Investigating the effect of learning in set-up cost for imperfect production systems by utilizing two-way inspection plan for rework under screening constraints

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Abstract- In the modern industrial environment, there is a continuous need for the advancement and improvement of the organization’s operations. Learning is an inherent property which is time-dependent and comes with experience. In view of this, the present framework considers the process of learning for an imperfect production system which aids in reducing the setup cost with the level of maturity gained, hence, providing positive results for the organization. Because of machine disturbances/ malfunctions, defectives are manufactured with a known probability density function. To satisfy the demand with good products only, the manufacturer invests in a two-way inspection process with multiple screening constraints. The first inspection misclassifies some of the items and delivers inaccuracies, viz., Type-I and Type–II. The loss due to inspection at the first stage is managed efficiently through a second inspection which is presumed to be free from errors. The study mutually optimizes the production and backordering quantities in order to maximize the expected total profit per unit time. Numerical analysis and detailed sensitivity analysis is carried out to validate the hypothesis and further cater to some valuable implications.

Keywords: Inventory; Imperfect-production; Two-way inspection; Sales-returns; Learning; Screening-constraints

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1. Introduction and literature overview

The following section showcases the inspiration behind the developed framework and presents the overview of literature in the field of inventory management relevant to the present study. In this way, the contribution of the present framework is established among the existing literature.

1.1. Motivation

Production systems prone to malfunctions have gained a lot of attention from researchers at some time or the other. These have been explored under imperfect environments where either the manufacturing process could be imperfect or the screening process or possibly both. In all cases, the result is a higher percentage of defective items that are generally preferred for rework so as to make these as good as new, while others are conventionally salvaged at a lower price without any further check due to the screening errors. So, many papers have adopted only a single inspection technique to separate out the defectives from the produced lot before sending them to the market even in imperfect inspection environment. In view of this, a lesser explored area of two-way inspection plans is emphasized in this paper in which the first one is prone to errors while the second is assumed to be error-free. The first inspection leads to Type-I and Type-II errors. Due to Type-I errors the revenue is directly affected as the non-defectives are classified as defectives. However, as a result of Type-II errors, the defectives are sold to the customers and it results into sales returns, thereby hampering the goodwill of the firm. Henceforth, through a second inspection plan, the outcome of Type-I error can be completely saved from scraping off by mistake, resulting in an increase in revenue. Also, second inspection plan categorizes items in three divisions namely, perfect, reparable, and scrap items, instead of two usual categories viz. reparable and non-reparable. Such a division aids in raising the count of perfect items in the inventory which are sold at the markup price, which ultimately leads to higher profits. Further, learning process is an inherent property of any organization and the maturity gained with time must be incorporated to fetch economic benefits. In lieu of this, learning in the case of production cost components is considered, which is indeed helpful in gaining profits for subsequent inventory cycles. Thus, the present paper considers a production run-time dependent set-up cost to moderate the overall expenses of the system. In this way, the current study fulfills the research gap by constructing an inventory model that deals with imperfect production systems, two-way inspection plans, inspection errors, rework, backorders and learning in setup cost.

1.2. Literature Review

**Imperfect Quality and Screening Errors:** While considering manufacturing systems it is important to take into account the malfunctions as they lead to defective items which directly result in an economic loss for the organization. Moreover, it is necessary to manage the defectives so as to extract the monetary value of the products as much as possible. The defectives can be categorized into reworkable and non-reworkable items. The non-reworkable items can either be vended to a subordinate market at a cheap rate or can be disposed of at some cost. The pioneer work in the field of imperfect production systems was done by Porteus [1], Rosenblatt and Lee [2], Lee and Rosenblatt [3], Kim and Hong [4], Ben-Daya and Hariga [5], Salameh and Jaber [6], Cardenas Barron [7], Huang [8], Chung and Hou [9], Yeh et al [10], Ben-Daya and Rahim [11], Huang [12], Hsieh and Lee [13], Chen and Lo [14], and Wee et al [15]. While dealing with imperfect quality items it is mandatory to adopt screening process, further the screening process can have some human errors thus, it is reasonable to incorporate screening errors in the inventory model so as to make it close to real time manufacturing process. Raouf et al. [16] were the pioneer contributors in the field of inspection; they incorporated the effect of human errors in their model. Later, Duffuaa and Khan [17] and Duffuaa and Khan [18] extended the work for the case of misclassifying the
good items into bad ones and vice-versa. Further, Zhou et al [19], Al-Salamah [20], Khan et al [21], Khanna et al [22], Pal and Mahapatra [23]. Sett et al [24] recently explored the area of inspection errors with other realistic scenarios of inventory management.

**Imperfect Quality and Rework:** To compensate the failure of raising profit margins majorly due to misclassifications which result in sale and salvage of defectives, it is many a times found useful to implement rework process apart from planning of backorders. Also, shortages are bound to occur whenever there is a difficulty of supply and demand especially in imperfect quality environment. Long back, Hayek and Salameh [25] looked into the significance of rework when there are defective items in the inventory system of finite production model with shortages. In several production systems, imperfect items are preferred for rework, which significantly reduces the overall costs of production and inventory. Some significant contributions made in this field are those of Chiu [26], Chiu et al [27], Chiu et al [28], Chiu et al [29], Sana [30], Sarkar et al [31], Sarkar et al [32], Dey and Giri [33]. Later, Chiu [34], Lin [35], Yoo et al [36] explored the area of inventory modeling with imperfect items, screening process and rework. Recently, Hsu and Hsu [37] and Wee et al [38] developed an optimal replenishment model with defective items, screening errors, shortages and sales returns. Cárdenas-Barrón et al. [39] put forth a pioneer work through a brief introduction to the inventory papers. Taleizadeh et al. [40] proposed an optimal order quantity model with partial backorders and reparation of imperfect products, Wang et al. [41] also proposed the optimal order quantity model with the consideration of screening constraints. Jaggi et al [42], Moussawi-Haidar et al [43], Liao [44], Pal et al [45], Shah et al [46], Sekar and Uthayakumar [47], Benkherouf et al [48], Chen [49], Shafiee-Gol [50], Jawla and Singh [51], Cárdenas-Barrón et al. [52], Nobil et al. [53], Chung et al. [54], Nobil et al. [55] have recently explored the area of inventory management by incorporating various rework scenarios.

**Learning in Set-Up Cost:** The process of learning is a time dependent process that comes into picture when the maturity phase of the organization and its workers occurs. In particular, learning in setup cost is a dynamic process through which setup costs can be reduced with the onset of learning in subsequent cycles. The earlier adoption of this process was carried out by Adler and Nanda [56], Sule [57], and also Urban [58]. The effect of learning and forgetting was incorporated in many articles thereafter. A wide review on the topic of learning can be studied in Jaber and Bonney [59] paper. Later, many other researchers, namely, Jaber and Bonney [60], Jaber [61], and also Darwish [62] followed the process of learning in their inventory modeling. Soon after, Khan et al [63] presented an inventory model with defective items and learning in inspection. Later, Konstantaras [64] extended the model of Khan et al [63] by incorporating shortages. Recently, Mukhopadhyay and Goswami [65] proposed an imperfect production inventory model for three kinds of defectives with rework and learning in setup. Table 1 gives a quick review of the literature and research gaps filled.

**Our Contribution:** Inventory management revolves around products and customers. The process of managing products is categorized into numerous parts implementing all the required activities viz. manufacturing, screening, rework, and sales refunds—when the end consumer is not satisfied. Therefore, it makes absolute sense to incorporate product management that includes the control of perfect and imperfect both kinds of products for realistic inventory modeling. In order to manage the whole lot efficiently, firstly an “error-prone” screening process is employed on the complete batch that discards some perfect products by mistake (an outcome of Type-I error) and also sells some defectives as perfect items erroneously (an outcome of Type-II error). In order to reduce the loss due to compromised screening at first hand, another screening process takes place simultaneously on the smaller lot of accumulated defectives (consisting of both actual and wrongly classified defectives) under rigorous
surveillance at a relatively higher cost than the first inspection process. As this re-inspection is considered to have been conducted with better quality, it comes out to be error-free and successfully nullifies the loss occurred due to Type-I error by extracting out the perfect items (that are wrongly classified as defectives) completely. Finally, this second inspection successfully categorizes the accumulated defectives into three parts viz. perfect, reparable, and non-reparable. Thus, it contributes in raising the count of perfect items and hence the revenue against an additional investment in a second inspection process. Post these two simultaneous screening processes, the rework process begins on the reparable lot which further adds to the revenue, at a marginal cost of rework. So, contrary to the previous research practices that assume single and perfect inspection to handle the defectives, the present paper investigates the imperfect production systems under the conditions of two-way inspection processes with screening errors in the first stage only along with sales returns. A rework process is then employed for accumulated defectives under various screening constraints for achieving higher standards of quality and thereby revenue. Additionally, learning in setup cost is considered in the model to gain some maturity with time to elevate the profit values. Two mathematical models are proposed for imperfect production system with and without the effects of learning respectively. The numerical analysis and sensitivity analysis are carried out to showcase key features. Further, the importance of two-way inspection plan is signified through a comparative study. The model is applicable to a variety of manufacturing industries that target high standards of quality, customer satisfaction, encourage teamwork, and prefer to invest in learning.

2. Model development
The present segment gives the notations, assumptions and constraints under which the model is developed.

2.1. Notations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Λ</td>
<td>Demand rate in units per unit time</td>
</tr>
<tr>
<td>Φ</td>
<td>Production rate</td>
</tr>
<tr>
<td>φ₁</td>
<td>Rework rate</td>
</tr>
<tr>
<td>x</td>
<td>Inspection rate in units per unit time, λ&gt;D</td>
</tr>
<tr>
<td>α</td>
<td>Proportion of imperfect items (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>q₁</td>
<td>Proportion of Type-I imperfection error (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>q₂</td>
<td>Proportion of Type-II imperfection error (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>p₁</td>
<td>Proportion of non-reparable/ scrap items (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>p₂</td>
<td>Proportion of reparable items (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>p₃</td>
<td>Proportion of perfect items, secluded from second inspection process (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>r</td>
<td>Proportion of rework items (a random variable with known p.d.f.)</td>
</tr>
<tr>
<td>T</td>
<td>Cycle length</td>
</tr>
<tr>
<td>E(.)</td>
<td>Expected value operator</td>
</tr>
<tr>
<td>E(θ)</td>
<td>Expected value of θ</td>
</tr>
<tr>
<td>K₀</td>
<td>Production set-up cost for each cycle</td>
</tr>
<tr>
<td>cₚ</td>
<td>Purchase cost per item ($/ item)</td>
</tr>
<tr>
<td>i₁</td>
<td>Inspection cost per item during production ($/ item)</td>
</tr>
<tr>
<td>i₂</td>
<td>Inspection cost per item after production ($/ item)</td>
</tr>
<tr>
<td>s</td>
<td>Selling price ($/ item)</td>
</tr>
</tbody>
</table>
Disposal cost of defectives ($/item)

Unit discounted price of each defective item ($/item)

Shortage cost per unit per unit time

Cost of obligating Type-I error ($/item)

Cost of obligating Type-II error ($/item)

Holding cost per unit time

Holding cost of reworked items per unit time

Decision variables

Production lot size

Backorder level

Functions

p.d.f. of defective items

p.d.f. of Type-I error

p.d.f. of Type-II error

p.d.f. of rework items

p.d.f. of scarp items

p.d.f. of non-repairable items

p.d.f. of repairable items

Total cost

Expected total cost per unit time

Total revenue

Expected total revenue per unit time

Total profit per unit time for j = 1,2

Expected total profit per unit time for j = 1,2

Optimal cycle length

Optimal order quantity per cycle

Optimal Backorder level

Optimal expected total profit per unit time

2.2. Assumptions

The mathematical model has been proposed under the following assumptions.

1) Demand rate is constant, uniform and deterministic. Also demand is satisfied by perfect items only.

2) Production rate is finite and constant.

3) Production process produces only single product type and delivers some imperfect items as well.

4) First screening process leads to Type-I and Type-II misclassification errors.

5) Second stage inspection is error free and produces three types of items, namely, perfect, repairable, and non-repairable items.

6) Rework process is considered after the end of second inspection procedure.

7) The screening cost during production is higher than the screening cost after production.

8) The holding cost of defectives that are reworked is more than that of non-defective items.

9) The manufacturer is learning from past experience and is able to reduce the set-up time and cost eventually.

10) The non-learning model can be the trivial case of the present study, i.e., $\infty = 0$
11) Shortages are allowed and fully backlogged.
12) Time period is infinite and lead time is insignificant.

2.3. Model Constraints
1) Production rate is greater than screening rate \( (\phi > x) \).
2) Rate of screening is greater than the rate of demand \( (x > \lambda) \).
3) Rate of rework is greater than the demand rate \( (\phi > \lambda) \).
4) Since some perfect items will always be there in the inventory, so \( I - E[p_I] > 0 \).
5) In order to eliminate the backorders and maintain the positive inventory, we have \( (I - E[p_I])x - \lambda > 0 \).
6) In order to avoid the shortages during the screening process, the number of inspected perfect items should be at least equal to the demand during that period i.e. \( \frac{\lambda y}{x} \leq y(I - E[p_I]) \).
7) Production rate of perfect items satisfies the inequality: \( \phi(I - E[p_I]) > x \).

2.4. Screening Constraints
1) When the production rate is greater than the screening time, the screening process will continue even after the production process i.e. \( t_s > 0 \).
2) For smooth functioning of inventory, screening should finish before the completion of inventory i.e. \( t_s < (t_s + t_h) \).
3) The total expected cost of production along with screening should be less than the total sales revenue earned by selling both perfect and imperfect items i.e. \( c_p + d_1 + d_2 < s(1 - \alpha) - s\alpha q_s + s\alpha r + v\alpha (1 - r) \).

3. Mathematical modeling
The present segment describes the problem definition so as to give the clear picture of the problem under consideration and formulates the mathematical model that fits the above discussed assumptions and constraints.

3.1. Problem Definition
Manufacturing systems are loaded with a number of sub units and due to reasons like interrupted supply of power, age based issues, tools malfunction, overheating of the machine etc. the system can end up producing defectives. The defective items should be removed from the lot by a vigilant screening process. However, the screening is not always error-free because of numerous reasons like lack of good work instructions, some human errors, weak control over the entire screening process etc. which ultimately leads to misclassification errors of two types namely Type-I and Type-II. The first type of error results in direct loss of revenue for the manufacturer and due to the second type of error, the defective items are delivered to the customers who bring sales returns. To reduce the harm related to the goodwill of the manufacturer in the market, sales returns are also legitimate for full price refunds. For the effective management of the whole inventory system, the management of all the items that are either actually defective or wrongly classified as defective or that are returned from the customers is vital. Due to the outcome of Type-I error, there arises a need for another inspection process that is capable of not only saving the perfect items from getting scrapped at a reduced price or getting them reworked for no use, but also extracting them successfully at a marginal cost of second inspection process only. In view of this, the
second inspection process is considered to be error-free and thus it caters the manufacturer with three types of products viz. non-reparable \((p_1 = a (1-r))\), reparable \((p_2 = ar)\), and the perfect ones \((p_3 = (1-a)q_1)\), which were particularly victimized by the Type-I error earlier. Further, a rework process is employed in order to transform the items of reparable category into perfect ones at some rework cost. Also, the process of learning is an ongoing time-dependent process and learning in the setup of the production process is of great importance for the manufacturer. With time, the organization is able to learn numerous things which should be incorporated in the subsequent production cycles in order to increase the efficiency of the firm. In light of this, the present framework develops two cases for an inventory model with and without the effects of learning in setup cost in the presence of defectives, two stage inspection process, screening errors in the first inspection procedure, perfect rework process, disposal of non-reparable stock, and also incorporates fully backlogged shortages. Thus, the proposed model has been explored under many realistic model constraints as well as screening constraints so as to attain more practical results. The sequential flow of above described events is given in Figure 1.

The problem of the manufacturer is formulated by jointly optimizing the optimal production batch size and backorder size. Profit is obtained by subtracting all the cost components viz. cost of production, inspection, misclassification, rework, shortage, disposal and holding from the revenue which is obtained by the sales of good items, imperfect items, and reworked items. Further, the fractions of Type-I and Type-II errors are considered independent of defect proportion.

3.2. Mathematical Formulation

The present fragment formulates a mathematical model which fits the problem description and assumptions of the model. The graphical representation of the inventory is given in Figure 2.

Total outcome of perfect items sorted after the combined effect of inspection and rework processes are \((1-p_1)y\)

where \(p_1\) is expanded in APPENDIX A

Since demand is satisfied from perfect items only so the length of total cycle is defined as the total number of perfect items sold as per the demand rate i.e.

\[
T = (1 - p_1) \frac{y}{\lambda}
\]  

(1)

Shortages start building up from the beginning of the inventory cycle i.e. from time 0 to \(A_1\), so the total backorder building time is calculated as \(t_1 = \frac{B}{\lambda}\)

(2)

After time point \(A_1\), shortages begin to reduce and get completely eliminated by \(A_2\), so the complete shortage elimination time is determined by \(t_2 = \frac{B}{(1-p_1)x-\lambda}\)

(3)
Also, the whole inventory of defectives built during the shortage removal time \((A_1, A_2)\) is estimated as:

\[
z_1 = \left[ \phi - (1 - p_j) x \right] t_2 = \left[ \phi - (1 - p_j) x \right] \frac{B}{(1 - p_j) x - \lambda} \tag{4}\]

The overall inventory during time period \((A_2, A_3)\) is constructed by the accumulation defectives as well as the unsold perfect items. The uptime of this inventory is calculated as:

\[
t_3 = \frac{z_5 - z_4}{\phi - \lambda} = \frac{1}{\phi - \lambda} \left( z_5 - \frac{\phi - (1 - p_j) x \cdot B}{(1 - p_j) x - \lambda} \right) \tag{5}\]

Also, the entire production runs for a period of \(A_1\) up to \(A_3\) so its duration can be obtained by adding lengths \(t_2, t_3\) i.e.

\[
t_2 + t_3 = \frac{y}{\phi} \tag{6}\]

By substituting the value of equation (3) in above equation, \(t_3 = \frac{y}{\phi} \frac{B}{(1 - p_j) x - \lambda}\)

On equating equations (5) and (7), the value of \(z_5\) is obtained as:

\[
z_5 = \frac{\left[ \phi - (1 - p_j) x \right] \cdot B}{(1 - p_j) x - \lambda} + \frac{\phi - \lambda}{\phi} \left( \frac{y}{x} - \frac{B}{(1 - p_j) x - \lambda} \right) \tag{8}\]

As the total screening time is presumed to exceed the production time, therefore, it varies from time point \(A_1\) to \(A_4\) and is given as \(t_2 + t_3 + t_4 = \frac{y}{x}\)

By substituting the value of equation (6) in above equation, \(t_4 = \frac{y - \frac{y}{\phi}}{x}\)

Also, the total inventory depleted during the post production screening time is determined as:

\[
z_5 - z_4 = \lambda t_4 \tag{11}\]

By using equations (8), (10), and (11), the value of \(z_4\) is obtained as:

\[
z_4 = \left[ \phi - (1 - p_j) x \right] \frac{B}{(1 - p_j) x - \lambda} + \frac{\phi - \lambda}{\phi} \left( \frac{y}{x} - \frac{B}{(1 - p_j) x - \lambda} \right) - \lambda \left( \frac{y - \frac{y}{\phi}}{x} \right) \tag{12}\]

The defectives which are scrapped add up to \(p_j y\) and these are disposed right after the end of screening process i.e. at \(A_n\), so the effective inventory gets reduced instantaneously and is given by:

\[
z_3 = z_4 - p_j y = \left[ \phi - (1 - p_j) x \right] \frac{B}{(1 - p_j) x - \lambda} + \frac{\phi - \lambda}{\phi} \left( \frac{y}{x} - \frac{B}{(1 - p_j) x - \lambda} \right) - \lambda \left( \frac{y - \frac{y}{\phi}}{x} \right) - p_j y \tag{13}\]

Next, the rework process of a fraction from total accumulated defectives begins subsequent to the second inspection procedure. The second inspection is done on hand-to-hand basis and so its time period is not taken into account. So, the rework runtime which begins from \(A_4\) and continues till \(A_5\) is determined as:
\[ t_5 = \frac{z_2 - z_1}{\phi - \lambda} = \frac{I}{\phi - \lambda} \left( \frac{\phi - (1 - p_1)x}{(1 - p_1)x - \lambda} B + (\phi - \lambda) \left[ y - \frac{B}{(1 - p_1)x - \lambda} \right] - \lambda \left( \frac{y}{x} - \frac{y}{\phi} \right) - p_1y - z_2 \right) \] (14)

Also, out of the entire defective inventory \( \alpha y \), the count of reworkables is \( p_2y \), so the rework processing time can also be rewritten as \( t_5 = \frac{p_2y}{\phi} \) (15)

On equating equations (14) and (15), the value of \( z_2 \) is obtained as:

\[ z_2 = \frac{\phi - (1 - p_1)x}{(1 - p_1)x - \lambda} B + (\phi - \lambda) \left[ y - \frac{B}{(1 - p_1)x - \lambda} \right] - \lambda \left( \frac{y}{x} - \frac{y}{\phi} \right) - p_1y - \frac{p_2y(\phi - \lambda)}{\phi} \] (16)

Lastly, the remaining perfect items coming out of the rework process \( p_2y \) and also those which are directly segregated from the second inspection process \( p_3y \) get sold till the inventory completely depletes to zero. So, we have,

\[ t_6 = \frac{z_2 - \lambda}{\phi} = \frac{I}{\phi} \left( \frac{\phi - (1 - p_1)x}{(1 - p_1)x - \lambda} B + (\phi - \lambda) \left[ y - \frac{B}{(1 - p_1)x - \lambda} \right] - \lambda \left( \frac{y}{x} - \frac{y}{\phi} \right) - p_1y - \frac{p_2y(\phi - \lambda)}{\phi} \right) \] (17)

Since whole screening process is covered in two parts, one part ends with the production procedure and thus runs for a length of \( (t_2 + t_3) \) while the second part begins after the production process is over and runs for a length of \( (t_4) \) i.e. till all the remaining items are screened. So, the number of units screened during the first interval \( (A_1, A_2) \) is obtained in the following manner:

\[ = \left[ \lambda + \lambda \alpha + \lambda \alpha^2 + ... \right] (t_2 + t_3) = \frac{\lambda (t_2 + t_3)}{1 - \alpha} = \frac{\lambda y}{\phi (1 - \alpha)} \] (18)

\[ = Ay \text{, where, } A = \frac{\lambda}{\phi (1 - \alpha)} \] (19)

To estimate the total number of units screened during the second interval \( (A_3, A_4) \), we first determine the complete count of defectives accumulated by timepoint \( A_3 \) and then subtract these from the maximum inventory level present at that time i.e. \( z_5 \). The total number of defectives accumulated during \( (A_3, A_4) \) is obtained by the total units screened by time point \( A_3 \) minus the demand satisfied by this time i.e. \( = Ay - \lambda t_2 \)

(20)

Hence, by using equation (20), the total number of items screened between \( (A_3, A_4) \) is as follows:

\[ = z_5 - (Ay - \lambda t_2) \] (21)

\[ = \frac{\phi - (1 - p_1)x}{(1 - p_1)x - \lambda} B + (\phi - \lambda) \left[ y - \frac{B}{(1 - p_1)x - \lambda} \right] - \frac{\lambda y}{\phi (1 - \alpha)} - B \] (22)
Furthermore, the model also incorporates the effect of learning in setup cost by assuming a variable setup cost function dependent on the production run length $T_p$,

where, $T_p = t_2 + t_3$

\[
C_0(T_p) = \begin{cases} 
C_0(T_p)^\in & T_p < T_M \\
C_{max} & T_p \geq T_M 
\end{cases} 
\]  
(23)

\[
\therefore C_0(T_p) = \begin{cases} 
C_0\left(\frac{y}{\phi}\right)^\in & T_p < T_M \\
C_{max} & T_p \geq T_M 
\end{cases} 
\]  
(24)

where $\in$ is the shape factor, $C_0$ is the setup cost related to the basic production quantity model when the shape factor is zero, $T_M$ is the minimum run length after which setup process requires maximum cost ($C_{max}$).

The cost ($C_{max}$) acts as an upper limit to the setup cost. The shape factor $\in$ is estimated on the past data of earlier manufacturing practices by adopting the curve-fitting method. (Darwish [15]) represented the setup cost against the production run length for the different values of the shape factor ($\in$). Its graphical representation is shown in Figure 3.

<Insert Figure 3>

### 3.3. Components of Sales Revenue

The components of revenue are evaluated as follows:

$R_1$  Sales from only good quality items = $s(1-\alpha)(1-q_f)y + s\alpha q_2 y$

(26)

$R_2$  Loss of revenue from sales returns = $-s\alpha q_2 y$

(27)

$R_3$  Sales from reworked items = $s\alpha r y$

(28)

$R_4$  Sales from misclassified perfect items (Outcome of Type-I error) = $s(1-\alpha)q_1 y$

(29)

$R_5$  Sales from scrap/ non-reworkable items = $v\alpha (1-r) y$

(30)

By adding the sales from good quality items, with revenue loss in sales returns, sales from reworked items, sales from scrap items and finally from the sales of non-reworkable items the total revenue of the manufacturer is obtained as follows:

\[
T.R. = R_1 + R_2 + R_3 + R_4 + R_5 \\
= s(1-\alpha)y + s\alpha r y + v\alpha (1-r) y 
\]  
(31)

### 3.4. Components of Inventory System Costs

Following are the cost components that are incurred in the present inventory scenario:
Setup cost, obtained by introducing the effects of learning in setup costs:

\[
C_o(T_p) = \begin{cases} 
C_0 \left( \frac{Y}{\phi} \right)^{\alpha} & T_p < T_M \\
C_{\text{max}} & T_p \geq T_M 
\end{cases}
\] (32)

Purchase cost including the variable cost per cycle: \( C_p y \) (33)

Screening cost of first inspection process during production runtime: \( d_1 \left( \frac{\lambda y}{\phi(1-\alpha)} \right) \) (34)

Screening cost of first inspection process post production:

\[
d_2 \left[ \frac{\phi - (1 - p_1) x}{B} (\phi - \lambda) + \frac{y B}{\phi (1 - p_1) x - \lambda} - \frac{\lambda y}{\phi(1-\alpha) - B} \right] (35)
\]

Screening cost of second inspection process:

\[
d_3 \left[ \alpha y + (1-\alpha) q_1 y \right] (36)
\]

Rework Cost: \( C_w \alpha r y \) (37)

Cost of Type-I error which incurs because the inspector has wrongly classified some fraction of non-defectives as defectives: \( C_r (1-\alpha) q_1 y \) (38)

Cost of Type-II error which incurs because the inspector has misclassified a portion of defectives as non-defectives: \( C_a \alpha q_2 y \) (39)

Disposal Cost of non-reworkable items: \( u\alpha (1-r) y \) (40)

Holding cost of the defective, non-defectives and sales returns in a cycle:

\[
h \left[ \frac{l}{2} z_1 t_2 + \frac{l}{2} z_2 \left( z_1 + z_3 \right) + \frac{l}{2} t_4 \left( z_3 + z_5 \right) + \frac{l}{2} t_6 z_2 + \frac{l}{2} \alpha q_2 y T \right] + h_1 \frac{l}{2} (z_3 + z_2) t_5 (41)
\]

Shortage Cost: \( C_B \frac{l}{2} (t_1 + t_2) B \) (42)

As the present model is developed under the assumption of learning in setup cost, thus, depending upon the learning effects two cases are established for manufacturer’s total cost:

**Case I:** \( T_p < T_M \) (Under the Effects of Learning)

Considering the case when \( T_p < T_M \) the following value is obtained for the total cost of the manufacturer by substituting the appropriate value of \( C_o(T_p) \).
The manufacturer’s total profit for **Case I** is expressed as follows:

\[
T.P.I. = s(1-\alpha)y + sary + \alpha(1-r)y - C_0\left(\frac{y}{\phi}\right)^{\ddagger} - C_yy - d_1\left(\frac{\lambda y}{\phi(1-\alpha)}\right) - d_2\left[\frac{\phi-(1-p_1)x}{(l-p_1)x-\lambda} + \left(\frac{\phi}{\phi(1-\alpha)}\right)\right] \nonumber
\]

\[
-\frac{h}{2}(\phi-\lambda)\left[\frac{\phi-(1-p_1)x}{(l-p_1)x-\lambda}\right] + \frac{h}{2}\left(\frac{\phi-(1-p_1)x}{(l-p_1)x-\lambda}\right) + \left(\frac{\phi}{\phi(1-\alpha)}\right) - \lambda\left(\frac{y}{x}\right)\nonumber
\]

\[
- \frac{h}{2}\left[\frac{\phi-(1-p_1)x}{(l-p_1)x-\lambda}\right] + \left(\frac{\phi}{\phi(1-\alpha)}\right) - \lambda\left(\frac{y}{x}\right)\nonumber
\]

\[
- \frac{h}{2}\left[\frac{\phi-(1-p_1)x}{(l-p_1)x-\lambda}\right] + \left(\frac{\phi}{\phi(1-\alpha)}\right) - \lambda\left(\frac{y}{x}\right)\nonumber
\]

\[
+ C_y y^{\ddagger} + \frac{1}{\lambda} + \frac{1}{(l-p_1)x-\lambda}\right]B^2
\]

\[= yG_1 - BG_2 - y^2G_3 - B^2G_4 - yBG_5 - \frac{C_0y^{\ddagger}}{\phi^{\ddagger}} \quad (45)\]

where, \( p_1, p_2, p_3, G_1, G_2, \ldots, G_5 \) are expanded in **APPENDIX A**

The manufacturer’s total profit per unit time for **Case I** can be expressed as follows:

\[
Z_1(y, B) = \frac{\lambda}{(l-p_1)}\left[ G_1 - \frac{BG_2}{y} - yG_3 - \frac{B^2G_4}{y} - BG_5 - \frac{C_0y^{\ddagger}}{\phi^{\ddagger}}\right] - \frac{\alpha q_2y}{2} \quad (46)
\]

Therefore, the expected total profit per unit time can be written as:

\[
E[Z_1(y, B)] = \frac{\lambda}{(1-E[p_1])}\left[ E[G_1] - \frac{BE[G_2]}{y} - yE[G_1] - \frac{B^2E[G_4]}{y} - BE[G_3] - \frac{C_0y^{\ddagger}}{\phi^{\ddagger}}\right] - \frac{E[\alpha]E[q_2]}{2} \quad (47)
\]

where, \( E[p_1], E[p_2], E[p_3], E[G_1], E[G_2], \ldots, E[G_5] \) are expanded in **APPENDIX B**

The above formulation of the function makes clear that when \( \varepsilon \geq 1 \), the profit function is monotonically decreasing in \( y \), which shows that the total cost function will be minimum when \( y = 0 \); which is
reasonably impractical. In practice, it recommends that $\gamma$ should be minimum i.e. it should be as much as required which closely follows the JIT (Just-in-Time) manufacturing philosophy. So, it is recommended that $\varepsilon < 1$. Moreover, $C_0(T_p)$ is a concave function which is increasing for $0 < \varepsilon \leq 1$ however decreasing for $\varepsilon < 0$.

The values indicating $\varepsilon < 0$ signify the state when the effect of learning disables the effect of forgetting and deterioration which results in the reduction of the setup costs with time.

**Case II:** $T_p \geq T_{yd}$ (Without the Effects of Learning)

When $T_p \geq T_{yd}$ the following expression for the manufacturer's total cost is obtained:

$$T.C.2 = C_{mx} + C_p y + d_i \left( \frac{\lambda y}{\phi(1-\alpha)} \right) + d_2 \left\{ \frac{\phi - (1-p_l)x}{(1-p_l)x - \lambda} \right\} + \left( \phi - \lambda \right) \left\{ \frac{y}{\phi} - \frac{B}{(1-p_l)x - \lambda} \right\} - \frac{\lambda y}{\phi(1-\alpha)} - B$$

$$+ d_3 \left[ ay + (1-\alpha)q_y \right] + C_a a y + C_r (1-\alpha) q_y + C_a a q_y + u a (1-r) y + h \left[ \frac{1}{2} z_t + \frac{1}{2} t_s (z_j + z_s) + \frac{1}{2} t_r (z_j + z_s) \right]$$

$$+ h_1 \left( z_j + z_s \right) t_s + C_y \frac{1}{2} \left( t_s + t_r \right) B$$

The manufacturer’s total profit for **Case II** is expressed as follows:

$$T.P.2 = s (1-\alpha) y + s a r y + v a (1-r) y - C_{mx} - C_p y - d_i \left( \frac{\lambda y}{\phi(1-\alpha)} \right) - d_2 \left\{ \frac{\phi - (1-p_l)x}{(1-p_l)x - \lambda} \right\} - \left( \phi - \lambda \right) \left\{ \frac{y}{\phi} - \frac{B}{(1-p_l)x - \lambda} \right\}$$

$$- d_3 \left[ ay + (1-\alpha)q_y \right] - C_s a r y - C_r (1-\alpha) q_y - C_a a q_y - u a (1-r) y - h \left[ \frac{1}{2} \left( \frac{\phi - (1-p_l)x}{(1-p_l)x - \lambda} \right) \right]$$

$$- h_1 \left( z_j + z_s \right) t_s + C_y \frac{1}{2} \left( t_s + t_r \right) B$$

$$- d_3 \left[ ay + (1-\alpha)q_y \right] - C_s a r y - C_r (1-\alpha) q_y - C_a a q_y - u a (1-r) y - h \left[ \frac{1}{2} \left( \frac{\phi - (1-p_l)x}{(1-p_l)x - \lambda} \right) \right]$$

$$- h_1 \left( z_j + z_s \right) t_s + C_y \frac{1}{2} \left( t_s + t_r \right) B$$

$$- d_3 \left[ ay + (1-\alpha)q_y \right] - C_s a r y - C_r (1-\alpha) q_y - C_a a q_y - u a (1-r) y - h \left[ \frac{1}{2} \left( \frac{\phi - (1-p_l)x}{(1-p_l)x - \lambda} \right) \right]$$

$$- h_1 \left( z_j + z_s \right) t_s + C_y \frac{1}{2} \left( t_s + t_r \right) B$$

$$- d_3 \left[ ay + (1-\alpha)q_y \right] - C_s a r y - C_r (1-\alpha) q_y - C_a a q_y - u a (1-r) y - h \left[ \frac{1}{2} \left( \frac{\phi - (1-p_l)x}{(1-p_l)x - \lambda} \right) \right]$$

$$- h_1 \left( z_j + z_s \right) t_s + C_y \frac{1}{2} \left( t_s + t_r \right) B$$

Thus, the manufacturer’s total profit can be written as:
\[ yG_1 - BG_2 - y^2G_3 - B^2G_4 - yBG_5 - C_{\max}, \]  

(50)

where, \( p_1, p_2, p_3, G_1, G_2, \ldots, G_5 \) are expanded in APPENDIX A.

The manufacturer’s total profit per unit time can be written as:

\[ Z_2(y, B) = \frac{\lambda}{(1 - p_1)} \left[ G_1 - \frac{BG_2}{y} - yG_3 - \frac{B^2G_4}{y} - yBG_5 - C_{\max} \right] - \frac{\alpha q_2 y}{2} \]  

(51)

Therefore, the expected total profit per unit time can be written as:

\[ E[Z_2(y, B)] = \frac{\lambda}{(1 - E[p_1])} \left[ E[G_1] - \frac{BE[G_2]}{y} - yE[G_3] - \frac{B^2E[G_4]}{y} - yBE[G_5] - C_{\max} \right] - \frac{\alpha E[\alpha]E[q_2]}{2} \]  

(52)

where, \( E[p_1], E[p_2], E[p_3], E[G_1], E[G_2], \ldots, E[G_5] \) are expanded in APPENDIX B.

4. Optimal policy

The manufacturer aims to maximize the expected total profit per unit time by jointly optimizing the production amount and the backorder quantity. In the following section, concavity of the objective function is proved in the form of two lemmas.

Case I: \( T_P < T_M \)

**Lemma 1.** The function of manufacturer’s expected total profit per unit time for Case I is concave.

**Proof.** In order to prove the global concavity of the expected profit function for this case, the following two second-order sufficient conditions of optimality should be satisfied:

\[
\left( \frac{\partial^2 E[Z_1(y, B)]}{\partial y^2} \right) \leq 0; \quad \left( \frac{\partial^2 E[Z_1(y, B)]}{\partial B^2} \right) \leq 0
\]

and

\[
\left( \frac{\partial^2 E[Z_1(y, B)]}{\partial y \partial B} \right)^2 - \left( \frac{\partial^2 E[Z_1(y, B)]}{\partial y^2} \right) \left( \frac{\partial^2 E[Z_1(y, B)]}{\partial B^2} \right) \leq 0
\]

(4.1)

By taking first order partial derivative of \( E[Z_1(y, B)] \) with respect to \( y \), we obtain

\[
\frac{\partial}{\partial y} E[Z_1(y, B)] = \frac{\lambda}{1 - E[p_1]} \left[ E[G_2] \frac{B}{y^3} - E[G_3] + E[G_4] \frac{B^2}{y^3} - \frac{c_0 (\epsilon - 1) \gamma^{(e - 2)}}{\varphi^e} \right] - \frac{\alpha E[\alpha]E[q_2]}{2}
\]

(53)

Next, by taking second order partial derivative of \( E[Z_1(y, B)] \) with respect to \( y \), we obtain

\[
\frac{\partial^2}{\partial y^2} E[Z_1(y, B)] = \frac{\lambda}{1 - E[p_1]} \left[ -2E[G_2] \frac{B}{y^3} - 2E[G_3] \frac{B^2}{y^3} - \frac{c_0 (\epsilon - 1) (\epsilon - 2) \gamma^{(e - 3)}}{\varphi^e} \right]
\]

(54)

Again, by taking first order partial derivative of \( E[Z_1(y, B)] \) with respect to \( B \), we obtain
Again, by taking second order partial derivative of \( E[Z_i(y, B)] \) with respect to \( B \), we obtain

\[
\frac{\partial^2}{\partial B^2} E[Z_i(y, B)] = -\frac{\lambda}{1-E[p_i]} \frac{2E[G_4]}{y}
\]

(56)

Also, \( \frac{\partial^2}{\partial y \partial B} E[Z_i(y, B)] = \frac{\lambda}{1-E[p_i]} \left\{ \frac{E[G_z]}{y^3} + \frac{2E[G_4]B}{y^2} \right\} \)

(57)

Therefore, by using equations (57), (54) and (56) we obtain:

\[
\left( \frac{\partial^2}{\partial y \partial B} E[Z_i(y, B)] \right)^2 - \left( \frac{\partial^2}{\partial y^2} E[Z_i(y, B)] \right) \left( \frac{\partial^2}{\partial B^2} E[Z_i(y, B)] \right) = \left( \frac{\lambda}{1-E[p_i]} \right)^2 \left\{ \frac{E[G_z]}{y^3} - \frac{2c_0(\epsilon-1)(\epsilon-2)E[G_4]}{y^{\phi \epsilon}} \right\}
\]

(58)

The three conditions of concavity are derived in APPENDIX C.

**Lemma 2.** The optimal solution \((y^*, B^*)\) that maximizes the manufacturer’s expected total profit per unit time for Case I is written as:

\[
y^* = \left[ \frac{4(1-\epsilon)c_0E[G_4]}{p^\epsilon \left\{ 4E[G_4] \left\{ E[G_3] + \frac{E[\alpha]E[G_z]}{2} \right\} - E^2[G_3] \right\} } \right]^{\frac{1}{\epsilon-2}}
\]

and

\[
B^* = -\frac{E[G_z] + E[G_5]y}{2E[G_4]}
\]

**Proof.** In order to find the optimal values of \( y \) and \( B \), say \( y^* \) and \( B^* \), that maximize \( E[Z_i(y, B)] \), the first-order necessary condition of optimality must be equated to zero i.e.

\[
\frac{\partial}{\partial y} E[Z_i(y, B)] = 0 \quad \text{and} \quad \frac{\partial}{\partial B} E[Z_i(y, B)] = 0
\]

(4.2)

On setting equation (53) equal to zero, we get:

\[
\frac{\lambda}{1-E[p_i]} \left\{ -2E[G_z] \frac{B}{y^3} - 2E[G_4] \frac{B^2}{y^3} - \frac{c_0(\epsilon-1)(\epsilon-2)y^{(\epsilon-3)}}{\varphi^\epsilon} \right\} = 0
\]

(59)

On setting equation (55) equal to zero, we get:

\[
\frac{\lambda}{1-E[p_i]} \left\{ \frac{E[G_z]}{y} - \frac{2E[G_4]B}{y} - E[G_5] \right\} = 0
\]

(60)
\[ B' = -\frac{E[G_2] + E[G_5]y}{2E[G_4]} \]  

(61)

Putting this value of \( B \) in equation (59) to attain the value for \( y \) i.e.:

\[
y^* = \left[ \frac{4(1-\varepsilon)c_0E[G_4]}{p^* \left( 4E[G_4] \left( E[G_3] + \frac{E[\alpha]E[q_2]}{2} \right) - E^2[G_5] \right)} \right]^{\frac{1}{2-\varepsilon}}
\]

(62)

Hence, \( y^* \) and \( B^* \) are the optimal values of \( y \) and \( B \) for Case I.

Case II: \( T_M \geq T_p \)

Lemma 3. The function of manufacturer’s expected total profit per unit time for Case II is concave.

Proof. In order to prove the global concavity of the expected profit function, the following two second-order sufficient conditions of optimality for this case should also be satisfied:

\[
\left( \frac{\partial^2 E[Z_2(y,B)]}{\partial y^2} \right) \leq 0; \quad \left( \frac{\partial^2 E[Z_2(y,B)]}{\partial B^2} \right) \leq 0
\]

and

\[
\left( \frac{\partial^2 E[Z_2(y,B)]}{\partial y \partial B} \right)^2 - \left( \frac{\partial^2 E[Z_2(y,B)]}{\partial y^2} \right) \left( \frac{\partial^2 E[Z_2(y,B)]}{\partial B^2} \right) \leq 0
\]

(4.3)

By taking first order partial derivative of \( E[Z_2(y,B)] \) with respect to \( y \), we obtain

\[
\frac{\partial}{\partial y} E[Z_2(y,B)] = \frac{\lambda}{1-E[p_1]} \left\{ E[G_3] \frac{B}{y^3} - E[G_5] + E[G_4] \frac{B^2}{y^3} + \frac{C_{\max}}{y^3} \right\} - \frac{E[\alpha]E[q_2]}{2}
\]

(63)

Next, by taking second order partial derivative of \( E[Z_2(y,B)] \) with respect to \( y \), we obtain

\[
\frac{\partial^2}{\partial y^2} E[Z_2(y,B)] = \frac{\lambda}{1-E[p_1]} \left\{ -2E[G_3] \frac{B}{y^3} - 2E[G_4] \frac{B^2}{y^3} - \frac{2C_{\max}}{y^3} \right\} \leq 0
\]

(64)

Again, by taking first order partial derivative of \( E[Z_2(y,B)] \) with respect to \( B \), we obtain

\[
\frac{\partial}{\partial B} E[Z_2(y,B)] = \frac{\lambda}{1-E[p_1]} \left\{ -E[G_3] \frac{B}{y} - 2E[G_4] \frac{B}{y} - E[G_5] \right\}
\]

(65)

Again, by taking second order partial derivative of \( E[Z_2(y,B)] \) with respect to \( B \), we obtain
\[
\frac{\partial^2}{\partial B^2} E[Z_2(y, B)] = -\frac{\lambda}{1-E[p_1]} \frac{2E[G_1]}{y} \leq 0
\]  
(66)

Also, 
\[
\frac{\partial^2}{\partial y \partial B} E[Z_2(y, B)] = \frac{E[G_2]}{y^2} + \frac{2E[G_4]B}{y^2} 
\]  
(67)

Therefore, by using equations (67), (64) and (66) we obtain
\[
\left(\frac{\partial^2}{\partial y \partial B} E[Z_2(y, B)]\right)^2 - \left(\frac{\partial^2}{\partial y^2} E[Z_2(y, B)]\right)\left(\frac{\partial^2}{\partial B^2} E[Z_2(y, B)]\right) = \frac{E^2[G_2]}{y^4} - \frac{4E[G_4]C_{\max}}{y^4} 
\]  
(68)

The conditions of concavity have been derived in APPENDIX D.

**Lemma 4.** The optimal solution \((y^*, B^*)\) that maximizes the manufacturer’s expected total profit per unit time for is written as
\[
y^* = \sqrt{\frac{2\lambda \left(BE[G_2] + B^2E[G_1] + C_{\max}\right)}{2\lambda E[G_3] + E[\alpha]E[q_2](1-E[p_1])}} \quad \text{and} \quad B^* = -\frac{E[G_2] + E[G_3]y}{2E[G_4]} 
\]  
Proof. In order to find the optimal values of \(y\) and \(B\), say \(y^*\) and \(B^*\), that maximize \(E[Z_2(y, B)]\), the first-order necessary condition of optimality must be equated to zero i.e.
\[
\frac{\partial}{\partial y} E[Z_2(y, B)] = 0 \quad \text{and} \quad \frac{\partial}{\partial B} E[Z_2(y, B)] = 0 
\]  
(4.4)

On setting equation (63) equal to zero, we get:
\[
\frac{\lambda}{1-E[p_1]} \left\{ E[G_2] \frac{B}{y^2} - E[G_3] + E[G_4] \frac{B^2}{y^2} + C_{\max} \right\} - \frac{E[\alpha]E[q_2]}{2} = 0 
\]  
(69)

\[\Rightarrow y^* = \sqrt{\frac{2\lambda \left(BE[G_2] + B^2E[G_1] + C_{\max}\right)}{2\lambda E[G_3] + E[\alpha]E[q_2](1-E[p_1])}} 
\]  
(70)

On setting equation (65) equal to zero, we get:
\[
\frac{\lambda}{1-E[p_1]} \left\{ -\frac{E[G_2]}{y} - \frac{2E[G_4]B}{y} - E[G_3] \right\} = 0 
\]  
(71)

\[\Rightarrow B^* = -\frac{E[G_2] + E[G_3]y}{2E[G_4]} 
\]  
(72)

Putting this value of \(B\) in equation (69) to attain the value for \(y\) i.e.:
\[ y^* = \sqrt{\frac{2\lambda \left( \frac{-E[G_2] + E[G_3]y}{2E[G_3]} E[G_2] + \frac{E[G_5] + E[G_3]y^2}{4E[G_4]} + C_{\text{max}} \right)}{2\lambda E[G_3] + E[\alpha]E[q_2](1 - E[p_1])}} \]  

(73)

Hence, \( y^* \) and \( B^* \) are the optimal values of \( y \) and \( B \) for Case II.

5. Numerical analysis

In order to validate the developed formulation, the following section presents the numerical analysis by illustrating two examples and solving them for each of the two cases viz. with and without the effects of learning in set up cost. Further, this section also draws a comparison for the change in optimal profit values between one way v/s two way inspection plans at the manufacturer’s end.

5.1 Examples

This subsection authenticates the hypothesis with the help of two examples, each solved for the above discussed cases of the learning effects. Table 2 and Table 3 demonstrate the parameter values of two numerical examples taken similar or different from Wee et al [38] paper respectively. Table 4 depicts the optimal values of the two examples.

\(<\text{Insert Table 2}>\)

\(<\text{Insert Table 3}>\)

\(<\text{Insert Table 4}>\)

5.2 Comparison of profit values between one way v/s two way inspection plans

This subsection derives a contrast regarding the expected total profit per unit time for the above discussed two cases when both are solved with and without the two way inspection plans respectively.

\(<\text{Insert Table 5}>\)

Table 5 reflects that it is of great financial interest for the manufacturer to practice the second inspection before taking the final decision of salvaging the defectives or sending the lot for rework. Another intuitive reason to opt for second inspection plan is that it is easier to deliver quality/ error-free inspection to a smaller batch of only accumulated defectives rather than to whole production lot. So, a small investment in this direction pays back with higher returns. By extracting the complete fraction of wrongly classified perfect items as defectives, the manufacturer is able to reduce his financial losses caused by committing the Type-I error as its effect gets nullified by the re-inspection. So, even by assuming inspection errors in the model, their impacts have been controlled proficiently.

6. Sensitivity analysis

The present section presents the robustness of the developed model by observing the change in the objective values when the model parameters are altered. The outcome of changes in the defect parameters \( \alpha, q_1, q_2, r \) and also on the shape parameter \( \epsilon \) are observed on the optimal production quantity (\( y^* \)), optimal backorder quantity (\( B^* \)), optimal cycle length
(\(T^*\)), optimal total cost per unit time (\(T.C.U.\)), and optimal expected total profit per unit time \(E[Z^*(y,B)]\) in tabular form below.

**Tables 6-10** provide the following **managerial insights**:

\[ < Insert Table 6> \]
\[ < Insert Table 7> \]

**Table 6** shows that with an increase in the value of shape factor, \(\epsilon\), the setup of production process is done frequently which results in decreasing the backorders (\(B^*\)) as there is sufficient quantity to satisfy the demand of the retailer. The production run length (\(Tp^*\)), cycle time (\(T^*\)), lot size (\(y^*\)) and the expected total cost (\(T.C.U.\)) per unit time are quite sensitive to the changes in the shape factor. The reason behind this decrease in the loss percentage as compared to the traditional EOQ models can be explained by the extensive crashing in the duration of the production run length and henceforth the set up cost.

As exhibited from **Table 7**, the value of optimal expected total profit per unit time \(E[Z^*(y,B)]\) shows declining trend along with optimal backorder level (\(B^*\)), while the optimal values of production quantity (\(y^*\)), cycle length (\(T^*\)), total revenue per unit time(\(T.R.U.\)), and total cost per unit time (\(T.C.U.\)) rise with the increase in defect proportion (\(a\)). With an increased proportion of defects in the system, there will be a higher sale of defectives as well as return of defectives. This not only brings considerable monetary loss to the firm but also harms reputation. However, due to more sales of scrap items, there is an increase of revenue but it is not able to compensate the losses incurred, so the overall profit of the system decreases. Since demand is satisfied through perfect items only, the manufacturer needs to produce more than the demand and hence the production quantity is observed to increase. So, it is advisable for the operations manager to improvise his production system so as to reduce the defect proportion substantially.

\[ < Insert Table 8> \]
\[ < Insert Table 9> \]

From **Table 8**, it is observed that with increase in the proportion of Type I error (\(q_1\)), the optimal production quantity (\(y^*\)), and expected value of total profit per unit time \(Z^*(y,B)\) show declining trends while total revenue per unit time(\(T.R.U.\)), and total cost per unit time (\(T.C.U.\)) reflect elevation in their values. Due to Type-I error, there is a direct financial loss to the manufacturer as the screening process and the inspection team is not competent enough to carry out the process without errors. Resultantly, there is significant fall in the profit values as the non-defectives are being sold at a reduced price by mistake. In this scenario, the manufacturer is unable to achieve maximum possible sales, so, it is beneficial for him to reduce his production quantity so as to minimize losses related to misclassification. Revenue is indirectly increasing by salvaging of non-defectives in a lesser restrictive inventory. However, the increase in total cost dominates the increase in revenue, so there is a decrease in overall profit values.

It is clear from **Table 9** that with increment in the proportion of Type II error(\(q_2\)), there is noteworthy decline in the optimal values of expected total profit per unit time \(Z^*(y,B)\) and total revenue per unit time(\(T.R.U.\)) while the cost values show considerable increase. No significant change is observed in the values of optimal backorder level (\(B^*\)), optimal production quantity (\(y^*\)), and cycle length (\(T^*\)). Since Type-II error leads to sales returns which are either entertained with full price refunds or replacement
with perfect products, so it impacts the revenue part majorly. Moreover, it also causes penalty and goodwill loss to the firm which is of serious concern to the managers it also leads to increase in the cost components. The constancy of demand and shortages is explained by the fact that the manager is able to maintain the demand despite some sales returns. In this particular scenario, it is beneficial for the operations manager to strengthen his screening team so as to reduce such damaging screening errors.

As evident from Table 10, with the increase in the proportion of reworked items ($r$), the optimal production quantity ($y^*$), cycle length ($T^*$), total cost per unit time ($T.C.U. ^*$), and total revenue per unit time ($T.R.U. ^*$) exhibit decreasing values while the optimal value of expected total profit per unit time ($Z^*(y,B)$) shows increase along with optimal backorder level ($B^*$). It is quite logical and intuitive that there is a cost associated with the rework process but here it also gets compensated with the rise in the sale of perfect items. So, the overall profit of the system increases. The production quantity gets lowered owing to the fact that due to the rework process, the count of perfect items gets increased considerably, thereby, decreasing the need to produce more items so as to meet the demand. Consequently, the cycle length also decreases.

7. **Concluding remarks**

In the current manufacturing scenario, the main challenge is to establish an efficient inventory model that takes care of the major as well as minor concerns of the system. The problems associated with the production system is majorly related to the production of defectives, their screening and management thereafter. In lieu of this, the present paper develops an inventory model for finite production system which is presumed to be imperfect and hence produces defectives at a uniform rate. To supply only good products to the customers, the screening process plays an eminent role for the manufacturer. In contrast to the previous studies, the present model is explored under two stages of inspection practices, with screening errors incorporated only at the first stage. To validate the hypothesis, the numerical section carries out the comparison of the optimal profit values in the presence of one-way and two-way inspection techniques. Various managerial implications obtained are as follows:

- The results show that the losses which were traditionally borne by the manufacturer by discarding the perfect items by mistake (an outcome of Type-I error) in the first inspection process are compensated with the help of small investment in the second inspection process which is error-free.
- Due to closer scrutiny in the second stage of inspection, the manufacturer is able to completely extract the misclassified perfect items before taking the final decision of rework and hence some undue expenses related to rework cost or unnecessary salvaging get avoided.
- Even in the prevailing imperfect quality and screening environment, the manufacturer is able to raise the revenue by selling a larger amount of perfect items at the mark-up price with respect to the scenario of no second time inspection. So, the model successfully diminishes the relatively harmful impact of Type-I error as compared to the effects of Type-II error.
- Also, in an attempt to cut down on these escalating cost components to some extent, the process of learning aids the manufacturer with the reduction in set-up cost of the production system in the present model. Therefore, it makes economic sense to employ the process of learning for the betterment of the organization as also authenticated by the decreasing loss percentage.
Overall, it advisable for the manufacturer to reduce the percentage of defectives through a correct/careful production process as these elevates the defect-related costs significantly. The model puts forth some of the very interesting and useful scenarios viz. inspection errors of Type-I and Type-II, revenue management through two-way inspection, reduction of setup costs through time-dependent learning, rework operations etc. Thus, the present model is preferable over the models without the inclusion of such practical settings and it holds wide applicability in real-time manufacturing industries.

8. Future research guidance and limitations
The model can be extended in a number of ways by adopting varying demand patterns. Also, it would be interesting to develop an integrated vendor-buyer model in this direction. The model can also be extended for deteriorating products, taking into account the impact of preservation technology. Consideration of environmental factors while transportation and production, would be another worthy contribution in this track.

The model is restricted to the scope of limited storage space for the manufacturer, also the exactness of various cash flows in it.

References


**APPENDIX A**

\[ p_1 = \alpha (1 - r) \]  

(A1)

\[ p_2 = ar \]  

(A2)

\[ p_3 = (1 - \alpha)q_i \]  

(A3)

\[ G_i = s(1-\alpha) + sar + \alpha(1-r) - C_a - d_i \left( \frac{\lambda}{\phi(1-\alpha)} \right) - d_r \left( \frac{\phi - \lambda}{\phi} \right) - d_r \left( \alpha + (1-\alpha)q_i \right) \]  

(A4)

\[ G_2 = -d_i \left[ \left( \frac{\phi - (1-p_1)x}{(1-p_1)x - \lambda} \right) \frac{(\phi - \lambda)}{(1-p_1)x - \lambda} \right] \]  

(A5)

\[ G_3 = \frac{h}{2} \frac{(\phi - \lambda)}{\phi} \frac{(1 - x)}{\phi} - h \frac{(1 - x)}{\phi} \left( \frac{\phi - \lambda}{\phi} \right) - \frac{h}{2\lambda} \left( \frac{\phi - \lambda}{\phi} \right) \left( \frac{1}{\phi} - p_i \right) \left( \frac{p_2}{\phi_i} \right) \]  

(A6)

\[ G_i = \frac{h}{2} \frac{(\phi - (1-p_1)x)}{(1-p_1)x - \lambda} + h \frac{(\phi - (1-p_1)x)}{(1-p_1)x - \lambda} - \frac{h}{2\lambda} \left( \frac{(\phi - (1-p_1)x)}{(1-p_1)x - \lambda} + \frac{(\phi - \lambda)}{(1-p_1)x - \lambda} \right)^2 \]  

(A7)

\[ + C_a \left( \frac{1}{2} \frac{1}{\lambda} + \frac{1}{(1-p_1)x - \lambda} \right) \]  

\[ G_i = \frac{d_i}{\phi(1-\alpha)} - h \frac{(\phi - (1-p_1)x)}{(1-p_1)x - \lambda} + h \frac{(\phi - \lambda)}{\phi} \frac{1}{(1-p_1)x - \lambda} - \frac{h}{2\lambda} \left( \frac{(\phi - (1-p_1)x)}{(1-p_1)x - \lambda} + \frac{(\phi - \lambda)}{(1-p_1)x - \lambda} \right) \]  

(A8)

\[ \frac{h}{2\lambda} \left( \frac{(\phi - (1-p_1)x)}{(1-p_1)x - \lambda} + \frac{(\phi - \lambda)}{(1-p_1)x - \lambda} \right) \left( \frac{1}{\phi} - p_i \right) \left( \frac{p_2}{\phi_i} \right) \]

**APPENDIX B**

\[ E[p_1] = E[\alpha](1 - E[r]) \]  

(B1)
\[ E[p_2] = E[\alpha] E[r] \] (B2) \n\[ E[p_3] = (I - E[\alpha]) E[q_i] \] (B3) \n\[ E[G_j] = s(I - E[\alpha]) + sE[\alpha]E[r] + vE[\alpha](I - E[r]) - C_\nu - d_1 \left( \frac{\lambda}{\phi(I - E[\alpha])} \right) - d_2 \left( \frac{\phi - \lambda}{\phi} \right) \] (B4) \n\[-d_1 \{ E[\alpha] + (I - E[\alpha]) E[q_i] \} - C_\nu E[\alpha] E[r] - C_r (I - E[\alpha]) E[q_i] - C_s E[\alpha] E[q_i] - uE[\alpha] (I - E[r]) \] \n\[ E[G_2] = -d_1 \left[ \frac{(\phi - (I - E[p_i]) x)}{(I - E[p_i]) x - \lambda} \left( \frac{\phi - \lambda}{(I - E[p_i]) x - \lambda} \right) \right] \] (B5) \n\[ E[G_3] = \frac{-h}{2} \left( \frac{\phi - \lambda}{\phi} \right) + \frac{hE[p_i]}{2} \left( \frac{I - 1}{x - \phi} \right) + \frac{h}{2} \left( \frac{I - 1}{x - \phi} \right) - \frac{h}{2\lambda} \left( \frac{\phi - \lambda}{\phi} \right) - \frac{h}{2\lambda} \left( \frac{I - 1}{x - \phi} \right) \] (B6) \n\[ E[G_j] = \frac{h}{2} \left[ (I - E[p_i]) x - \lambda \right] + h \left[ \frac{(\phi - (I - E[p_i]) x)}{(I - E[p_i]) x - \lambda} \right] \] (B7) \n\[ E[G_i] = \frac{d_1}{2} \lambda \left( \frac{I - 1}{x - \phi} \right) + \frac{h}{2} \left( \frac{I - 1}{x - \phi} \right) - \frac{h}{2\lambda} \left( \frac{\phi - \lambda}{\phi} \right) \] (B8) \n\[ \text{APPENDIX C} \] \n\[ \text{Case I: } T_p < T_M \] \n\[ \text{C1: Proof of first condition of concavity: } \frac{\partial^2}{\partial y^2} E[Z_1(y, B)] \leq 0 \] \n\[ \lambda \left( 2d_2 \frac{B^2}{y^3} + 2h \frac{B^2}{y^3} \left( \frac{1}{1 - \left( E[\alpha] (1 - E[r]) \right)} \right) \right) \] \n\[ \leq 0 \]
Proof: As \( 0 \leq E[\alpha], E[r] \leq 1, \) the following inequalities hold true:
\[
(1 - E[r]) \geq 0 \Rightarrow 1 - \{E[\alpha](1 - E[r])\} \geq 0
\]
As \( 0 \leq \varepsilon \leq 1, \) the following inequalities hold true:
\[
(\varepsilon - 1) \leq 0; \ (\varepsilon - 2) \leq 0
\]
Also, from model constraints (5) and (7), we have
\[
\left[ \phi - (1 - E[p_i]) \right] \geq 0 \quad \text{and} \quad \left[ (1 - E[p_i]) x - \lambda \right] \geq 0
\]
i.e. \( \phi - \{1 - \{E[\alpha](1 - E[r])\}\} x \geq 0; \ \{1 - \{E[\alpha](1 - E[r])\}\} x - \lambda \geq 0 \)
So, the first condition of concavity holds true under the condition of \( C_B \geq h \)

C2: Proof of second condition of concavity\( : \frac{\partial^2 Z_i(y, B)}{\partial B^2} \leq 0 \)

\[
\frac{2\lambda B^2}{\{1 - \{E[\alpha](1 - E[r])\}\} y^4} \left( 2h \left[ 1 - \{E[\alpha](1 - E[r])\} x \right] \right) \leq 0
\]

Proof: As \( 0 \leq E[\alpha], E[r] \leq 1, \) the following inequalities hold true:
\[
(1 - E[r]) \geq 0 \Rightarrow 1 - \{E[\alpha](1 - E[r])\} \geq 0
\]
Also, from model constraints (5) and (7), we have
\[
\left[ \phi - (1 - E[p_i]) \right] \geq 0 \quad \text{and} \quad \left[ (1 - E[p_i]) x - \lambda \right] \geq 0
\]
i.e. \( \phi - \{1 - \{E[\alpha](1 - E[r])\}\} x \geq 0; \ \{1 - \{E[\alpha](1 - E[r])\}\} x - \lambda \geq 0 \)
So, the second condition of concavity holds true under the condition of
\[
\frac{C_B}{h} \leq \left[ 1 - \frac{2\lambda \left[ \phi - \{E[\alpha](1 - E[r])\} x \right]}{\{1 - \{E[\alpha](1 - E[r])\}\} x \{1 - \{E[\alpha](1 - E[r])\}\} x - \lambda} \right]
\]
C3: Proof of third condition of concavity:
\[
\left( \frac{\partial^2}{\partial y \partial B} E[Z_i(y,B)] \right)^2 - \left( \frac{\partial^2}{\partial y^2} E[Z_i(y,B)] \right) \leq 0
\]
\[
i.e. \left( \frac{\lambda}{y^2 \{1 - \{E[\alpha](1 - E[r])\}\}} \right)^2 \cdot \left( \frac{d_z^2 (\varepsilon - 1)(\varepsilon - 2) y^2 B^2}{y^3 \varphi^2} + \frac{2 \phi - 1 \{E[\alpha](1 - E[r])\} x}{\left\{1 - \{E[\alpha](1 - E[r])\}\right\} x - \lambda} \right) \leq 0
\]
Proof: As \(0 \leq E[\alpha], E[r] \leq 1\), the following inequalities hold true:
\[(1 - E[r]) \geq 0 \Rightarrow 1 - \{E[\alpha](1 - E[r])\} \geq 0\]
As \(0 \leq \varepsilon \leq 1\), the following inequalities hold true:
\[(\varepsilon - 1) \leq 0; (\varepsilon - 2) \leq 0\]
Also, from model constraints (5) and (7), we have
\[\left[ \phi - (1 - E[p_i]) x \right] \geq 0 \text{ and } \left[ (1 - E[p_i]) x - \lambda \right] \geq 0\]
i.e. \(\phi - \{E[\alpha](1 - E[r])\} x \geq 0; \{E[\alpha](1 - E[r])\} x - \lambda \geq 0\)
So, the third condition of concavity holds true under the condition of
\[
\left( \frac{d_z^2}{B} \right)^2 \leq \frac{2 \phi - 1 \{E[\alpha](1 - E[r])\} x}{\left\{1 - \{E[\alpha](1 - E[r])\}\right\} x - \lambda}
\]
Hence, the global concavity of the objective function for **Case I** has been proved mathematically.

**APPENDIX D**

**Case II:** \(T_M \geq T_p\)

D1: Proof of first condition of concavity:
\[
\frac{\partial^2}{\partial y^2} E[Z_i(y,B)] \leq 0
\]
\[
i.e. \left( \frac{\lambda}{1 - \{E[\alpha](1 - E[r])\}} \right)^2 \cdot \left( \frac{-2 d_z B}{y^3} - \frac{2 h B^2}{y^3} \frac{2 \phi - 1 \{E[\alpha](1 - E[r])\} x}{\left\{1 - \{E[\alpha](1 - E[r])\}\right\} x - \lambda} \right) \leq 0
\]
Proof: As $0 \leq E[\alpha], E[r] \leq 1$, the following inequalities hold true:

$$(1 - E[r]) \geq 0 \Rightarrow 1 - \{E[\alpha](1 - E[r])\} \geq 0$$

Also, from model constraints (5) and (7), we have

$$\left[ \phi - (1 - E[p_i]) x \right] \geq 0$$

and

$$\left[ (1 - E[p_i]) x - \lambda \right] \geq 0$$

i.e. $\phi - \{E[\alpha](1 - E[r])\} x \geq 0$;

$$\{1 - \{E[\alpha](1 - E[r])\}\} x - \lambda \geq 0$$

So, the first condition of concavity holds true under the condition of $C_B \geq h$

D2: Proof of second condition of concavity:

$$\frac{\partial^2}{\partial B^2} E[Z_i(y, B)] \leq 0$$

i.e. $2 \lambda B^2 \frac{\partial^2}{\partial B^2} E[Z_i(y, B)] \leq 0$

Proof: As $0 \leq E[\alpha], E[r] \leq 1$, the following inequalities hold true:

$$(1 - E[r]) \geq 0 \Rightarrow 1 - \{E[\alpha](1 - E[r])\} \geq 0$$

Also, from model constraints (5) and (7), we have

$$\left[ \phi - (1 - E[p_i]) x \right] \geq 0$$

and

$$\left[ (1 - E[p_i]) x - \lambda \right] \geq 0$$

i.e. $\phi - \{E[\alpha](1 - E[r])\} x \geq 0$;

$$\{1 - \{E[\alpha](1 - E[r])\}\} x - \lambda \geq 0$$
So, the second condition of concavity holds true under the condition of

\[
\frac{C_h}{\eta} \leq \left( 1 - \frac{2\lambda \left[ \phi - \left\{ \left[ 1 - E[\alpha] \right] (1 - E[r]) \right\} x \right]}{\left[ 1 - E[\alpha] (1 - E[r]) \right] x \left[ \left[ 1 - E[\alpha] (1 - E[r]) \right] x - \lambda \right]} \right)
\]

D3: Proof of third condition of concavity:

\[
\left( \frac{\partial^2}{\partial y^2} E[Z_i(y,B)] \right)^2 - \left( \frac{\partial^2}{\partial y^2} E[Z_i(y,B)] \right) \leq 0
\]

\[
i.e. \left( \frac{\lambda}{y^2 \left[ 1 - E[\alpha] (1 - E[r]) \right]} \right)^2 \left( \frac{d_2^2 + 4C_{\text{max}} B^2}{y} \right) \leq 0
\]

\[
\text{Proof: As } 0 \leq E[\alpha] \leq 1, \ldots, \text{the following inequalities hold true:}
\]

\[
(1 - E[r]) \geq 0 \Rightarrow 1 - \left\{ E[\alpha] (1 - E[r]) \right\} \geq 0
\]

Also, from model constraints (5) and (7), we have

\[
\left[ \phi - \left( 1 - E[p_i] \right) x \right] \geq 0 \text{ and } \left[ \left( 1 - E[p_i] \right) x - \lambda \right] \geq 0
\]

\[
i.e. \phi - \left\{ E[\alpha] (1 - E[r]) \right\} x \geq 0; \left\{ 1 - E[\alpha] (1 - E[r]) \right\} x - \lambda \geq 0
\]

So, the third condition of concavity holds true under the condition of

\[
\left( \frac{d_2}{B} \right) \leq \left( \frac{4C_{\text{max}}}{y^3} \right) \left( \frac{\left( \frac{1}{\lambda} + \left\{ \left[ 1 - E[\alpha] (1 - E[r]) \right] x - \lambda \right\} \right)}{\left[ \left[ 1 - E[\alpha] (1 - E[r]) \right] x - \lambda \right]^2} \right)
\]

Hence, the global concavity of the objective function for Case II has been proved mathematically.
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Production lot $y$

Inspection

$\alpha y$ (Imperfect Proportion)

$\alpha (1-q_2)y$ (Type-II error)

$\alpha q_2y = Sales Returns$

$1 - \alpha y$ (Perfect Proportion)

$\alpha q_1y$ (Type-I error)

$1 - \alpha (1-q_2)y = Re-inspection of Total Imperfect Items$

$\alpha y + (1 - \alpha)q_1y$

$\alpha (1-q_1)y = p_1y$

$\alpha ry = p_2y$

$\alpha (1-q_1)y = p_3y$

$1 - \alpha (1-q_2)y = Re-worked and Screened Perfect Items$

Re-inspection of Total Imperfect Items

Salvaged Items

Reparable Items

Perfect Items

Figure 1
Sales Returns = \alpha q_2 y

Inventory Level

\begin{align*}
\text{Time} \ (1-p_1) x - \lambda \\
\Phi_1 - \lambda \\
A_1 A_2 A_3 A_4 A_5 A_6 \\
\text{Time} \\
& \text{Sales Returns} = \alpha q_2 y
\end{align*}

Figure 2

\begin{align*}
C_{max} \\
C_0 \\
C_0(T_p) \\
\epsilon=0 \\
\epsilon=0.1 \\
\epsilon=0.5 \\
\epsilon=1
\end{align*}

Figure 3
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<tr>
<td>Type-2 error proportion</td>
<td>( q_2 )</td>
<td>U ~ (0.1,0.3)</td>
<td>U ~ (0.2,0.8)</td>
<td></td>
</tr>
<tr>
<td>Rework proportion</td>
<td>( R )</td>
<td>U ~ (0.5,0.7)</td>
<td>U ~ (0.5,0.8)</td>
<td></td>
</tr>
<tr>
<td>Probability density function</td>
<td>( f(\alpha) )</td>
<td>1/(0.6 - 0.4)</td>
<td>1/(0.15 - 0.5)</td>
<td></td>
</tr>
<tr>
<td>Probability density function</td>
<td>( f(q_1) )</td>
<td>1/(0.3 - 0.1)</td>
<td>1/(0.8 - 0.2)</td>
<td></td>
</tr>
<tr>
<td>Probability density function</td>
<td>( f(q_2) )</td>
<td>1/(0.3 - 0.1)</td>
<td>1/(0.8 - 0.2)</td>
<td></td>
</tr>
<tr>
<td>Probability density function</td>
<td>( f(r) )</td>
<td>1/(0.7 - 0.5)</td>
<td>1/(0.8 - 0.5)</td>
<td></td>
</tr>
<tr>
<td>Demand Rate</td>
<td>( \lambda )</td>
<td>90,000</td>
<td>50,000</td>
<td>units / year</td>
</tr>
<tr>
<td>Shape Factor</td>
<td>( \epsilon )</td>
<td>0.2</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Rework Rate</td>
<td>( \varphi_1 )</td>
<td>80,000</td>
<td>40,000</td>
<td>units / cycle</td>
</tr>
<tr>
<td>Selling Price</td>
<td>( S )</td>
<td>60</td>
<td>70</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Screening Cost post production</td>
<td>( d_2 )</td>
<td>0.6</td>
<td>0.7</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Second Screening Cost</td>
<td>( d_3 )</td>
<td>0.7</td>
<td>0.8</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Type-I error Cost</td>
<td>( c_r )</td>
<td>10</td>
<td>12</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Type-II error Cost</td>
<td>( c_a )</td>
<td>12</td>
<td>12</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Rework Cost</td>
<td>( c_W )</td>
<td>8</td>
<td>9</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Disposal Cost</td>
<td>( u )</td>
<td>2</td>
<td>3</td>
<td>$ / unit</td>
</tr>
<tr>
<td>Maximum Set-Up Cost</td>
<td>( C_{\text{max}} )</td>
<td>-</td>
<td>-</td>
<td>$ / cycle</td>
</tr>
<tr>
<td>Holding Cost during Rework</td>
<td>( h_j )</td>
<td>6</td>
<td>6</td>
<td>$ / unit</td>
</tr>
</tbody>
</table>
Table 4

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Numerical 1</th>
<th>Numerical 2</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order size</td>
<td>$y^*$</td>
<td>1,648</td>
<td>701</td>
<td>units/cycle</td>
</tr>
<tr>
<td>Backorder Quantity</td>
<td>$B^*$</td>
<td>267</td>
<td>109</td>
<td>units/cycle</td>
</tr>
<tr>
<td>Expected total profit per unit time</td>
<td>$E [ Z^*(y, B)]$</td>
<td>30,55,994</td>
<td>18,60,999</td>
<td>$/year</td>
</tr>
<tr>
<td>Cycle Length</td>
<td>$T^*$</td>
<td>6.55</td>
<td>4.94</td>
<td>Days</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Numerical 1</th>
<th>Numerical 2</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>One way Inspection</td>
<td>$E [ Z^*(y, B)]$</td>
<td>30,39,578</td>
<td>18,10,119</td>
<td>$/year</td>
</tr>
<tr>
<td>Two way Inspection</td>
<td>$E [ Z^*(y, B)]$</td>
<td>30,55,994</td>
<td>18,60,999</td>
<td>$/year</td>
</tr>
</tbody>
</table>

Table 6

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$Tp^*$</th>
<th>$y^*$</th>
<th>$B^*$</th>
<th>$T^*$</th>
<th>$E[Z^*(y,B)]$</th>
<th>% Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>4.12</td>
<td>2,258.45</td>
<td>366.89</td>
<td>8.98</td>
<td>30,54,899.54</td>
<td>-25.05</td>
</tr>
<tr>
<td>0.2</td>
<td>3.01</td>
<td>1,648.97</td>
<td>267.88</td>
<td>6.55</td>
<td>30,55,994.14</td>
<td>-25.09</td>
</tr>
<tr>
<td>0.3</td>
<td>2.10</td>
<td>1,149.54</td>
<td>186.74</td>
<td>4.57</td>
<td>30,56,944.69</td>
<td>-25.13</td>
</tr>
<tr>
<td>0.4</td>
<td>1.38</td>
<td>756.22</td>
<td>122.85</td>
<td>3.01</td>
<td>30,57,746.91</td>
<td>-25.17</td>
</tr>
<tr>
<td>0.5</td>
<td>0.84</td>
<td>461.71</td>
<td>75.01</td>
<td>1.84</td>
<td>30,58,400.44</td>
<td>-25.19</td>
</tr>
<tr>
<td>0.6</td>
<td>0.47</td>
<td>255.16</td>
<td>41.45</td>
<td>1.01</td>
<td>30,58,909.70</td>
<td>-25.21</td>
</tr>
<tr>
<td>0.7</td>
<td>0.46</td>
<td>251.86</td>
<td>41.64</td>
<td>1.01</td>
<td>30,60,737.84</td>
<td>-25.29</td>
</tr>
<tr>
<td>0.8</td>
<td>0.09</td>
<td>47.16</td>
<td>7.66</td>
<td>0.19</td>
<td>30,59,541.11</td>
<td>-25.24</td>
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Table 7

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$y^*$</th>
<th>$B^*$</th>
<th>$T^*$</th>
<th>E.T.C.U.*</th>
<th>E.T.R.U.*</th>
<th>$E[Z^*(y,B)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1,681.62</td>
<td>272.75</td>
<td>6.79</td>
<td>23,07,399.23</td>
<td>54,04,698.80</td>
<td>30,97,299.57</td>
</tr>
<tr>
<td>0.03</td>
<td>1,664.67</td>
<td>270.26</td>
<td>6.67</td>
<td>23,37,406.30</td>
<td>54,14,210.53</td>
<td>30,76,804.23</td>
</tr>
<tr>
<td>0.05</td>
<td>1,648.98</td>
<td>267.88</td>
<td>6.55</td>
<td>23,67,883.41</td>
<td>54,23,877.55</td>
<td>30,55,994.14</td>
</tr>
<tr>
<td>0.07</td>
<td>1,634.43</td>
<td>265.60</td>
<td>6.44</td>
<td>23,98,841.05</td>
<td>54,33,703.70</td>
<td>30,34,862.66</td>
</tr>
<tr>
<td>0.09</td>
<td>1,620.97</td>
<td>263.41</td>
<td>6.34</td>
<td>24,30,289.94</td>
<td>54,43,692.95</td>
<td>30,13,403.01</td>
</tr>
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</table>

Table 8

<table>
<thead>
<tr>
<th>$q_1$</th>
<th>$y^*$</th>
<th>$B^*$</th>
<th>$T^*$</th>
<th>E.T.C.U.*</th>
<th>E.T.R.U.*</th>
<th>$E[Z^*(y,B)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,648.97</td>
<td>267.88</td>
<td>6.55</td>
<td>23,58,548.20</td>
<td>54,23,877.55</td>
<td>30,65,329.35</td>
</tr>
<tr>
<td>0.01</td>
<td>1,648.98</td>
<td>267.88</td>
<td>6.55</td>
<td>23,67,883.41</td>
<td>54,23,877.55</td>
<td>30,55,994.14</td>
</tr>
<tr>
<td>0.02</td>
<td>1,648.97</td>
<td>267.88</td>
<td>6.55</td>
<td>23,77,218.61</td>
<td>54,23,877.55</td>
<td>30,46,658.94</td>
</tr>
<tr>
<td>0.03</td>
<td>1,648.97</td>
<td>267.88</td>
<td>6.55</td>
<td>23,86,553.82</td>
<td>54,23,877.55</td>
<td>30,37,323.74</td>
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Table 9

<table>
<thead>
<tr>
<th>$q_2$</th>
<th>$y^*$</th>
<th>$B^*$</th>
<th>$T^*$</th>
<th>$E.T.C.U.*$</th>
<th>$E.T.R.U.*$</th>
<th>$E[Z*(y,B)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,651.19</td>
<td>268.24</td>
<td>6.56</td>
<td>23,66,777.24</td>
<td>54,29,387.76</td>
<td>30,62,610.51</td>
</tr>
<tr>
<td>0.01</td>
<td>1,650.08</td>
<td>268.06</td>
<td>6.56</td>
<td>23,67,330.33</td>
<td>54,26,632.65</td>
<td>30,59,302.33</td>
</tr>
<tr>
<td>0.03</td>
<td>1,647.87</td>
<td>267.70</td>
<td>6.55</td>
<td>23,68,436.49</td>
<td>54,21,122.45</td>
<td>30,52,685.96</td>
</tr>
<tr>
<td>0.02</td>
<td>1,648.98</td>
<td>267.88</td>
<td>6.55</td>
<td>23,67,883.41</td>
<td>54,23,877.55</td>
<td>30,55,994.14</td>
</tr>
<tr>
<td>0.04</td>
<td>1,646.77</td>
<td>267.52</td>
<td>6.54</td>
<td>23,68,989.57</td>
<td>54,18,367.35</td>
<td>30,49,377.78</td>
</tr>
</tbody>
</table>

Table 10

<table>
<thead>
<tr>
<th>$r$</th>
<th>$y^*$</th>
<th>$B^*$</th>
<th>$T^*$</th>
<th>$E.T.C.U.*$</th>
<th>$E.T.R.U.*$</th>
<th>$E[Z*(y,B)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,752.79</td>
<td>254.91</td>
<td>6.75</td>
<td>24,25,417.21</td>
<td>54,70,105.26</td>
<td>30,44,688.05</td>
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<tr>
<td>0.5</td>
<td>1,665.80</td>
<td>265.83</td>
<td>6.59</td>
<td>23,77,225.96</td>
<td>54,31,384.62</td>
<td>30,54,158.66</td>
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<tr>
<td>0.6</td>
<td>1,648.97</td>
<td>267.88</td>
<td>6.55</td>
<td>23,67,883.41</td>
<td>54,23,877.55</td>
<td>30,55,994.14</td>
</tr>
<tr>
<td>0.7</td>
<td>1,632.35</td>
<td>269.88</td>
<td>6.52</td>
<td>23,58,635.91</td>
<td>54,16,446.70</td>
<td>30,57,810.79</td>
</tr>
<tr>
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<td>1,615.93</td>
<td>271.85</td>
<td>6.49</td>
<td>23,49,482.00</td>
<td>54,09,090.91</td>
<td>30,59,608.90</td>
</tr>
</tbody>
</table>

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DSL, YUJOR etc. He is a reviewer of more than 25 International/National journals. He has guided 10 Ph. D. and 20 M. Phil. in Operations Research. He is Ex-Editor-in-Chief of IJICM, Associate Editor of IJSAEM, Springer, and on the Editorial Board of the IJSS: Operations & Logistics, IJSOI, AJOR, IJECBS, JASR, AJBAS.