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Structural damage identification based on incomplete static responses as an optimization problem

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Abstract. Damage detection and estimation in structures using incomplete static responses are presented in this study. In the proposed approach, damage location and severity is determined by solving an optimization problem using a pattern search algorithm. Therefore, an objective function is formulated using incomplete static responses. Because of limitations in using sensors and difficulties in sensing all degrees of freedom, the effect of using incomplete responses has been evaluated. The performance of the proposed method was evaluated using three numerical examples, namely, a simply supported beam, a three-story plane frame, and a plane bridge with and without noise in measured displacement and containing one or several damages. The results indicate that the proposed method is effective and robust in the detection and estimation of damage in spite of the incomplete responses.

1. Introduction

Structural damage detection in civil and mechanical engineering structures during their service life has drawn wide attention during last few decades. Structural damage can be identified as a weakening of the structure that causes negative changes in its performance. Damage may also be considered as any change in the property of the material and the original geometry of the structure that creates undesirable stress, displacement and vibrations in the structure. Consequently, most damage detection methods are based on changes in dynamic characteristics and static responses [1].

Static responses are more sensitive to damage than dynamic responses [2,3], and the equipment used for static testing and for precise static displacement of structures can be obtained rapidly and economically [1]. However, there are two main drawbacks in the static damage identification methods: (1) Static testing provides less information compared to dynamic testing; (2) The effect of damage on static responses for damage detection may be cryptic due to limited load paths [1].

Some researchers have used static responses for the damage detection of structures. To identify damage error, force error and displacement error estimators for a static parameter grouping scheme, using least square minimization, was presented by Banan et al. [4]. Hjelmstad and Shin [3] proposed a data perturbation scheme for the baseline structure to establish the damage threshold between noise and the damaged structure to compare the damage indices. Hwu and Liang [5] used static strain measurement from multiple loading models for identification of the holes and cracks in linear anisotropy elastic materials with nonlinear opti-

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mization. Hajela and Soeiro [6] presented a damage detection algorithm based on static displacements, mode shapes, and frequencies. To solve an unconstrained optimization problem, an iterative non-linear programming method was developed. Paola and Bilello [7] proposed a damage identification procedure based on a least-square constrained nonlinear minimization problem for Euler-Bernoulli beams under static loads. Hwu and Liang [8] used static strain measurement from multiple loading models for identification of the hole and cracks in linear anisotropy elastic materials with nonlinear optimization. Yam et al. [9] proposed sensitivity analyses in static and dynamic parameter damage indices quantification for their identification capabilities over plate-like structures. Hua et al. [10] proposed a new damage detection procedure for cable-stayed bridges by changes in cable forces. Also, Lee et al. [11] developed a method using continuous strain data from fiber optic sensors and neural network models. Recently, Cao et al. [12] investigated the sensitivity of fundamental mode shapes and static deflection for damage identification in cantilever beams whose features are extremely similar in configuration.

In this paper, a new method for localizing and estimating the severity of structural damage is introduced. Damage identification is carried out through applying a pattern search algorithm to minimize the objective function derived from incomplete static characteristics of the damaged structure. Numerical examples show that the proposed method can be considered a flexible and robust approach to the damage identification of structures.

2. Proposed method

In this section, the proposed method for structural damage detection and estimation is illustrated. In the presented method, an objective function is formulated using a static residue force vector. Then, a pattern search optimization algorithm for minimizing the objective functions is presented.

2.1. Formulation of objective function

The static equilibrium equation of a structure in a displacement based finite element framework can be expressed as follows:

$$\lfloor \mathbf{K}^{ud} \rfloor \{\mathbf{x}\} = \{\mathbf{F}\}, \tag{1}$$

where $\mathbf{K}^{ud}$, $\mathbf{F}$, and $\mathbf{x}$ are undamaged stiffness matrix, and the force and displacement vectors, respectively.

One of the simplest techniques to determine damage-induced alteration stiffness is the degradation in Young's modulus of an element as follows:

$$E_j^d = E_j^{\mathrm{ud}}(1 - d_j), \tag{2}$$

where E_j^d and E_j^{ud} are the damaged and undamaged Young's modulus of the jth element in the finite element model, respectively, and d_j indicates the damage severity at the jth element in the finite element model whose values are between 0 for an element without damage and 1 for a ruptured element.

Moreover, it is assumed that no change will occur after damage in the mass matrix, which seems to be reasonable in most real problems.

From Eq. (1), the static equilibrium equation of a damaged structure can be obtained as:

$$\lfloor \mathbf{K}^d \rfloor \{\mathbf{x}^d\} = \{\mathbf{F}\}, \tag{3}$$

where superscript d is noted as the damage state. As the number of sensors used to measure static responses is normally limited and is usually less than the number of DOFs in the finite element model, the model reduction method should be used to match the incomplete measured static responses. In fact, not all displacements in $\mathbf{x}^d$ can be measured. Therefore, Eq. (3) is partitioned into the master and slave coordinates as follows:

$$\begin{bmatrix} \mathbf{K}_{mm}^d & \mathbf{K}_{ms}^d \\ \mathbf{K}_{sm}^d & \mathbf{K}_{ss}^d \end{bmatrix} \begin{Bmatrix} \mathbf{x}_m^d \\ \mathbf{x}_s^d \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m \\ \mathbf{F}_s \end{Bmatrix}, \tag{4}$$

in which the subscripts m and s are the master and slave coordinates, respectively. The vector of slaved displacements, x_s^d, is condensed out following static condensation, and Eq. (3) reduces to the following:

$$\left[\mathbf{K}_r^d\right] \{\mathbf{x}_m^d\} = \{\mathbf{F}_r\}, \tag{5}$$

where:

$$[\mathbf{K}_r^d] = \left([\mathbf{K}_{mm}^d] - [\mathbf{K}_{ms}^d][\mathbf{K}_{ss}^d]^{-1}[\mathbf{K}_{\mathbf{sm}}^{\mathbf{d}}]\right)\{\mathbf{x}_{\mathbf{m}}^{\mathbf{d}}\}, \tag{6}$$

$$\{\mathbf{F}_r\} = \{\mathbf{F}_m\} - [\mathbf{K}_{ms}^d][\mathbf{K}_{ss}^d]^{-1}\{\mathbf{F}_m\}, \tag{7}$$

in which $\mathbf{K}_r^d$ and $\mathbf{F}_r$ are the condensed stiffness matrix and the condensed load vector of the damaged structure, respectively.

Finally, the objective function is formed as a static residue force vector as follows:

$$f(d) = \|(F_r - K_r^d X_m^d)\|^2$$
$$0 \le d_1 \le 1,\ \ 0 \le d_2 \le 1,\ \ ...,\ \ 0 \le d_{N_e} \le 1, \tag{8}$$

where $\|\ \|$ represents the Euclidean length, and N_e is the number of elements.

2.2. Optimization using pattern search method

The pattern search method is a subclass of direct search methods that was first introduced in the 1950s [13].

However, in 1991, there was a growth of interest in the direct search method. Since then, two things have become increasingly clear [14]:

1. Direct search methods stay an effective option, and sometimes the only choice for several varieties of difficult optimization problems.
2. For a large number of direct search methods, it is possible to provide a thorough guarantee of convergence.

The pattern search method is a derivative-free method for solving a variety of optimization problems, where typical optimization methods are not so effective. The main idea of this procedure is to generate a sequence of iterates that considers the behavior of the objective function at a pattern of points, all of which lie on a logical lattice without utilizing any information about derivatives, including gradient and second-order derivatives of the objective function.

The pattern search method can be briefly explained in such a way that starts by establishing a set of points, called mesh, around a given point, which can be computed from previous steps of the iteration or from the initial starting point provided by the user. The mesh is created by adding a scalar multiple set of vectors, called a pattern, to the current point, which, then, searches a set of points (mesh) around the current point of the parameters to find a point where the objective function has a lower value. After a point with a lower objective function value is detected, the algorithm sets the point as its current point and the iteration can be considered successful. Then, the algorithm goes on to the next iteration with extended mesh size, which is induced by an expansion factor. If the algorithm does not find a point that improves the objective function, the iteration is called unsuccessful. The current points stay the same in the next iteration and the mesh size decreases due to the contraction factor [15]. The pattern search optimization algorithm stops when any of the following situations occur [16]:

- The number of iterations or the evaluation of the objective function reaches the max value.
- The mesh size becomes less than mesh tolerance.
- The distance between two successful points obtained in two consecutive iterations is less that the given tolerance.
- Alteration in the improvement of the objective function is less than the function tolerance.

The optimization problem is formulated as a minimization of the objective function. The pattern search method is applied to Eq. (8) to find an optimal solution using incomplete static responses, which leads to localizing and quantifying damage. Figure 1 shows the flowchart of the proposed method for estimation and localization of the damage via a pattern search method.

3. Verification examples

In this section, the efficiency and effectiveness of the proposed method is evaluated through some numerically damaged identification examples, using incomplete static responses. A simply supported beam, a three-story plane frame, and a plane bridge are chosen, with three different scenarios of damage for each of them for this purpose.

3.1. Simply supported beam

A simply supported beam, as illustrated in Figure 2, with a finite-element model consisting of 10 beam elements and 11 nodes is considered. For the considered concrete beam, the material properties include a Young's modulus of $E = 25$ GPa, and mass density of $\rho = 2500$ kg/m^3. The cross-sectional area and the second moment of inertia of the beam are $A = 0.12$ m^2 and $I = 0.0016$ m^4, respectively.

In this example, three damage scenarios are represented as elements with a reduction in Young's modulus. The damage severity in each element is given by the reduction factor listed in Table 1. In this case, only 9 translational DOFs are selected as measured DOFs.

Damage in the beam can be determined by using the proposed method. The pattern search method input parameters adopted for the following analyses are summarized in Table 2.

To be more suited to real cases, an examination has been performed in which the measured displace-

Table 1. Damage scenarios for the simply supported beam.

Scenario 1		Scenario 2		Scenario 3	
Element 6	50%	Element 1	35%	Element 1	45%
		Element 7	50%	Element 6	50%
				Element 9	20%

Table 2. Input parameters for the pattern search method.

Maximum iteration	200-20000
Maximum function evaluations	40000-100000
Bind tolerance	0.001
X tolerance	1.00E-20
Function tolerance	1.00E-20
Nonlinear constrain tolerance	1.00E-20
Expansion factor	2
Contraction factor	0.5
Mesh tolerance	1.00E-20

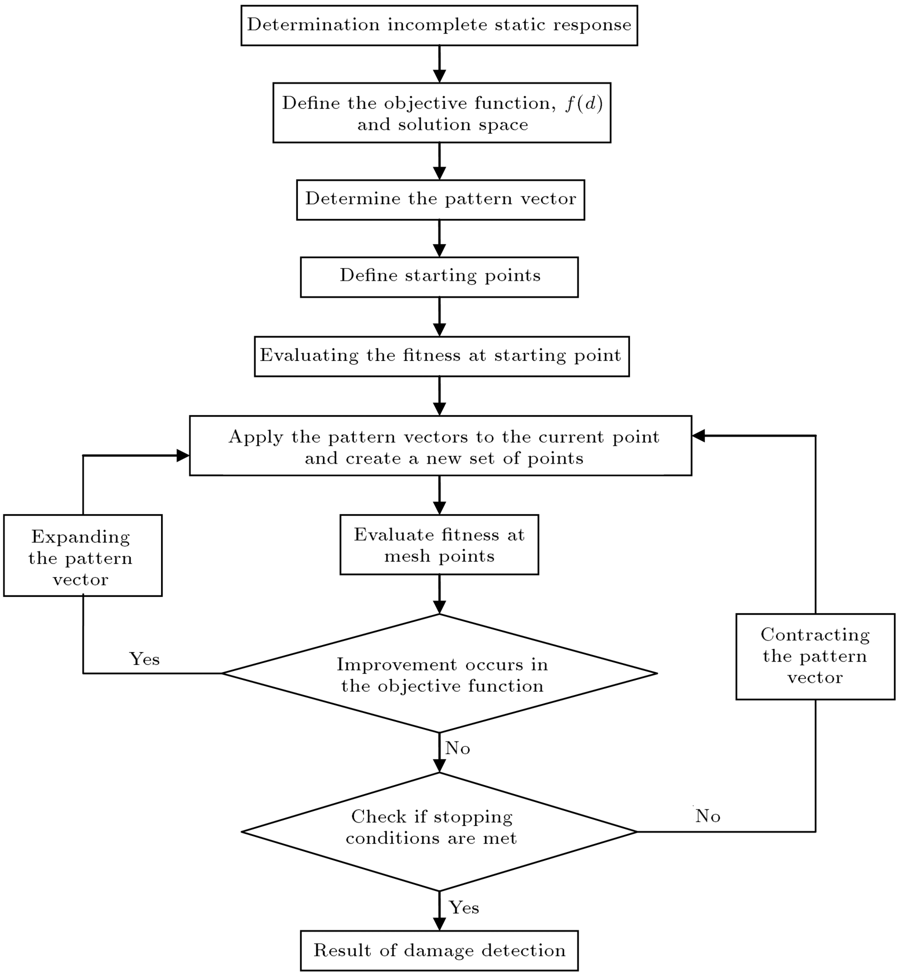

Figure 1. Flowchart of the damage detection method using the pattern search method.

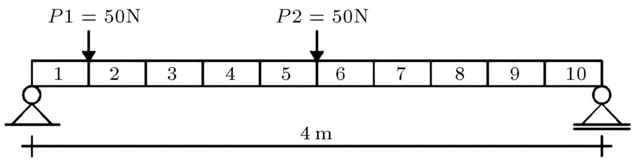

Figure 2. The simply supported beam with the finite element model.

ments of the damaged structure with 2% noise are utilized for damage identification considering the same patterns mentioned before. To perform this, some random noise has been added to the theoretically calculated measured displacements. The contaminated displacement with noise can be obtained from the displacement without noise using the following equation:

$$(x_m^d)_{\text{noisy}} = (x_m^d)(1 + \beta \text{ rand}[-1, 1]), \tag{9}$$

where $(x_m^d)_{\text{noisy}}$ and (x_m^d) are the measured displacements of the damaged structure contaminated with noise and without noise, respectively; β is the noise level (e.g., 0.02 relates to a 2% noise level), and rand is a random number in the range [-1 1].

The obtained results of damage detection and quantification using the proposed objective function, which are based on incomplete static responses of the structure, are shown in Figure 3. The results show that the proposed method is robust and promising in localizing and quantifying different damage scenarios.

3.2. Three-story plane frame

A three-story plane steel frame, as illustrated in Figure 4, with a finite-element model consisting of nine elements (six columns and three beams) and six free nodes is considered. For the considered steel frame, the material properties of the steel include a Young's modulus of $E = 200$ GPa, and mass density of $\rho = 7850$ kg/m^3. The mass per unit length, the second moment of inertia, and cross-sectional area of the columns are: $m = 117.75$ kg/m, $I = 3.3 \times 10^{-4}$ m^4 and $A = 1.5 \times 10^{-2}$ m^2, respectively. For the beams, they are: $m = 119.71$ kg/m, $I = 3.69 \times 10^{-4}$ m^4 and $A = 1.52 \times 10^{-2}$ m^2. Also, the damage severity in each element is given by the reduction factor listed in Table 3. In this case, only 6 translational DOFs are

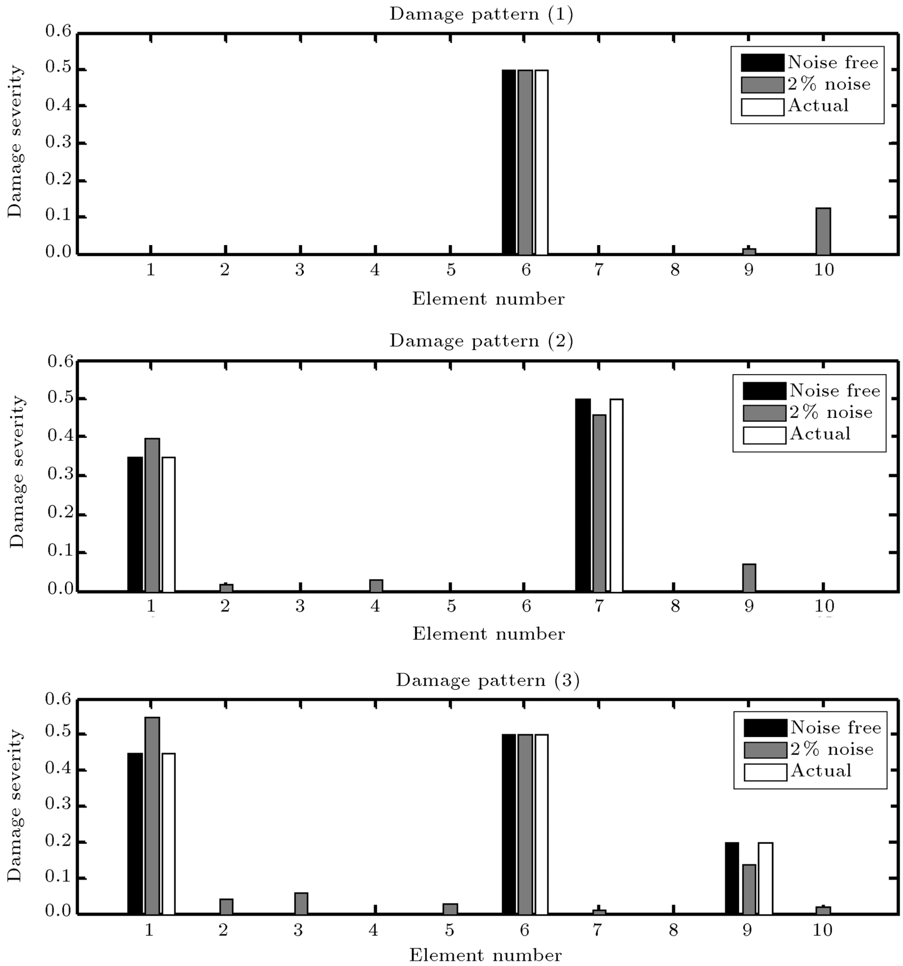

Figure 3. The obtained results for three damage patterns of the simply supported beam.

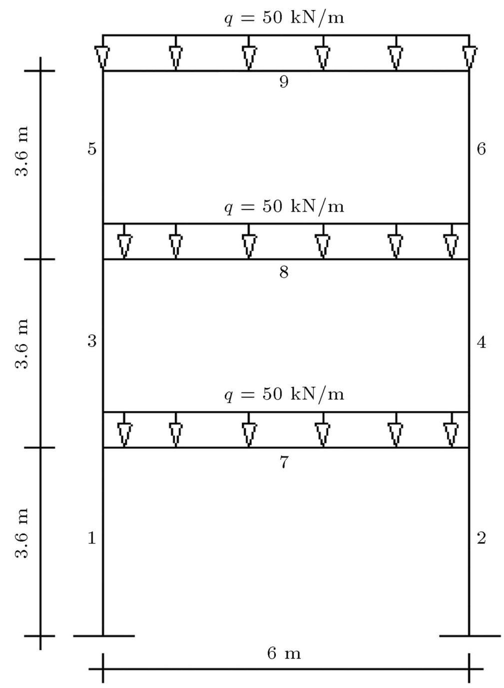

Figure 4. The three-story plane frame with the finite element model.

Table 3. Damage scenarios for the three-story plane frame.

Scenario 1		Scenario 2		Scenario 3	
Element 5	15%	Element 1	30%	Element 1	40%
		Element 7	20%	Element 6	30%
				Element 8	45%

selected as measured DOFs in the process of damage detection and quantification.

Figure 5 shows the identified damaged elements using the proposed objective function. It can be seen that the damage severity and locations can be obtained correctly for the three different scenarios.

3.3. Plane steel bridge

A plane steel bridge, as illustrated in Figure 6, with a finite-element model consisting of elements 1 to 4, as beam-column elements, 5 to 8 as beam elements, and 9 to 11 as column elements is considered. A uniformly distributed load of 50 kN/m has been used on the beam elements. For the considered steel bridge, the material properties of the steel include a Young's modulus of $E = 200$ GPa. The cross-sectional area of the columns

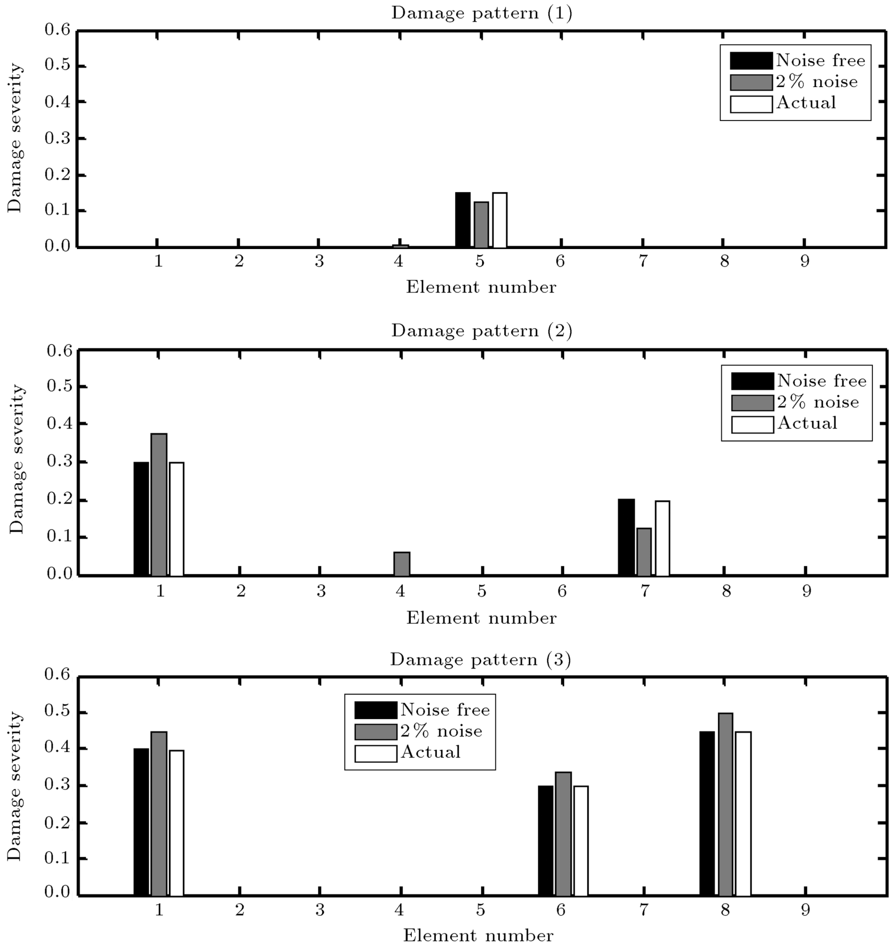

Figure 5. The obtained results for three damage patterns of the three-story plane frame.

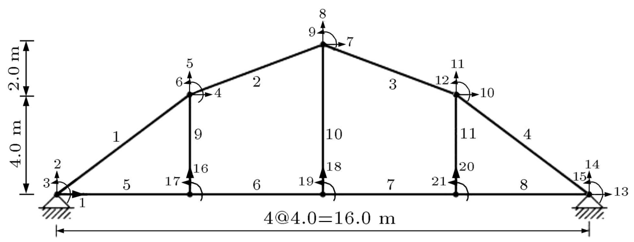

Figure 6. The plane bridge with the finite element model.

is $A = 150 \times 10^{-4}$ m^4. The second moment of inertia and the cross-sectional area of the beam-columns are $I = 189813.3 \times 10^{-8}$ m^4 and $A = 280 \times 10^{-4}$ m^2, respectively. The moment of inertia of the beams is $I = 189813.3 \times 10^{-8}$ m^4. Also, the damage severity in each element is given by the reduction factor listed in Table 4. In this case, only the last 6 DOFs of the bridge are selected as measured DOFs in the process of damage detection and quantification.

The proposed method was applied to detect the damage in the plane steel bridge. Figure 7 shows the capability of the proposed method for detection and estimation of damage in the plane steel bridge for three different damage patterns. The obtained results for damage detection present a good agreement between actual and estimated damage in the structure. The results indicate that the proposed method can be characterized as a robust and viable method for damage detection in bridge structures.

Table 4. Damage scenarios for the plane bridge.

Scenario 1		Scenario 2		Scenario 3	
Element 2	20%	Element 5	15%	Element 2	30%
		Element 10	35%	Element 5	45%
				Element 10	35%

4. Conclusions

In this paper, a method was developed for the detection and estimation of damage in structures on the basis of the incomplete static responses of the damaged structure using an optimization problem. In this method, a pattern search algorithm was used to determine the damage in structures by optimizing a cost function.

For damage detection and estimation, the proposed method was applied to three different problems,

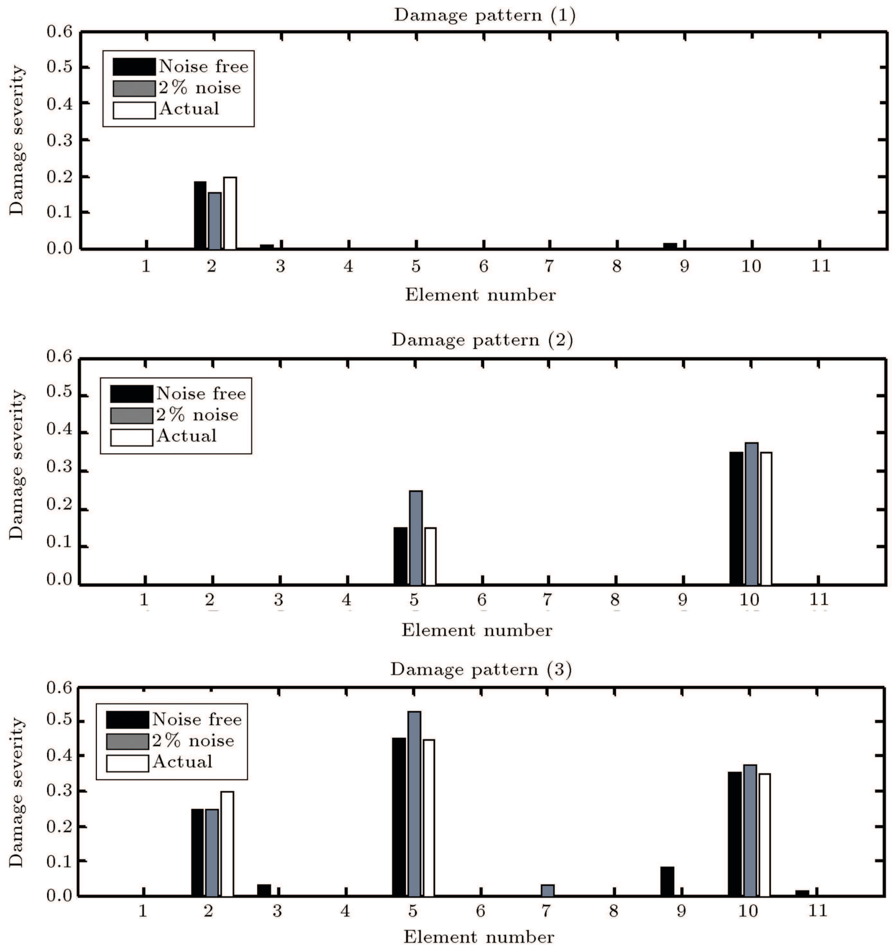

Figure 7. The obtained results for three damage patterns of the plane bridge.

namely, a simply supported concrete beam, a three-story plane steel frame, and a plane steel bridge, with and without noise in measured displacement and containing one or several damages. The obtained results indicate that the proposed method is a strong and viable method in the problem of detection and estimation of damage in structures. The results revealed the high sensitivity of the proposed method to damage, in spite incomplete measurements.

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Mohsen Ghafory-Ashtiany received BS, MS and PhD degrees (honors) in Structural Dynamics and Earthquake Engineering from VA. Tech, USA. He founded the International Institute of Earthquake Engineering and Seismology (IIEES) in Iran in 1989, and was its president until December 2006. He is the author of more than 250 papers, 3 books and 40 research reports in the field of seismic hazard and risk analysis, risk modeling, risk management, risk reduction policy development, socio-economic vulnerability and resilience, and as well as random vibration, earthquake engineering, and strong ground motion seismology. He was Chief Editor of JSEE and Chairman of the International Conference on Seismology and Earthquake Engineering from 1991-2007. He is now Co-Editor of IDRiM Journal, Editor of the Earthquake Engineering Journal (EEJ) and member of the Editorial Board of many other journals. He is one of the founders of the Iranian Earthquake Engineering Association (IEEA) and a pioneer in Risk mitigation activities in Iran. He has served as a member of Iran's Natural Disaster Prevention and Management Headquarters, Iran's Risk Reduction Commission, Iran Scientific Research Council, and the National Building Code Council, etc. Currently, he is Professor of Earthquake Engineering and Risk Management at IIEES, Adjunct Professor and Associate Director of the World Institute of Disaster Risk Management in Virginia Tech (USA), since 2012, Affiliate Member of Iran Academy of Sciences, Chairman of SP Insurance Risk Management Institute. President of IEEA, Director of the International Association of Earthquake Engineering (IAEE), Chairman of Earthquake Strong Ground Motion, Hazard and Risk Commission of IASPEI, member of UNISDR Science and Technical Commission, and member of EERI, ASCE, IASBEI, GEM, EMME, etc.