A game theoretic approach to coordinate pricing, ordering and co-op advertising in supply chains with stochastic demand

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Abstract

This paper combines the newsboy problem with the cooperative advertisement problem in the presence of uncertain demand which depends on retail price as well as both local and national advertising expenditures to coordinate pricing, ordering, and advertising decisions in a manufacturer-retailer supply chain. A game theoretic approach is adopted to determine the equilibrium values of the decisions. Three different game scenarios based on the newsboy problem model are developed and analyzed: 1) Stackelberg manufacturer game in which manufacturer as the dominant power plays the role of leader in the market and the follower retailer makes its own best decisions after observing the leader decisions, 2) Nash game wherein both manufacturer and retailer have equal power in the market and make their decisions simultaneously to find their own best strategies and 3) centralized scenario in which retailer and manufacturer make the best decisions by information sharing and joint cooperation. The equilibrium decisions are obtained exactly in the three scenarios. Some corollaries are also presented and theoretically proved to show the relationships among the variables in centralized vs. decentralized supply chain. Finally, some numerical examples are randomly generated and a sensitivity analysis is carried out to show capabilities of the proposed models.

Keywords: supply chain coordination, newsboy problem, pricing, cooperative advertising, ordering, uncertainty, game theory.

1- Introduction

The newsboy problem is a mathematical model to optimize the inventory levels. The supply chain outlook, the need for expanding into new markets and the decline in demand has forced production and operations management to explore profit of the entire supply chain and make coordinated ordering decisions among all supply chain members. In this regard, pricing and advertising are effective tools for making coordination among the separate property companies as independent members. Decisions in a supply chain can be made in either centralized or decentralized manner. In a centralized structure, a central authority is responsible for coordinating supply chain activities to optimize the entire supply chain performance whereas in a decentralized scheme, the different entities compete with each other to improve their individual performance. In the latter case, making coordinated decisions is an imperative issue which is

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usually dealt with a game theoretic approach. In the decentralized arrangement, different members may be able to simultaneously optimize both their individual and the whole supply chain performance through coordinating their strategies.

Cooperative advertising between manufacturer and retailer is an interactive relationship in which the manufacturer pays a proportion of the advertisement expenditures in order to achieve higher potential market share, build brand equity and create motivation at the retail level while the retailer pays for local advertising costs in order to increase the local demand [1]. In fact, each member undertakes to pay a part of advertising costs for different objectives. Berger [2] carried out the first study on a two-member supply chain wherein a retailer was given a discount on the wholesale price from a manufacturer as a cooperative advertisement. Thenceforth extensive researches are conducted on cooperative advertising in two-echelon supply chains. Aust and Buscher [3] conducted a comprehensive review of articles published on cooperative advertising in supply chains since 40 years ago. In addition to retail advertising expenditures, cooperation rate with the aim of maximizing total supply chain profit and making coordination among members has been an important issue. In this regard, Huang and Li [4] included investment in advertising as well as local and national advertising expenditures in their model while taking bargaining and members' risk attitudes into account. They addressed the shift of power from wholesaler to retailer in new markets like Walmart and developed three game models including a Stackelberg manufacturer game, a Stackelberg retailer game and a cooperative game to examine the cooperative advertising efficiency with respect to transactions between wholesaler and retailer through brand name investments.

Pricing problem, that includes determining retailer and wholesaler prices, is another important decision in supply chain coordination. In most studies regarding advertisement decisions, it is assumed that demand is only a function of advertising expenditures which is not influenced by retail price. However, making pricing decisions is the core task in marketing and distribution channels where pricing and advertising costs conflict with each other in the way of attaining supply chain coordination [5]. With the increasing role of game theory in analyzing supply chain approach, a great body of literature has investigated the role of advertising and pricing in the balance of power among members using game theoretic approaches. Karray and Amin [6] considered cooperative advertising in a network of competing retailers. In their work, the effects of cooperative advertising under two game scenarios (with and without cooperation) were evaluated while both price and advertising costs were considered as decision variables. They showed that in supply chains when retailers are in competition for low price levels and high advertising, the cooperative advertising may not be profitable for the retailer or the network. Yan et al. [7] studied the value of manufacturer's cooperative advertising and its strategic impact on information sharing between the manufacturer and the retailer under demand uncertainty. They indicated that information had a remarkable effect on both retailer and manufacturer decisions for investing in advertisement. Thus, they concluded that using an advertising agency could contribute to better information sharing and eliminating information distortion while helped both members make the optimal investing decisions. Jorgensen and Zaccour [8] studied game-theoretic models of cooperative advertising in both static and dynamic environments. Their research included two main parts. In the first part, a simple manufacturer-retailer supply chain is studied while in the second part, more complex supply chains with more than one supplier and retailer with horizontal interactions are investigated. Alirezae and Khoshalhan [9] investigated optimal pricing, optimal advertising expenditures and cooperative advertising decisions in a two-echelon supply chain. They employed Nash game, Stackelberg manufacturer game, Stackelberg retailer game and cooperative game for analyzing the supply chain. Finally, the best settings for achieving coordination was determined using a bargaining model.

There are several researches on the joint area of pricing and advertising decisions. He et al. [10] developed a new contract scheme in a two-echelon supply chain when demand is influenced by both retail price and advertising costs. The proposed scheme was a combination of return policy and revenue sharing contract embodied sales rebate and penalty. They showed that in the proposed situation achieving win-win situation is possible. Szmerekovsky and Zhang [11] studied pricing decisions in a manufacturer-retailer supply chain when demand was stochastic and was influenced by both retail price and advertisement. The model was then analyzed by three games including Stackelberg, Nash and cooperative game. The obtained results indicated that in a two-tier distribution channel, local advertising costs was not appropriate for making decisions. Instead, national advertisements by the manufacturer along with a wholesale price discount to the retailer could produce superior results. Xie and Neyret [12] and Xie and Wei [5] followed the same approach and analyzed the effects of changing price and advertising functions under Stackelberg, Nash and cooperative games. Chen [13] examined the effects of advertisement with return policy for a sales problem in a manufacturer-retailer supply chain to determine the values of inventory and advertising decision variables in both cooperative and non-cooperative situations. SeyedEsfahani et al. [14] investigated the role of coordination in a two-echelon supply network with advertising and price dependent demand using four game scenarios of cooperative game, Nash equilibrium, Stackelberg manufacturer game and Stackelberg retailer game. The authors concluded that in the cooperative condition, advertisements play a strategic role in making critical decisions. A non-linear demand function was applied in the model. Dridi and Youssefi [1] considered a supply chain with competing retailers and a demand dependent on both price and national advertising expenditures. Three games including full cooperative, parallel cooperative and non-cooperative games were developed to assess the surplus of customer demands. Also, they considered a new linear additive form of demand function in a cooperative advertising supply chain consisting of a monopolistic manufacturer and two competing retailers. In order to maximize total profit, three game scenarios were presented and the profits were compared. Naimi Sadigh et al. [15] took into account a multi-echelon distribution channel consisting of multiple suppliers, one manufacturer and several retailers while demand is affected by both price and advertisement. They used a non-cooperative Nash game to obtain the optimal values of decision variables including economic order quantity, price, inventory level, and advertising expenditures.

There are many articles that study the coordination of pricing and advertising decisions, albeit few of them have explored the uncertainty. Choi [16] suggested that changing the demand function can yield different outcomes. Aust [17] analyzed cooperative advertising model and pricing decisions by four game theoretic scenarios including Nash game, Stackelberg manufacturer game, Stackelberg retailer game and cooperative game. The authors extended the existing models and took the role of bargaining and risk into account. It was shown that in the cooperative condition total profit is maximized. Moreover, they showed that customer satisfaction could be enhanced by decreasing price and increasing advertising expenditures provided that a coordinated structure between the manufacturer and retailer be present. Ke et al. [18] studied pricing competition in a two-echelon supply chain consisting of one manufacturer and two competing retailers while the manufacturing costs, sales costs, and market bases are all characterized as uncertain fuzzy variables. They investigated the equilibrium behaviors of the supply chain members in three decentralized game models.

<<Insert Table 1 >>

Table 1 compares the general settings of this paper with the related literature. As can be seen in table 1, the combination of newsboy problem and cooperative advertisement, to determine joint ordering and advertising decisions, has been rarely studied in the related literature (e.g. [20] and [21]). Amirtaheri et al. [20] studied a joint pricing, ordering and cooperative advertising problem in a bi-level decentralized supply chain consisting of one manufacturer and one distributer considering a deterministic demand function. Zhou et al. [21] addressed a joint cooperative advertising and ordering problem in a two-echelon supply chain consisting of a risk-averse leader manufacturer and a risk-averse follower retailer. They assumed a random demand which depends on the manufacturer's global advertising as well as the retailer's local advertising. However, their proposed demand function has not considered the pricing impacts. The simultaneous relationship between the manufacturer and the retailer was also neglected. The main contribution of this paper is combining the newsboy problem and the cooperative advertisement problem considering stochastic demand to coordinate ordering, pricing, and advertising decisions in a two-echelon supply chain. For this purpose, we present a new additive demand function which depends on retail price as well as both local and national advertising expenditures and affected by a random factor with a uniform distribution. Three different scenarios are analyzed: 1- the centralized scenario, in which the retailer and the manufacturer cooperate and make the global best decisions, 2- the manufacturer-leader Stackelberg scenario in which the manufacturer as the dominant power plays the role of the leader and makes his decisions first. Then the retailer as the follower choose his own best strategies after observing the leader decisions, 3- the Nash game scenario wherein both the manufacturer and the retailer have the same power and make their decisions simultaneously. The rest of the paper organized as follows. In section 2, the main assumptions of the under studied problem are stated and the notations are introduced. The problem under centralized channel scenario is modeled in section 3 and the optimal decisions are calculated. Section 4 presents the mathematical models of the problem under two decentralized channel scenarios and the equilibrium decisions are obtained. In section 5, some numerical examples are randomly generated and the results are discussed. A sensitivity analysis is also carried out on main parameters of the proposed demand function. Eventually, section 6 is devoted to concluding remarks.

2- **Problem Definition**

Consider a single-product two-echelon supply chain including one manufacturer and one retailer. Demand is dependent on both price and advertising expenditures and the retailer is able to boost product demand by price discount offers and/or advertisement. On the contrary, advertisement expenditures reduce the retailer profit which is a demotivating factor from the retailer’s viewpoint. As a result, the manufacturer undertakes to pay for a proportion of advertising costs in order to inspire the retailer to advertise its products and stimulate product demand throughout the chain that can improve the retailer's profit margin as well. Moreover, it is assumed that demand is stochastic. In other words, demand not only is influenced by both pricing and advertising decisions but also is affected by a stochastic factor.

The problem concerned in this study takes into account the assumptions of newsboy model. The objective is to determine the optimal values of key decision variables in a two-echelon supply chain for which establishing coordination between members is sought. The key variables include economic order quantity, the manufacturer's and retailer's participation rates in advertising expenditures and both the retail price and the wholesale price. In the newsboy problem, before the sales period, the retailer decides on retail price, local advertising costs, and its economic lot size while the manufacturer determines the national advertising expenditures and the wholesale price.

2-1- Assumptions

The following assumptions are considered for the manufacturer's and retailer's models:

1- The channel produces a single commodity.
2- The demand of the commodity is considered as a function which depends linearly on retail price and nonlinearly on local and national advertising expenditures.
3- The demand is stochastic.
4- Ordering decisions follow the assumptions of newsboy problem.
5- The planning horizon is single period and the surplus inventory at the end of this period has no salvage value.
6- No shortage is allowed.

2-2- Notations

The following notations are used to formulate the problem.

$F(x)$	Cumulative distribution function of demand
$f(x)$	Probability density function of demand
μ	Expected value of demand
c	Unit production cost
q	Ordering quantity
p	Retail price
n	Manufacturer's advertising expenditure
e	Retailer's advertising expenditure
$D(p,n,e,\varepsilon)$	Stochastic demand function

$h(n,e)$	Advertising dependent definite function
$d(p)$	price dependent definite function
ε	Stochastic variable
z	Stocking factor
$\Theta(z)$	Expected value of shortage
π_s	Manufacturer's benefit
π_r	Retailer's benefit
π_c	Supply chain benefit

It is assumed that demand function is an additive stochastic function which is dependent on price and both local and national advertising expenditures and is defined as follows.

$$D(p,n,e,\varepsilon)=d(p)+h(n,e)+\varepsilon \qquad (1)$$

Where, $d(p)$ shows demand dependency on retail price and is defined as a linear function $d(p)=a-bp$ in which a is the market scaling factor, b is the price elasticity of demand coefficient and $h(n,e)$ represents the dependency between advertising expenditures and demand and is given by $h(n,e)=k_1\sqrt{n}+k_2\sqrt{e}$ as a nonlinear function where k_1 is the demand to manufacturer advertising costs elasticity coefficient and k_2 is the demand to retailer advertising costs elasticity coefficient. ε is a random variable with uniform distribution function in $[\mathrm{A,B}]$ interval.

According to the notations and problem assumptions explained above, in the following sections, the problem is modeled and the optimal values of decision variables are explored for three different power scenarios.

3- Centralized Channel

In this scenario, the manufacturer and the retailer completely cooperate together and form an integrated centralized supply chain. In this condition, the global optimum solution of the system is obtained by maximizing total profit of the entire supply chain while no motivation obstacle is in the way of obtaining the solution. The profit function of the entire supply chain is formulated as follows:

$$\pi_c(q,p,n,e)=pE[\min(D,q)]-cq-n-e \qquad (2)$$

$$(q,p,n,e)\in\arg\max\ \pi_c$$

Where, $E[\min(D,q)]$ indicates the expected retail sales. In order to calculate the expected sales value, via an approach similar to the work of [22], the following stocking factor z is initially defined by variable change:

$$z = q - d(p) - h(n,e) \tag{3}$$

Now, using the variable change, the set of decision variables (q, p, n, e) is converted to the set of variables (z, p, n, e) and the expected sales value is obtained as a function of the stochastic demand:

$$\begin{aligned} &E\left[\min(D,q)\right] = E\left[\min(d(p)+h(n,e)+\varepsilon, z+d(p)+h(n,e))\right] = \\ &= E\left[\min(z,\varepsilon)\right] + d(p) + h(n,e) \end{aligned} \tag{4}$$

Such that $E\left[\min(z,\varepsilon)\right]$ is computed as follows:

$$E\left[\min(z,\varepsilon)\right] = \int_A^Z xf(x) + \int_z^B zf(x)dx = \int_A^B xf(x)dx - \int_Z^B (x-z)f(x)dx \tag{5}$$

$$E\left[\min(z,\varepsilon)\right] = \int_A^Z xf(x) + \int_z^B zf(x)dx = \int_A^B xf(x)dx - \int_z^B (x-z)f(x)dx = \mu - \Theta(z) \tag{6}$$

where $\Theta(z)$ is given by:

$$\Theta(z) = \int_z^B (x\text{-}z)f(x)dx \tag{7}$$

As a result, the profit function of the entire supply chain in the centralized scenario is rewritten like this:

$$\begin{aligned} &\pi_c(p,z,n,e) = p(E[\min(z,\varepsilon)] + d(p) + h(n,e)) - cq - n - e \\ &= p(E[\min(z,\varepsilon)] + d(p) + h(n,e)) - c\left[z + d(p) + h(n,e)\right] - n - e \end{aligned} \tag{8}$$

Lemma 1: In the centralized situation, the supply chain’s total profit function is pseudo-concave.

Proof: See Appendix A.

The optimal values of manufacturer’s and retailer’s decision variables are calculated through taking derivatives of the profit function.

$$\frac{\partial \pi_c}{\partial z} = 0 \Rightarrow F(z) = 1 - \frac{c}{p} \tag{9}$$

$$\frac{\partial \pi_c}{\partial p} = 0 \Rightarrow p = \frac{\mu - \Theta(z) + a + h(n,e) + cb}{2b} \tag{10}$$

Then, through solving a two-equation system with two variables considering equations (9) and (10) for the given z and p, the optimal value z^* is obtained as follows:

$$\left[\mu-\Theta(z*)+a\right]\left[1-F(z*)\right]+\left[\frac{k_1^2+k_2^2}{2}\right]\times\left[cF(z*)\right]+cb\left[1-F(z*)\right]-2cb=0 \tag{11}$$

Since equation (11) holds for optimal value of z^*, the optimal values of the other decision variables including retail price, lot size, the retailer's local advertising expenditures and the manufacturer's national advertising costs could be obtained. The optimal retail price is given in equation (12):

$$p_c^*=\frac{c}{1-F(z^*)} \tag{12}$$

When partial derivative of the total profit function with respect to local advertising variable equals zero, the retailer's local advertising expenditures is calculated as follows:

$$\frac{\partial\pi_c}{\partial e}=0\Rightarrow \quad e_c^*=\left[\frac{k_2(p^*-c)}{2}\right]^2 \tag{13}$$

The optimal value of the manufacturer national advertising expenditures can be calculated through the same approach:

$$\frac{\partial\pi_c}{\partial n}=0\Rightarrow \quad n_c^*=\left[\frac{k_1(p^*-c)}{2}\right]^2 \tag{14}$$

Finally, the optimal value of the ordering lot size is:

$$q_c^*=z^*+d(p)+h(n,e)=z^*+\left[a-bp^*+k_1\sqrt{n^*}+k_2\sqrt{e^*}\right] \tag{15}$$

4- Decentralized Channel

In this section, we study the problem in a decentralized channel where manufacturer and retailer decide on their strategies regarding to their individual profits. Two different scenarios are considered: The Stackelberg manufacturer-leader game and the Nash equilibrium game.

4-1- Stackelberg Manufacturer-Leader Equilibrium

In this scenario, the manufacturer as the leader makes its own decisions in the beginning. Subsequently, the retailer as the follower chooses the best reaction to the manufacturer's strategy and decides on the variables under its control. Hence, the problem model in the Stackelberg game scenario is a bi-level programming model while the manufacturer sub-model and the retailer sub-model are in the higher and the lower levels, respectively.

$$Max \quad \pi_s(w,n) = (w-c)q - n \tag{16}$$

$$s.t. \quad (p,q,e) \in \arg\max\ \pi_r = p\ \ E(\min(D(p,n,e),q)\ \ - wq - e$$

In order to solve the obtained bi-level programming model in the Stackelberg manufacturer game scenario, the optimality conditions of the lower level model are primarily added to the upper level model's constraints space and then the profit function of the leader player is maximized considering the best reaction of the follower player. Consequently, the bi-level programming model is converted to its equivalent single level programming model.

The retailer profit function is rewritten by variable changing of equation (3) as follows:

$$\begin{aligned} &\pi_r(p,q,e) = p\ \ E(\min(D(p,n,e),q)\ \ - wq - e = \\ &p(E[\min(z,\varepsilon)] + d(p) + h(n,e)) - w\ \ z + d(p) + h(n,e)\ \ - e \end{aligned} \tag{17}$$

Lemma 2: The retailer's profit function is pseudo-concave according to the retailer's decision variables.

Proof: See Appendix B.

Taking Lemma 2 into account, it is concluded that the optimal values of the retailer's decision variables including retail price, local and national advertising costs and economic order quantity uniquely exist and are calculated in the following manner:

$$\frac{\partial \pi_r}{\partial p} = 0 \Rightarrow p(w) = \frac{w}{1-F(z)} \tag{18}$$

$$\frac{\partial \pi_r}{\partial e} = 0 \Rightarrow e(w) = \left[\frac{k_2^2 wF(z)}{2(1-F(z)}\right]^2 \tag{19}$$

$$\begin{aligned} z &= q - \left[d(p) + h(n,e)\right] = q - \left[a - bp + k_1\sqrt{n} + k_2\sqrt{e(w)}\right] \\ &= q(w) = z + (a - b(\frac{w}{1-F(z)}) + k_1\sqrt{n} + k_2\sqrt{e(w)} \\ &= q(w) = z + (a - b(\frac{w}{1-F(z)}) + k_1\sqrt{n} + \left[\frac{k_2^2 wF(z)}{2(1-F(z)}\right] \end{aligned} \tag{20}$$

Thus, we can get that:

$$p(w) = \frac{w}{1-F(z)} \tag{21}$$

$$e(w) = \left[\frac{k_2^2 wF(z)}{2(1-F(z)} \right]^2 \tag{22}$$

$$q(w) = z + (a - b(\frac{w}{1-F(z)}) + k_1\sqrt{n} + \left[\frac{k_2^2 wF(z)}{2(1-F(z)} \right] \tag{23}$$

Incorporating the optimal values of the retailer's decision variables into the manufacturer's model, the manufacturer's profit function can be rewritten in the following way:

$$\pi_s(w,n) = (w-c)q - n \tag{24}$$

$$\pi_s(w,n) = (w-c)q - n \Rightarrow (w-c)(z + a - b(\frac{w}{1-F(z)}) + k_1\sqrt{n} + \left[\frac{k_2^2 wF(z)}{2(1-F(z)} \right]) \tag{25}$$

Lemma 3: In Stackelberg game, the manufacturer's profit function is pseudo-concave with respect to the manufacturer's decision variables.

Proof: See Appendix C.

Considering the concavity property of the manufacturer's profit function, one can obtain the optimal decisions of the manufacturer by taking derivatives of the manufacturer's profit function with respect to its decision variables.

$$\frac{\partial \pi_s}{\partial n} = 0 \Rightarrow n = \left[\frac{(w-c)k_1}{2} \right]^2 \tag{26}$$

$$\begin{aligned} \frac{\partial \pi_s}{\partial w} = 0 \Rightarrow (w-c)q - n &\Rightarrow (w-c)(z + a - b(\frac{w}{1-F(z)}) + k_1\sqrt{n} + \left| \frac{k_2^2 wF(z)}{2(1-F(z)} \right|) \\ &= (w-c)(z + a - b(\frac{w}{1-F(z)}) + \frac{(w-c)k_1^2}{2} + \left| \frac{k_2^2 wF(z)}{2(1-F(z)} \right|) \\ &= w^* = \frac{2(1-F(z)(z+a) + 2bc - 2ck_1^2(1-F(z) - ck_2^2 F(z)}{4b - 2k_1^2(1-F(z)) - 2k_2^2 F(z)} \end{aligned} \tag{27}$$

The optimal values of retail price, local and national advertising expenditures and economic order quantity can be computed by solving the Stackelberg model as follows.

$$w^* = \frac{2(1-F(z)(z+a) + 2bc - ck_1^2(1-F(z) - ck_2^2 F(z)}{4b - 2k_1^2(1-F(z) - 2k_2^2 F(z)} \tag{28}$$

$$n^* = \left[\frac{(w^* - c)k_1}{2} \right]^2 \tag{29}$$

$$q^* = z + (a - b(\frac{w^*}{1-F(z)}) + \frac{(w^* - c)k_1^2}{2} + \left[\frac{k_2^2 w^* F(z)}{2(1-F(z)} \right] \tag{30}$$

$$p^* = \frac{w^*}{1-F(z)} \tag{31}$$

$$e^* = \left[\frac{k_2 w^* F(z)}{2(1-F(z)}\right]^2 \tag{32}$$

4-2- Nash Equilibrium

It is assumed in this scenario that the manufacturer and the retailer are separate property companies that independently take their strategies without cooperation. Moreover, the manufacturer and the retailer have equal power in the market and reveal their decisions to the market at the same time. In this situation, both players concurrently pursue the maximization of their individual profits while the supply chain's total profit is ignored. Thus, the Nash equilibrium solution is obtained when the profit functions of both players are maximized simultaneously.

The optimality conditions of the manufacturer's sub-model:

Using the variable change z in relation (3), the manufacturer's profit function can be formulated as follows:

$$\pi_s(w,n) = (w-c)(z+d(p)+h(n,e))-n \tag{33}$$

Considering Lemma 3, because of the concavity property of the manufacturer's profit function, the optimal values of the manufacturer's decision variables can be obtained by setting partial derivatives of the profit function with regard to decision variables equal to zero.

$$\frac{\partial \pi_s}{\partial n} = 0 \Rightarrow n = \left[\frac{(w-c)k_1}{2}\right]^2 \tag{34}$$

The derivative relation cannot be used for computing the optimum wholesale price. Thus, it is assumed that the wholesale price w equals the production cost C multiplied by a constant factor β . In other words, $w=\beta$c such that $\beta>1$. By replacing w, the optimal advertising expenditures is calculated in the following manner:

$$n = \left[\frac{(w-c)k_1}{2}\right]^2 = \left[\frac{c(\beta-1)k_1}{2}\right]^2 \tag{35}$$

The retailer's profit function is formulated as follows:

$$\begin{aligned} \pi_r(p,q,e) &= p\ \ E(\min(D(p,n,e),q)\ \ -wq-e = p(E[\min(z,\varepsilon)]+d(p)+h(n,e))-w\ \ z+d(p)+h(n,e)\ \ -e \\ \pi_r(p,q,e) &= p(E[\min(z,\varepsilon)]+d(p)+h(n,e))-w\ \ z+d(p)+h(n,e)\ \ -e \end{aligned} \tag{36}$$

Regarding Lemma 2, since the retailer's profit function is concave, the optimal values of the retailer's decision variables are accessible through setting partial derivatives of the profit function with regard to decision variables equal to zero:

$$\frac{\partial \pi_r}{\partial p} = 0 \Rightarrow p(w) = \frac{w}{1-F(z)} \Rightarrow p = \frac{\beta c}{1-F(z)} \tag{37}$$

$$\frac{\partial \pi_r}{\partial e} = 0 \Rightarrow e = \left[\frac{\beta k_2 F(z)}{2(1-F(z)} \right]^2 \tag{38}$$

$$q = z + (a - b(\frac{\beta c}{1-F(z)}) + ck_1^2 + \left[\frac{\beta k_2^2 F(z)}{2(1-F(z)} \right] \tag{39}$$

As a result, the Nash equilibrium point of the problem is obtained through solving both the manufacturer's and retailer's sub-models, simultaneously. The calculation is shown below.

$$w^* = \beta c \tag{40}$$

$$n^* = \left[\frac{c(\beta - 1)k_1}{2} \right]^2 \tag{41}$$

$$p^* = \frac{\beta c}{1-F(z)} \tag{42}$$

$$e^* = \left[\frac{\beta k_2 F(z)}{2(1-F(z)} \right]^2 \tag{43}$$

$$q^* = z + (a - b(\frac{\beta c}{1-F(z)}) + ck_1^2 + \left[\frac{\beta k_2^2 F(z)}{2(1-F(z)} \right] \tag{44}$$

Comparing the obtained solutions of the decentralized channels with those of the centralized channel, the following outcomes are achieved:

Corollary 1:
$e_d^* < e_c^*, n_d^* < n_c^*, p_d^* > p_c^* \quad and \quad q_d^* < q_c^*$

Proof: See Appendix D.

Corollary 1 indicates that in the decentralized channels (Stackelberg Manufacturer-Leader and Nash scenarios) retail price value is always greater than its value in the centralized channel scenario. In fact, the retailer is able to decrease retail price because he does not have to pay wholesale price to the manufacturer under the centralized scenario. Besides, both local and national advertising expenditures of the decentralized channels are lower than those of the centralized channel. The reason is the competition between manufacturer and retailer in the decentralized scenarios which makes the two parties have no interest in raising their own share of advertising. As a result, the whole supply chain earns more demand and consequently more profit in the centralized channel which leads to an increase in the economic order size.

Corollary 2: $n_d^* / e_d^* < n_c^* / e_c^*$

Proof: See Appendix E.

Corollary 2 implies that the retailer always has to spend a greater share of advertising expenditure to increase the demand and consequently his individual profit under the decentralized channel scenarios, while the manufacturer gets motivated to spend more on advertising in the centralized channel to increase profit of the whole supply chain. Thereupon, the proportion of local advertising expenditure to national advertising expenditure in the decentralized channel scenarios is always lower than those of the centralized scenario.

Corollary 3: $dn_d^* / dk_1 > 0, de_d^* / dk_2 > 0,\ \partial q_d^* / dk_2 > 0\ and\ \partial q_d^* / dk_1 > 0$

Proof: See Appendix F.

Corollary 3 shows that the greater the national/local advertising elasticity of demand the more national/local advertising expenditures by the manufacturer/retailer in a decentralized channel. Moreover, when demand is elastic an increase in the advertising expenditures of the two parties leads to increasing total demand. In consequence, the retailer orders more because selling a higher quantity. In the other words, a uniform increment in k_1 and k_2 results in the increment in national and local advertising expenditures, respectively. This outcome can be also extended to the centralized channel.

Corollary 4: $: dw / d\beta > 0 \quad and \quad dp^* / d\beta > 0$

Proof: See Appendix G.

Since the factor β determines the profit margin of the manufacturer, corollary 4 indicates that the more margin the manufacturer chooses the higher is the wholesale price and therefore the higher is the price that the retailer can offer. It means an increment in β leads to the increment in wholesale as well as retail prices.

Bringing together all we discussed above, Table 2 shows the optimal values of decision variables including retail price, wholesale price, local and national advertising expenditures and the economic order quantity for the optimum solutions of the three considered games i.e., the Stackelberg game, the Nash equilibrium and the centralized channel scenario.

<<Insert Table 2 >>

5- Numerical results

In order to show the performance of the proposed models and evaluate the obtained results, several problem instances are investigated in this section. It is assumed that random variable ε has a uniform distribution function in [-1000, 1000] interval and $\beta = 1.1$. The other parameters are randomly selected from the defined ranges provided in Table 3. The optimum solutions for the five problem instances presented in Table 4 are calculated for the three scenarios (manufacturer-leader, Nash and centralized channel) and are shown in Table 5.

<<Insert Table 3 >>

<<Insert Table 4 >>

<<Insert Table 5 >>

Considering the results summarized in table 5, we can discuss the influence of scenario on the prices, the local and the national advertising expenditures, and the profits. the highest retail prices occur at both the Manufacturer-Leader and Nash equilibria, whereas the lowest retail prices occur for centralized channel. Comparing the two decentralized scenario, higher retail prices often happen at Manufacturer-Leader equilibria due to the fact that the manufacturer as the leader imposes higher wholesale prices to the retailer who consequently has to set high retail prices.
In terms of the local and the national advertising expenditures, they are the highest under the centralized scenario. Considering the two decentralized scenarios, the retailer's local advertising expenditure is higher at the Manufacturer-Leader equilibria compared to the Nash equilibria because the retailer has the lower position in the channel. It is worth mentioning when the advertising elasticity coefficient is equal for both the manufacturer and the retailer (instance 1), the advertising expenditure is divided equally between the two parties under centralized scenario, but the retailer yet has to spend significantly more share on advertising at the Manufacturer-Leader equilibria.

Comparing the retailer's profit under different scenarios, the retailer gains the lowest profit being the manufacturer's follower. On the other hand, the manufacturer's profit under decentralized scenarios varies based on the value of the constant β. When the manufacturer and the retailer cooperate, the whole supply chain always gains the highest profit. In order to compare the supply chain's efficiency of the Nash and the Manufacturer-Leader equilibria against the global optima of the centralized case, a deviation value is defined as Equation (51).

$$\Delta = \frac{\pi_c - (\pi_r + \pi_s)}{\pi_r + \pi_s} \tag{51}$$

Table 6 illustrates the values of Δ for the five problem instances. We notice that the amount of improvement that the cooperation of the manufacturer and the retailer yields can be influenced by the parameters of the demand function. Nevertheless, the benefits of cooperation are often higher when the two parties do not have the same power position.

<<Insert Table 6 >>

Sensitivity analysis

Here, a sensitivity analysis is performed to show the impacts of the changes in the main parameters of the proposed demand function on the model outcomes.

First, we analyze how sensitive the value of national advertising expenditure is by the changes in demand elasticity parameter k_1 while keeping the other parameters constant. Figure 1 indicates that increasing k_1 motivates the manufacturer to spend more on advertising and this motivation is always greater under Manufacturer-Leader scenario where the manufacturer acts as the leader and choose his strategies at first to maximize his profit.

<<Insert Figure 1 >>

Second, in order to show the impact of local advertising expenditure elasticity of demand, we change k_2 while keeping the other parameters constant and illustrate the results in figure 2. As expected, an increment in the value of k_2 makes the demand more elastic to local advertising expenditure and therefore the retailer raises his expenditure on advertising in order to get higher demand. Furthermore, the advertising share of the retailer is higher under the Manufacturer-Leader scenario in comparison with the simultaneous game scenario. The reason is that the retailer, as the follower, observing the decisions of the manufacturer as the leader and then does his best to increase his profit. Therefore, the retailer spends more on advertising in order to obtain more demand.

<<Insert Figure 2 >>

Third, to analyze the sensitivity of the outcomes to the changes in price elasticity of demand, we increase the value of parameter *b* and depict the outcomes in figures 3 and 4. Figure 3 illustrates that the greater the parameter *b* is the lower price the retailer offers. When *b* is high the demand is more elastic to price, therefore the retailer has to reduce the price of commodity to gain more demand. As an immediate result, the whole supply chain earns less profit at a lower retail price. This result can be observed in all the three game scenarios.

<<Insert Figure 3 >>

Figure 4 depicts the impact of price f and national advertising expenditures raise by increasing parameter *b*. In fact, the retailer needs to reduce his costs to be able to offer lower retail price and therefore he spends less on local advertising. In a decentralized channel, the manufacturer competes with the retailer and sets lower national advertising expenditure in response to the retailer's strategies. Advertising expenditures drop due to price elasticity factor growth is also observed in centralized scenario. Under this scenario, the two parties agree on lower advertising expenditures to reduce the whole supply chain costs. In fact, low retail prices do not allow high expenditures on advertising.

<<Insert Figure 4 >>

6- Conclusion

In this paper, coordination of supply chain decisions in a manufacturer-retailer network was investigated using three different game theoretic approaches including Stackelberg manufacturer game, Nash game and centralized scenario. Afterwards, the retail price, the wholesale price, economic order quantity, the optimal retailer advertising costs, and the manufacturer national advertising expenditures were determined in newsboy model. Demand was defined to be stochastic and influenced by both retail price and advertising expenditures. The equilibrium solutions were obtained exactly in three different market structures. Some corollaries are presented and theoretically proved to show the relations between optimal decision variables in centralized vs. decentralized scenarios.

The obtained results showed that an increase in demand elasticity coefficient leads to remarkable reductions in the retail price, the economic order quantity, local and national advertising expenditures and the supply chain's profit. On the other hand, both local and national advertising costs are increased when advertising function coefficient is also increased. The ratio of national advertising costs to local advertising costs under the centralized channel condition is higher than that of the Stackelberg or Nash games. In fact, the proportion of advertising expenses in centralized channel is more than that of its rivals. Also, it was found that in the centralized structure advertising expenditures are always higher than the decentralized setting. In centralized structure, the order quantity is higher than the decentralized state while it's lower for the retail price. However, the profit of members in centralized scenario is always more than the proposed Nash and Stackelberg games.

As suggestions for future research, one can consider other types of contracts such as revenue sharing contract and return policy using the newsboy problem approach with stochastic demand. Besides, applying Stackelberg retailer game to a three-tier supply chain with advertising and price dependent demand when the manufacturer is the dominant power can be an interesting research area.

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Appendix A.

Theorem 1: If the Hessian matrix is negative semi-definite, *f(x,y)* is pseudo-concave.

Theorem 2: The Hessian matrix H is negative semi-definite if and only if:

$$\left[x, y\right]\times \mathrm{H}\times \begin{bmatrix} x \\ y \end{bmatrix} \le 0 \tag{A-1}$$

The Hessian matrix of the centralized supply chain's objective function is derived as follows:

$$\mathrm{H}=\begin{bmatrix} \frac{\partial \pi}{\partial p^2} & \frac{\partial \pi}{\partial p \partial z} & \frac{\partial \pi}{\partial p \partial n} & \frac{\partial \pi}{\partial p \partial e} \\ \frac{\partial \pi}{\partial z \partial p} & \frac{\partial \pi}{\partial z^2} & \frac{\partial \pi}{\partial z \partial n} & \frac{\partial \pi}{\partial z \partial e} \\ \frac{\partial \pi}{\partial n \partial p} & \frac{\partial \pi}{\partial n \partial z} & \frac{\partial \pi}{\partial n^2} & \frac{\partial \pi}{\partial n \partial e} \\ \frac{\partial \pi}{\partial e \partial p} & \frac{\partial \pi}{\partial e \partial z} & \frac{\partial \pi}{\partial e \partial n} & \frac{\partial \pi}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2b & 1-F(z) & \frac{k_1}{2\sqrt{n}} & \frac{k_2}{2\sqrt{e}} \\ 1-F(z) & -pf(z) & 0 & 0 \\ \frac{k_1}{2\sqrt{n}} & 0 & -\frac{k_1(p-c)}{4\sqrt{n^3}} & 0 \\ \frac{k_2}{2\sqrt{e}} & 0 & 0 & -\frac{k_2(p-c)}{4\sqrt{e^3}} \end{bmatrix} \tag{A-2}$$

Using Theorem 1 and Theorem 2 we have:

$$[p \quad z \quad n \quad e]\begin{bmatrix} \frac{\partial\pi}{\partial p^2} & \frac{\partial\pi}{\partial p\partial z} & \frac{\partial\pi}{\partial p\partial n} & \frac{\partial\pi}{\partial p\partial e} \\ \frac{\partial\pi}{\partial z\partial p} & \frac{\partial\pi}{\partial z^2} & \frac{\partial\pi}{\partial z\partial n} & \frac{\partial\pi}{\partial z\partial e} \\ \frac{\partial\pi}{\partial n\partial p} & \frac{\partial\pi}{\partial n\partial z} & \frac{\partial\pi}{\partial n^2} & \frac{\partial\pi}{\partial n\partial e} \\ \frac{\partial\pi}{\partial e\partial p} & \frac{\partial\pi}{\partial e\partial z} & \frac{\partial\pi}{\partial e\partial n} & \frac{\partial\pi}{\partial e^2} \end{bmatrix}\begin{bmatrix} p \\ z \\ n \\ e \end{bmatrix} = [p \quad z \quad n \quad e]\begin{bmatrix} -2b & 1-F(z) & \frac{k_1}{2\sqrt{n}} & \frac{k_2}{2\sqrt{e}} \\ 1-F(z) & -pf(z) & 0 & 0 \\ \frac{k_1}{2\sqrt{n}} & 0 & -\frac{k_1(p-c)}{4\sqrt{n^3}} & 0 \\ \frac{k_2}{2\sqrt{e}} & 0 & 0 & -\frac{k_2(p-c)}{4\sqrt{e^3}} \end{bmatrix}\begin{bmatrix} p \\ z \\ n \\ e \end{bmatrix} = \qquad \text{(A-3)}$$

$$= 8zc(1-F(z))^2 + 4((k_1+k_2)c^2F(z) - (k_1+k_2)c^2F^2(z) - 8bc^2 - f(z)cz^2(1-F(z) \leq 0$$

Analyzing numerical results given that $\mathrm{A} < z^* < \mathrm{B}$ shows that (A2) is always non-positive and the Hessian matrix is negative pseudo-definite. Thus, optimality conditions are elicited by taking derivatives of the profit function.■

Appendix B.

The Hessian matrix of the retailer's profit function is given by:

$$\mathrm{H} = \begin{bmatrix} \frac{\partial\pi}{\partial p^2} & \frac{\partial\pi}{\partial p\partial z} & \frac{\partial\pi}{\partial p\partial e} \\ \frac{\partial\pi}{\partial z\partial p} & \frac{\partial\pi}{\partial z^2} & \frac{\partial\pi}{\partial z\partial e} \\ \frac{\partial\pi}{\partial e\partial p} & \frac{\partial\pi}{\partial e\partial z} & \frac{\partial\pi}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2b & 1-F(Z) & \frac{k_2}{2\sqrt{e}} \\ 1-F(Z) & -pf(z) & 0 \\ \frac{k_2}{2\sqrt{e}} & 0 & -\frac{k_2(p-w)}{4\sqrt{e^3}} \end{bmatrix} \qquad \text{(B-1)}$$

Using Theorem 1 and Theorem 2:

$$[p \quad z \quad e]\begin{bmatrix} \frac{\partial\pi}{\partial p^2} & \frac{\partial\pi}{\partial p\partial z} & \frac{\partial\pi}{\partial p\partial e} \\ \frac{\partial\pi}{\partial z\partial p} & \frac{\partial\pi}{\partial z^2} & \frac{\partial\pi}{\partial z\partial e} \\ \frac{\partial\pi}{\partial e\partial p} & \frac{\partial\pi}{\partial e\partial z} & \frac{\partial\pi}{\partial e^2} \end{bmatrix}\begin{bmatrix} p \\ z \\ e \end{bmatrix} \qquad \text{(B-2)}$$

$$= 2wz + \frac{w^2k^2{}_2F(z)c}{2(1-F(z)} + \frac{w^2k_2^3F(z)}{2(1-F(z))^2} - \frac{2bw^2}{(1-F(z))^2} - \frac{wz^2f(z)}{(1-F(z)} - \frac{k_2}{2} \leq 0$$

Analyzing numerical results given $A < z^* < B$ shows that the relation (B-2) holds, the solution is unique and the Hessian matrix is negative pseudo-definite. Thus, optimality conditions are extracted by taking derivatives of the profit function.■

Appendix C.

The Hessian matrix of the manufacturer's profit function is obtained as follows:

$$H = \begin{bmatrix} \frac{\partial \pi}{\partial w^2} & \frac{\partial \pi}{\partial w \partial n} \\ \frac{\partial \pi}{\partial n \partial w} & -\frac{k_1(w-c)}{4\sqrt{n^3}} \end{bmatrix} = \begin{bmatrix} 0 & \frac{k_1}{2\sqrt{n}} \\ \frac{k_1}{2\sqrt{n}} & -\frac{k_1(w-c)}{4\sqrt{n^3}} \end{bmatrix} \tag{C-1}$$

Using theorems 1 and 2 we have:

$$\begin{aligned} &\begin{bmatrix} w & n \end{bmatrix} H \begin{bmatrix} w \\ n \end{bmatrix} = \\ &\begin{bmatrix} w & n \end{bmatrix} \begin{bmatrix} 0 & \frac{k_1}{2\sqrt{n}} \\ \frac{k_1}{2\sqrt{n}} & -\frac{k_1(w-c)}{4\sqrt{n^3}} \end{bmatrix} \begin{bmatrix} w \\ n \end{bmatrix} \\ &= \frac{wnk_1}{\sqrt{n}} - \frac{nk_1(w-c)}{4\sqrt{n}} \le 0 \;\; \Rightarrow c \le w(n-1) \end{aligned} \tag{C-2}$$

Since inequality $c \le w(n-1)$ always holds, the Hessian matrix is negative semi-definite. Therefore, the optimality conditions of the wholesaler's profit function are derived by taking derivatives.■

Appendix D.

Stackelberg game vs. centralized channel scenario:

Since the retail price value p is always bigger than the wholesale price w, $p-c$ will be always bigger than $w-c$ as well. Consequently, national advertising expenditures in the centralized supply chain is bigger than national advertising costs in Equation 32 in the decentralized scheme, and so we have $n_d^* < n_c^*$.

Since the retail price p is always bigger than the wholesale price w and the numerator is multiplied by F(z) which is a real number between 0 and 1, and the below relation holds:

$$w * \frac{F(Z)}{1-F(Z)} \le p - c$$

And so we have $e_d^* < e_c^*$.

The retail price in the decentralized channel is given in Equation 35. Since the wholesale price w is always greater than the unit production cost c, considering Equation 16 it can be easily proved that $p_d^* > p_c^*$.

Regarding the equation $q_c^* = z^* + d(p) + h(n,e) = z^* + \left[a - bp^* + k_1\sqrt{n^*} + k_2\sqrt{e^*}\right]$, since in the decentralized configuration *n, e* are less and *p* is more, the economic order quantity, which is obtained from algebraic addition of the local and national advertising costs when the retail price is subtracted, will be always smaller than that of the centralized structure and $q_d^* < q_c^*$.

Nash game vs. centralized channel scenario:

$n_d^* < n_c^*$: For $\beta > 1$, $p - c$ is always bigger than $c(\beta - 1)$ because the retail price *p* is always bigger than the production cost *c*. Hence, the national advertising costs of the centralized structure is larger than the national advertising costs of Equation 41 in the decentralized setting.

$e_d^* < e_c^*$: Since the retail price *p* is always bigger than *c* and the numerator is multiplied by *F(Z)* which is a real number between 0 and 1, the following relation holds:

$$\frac{\beta F(Z)}{1 - F(Z)} \leq p - c$$

and so we have $e_d^* < e_c^*$.

$p_d^* > p_c^*$: Considering the retail price in the Nash structure (Equation 42), $0 \leq F(Z) \leq 1$, and the fact that the wholesale price *w* is always bigger than the unit production cost *c* ($w = \beta c$ *where* $\beta > 1$), It can be straightforwardly proved that the inequality $p_d^* > p_c^*$ holds.

$q_d^* < q_c^*$: Regarding the three previous properties and the relation $q_c^* = z^* + d(p) + h(n,e) = z^* + \left[a - bp^* + k_1\sqrt{n^*} + k_2\sqrt{e^*}\right]$, since in the Nash setting *n* and *e* are less and *p* is more, the order quantity, which is obtained from algebraic addition of the local and national advertising costs when the retail price is subtracted, will be always smaller than that of the centralized structure.■

Appendix E.

Stackelberg game vs. centralized channel scenario:

$$n_d^*/e_d^* \Rightarrow \frac{n^* = \left[\dfrac{(w^*-c)k_1}{2}\right]^2}{e^* = \left[\dfrac{k_2 w^* F(z)}{2(1-F(z)}\right]^2} = \frac{k_1}{k_2}(1-\frac{c}{w})(1-F(z)$$

$$n_c^*/e_c^* \Rightarrow \frac{n_c^* = \left[\dfrac{k_1(p^*-c)}{2}\right]^2}{e_c^* = \left[\dfrac{k_2(p^*-c)}{2}\right]^2} = \frac{k_1}{k_2} \qquad \text{(E-1)}$$

Taking relations (E-1) into consideration, since $(1-\frac{c}{w})(1-F(z))$ is always smaller than one, corollary 2 always holds. This property indicates that the ratio of local to national advertising costs in the Stackelberg game is always smaller than the centralized scenario.

Nash game vs. centralized channel scenario:

$$n_d^*/e_d^* \Rightarrow \frac{n^* = \left[\dfrac{c(\beta-1)k_1}{2}\right]^2}{e^* = \left[\dfrac{\beta k_2 F(z)}{2(1-F(z)}\right]^2} = \frac{k_1}{k_2}(\frac{\beta-1}{\beta})(\frac{1-F(z)}{F(z)}) \quad where\ \beta \geq 1\ and\ 0 \leq F(Z) \leq 1$$

$$n_c^*/e_c^* \Rightarrow \frac{n_c^* = \left[\dfrac{k_1(p^*-c)}{2}\right]^2}{e_c^* = \left[\dfrac{k_2(p^*-c)}{2}\right]^2} = \frac{k_1}{k_2} \qquad \text{(E-2)}$$

■

Appendix F.

Stackelberg game vs. centralized channel scenario:

$$dn_d^*/dk_1 > 0 \quad where\ n^* = \left[\frac{(w^*-c)k_1}{2}\right]^2 \Rightarrow \frac{dn_d^*}{dk_1} = (w^*-c)k_1 > 0$$

$$de_d^* / dk_2 > 0 \;\; where \;\; e^* = \left[\frac{k_2 w^* F(z)}{2(1-F(z)}\right]^2 \Rightarrow \frac{de_d^*}{dk_2} = \frac{k_2 w^* F(z)}{(1-F(z)} > 0$$

$$\partial q_d^* / dk_2 > 0 \;\; where \;\; q = z + \left[a - bp + k_1\sqrt{n} + k_2\sqrt{e}\right] \Rightarrow \frac{\partial q_d^*}{dk_2} = \sqrt{e} > 0$$

$$\partial q_d^* / dk_1 > 0 \;\; where \;\; q = z + \left[a - bp + k_1\sqrt{n} + k_2\sqrt{e}\right] \Rightarrow \frac{\partial q_d^*}{dk_2} = \sqrt{n} > 0$$

Nash game vs. centralized channel scenario:

$$dn_d^* / dk_1 > 0 \;\; where \;\; n = \left[\frac{c(\beta-1)k_1}{2}\right]^2 \Rightarrow \frac{dn_d^*}{dk_1} = \frac{1}{2} c^2 (\beta-1)^2 k_1 > 0$$

$$de_d^* / dk_2 > 0 \;\; where \;\; e_d^* = \left[\frac{\beta k_2 F(z)}{2(1-F(z)}\right]^2 \Rightarrow \frac{de_d^*}{dk_2} = \frac{k_2(\beta F(z))^2}{2(1-F(z))^2} > 0$$

$$\partial q_d^* / dk_2 > 0 \;\; where \;\; q = z + \left[a - bp + k_1\sqrt{n} + k_2\sqrt{e}\right] \Rightarrow \frac{\partial q_d^*}{dk_2} = \sqrt{e} > 0$$

$$\partial q_d^* / dk_1 > 0 \;\; where \;\; q = z + \left[a - bp + k_1\sqrt{n} + k_2\sqrt{e}\right] \Rightarrow \frac{\partial q_d^*}{dk_2} = \sqrt{n} > 0$$

.■

Appendix G.

According to Equations 40 and 42, it can be easily shown that $dw / d\beta > 0$ and $dp^* / d\beta > 0$. It is due to the following relations:

$$dw / d\beta \quad where \;\; w = \beta c \Rightarrow dw / d\beta = w > 0$$

$$dp^* / d\beta \quad where \quad p = \frac{\beta c}{1-F(z)} \Rightarrow dp^* / d\beta = \frac{c}{1-F(z)} > 0$$

It can be simply observed that an increase in β value leads to the increase in both retail price and wholesale price.■

Table 1. Comparison of the general settings of the current paper and the related literature

Table 2. The equilibrium solution in the three game setting

Table 3. Rang of parameters

Table 4. The problem instances

Table 5. The equilibrium solution in the three game settings

Table 6. Improvement of supply chain benefit in centralized scenario

Figure 1. Effect of k_1 on the manufacturer's national advertising expenditure

Figure 2. Effect of k_2 on the retailer's local advertising expenditure

Figure 3. Effect of price elasticity factor b on the retail prices and the supply chain total profit

Figure 4. Effect of price elasticity factor b on the local advertising expenditure and the national advertising expenditure

Table1. Comparison of the general settings of the current paper and the related literature

Reference	Channel structure	Decisions	Demand type	Uncertainty	Advertising type	Centralized structure	Decentralized structure
[10]	Two-echelon	• Pricing • Ordering • Advertising	Price and advertising dependent	Uniform distribution	Local advertising	No	• Stackelberg equilibrium
[12]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	No	Cooperative advertising	Yes	• Nash equilibrium • Stackelberg equilibrium
[5]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	No	Cooperative advertising	Yes	• Stackelberg equilibrium
[14]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	No	Cooperative advertising	Yes	• Nash equilibrium • Stackelberg equilibrium
[19]	Two-echelon	• Pricing • Advertising • Ordering	Price and advertising dependent	No	Local advertising	Yes	• Stackelberg equilibrium
[9]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	No	Cooperative advertising	Yes	• Nash equilibrium • Stackelberg equilibrium
[13]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	No	Cooperative advertising	No	• Stackelberg equilibrium
[1]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	No	Local advertising	Yes	• Stackelberg equilibrium
[6]	Two-echelon	• Pricing • Advertising	Advertising dependent	No	Local advertising	Yes	• Stackelberg equilibrium
[17]	Two-echelon	• Pricing • Advertising	Price and advertising	No	Coop	Yes	• Nash equilibrium • Stackelberg

			dependent				equilibrium
[7]	Two-echelon	• Pricing • Advertising	Price and advertising dependent	Normal distribution	Local advertising	No	• Stackelberg equilibrium
[15]	Three-echelon	• Pricing • Advertising	Price and advertising dependent	No	Local advertising	No	• Nash equilibrium
[18]	Two-echelon	• Pricing	Price dependent	Fuzzy	No	No	• Stackelberg equilibrium
[20]	Two-echelon	• Pricing • Ordering • Advertising	Price and advertising dependent	No	Cooperative advertising	No	• Stackelberg equilibrium
[21]	Two-echelon	• Ordering • Advertising	Advertising dependent	Uniform and normal distribution	Cooperative advertising	Yes	• Stackelberg equilibrium
Current paper	Two-echelon	• Pricing • Ordering • Advertising	Price and advertising dependent	Uniform distribution	Cooperative advertising	Yes	• Nash equilibrium • Stackelberg equilibrium

Table 2. The equilibrium solution in the three game setting

	Centralized	Stackelberg manufacturer-leader game	Nash game
p	$p_c^* = \dfrac{c}{1-F(z^*)}$	$p^* = \dfrac{w^*}{1-F(z)}$	$p = \dfrac{\beta c}{1-F(z)}$
n	$n_c^* = \left[\dfrac{k_1(p^*-c)}{2}\right]^2$	$n^* = \left[\dfrac{(w^*-c)k_1}{2}\right]^2$	$n = \left[\dfrac{c(\beta-1)k_1}{2}\right]^2$
e	$e_c^* = \left[\dfrac{k_2(p^*-c)}{2}\right]^2$	$e^* = \left[\dfrac{k_2 w^* F(z)}{2(1-F(z)}\right]^2$	$e = \left[\dfrac{\beta k_2 F(z)}{2(1-F(z)}\right]^2$
q	$q^* = z^* + \left[a - bp^* + k_1\sqrt{n^*} + k_2\sqrt{e^*}\right]$	$q^* = z + (a - b(\dfrac{w^*}{1-F(z)}) + \dfrac{(w^*-c)k_1^2}{2} + \left[\dfrac{k_2^2 w^* F(z)}{2(1-F(z)}\right]$	$q = z + (a - b(\dfrac{\beta c}{1-F(z)}) + ck_1^2 + \left[\dfrac{\beta k_2^2 F(z)}{2(1-F(z)}\right]$
w	-	$w^* = \dfrac{2(1-F(z)(z+a) + 2bc - ck_1^2(1-F(z) - ck_2^2 F(z)}{4b - 2k_1^2(1-F(z) - 2k_2^2 F(z)}$	$w = \beta c$

Table 3. Rang of parameters

Parameters	a	b	k_1	k_2
Ranges	$2000-3500$	$0.010-0.015$	$[0.01-0.05]$	$[0.01-0.05]$

Table 4. The problem instances

Problems	a	b	k_1	k_2
Instance 1	2,000	0.010	0.01	0.01
Instance 2	3,000	0.015	0.02	0.05
Instance 3	3,000	0.012	0.04	0.02
Instance 4	3,000	0.010	0.02	0.04
Instance 5	3,500	0.010	0.05	0.05

Table 5. The equilibrium solution in the three game settings

	Problems	p	q	n	e	w	π_s	π_r	π_T
Centralized	Instance 1	197,531	408	35,214	35,214				8,127,9 80
	Instance 2	191,617	502	99,962	624,762				9,917,2 52
	Instance 3	229,226	923	1,916,916	479,229				40,559, 507
	Instance 4	267,335	1,237	1,152,080	4,608,321				83,916, 380
	Instance 5	310,680	1,740	14,190,216	14,190216		,		160,707 ,852
act ur er-	Instance 1	217,961	203	6,847	34,346	176,549	3,353,65	506,776	3,860,4

							0		27
	Instance 2	208,547	244	19,985	612,938	174,137	3,426,094	2,097,458	5,523,552
	Instance 3	267,321	431	282,815	440,531	186,590	11,169,377	9,948,964	21,118,341
	Instance 4	330,914	581	144,796	3,959,357	198,052	21,965,825	19,897,599	41,863,424
	Instance 5	399,340	698	1,303,022	10,836,721	205,660	30,580,115	30,469,917	61,050,031
Nash	Instance 1	217284	210	6400	34133	176000	3,356,848	957,447	4,314,295
	Instance 2	210,778	211	25,600	626,125	176,000	3,377,238	1,651,213	5,028,451
	Instance 3	252149	604	102400	391944	176000	9,656,210	25,797,008	35,453,218
	Instance 4	294069	936	25600	3126735	176000	14,979,764	60,950,857	75,930,621
	Instance 5	341748	1213	160000	7936400	176000	19,413,718	119,626,159	139,039,877

Table 6. Improvement of supply chain benefit in centralized scenario

Problems	$\Delta_{Manufacturer-Leader}$	Δ_{Nash}
Instance 1	111%	88%
Instance 2	80%	97%
Instance 3	92%	14%
Instance 4	100%	11%
Instance 5	163%	16%

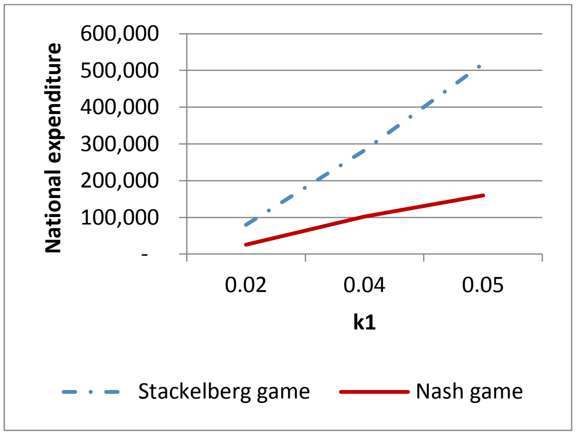

Figure 1. Effect of k_1 on the manufacturer's national advertising expenditure

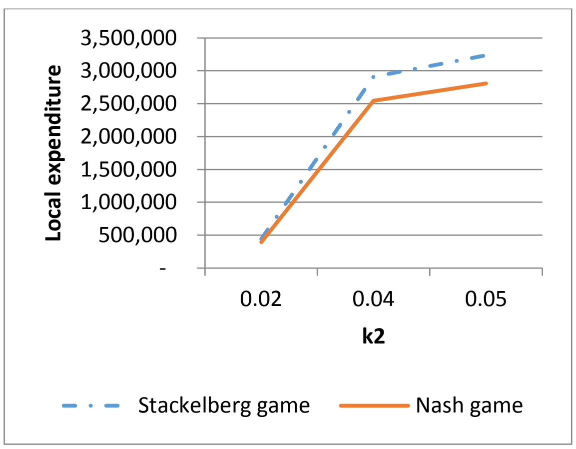

Figure 2. Effect of k_2 on the retailer's local advertising expenditure

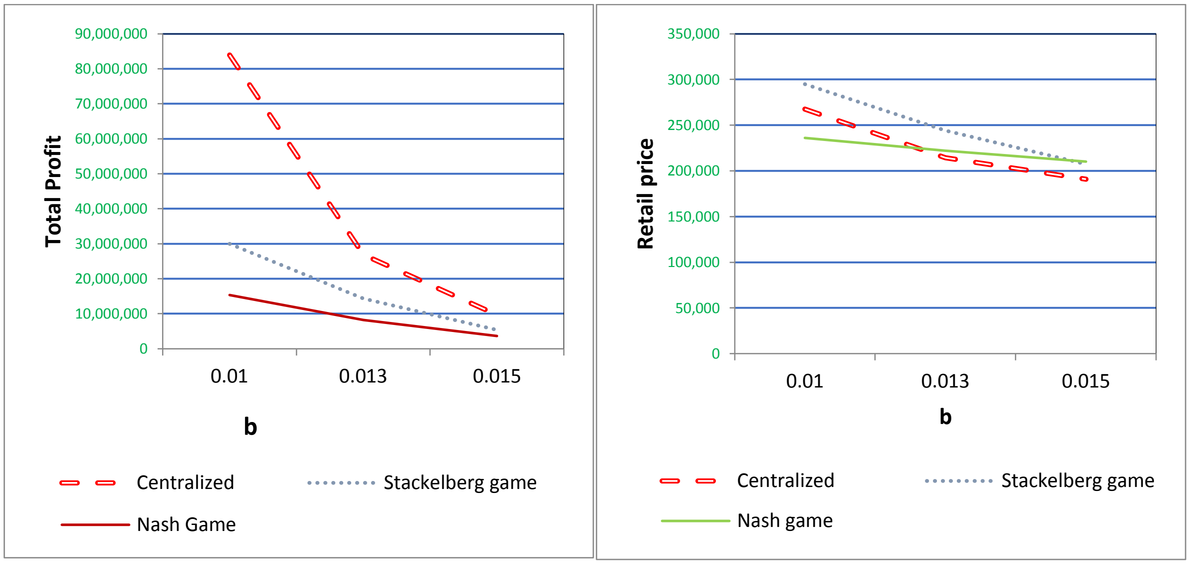

Figure 3. Effect of price elasticity factor b on the retail prices and the supply chain total profit

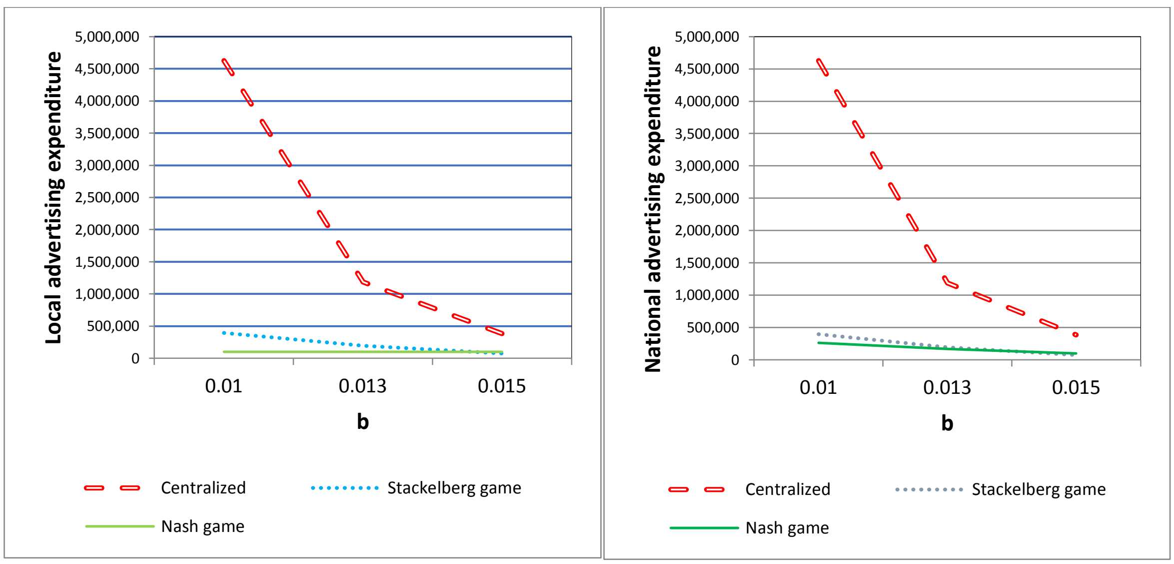

Figure 4. Effect of price elasticity factor b on the local advertising expenditure and the national advertising expenditure

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