Scientia Iranica A (2016) 23(2), 508–519

Sharif University of Technology
Scientia Iranica
Transactions A: Civil Engineering
www.scientiairanica.com

A hybrid optimization algorithm for the optimal design of laterally-supported castellated beams

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Received 10 January 2015; received in revised form 19 May 2015; accepted 22 June 2015

KEYWORDS
Colliding Bodies Optimization (CBO);
Particle Swarm Optimization (PSO);
A hybrid CBO-PSO algorithm;
Optimum design of castellated beams;
Hexagonal opening beam;
Cellular opening beam.

Abstract. In this paper, a hybrid method is developed for optimum design of castellated beams by combining two meta-heuristic algorithms, namely the Colliding Bodies Optimization (CBO) and Particle Swarm Optimization (PSO). In this hybrid algorithm (CBO-PSO), positive features of PSO are added to the CBO. Two common types of laterally supported castellated beams are considered to be the design problems: beams with hexagonal openings and beams with circular openings. These beams have found widespread usage in buildings because of great savings in materials and construction costs. Here, the minimum cost is taken as the design objective function and the new hybrid method is utilized for obtaining the solution of some benchmark problems. Comparison of the results of the CBO-PSO with those of some other meta-heuristics demonstrates the capability of the presented optimization algorithm. For most of the examples, the results obtained by CBO-PSO have less cost than those of the other considered methods. It is also concluded that the beams with hexagonal openings require smaller amount of steel, hence being superior to the beams with cellular openings.

1. Introduction

Nowadays, due to huge extension in size and dimension of the structures, there has been a great increase in weight and cost of construction materials used for structures. Therefore, it is not surprising that a lot of attention is being paid by engineers to optimal design of the structures which lead to a significant decrease in their cost.

Since the 1940s, the production of structural beams with higher strength and lower cost has been an asset to engineers in their efforts to design more efficient steel structures. Due to the limitations on maximum allowable deflections, the use of sections with heavy weight and high capacity in the design problem cannot always be beneficial. As a result, several new methods have been created for increasing the stiffness of steel beams without increase in weight of the required steel. Hence, castellated and cellular beams have been utilized extensively in recent years [1].

In design of steel structures, beams with web-opening are widely used to pass the underfloor services ducts such as water pipes and air ducts. Castellated beam is created from a standard wide-flange beam by cutting it longitudinally in a zigzag or semi-circular pattern, separating and offsetting the two halves, and welding them back together. The resulting holes in the webs permit mechanical ducts, plumbing, and electrical lines to pass through the beam rather than beneath the beam. Web-openings have been used for many years in structural steel beams in a great variety of applications because of the necessity and economic advantage. The important advantage of the steel beam castellation process is that the designer can increase the depth of

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a beam to raise its strength without adding steel. The resulting castellated beam is approximately 50% deeper and much stronger than the original unaltered beam [2-5].

In recent years, a great deal of progress has been made in the design of steel beams with web-openings and cellular beam is one of them. Cellular beam is the modern form of the traditional castellated beam, but with a far wider range of applications for floor beams, in particular. Cellular beams are steel sections with circular openings that are made by cutting a rolled beam web in a half-circular pattern along its centerline and re-welding the two halves of hot rolled steel sections as shown in Figure 1. This opening increases the overall

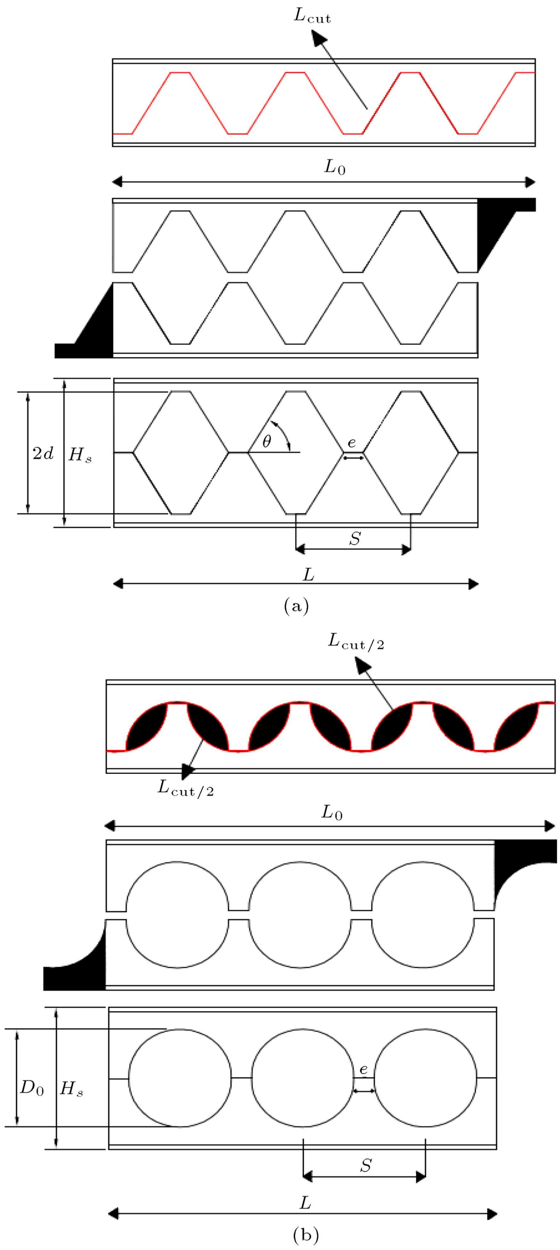


Figure 1. (a) A castellated beam with hexagonal opening. (b) A castellated beam with circular opening.

beam depth, moment of inertia, and section modulus without increasing the overall weight of the beam [6].

The Colliding Bodies Optimization method (CBO) is one of the recently developed meta-heuristic algorithms that utilizes simple formulation and it requires no parameter tuning. This algorithm is based on one-dimensional collisions between two bodies, where each agent solution is modeled as a body [7,8].

Particle Swarm Optimization (PSO) is based on the behavior of a colony or swarm of insects (such as ants, termites, bees, and wasps), a flock of birds, or a school of fish. This algorithm was originally proposed by Kennedy and Eberhart in 1995. A basic variant of the PSO algorithm works by having a population (swarm) of candidate solutions (particles). These particles move around in the search-space using a few simple formulas. Each particle iteratively moves across the search-space and is attracted to the position of the best fitness (evaluation of the objective function) historically achieved by the particle itself (local best) and by the best among the neighbors of the particle (global best) [9,10].

The main objective of this paper is to present a hybrid CBO-based algorithm (combined with the PSO) to find an optimum design of castellated beams. For this purpose, the positive properties of the PSO will be added to the CBO.

According to the above-mentioned content, the present study is organized as follows: In Section 2, the design of castellated beam is introduced. Statement of the optimization design problem is formulated in Section 3, based on The Steel Construction Institute Publication Number 100 and Euro code 3. In Section 4, the CBO algorithm and the PSO approach are briefly introduced. Also, the new hybrid method is presented in this section. In Section 5, the cost of castellated beam as the design objective function is minimized, and finally, Section 6 concludes the paper.

2. Design of castellated beams

Beams must be sufficiently strong to carry the applied bending moments and shear forces. The performance of any beam is dependent upon the physical dimensions as well as the cross-section geometry and shape. Due to the presence of holes in the web, the structural behavior of castellated steel beam is different from that of the solid web beams. At present, there is not a prescribed design method due to the complexity of the behavior of castellated beams and their associated modes of failure [2]. The strength of a beam with various web openings is determined by considering the interaction of the flexure and shear at the openings. There are many failure modes to be considered in the design of a beam with web opening consisting of lateral-torsional buckling, Vierendeel mechanism, flexural mechanism, rupture of welded joints, and web post buckling. Lateral-torsional buckling may occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. In this paper, it is assumed that the compression flange of the castellated beam is restrained by the floor system. Therefore, the overall buckling strength of the castellated beam is omitted from the design considerations. These modes are closely associated with beam geometry, shape parameters, type of loading, and provision of lateral supports. In the design of castellated beams, these criteria should be considered [11-17].

2.1. Overall beam flexural capacity

This mode of failure can occur when a section is subjected to pure bending. In the span subjected to pure bending moment, the tee-sections above and below the holes yield in a manner similar to that of a plain webbed beam. Therefore, under applied load combinations, the castellated beam should have sufficient flexural capacity to be able to resist the external loading [12,13]:

$$M_U \leq M_P = A_{LT} P_Y H_U, \tag{1}$$

where A_{LT} is the area of lower tee, P_Y is the design strength of steel, and H_U is the distance between centers of gravities of upper and lower tees.

2.2. Beam shear capacity

In the design of castellated beams, it is necessary to control two modes of shear failure. The first one is the vertical shear capacity and the upper and lower tees should undergo it. The sum of the shear capacities of the upper and lower tees is checked using the following equations [2,13]:

$$P_{VY} = 0.6 P_Y (0.9 A_{WUL}) \quad \text{for circular opening,}$$

$$P_{VY} = \frac{\sqrt{3}}{3} P_Y (A_{WUL}) \quad \text{for hexagonal openning,} \tag{2}$$

where A_{WUL} is the total area of the webs of tees.

The second one is the horizontal shear capacity. It is developed in the web post due to the change in axial forces in the tee-section as shown in Figure 2. Web post with too short mid-depth welded joints may fail prematurely when horizontal shear exceeds the yield strength. The horizontal shear capacity is checked using the following equations [2,13]:

$$P_{VH} = 0.6 P_Y (0.9 A_{WP}) \quad \text{for circular opening}$$

$$P_{VH} = \frac{\sqrt{3}}{3} P_Y (A_{WP}) \quad \text{for hexagonal openning,} \tag{3}$$

where A_{WP} is the minimum area of web post.

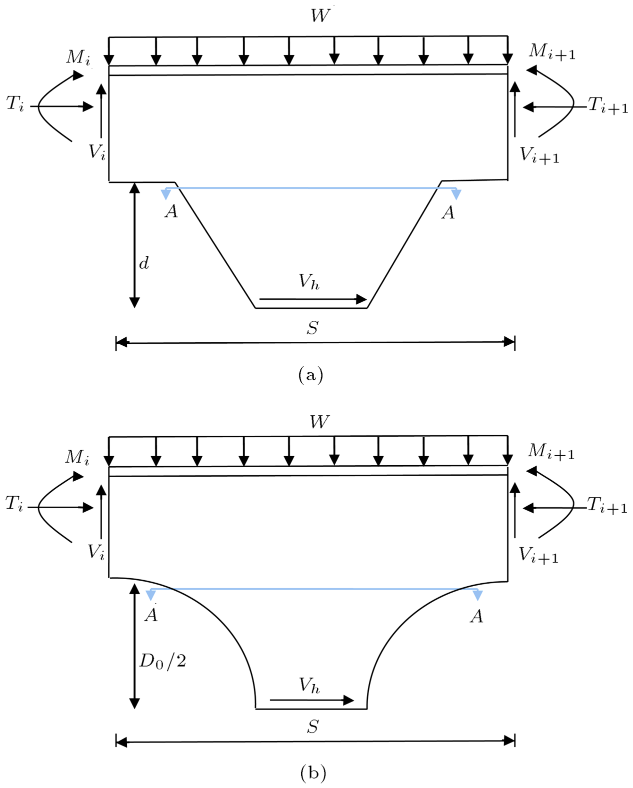


Figure 2. Horizontal shear in the web post of castellated beams: (a) Hexagonal opening; and (b) circular opening.

2.3. Flexural and buckling strength of web post

As mentioned above, it is assumed that the compression flange of the castellated beam is restrained by the floor system. Thus, the overall buckling of the castellated beam is omitted from the design considerations. The web post buckling capacity in castellated beam is given by [2,13]:

$$\frac{M_{\text{MAX}}}{M_E} = \left[C_1.\alpha - C_2.\alpha^2 - C_3 \right], \tag{4}$$

where $\alpha = \frac{S}{2d}$ for hexagonal openings, and $\alpha = \frac{S}{D_0}$ for circular openings, also M_{MAX} is the maximum allowable web post moment and M_E is the web post capacity at critical section A-A shown in Figure 2. C_1, C_2, and C_3 are constants obtained by the following expressions:

$$C_1 = 5.097 + 0.1464(\beta) - 0.00174(\beta)^2, \tag{5}$$

$$C_2 = 1.441 + 0.0625(\beta) - 0.000683(\beta)^2, \tag{6}$$

$$C_3 = 3.645 + 0.0853(\beta) - 0.00108(\beta)^2, \tag{7}$$

where $\beta = \frac{2d}{t_w}$ for hexagonal openings, and $\beta = \frac{D_0}{t_w}$ for circular openings, S is the spacing between the centers of holes, d is the cutting depth of hexagonal opening, D_0 is the holes diameter, and t_w is the web thickness.

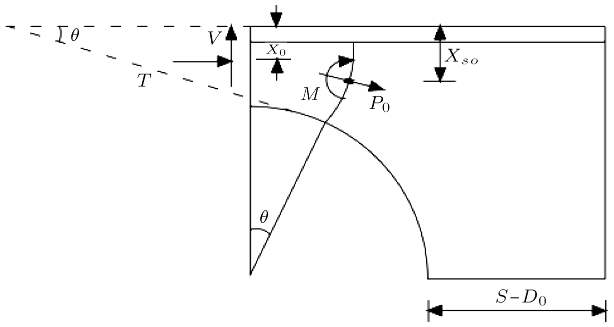


Figure 3. Olander's curved beam approach.

2.4. Vierendeel bending of upper and lower tees

Vierendeel mechanism is the most common failure for perforated steel beams that results in the formation of four plastic hinges above and below the web opening. It is always critical in steel beams with web openings, where global shear force is transferred across the opening length, and the Vierendeel moment is resisted by the local moment resistances of the tee-sections above and below the web openings.

The overall Vierendeel bending resistance depends on the local bending resistance of the web-flange sections. This mode of failure is associated with high shear forces acting on the beam. The Vierendeel bending stresses in the circular opening is obtained by using the Olander's approach in Figure 3. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as follows [13]:

$$\frac{P_0}{P_U} + \frac{M}{M_P} \leq 1.0, \tag{8}$$

where P_0 and M are the force and the bending moment on the section, respectively; P_U is equal to the area of critical section $\times P_Y$; M_P is calculated as the plastic modulus of critical section in plastic section or elastic section modulus of critical section for other sections.

The plastic moment capacity of the tee-sections in castellated beams with hexagonal opening are calculated independently. The total of the plastic moments is equal to the sum of the Vierendeel resistances of the above and below tee-sections [2]. The interaction between Vierendeel moment and shear forces should be checked by the following expression:

$$V_{\text{OMAX}}.e - 4M_{TP} \leq 0, \tag{9}$$

where V_{OMAX} and M_{TP} are the maximum shear force and the moment capacity of tee-section, respectively.

2.5. Deflection of castellated beam

Serviceability checks are of high importance in design, especially for beams with web opening where the deflection due to shear forces is significant. The deflection of a castellated beam under applied load combinations should not exceed span/360. In castellated beams with circular opening, the deflection at each point is calculated by the following expression:

$$Y_{TOT} = Y_{MT} + Y_{WP} + Y_{AT} + Y_{ST} + Y_{SWP}, \tag{10}$$

where Y_{MT}, Y_{WP}, Y_{AT}, Y_{ST}, and Y_{SWP} are deflection due to bending moment in tee, deflection due to bending moment in web post of beam, deflection due to axial force in tee, deflection due to shear in tee, and deflection due to shear in web post, respectively. These equations are provided in [13].

For a castellated beam with hexagonal opening and length L subjected to transverse loading, the total deflection is composed by two terms: the first term corresponds to pure moment action, f_b, and the second one corresponds to shear action, f_s. Thus, the total deflection can be calculated by the following expression:

$$f = f_b + f_s = c_1 L^3 + c_2 L, \tag{11}$$

c_1 and c_2 are determined by means of a curve fitting technique [15].

3. Formulation of the optimization problem

The main initiative for producing and using castellated beam is to suppress the cost of material by applying more efficient cross sectional shapes made from standard profiles in combination with aesthetic and architectural design considerations.

In a castellated beam, there are many factors that require special considerations when estimating the cost of beam, such as man-hours of fabrication, weight, price of web cutting, and welding process. At this study, it is assumed that the costs associated with man-hours of fabrication for hexagonal and circular opening are identical. Thus, the objective function includes three parts: the beam weight, price of the cutting, and price of the welding. The objective function can be expressed as:

$$F_{\text{cost}} = \rho A_{\text{initial}}(L_0).P_1 + L_{\text{cut}}.P_2 + L_{\text{weld}}.P_3, \tag{12}$$

where P_1, P_2, and P_3 are the price of the weight of the beam per unit weight, length of cutting and welding for per unit length, respectively, L_0 is the initial length of the beam before castellation process, ρ is the density of steel, A_{initial} is the area of the selected universal beam section, L_{cut} and L_{weld} are the cutting length and welding length, respectively. The length of cutting is different for hexagonal and circular web-openings. The dimension of the cutting length is described by the following equations:

For circular opening:

$$L_{\text{cut}} = \pi D_0.NH + 2e(NH+1) + \frac{\pi D_0}{2} + e. \tag{13}$$

For hexagonal opening:

$$L_{\text{cut}} = 2NH\left(e + \frac{d}{\sin(\theta)}\right) + 2e + \frac{d}{\sin(\theta)}, \tag{14}$$

where NH is the total number of holes, e is the length of horizontal cutting of web, D_0 is the diameter of holes, d is the cutting depth, and θ is the cutting angle.

Also, the welding length for both circular and hexagonal openings is determined by Eq. (15):

$$L_{\text{weld}} = e(NH+1). \tag{15}$$

As an example, in Figure 1(a), the number of holes is equal to 3. Therefore, the total length of cutting can be expressed by the following equation:

$$L_{\text{cut}} = 8e + 7\left(\frac{d}{\sin(\theta)}\right). \tag{16}$$

Similarly, for cellular beams, the same equations can be obtained. L_{cut} is shown for both circular and hexagonal openings in Figure 1.

3.1. Design of castellated beam with circular opening

Design process of a cellular beam consists of three phases: selection of a rolled beam, selection of a diameter, and spacing between the centers of holes or total number of holes in the beam as shown in Figure 1, [13,14]. Hence, the sequence number of the rolled beam section in the standard steel sections tables, the circular holes diameter, and the total number of holes are taken as design variables in the optimum design problem. The optimum design problem formulated by considering the constraints explained in the previous sections can be expressed as follows.

Find an integer design vector $\{X\} = \{x_1, x_2, x_3\}^T$ where x_1 is the sequence number of the rolled steel profile in the standard sections list, x_2 is the sequence number for the hole diameter which contains various diameter values, and x_3 is the total number of holes for the cellular beam [13]. Hence, the design problem can be expressed as:

Minimize Eq. (12):

Subjected to:

$$g_1 = 1.08 \times D_0 - S \leq 0, \tag{17}$$

$$g_2 = S - 1.60 \times D_0 \leq 0, \tag{18}$$

$$g_3 = 1.25 \times D_0 - H_s \leq 0, \tag{19}$$

$$g_4 = H_s - 1.75 \times D_0 \leq 0, \tag{20}$$

$$g_5 = M_U - M_P \leq 0, \tag{21}$$

$$g_6 = V_{\text{MAXSUP}} - P_V \leq 0, \tag{22}$$

$$g_7 = V_{\text{OMAX}} - P_{VY} \leq 0, \tag{23}$$

$$g_8 = V_{\text{HMAX}} - P_{VH} \leq 0, \tag{24}$$

$$g_9 = V_{\text{A-AMAX}} - M_{\text{WMAX}} \leq 0, \tag{25}$$

$$g_{10} = V_{\text{TEE}} - 0.50 \times P_{VY} \leq 0, \tag{26}$$

$$g_{11} = \frac{P_0}{P_U} + \frac{M}{M_P} - 1.0 \leq 0, \tag{27}$$

$$g_{12} = Y_{\text{MAX}} - L/360 \leq 0, \tag{28}$$

where t_W is the web thickness; H_s and L are the overall depth and the span of the cellular beam; S is the distance between centers of holes; M_U is the maximum moment under the applied loading; M_P is the plastic moment capacity of the cellular beam; V_{MAXSUP} is the maximum shear at support; V_{OMAX} is the maximum shear at the opening; V_{HMAX} is the maximum horizontal shear; $M_{\text{A-AMAX}}$ is the maximum moment at A-A section shown in Figure 2. M_{WMAX} is the maximum allowable web post moment; V_{TEE} represents the vertical shear on the tee at $\theta = 0$ of web opening; P_0 and M are the internal forces on the web section as shown in Figure 3; and Y_{MAX} denotes the maximum deflection of the cellular beam [13,17].

3.2. Design of castellated beam with hexagonal opening

In design of castellated beams with hexagonal openings, the design vector includes four design variables: selection of a rolled beam, selection of a cutting depth, spacing between the centers of holes or total number of holes in the beam, and cutting the angle as shown in Figure 2. Hence, the optimum design problem can be expressed as follows.

Find an integer design vector $\{X\} = \{x_1, x_2, x_3, x_4\}^T$, where x_1 is the sequence number of the rolled steel profile in the standard sections list, x_2 is the sequence number for the cutting depth which contains various values, x_3 is the total number of holes for the castellated beam, and x_4 is the cutting angle. So, the design problem turns out to be as follows:

Minimize Eq. (12).
Subjected to:

$$g_1 = d - \frac{3}{8}.(H_s - 2t_f) \leq 0, \tag{29}$$

$$g_2 = (H_s - 2t_f) - 10 \times (d_T - t_f) \leq 0, \tag{30}$$

$$g_3 = \frac{2}{3}.d.\cot\phi - e \leq 0, \tag{31}$$

$$g_4 = e - 2d.\cot\phi \leq 0, \tag{32}$$

$$g_5 = 2d.\cot\phi + e - 2d \leq 0, \tag{33}$$

$$g_6 = 45° - \phi \leq 0, \tag{34}$$

$$g_7 = \phi - 64° \leq 0, \tag{35}$$

$$g_8 = M_U - M_P \leq 0, \tag{36}$$

$$g_9 = V_{\text{MAXSUP}} - P_V \leq 0, \tag{37}$$

$$g_{10} = V_{\text{OMAX}} - P_{VY} \leq 0, \tag{38}$$

$$g_{11} = V_{\text{HMAX}} - P_{VH} \leq 0, \tag{39}$$

$$g_{12} = M_{\text{A-AMAX}} - M_{\text{WMAX}} \leq 0, \tag{40}$$

$$g_{13} = V_{\text{TEE}} - 0.50 \times P_{VY} \leq 0, \tag{41}$$

$$g_{14} = V_{\text{OMAX}}.e - 4M_{TP} \leq 0, \tag{42}$$

$$g_{15} = Y_{\text{MAX}} - L/360 \leq 0, \tag{43}$$

where t_f is the flange thickness, d_T is the depth of the tee-section, M_P is the plastic moment capacity of the castellated beam, $M_{\text{A-AMAX}}$ is the maximum moment at A-A section, shown in Figure 2, M_{WMAX} is the maximum allowable web post moment, V_{TEE} represents the vertical shear on the tee, M_{TP} is the moment capacity of tee-section, and Y_{MAX} denotes the maximum deflection of the castellated beam with hexagonal opening [2].

4. A hybrid colliding bodies optimization and particle swarm optimization

In this section, the hybrid CBO and PSO methods is presented. In order to create the hybrid approach, the CBO is used as the main algorithm and the positive features of the PSO are added to it. The hybrid algorithm utilizes the location of the global and local best points to improve the searching process. A summary of these methods is described in the following subsections.

4.1. Particle swarm optimization algorithm

Particle swarm optimization, first developed by Kennedy and Eberhart [9], is a population-based meta-heuristic method. The development of this algorithm follows from observations of social behaviors of animals, such as bird flocking and fish schooling. The theory of PSO describes a solution process in which each particle flies through the multidimensional search space while the velocity and position of the particle are constantly updated according to the best previous performance of the particle or of the particle's neighbors, as well as the best performance of the particles in the entire population [18].

The velocity vector is used to update the current position of each particle in the swarm. Likewise, the velocity vector is updated utilizing a memory in which the best position of each particle and the best position among all particles are stored.

The position of the ith particle at iteration $k+1$ is calculated using the following equation:

$$x^i_{k+1} = x^i_k + v^i_{k+1}, \tag{44}$$

where, x^i_{k+1} is the new position, x^i_k is the position at the kth iteration, and v^i_{k+1} is the updated velocity vector of the ith particle. The velocity vector of each particle is determined by:

$$v^i_{k+1} = \omega.v^i_k + c_1.r_1.\left(P^i_k - X^i_k\right) + c_2.r_2.\left(P^g_k - X^i_k\right), \tag{45}$$

where, v^i_k is the velocity vector at iteration k; r_1 and r_2 are two random numbers between 0 and 1; P^i_k represents the best ever position of particle i, local best; P^g_k is the best position among all particles in the swarm up to iteration k; c_1 and c_2 are two acceleration constants; and ω is the inertia weight.

4.2. Colliding bodies optimization algorithm

The Colliding bodies optimization algorithm is one of the recently developed meta-heuristic search algorithms [7]. It is a population-based search approach, where each agent is considered as a Colliding Body (CB) with mass m. The idea of the CBO algorithm is based on observation of a collision between two objects in one dimension, in which one object collides with another object and they move toward minimum energy level [8].

In the CBO algorithm, each solution candidate, X_i, is considered to be a Colliding Body (CB). The massed objects are composed of two main equal groups, i.e. stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects; (ii) to push stationary objects towards better positions. After the collision, the new positions of the colliding bodies are updated based on their new velocities.

The pseudo-code for the CBO algorithm can be summarized as follows:

Step 1. *Initialization.* The initial positions of CBs are determined randomly in the search space:

$$x^0_i = x_{\min} + \text{rand.}\left(x_{\max} - x_{\min}\right) \qquad i = 1, 2, ..., n, \tag{46}$$

where x^0_i determines the initial value vector of the ith CB; $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable value vectors of variables, respectively; rand is a random number in the interval [0,1], and n is the number of CBs.

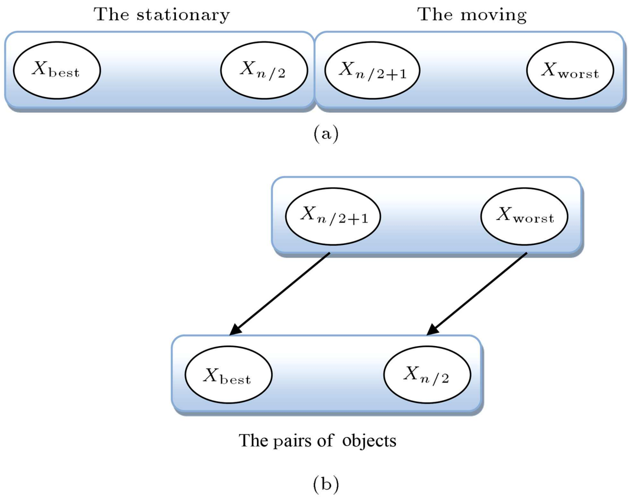


Figure 4. (a) The sorted CBs in an increasing order. (b) The pairs of objects for the collision.

Step 2. *Determination of the body mass for each CB.* The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \qquad k = 1, 2, ..., n, \tag{47}$$

where $fit(i)$ represents the objective function value of the agent i, and n is the population size. Obviously, a CB with good values exerts a larger mass than the bad ones. Also, for maximizing the objective function, the term $\frac{1}{fit(i)}$ is replaced by $fit(i)$.

Step 3. *Arrangement of the CBs.* The arrangement of the CBs objective function values is performed in ascending order (Figure 4(a)). The sorted CBs are equally divided into two groups:

- The lower half of CBs (stationary CBs): These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0 \qquad i = 1, 2, ..., \frac{n}{2}. \tag{48}$$

- The upper half of CBs (moving CBs): These CBs move toward the lower half. Then, according to Figure 4(b), the better and worse CBs, i.e. agents with upper fitness value in each group, will collide with each other. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_i - x_{i-\frac{n}{2}} \qquad i = \frac{n}{2} + 1, ..., n, \tag{49}$$

where v_i and x_i are the velocity and position vectors of the ith CB in this group, respectively; and $x_{i-\frac{n}{2}}$ is the ith CB pair position of x_i in the previous group.

Step 4. *Calculation of the new position of the CBs.* After the collision, the velocity of bodies in each group is evaluated using collision laws and the velocities before collision. The velocity of each moving CB after the collision is:

$$v'_i = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}} \qquad i = \frac{n}{2} + 1, ..., n, \tag{50}$$

where v_i and v'_i are the velocity of the ith moving CB before and after the collision, respectively; m_i is the mass of the ith CB; and $m_{i-\frac{n}{2}}$ is mass of the ith CB pair. Also, the velocity of each stationary CB after the collision is:

$$v'_i = \frac{\left(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}}\right) v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}}, \qquad i = 1, ..., \frac{n}{2}, \tag{51}$$

where $v_{i+\frac{n}{2}}$ and v_i are the velocity of the ith moving CB pair before and the ith stationary CB after the collision, respectively; m_i is mass of the ith CB; $m_{i+\frac{n}{2}}$ is mass of the ith moving CB pair. As mentioned previously, ε is the Coefficient Of Restitution (COR) and for most of the real objects, its value is between 0 and 1. It is defined as the ratio of the separation velocity of two agents after collision to the approach velocity of two agents before collision. In the CBO algorithm, this index is used to control the exploration and exploitation rates. For this goal, the COR decreases linearly from unit to zero. Thus, ε is defined as:

$$\varepsilon = 1 - \frac{\text{iter}}{\text{iter}_{\max}}, \tag{52}$$

where iter is the actual iteration number, and $\text{iter}_{\max}$ is the maximum number of iterations, with COR being equal to unit and zero representing the global and local search, respectively.

New positions of CBs are obtained using the generated velocities after the collision in position of stationary CBs.

The new position of each moving CB is:

$$x_i^{\text{new}} = x_{i-\frac{n}{2}} + \text{rand} \circ v'_i \qquad i = \frac{n}{2} + 1, ...n, \tag{53}$$

where x_i^{new} and v'_i are the new position and the velocity after the collision of the ith moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of the ith stationary CB pair. Also, the new positions of stationary CBs are obtained by:

$$x_i^{\text{new}} = x_i + \text{rand} \circ v'_i \qquad i = 1, ..., \frac{n}{2}, \tag{54}$$

where x_i^{new}, x_i, and v'_i are the new position, old position, and velocity after the collision of the ith stationary CB, respectively. rand is a random vector uniformly distributed in the range (-1,1) and the sign "$\circ$" denotes an element-by-element multiplication.

Step 5. *Termination criterion control.* Steps 2-4 are repeated until a termination criterion as the maximum number of iterations is satisfied.

4.3. Hybrid CBO and PSO

Both of the previously introduced methods (CBO and PSO) are population-based algorithms and find optimum solutions by changing the position of the agents. However, the movement strategies are different for the CBO and PSO. The PSO algorithm utilizes the local best and the global best to determine the direction of the movement, while the CBO approach uses the collision laws and the velocities before collision to determine the new positions. Using the local best and the global best are the main reasons for the success of PSO.

However, in spite of having the above-mentioned advantages, the standard PSO is infamous for premature convergence. This algorithm has some problems in controlling the balance between the exploration and exploitation due to ignoring the effect of other agents.

Similar to the PSO method, the CBO algorithm uses the previous velocities, when the upper half of CBs (moving CBS) moves toward the stationary CBs. As it is mentioned in the previous section, after the collision, the velocity of all CBs is evaluated using the velocity of moving CBs before the collision and the mass of the paired CBs. This will lead to loss of the best position of particles which is found in the previous iteration. Therefore, in the present hybrid algorithm, the advantages of the PSO consist of the local best and the global best is added to the CBO algorithm. For this purpose, the best position of the stationary particles is saved in a memory called Stationary Bodies Memory (SBM). Also, another memory is considered to save the better position of each particle that has been found so far. This memory, so-called Particles Memory (PM), is treated as the local best in the PSO, and it is updated by the following expression:

$$PM_{k+1}^i = \begin{cases} PM_k^i \rightarrow F(X_{k+1}^i) \geq F(X_k^i) \\ X_{k+1}^i \rightarrow F(X_{k+1}^i) \leq F(X_k^i), \end{cases} \tag{55}$$

in which the first term identifies that when the new position is not better than the previous one, updating will not be performed, while when the new position is better than the so far stored good position, the new solution vector is replaced. With the above definitions, and considering the above-mentioned new memories, the velocity of CBs after collision is modified by the following equations:

For moving particles:

$$v_i' = \frac{\left(m_i - \varepsilon m_{i-\frac{n}{2}}\right) v_i}{m_i + m_{i-\frac{n}{2}}} + r_1.c_1.\left(PM_i - x_i\right)$$

$$+r_2.c_2.\left(\mathrm{SPM} - x_i\right) \qquad i = \frac{n}{2} + 1, ..., n. \tag{56}$$

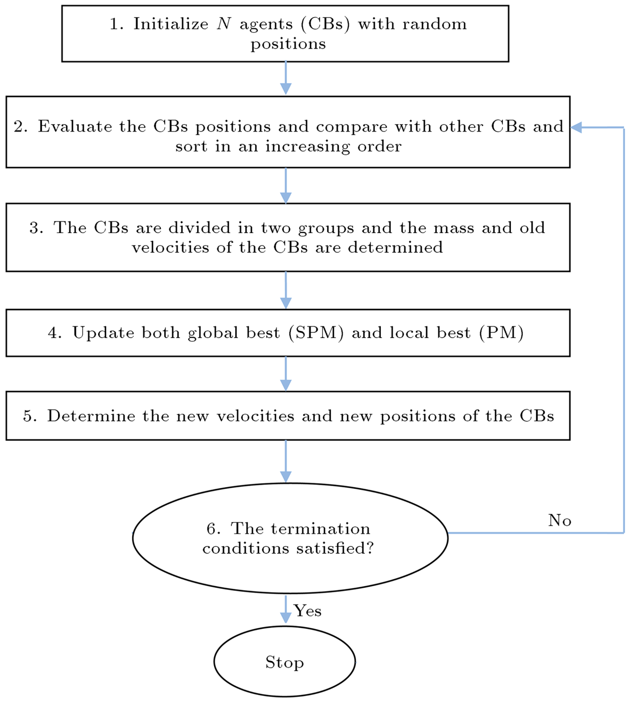


Figure 5. Flowchart of the CBO.

For stationary particles:

$$v_i' = \frac{\left(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}}\right) v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}} + r_1.c_1.\left(PM_i - x_i\right)$$

$$+ r_2.c_2.\left(\mathrm{SPM} - x_i\right) \qquad i = 1, ..., \frac{n}{2}. \tag{57}$$

Figure 5 shows the schematic procedure of the CBO-PSO algorithm.

5. Design examples

In this section, in order to compare fabrication costs of the castellated beams with circular and hexagonal holes, three benchmark examples from the literature are selected. Also, these beams are used in this section to show the efficiency of the new optimization algorithm. Among the steel section list of British Standards, 64 Universal Beam (UB) sections starting from 254 × 102 × 28 UB to 914 × 419 × 388 UB are chosen to constitute the discrete set for steel sections from which the design algorithm selects the sectional designations for the castellated beams. In the design pool of holes diameters, 421 values are arranged which vary between 180 and 600 mm with increment of 1 mm. Also, for cutting depth of hexagonal opening, 351 values are considered which vary between 50 and 400 mm with increment of 1 mm and cutting angle changes from 45 to 64. Another discrete set is arranged for the

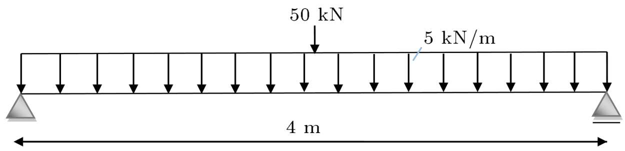


Figure 6. Simply supported beam with 4 m span.

number of holes. Likewise, in all the design problems, the modulus of elasticity is equal to 205 kN/mm^2 and Grade 50 is selected for the steel of the beam which has the design strength of 355 MPa [13,14]. The coefficients P_1, P_2, and P_3 in the objective function are considered 0.85, 0.30, and 1.00, respectively.

5.1. Castellated beam with 4 m span

A simply supported beam with 4 m span, shown in Figure 6, is selected as the first design example. The beam is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 kN also acts at mid-span of the beam and the allowable displacement of the beam is limited to 12 mm. The number of CBs is taken as 50 and the maximum number of iterations is considered 200.

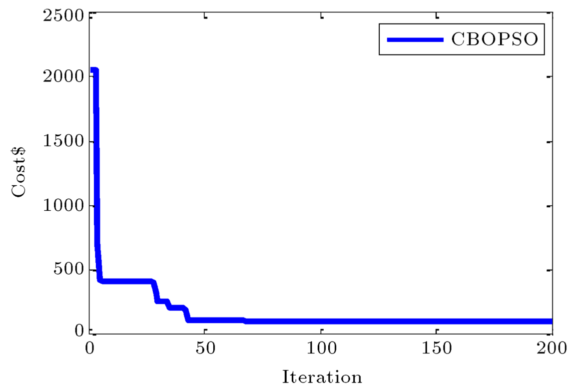
Castellated beams with hexagonal and circular openings are separately designed by using the new algorithm. The optimum results obtained by CBO-PSO are given in Table 1. It is apparent from the same table that the optimum cost for castellated beam with hexagonal hole is equal to 89.78$, which is obtained by three methods, but the CBO-PSO algorithm gives better results than those of other approaches for cellular beams [19-20]. Also, these results indicate that the castellated beam with hexagonal opening has low cost as compared to the cellular beam. In this problem, the length of the span is short; hence, shear capacity is very important in optimum design of this beam and it is the most effective factor in the design of this example.

Figure 7 shows the convergence of CBO-PSO algorithm for design of castellated beams with different openings.

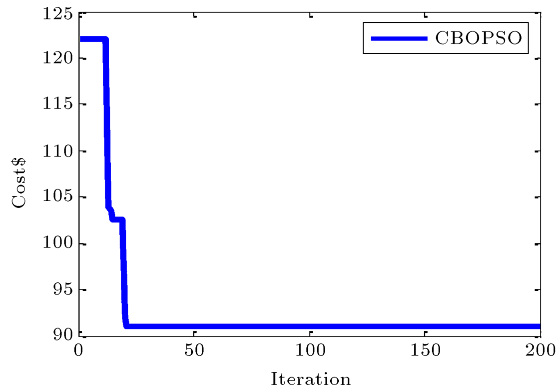
5.2. Castellated beam with 8 m span

In the second problem, the CBO-PSO algorithm is used to design a simply supported castellated beam with 8 m span. The beam carries a uniform dead load 0.40 kN/m, which includes its own weight. In addition, it is subjected to two concentrated loads; dead load of 70 kN and live load of 70 kN, as shown in Figure 8. The allowable displacement of the beam is limited to 23 mm. The number of CBs is taken as 50. The maximum number of iterations is considered 200.

This beam is also designed by three optimization methods and the optimum results are given in Table 2. In design of the beam with hexagonal hole, the corresponding costs obtained by the ECSS and



(a)



(b)

Figure 7. Variation of minimum cost versus the number of iterations for 4 m span castellated beam: (a) Castellated beam with hexagonal opening; and (b) castellated beam with circular opening.

Table 1. Optimum designs of the castellated beams with 4 m span.

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO-PSO algorithm	**UB 305×102×25**	**125**	**14**	**57**	**89.78**	Hexagonal
CBO algorithm [20]	UB 305×102×25	125	14	57	89.78	
ECSS algorithm [19]	UB 305×102×25	125	14	57	89.78	
CBO-PSO algorithm	UB 305×102×25	243	14	–	91.08	Circular
CBO algorithm [20]	UB 305×102×25	244	14	–	91.14	
ECSS algorithm [19]	UB 305×102×25	248	14	–	96.32	

Table 2. Optimum designs of the castellated beams with 8 m span.

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO-PSO algorithm	**UB 610×229×101**	**244**	**14**	**55**	**718.33**	
CBO algorithm [20]	UB 610×229×101	243	14	59	718.93	Hexagonal
ECSS algorithm [19]	UB 610×229×101	246	14	59	719.47	
CBO-PSO algorithm	UB 610×229×101	487	14	–	721.55	
CBO algorithm [20]	UB 610×229×101	487	14	–	721.55	Circular
ECSS algorithm [19]	UB 610×229×101	487	14	–	721.55	

Table 3. Optimum designs of the castellated beams with 9 m span.

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO-PSO algorithm	**UB 684×254×125**	**230**	**16**	**56**	**990.33**	
CBO algorithm [20]	UB 684×254×125	233	15	64	993.79	Hexagonal
ECSS algorithm [19]	UB 684×254×125	277	13	56	995.97	
CBO-PSO algorithm	UB 684×254×125	538	14	–	998.58	
CBO algorithm[20]	UB 684×254×125	538	14	–	997.57	Circular
ECSS algorithm [19]	UB 684×254×125	539	14	–	998.94	

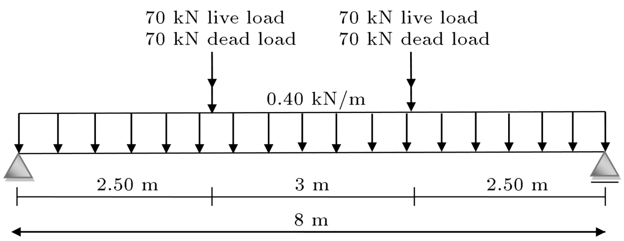


Figure 8. A simply supported beam with 8 m span.

CBO are equal to 719.47$ and 718.93$, respectively, while this value is equal to 718.33$ for the CBO-PSO algorithm. As a result, the performance of the CBO-PSO method is better than other ways in this design example. According to the obtained results, the designed beam with hexagonal opening in comparison with the cellular beam has low cost, and it is a more appropriate option in this case. Also, the maximum value of the strength ratio is equal to 0.99 for both hexagonal and circular beams, and it is shown that these constraints are dominant in the design.

Figure 9 shows the convergence history for optimum design of hexagonal beam, which is obtained by three methods. As can be observed, the convergence rate of the CBO-PSO is nearly the same as that of the CBO and higher than ECSS.

5.3. Castellated beam with 9 m span

The beam with 9 m span is considered to be the last example of this study in order to compare the minimum cost of the castellated beams. The beam

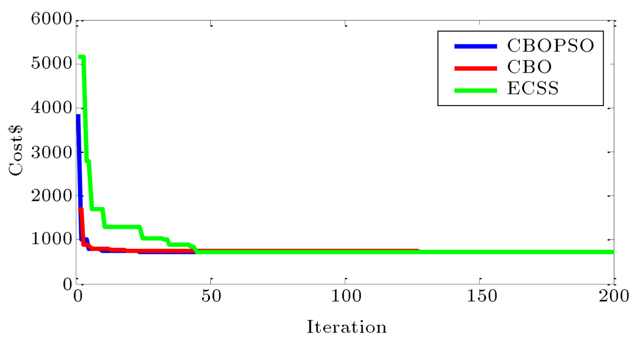


Figure 9. Convergence history of the hexagonal beam with 8 m span.

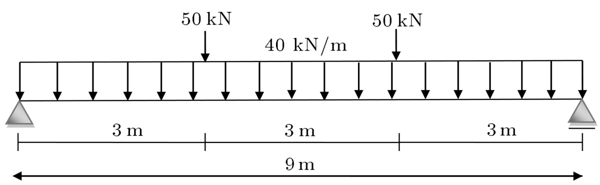


Figure 10. Simply supported beam with 9 m span.

caries a uniform load of 40 kN/m including its own weight and two concentrated loads of 50 kN, as shown in Figure 10. The allowable displacement of the beam is limited to 25 mm. Similar to the two previous examples, the number of CBs is taken as 50 and the maximum number of iterations is considered 200.

Table 3 compares the results obtained by the CBO-PSO with those of the other algorithms. In the optimum design of castellated beam with hexagonal

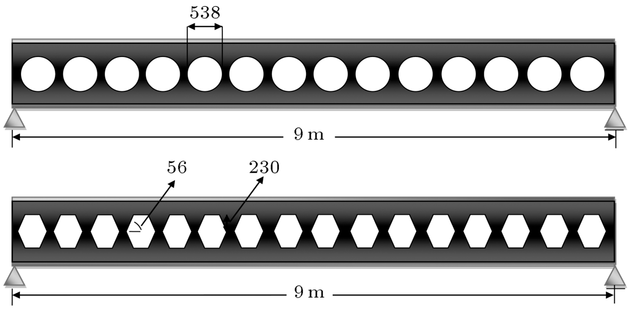


Figure 11. Optimum profiles of the castellated beams with cellular and hexagonal openings.

hole, CBO-PSO algorithm selects 684 × 254 × 125 UB profile, 16 holes, and 230 mm for the cutting depth and 56 degree for the cutting angle. The minimum cost of design is equal to 990.33$. Also, in the optimum design of cellular beam, the CBO-PSO algorithm selects 684 × 254 × 125 UB profile, 14 holes, and 538 mm for the holes diameter. It is observed from Table 3 that the optimal design has the minimum cost of 990.33$ for the beam with hexagonal holes, which is obtained by the CBO-PSO algorithm; however, the CBO method gives better results for cellular beam. In cellular beam, the maximum value of deflection of the beam is smaller than that of its upper bound. This shows that the strength criteria are dominant in the design of this beam and they are related to the Vierendeel mechanism. Similar to the cellular beam, in castellated beam with hexagonal opening, the strength constraints are dominant in the design process. The maximum ratio of these criteria is equal to 0.99 for the Vierendeel mechanism.

The optimum shapes of the hexagonal and circular openings are illustrated separately as shown in Figure 11.

6. Concluding remarks

In this paper, a new hybrid algorithm, called CBO-PSO, has been proposed for optimum design of castellated beams. This algorithm consists of hybridization of the CBO and PSO methods and it synthesizes their merits. For this purpose, the positive features of the PSO algorithm are added to the CBO. Three castellated beams are selected from literature to design by the presented algorithm. Beams with hexagonal and circular openings are considered as web-openings of castellated beams. Also, the cost of the beam is considered as the objective function. Comparing the results obtained by CBO-PSO with those of other optimization methods demonstrates that the proposed approach is superior to the other methods in the ability to find the optimum solution. It is observed that the optimization results obtained by the CBO-PSO algorithm for more design examples have low cost in comparison to the results of the CBO and ECSS algorithms. Also, from the results obtained in this paper, it can be concluded that the use of the beam with hexagonal opening can lead to the use of less steel material and it is better than cellular beam from the point of view of the cost.

Acknowledgment

The first author is grateful to Iran National Science Foundation for the support.

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