On the Performance of Median Based Tukey and Tukey-EWMA Charts Under Rational Subgrouping

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Abstract

Control chart (CC) is used to monitor the special causes that arise during the process monitoring. These special causes produce continual shifts in the process parameters that last until it is identified and removed. There is a need for such techniques, which present the true representation of the entire process. Rational subgrouping is an essential concept in Statistical Process Control (SPC) which is seldom overlooked by the practitioner. Hence, most of the manufacturing, engineering, and production processes give output products in the form of batches over smaller intervals of time. The aim of this study is to provide a median based design for Tukey and Tukey-EWMA control charts under subgrouping. It will use the idea of boxplot to monitor the process behavior. This study also provides a brief discussion regarding selecting and forming subgroups from the process data. The performance of the median based Tukey and Tukey-EWMA charts are judged using Average, Median and Standard-Deviation run-length as performance measures. We have considered subgroup sizes of m=1,5 &10 at pre-specified ARL0 equal to 370. To real-life applications of the median based tukey designs are also presented to show their implementation in food manufacturing and hard-bake processes.

Keywords: Average Run Length; Median; Rational-Subgroups; Tukey chart, Tukey EWMA Chart.

1. Introduction

Tukey control (TCC) chart is well-known individual control chart designed to monitor the skewed data using the concept of boxplot, and is mainly based on the individual observations per period. The rational subgrouping (R.S) is an essential concept in SPC but is frequently overlooked in some process. In R.S, all the goods and products are manufactured under the condition in which only random effects are responsible for observed variations. It is the process of organizing a similar group of products that are produced under similar circumstances. It helps measuring the variations between the subgroups instead of within subgroups, which are considered background noises. The subgroup size should be large enough to represent the overall variation when process is in control. It is concerned with the collection and organization of numerical data. There is variety of literature available on tukey designs and many of its modifications. We provide here a brief review of some useful literature on the topic.

Literature Review

The TCC is mainly an individual observations based control chart proposed by Alemi [1]. Borckardt et al. [2, 3] applied the tukey chart to serially dependent data. Torng and Lee [4] calculated the ARL values of the Tukey chart under several distributions. For small sample, TCC performed very well. Torng et al. [5] introduced the economic design of TCC. Lee [6] applied asymmetrical control limits (ACL) to TCC. In skewed distribution, ACL-TCC offered lower ARL values than SCL-TCC (symmetrical Control limits). Sukparungsee [7] evaluated the performance of TCC; TCC performance is superior to classical EWMA and Shewhart control charts. Lee at el. [8] applied the ACL to economic design of TCC to get the optimum performance. Sukparungsee [9] applied ACL to Tukey chart to evaluate its performance under non normal distributed data and Tukey chart has better ARL_1 performance.

Khaliq at el.[10] did the comparative analysis to judge the performance of Tukey chart versus X/MR chart under the several probability models and Tukey chart was the best choice in many cases. The study revealed that this charts is a good alternative to Shewhart and X/MR chart for monitoring when data is of skewed form. Tercero-Gomez at el. [11] designed the Modified Tukey control Chart (MTCC), which has smaller ARL_1 values than TCC. Mekparyup at el. [12] introduced the adjusted design for Tukey chart. Mekparyup at el. [13] combined the feature of the adjusted Tukey chart with an ARIMA (Auto-Regressive Integrated Moving Averages) model to monitor the Dengue Hemorrhagic Fever (DHF).

Saithanu at el. [14] applied the sensitizing runs rules scheme to Tukey chart. The results indicated that the run length performance of this scheme improves over the Tukey chart. Khaliq et al. [15] introduced the EWMA design for Tukey chart. The Tukey-EWMA design was more sensitive to small sustained shifts in process location than Tukey chart. This design is a good alternate to classical EWMA, when data follows a skewed distribution. Khaliq and Riaz [16] designed the CUSUM structure of Tukey chart for small and sustained shifts. This design is best alternate to classical CUSUM chart, when data follow the skewed distribution. For symmetric distribution, this design has similar run length performance to classical CUSUM. Riaz at el.[17] introduced mixed Tukey EWMA CUSUM design which was more sensitive to small and moderate amount of shifts.

All of the above-mentioned charts are designed to deal with the individual observations. In the current article, we have designed median based Tukey control chart (TCC) and Tukey-EWMA control chart (EWMA-TCC) for subgroups based observations. These median based TCC and EWMA-TCC charts have a variety of applications in different areas such as business, technology, management, education, manufacturing, accounting, finance, engineering and service sectors. We will discuss two applications in details in Section 5 mainly focusing on food manufacturing and hard-bake processes.

The rest of the article is organized as follows: Section 2 discusses the significance of the subgrouping in control chart; Section 3 presents median based tukey and tukey ewma control charts under rational subgrouping; Section 4 provides performance analysis of the charts; Section 5 includes two real applications; and finally Section 6 summarizes and concludes the study.

2. Significance of subgrouping in control charts

Rational Subgrouping (R.G) is fundamental spirit of any application of process behavior chart. It does not follow procedure to form the subgroups. It will be nothing more than wall-paper. As an alternative, the observations compromising the subgroup must be display of the process at short time interval and show how it has fluctuations over the time. The size time interval established an individual process basis to reduce the chance of a special cause happening in the subgroup. Control charts are generally based on more than one sample observations, m>1, selected at fixed length of sampling interval say l, the rational-subgroups idea seems for sampling implies that sampling must be done subsequently, that any change in the process will occur among samples and affect an entire sample. However, if the length of a transient shift t is less than l, then it seems that it might be useful to disperse the samples over the interval *l*, Here is chance of occurring the transient shift will increase. Sampling and subgrouping should be carried out with care. According to Nelson [18] “The rational subgroup is basically a sample, random effects are responsible for the observed variation during the product production “.

Sefik [19] provided complete discussion regarding to importance of subgrouping in process control chart. Nelson [20] discussed the properties of rational subgrouping as follows: The observations among a subgroup are from single and stable process. If subgroups following the multiple process stream and special cause happen continually within subgroup, it have more variation within sample rather than between subgroup averages, these variations may lead the control limits more wider and resulting in a lack of sensitivity to process shifts. “Western Electric Role Test-vii fifteen-successive points within one sigma of center line are helpful in detecting this condition. The subgroups are formed non-randomly from the observations taken in a time-ordered sequence. As an alternative, the observations compromising the subgroup must be display of the process at short time interval and show how it has fluctuations over the time. The size time interval demonstrated an individual process basis to reduce the chance of a special cause happening in the subgroup. Hillier [21,22] designed a adjusted limits for Shewhart type chart for both retrospective and future testing stages. These limits were used to ensure the each future desired subgroup size using at pre-specified type-I error. According to Wheeler [23] subgroups should be logically homogenous by minimizing the variation within subgroup by trying to maximize the variations between subgroups. It must be remember before forming the subgroup that samples which comprising the subgroup must be homogenous and independent over the time. They must be collected time order sequence and come from stable process. Quenvedo at el. [24] applied the iterative procedure to classical X bar and R chart for setting the control limits of respective chart in better way. Djauhari at el. [25] evaluated the performance of the multivariate chart, when subgroup size is too small. Abbasi at el. [26] presented some modification to classical EWMA design to improve the sensitivity of memory chart. Mahmood at el. [27] introduced the joint memory structure of control chart to monitor location and spread. Ajadi and Riaz [28] designed the memory type control charts to monitor the subgroup data. Abujiya at el.[29] developed the variance chart to monitoring dispersion in process monitoring. Mukherjee [30] applied the joint monitoring scheme of location and spread to non-parametric EWMA chart based on the subgroup samples. Ansorena [31] used the SPC control design to monitor quality of seaport services. Hussain at el. [32] designed the Interquartile range base EWMA chart to monitor the continuous tank rector process. Abtew at el. [33] applied the SPC chart in the sewing section of garment industry. Rational subgroups of the samples were first formed to evaluate the process performance. Huberts at el. [34] introduced the method of continuously updating the control limits of control chart, when data is provided in subgroup form.

Two common methods to constructing rational subgroups

i. This method to rational subgrouping provides a pictorial display of the system at each point in time wherein an observation is collected. It is used when the main objective of the control chart is identify the process shift. It diminishes the chance of inconsistency due to special causes within a sample and it increases the chance of variability among the samples if assignable cause is present. It additionally offers a higher estimate of the standard deviation of the manner within the case of variables control charts.

ii. Each sample comprises of units of product that are illustrative of all the units that have been produced since the last sample was taken. Basically, every subgroup is an unsystematic sample of entire process output over the sampling interval. This technique of rational subgrouping is frequently used, when the control chart is employed to draw conclusions about the acceptance of all units of product that have been produced since the last sample. Indeed, if the process shifts to an out-of-control state and then back in control again between samples, it is sometimes argued that the picture technique of rational subgrouping will be unsuccessful against these types of changes, and so the technique sample scheme must be used. (cf. Montgomery [35]).

3. Median Based Tukey and Tukey EWMA Control Charts under Rational subgroup

Let X_{i_j} be independent observations collected over the time from a normal process, $i = 1,2,3,\ldots.n$ and $j = 1,2,3,\ldots.m$. That is we have m subgroups, each of size n. Now we compute three quartiles (q_1, q_2, q_3) and interquartile range (iqr) for all the m subgroups that may be presented as:

$$q_1 = \begin{matrix} q_{1_1} \\ q_{1_2} \\ q_{1_3} \\ q_{1_4} \\ \vdots \\ q_{1_m} \end{matrix} \; ; \; q_2 = \begin{matrix} q_{2_1} \\ q_{2_2} \\ q_{2_3} \\ q_{2_4} \\ \vdots \\ q_{2_m} \end{matrix} \; ; \; q_3 = \begin{matrix} q_{3_1} \\ q_{3_2} \\ q_{3_3} \\ q_{3_4} \\ \vdots \\ q_{3_m} \end{matrix} \quad \text{and} \quad iqr = \begin{matrix} (iqr)_1 \\ (iqr)_2 \\ (iqr_1)_3 \\ (iqr_1)_4 \\ \vdots \\ (iqr)_m \end{matrix} \; ;$$

Let $\tilde{q}_1$, $\tilde{q}_2$, $\tilde{q}_3$ and $i\tilde{q}r$ are the medians of first, second, third and interquartile range of the subgroup data set, as described above.

Then, the control limits of TCC are as follows.

$$LCL = \tilde{q}_1 - L\left(i\tilde{q}r\right) \quad (1)$$

$$CL = \tilde{q}_2 \quad (2)$$

$$UCL = \tilde{q}_3 + L\left(i\tilde{q}r\right) \quad (3)$$

where L is the control limits coefficient and it is set according to pre-specified ARL_0. The median ($q_2 = \tilde{x}_j$, = 1,2,3, ...) of these values will be used as plotting statistic for the TCC chart.

The Plotting statistic for EWMA-TCC is:

$$G_j = \lambda \tilde{x}_j + (1-\lambda) G_{j-1} \quad (5)$$

The variance of EWMA-TCC statistic as follows

$$Var(G_j) = \frac{i\tilde{q}r(\lambda(1-(1-\lambda)^{2j}))}{2-\lambda} \quad (6)$$

where λ is the weighting parameter and it lies between 0 and 1. For λ = 1, it exhibits the most recent observation and it becomes the special case of TCC. The initial value of G_i (i.e. G_0) is set equal to the overall median.

The time varying control limits of EWMA-TCC are given as:

$$LCL = \tilde{q}_1 - L_t\left(i\tilde{q}r\right)\sqrt{\frac{\lambda(1-(1-\lambda)^{2j})}{2-\lambda}} \quad (7)$$

$$CL = \tilde{q}_2 \quad (8)$$

$$UCL = \tilde{q}_3 + L_t\left(i\tilde{q}r\right)\sqrt{\frac{\lambda(1-(1-\lambda)^{2j})}{2-\lambda}} \tag{9}$$

where L_t is the control limits coefficient taking into account the time varying feature. These control limits are known as time varying control limits it depend upon the *j*. When $j \rightarrow \infty$, $(1-(1-\lambda)^{2j}) \rightarrow 1$ and the time varying limits turns into asymptotic limits given as:

$$LCL = \tilde{q}_1 - L_t\left(i\tilde{q}r\right)\sqrt{\frac{\lambda}{2-\lambda}} \tag{10}$$

$$CL = \tilde{q}_2 \tag{11}$$

$$UCL = \tilde{q}_3 + L_t\left(i\tilde{q}r\right)\sqrt{\frac{\lambda}{2-\lambda}} \tag{12}$$

4. Performance Evaluations and Analysis

A sequence of points plotted on a chart until an out of control signal is identified is known as a run and a series of points in a run is named as Run Length (RL). Typically in control RL is expected to be higher while out of control RL anticipated to be as small as possible. One may see Chakraborti[36] for more useful discussion on RL.

Several measures based on RL are presented in literature to assess the performance of the chart, some are used for specific shifts and others are use for overall of shifts in a process. For this study, only specific shifts measures are considered including Average, Standard deviation and Median RL. The details of these measures are provided in sub succeeding text.

Average Run Length (ARL): It is generally used to assess the performance of a chart for a specific shift value. It refers to the average number of points plotted on a chart until an out of control signal is identified. There are two famous terms used in the control chart for the said purpose, termed as in control and out of control ARL. The in and out of control measures are denoted by ARL_0 and ARL_1. A chart which shows smaller ARL_1 on a particular shift value is considered to be more efficient than other competing charts. An estimate of ARL may be given as:

$$ARL = \sum_{j}^{k} (RL)_j / k \; ; \qquad (13)$$

Standard Deviation Run Length (SDRL): The dispersion of RL may be observed by Variance and Standard Deviation of RL. This shows how much average variation is present in the particular control chart RL values. An estimate of SDRL may be given as:

$$SDRL = \sqrt{\sum_{j}^{k} (RL)_j^2 / k - \left\{ \left(\sum_{j}^{k} (RL)_j / k \right)^2 \right\}} \qquad (14)$$

Median Run Length (MRL): The distribution of RL is mostly skewed and hence Median of RL is another most suitable choice. Median, being wonderful and robust measure to outliers, is a more detailed performance indicator of a chart. We may define it as:

$$MRL = \text{Median(RL)} \qquad (15)$$

Using Monte carlo simulations, we have computed the aforementioned RL properties of both median based TCC and EWMA-TCC charts. The results are reported in Tables 1, 2 & 3 in the form of ARL, MRL and SDRL using several subgroup values at ARL_0=370. Figure 1 presents

RL curves of Tukey and Tukey-EWMA at several subgroup sizes m with fixed λ. Moreover, Figure 2 presents ARL curves of TCC and EWMA-TCC charts at several λ and m. The Performance analysis of these control charts advocate the following (cf. Tables 1-3 and Figures 1-2):

TCC Analysis

- With an increase in subgroups size the ARL_1 of median based TCC chart exhibits a decreasing pattern. For example, TCC having ARL_1 values are 178.13 & 102.38 for m=5 & 10 at δ=.25. Similar pattern may be observed from Figure 1(i).
- The larger the subgroup size gives the smaller MRL and SDRL values for median based TCC. For instance, MRL values of proposed chart are 191,99 & 65 while for SDRL values are 282.91, 279.72 & 129.18 at δ=.25and m=1,5 & 10(cf. Table 2 & 3). It is evident from these results that when subgroup size varies from 1 to 10, we MRL and SDRL values keep getting smaller.

EWMA-TCC Analysis

- The subgroup size and λ affects the performance of the median based EWMA-TCC. As the value subgroup size m increases the median based EWMA-TCC chart exhibits more sensitivity towards shifts. As an example, ARL_1 values of EWMA-TCC are 243.29,125.41 & 63.016 at δ=.25, λ=.75 using subgroup size 1,5 & 10. It is obvious that the subgroup size increases the sensitivity of EWMA-TCC for small to moderate shifts (cf. Table 1). Moreover, in Figure 1(ii &iii) EWMA-TCC design offers more steeper ARL_1 curve at λ=.5 &m=10.

- The large subgroup size ensures the smaller ARL, MRL and SDRL values median based EWMA-TCC. For example, MRL values of EWMA-TC are 164, 89.5 & 41 while the SDRL values are 246.97, 138.16 & 73.82 at δ=.25, λ= .75 and m=1,5 & 10. The similar outcome may be observed for the other values of λ (cf. Tables 2 & 3).
- The median based EWMA-TCC (at λ=.75 & .5) design shows the smaller ARL_1, MRL and SDRL values than median based TCC for small to moderate shifts. It may be seen in Tables 1-3 and Figure 2.

[Insert Tables 1-3 and Figures 1-2]

5. The Real Applications

In this section, we consider two real life data sets for illustration of subgroups based Tukey control charts. The details and application of both data sets is given below.

Application 1: Food manufacturing process

The food industry has started showing tendency towards the utilization of nanotechnology. The nanotechnology plays its role in food ingredients, food packaging, water purification, improving mechanical strength; reducing weight; increasing heat resistance; improving barrier against oxygen, carbon dioxide, ultra-violet radiation, moisture, and volatiles of food packaging materials. Packaging is the process of enclosing the meals material in a container to ensure the delivery of product in fine circumstances to the customer for final use. Therefore, proper packaging performs an essential role in growing the image of company product. Packaging protects the meals and permits it to reach the customer in hygienic and safe and secure condition.

The use of protecting coatings and appropriate packaging by using food industry can surely increase the shelf life of food product (cf. Sekhon[37]). Ahmed at el[38] and Razzaq at el.[39] discussed the importance of Nano-technology in several manufacturing industries.

Figure 3 displays manufacturing setting of different nanotechnology materials that are produced by five different machines. The different subgroup arrangements are found after packing of the food product. We have considered the subgroups formed during food packaging process and collected data from an ongoing process in a firm (cf. Figure 3) located in RWP Pakistan. A group of 50 sample is selected from the process, each batch size ten. For this datasets, we have constructed two control charts (TCC and EWMA-TCC (at λ=.25)) displayed in Figure 4.

It is evident from the display that TCC and EWMA-TCC offer six and ten out of control signals respectively. It shows that the process has a mix up of both small and large shifts and our two charts are helpful in alarming these changes in the process.

[Insert Figures 3&4]

Application 2: hard-bake process

A hard-bake process is used in conjunction with the photolithography in semiconductor manufacturing. Our objective is to establish statistical control of the flow width of the resist in this process. The data on the said variable is taken from Montgomery [35] and we intend to construct TCC and EWMA-TCC. Twenty-five samples, each of size five wafers have been taken when the process is in-control. The interval of time between subgroups is one hour. The display of the wafer data is given in Figure 5. We have implemented both TCC and EWMA-TCC (λ=.25) charts for the said dataset. TCC offered no out of control points while EWMA-TCC detected eight points in an out of control state. This advocates that there are shifts of smaller amount that are not captured by TCC but EWMA-TCC successfully signaled them.

[Insert Figure 5]

6. Summary and Conclusions

This study investigates the performance of median based Tukey and Tukey-EWMA charts using rational subgrouping concept. The manufacturing and industrial data is mostly formed in form of batches at equal time intervals. Traditional Tukey type charts are designed to monitor the individual observations over the time. In the current study, the design of the median based Tukey and Tukey-EWMA (TCC and EWMA-TCC) charts are presented using subgroups. The ARL, SDRL and MRL measures are used to evaluate the performance ability. These median based TCC and EWMA-TCC charts have exhibited very effective performance under rationale subgrouping. Two real life applications are also presented to show the practical demonstration in real processes such as manufacturing, production and packaging, under the subgroups data. The scope of the idea may be extended easily to the Tukey-CUSUM design as well following the same line of action.

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Nomenclature

Acronym/ Symbol	Description	Acronym /Symbol	Description	Acronym/ Symbol	Description
ACL	Asymmetrical Control limits	SDRL	standard deviation run length	ARL_0	In control *ARL*
ARL	Average Run Length	SPC	statistical process control	ARL_1	Out of control *ARL*
CUSUM	Cumulative Sum	SDRL	standard deviation run length	$\tilde{q}_1$	Median of first quartile
CL	Control Limits	MDRL	Median RL	$\tilde{q}_2$	Median of second quartile
EWMA-TCC	Exponentially Weighted Moving Average Tukey Chart	TCC	Tukey Control Chart	$i\tilde{q}r$	Median Interquartile range
Iqr	interquartile range	SPC	Statistical Process Control	X	quality characteristics
LCL	Lower Control Limit	MTCC	Modified TCC	$\tilde{q}_3$	Median third quartile
MEC-TCC	Mixed Tukey EWMA-CUSUM	RL	Run Length	L	Control limits coefficient
S.G	Sub-Grouping	DHF	Dengue Hemorrhagic Fever	R.S	Rational Subgrouping
ARIMA	Auto-Regressive Integrated Moving Averages	SCL	Symmetrical Control Limits	L_t	EWMA's control limits coefficient

References

1. Alemi, F., “ Tukey's control chart. Quality Management in Healthcare”, 13(4), pp.216-221(2004).
2. Borckardt, J.J., Nash, M.R., Hardesty, S., Herbert, J., Cooney, H. and Pelic, C.,” An empirical evaluation of Tukey's control chart for use in health care and quality management applications”, Quality Management in Healthcare, 14(2), pp.112-115(2005).
3. Borckardt, J.J., Pelic, C., Herbert, J., Borckardt, D., Nash, M.R., Cooney, H. and Hardesty, S., “ An Autocorrelation-corrected Nonparametric Control Chart Technique for Health Care Quality Applications”. Quality Management in Healthcare, 15(3), pp.157-162 (2006).
4. Torng, C.C. and Lee, P.H., 2008. ARL performance of the Tukey's control chart. Communications in Statistics-Simulation and Computation, 37(9), pp.1904-1913 (2012).
5. Torng, C.C., Lee, P.H. and Tseng, C.C., “ An Economic Design of Tukey's Control Chart. Journal of the Chinese Institute of Industrial Engineers”, 26(1), pp.53-59 (2009).
6. Lee, P.H., “The effects of Tukey’s control chart with asymmetrical control limits on monitoring of production processes”. African Journal of Business Management, 5(11), pp.4044-4050 (2011).
7. Sukparungsee, S.,” Robustness of Tukey's Control Chart in Detecting a Changes of Parameter of Skew Distributions. International journal of Applied Physics and Mathematics, 2(5), p.379 (2012).
8. Lee, P.H., Kuo, T.I. and Lin, C.S.,” Economically optimum design of Tukey’s control chart with asymmetrical control limits for controlling process mean of skew population distribution”. European Journal of Industrial Engineering, 7(6), pp.687-703(2013).
9. Sukparungsee, S., “Asymmetric Tukey’s control chart robust to skew and non-skew process observation”. International Journal of Mathematical, Computational Science and Engineering, 7(8), pp.51-54 (2013).
10. Khaliq, Q.U.A., Riaz, M. and Alemi, F., “Performance of Tukey's and Individual/Moving Range Control Charts”. Quality and Reliability Engineering International, 31(6), pp.1063-1077 (2014).
11. Tercero-Gomez, V., Ramirez-Galindo, J., Cordero-Franco, A., Smith, M., & Beruvides, M. (2012). “Modified Tukey's control chart”. Communications in Statistics-Simulation and Computation, 41(9), 1566-1579.
12. Mekparyup, J., Kornpetpanee, S. and Saithanu, K., “ The performance of the adjusted Tukey’s control chart under asymmetric distributions”. Global Journal of Pure and Applied Mathematics, 10(5), pp.719-724 (2014).
13. Mekparyup, J., Kornpetpanee, S. and Saithanu, K., “The adjusted Tukey’s control chart with MADM”. International Journal of Applied Environmental Sciences, 9(4), pp.2063-2075 (2014).

14. Saithanu, K., Nojit, T. and Mekparyup, J., “Integration of sensitizing runs rules and Tukey’s control chart in monitoring gamma distribution”. Global Journal of Pure and Applied Mathematics, 11(3), pp.269-272 (2015).
15. Khaliq, Q.U.A., Riaz, and Ahmad, S., “On designing a new Tukey-EWMA control chart for process monitoring”. The International Journal of Advanced Manufacturing Technology, 82(1-4), pp.1-23 (2016).
16. Khaliq, Q.U.A. and Riaz, M.,“Robust Tukey–CUSUM control chart for process monitoring”. Quality and Reliability Engineering International, 32(3), pp.933-948 (2016).
17. Riaz, M., Khaliq, Q.U.A. and Gul, S., “ Mixed Tukey EWMA-CUSUM control chart and its applications”. Quality Technology & Quantitative Management, pp.1-34(2017).
18. Nelson, L.S., “Setting up a control chart using subgroups of varying sizes”. Journal of quality technology, 22(3), pp.245-246 (1990).
19. Sefik, M., “Importance of the rational subgroups in designing control charts”. Computers & Industrial Engineering, 35(1-2), pp.205-208 (1998).
20. Nelson, L.S., “Control charts-rational subgroups and effective applications”. Journal of Quality Technology, 20(1), pp.73-75 (1988).
21. Hillier, F.S., “X chart control limits based on a small number of subgroups”. Applied Mathematics and Statistics Laboratories, Stanford University (1962).
22. Hillier, F.S., “Small sample probability limits for the range chart”. Journal of the American Statistical Association, 62(320), pp.1488-1493 (1967).
23. Wheeler, Donald J., and David S., "Understanding Statistical Process Control." SPC PRESS (Statistical Process Control); 3rd Edition, ISBN-13: 978-0945320692,(1992).
24. Quevedo, V., Vegas, S. and Vining, G.,.” A tutorial on an iterative approach for generating Shewhart control limits”. Quality Engineering, 28(3), pp.305-312(2016)
25. Djauhari, M.A., Sagadavan, R. and Li, L.S., 2016. “Monitoring multivariate process variability when sub-group size is small”. Quality Engineering, 28(4), pp.429-440(2016).
26. Abbasi, S.A., Ahmad, S. and Riaz, M.,. “On enhanced sensitivity of nonparametric EWMA control charts for process monitoring”. Scientia Iranica. Transaction E, Industrial Engineering, 24(1), p.424(2017).
27. Mahmood, T., Nazir, H.Z., Abbas, N., Riaz, M. and Ali, A., 2017. Performance evaluation of joint monitoring control charts. Scientia Iranica, 24(4), pp.2152-2163(2017).
28. Ajadi, J.O. and Riaz, M., . “New memory-type control charts for monitoring process mean and dispersion”. Scientia Iranica. Transaction E, Industrial Engineering, 24(6), pp.3423-3438(2017).
29. Abujiya, M.R., Lee, M.H. and Riaz, M., “ New EWMA S^ sup 2^ control charts for monitoring of process dispersion”. Scientia Iranica. Transaction E, Industrial Engineering, 24(1), p.378 (2017).
30. Mukherjee, A., “ Distribution-free phase-II exponentially weighted moving average schemes for joint monitoring of location and scale based on subgroup samples”. The

International Journal of Advanced Manufacturing Technology, 92(1-4), pp.101-116(2017).
31. Ansorena, Í. L. " Statistical process control and quality of service at seaports." International Journal of Productivity and Quality Management, 24(2), 165-176(2018).
32. Hussain, S., Song, L., Ahmad, S., & Riaz, M. (2018). "New Interquartile Range EWMA Control Charts with Applications in Continuous Stirred Tank Rector Process". Arabian Journal for Science and Engineering, 1-19(2018).
33. Abtew, M. A., Kropi, S., Hong, Y., & Pu, L. " Implementation of Statistical Process Control (SPC) in the Sewing Section of Garment Industry for Quality Improvement". Autex Research Journal, 18(2), 160-172(2018).
34. Huberts, L. C., Schoonhoven, M., & Does, R. J. " The effect of continuously updating control chart limits on control chart performance". Quality and Reliability Engineering International. https://doi.org/10.1002/qre.2448 (2019).
35. Montgomery, D.C., "Introduction to statistical quality control". John Wiley & Sons (New York, (2009).
36. Chakraborti, S., " Run length, average run length and false alarm rate of Shewhart X-bar chart: exact derivations by conditioning". Communications in Statistics-Simulation and Computation, 29(1), pp.61-81 (2000).
37. Sekhon, B.S., "Food nanotechnology–an overview". Nanotechnology, science and applications, 3, p.1(2010).
38. Ahmed, T., Imdad, S., Yaldram, K. and Raza, S.M.," Awareness and attitude about nanotechnology in Pakistan." Journal of Nano Education, 7(1), pp.44-51(2015).
39. Razzaq, A., Ammara, R., Jhanzab, H. M., Mahmood, T., Hafeez, A., and Hussain, S., "Journal of Nano science and Technology." Journal of Nano science and Technology, 2(1), 55-58.,(2016).

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		TCC			EWMA-TCC(λ=.75)			EWMA-TCC(λ=.5)	
δ/m	**m=1**	**m=5**	**m=10**	**m=1**	**m=5**	**m=10**	**m=1**	**m=5**	**m=10**
0	370.12	370.1404	370.52	370.00	370.14	370	369.124	371.3	370.41
0.25	283.13	178.69	102.38	235.29	125.41	63.01	196.62	83.83	40.17
0.5	158.21	56.10	21.72	109.47	35.06	10.68	71.959	19.02	8.08
0.75	80.33	19.57	6.38	51.01	11.18	4.042	29.88	7.0938	3.53
1	44.27	8.15	2.69	25.04	5.57	2.14	15.34	3.87	2.17
1.25	25.10	4.06	1.59	13.94	2.90	1.46	8.97	2.65	1.621
1.5	15.03	2.39	1.18	8.55	2.04	1.18	5.92	2.0135	1.29
2	6.28	1.29	1.009	4.11	1.31	1.01	3.42	1.405	1.0307
2.5	2.55	1.05	1	2.99	1.05	1	2.11	1.16	1.00
3	1.97	1	1	1.79	1	1	1.861	1.01	1
4	1.19	1	1	1.18	1	1	1.29	1	1

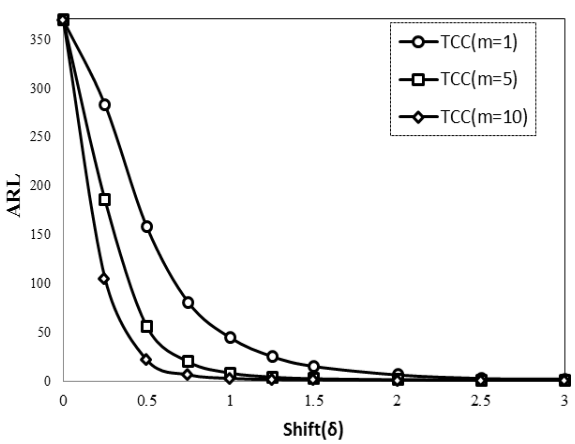
Table 1: ARL performance of TCC and EWMA-TCC using different subgroup sizes (m)

		TCC			EWMA-TCC(λ=.75)			EWMA-TCC(λ=.5)	
δ/m	m=1	m=5	m=10	m=1	m=5	m=10	m=1	m=5	m=10
0	256	195	156	257	201	166	256	188	155
0.25	191	99	65	164	89.5	41	133	51	28
0.5	108	31	14	76	24	7	50	12	6
0.75	57	12	4	36	8	3	21	5	3
1	30	5	2	18	4	2	11	3	2
1.25	17	3	1	10	3	1	7	2	2
1.5	11	2	1	6	2	1	5	2	1
2	4	1	1	3	1	1	3	1	1
2.5	2	1	1	3	1	1	2	1	1
3	1	1	1	2	1	1	2	1	1
4	1	1	1	1	1	1	1	1	1

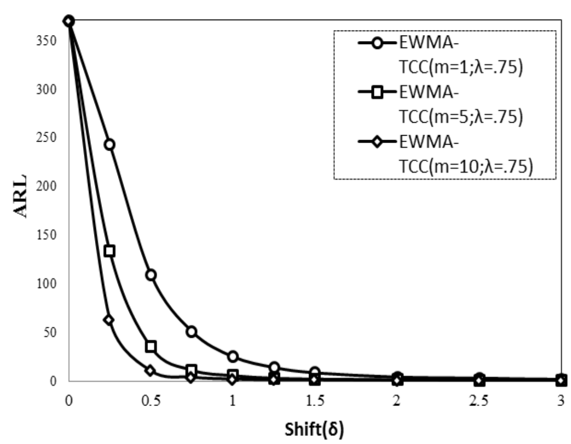
Table 2: MRL performance of TCC and EWMA-TCC using different subgroup sizes (m)

Table 3: SDRL performance of TCC and EWMA-TCC using different subgroup sizes (m)

	TCC			EWMA-TCC(λ=.75)			EWMA-TCC(λ=.5)		
δ/m	m=1	m=5	m=10	m=1	m=5	m=10	m=1	m=5	m=10
0	376.64	374.37	370.173	367.74	528.70	369.21	373.40	564.80	469.25
0.25	279.72	282.92	129.18	246.97	138.16	76.82	194.63	131.42	53.33
0.5	157.79	77.96	23.47	110.36	34.32	10.99	70.062	20.30	6.88
0.75	81.67	25.82	6.24	50.78	9.59	3.079	28.52	5.86	2.049
1	42.73	9.17	2.23	23.84	4.17	1.23	13.41	2.40	0.97
1.25	25.21	4.07	0.99	12.86	1.78	0.69	6.89	1.31	0.64
1.5	14.65	2.00	0.46	7.52	1.187	0.39	4.233	0.87	0.47
2	5.84	0.62	0.09	3.038	0.53	0.099	1.87	0.53	0.17
2.5	4.52	0.24	0.08	1.56	0.235	0	0.98	0.33	0.02
3	1.43	0.08	0.05	0.95	0.099	0	0.74	0.13	0.01
4	0.47	0	0.014	0.42	0	0	0.47	0	0



(i)



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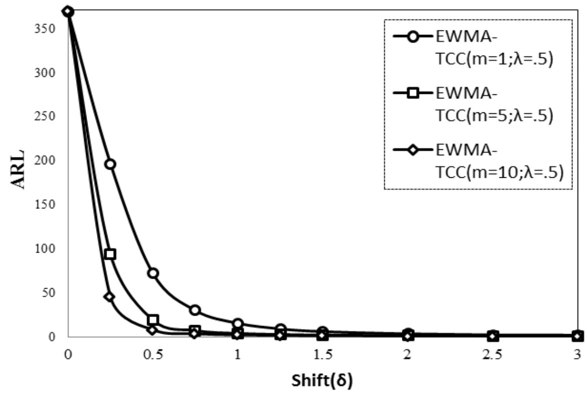
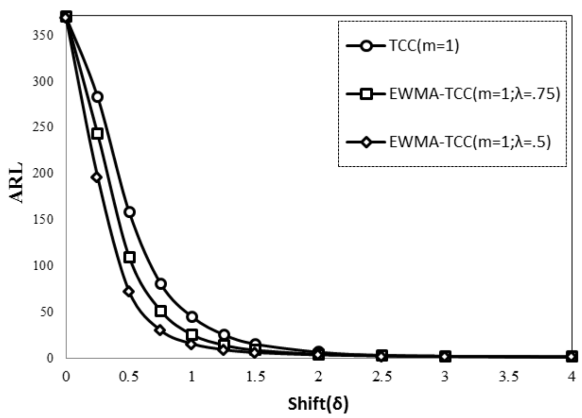


(iii)

Figure1: ARL performance of TCC and EWMA-TCC at different subgroup sizes (m=1,5,10) for: i) TCC; ii) EWMA-TCC at λ=.7; iii). EWMA-TCC at λ=.5



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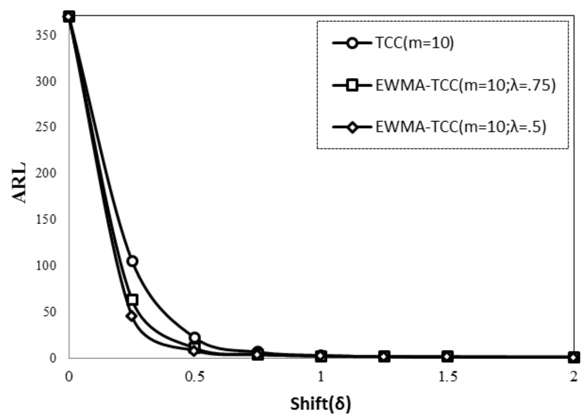

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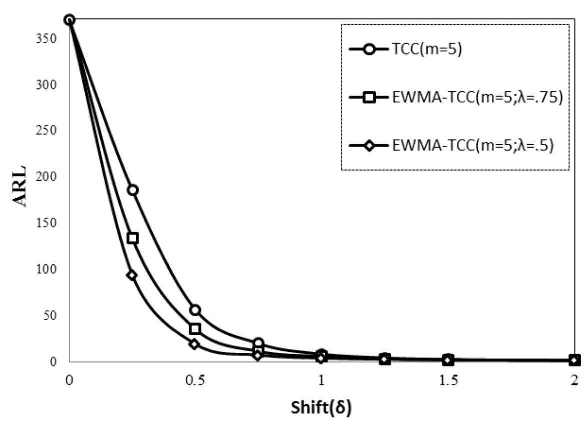


(iii)

Figure 2:Comparative performance of EWMA-TCC and TCC at several λ and subgroup size for: i).TCC and EWMA-TCC (λ=.75 &.5) designs at m=1; ii). TCC and EWMA-TCC (λ=.75 &.5) designs at m=5; iii).TCC and EWMA-TCC (λ=.75 &.5) designs at m=10

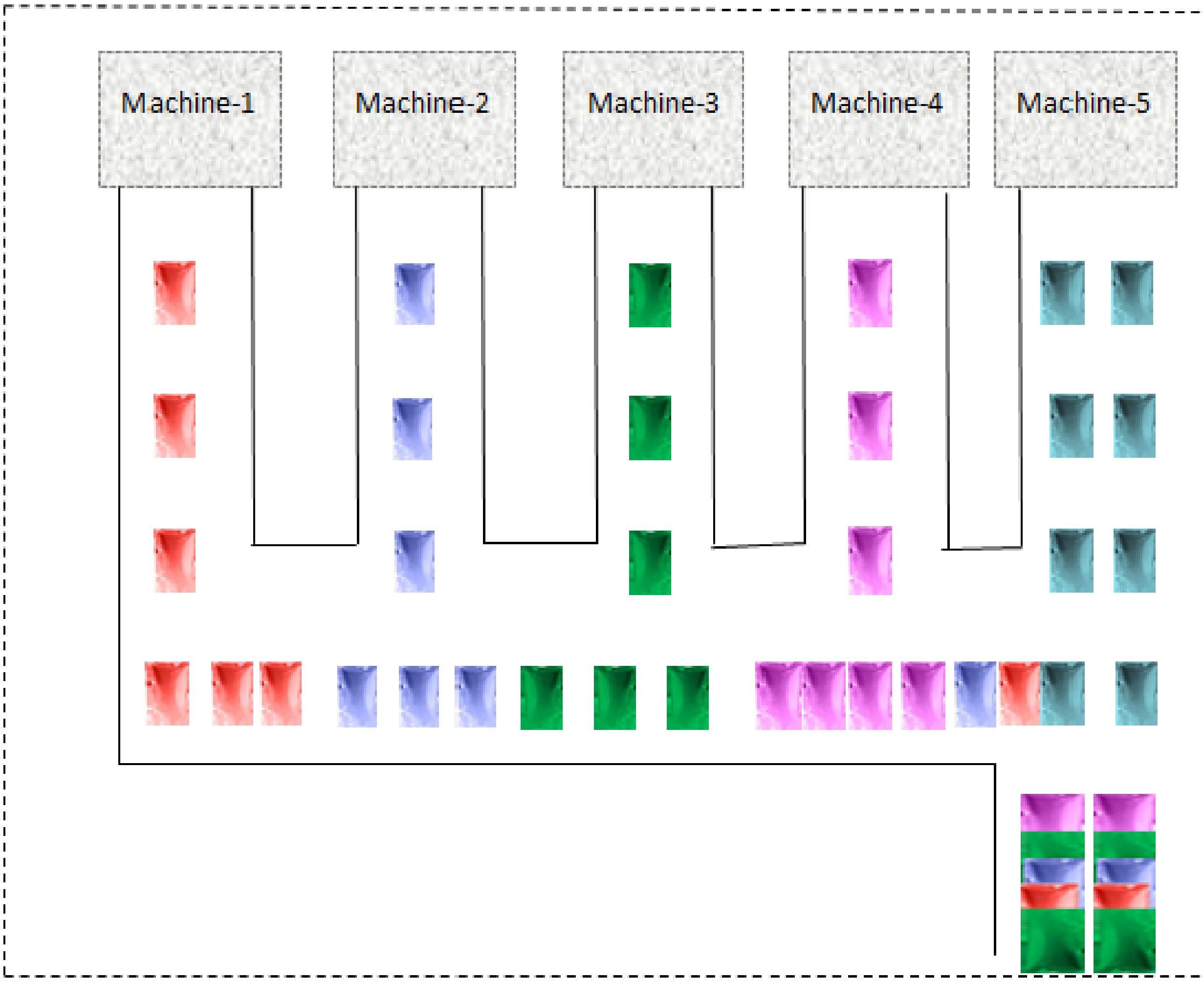
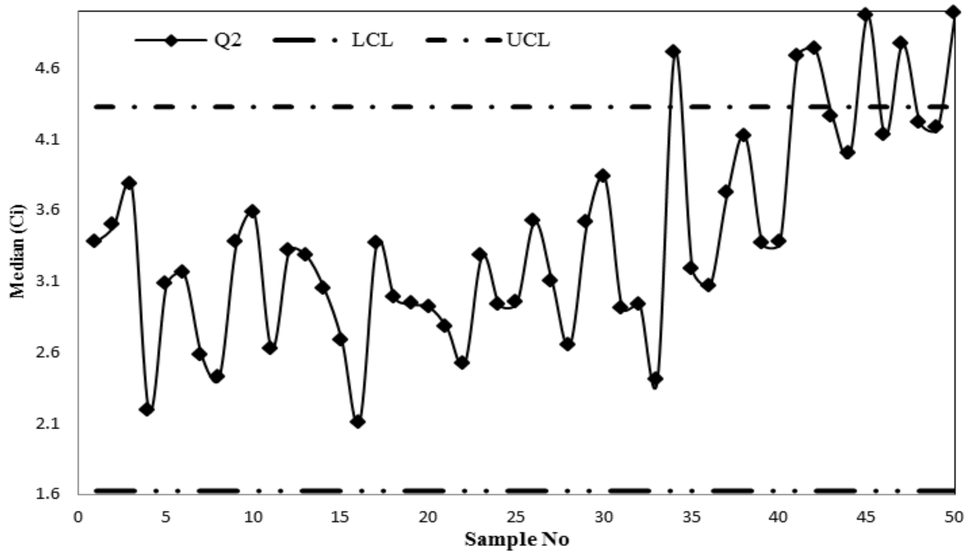
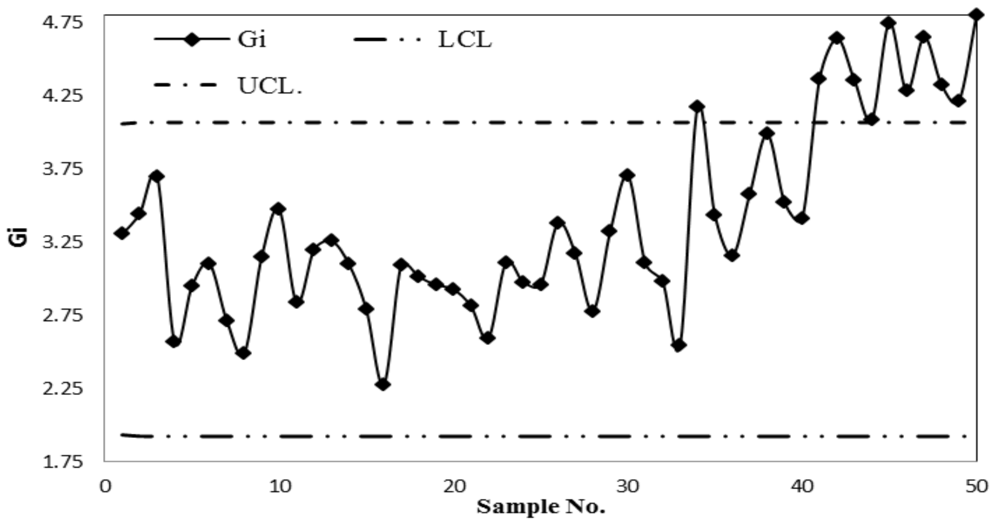


Figure 3: Procedural flow of forming subgroup during packaging process

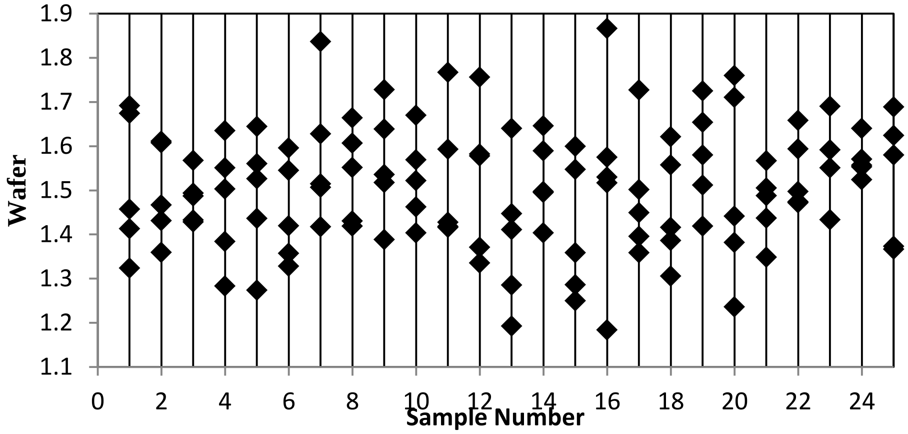


(i)



(ii)

Figure 4: Implementation of Charts on manufacturing data i). TCC; ii). EWMA-TCC



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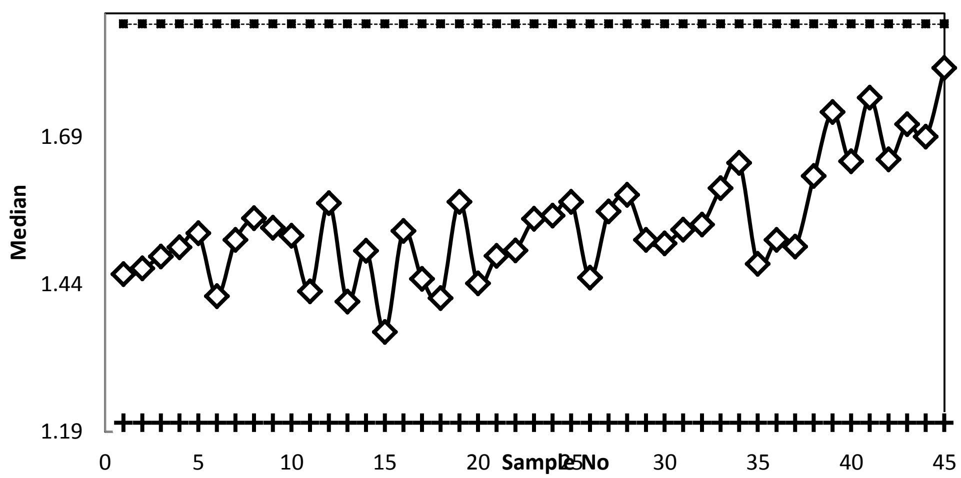

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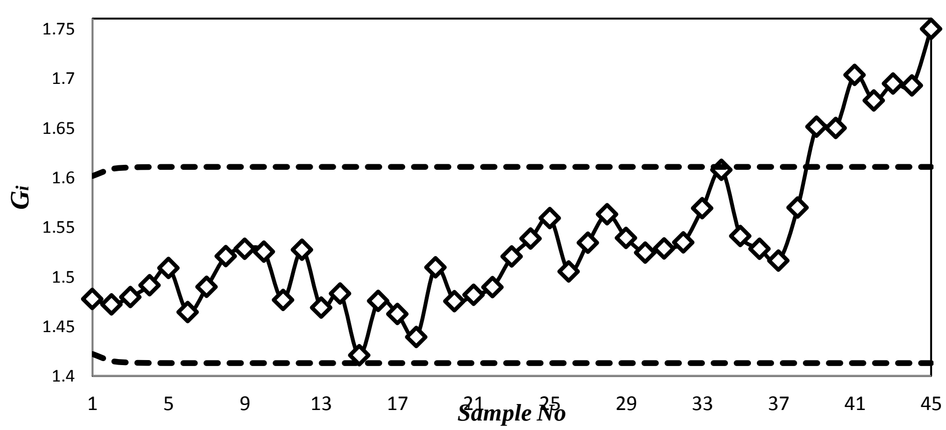

(iii)**Figure 5: Implementation of Charts on Wafer data for: i) Display of data; ii). TCC; iii). EWMA-TCC**