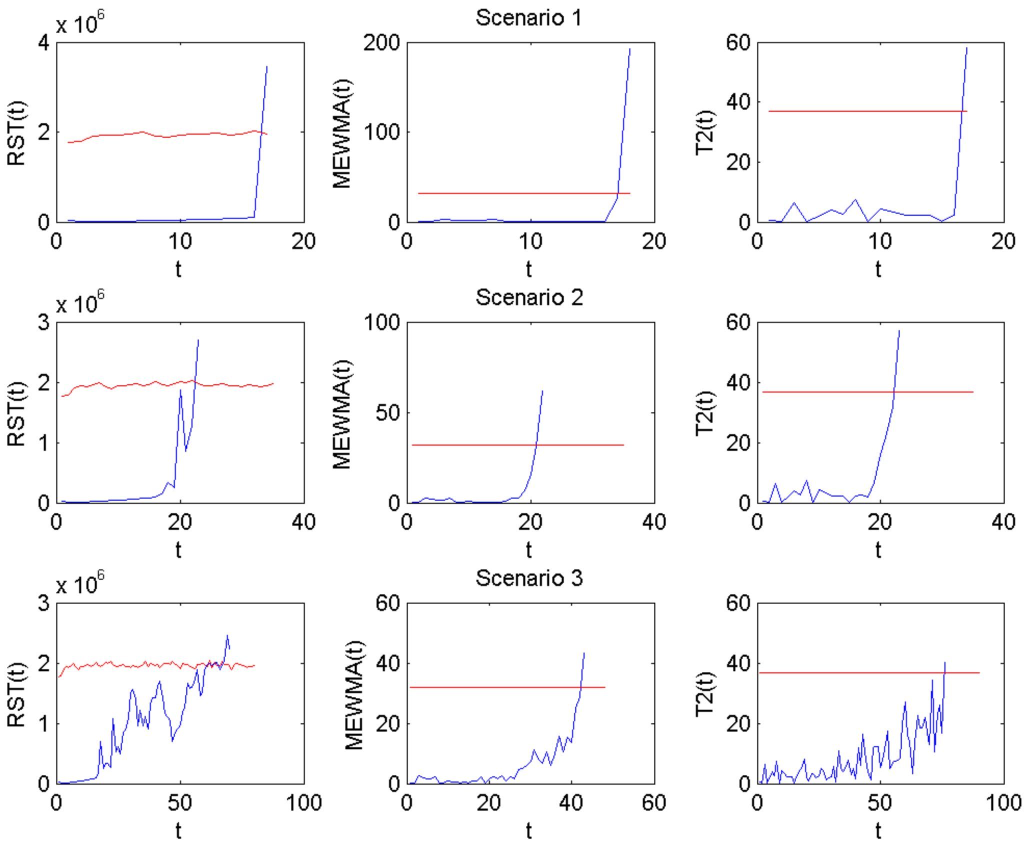

Figure 2) Control charts for real-data analysis

Appendix B (Tables)

Table 1. ARL for isotonic drift in intercept of binomial profile

δ_1	RST (Window=20)	RST (Window=50)	RST (Window=100)	Hotelling T^2	MEWMA λ=0.05	MEWMA λ=0.2	MEWMA λ=0.4
0.001	95.8035 (63.06388)	93.1838 (59.71108)	90.5993 (57.3664)	367.0953 (333.1151)	165.4975 (85.42436)	200.363 (117.1865)	250.1403 (165.8289)
0.005	42.7568 (20.78963)	41.3498 (19.76968)	41.3548 (19.54215)	163.8714 (68.62347)	61.8955 (19.43502)	72.2004 (24.62263)	93.2443 (32.86928)
0.01	28.5414 (12.36723)	27.9485 (12.08684)	27.9149 (11.93115)	97.516 (30.76698)	40.3651 (10.5095)	43.9174 (12.56879)	55.6392 (15.826645)
0.05	10.5216 (3.699964)	10.4708 (3.803413)	10.4925 (3.724882)	25.8201 (5.102287)	15.8617 (2.7960)	14.1949 (2.848072)	15.9725 (3.325168)
0.1	6.8208 (2.221821)	6.8206 (2.238262)	6.8363 (2.237611)	14.2792 (2.521239)	10.8136 (1.7164)	9.0160 (1.582259)	9.4987 (1.787232)
0.5	2.4638 (0.689122)	2.4664 (0.68518)	2.4648 (0.688739)	3.6675 (0.656311)	4.6202 (0.6744)	3.4937 (0.564589)	3.212 (0.527879)
1	1.6466 (0.481569)	1.6480 (0.479475)	1.6487 (0.479467)	2.0833 (0.362162)	3.2505 (0.5266)	2.4007 (0.500739)	2.0708 (0.302634)
1.5	1.1592 (0.365862)	1.1638 (0.370094)	1.1567 (0.363518)	1.6528 (0.47608)	2.7079 (0.4604)	1.97 (0.194165)	1.8744 (0.331398)
2	1.0062 (0.078496)	1.0057 (0.075283)	1.0052 (0.071923)	1.121 (0.326127)	2.2599 (0.4554)	1.8458 (0.36114)	1.5147 (0.499784)
2.5	1 (0)	1 (0)	1 (0)	1.0021 (0.045778)	2.0029 (0.2247)	1.5922 (0.491426)	1.1359 (0.342682)
3	1 (0)	1 (0)	1 (0)	1 (0)	1.9484 (0.22167)	1.2788 (0.448409)	1.0114 (0.10616)

Table 2. Change-point estimation (Isotonic drift in intercept of binomial profile)

δ_1	Window=20		Window=50		Window=100	
	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$
0.001	119.8958	62.18153	112.6677	56.93061	104.8658	52.83577
0.005	65.0683	20.78963	57.9071	19.16977	56.3100	19.54215
0.01	50.4597	12.13221	46.1548	12.08684	45.8965	11.93115
0.05	34.7407	5.126818	34.1740	6.237445	34.2766	6.211899
0.1	32.4584	3.981491	32.2473	4.606446	32.2701	4.622764
0.5	30.3311	2.385932	30.2677	2.60765	30.2682	2.660013
1	30.0730	1.917517	30.0787	2.032512	30.0470	0.479467
1.5	29.9599	1.270548	29.9419	1.366793	29.9429	1.333731
2	29.9426	0.603246	29.9327	0.663303	29.9402	0.600187
2.5	29.9817	0.280651	29.9715	0.378798	29.9761	0.325467
3	29.9931	0.158279	29.9890	0.21466	29.9931	0.150507

Table 3. ARL for isotonic multiple change in intercept of binomial profile

$(\delta_{11}, \delta_{12})$	RST (Window=20)	RST (Window=50)	RST (Window=100)	MEWMA λ=0.05	MEWMA λ=0.2	MEWMA λ=0.4
(0.1,0.2)	35.4825 (30.38748)	31.3287 (24.69028)	30.4503 (22.42146)	87.2043 (63.5371)	249.3337 (242.0212)	622.2976 (610.2426)
(0.1,0.3)	19.1274 (12.88038)	18.1492 (10.95447)	18.1014 (10.83084)	37.2727 (17.7532)	67.8792 (57.2167)	292.2946 (289.1756)
(0.1,0.4)	13.0761 (6.754236)	12.9803 (6.47031)	12.9902 (6.524163)	24.3403 (8.6288)	28.7672 (18.4522)	92.6143 (86.0084)
(0.1,0.5)	10.3039 (4.444361)	10.3657 (4.428178)	10.3195 (4.430533)	18.7677 (5.4528)	17.7730 (8.1244)	37.0998 (29.2889)
(0.1,0.6)	8.8622 (3.246754)	8.8922 (3.218288)	8.8538 (3.21612)	15.7622 (3.9346)	13.3857 (4.4544)	20.2450 (12.4547)
(0.1,0.7)	7.9498 (2.50533)	7.9072 (2.630359)	7.9525 (2.543628)	13.8507 (3.0414)	11.2853 (2.9839)	13.8790 (6.313)
(0.1,0.8)	7.2955 (2.109545)	7.3186 (2.158447)	7.2930 (2.122346)	12.4965 (2.5694)	10.0001 (2.1770)	10.9394 (3.5871)
(0.1,0.9)	6.8901 (1.855915)	6.8799 (1.87512)	6.8723 (1.900051)	11.5568 (2.1714)	9.1583 (1.7149)	9.4649 (2.4113)
(0.1,1)	6.5446 (1.659406)	6.5754 (1.693669)	6.5639 (1.6764)	10.8136 (1.9273)	8.5933 (1.4045)	8.5968 (1.7331)
(0.1,1.1)	6.3227 (1.524849)	6.3221 (1.541347)	6.3354 (1.528367)	10.2322 (1.6928)	8.1681 (1.2038)	7.9762 (1.3479)

Table 4. Change-point estimation (Isotonic multiple change in intercept of binomial profile)

$(\delta_{11}, \delta_{12})$	Window=20		Window=50		Window=100	
	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$
(0.1,0.2)	57.7286	30.29054	47.1392	23.52195	43.2806	20.43896
(0.1,0.3)	40.8859	12.58369	36.4500	10.64466	36.1211	10.52653
(0.1,0.4)	35.7408	6.588734	34.4359	7.410512	34.3527	7.343862
(0.1,0.5)	34.3025	4.85477	33.7665	6.049163	33.7434	5.901471
(0.1,0.6)	33.9021	4.26712	33.7055	5.062467	33.5491	5.172503
(0.1,0.7)	33.8740	3.775781	33.5166	4.771658	33.6375	4.522731
(0.1,0.8)	33.7937	3.636776	33.5818	4.35579	33.5708	4.374127
(0.1,0.9)	33.8383	3.450703	33.6934	4.021442	33.6544	4.11227
(0.1,1)	33.8549	3.389638	33.7509	3.845237	33.6924	3.985923
(0.1,1.1)	33.9336	3.27249	33.8121	3.671974	33.8363	3.73067

Table 5. Result for effect of number of predictors and observations (standard deviations are given in parenthesis)

δ_1	RST		T^2
	ARL	Change-point	ARL
0.001	81.8060 (45.6467)	94.1230 (37.4829)	165.7940 (137.5765)
0.005	32.1920 (12.5176)	45.5820 (12.5241)	107.7130 (68.9080)
0.01	21.6800 (7.6661)	38.6630 (8.9933)	79.7070 (43.4161)
0.05	8.0310 (2.3966)	32.2180 (4.3323)	30.9970 (13.2006)
0.1	5.3520 (1.4430)	31.1470 (3.1638)	19.7210 (7.3898)
0.5	1.9090 (0.3935)	29.8060 (2.5642)	6.5260 (2.1391)
1	1.1440 (0.3513)	29.8800 (1.4421)	4.0500 (1.2685)
1.5	1 (0)	29.9560 (0.4902)	3.0420 (0.9743)
2	1 (0)	29.9870 (0.1512)	2.4830 (0.8357)
2.5	1 (0)	29.9810 (0.3045)	2.1580 (0.7011)
3	1 (0)	29.9960 (0.0774)	1.8900 (0.6483)

Table 6. ARL for antitonic drift in intercept of Poisson profile

δ_1	RST (Window=20)	RST (Window=50)	RST (Window=100)	Hotelling T^2	MEWMA λ=0.05	MEWMA λ=0.2	MEWMA λ=0.4
0.001	102.1630 (65.0406)	97.6786 (59.8645)	91.6726 (55.2262)	122.5580 (84.5574)	77.8625 (41.9650)	90.4725 (52.4263)	100.5746 (61.7167)
0.005	42.6454 (19.5507)	40.6931 (18.3308)	40.4171 (17.9133)	59.3012 (29.7561)	35.8102 (13.9485)	39.1006 (16.3133)	44.7450 (19.7063)
0.01	27.8681 (11.4363)	27.3876 (10.9084)	27.2422 (10.9074)	39.6646 (17.1881)	24.9405 (8.4750)	25.9056 (9.6675)	29.3069 (11.4395)
0.05	10.4254 (3.4140)	10.4066 (3.5638)	10.4094 (3.4250)	13.5452 (4.5513)	10.8028 (2.8830)	9.8563 (2.7702)	10.3102 (3.1317)
0.1	6.8028 (2.0960)	6.8258 (2.1055)	6.8451 (2.1027)	8.2083 (2.5387)	7.5457 (1.8817)	6.6082 (1.6707)	6.6275 (1.8213)
0.5	2.5733 (0.6901)	2.5679 (0.6824)	2.5857 (0.6782)	2.5374 (0.7294)	3.3113 (0.7840)	2.6995 (0.6152)	2.4756 (0.6093)
1	1.7318 (0.4570)	1.7407 (0.4546)	1.7514 (0.4454)	1.5783 (0.5029)	2.3713 (0.5807)	1.8991 (0.3951)	1.7460 (0.4413)
1.5	1.2753 (0.4467)	1.2788 (0.4484)	1.2843 (0.4511)	1.1369 (0.3438)	1.9288 (0.4349)	1.6055 (0.4888)	1.3342 (0.4717)
2	1.0302 (0.1711)	1.0314 (0.1744)	1.0330 (0.1786)	1.0066 (0.0810)	1.7440 (0.4410)	1.2933 (0.4553)	1.0721 (0.2587)
2.5	1.0007 (0.0264)	1.0008 (0.0283)	1.0014 (0.0374)	1 (0)	1.5820 (0.4933)	1.0849 (0.2787)	1.0061 (0.0779)
3	1 (0)	1.0001 (0.0100)	1 (0)	1 (0)	1.4059 (0.4911)	1.0136 (0.1158)	1 (0)

Table 7. Change-point estimation (Antitonic drift in intercept of Poisson profile)

δ_1	Window=20		Window=50		Window=100	
	$\bar{\hat{\tau}}$	s.e ($\hat{\tau}$)	$\bar{\hat{\tau}}$	s.e ($\hat{\tau}$)	$\bar{\hat{\tau}}$	s.e ($\hat{\tau}$)
0.001	124.1868	63.3619	111.9734	55.0070	98.2496	48.1345
0.005	62.7816	18.3835	53.4607	17.6813	51.3865	18.8041
0.01	47.8178	11.1994	43.1857	12.4164	42.5046	13.0578
0.05	33.6446	4.9518	32.9300	6.3049	32.9654	6.2887
0.1	31.6196	3.9092	31.3930	4.8558	31.3608	4.8428
0.5	30.0230	2.4248	29.9161	2.9887	29.9218	2.9734
1	29.8152	2.0916	29.7412	2.4162	29.7918	2.3062
1.5	29.8052	1.6822	29.7457	2.0124	29.7564	2.0091
2	29.8415	1.2015	29.8212	1.3430	29.8537	1.1899
2.5	29.9039	0.8001	29.8973	0.8731	29.9180	0.7137
3	29.9455	0.607396	29.9545	0.5001	29.9454	0.5292

Table 8. ARL for antitonic multiple change in intercept of Poisson profile

$(\delta_{11}, \delta_{12})$	RST (Window=20)	RST (Window=50)	RST (Window=100)	Hotelling T^2	MEWMA λ=0.05	MEWMA λ=0.2	MEWMA λ=0.4
(0.1,0.2)	33.0810 (27.5027)	28.6003 (19.9843)	27.6637 (18.2534)	76.9551 (74.3105)	21.2364 (11.5879)	25.8082 (19.5560)	36.7855 (31.4660)
(0.1,0.3)	17.3729 (10.3399)	16.6281 (8.8489)	16.4788 (8.7580)	46.0258 (42.7766)	14.8859 (6.2077)	15.1887 (8.2943)	19.6177 (13.7646)
(0.1,0.4)	12.2378 (5.4792)	12.2963 (5.3827)	12.1655 (5.3349)	28.5567 (23.9756)	12.0527 (4.1468)	11.3892 (4.6050)	12.9013 (6.8018)
(0.1,0.5)	10.0138 (3.7808)	10.0327 (3.8308)	10.0225 (3.7681)	19.3053 (14.5890)	10.6090 (3.2261)	9.5737 (3.0303)	10.0674 (4.0634)
(0.1,0.6)	8.7051 (2.9399)	8.7580 (2.9375)	8.7518 (2.8576)	13.8968 (9.0027)	9.6099 (2.5577)	8.5186 (2.2459)	8.6840 (2.6887)
(0.1,0.7)	7.9212 (2.3745)	7.9552 (2.4096)	7.9992 (2.3449)	10.7440 (5.6346)	8.9306 (2.1335)	7.9273 (1.7415)	7.8477 (2.0036)
(0.1,0.8)	7.3439 (2.0637)	7.3938 (2.0668)	7.3812 (2.0530)	8.9828 (3.8050)	8.4001 (1.8876)	7.4613 (1.4778)	7.3397 (1.5578)
(0.1,0.9)	6.9886 (1.8006)	7.0202 (1.7799)	7.0362 (1.7580)	7.8999 (2.6537)	8.0562 (1.6717)	7.1758 (1.2929)	6.9960 (1.3255)
(0.1,1)	6.7054 (1.6333)	6.6985 (1.6996)	6.7278 (1.6435)	7.2182 (1.9965)	7.7799 (1.5107)	6.9349 (1.1899)	6.7302 (1.1539)
(0.1,1.1)	6.4822 (1.5443)	6.5006 (1.4901)	6.4893 (1.5672)	6.7191 (1.4712)	7.5433 (1.4035)	6.7422 (1.1216)	6.5400 (1.0511)

Table 9. Change-point estimation (Antitonic multiple change in intercept of Poisson profile)

$(\delta_{11}, \delta_{12})$	Window=20		Window=50		Window=100	
	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$	$\bar{\hat{\tau}}$	s.e $(\hat{\tau})$
(0.1,0.2)	52.9091	27.0831	40.5922	18.2721	37.5572	15.7719
(0.1,0.3)	37.5309	9.7858	33.6154	8.9670	33.5525	8.9150
(0.1,0.4)	33.9825	5.4301	32.8955	6.6706	32.8347	6.8298
(0.1,0.5)	33.4308	4.4555	32.7501	5.8495	32.6726	6.0940
(0.1,0.6)	33.2084	4.2335	32.8438	5.3553	32.8973	5.1945
(0.1,0.7)	33.3253	3.9451	33.0012	4.9768	33.0551	4.8684
(0.1,0.8)	33.3672	3.8647	33.1313	4.6572	33.1544	4.6130
(0.1,0.9)	33.4746	3.6598	33.2840	4.4724	33.3692	4.1998
(0.1,1)	33.6080	3.5141	33.3649	4.3120	33.4304	4.2077
(0.1,1.1)	33.6382	3.5171	33.5438	4.0952	33.4695	4.1614

Table 10. ARL comparison of the proposed method with T^2

δ_1	Bias Reduced T^2		T^2	
	Isotonic	Antitonic	Isotonic	Antitonic
0.8	46.3708 (45.4409)	8.5361 (7.9815)	147.8055 (148.8448)	8.2820 (7.8097)
1	13.4137 (12.7153)	4.9672 (4.4272)	34.4333 (33.8360)	4.7895 (4.2527)
1.1	7.9368 (7.3582)	3.9600 (3.4593)	18.4573 (18.0038)	3.8633 (3.3482)
1.2	5.1139 (4.6667)	3.2107 (2.6734)	10.4202 (9.8837)	3.1227 (2.5878)
1.3	3.4750 (2.9379)	2.6992 (2.1582)	6.1981 (5.6435)	2.6422 (2.0693)
1.4	2.5291 (1.9569)	2.2879 (1.7030)	4.2043 (3.6496)	2.2458 (1.6594)
1.5	1.9047 (1.3017)	1.9858 (1.4171)	2.8906 (2.3483)	1.9450 (1.3494)
2	1.0534 (0.2374)	1.2604 (0.5634)	1.1296 (0.3821)	1.2444 (0.5460)
2.5	1.0004 (0.0200)	1.0545 (0.2382)	1.0014 (0.0374)	1.0579 (0.2453)
3	1 (0)	1.0093 (0.0960)	1 (0)	1.0097 (0.1010)

Table 11. . Alarm time for different scenarios in real-data analysis (CP is the estimated change-point)

	RST	MEWMA	T^2
Scenario 1	17 (CP=16)	18	17
Scenario 2	23 (CP=19)	22	23
Scenario 3	61 (CP=47)	43	76

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