Efficiency Evaluation of a Three-Stage Leader-Follower Model by the Data Envelopment Analysis with Double-Frontier Viewpoint

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Abstract

In this paper, a three-stage network with optimal desirable and undesirable inputs and outputs has been taken into consideration by us. This network comprises of a leader and two followers. Four diverse models of Data Envelopment Analysis (DEA) to measure the efficiency or the performance, of this three-stage network have been taken under contemplation; these are namely, a Black Box Model and three Stackelberg Game (Theory) Models. A multiplicative DEA, with a double-frontier approach, to measure the efficiency of the entire system and the performances of the decision making units (DMUs), from both the optimistic and pessimistic views have been utilized. In this paper attempts have been made to present the goals of the managers in the models. Hence, aspects of goal programming have been manipulated so as to define cooperation between the leader and followers, such that, we are able to include the objectives of the managers in the models. In actual fact, a non-cooperative collaboration is deliberated upon. In addition to which, in the second and third scenarios, the leader-follower, nonlinear models are present. Thereby, a heuristic approach is suggested to convert the nonlinear models into linear ones.

Keywords: Data envelopment analysis; Three-stage processes; Game theory; Goal programming; Double-frontier; Undesirable output.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric method to measure the relative efficiency of a set of analogous decision-making units (DMUs), with multiple inputs and outputs [1]. This method considers a frontier or boundary function, which surrounds and involves factors, such as, inputs and outputs. It not only determines the most efficient units, but also analyses the inefficient ones [2]. Charnes et al. [3] developed the initial DEA task of Farrel [4]. The said model was known as (Charnes-Cooper-Rhodes) or the "CCR Model". Banker et al. [5] expanded on the DEA models and presented the (Banker-Charnes-Cooper) or the "BCC Model". Classical data envelopment analysis models, such as, the CCR and BCC, assume that the systems are considered as black boxes; and due to the shortcomings in considering the intermediary variables and the internal interactions of the system, valuable information is eliminated [6]. Fare and Grosskopf [7] specified the disadvantages and weak points of the classical DEA models and referred to the Network Data Envelopment Analysis Model (NDEA). NDEA systems can simulate the internal structures of systems by taking the stages and sub-divisions into consideration, thereby, elevating the efficiency measurement [8, 9]. After the introduction of NDEA models, ample studies have been carried out and rendered on the grounds of various models. In the past few years, new discussions have been contributed in relevance to network analysis, in view of the game theory, such that this theory has become one of the vital methods in analyzing NDEA, or have been converted into multi-stage models [10]. Li et al. [11] presented a model for a two-stage structure, a phase which holds a more important standpoint for managers; and they have named this phase as "leader" and the other is known as "follower". In order to calculate the efficiency, initially, the efficiency of the leader phase was maximized to the optimum and thus, the efficiency of the follower phase was secured by maintaining a constant efficiency in the phase of the leader. This exemplary, was designated as a decentralized control or a Stackelberg Game, which has been widely utilized by researchers in recent years. An et al. [12] procured a network which comprised of two stages, having a collaborative condition between them, into consideration and computed the efficiency of this network, under cooperative and non-cooperative conditions on a (leader-follower) basis. Results demonstrated that, the overall efficiency in cooperative conditions was higher than that of the non-cooperative one. In another similar research, Wu et al. [13] contemplated on and computed the efficiency of a two-stage network, with undesirable outputs in cooperative and non-cooperative conditions. The results of this research, which considers the total efficiency as the sum of the efficiency component, denotes that, the efficiency of the sub-DMUs is in the condition of a leader in the maximal and as a follower in the minimal. In yet another research by Zhou et al. [14], a network consisting of a leader and some followers were evaluated in a black box and non-cooperative modes and the results were compared. In this study which aimed at minimizing costs, the CCR data envelopment analysis model was utilized. A research performed by Du et al. [15] in the grounds of leader-follower, can also be designated, in which they analyzed a parallel network in the cooperative and non-cooperative modes. In recent years, the Stackelberg Game was utilized by several researchers such as, Fard and Hajaghaei-Keshteli [16, 17], Fathollahi-Fard et al. [18, 19], including Hajaghaei-Keshteli and Fathollahi-Fard [20].

Undesirable factors are one of the critical arenas that are accounted for DEA. For the first time, Fare et al. [21] took the undesirable factors under consideration to evaluate the efficiency in DEA models. Lu and Lo [22] classified the undesirable outputs within a framework of three modes; the first method was to overlook all the undesirable outputs. The second method was to restrict the expansion of the undesirable outputs, or that, these undesirable outputs were to be considered as a nonlinear DEA model; whereas, the third technique, which was taken under contemplation, for the undesirable outputs, was as an input, signified with a negative sign, as an output and or was handled by imposing a single downward conversion. In the past few years, the role of the undesirable factors in DEA models has made considerable progress and the tasks of Wang et al. [23] and Wu et al. [24] are significant in this respect.

Most of the research carried in the field of DEA is in static environments; and for the very first time, Sengupta [25] performed efficiency evaluations in dynamic environments. In dynamic models, each time period is considered as a decision-making unit. Similarly, the correlation between the time periods in these models is contemplated by using additional inputs and outputs in between these periods [26]. Since the era of Sengupta's tasks, till date, many articles have been published in the sphere of dynamic networks; the difference of which, lies in the case studies and the manner in which, the efficiency of the DMUs are computed. These include the Kawaguchi et al. [27] and Wang et al. [28] Models which can be indicated to in a dynamic mode respectively, for the evaluation of performance in hospital environments and banks.

DEA with a double-frontier contemplates on two frontiers to compute the efficiency for each DMU. One is the efficient frontier and the other, the inefficient one. The efficiency calculated by the efficient frontier is called the optimistic efficiency, whereas, the efficiency computed by the inefficient frontier is known as the pessimistic efficiency [29]. In the optimistic view, each DMU is compared with a set of efficient units that are on the efficient frontier and in the pessimistic view each DMU is compared with a set of inefficient units that are on the inefficient frontier [30]. In the optimistic view the amount of efficiency which has come to hand is less than (1), on the condition that, the DMU under evaluation, is not on the efficient frontier and equates to (1) when the DMU under evaluation is on the efficient frontier. The efficiency value that has been gained in the pessimistic view is more than (1), on condition that, the DMU under evaluation is not on the inefficient frontier and is equivalent to (1), if the DMU under evaluation is on the inefficient frontier [31]. Though, when calculating the optimistic efficiency, the nearer the DMU proves to be to the efficiency frontier, the more desirable, whereas, in the case of computing the pessimistic efficiency, the further the distance of the DMU, the better and has an additional desirability [32]. For the first time, Doyle et al. [33] computed the efficiency of DMUs from the optimistic and pessimistic viewpoints. In the recent years, the double-frontier was utilized by numerous researchers such as, Badiezadeh et al. [34]; Azizi et al. [35] and Wang and Lan [36].

A multiple criteria decision-making can be divided into two groups, consisting of multi-criterion and multi-objective decision-making. A goal programming (GP) is one of the multi-objective decision-making techniques, which assists in encompassing several aims synchronously; and by minimizing the deviation between these objectives, the optimal solution can be determined [37]. In this method, the objective function of the key problem is somehow formulated by the auxiliary variables that are namely, deviations from the goal condition, so that the total set of undesirable deviations of the ideals are minimized. This technique specifies the goals achieved and the ones which have not been so. In addition to which, by utilizing a GP, the amount of deviation for each of these goals, comes to hand from their ideal level [38-40]. A GP was performed by Charnes and Cooper [41] in 1961. Yousefi et al. [42] suggested a hybrid GP-DEA model in a network to present an improvement in solutions and to rank units (all efficient and inefficient ones) based on the requirements of experts. In the past few years, numerous researchers have used the GP method and rendered new models and for such paradigms, one can refer to Maiti and Roy [43, 44]; Chen et al. [45]; Trivedi and Singh [46]; He et al. [47] and Roy et al. [48, 49]. Methods in relevance to GP modes are extremely diverse and even make provisions to optimize contradictory goals. Julia et al. [50] set up and utilized goal programming for three kinds of analysis: 1-Specify the essential resources to fulfill a set of goals under consideration. 2-Determine the intensity of attaining the goals. 3-Determining the optimal and substantial response with due attention to the amount of resources available and the priority of objectives or goals. Table 1 reviews the studies which have applied the game theory methods in DEA. The last row of Table 1 presents characteristics of the current paper.

The main objective of this paper is to expand DEA models, utilizing goal programming concepts. We perform this task for non-cooperative models (leader-follower). The weakness present in the non-cooperative models is that, the leader maximizes the performance on its own. Hence, there is a probability that, this may cause an annihilation of the follower. The flaw or weakness of the non-cooperative models in the case, where, the network is a series, illustrates itself more prominently, as in such conditions a failure of one stage leads to a collapse of the entire system. The other reason is that, in this case the efficiency or performance is computed from the multiplicative angle which is devoid of compensation properties. In order to overpower this weakness we shall define a level of goals for the followers. In actual fact, in this paper and in terms with the non-cooperative game, collaboration is proposed between the leader and follower. In this manner we induce the views of the managers into DEA models. A comparison is made by us, in view of our proposed model, with the other two non-cooperative common models and that of the black box model, so that the managers can accurately analyze and secure the level of goals and the results of the proposed model. Hence, it is for this purpose that in this paper we attain the efficiency of a three-stage performance on the basis of four scenarios, a black box scenario and three non-cooperative scenarios. An optimistic and pessimistic approach is utilized to secure efficiency and increase accuracy. In the second and third scenarios, the non-cooperative models, from the optimistic and pessimistic views, cannot be turned into linear models, because of the additional inputs and outputs in the first, second and third stages. Therefore, we use a heuristic technique, to convert the nonlinear models into linear models. In accordance with the points mentioned, most of the researches performed in the network deliberate on two stages, but the current research takes a three-stage process into consideration, which, in addition to the intermediary variables, has additional and undesirable inputs and outputs as well. As a summarization, contributions of this paper are as follows:

- A three-stage network is taken under consideration in respect to the additional desirable and undesirable inputs and outputs
- The suggested model is a hybrid data envelopment analysis and goal programming model in a network structure
- We define a kind of cooperation between the leader and followers, so that the objectives of the managers are capable of being inserted into the models
- A Double Frontier Approach is utilized to evaluate efficiency, in order to make results more realistic
- A heuristic technique is proposed to convert non-linear models into linear models
- Implementation of the suggested model on an authentic example. (We simulate a factory in a real world with a production area, a warehouse for goods and a delivery point)
- The said factory is considered as a dynamic network.

The structure of this paper has been rendered as follows: Section (2) describes the model and in this section, after introducing the structure of the network, the modeling is performed from the black box perspective and three non-cooperative scenarios. Section (3) renders the model solution and presents the same according to the heuristic method. In the Section (4) a case-history is described, where a factory in the real world has been rendered and illustrated as an example and finally, Section (5) concludes the paper.

2. Model Description

We consider a set of n homogeneous decision making units (DMUs) that are denote by DMU_j (j=1,..., n), and each DMU_j has three-stage, as shown in Fig. 1, where all the stages are connected together in series. We denote, the inputs of the first stage by $x^1_{i_1j}$ ($i_1=1,\ldots,I_1$) and the undesirable outputs of the first stage by $y^1_{r_1j}$ ($r_1=1,\ldots,R_1$). We denote, the intermediate measures between first stage and second stage by $z^1_{d_1j}$ ($d_1=1,\ldots,D_1$) and between second stage and third stage by $z^2_{d_2j}$ ($d_2=1,\ldots,D_2$). The additional inputs and outputs of the second stage are denoted by $x^2_{i_2j}$ ($i_2=1,\ldots,I_2$) and $y^2_{r_2j}$ ($r_2=1,\ldots,R_2$), respectively. Finally, we denote, the additional inputs of the third stage by $x^3_{i_3j}$ ($i_3=1,\ldots,I_3$) and the outputs of the third stage by $y^3_{r_3j}$ ($r_1=1,\ldots,R_3$). We adopt $v^1_{i_1}$, $v^2_{i_2}$ and $v^3_{i_3}$ as the weights of the inputs to the first, second and third stages, respectively. We adopt $w^1_{d_1}$ and $w^2_{d_2}$ as the weights of the intermediate measures between stages 1, 2 and 3, respectively. The weights of the outputs for the first, second and third stages are denoted by $u^1_{r_1}$, $u^2_{r_2}$ and $u^3_{r_3}$, respectively.

In relevance to analyzing the abovementioned network (Fig. 1), we have considered a black box and three non-cooperative viewpoints. In this section we shall perform the modeling for these four approaches correspondingly. Researchers are more inclined to utilize input-oriented models for efficiency analysis, for three reasons mainly. The first is that, demand reveals a growing trend, the estimation of which is an intricate matter. The second reason encompasses the fact that, managers have a better control on the inputs, rather than the outputs and the third motive entails the point that, the model reflects the initial objectives of policy-makers, on the basis of being responsible in responding to the demands of the people. Furthermore, the units must reduce costs and or restrict the use of resources. Thereby, in this research, an input- oriented model is utilized.

2.1. Black Box Approach

As indicated in the first section, the black box approach is utilized towards alleviating complex networks and ignores the intermediary variables. The following figure, (Fig. 2) illustrates the black box model of the network and the inputs and outputs, which are the inputs and outputs of the three stages respectively.

It should be noted that, $y^1_{r_1j}$ is an undesired output. Therefore, we shall describe the efficiency of the black box approach from the optimistic view as follows:

Model 1:

$$\theta_o^{overall}=\max\sum_{r_2=1}^{R_2}u^2_{r_2}y^2_{r_2o}+\sum_{r_3=1}^{R_3}u^3_{r_3}y^3_{r_3o}-\sum_{r_1=1}^{R_1}u^1_{r_1}y^1_{r_1o}$$

$$\text{s.t.}\ \sum_{i_1=1}^{I_1}v^1_{i_1}x^1_{i_1o}+\sum_{i_2=1}^{I_2}v^2_{i_2}x^2_{i_2o}+\sum_{i_3=1}^{I_3}v^3_{i_3}x^3_{i_3o}=1$$

(1)

$$\sum_{r_2=1}^{R_2}u^2_{r_2}y^2_{r_2j}+\sum_{r_3=1}^{R_3}u^3_{r_3}y^3_{r_3j}-\sum_{r_1=1}^{R_1}u^1_{r_1}y^1_{r_1j}-\sum_{i_1=1}^{I_1}v^1_{i_1}x^1_{i_1j}-\sum_{i_2=1}^{I_2}v^2_{i_2}x^2_{i_2j}-\sum_{i_3=1}^{I_3}v^3_{i_3}x^3_{i_3j}\le 0,\quad j=1,\ldots,n.$$

$$u^1_{r_1},u^2_{r_2},u^3_{r_3}\ge\varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;r_3=1,\ldots,R_3;$$

$$v^1_{i_1},v^2_{i_2},v^3_{i_3}\ge\varepsilon;\ i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3.$$

On the basis of the Wang et al. [51] we shall modify model (1) and describe the efficiency of the black box approach from the pessimistic view as follows:

Model 2:

$$\varphi_o^{overall}=\min\sum_{r_2=1}^{R_2}u^2_{r_2}y^2_{r_2o}+\sum_{r_3=1}^{R_3}u^3_{r_3}y^3_{r_3o}-\sum_{r_1=1}^{R_1}u^1_{r_1}y^1_{r_1o}$$

$$\text{s.t.}\sum_{i_1=1}^{I_1}v^1_{i_1}x^1_{i_1o}+\sum_{i_2=1}^{I_2}v^2_{i_2}x^2_{i_2o}+\sum_{i_3=1}^{I_3}v^3_{i_3}x^3_{i_3o}=1$$

(2)

$$\sum_{r_2=1}^{R_2}u^2_{r_2}y^2_{r_2j}+\sum_{r_3=1}^{R_3}u^3_{r_3}y^3_{r_3j}-\sum_{r_1=1}^{R_1}u^1_{r_1}y^1_{r_1j}-\sum_{i_1=1}^{I_1}v^1_{i_1}x^1_{i_1j}-\sum_{i_2=1}^{I_2}v^2_{i_2}x^2_{i_2j}-\sum_{i_3=1}^{I_3}v^3_{i_3}x^3_{i_3j}\ge 0,\quad j=1,\ldots,n$$

$$u^1_{r_1},u^2_{r_2},u^3_{r_3}\ge\varepsilon;\ r_1=1,\ldots,R_1;r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;$$

$$v^1_{i_1},v^2_{i_2},v^3_{i_3}\ge\varepsilon;\ i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3.$$

Models (1) and (2) attain the efficiency values on the basis of the distance from the efficient and inefficient frontiers respectively. In respect to the Wang and Chin Approach [52], the overall efficiency of Fig. 2, by taking into consideration, the optimistic and pessimistic views, based on the results of models (1 and 2), which we describe in equation (3) is as follows:

$$\varnothing_o=\sqrt{\theta_o^{overall}.\varphi_o^{overall}} \tag{3}$$

2.2. Non-Cooperative Approach

2.2.1. First Scenario

In the first scenario, we will consider the first, second and third stages as the role of a “leader”, “first follower” and “second follower” respectively. Though, the second stage is given more importance due to its position, hence, in this scenario, two roles are taken into contemplation for the second stage: 1-Follower of the first stage 2-Leader of the third stage. We initially maximize the leader’s stage and then by assuming that the efficiency of leader’s stage is constant or fixed, subsequently, the efficiency of the second stage is maximized. Eventually, with the assumption that, the two previous stages have remained constant, we gain the efficiency of the third stage. Thence, the optimistic efficiency of the leader’s stage is demonstrated by θ_o^L and the optimistic efficiencies of the second and third stages by θ_o^{1F} and θ_o^{2F} respectively. We have defined the maximal efficiencies of the leader, second and third stages, from the optimistic standpoint, based on the approach of Li et al. according to the following:

Model 3:

$$\theta_o^{L*}=\max\left\{\theta_o^L \,\middle|\, \theta_j^L \le 1,\ \theta_j^{1F} \le 1,\ \theta_j^{2F} \le 1,\ j=1,\ldots,n\right\}$$

(4)

Model 4:

$$\theta_o^{1F*}=\max\left\{\theta_o^{1F} \,\middle|\, \theta_j^L \le 1,\ \theta_j^{1F} \le 1,\ \theta_j^{2F} \le 1,\ \theta_o^L=\theta_o^{L*},\ j=1,\ldots,n\right\} \qquad (5)$$

Model 5:

$$\theta_o^{2F*}=\max\left\{\theta_o^{2F} \,\middle|\, \theta_j^L \le 1,\ \theta_j^{1F} \le 1,\ \theta_j^{2F} \le 1,\ \theta_o^L=\theta_o^{L*},\ \theta_o^{1F}=\theta_o^{1F*},\ j=1,\ldots,n\right\} \qquad (6)$$

All the variables in models (3, 4 and 5) are non-negative and the optimum efficiency has been demonstrated with the symbol (*). By means of model (3), the maximum efficiency was attained by the leader’s stage, on conditions that, the efficiencies of the other stages which came to hand, are less than (1). In employing model (4), the maximum efficiency was gained for the second stage under circumstances that, the efficiencies of the other stages are less than (1) and that the efficiency of the leader stage for DMU_O is constant. In actual fact under such conditions, the second stage plays the role of a “follower”. Finally, by utilizing model (5), we obtain the maximum efficiency of the third stage, under such circumstances that, the efficiencies of the other stages are less than (1) and the efficiencies of the leader and second stages remain constant for DMU_O. Under such conditions, the second stage plays the role of a leader for the third stage. Models (3, 4 and 5) are fractional and by utilizing the Charnes-Cooper conversion (1962), such as illustrated hereunder, they are converted into linear models.

Model 6:

$$\theta_o^{L*} = \max \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1$$

$$\text{s.t.} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \leq 0, \quad j=1,\ldots,n \tag{7}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \leq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_1=1}^{I_1} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \leq 0, \quad j=1,\ldots,n$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon;\ i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2.$$

Model 7:

$$\theta_o^{1F*} = \max \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^2$$

$$\text{s.t.} \sum_{i_1=1}^{I_1} v_{i_2}^2 x_{i_2 o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \leq 0, \quad j=1,\ldots,n \tag{8}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \leq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_1=1}^{I_1} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \leq 0, \quad j=1,\ldots,n$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1 - \theta_o^{L*} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1 = 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;$$

$$v^1_{i_1}, v^2_{i_2}, v^3_{i_3} \geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w^1_{d_1}, w^2_{d_2} \geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

Model 8:

$$\theta_o^{2F*} = \max \sum_{r_3=1}^{R_3} u^3_{r_3} y^3_{r_3 o}$$

$$\text{s.t.} \sum_{i_1=1}^{I_1} v^3_{i_3} x^3_{i_3 o} + \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2 o} = 1$$

$$\sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1 j} - \sum_{r_1=1}^{R_1} u^1_{r_1} y^1_{r_1 j} - \sum_{i_1=1}^{I_1} v^1_{i_1} x^1_{i_1 j} \leq 0, \quad j=1,\ldots,n \tag{9}$$

$$\sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2 j} + \sum_{r_2=1}^{R_2} u^2_{r_2} y^2_{r_2 j} - \sum_{i_2=1}^{I_2} v^2_{i_2} x^2_{i_2 j} - \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1 j} \leq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u^3_{r_3} y^3_{r_3 j} - \sum_{i_1=1}^{I_1} v^3_{i_3} x^3_{i_3 j} - \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2 j} \leq 0, \quad j=1,\ldots,n$$

$$\sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1 o} - \sum_{r_1=1}^{R_1} u^1_{r_1} y^1_{r_1 o} - \theta_o^{L*} \sum_{i_1=1}^{I_1} v^1_{i_1} x^1_{i_1 o} = 0$$

$$\sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2 o} + \sum_{r_2=1}^{R_2} u^2_{r_2} y^2_{r_2 o} - \theta_o^{1F*} \sum_{i_1=1}^{I_1} v^2_{i_2} x^2_{i_2 o} - \theta_o^{1F*} \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1 o} = 0$$

$$u^1_{r_1}, u^2_{r_2}, u^3_{r_3} \geq \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v^1_{i_1}, v^2_{i_2}, v^3_{i_3} \geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w^1_{d_1}, w^2_{d_2} \geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

In accordance with the task of Wang et al., we modify models (6, 7 and 8) as given below, so that the minimum efficiencies of the first, second and third stages are gained respectively, from the pessimistic view.

Model 9:

$$\varphi_o^{L*} = \min \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1 o} - \sum_{r_1=1}^{R_1} u^1_{r_1} y^1_{r_1 o}$$

$$\text{s.t.} \sum_{i_1=1}^{I_1} v^1_{i_1} x^1_{i_1 o} = 1$$

$$\sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1 j} - \sum_{r_1=1}^{R_1} u^1_{r_1} y^1_{r_1 j} - \sum_{i_1=1}^{I_1} v^1_{i_1} x^1_{i_1 j} \geq 0, \quad j=1,\ldots,n \tag{10}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_1=1}^{I_1} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \geq 0, \quad j=1,\ldots,n$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon; i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

Model 10:

$$\varphi_o^{1F*} = \min \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^2$$

$$\text{s.t.} \sum_{i_1=1}^{I_1} v_{i_2}^2 x_{i_2 o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \geq 0, \quad j=1,\ldots,n \tag{11}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_1=1}^{I_1} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1 - \varphi_o^{L*} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1 = 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon; i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

Model 11:

$$\varphi_o^{2F*} = \min \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 o}^3$$

$$\text{s.t.} \sum_{i_1=1}^{I_1} v_{i_3}^3 x_{i_3 o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \geq 0, \quad j=1,\ldots,n \tag{12}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_1=1}^{I_1} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1 - \varphi_o^{L*} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1 = 0$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^2 - \varphi_o^{1F*} \sum_{i_1=1}^{I_1} v_{i_2}^2 x_{i_2 o}^2 - \varphi_o^{1F*} \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 = 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;\ j=1,\ldots,n.$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

It must be observed that, the stages of Fig. 1, are in series, based on the views of Kao and Hwang [53], and a multiplicative approach is used to compute the overall efficiency. Hence, by utilizing the results of the abovementioned models (6-11), the maximum overall optimistic efficiency ($\theta_o^{overall}$), the minimum overall pessimistic efficiency ($\varphi_o^{overall}$) as well as the overall efficiency, based on the double-frontier ($\varnothing_o^{overall}$) are defined respectively, as follows, in equations (13):

$$\theta_o^{overall} = \theta_o^{L*} . \theta_o^{1F*} . \theta_o^{2F*}, \qquad \varphi_o^{overall} = \varphi_o^{L*} . \varphi_o^{1F*} . \varphi_o^{2F*}, \qquad \varnothing_o^{overall} = \sqrt{\theta_o^{overall} . \varphi_o^{overall}} \tag{13}$$

2.2.2. Second Scenario

In the second scenario, we depict the first stage as the leader and the second and third stages together, in the form of a follower. On these bases, the efficiency of the optimistic leader stage is illustrated as θ_o^{L}, whereas, the optimistic efficiencies of the second and third stages are shown as θ_o^{1F} and θ_o^{2F} respectively; and the optimistic efficiency of the second and third stages together is demonstrated as θ_o^{12F}. In fact, the difference between this scenario and the first scenario lies in the role of the second and third stages. Therefore, the maximal optimistic efficiency of the leader stage (θ_o^{L*}) and the minimal pessimistic efficiency of the leader stage (φ_o^{L*}) are brought to hand respectively, from models (6 and 9) in the first scenario. Similarly, we hybrid the efficiencies of the second and third stages, being attentive to the fact that they are in series and define them as figures $\theta_o^{12F} = \theta_o^{1F} .\theta_o^{2F}$, according to the tasks of Kao and Hwang. Hence, the maximal efficiency for the follower stage (θ_o^{12F}) from the optimistic viewpoint is brought to hand and rendered as hereunder:

Model 12:

$$\theta_o^{12F*}=\max \frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1} . \frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}$$

$$\text{s.t.} \quad \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1} \le 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1} \le 1, \quad j=1,\ldots,n$$

(14)

$$\frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2} \le 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1} = \theta_o^{L*}$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

The maximum and overall efficiency of the follower stage (θ_o^{12F}) is gained by model (12), on condition that, the efficiencies of the other stages are less than (1); and according to the approach of Li et al. the efficiency of the leader's stage should remain constant. On the founding's of the tasks of Wang et al., we describe model (12) as given below, in order to attain the minimum efficiency of the follower stage (φ_o^{12F}) from the pessimistic viewpoint.

Model 13:

$$\varphi_o^{12F*}=\min \frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1} . \frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}$$

$$\text{s.t.} \quad \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1} \geq 1, \qquad j=1,\ldots,n$$

$$\frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1} \geq 1, \qquad j=1,\ldots,n$$

(15)

$$\frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 j}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2} \geq 1, \qquad j=1,\ldots,n$$

$$\frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1} = \varphi_o^{L*}$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

Models (12 and 13) are nonlinear and in the third section of this paper, an innovative approach in resolving them is utilized. In assuming that, the models (12 and 13) are solved and given that the stages are in series (Fig. 1), we define the total and maximal optimistic efficiency($\theta_o^{overall}$), the minimal and total pessimistic efficiency ($\varphi_o^{overall}$) and the overall efficiency with the double-frontier ($\varnothing_o^{overall}$) and these are respectively, specified below, in equations (16):

$$\theta_o^{overall} = \theta_o^{L*} . \theta_o^{12F*}, \qquad \varphi_o^{overall} = \varphi_o^{L*} . \varphi_o^{12F*}, \qquad \varnothing_o^{overall} = \sqrt{\theta_o^{overall} . \varphi_o^{overall}}$$

(16)

2.2.3. Third Scenario

In this scenario, which comprises of the proposed approach of this paper, efforts have been made to insert the goals of the managers into the models. In the third scenario we designate the first stage as the "leader" and assume that the second and third stages together, are in the form of a "follower". The dissimilarity between this scenario and the second scenario is that, in such circumstances collaboration between the leader and followers is taken into consideration. In the second scenario, the leader stage concentrates only on maximizing its efficiency, which causes deterioration in the efficiencies of the followers. Though, in this scenario and under such circumstances, the leader optimizes its efficiency, so that the efficiencies of the followers do not reduce from a certain level, or in actual fact, the leader maximizes its efficiency to forestall the eradication of the followers. Actually, the leader-follower characteristic is a non-cooperative game, which we hybrid with a cooperative approach in this section. In accordance with this, we describe the maximal efficiency of the leader stage from the optimistic viewpoint as hereunder:

Model 14:

$$\theta_o^{L*}=\max\left\{\theta_o^L \,\middle|\, \theta_o^{1F} \geq c_1, \theta_o^{2F} \geq c_2, \theta_j^L \leq 1, \theta_j^{1F} \leq 1, \theta_j^{2F} \leq 1, j=1,\ldots,n \right\} \tag{17}$$

All the variables in model (14) are non-negative. Model (14) secures the maximal efficiency of the leader stage, on condition that, the efficiencies of the other stages are less than (1) and for DMU_o, the efficiencies of the second and third stages are not lower than the values of c_1 and c_2 respectively. The values of c_1 and c_2 are actually the minimal efficiencies of the second and third stages which are numerals at intervals of (0 and 1) in accordance with the goals of managers. It should be noted that if the values of $c_1 = c_2 = \varepsilon$ are considered such, so that they are closer to (0), then the two constraints $\theta_o^{1F} \geq c_1$, $\theta_o^{2F} \geq c_2$ are simply redundant. So the model (14) is feasible. But there could be a possibility that in reality, the goals of managers is not capable of being attained and the model turns into a superfluous one. Hence, we utilized the concept of 'goal programming' and the two assigned values α_1 , α_2 are reduced $(\theta_o^{1F} \geq c_1\text{-}\alpha_1 , \theta_o^{2F} \geq c_2\text{-}\alpha_2)$ from the opinion of managers under contemplation, so that by using model (15), conditions for securing the goal of the managers is surveyed. Model (14) is a fractional model and by utilizing the Charnes-Cooper conversion (1962), as well as contemplating on the goal programming concept, as illustrated hereunder, it is converted into a linear model.

Model 15:

$$\theta_o^{L*}= \max\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1 - M(\alpha_1+\alpha_2)$$

$$s.t.\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \leq 0, \quad j=1,\ldots,n \tag{18}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \leq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \leq 0, \quad j=1,\ldots,n$$

$$(c_1 - \alpha_1)\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + (c_1 - \alpha_1)\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 - \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 \leq 0$$

$$(c_2 - \alpha_2)\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + (c_2 - \alpha_2)\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 - \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3 \leq 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon; i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2.$$

In the model (15) "M" denotes a large numeral, which factually is a penalty that causes the manager's goal to be achievable. It should be mentioned that in the case where, $\alpha_1=0, \alpha_2=0$, model (15) is feasible from the point of the manager's goal and if this is not the issue, we request the manager to reduce his goals (c_1, c_2) to the measurement of α_1 and α_2 to make the model possible. On the basis of the task of Wang et al., we modify model (15), as hereunder, to obtain the efficiency of the leader stage from the pessimistic view. Similar to our optimistic approach, we obtain the pessimistic efficiency of the leader stage, under conditions where the follower stages are at a distance from the inefficient frontier, *i.e.* $\varphi_o^{1F} \geq c_3 - \alpha_3$, $\varphi_o^{2F} \geq c_4 - \alpha_4$ in which case, $c_3, c_4 \geq 1$.

Model 16:

$$\varphi_o^{L*} = \min \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1 + M(\alpha_3+\alpha_4)$$

$$s.t.\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \geq 0, \quad j=1,\ldots,n \tag{19}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^2 - (c_3 - \alpha_3) \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - (c_3 - \alpha_3) \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 o}^2 \geq 0$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 o}^3 - (c_4 - \alpha_4) \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 o}^3 - (c_4 - \alpha_4) \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 \geq 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon; r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon; i_1=1,\ldots,I_1; \ i_2=1,\ldots,I_2; \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon; d_1=1,\ldots,D_1; \ d_2=1,\ldots,D_2.$$

Analogous to the optimistic approach of "M" that is a large numerical, which, in this circumstance and with due attention to the type of objective function, has been supplemented to the model, in order to fulfill the manager's goal. It should be observed that in the case where $\alpha_1=0$, $\alpha_2=0$, model (16) is feasible, in respect to the opinion of the manager, or else, we shall request the manager to reduce his goals (c_3 , c_4) to the measurement of a_3 , a_4 to make the model possible.

To compute the maximal efficiency for the follower stage (θ_o^{12F}) from the optimistic viewpoint and the minimal efficiency for the follower stage (φ_o^{12F}) from the pessimistic viewpoint, we shall operate according to the second scenario; and assume the second and third stages as one stage and obtain the efficiency of the follower stage on the basis of models (12 and 13). It should be observed that, in such conditions the efficiencies values of the leader stage (θ_o^{L*} and φ_o^{L*}), are found in models (12 and 13) on the founding's of the results of models (15 and 16); and likewise, two constraints $\theta_o^{1F} \geq c_1\text{-}\alpha_1$ and $\theta_o^{2F} \geq c_2\text{-}\alpha_2$ are supplemented to model (12), including two constraints $\varphi_o^{1F} \geq c_3 - \alpha_3$ and $\varphi_o^{2F} \geq c_4 - \alpha_4$ are which added to model (13). The values of α_i, i=1,2,3,4 for these 4 constraints comes to hand from models (15 and 16). Finally, by utilizing equations (16) in this scenario, the maximal overall optimistic efficiency, the minimal overall pessimistic efficiency and the total efficiency in relative to the double-frontier is attained.

3. Heuristic approach to solve nonlinear models

Two exploratory approaches are proposed for the optimistic and pessimistic views relatively. It is for the first time that we have developed an exploratory approach, in relevance with the pessimistic perspective or condition, which, with the best of our findings, we had not been able to perform the modeling, as to this conceptual approach under pessimistic conditions, till date. Similarly, we have also implemented an exploratory approach from the optimistic standpoint, in relative to the leader-follower concept. Thereby, the exploratory approach from the pessimistic angle is proposed in this paper. In this section we will use a solution to gain the efficiencies of the follower (θ_o^{12F} , φ_o^{12F}) in the second and third scenarios, which will be described in accordance to the second scenario. Due to the presence of additional inputs and outputs in the first, second and third stages, models (12 and 13) are nonlinear. To solve these models we use a heuristic approach as hereunder:

3.1. A heuristic method from optimistic viewpoint

We are aware that the objective function of model (12) is a multiplicative efficiency of the two-stages, i.e. $\theta_o^{12F*} = \max \theta_o^{1F} . \theta_o^{2F}$. Let us consider the first stage (θ_o^{1F}) as a variable that changes between the interval $\left[0, \theta_o^{1F\text{-max}}\right]$. By using the equation (20), we are able to move this variable into its interval, as given below:

$$\theta_o^{1F} = \theta_o^{1F\text{-max}} - k_1 \Delta\varepsilon , \qquad k_1 = 0,1,\ldots,\left[\frac{\theta_o^{1F\text{-max}}}{\Delta\varepsilon}\right]+1$$

(20)

In taking $\Delta\varepsilon$ as a step size, we consider it as an extremely small amount and describe $\theta_o^{1F\text{-max}}$ as the maximum efficiency of the first follower stage and its value is capable of being computed by the model below:

Model 17:

$$\theta_o^{1F\text{-max}} = \max\left\{\theta_o^{1F} \,\middle|\, \theta_j^{L} \le 1,\ \theta_j^{1F} \le 1,\ \theta_j^{2F} \le 1,\ \ j=1,\ldots,n\right\} \tag{21}$$

Model (17) secures the maximum efficiency of the first follower stage, under conditions where the efficiencies of the other stages are less than (1). In actual fact, this model irrespective, of the leader-follower correlations, attributes the highest efficiency to the second stage. This model is a fractional model and by utilizing the Charnes-Cooper conversion (1962), as illustrated hereunder, it is converted into a linear model:

Model 18:

$$\begin{aligned}
\theta_o^{1F\text{-max}} &= \max \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 \\
\text{s.t.}\quad & \sum_{i_1=1}^{I_1} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 = 1 \\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \le 0, \quad j=1,\ldots,n \\
& \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \le 0, \quad j=1,\ldots,n \\
& \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \le 0, \quad j=1,\ldots,n \\
& u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3; \\
& v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3; \\
& w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.
\end{aligned} \tag{22}$$

In determining the value of $\theta_o^{1F\text{-max}}$ by model (18), we convert model (12) into the following model:

Model 19:

$$\theta_o^{12F*} = \max\left\{\theta_o^{1F}.\theta_o^{2F} \,\middle|\, \theta_j^{L} \le 1,\ \theta_j^{1F} \le 1,\ \theta_j^{2F} \le 1,\ \theta_o^{L} = \theta_o^{L*}, \theta_o^{1F} = \frac{O_o^2}{I_o^2},\ \theta_o^{1F} \in \left[0,\ \theta_o^{1F\text{-max}}\right],\ j=1,\ldots,n\right\} \tag{23}$$

In model (19) we considered θ_o^{1F} in the objective function as a variable and the constraint which specified this variable, together with its interval of modification was added to the model. In this model, we have briefly demonstrated the efficiency of the second stage or θ_o^{1F}, in a form of output to an input. The mentioned model is a fractional one and by utilizing the Charnes-Cooper conversion (1962), as illustrated hereunder, it is converted into a linear model:

Model 20:

$$\theta_o^{12F*} = \max\ \theta_o^{1F}.\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 o}^3$$

$$\text{s.t.}\ \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \le 0, \quad j=1,\ldots,n \tag{24}$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \le 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \le 0, \quad j=1,\ldots,n$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1 - \theta_o^{L*} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1 = 0$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^2 - \theta_o^{1F*} \left(\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 \right) = 0$$

$$\theta_o^{1F} \in \left[0, \theta_o^{1F\text{-max}}\right]$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon; i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.$$

In model (20) and by utilizing equation (20), we increase the value of k_1 from (0) to its higher level, in order to solve the new model each time with θ_o^{1F}. For the entire conditions of k_1, model (20) is solved and the responses of the model are assigned as $\theta_o^{12F}(k_1)$. By comparing all the values of $\theta_o^{12F}(k_1)$, we define θ_o^{12F*} as the maximal efficiency of the follower stage from the optimistic view. It should be noted that, we have tested our proposed approach under two conditions and each time have considered a stage as a variable. Given that the efficiency of a stage is somewhat unique, thereby, the results of these two methods have come to hand with an extremely good approximation and in order to explain our approach, we have denoted one of these two conditions above.

3.2. A heuristic method from pessimistic viewpoint

We know that the objective function of model (13) is the multiplicative efficiency of two stages, *i.e.* $\varphi_o^{12F*} = \min \varphi_o^{1F}.\varphi_o^{2F}$. Similar to our optimistic view, we take φ_o^{1F} as a variable in the objective function that modifies between the $\left[\varphi_o^{1F\text{-min}}, M\right]$ interval. We describe φ_o^{1F} as rendered below so that we can move it within the interval.

$$\varphi_o^{1F} = \varphi_o^{1F\text{-min}} + k_1\Delta\varepsilon, \qquad k_1=0,1,\ldots,\left[\frac{M\text{-}\varphi_o^{1F\text{- min}}}{\Delta\varepsilon}\right]+1$$

(25)

We consider "M" to be a large amount and alike the optimistic approach, $\Delta\varepsilon$ as a step size and an extremely small amount. $\varphi_o^{1F\text{-min}}$ is described as the minimum efficiency of the first follower stage and its sum can be computed by the following model.

Model 21:

$$\varphi_o^{1F\text{-min}}=\min\left\{\varphi_o^{1F} \,\middle|\, \varphi_j^{L} \geq 1,\ \varphi_j^{1F} \geq 1,\ \varphi_j^{2F} \geq 1,\quad j=1,\ldots,n\right\} \tag{26}$$

Model (21), secures the minimum efficiency of the first follower stage, on condition that the efficiencies of the other stages are more than (1). In fact, this model, regardless to the leader-follower correlation, attributes the least amount of efficiency to the second stage. This model is a fractional model and by employing the Charnes-Cooper conversion (1962), it is converted into a linear model as given hereunder:

Model 22:

$$\begin{aligned}
\varphi_o^{1F\text{-min}} &= \min \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 \\
\text{s.t.} \quad & \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 = 1 \\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \geq 0, \quad j=1,\ldots,n \\
& \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \geq 0, \quad j=1,\ldots,n
\end{aligned} \tag{27}$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \geq 0, \quad j=1,\ldots,n$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon; i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2.$$

In specifying the value of $\varphi_o^{1F\text{-min}}$ by model (22), model (13) is modified and converted to the model below:

Model 23:

$$\varphi_o^{12F*} = \min\left\{ \varphi_o^{1F}.\varphi_o^{2F} \,\middle|\, \varphi_j^L \geq 1,\ \varphi_j^{1F} \geq 1,\ \varphi_j^{2F} \geq 1,\ \varphi_o^L = \varphi_o^{L*},\ \varphi_o^{1F} = \frac{O_o^2}{I_o^2},\ \varphi_o^{1F} \in \left[\varphi_o^{1F\text{-min}}, M\right],\ j=1,\ldots,n \right\} \quad (28)$$

It should be brought to attention that, in the model (23), we take φ_o^{1F} . in the objective function as aariable and alike the optimistic approach, constraints which specify this variable, along with its interval of modification is supplemented to the model. The model (23) is a fractional model and by using the Charnes-Cooper conversion (1962), it is converted into a linear model as given hereunder:

Model 24:

$$\varphi_o^{12F*} = \min\ \varphi_o^{1F}.\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 o}^3$$

$$\text{s.t.} \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 j}^1 \geq 0, \quad j=1,\ldots,n \quad (29)$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 j}^1 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 j}^2 \geq 0, \quad j=1,\ldots,n$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^1 - \varphi_o^{L*} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^1 = 0$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2 o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^2 - \varphi_o^{1F*} \left(\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1 o}^1 \right) = 0$$

$$\varphi_o^{1F} \in \left[\varphi_o^{1F\text{-min}}, M\right]$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \geq \varepsilon;\ r_1=1,\ldots,R_1; r_2=1,\ldots,R_2; r_3=1,\ldots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \geq \varepsilon; i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;$$

$$w_{d_1}^1, w_{d_2}^2 \geq \varepsilon;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2.$$

In model (24) and by employing equation (25), we increment the value of k_1 to its utmost level, in order to solve the model each time with the new φ_o^{1F}. For the entire conditions of k_1, model (24) is solved and the responses of the model are assigned as $\varphi_o^{12F}(k_1)$. By comparing all the values of $\varphi_o^{12F}(k_1)$, we define φ_0^{12F*} as the minimal efficiency of the follower stage from the pessimistic view. It should be noted that, similar to the optimistic approach, we have tested our proposed approach under two conditions and each time have considered a stage as a variable; with due attention to the fact that, the efficiency of a stage is somewhat unique, thereby, the results of these two methods have come to hand with an extremely good approximation and in order to explicate our approach, we have represented one of these two conditions above.

4. Case Study Description

Throughout the past years, an increment in the importance of the production sector; and an anxiety in regard the development and efficiency growth of this segment is directly correlated with that of the economic system. A rise in costs, has led to haul, the production units towards incrementing their organizational performance. The optimal mode which would increase efficiency is to logically utilize, adopt and modify the available resources. This could only be achieved by ensuring a correct managerial performance, including a rational evaluation of the returns attained [54, 55]. The statistical population of this research includes the production, maintenance and distribution network of a factory (Nasiri Dairy factory), which is defined as an annual planning horizon in 24 periods. The factory is located in Nazar Abad Industrial Estate (in Iran), and its brand name is "Aramesh-e-Paitakht". This factory has a production area, a warehouse area and a delivery point; each of which is considered as a stage. The production area plays the role of the "leader" and the other two, the role of "followers". We have contemplated on this factory for within a length of 24 time periods and as a dynamic network. In this network, a number of outputs during a time period of t in the second stage are converted to a number of inputs in the second stage during a time period of t+1. We assume each time period to be a DMU. Hence, the inputs and outputs of each DMU are according to the following. We assign the production costs of the three products produced as inputs of the first stage and denote them as (x_1^1, x_2^1, x_3^1). The transport costs for produce from the first to the second stage is described as an undesirable output of the first stage, which we show as y_1^1. The intermediary products between the first and second stages, are the quantity of produce of each commodity, which are demonstrated as (z_1^1, z_1^2, z_1^3). The additional outputs for the second stage are respectively, the cost of reserving storage location x_1^2 , cost of holding goods x_2^2 and the goods remaining in the warehouse from the previous period, which are illustrated as (x_3^2, x_4^2, x_5^2). We define the outputs of the second stage as the quantity of the goods remaining in the warehouse for the subsequent period of time, and represent them with (y_1^2, y_2^2, y_3^2). The intermediary products between the second and third stages are the quantities of delivery of each commodity, which are demonstrated by (z_1^4, z_1^5, z_1^6). We describe the additional input of the third stage as the transport costs of goods to the third stage and this is illustrated as x_1^3. Finally, the output of the third stage is the profit from the sale of goods, which is indicated by y_1^3. In continuation, we illustrate the input values for the 24 time periods in Table 2 and the mean values and outputs in Table 3.

In Table 2, the values of (0), for each period indicate that, the goods have not remained in the warehouse since the previous phase (Columns 7 to 9). The following Table 3, also shows values with (0), which illustrate that the goods for the subsequent period have not remained in the warehouse (Columns 9 to 11).

In continuation, we secure the efficiency of the factory from a black box approach and three leader-follower scenarios. For this purpose and in the third scenario, $c_1 = c_2 = 0.6$ and $c_3 = c_4 = 1.05$ are considered as the goals of managers. The values of α_i have come to hand from models (15 and 16) and show that the goals of the managers have been attained ($\alpha_i = 0, i = 1,2,3,4$). In the second and third scenarios, on the basis of the opinions of managers, $\Delta\varepsilon = 0.01$ and $M = 5$ have been contemplated. Similarly, the value for ε in all the models has been considered as 0.05 by the managers. We have executed the heuristic method expressed in the section (3) for both, the second and third scenarios. The values achieved for k_1 together with the maximal optimistic efficiency and the minimal pessimistic efficiency of the first stage for the second and third scenarios have been illustrated in the Table 4.

In studying the values of k_1, we were aware that, in this case-study, that the pessimistic efficiency values of all the followers in the second and third scenarios, in most of the cases, are optimized, when the values of k_1 are low (Columns 4 and 8). This signifies that, the optimal efficiency values of the second stage or the first follower are proximate to their minimum values (Columns 5 and 8), whereas, in the case of the optimistic efficiency values of the second stage, are far from their maximum values, in most circumstances (Columns 3 and 7). It should be noted that, the closer the efficiency value is closer to (1), from the optimistic standpoint and the farther the efficiency value is from (1), from the pessimistic standpoint, the DMU proves to have a better condition. Likewise, according to the expectations, the entire efficiency values, both, from the optimistic and pessimistic perspective in the third leader-follower scenario are higher than the level of goals (0.6, 1.05). Tables (5 and 6) render the optimistic and pessimistic efficiency values of the stages for the three "leader-follower" scenarios respectively.

In the Table 5, columns (2, 5 and 8) demonstrate the efficiency values of the leader stage from the optimistic approach. In this case study, the efficiency values of the leader stage for all the DMUs, in each of the three leader-follower scenarios equate to (1). In general and with due attention to the two constraints supplemented to the third scenario, in respect to the optimistic efficiency values of the leader stage, the equation $\theta_o^{Lscenario3} \leq \theta_o^{Lscenario1} = \theta_o^{Lscenarioo2}$ is expected. In comparing the efficiency values of the followers in this case study, we observed that the optimistic efficiency values gained for the first scenario (including all the DMUs) are more than the second and third scenarios ($\theta_o^{1Fscenario2}$, $\theta_o^{1Fscenario3} \leq \theta_o^{1Fscenarioo1}$ and $\theta_o^{2Fscenario2}$, $\theta_o^{2Fscenario3} \leq \theta_o^{2Fscenarioo1}$). Likewise, as expected, results show that, the optimistic efficiency values of the followers for all the DMUs in the second scenario are more than the third scenario ($\theta_o^{12Fscenario3} \leq \theta_o^{12Fscenarioo2}$); and this is because of the two additional constraints of the third scenario. Similarly, results also illustrate that the average optimistic efficiency values of the leader, first follower and second follower in the three leader-follower scenarios are of the following values respectively, (1, 0.98, 0.91), (1, 0.82, 0.63) and (1, 0. 80, 0.64). As a result, in all the three scenarios, the average optimistic efficiency values of the leader are higher than that of the followers and likewise, the average optimistic efficiency values of the first leader, which has come to hand, are more than the second follower. These results are acceptable or appropriate due to the priority status of the stages.

In the Table 6, columns (2, 5 and 8) demonstrate, the pessimistic efficiency values of the leader stage in each of the three scenarios are equal to (1). The objective function of the pessimistic efficiency is minimizing, in general, we expect that, by being attentive, in respect to the two constraints added to the third scenario, the equation $\varphi_o^{Lscenario1} = \varphi_o^{Lscenario2} \leq \varphi_o^{Lscenarioo3}$ to be confirmed. In comparing the efficiency values of the first follower in this case study, we observed that, the pessimistic efficiency values obtained from the first scenario for all the DMUs are less than the second and third scenarios $\left(\varphi_o^{1Fscenario1} \leq \varphi_o^{1Fscenario2} \text{ and } \varphi_o^{1Fscenarioo3}\right)$. The pessimistic efficiency values attained from the second scenario for the second follower are less than the first and third scenarios ($\varphi_o^{2Fscenario2} \leq \varphi_o^{2Fscenario1}$ and $\varphi_o^{2Fscenarioo3}$). As expected, results show that the pessimistic efficiency values of the followers for all the DMUs in the second scenario are less than the third scenario and this is due to the two additional constraints of the third scenario ($\varphi_o^{12Fscenario2} \leq \varphi_o^{12Fscenarioo3}$). Results demonstrate that the average pessimistic efficiency values of the leader and the first and second followers in the three leader-follower scenarios are (1, 1.05, 1.04), (1, 1. 08, 1. 007) and (1, 1.10, 1.05) respectively.

This results in the fact that, in each of the three scenarios, the pessimistic average efficiency values of the leader are less than the followers and similarly, the average pessimistic efficiency values attained for the first follower, are more, than that, of the second follower. As expected, in the third scenario, the pessimistic efficiency values, in the entire followers are higher than the level of goal by (1. 05). For each view and scenario we have described the optimistic, pessimistic and dual-frontier overall efficiency values and the results of which have been rendered in Table 7.

In comparing the optimistic overall efficiency of the black box view and the three leader-follower scenarios, by utilizing the Table 7, it can be observed that the black box view had the maximal efficiency for all the DMUs. This arises because the black box view fails to consider the intermediary variables. The first scenario of the leader-follower aspects shows the optimistic overall efficiency with a slight difference and decreases than the black box view and is in second place. The optimistic overall efficiency computed by the second and third leader-follower scenarios are in the third and fourth positions respectively. But in most of the cases, have an outstanding difference from the viewpoint of the optimistic overall efficiency performance with the two scenarios, *i.e.* the first and second scenarios. The reason as to why the third leader-follower scenario is lower than the second scenario is because the goals of the managers are imposed in the models of the third scenario. Hence, by comparing the columns (2, 5, 8 and 11) in Table 7, it can be noted that, the optimistic overall efficiency performance computed in this case study for 4 viewpoints are accordingly, $\theta_o^{\text{overall-scenario3}} \leq \theta_o^{\text{overall-scenario2}} \leq \theta_o^{\text{overall-scenarioo1}} \leq \theta_o^{\text{overall-black box}}$.

In Table 7, by comparing the columns (3, 6, 9 and 12) which, demonstrates the pessimistic overall efficiency, we are aware that in this case study, the results of the black box view has the minimal distance to the inefficient frontier in comparison to the three other scenarios, which contrary to the optimistic view, causes it to earn the poorest or worst position. Likewise, in comparing the pessimistic overall efficiency of the leader-follower in the second and third scenarios and to learn that, the pessimistic overall efficiency which has come to hand from the third scenario, is more than that of the second one and this is caused due to the goals of managers; which is in contrast to the optimistic approach of the third scenario, holds a better condition in comparison to the second scenario. But from comparing these two scenarios with the first leader-follower scenario, a specific result cannot be outlined. Given the differences in the results of the optimistic and pessimistic approaches, for a final summarization of these 4 viewpoints, we seek columns (4, 7, 10 and 13) of Table 7, which illustrates the efficiency by taking the double-frontier into consideration. We observed that in this case study, the black box view and the leader-follower of the first scenario provided close results for the network shown in Fig. 1; but with this variance that the first leader-follower scenario, takes the intermediate variables under consideration, though in the black box view, in order to alleviate matters, it is neglected. Similarly, by considering the goals of managers, the results of the second and third leader-follower scenarios are in approximation to each other, whilst there is a significant difference with the two other scenarios.

Given, that this paper proposes a double-frontier approach in order to measure the overall efficiency, thereby, the best and poorest DMUs in accordance with the black box scenario are units 24 and 1 with overall efficiencies of 1.12901 and 0.97214 respectively. On the basis of the first leader-follower scenario, unit 12, with a total efficiency of 1.18253 is the best; whereas, units 2 and 14 display a common and overall efficiency of 0.91598 are the worst units. It should be noted that, in some cases, for example, in DMU_2 and DMU_{14}, the rank of the DMUs are equal. This is due to the fact that the demand, the amount of production of each good, the amount of delivery and maintenance of each good and other item during periods 2 and 14 were absolutely equivalent. In accordance with the second scenario, the leader-follower aspect of units 8 and 23, with overall efficiencies of 0.90003 and 0.60246 are the optimum and the poorest units respectively. Ultimately, based on the leader-follower facet of the third scenario, unit 22 with a total efficiency of 0.9337 is the best and unit 23 with an overall efficiency of 0.63 is the worst unit.

5. Conclusion

In this paper a three-stage network has been considered with additional inputs and outputs that are desirable and undesirable. We have computed the efficiency of this network from the black box approach and three leader-follower scenarios. In the black box view, the entire system is considered as a leader without any followers. In the first leader-follower scenario, the first, second and third stages were contemplated as “leader”, “first follower” and “second follower” respectively. In this scenario, two roles are considered for the second stage: 1- Follower of the first stage, 2-Leader of the third stage.

In the second and third scenarios, assigned as leader-follower, we considered the first stage as a leader and the other two stages together, as a follower. In this paper, a manufacturing factory with a production and a warehouse area, including a delivery point for goods, was taken under consideration. The total costs, comprising of production costs, storage costs, reservation of warehouse (space) costs, and transportation costs from both, the production area to the warehouse and from the warehouse to the delivery point of goods, including the profits from the sale of commodities have been simulated. We have considered this factory as a dynamic network consisting of 24 intervals. In order to achieve a better accuracy, the optimistic and pessimistic approaches are utilized to evaluate efficiency. In this paper a heuristic method is used to solve complex models in the leader-follower aspect of the second and third scenarios.

In this paper, we have made efforts to assist managers in network analysis by utilizing diverse non-cooperative approaches; and also insert the goals of managers in the models. It was for this purpose that we pursued the goal programming (GP) concept in the third leader-follower scenario; and defined a leader-follower collaboration based on the goals of managers, which distinguishes this scenario with that of the second scenario. The black box view was observed and the first leader-follower scenario showed close results for the network, with this disparity that, in this scenario, the intermediary variables were taken into consideration, whereas, in the black box view this was not so. Likewise, due to the goals of managers which have been contemplated in the second scenario, the results of the second and third leader-follower scenarios are in approximation with each other, but have significant differences with the other two scenarios. It is quite evident that, if the values of the goals of managers (a_i, i=1,2,3,4) are modified, the results of the third leader-follower scenario can exhibit even more exceptional differences in respect to the second leader-follower scenario. The conditions necessary to ascertain the goals of managers are where the values of (c_i, i=1,2,3,4) are equivalent to (0).

The proposed heuristic approach in this paper was performed for three stages and due to the presence of additional inputs and outputs, the stages increased, thereby, making the model more complicated. As a result the solving period is extremely elevated. So as to decrease this period, step size $(\Delta\varepsilon)$ can be increased. Hence, the value of the step size $(\Delta\varepsilon)$, which determines the accuracy and the time for resolving the problem can be reflected on by managers. The results of this research are put at the disposal of the managers, so that they procure the best decision for the abovementioned factory. Modeling with inaccurate and random data is suggested for research in the future.

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List of captions

Tables with their numbers

Table 1. Classification of Studies on DEA-Game Theory method

Reference	Type of game	Structure Of Network	Additional inputs	Undesirable output	Type of modelling	Type of frontier	Dea-Gp	Dynamic
Hwang et al.[1]	Cooperative	One-stage	-	✓	Linear programming	Optimistic view	-	-
Kao and Hwang[53]	Cooperative	Two-stage	-	-	Linear programming	Optimistic view	-	-
Wang et al.[28]	Cooperative	Two-stage	-	-	Linear programming	Optimistic view	-	✓
Kou et al.[8]	Cooperative	Two-stage	✓	-	Linear programming	Optimistic view		✓
Li et al.[11]	Non-cooperative	Two-stage	✓	-	Linear programming	Optimistic view	-	-
Liang et al.[10]	Cooperative and Non-cooperative	Two-stage	-	-	Linear programming	Optimistic view	-	-
Wu et al.[24]	Cooperative	Two-stage	✓	✓	Linear programming	Optimistic view	-	-
An et al.[12]	Cooperative and Non-cooperative	Two-stage	✓	-	Non- linear programming	Optimistic view	-	-
Wu et al.[13]	Cooperative and Non-cooperative	Two - stage	✓	✓	Non- linear programming	Optimistic view	-	✓
Zhou et al.[14]	Non-cooperative	Two - stage	-	-	Non- linear programming	Optimistic view	-	-
Du et al.[15]	Cooperative and Non-cooperative	Three - stage	-	-	Linear programming	Optimistic view	-	-
Badiezadeh et al.[34]	Cooperative	Three - stage	✓	✓	Linear programming	Double-frontier	-	-
Shabanpour et al.[40]	Cooperative	one-stage	-	-	Linear programming	Double-frontier	✓	-
Yousefi et al.[42]	Cooperative	Three - stage	✓	-	Non- linear programming	Optimistic view	✓	-
current paper	Non-cooperative	Three - stage	✓	✓	Non- linear programming	Double-frontier	✓	✓

Table 2. The inputs of the factory for 24 period in 2016

DMU	Production cost			Cost of reserving storage location	Cost of holding goods	Goods remaining from last period			Cost of Transport goods to delivery points
	x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_4^2	x_5^2	x_1^3
1	29120000	36160000	51520000	1700000	1430000	0	0	0	3680000

2	50960000	63280000	77280000	1700000	1430000	0	0	0	6235000
3	80080000	99440000	128800000	1700000	1430000	0	0	0	9915000
4	101920000	126560000	180320000	1700000	1430000	0	0	0	12880000
5	43680000	54240000	77280000	1700000	1430000	0	0	0	5520000
6	50960000	63280000	103040000	1700000	1430000	0	0	0	6645000
7	94640000	126560000	154560000	1700000	1670000	0	0	0	11755000
8	145600000	180800000	257600000	1700000	3620000	0	2	0	15435000
9	145600000	180800000	257600000	1700000	3170000	6	8	4	19115000
10	145600000	180800000	257600000	1700000	1730000	4	6	4	20555000
11	145600000	180800000	257600000	1700000	1430000	0	0	2	19220000
12	145600000	180800000	257600000	1700000	1430000	0	0	0	16815000
13	87360000	99440000	128800000	1700000	1430000	0	0	0	10290000
14	50960000	63280000	77280000	1700000	1430000	0	0	0	6235000
15	50960000	63280000	103040000	1700000	1430000	0	0	0	6645000
16	43680000	54240000	77280000	1700000	1430000	0	0	0	5520000
17	80080000	99440000	128800000	1700000	1430000	0	0	0	9915000
18	94640000	117520000	154560000	1700000	1430000	0	0	0	11755000
19	72800000	90400000	128800000	1700000	1430000	0	0	0	9200000
20	87360000	108480000	154560000	1700000	1430000	0	0	0	11040000
21	87360000	108480000	128800000	1700000	1430000	0	0	0	10630000
22	109200000	135600000	180320000	1700000	3830000	0	0	0	9915000
23	145600000	180800000	257600000	1700000	1430000	8	8	4	22080000
24	145600000	180800000	257600000	1700000	1430000	0	0	0	18400000

Table 3. The outputs and the intermediate measures of the factory for 24 period in 2016

DMU	Quantity of goods produced			Quantity of goods delivered			Cost of Transport goods to warehouse	Goods remaining for next period			Profit
	z_1^1	z_2^1	z_3^1	z_1^2	z_2^2	z_3^2	y_1^1	y_1^2	y_2^2	y_3^2	y_1^3
1	8	8	4	8	8	4	1960000	0	0	0	31800000
2	14	14	6	14	14	6	3310000	0	0	0	51110000
3	22	22	10	22	22	10	5270000	0	0	0	82910000
4	28	28	14	28	28	14	6860000	0	0	0	111300000
5	12	12	6	12	12	6	2940000	0	0	0	47700000
6	14	14	8	14	14	8	3550000	0	0	0	60190000

7	26	28	12	26	26	12	6460000	0	2	0	98810000
8	40	40	20	34	34	16	9800000	6	8	4	130610000
9	40	40	20	42	42	20	9800000	4	6	4	162410000
10	40	40	20	44	46	22	9800000	0	0	2	177380000
11	40	40	20	40	40	22	9800000	0	0	0	166880000
12	40	40	20	34	40	20	9800000	0	0	0	153510000
13	24	22	10	24	22	10	5430000	0	0	0	83640000
14	14	14	6	14	14	6	3310000	0	0	0	51110000
15	14	14	8	14	14	8	3550000	0	0	0	60190000
16	12	12	6	12	12	6	2940000	0	0	0	47700000
17	22	22	10	22	22	10	5270000	0	0	0	82910000
18	26	26	12	26	26	12	6250000	0	0	0	98810000
19	20	20	10	20	20	10	4900000	0	0	0	79500000
20	24	24	12	24	24	12	5880000	0	0	0	95400000
21	24	24	10	24	24	10	5640000	0	0	0	86320000
22	30	30	14	22	22	10	7230000	8	8	4	82910000
23	40	40	20	48	48	24	9800000	0	0	0	190800000
24	40	40	20	40	40	20	9800000	0	0	0	159000000

Table 4. Results of the maximum and minimum efficiencies of the first stage and k values

DMU	Second leader-follower scenario				Third leader-follower scenario			
	Optimistic View		Pessimistic View		Optimistic View		Pessimistic View	
	k_1	$\theta_o^{1F\text{-max}}$	k_1	$\varphi_o^{1F\text{-min}}$	k_1	$\theta_o^{1F\text{-max}}$	k_1	$\varphi_o^{1F\text{-min}}$
1	26	0.94507	0	1	26	0.94507	0	1.05
2	17	0.96910	0	1	17	0.9691	0	1.05
3	22	0.98283	1	1.06867	19	0.98283	1	1.06867
4	13	0.97940	0	1.17167	17	0.9794	5	1.17167
5	20	0.95193	1	1.03433	20	0.95193	0	1.05
6	18	1	2	1	18	1	0	1.05
7	16	1	2	1.05053	17	1	2	1.05053
8	0	1	2	1.08231	0	1	0	1.08231
9	0	1	0	1	0	1	0	1.05
10	9	1	0	1	9	1	0	1.05
11	14	1	10	1.08356	39	1	7	1.08356

12	6	0.99375	22	1.09597	26	0.99375	20	1.09597
13	20	1	2	1	17	1	0	1.05
14	17	0.9691	0	1	17	0.9691	0	1.05
15	18	1	2	1	18	1	0	1.05
16	20	0.95193	1	1.03433	20	0.95193	0	1.05
17	22	0.98283	1	1.06867	19	0.98283	1	1.06867
18	16	0.98970	3	1.103	17	0.9897	2	1.103
19	23	0.96567	0	1.103	20	0.96567	2	1.103
20	18	0.97253	0	1.13734	15	0.97253	3	1.13734
21	20	1	2	1.02199	19	1	0	1.05
22	0	1	0	1	0	1	0	1.05
23	38	1	0	1	40	1	0	1.05
24	6	1	3	1.27467	33	1	13	1.27467

Table 5. The efficiency values of stages in the three leader-follower scenarios from the optimistic approach

DMU	First leader-follower scenario			Second leader-follower scenario			Third leader-follower scenario		
	θ_o^L	θ_o^{1F}	θ_o^{2F}	θ_o^L	θ_o^{1F}	θ_o^{2F}	θ_o^L	θ_o^{1F}	θ_o^{2F}
1	1	0.94507	0.91195	1	0.68507	0.72563	1	0.68507	0.72563
2	1	0.9691	0.86468	1	0.7991	0.61713	1	0.7991	0.61713
3	1	0.98283	0.88133	1	0.79283	0.63265	1	0.79283	0.63265
4	1	0.9794	0.91057	1	0.8794	0.56568	1	0.8094	0.60449
5	1	0.95193	0.91168	1	0.75193	0.67144	1	0.75193	0.67144
6	1	1	0.97593	1	0.82	0.68081	1	0.82	0.68081
7	1	1	0.90633	1	0.87	0.58235	1	0.83	0.60559
8	1	1	0.92294	1	1	0.72931	1	1	0.72931
9	1	1	0.92576	1	1	0.68483	1	1	0.68483
10	1	1	0.92942	1	0.91	0.67609	1	0.91	0.67609
11	1	1	0.93714	1	0.76	0.50927	1	0.61	0.60542
12	1	0.99375	0.96025	1	0.82375	0.54205	1	0.73375	0.60002
13	1	1	0.87972	1	0.84	0.5945	1	0.83	0.60109
14	1	0.9691	0.86468	1	0.7991	0.61713	1	0.7991	0.61713
15	1	1	0.97593	1	0.82	0.68081	1	0.82	0.68081
16	1	0.95193	0.91168	1	0.75193	0.67144	1	0.75193	0.67144
17	1	0.98283	0.88133	1	0.79283	0.63265	1	0.79283	0.63265

18	1	0.9897	0.8856	1	0.8497	0.58492	1	0.8197	0.60045
19	1	0.96567	0.91112	1	0.76567	0.66846	1	0.76567	0.66846
20	1	0.97253	0.91085	1	0.82253	0.62146	1	0.82253	0.62146
21	1	1	0.87839	1	0.83	0.59195	1	0.81	0.60153
22	1	1	0.91345	1	1	0.79076	1	1	0.79076
23	1	1	0.91335	1	0.57	0.63178	1	0.6	0.6
24	1	1	0.93563	1	0.83	0.51003	1	0.67	0.60471

Table 6. The efficiency values of stages in the three leader-follower scenarios from the pessimistic approach

DMU	First leader-follower scenario			Second leader-follower scenario			Third leader-follower scenario		
	φ_o^L	φ_o^{1F}	φ_o^{2F}	φ_o^L	φ_o^{1F}	φ_o^{2F}	φ_o^L	φ_o^{1F}	φ_o^{2F}
1	1	1	1.00781	1	1	1.00781	1	1.05	1.05
2	1	1	1.00128	1	1	1.00128	1	1.05	1.05
3	1	1.06867	1.0182	1	1.07867	1.00708	1	1.07867	1.05
4	1	1.17167	1.00785	1	1.17167	1.00785	1	1.22167	1.05
5	1	1.03433	1.00781	1	1.04433	1.00781	1	1.05	1.05
6	1	1	1.04651	1	1.02	1.00966	1	1.05	1.05
7	1	1.05182	1.04226	1	1.07053	1.01118	1	1.07053	1.05
8	1	1.08231	1.04466	1	1.10231	1.00763	1	1.08231	1.05
9	1	1	1.00626	1	1	1.00626	1	1.05	1.05
10	1	1	1.00775	1	1	1.00775	1	1.05	1.05
11	1	1.08356	1.20643	1	1.18356	1.00223	1	1.15356	1.0558
12	1	1.09597	1.33712	1	1.31597	1.02494	1	1.29597	1.05
13	1	1	1.03943	1	1.02	1	1	1.05	1.05
14	1	1	1.00128	1	1	1.00128	1	1.05	1.05
15	1	1	1.04651	1	1.02	1.00966	1	1.05	1.05
16	1	1.03433	1.00781	1	1.04433	1.00781	1	1.05	1.05
17	1	1.06867	1.0182	1	1.07867	1.00708	1	1.07867	1.05
18	1	1.103	1.05095	1	1.133	1.00734	1	1.123	1.05
19	1	1.103	1.00783	1	1.103	1.00783	1	1.123	1.05
20	1	1.13734	1.00784	1	1.13734	1.00784	1	1.16734	1.05

21	1	1.02216	1.15101	1	1.04199	1.00791	1	1.05	1.05
22	1	1	1.00397	1	1	1.00397	1	1.05	1.05
23	1	1	1.00789	1	1	1.00789	1	1.05	1.05
24	1	1.27467	1	1	1.30467	1.00787	1	1.40467	1.05

Table 7. A comparison of the black box view and the three leader-follower scenarios

	Black box view			First leader-follower scenario			Second leader-follower scenario			Third leader-follower scenario		
	$\theta_o^{overall}$	$\varphi_o^{overall}$	$\emptyset_o^{overall}$	$\theta_o^{overall}$	$\varphi_o^{overall}$	$\emptyset_o^{overall}$	$\theta_o^{overall}$	$\varphi_o^{overall}$	$\emptyset_o^{overall}$	$\theta_o^{overall}$	$\varphi_o^{overall}$	$\emptyset_o^{overall}$
1	0.94507	1	0.97214	0.86185	1.00781	0.93198	0.49711	1.00781	0.7078	0.49711	1.1025	0.74031
2	0.9691	1	0.98442	0.83796	1.00128	0.91598	0.49315	1.00128	0.70269	0.49315	1.1025	0.73735
3	0.98283	1.06867	1.02485	0.86619	1.08811	0.97083	0.50159	1.08631	0.73816	0.50159	1.1326	0.75372
4	0.9794	1.17167	1.07122	0.89181	1.18086	1.02621	0.49746	1.18087	0.76644	0.48928	1.28275	0.79222
5	0.95193	1.03433	0.99227	0.86785	1.0424	0.95113	0.50488	1.05249	0.72895	0.50488	1.1025	0.74607
6	1	1.01845	1.00918	0.97593	1.04651	1.0106	0.55827	1.02986	0.75824	0.55827	1.1025	0.78453
7	1	1	1	0.90633	1.09626	0.99678	0.50665	1.0825	0.74057	0.50264	1.12406	0.75166
8	1	1	1	0.92294	1.13064	1.02152	0.72931	1.11073	0.90003	0.72931	1.13643	0.91038
9	1	1	1	0.92576	1.00626	0.96517	0.68483	1.00626	0.83013	0.68483	1.1025	0.86892
10	1	1	1	0.92942	1.00775	0.96779	0.61525	1.00775	0.78741	0.61525	1.1025	0.82359
11	1	1	1	0.93714	1.30723	1.10682	0.38705	1.18621	0.67758	0.36931	1.21794	0.67066
12	1	1.11741	1.05707	0.95424	1.46544	1.18253	0.44652	1.3488	0.77605	0.44027	1.36077	0.77401
13	0.98753	1	0.99374	0.87972	1.03943	0.95624	0.49938	1.02	0.71369	0.49891	1.1025	0.74165
14	0.9691	1	0.98442	0.83796	1.00128	0.91598	0.49315	1.00128	0.70269	0.49315	1.1025	0.73735
15	1	1.01845	1.00918	0.97593	1.04651	1.0106	0.55827	1.02986	0.75824	0.55827	1.1025	0.78453
16	0.95193	1.03433	0.99227	0.86785	1.0424	0.95113	0.50488	1.05249	0.72895	0.50488	1.1025	0.74607
17	0.98283	1.06867	1.02485	0.86619	1.08811	0.97083	0.50159	1.08631	0.73816	0.50159	1.1326	0.75372
18	0.9897	1.103	1.04481	0.87647	1.15919	1.00797	0.49701	1.14132	0.75315	0.49219	1.17915	0.76181
19	0.96567	1.103	1.03205	0.87984	1.11163	0.98897	0.51182	1.11164	0.75429	0.51182	1.17915	0.77686
20	0.97253	1.13734	1.05171	0.88582	1.14625	1.00766	0.51117	1.14626	0.76546	0.51117	1.22571	0.79154
21	1	1	1	0.87839	1.17651	1.01658	0.49132	1.05024	0.71833	0.48724	1.1025	0.73292
22	1	1	1	0.91345	1.00397	0.95764	0.79076	1.00397	0.89101	0.79076	1.1025	0.9337
23	1	1	1	0.91335	1.00789	0.95945	0.36012	1.00789	0.60246	0.36	1.1025	0.63
24	1	1.27467	1.12901	0.93563	1.27467	1.09207	0.42333	1.31494	0.74609	0.40516	1.4749	0.77302

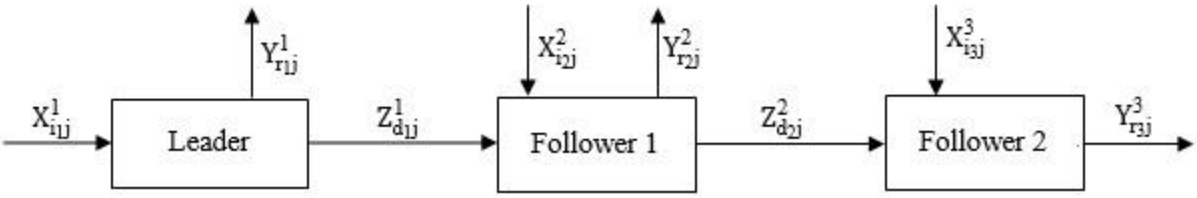

Fig. 1. Structure of three-stage leader-follower system with additional inputs and undesirable outputs

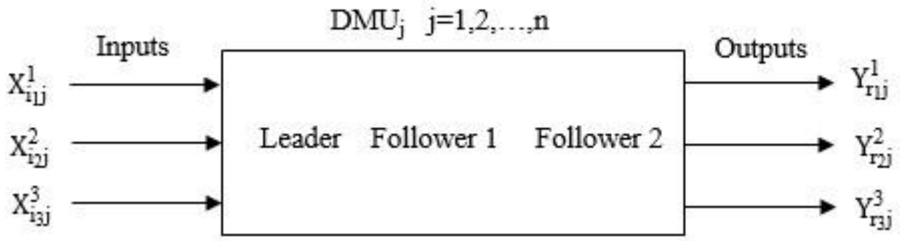

Fig. 2. Structure of a "black box" system

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