Designing a resource-constrained project scheduling model considering multiple routes for flexible project activities: meta-heuristic algorithms

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Abstract
Resource constrained project scheduling problem with multiple routes for flexible project activities (RCPSP-MR) is a generalization of the RCPSP, in which for the implementation of each flexible activity in main structure of the project, several exclusive sub-networks are considered. Each sub-network is regarded as a route for the flexible activity. The routes are considered for each flexible activity that are varied in terms of: 1) Number of activities required to execute; 2) Precedence relationship between activates; 3) Allocation of different renewable and nonrenewable resources to each activity; and 4) Effectiveness on the duration and cost of project completion. In this paper, a new mathematical formulation of RCPSP-MR is firstly presented. Then, two solving approaches based on particle swarm optimization (PSO) and genetic algorithm (GA) are proposed to minimize costs of project completion. To evaluate the effectiveness of these proposed approaches, 50 problems (in very small, small, medium, and large-sized test problems) are designed and then are solved. Finally, comparisons are provided. Computational results show that the proposed GA generates high-quality solutions in a timely fashion.

Keywords: Resource constrained project scheduling problem (RCPSP), flexible activities, multiple routes, particle swarm optimization (PSO), genetic algorithm (GA)
1. Introduction

Project scheduling problem (PSP) determines run times for a specific set of fixed activities with regard to precedent relationships and via an allocation of different resources in order to achieve predetermined goals [1-4]. Role of the PSP and baseline scheduling in the project management is crucial [5-8] and for this reason, Vanhoucke [9] presented nine time and eight cost forecasting methods for both project duration and cost. The project scheduling has attracted a lot of attention because of its critical role in the resource management as long as resource constrained project scheduling problem (RCPSP) is turned into a standard problem. RCPSP in classical viewpoint is defined as scheduling for a set of fixed activities with precedence constraints is known and must be implemented under the limited availability of resources to minimize the makespan [10, 11]. RCPSP is a basic scheduling problem which comprises activities, scarce resources required by activities for their executions, and precedence relations between activities. The goal is to find an optimal schedule satisfying the resource and precedence constraints. These scheduling problems have many applications, ranging from production planning to project management [12]. Therefore, RCPSP determines a proper sequence of activities so that we involve in two main constraints: 1) Resource constraints and 2) Precedence relationships are satisfied at fashion time, and measurement criteria, such as time, cost, and quality are optimized [1]. The classification of RCPSP is summarized as follows [13]:

1. Resource
   a. Renewable resource
   b. Non-renewable resource
   c. Doubly constrained resources (e.g., money and energy)

2. Characteristics of activity
   a. Preemption
   b. Execution mode
   c. Type of resource consumption during processing of an activity
   d. Other characteristics

3. Objective function
   a. Time-based
   b. Economic
   c. Resource-based
   d. Multi-objective
   e. Other objectives

4. Availability level of information
   a. Deterministic
   b. Non-deterministic

Since the RCPSP is NP-hard, and for more adaptation of the RCPSP with reality, some researchers have considered various extensions and different methods for solving the problem and working on exact methods using mixed-integer programming, constraint programming, and satisfiability modulo theories [14-22]. Bibiks et al. [23] presented a discrete cuckoo search (DCS) algorithm for RCPSP and improved the DCS algorithm in their study [24]. Fathallah and Najafi [25] discussed a fuzzy RCPSP with the aim of maximizing NPV of project cash flows. They proposed a mathematical formulation and a hybrid GA to solve this problem. Gonzalez-Pardo et al. [26] modeled the RCPSP by constraint satisfaction problems-graph and solved by ant colony optimization. Chand et al. [1] regarded a genetic programming based hyper-heuristic to produce efficient priority rules for the RCPSP. Kadri and Boctor [27] addressed the RCPSP with transfer times to be known and
deterministic, and proposed a GA for the solving. Coelho and Vanhoucke [28] developed a branch-and-bound procedure to solve the RCPSP.

Multi-mode resource-constrained project scheduling problem (MRCPS) is one of extensions of the RCPSP, and because of its more complexity, different common methods were presented in the literature [29-32]. Fernandes et al. [33] regarded a path-relinking (PR) algorithm for MRCPS to minimize makespan of the project. Van Den Eeckhout et al. [34] integrated multi-mode RCPSP and resource scheduling in order to introduce some flexibilities in the scheduling process to determine the optimal personnel budget that minimized the overall cost.

It is possible to use manpower with different skills in the implementation of projects that can be considered for the RCPSP as another generalization, namely MSRCPSP [35-38]. Javanmard et al. [39] integrated MSRCPSP with the resource investment problem to obtain concurrent optimal policy of project scheduling and skills’ recruitment. Wang and Zheng [40] proposed a multi-objective fruit fly optimization for the MSRCPSP with the criteria of minimizing the makespan and total cost simultaneously. Myszkowski et al. [41] designed a hybrid differential evolution and greedy algorithms for the MSRCPSP.

Since the choice of technology and implementing route of different parts of a project can be an important factor that will lead to success or failure of the project in reality, in this paper, the RCPSP is generalized and project flexibility is considered for the choice of different technologies. Regarding different types of technology, different routes are regarded for each section. Each route has independent activity set, precedence relationships, cost, and separated time for each activity. Therefore, in addition to sequencing of activities by limited operational resources, choosing the optimal route for the implementation of different parts of the project is important which leads to a more complicated problem. In this paper, such problems are summarized as RCPSP-MR. The major difference between RCPSP-MR and MRCPS is that in the MRCPS, the project activity network (i.e., the number of activities and precedence relationships) is fixed, duration of each activity will be varied with regard to number of resource allocations, and all project activities are essential; while in RCPSP-MR problem, the selection of a route might lead to basic changes in the project, so that each part of the project activity network is replaced with a different activity sub-network. This leads to some changes in the project, such as: 1) the number of activities, 2) precedence relationships between activities, 3) allocation of resources to the activities, 4) duration of each activity, and 5) cost of each activity, which increase the complexity of the problem into MRCPS.

RCPSP-MR problems can be regarded in the scheduling of projects, such as mechanizing various production line, launching SCADA systems, launching smart urban systems, dispatching liquid reservoirs fuel, launching different vehicle tracking systems, developing educational simulators, developing electronic Siebel for shooting training and reverse engineering of critical and important equipment. Also, RCPSP-MR is defined and described in this paper is indicated in flexible manufacturing cells (FMCs) according to the related literature [e.g., 42-55].

To solve the mathematical model in this paper, two meta-heuristic algorithms are proposed. Firstly, in the proposed GA, due to the importance of producing initial solutions, each chromosome in problem space is divided into seven parts, in which two parts are considered for the sequencing of total of project activities (e.g., fixed and flexible activities); one part is related to route is selected for flexible parts, and finally, four parts are regarded to assigned reassures and level of the implementation for activities. Solutions are reinforced with crossover and mutations operations frequently and at each iteration, a specification of high quality solution is saved. Secondly, considering that the solution space presented for the problem is discrete and the proposed PSO algorithm is used in continuous space; it is necessary to use an appropriate method to transform the production solutions into continuous space. The method used in this paper is as follows: To each of the number of project activities an equal interval between 0 to 1 is considered. Selection of numbers
inside each interval, at each iteration, indicates the selection of the corresponding activity. Also, in
this proposed algorithm, in order to produce quality solutions, using mutation is presented at GA
algorithm; initial solution is presented. According to velocity vector in each iteration, obtained
solutions with the proposed algorithm move to the optimal solution.

Afterwards, in Section 2, a mathematical model to solve the RCPSP-MR problem is presented. Since
software packages for the proposed model will not be able to find optimal solutions for medium
and large-sized problems, in Section 3, two solution algorithms based on GA and PSO are presented
to achieve near-optimal solutions. Then, in Section 4, 50 test problems are designed and solved with
the proposed algorithms and the results are compared with each other. Conclusions are provided in
Section 5.

2. RCPSP with multiple routes for flexible project activities

2.1. Problem description

In this paper, a mixed integer nonlinear programming (MINLP) model is proposed to find an
optimal solution for the RCPSP-MR. The activity network is CPM and activity on node (AON),
which is divided two main sections of flexible activities and fixed activities. The flexible activities are
parts of the project, which due to the ability of using various technologies in their implementations,
different routes can be assigned. The route selection leads to the addition of new activity sub-
networks to the project’s main network, which will lead to more extensive project main network and
more complicated decision-making on implementation.

A graph \( G = (J, A_j) \) is considered in witch \( J = \{0,1,2,...,N+1\} \) is list of activities (0 and
\( N+1 \) are artificial start and the end of the project) and \( \{A_j|\forall (i \to j) \in A_j\} \) is represented list of FS
precedence relationships.

The duration of each activity is shown with \( d \), which is not preemptive. Two sets of resources are
considered for the problem, non-renewable \( NR \) and multi-skill renewable \( R \). The resources are
available until the completion of the whole project, and it is not possible to transfer to another project.
Levels of the implementation for activity \( j \) by the renewable resource \( r \in R \) is shown with \( \ell_r \in L_r \),
where \( L_r \) is the total number of execution levels. The purpose of the problem introduced in this paper
is to minimize the cost of project completion, including: 1) total cost of the availability of renewable
resources until the project completion, 2) total cost of using renewable resources and its level of
implementation for any project activity, and 3) total cost of using non-renewable resources for project
activities. All the sets, indices, and variables are presented as follows:

**Sets and indices:**

- \( J \) Set of total project activities \( J = \{0,1,2,...,N+1\} \)
- \( J_1 \) Set of total project fixed activities, \( J_1 \subseteq J \)
- \( J_2 \) Set of total flexible project activities, \( J_2 = J - J_1 \)
- \( \lambda_r \) Set of routes of each flexible activity \( \rho \in J_2 \)
- \( \rho_h \) Set of total sub network for flexible activity \( \rho \in J_1 \)
\( A_J \)  Set of precedence relationships for \( J \) in graph \( G = (J, A_J) \)
\( A_{J_1} \)  Set of precedence relationships for \( J_1 \) in graph \( G = (J_1, A_{J_1}) \)
\( A_{\rho_h} \)  Set of precedence relationships for \( \rho_h \) in graph \( G_{\rho_h} = (\rho_h, A_{\rho_h}) \)
\( R \)  Set of renewable resource
\( NR \)  Set of non-renewable resource
\( L_i \)  Set of level of implementation for renewable resource \( r \)
\( i, j \)  Index of activity
\( \rho, \rho' \)  Index of flexible activities
\( h, h' \)  Index of route
\( l \)  Index of level of implementation

**Parameters:**

\( \gamma_{jnr} \)  Non-renewable resource requirement for fixed activity \( j \)
\( \gamma_{j\rho_hnr} \)  Non-renewable resource requirement for activity \( j \) of flexible activity \( \rho_h \)
\( C \)  Fixed cost of the availability of renewable resources per unit of time
\( C_{jrl} \)  Cost of using renewable resources and its level of implementation for activity \( j \)
\( C_{j\rho_hl} \)  Cost of using renewable resources and its level of implementation for activity \( j \) of flexible part \( \rho_h \)
\( C_{nr} \)  Fixed cost of using non-renewable resources
\( d_{jrl} \)  Duration of using renewable resources and its level of implementation for activity \( j \)
\( d_{j\rho_hl} \)  Duration of using renewable resources and its level of implementation for activity \( j \) of flexible part \( \rho_h \)

**Binary variables:**

\( Y_{jrl} = \{0,1\} \)  Level of implementation for activity \( j \) by renewable resource \( r \)
\( Y_{j\rho_hl} = \{0,1\} \)  Level of implementation for activity \( j \) by renewable resource \( r \) of flexible part \( \rho_h \)
\( W_{\rho_h} = \{0,1\} \)  Route selection \( h \) for flexible activity \( \rho \)
\( \alpha_{ij} = \{0,1\} \)  Sequencing of fixed activity \( i \) and \( j \) allocated to resource \( r \) considering the precedence relationships
\( \eta_{j\rho_hl} = \{0,1\} \)  Sequencing of flexible activity \( i \) and \( j \) of flexible part \( \rho_h \) allocated to resource \( r \) considering the precedence relationships
\( \psi_{j\rho_hl\rho'_l} = \{0,1\} \)  Sequencing of flexible activity \( j \) of flexible part \( \rho_h \) and flexible activity \( i \) of flexible part \( \rho'_l \) allocated to resource \( r \) considering the precedence relationships

**Positive variables:**
\[ F_{jrl} \] Finish time activity \( j \) allocated to resource \( r \) with level \( l \)
\[ S_{jrl} \] Start time activity \( j \) allocated to resource \( r \) with level \( l \)
\[ F_{jrlh} \] Finish time activity \( j \) of flexible part \( h \) allocated to resource \( r \) with level \( l \)
\[ S_{jrlh} \] Start time activity \( j \) of flexible part \( h \) allocated to resource \( r \) with level \( l \)
\[ TC \] Total cost of project

### 3.2. Mathematical formulation

Proposed mathematical formulation of the problem is given as follows:

\[
\begin{align*}
\text{Min } TC &= C_r \sum_{j \in J} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) + \sum_{j \in J} \sum_{l \in I} \left( S_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) + \\
&\quad \sum_{p \in P} \sum_{j \in J} \sum_{l \in L} \left( C_{jrl} Y_{jrl} - d_{jrl} W_{jrl} \right) + \\
&\quad \sum_{n \in N} \sum_{r \in R} \sum_{l \in L} \left( \gamma_{jnlr} W_{jnlr} + \gamma_{jnr} \right)
\end{align*}
\]

Subject to:

\[
\begin{align*}
\sum_{r \in R} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) &= \left( \sum_{r \in R} \sum_{l \in L} \left( F_{irl} X_{irl} \right) + \sum_{r \in R} \sum_{l \in L} \left( s_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) \right), \quad \forall (i, j) \in A_i, \\
\sum_{r \in R} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) &= \left( \sum_{r \in R} \sum_{l \in L} \left( F_{irl} X_{irl} \right) + \sum_{r \in R} \sum_{l \in L} \left( s_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) \right), \quad \forall (i, j) \in A_i, \\
\sum_{r \in R} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) &= \left( \sum_{r \in R} \sum_{l \in L} \left( F_{irl} X_{irl} \right) + \sum_{r \in R} \sum_{l \in L} \left( s_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) \right), \quad \forall (i, j) \in A_i, \\
\sum_{r \in R} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) &= \left( \sum_{r \in R} \sum_{l \in L} \left( F_{irl} X_{irl} \right) + \sum_{r \in R} \sum_{l \in L} \left( s_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) \right), \quad \forall (i, j) \in A_i,
\end{align*}
\]

\[
\begin{align*}
\sum_{r \in R} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) &= \left( \sum_{r \in R} \sum_{l \in L} \left( F_{irl} X_{irl} \right) + \sum_{r \in R} \sum_{l \in L} \left( s_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) \right), \quad \forall (i, j) \in A_i, \\
\end{align*}
\]

\[
\begin{align*}
\sum_{r \in R} \sum_{l \in L} \left( F_{jrl} X_{jrl} \right) &= \left( \sum_{r \in R} \sum_{l \in L} \left( F_{irl} X_{irl} \right) + \sum_{r \in R} \sum_{l \in L} \left( s_{jrl} Y_{jrl} t - s_{jrl} d_{jrl} \right) \right), \quad \forall (i, j) \in A_i,
\end{align*}
\]
\[
\begin{align*}
\left( \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (d_{jil} \cdot Y_{jil}) \right) \alpha_{ji} + \\
\left( \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (d_{jil} \cdot Y_{jil}) \right) \alpha_{ij} > 0, \\
\alpha_{ij} + \alpha_{ji} = \sum_{i \in L} \sum_{i \in L} Y_{jil}, \forall j \neq i \in J, \forall r \in R
\end{align*}
\]

(7)

\[
\begin{align*}
\left( \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (d_{jil} \cdot Y_{jil}) \right) \eta_{rij} + \\
\left( \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (d_{jil} \cdot Y_{jil}) \right) \eta_{nrij} > 0, \\
\eta_{rij} + \eta_{nrij} = \sum_{i \in L} \sum_{i \in L} Y_{jil}, \forall r \in R, \forall j \neq i \in \rho, \forall h \in \lambda, \forall \rho \in J_2
\end{align*}
\]

(8)

\[
\begin{align*}
\left( \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (d_{jil} \cdot Y_{jil}) \right) \psi_{nrij} + \\
\left( \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (F_{jil} \cdot Y_{jil}) - \sum_{i \in L} (d_{jil} \cdot Y_{jil}) \right) \psi_{rij} > 0, \\
\psi_{nrij} + \psi_{rij} = \sum_{i \in L} \sum_{i \in L} Y_{jil}, \forall r \in R, \forall j \in \rho, \forall i \in \rho', \forall h \in \lambda, \forall h' \in \lambda' \rho, \forall (\rho \neq \rho') \in J_2
\end{align*}
\]

(9)

\[
\sum_{r \in R} \sum_{i \in L} Y_{jil} = 1, \forall j \in J_1
\]

(10)

\[
\sum_{r \in R} \sum_{i \in L} Y_{jil} = W_{\rho}, \forall \rho \in J_2, \forall j \in \rho, \forall h \in \lambda
\]

(11)

\[
\sum_{h \in \lambda} W_{\rho} = 1, \forall \rho \in J_2
\]

(12)

Equation (1) is the objective function; the total cost of project completion is minimized. Relations (2-4) assure that the completion time of each activity is greater than (or equal to) the completion time of its predecessor activities. Equations (5) and (6) represent each activity must be implemented without interruption. Relations (7-9) define relations between the completion times of each pair of activities by each renewable resource. Equations (10) and (11) explain that a project activity will be carried out only by an executive level of renewable resource. Equation (12) guarantees that only one route out of the possible routes is selected for each flexible activity $\rho \in J_2$.

3. Proposed meta-heuristic algorithms

3.1. Genetic algorithm (GA)
Genetic algorithm (GA) is a competitive algorithm among other meta-heuristic algorithms in which the presence or absence of genes and their order in the chromosome decide the characteristics of a species. Different traits are passed from generation to the next through different biological processes that operate on the genetic structure [56, 57]. By the process of genetic change, the survival of the fittest population may well adapt to the environment [58, 59]. GA has attracted the attention of researchers to use for the PSP and its extensions [60, 61].

The first proposed meta-heuristic algorithm for the RCPSP-MR problem is presented based on the GA; hereafter, it is abbreviated as GA-MR. In this algorithm, with regard to the dimensions of the problem, a certain number of initial solutions are generated.

The initial solution generation is in a way that each solution (chromosome) is consisted of different parts:

1) C.Main (C.Flex): It determines a permutation from order of all fixed activities of the main project network (or all activities related to any flexible activity) by observing all precedence relations.
2) C.R (C.FR): It determines resource allocated to each fixed activity (or each sub-activity that is related to flexible activity network).
3) C.L (C.FL): It determines level of implementation any fixed activity (or any sub-activity that is related to flexible activity network) by allocated resources of the before part.
4) C.Route: It determines route selected for each flexible activity.

By calculating total cost of the project completion based on the roulette wheel, the fitness value or generation solution value will be calculated by Equation (13).

\[
P_i = 1 - \frac{TC_i}{\sum_{i=1}^{x} TC_j}
\]

where \(TC_i\) is objective function (total project cost) in solution \(i\). As shown in Equation (13), the goodness ratio of one solution or chromosome (\(P_i\)) is calculated based on the goodness value of all the obtained solutions. The higher ratio will raise the chance of chromosome selection for next solution production. The most valuable solutions will be selected. Then, crossover operation is implemented. The crossover location is randomly chosen and number of new solutions from each parent is in accordance to Equation (14).

\[
CH = (2 + J_2) \times 2
\]  

In Equation (14), \(CH\) indicates the number of new solutions and \(J_2\) indicates the number of flexible activities. For example, consider a network with number of 8 activities having a flexible activity. For the flexible activity, 2 separate routes are considered. The main project activity network and the sub-networks of flexible activity route are shown in Figure 1.

{Please insert Figure 1 here.}

The crossover operation are performed on C.R, C.L, C.Route, C.FR, and C.FL parts of one solution considered in Figure 2.

{Please insert Figure 2 here.}
In the next step, using the four different types of mutations on the most valuable solutions of crossover operation, the new solutions are generated and their values are calculated. The number of new solutions from mutation operations is in accordance to Equation (15):

\[ CH = 1 + 4J_2 \]  

(15)
The various types of mutations are: 1) swap mutation, 2) flip mutation, 3) reversion mutation, and 4) insertion mutation and performed on \( C.R \), \( C.L \), \( C.FR \), and \( C.FL \) parts of one solution considered in Figure 3.

1. **Swap mutation**: In this mutation, two positions of the chromosomes are randomly selected and their corresponding values are exchanged. In this paper, the mutation is done on \( C.Main \) and \( C.Flex \).

2. **Reversion mutation**: In this mutation, two points are randomly selected from the parent chromosome, and the numbers are written between these two points inversely from the end to the beginning. In this paper, the mutation is done on \( C.FR \) and \( C.FL \).

3. **Insertion mutation**: In the insertion mutation, two points of the chromosome are selected and their positions are changed. After random selection of two points of the parent chromosome, the second point is transferred next to the first point. In this paper, the mutation is done on \( C.FR \) and \( C.FL \).

4. **Flip mutation**: In this mutation, a small impact is given to the chromosome. As a result, all the numbers will move forward up to the impact. Obviously, the numbers that are located at the end of chromosome might transferred to the beginning of chromosome with regard to the flip rate. In this paper, the mutation is done on \( C.FR \) and \( C.FL \).

Finally, all the generation solutions are categorized and the certain number of best solutions are stored in a memory. The above-mentioned process is repeated to a certain number, and after acquisition of the stop conditions, the best selected route for each flexible activity, best permutation determined for network activities, best resource allocation conditions and level of implementing each activity, will be provided in the form of the best solution. In Figure 4, steps of the GA-MR algorithm are briefly presented in the form of a flowchart.

3.2. **Particle swarm optimization (PSO)**

Particle swarm optimization (PSO) was developed by a social psychologist named Kennedy along with an electrical engineer, named Eberhart [62]. The PSO algorithm consists of initial solutions or particles with separate position and velocity vectors [63]. The particles move frequently in the problem space with regard to the velocity vector in each iteration, and examine the new possible options by calculating the fitness value as a standard measurement. Two memories are assigned to store the best position of each particle in the past (the best local solution), and the best position among all particles (the best global solution) and finally, by considering the stopping algorithm conditions, the best global solution is determined as the result of the algorithm [64, 65].
The velocity and position vectors of each particle are updated as follows [66]:

\[
\begin{align*}
\vec{v}_k^i &= \delta \left( w \vec{v}_{k-1}^i + c_1 \vec{r}_1 \left( \vec{p}_k^i - \vec{x}_k^i \right) + c_2 \vec{r}_2 \left( \vec{p}_k^i - \vec{x}_k^i \right) \right) \\
\vec{x}_{k+1}^i &= \vec{x}_k^i + \vec{v}_k^i
\end{align*}
\]  

(16)

(17)

where for particle \(i\), \(\vec{x}_k^i\) is position vector, \(\vec{v}_k^i\) is velocity vector, \(\vec{p}_k^i\) is the best position vector, and \(\vec{p}_k^i\) is the best of position vector between all particles in iteration \(k\). The vectors \(\vec{r}_1\) and \(\vec{r}_2\) are random numbers randomly distributed between 0 and 1. \(c_1\) and \(c_2\) are acceleration constants for local and global searches [67]. Parameter \(w\) is the inertial weight factor and its values are different at each step of algorithm iteration and calculated by Equation (18):

\[
w = W_{\text{max}} - \left( \frac{W_{\text{max}} - W_{\text{min}}}{N} \right) \times k
\]  

(18)

where \(k\) is the current iteration index, and \(N\) is the total number of iterations in the algorithm. \(W_{\text{min}}\) and \(W_{\text{max}}\) are constants [66]. Parameter \(\delta\) is contraction coefficient for velocity control and is obtained from Equation (19):

\[
\delta = \left( \frac{2}{C - 2 + \sqrt{C^2 - 4C}} \right), \quad C = c_1 + c_2, \quad C \geq 4
\]  

(19)

The flowchart of the proposed improved algorithm is depicted in Figure 5, based on the PSO, called PSO-MR.

3.3. Numerical example

To appraise evaluate the performance of presented mathematical model, small-scale example is designed, and then, solved with the proposed model. In this example, consider an AON network of a project with three activities, \(J = \{0,1,2,3,4\}\), where activity 1 is flexible, \(J_1 = \{1\}\), and other activities are fixed, \(J_2 = \{0,2,3,4\}\). For flexible activity 1, three different routes are considered, \(\lambda_2 = \{1,2,3\}\). The network of activity of each route is shown in Figure 6. For the implementation of all project activities, only a renewable resource, \(R = \{1\}\) is considered with two level of implementation, \(L_1 = \{1,2\}\), and two types of non-renewable resources, \(NR = \{1,2\}\), and the value of each one for any activity is specified individually in Table 1. The cost of using any unit of non-renewable resources is \(C_1 = 10, C_2 = 8\), respectively.

Other information on the problem can be seen in Table 1.
The problem has been formulated using Equations (1)-(12) and solved by the GAMS optimization software. The number of equations and variables used to solve the problem are 973 and 528, respectively. The route 2 for flexible activity 1 is selected, and optimal solution 1654 has been obtained in 248.65s. In Figure 7, sequencing of activities is shown.

The numerical example has been solved with the proposed GA-MR and PSO-MR. The results obtained is shown in Figure 7. The optimal solution 1654 has been obtained by GA-MR in 7.23s and by PSO-MR in 11.51s, respectively.

4. Computational results

In this paper, in order to test the performance of proposed mathematical model and algorithms, a number of 50 problems in various scales (i.e., 20 very small problems, 10 small problems, 10 medium problems and 10 large sized problems) are designed and solved. As shown in Table 2, all the designed problems are marked with a certain (ID) from 001 to 050. Among 50 designed problems, 20 problems (001 to 020) are very small, 10 problems (021 to 030) are small, 10 problems (031 to 040) are medium, and 10 problems (041 to 050) are large sized problems. In order to show the designed problems, an ID is considered for each problem; the ID of each problem is described through a specific quadrant T.F.R.R.L. In this ID, T) is the least total number of activities when adding new activity networks from a selection route for flexible project activities, F) is the number of flexible activities considered for project, firstly, R) represents the number of applicable activity network for project (number of routes), secondly, R) represents the number of multi-skill renewable resources considered for the project, and finally, and L) represents number of application levels to perform any activities by renewable resources. In Table 2, characteristics of designed problems are presented.

As seen in Table 2, the framework for very small, small, medium, and large sized problems is in such a way that T is for very small problems in a range of 6-9, small problems in range of 10-17, medium problems in a range of 17-49, and large problems in a range of 55-95. For example, suppose that the problem ID is 8.1.2.1.1. In this case, there will be a project with at least 8 activities in the final network. In this project, there is also a flexible activity that 2 different routes can be considered for its performance, and a selection of each route leads to a new extra activity sub-network to the project main network. Therefore, the number of applicable activity network for project is 2. The number of available multi-skill resources for implementing the project is equal to 1, and the number of quality levels applied to each activity by renewable resources is considered equal to 1.

4.1. Comparing optimization software with proposed GA-MR and PSO-MR

In order to evaluate the performance of the mathematical model and meta-heuristics GA-MR and PSO-MR, 10 small problems are considered, and the results obtained from solving the problems are compared from the perspective of the time and quality of the obtained solutions. As shown in Table 3, the framework of the designed problems is in such a way that T is in the range of 6-9. Also, F
is equal to 1 and first $R$ is in range of 2-3, and second $R$ is in range of 1-2, and finally $L$ is in range of 1-2.

In Table 3, three columns are considered for the mathematical model, the proposed PSO-MR and GA-MR algorithms, in one of which the data related to the best final solution is registered (TC columns), and the other one represents the time required for reaching to the best solution (Time (s) columns). TC is the total estimated cost for an implementation of project, which is obtained from calculating the sum of three different costs. The project costs include: 1) the fixed cost of availability of resources during the project regardless of their uses for project activities; 2) cost of using available renewable resources to carry out project activities with regard to the selection of proper levels for their applications, and 3) cost of using non-renewable resources for project activities. Two other columns known as the gap (PSO-MR and GA-MR) are also considered. The gap represents the percentage of solution differed by PSO-MR and GA-MR compared to the optimal solution. For example, if the gap of the PSO-MR and GA-MR algorithms for a problem is 0.17% and 0.0%, respectively, then solution obtained by GA-MR is equal to the optimal solution, while PSO-MR obtains a solution with 0.17% worse than optimal solution.

As seen in Table 3 and Figure 8, the solution average obtained from the modeling and solving by GAMS software is equal to 1410.7 in time average 801.85s. The result obtained from the mathematical model is equal to result obtained from GA-MR and is better than PSO-MR with a difference of 0.148%. Therefore, it can be shown that presented mathematical model and two meta-heuristics algorithms are able to demonstrate high performance from the perspective of the solution in very small problems. As seen in Table 3, the PSO-MR in time average of 25.755s obtained near-optimal solutions and GA-MR in time average of 15.725s obtained optimal solutions. Since the PSO-MR and GA-MR by saving 776.095s and 786.125s compared to the average of time consumed by GAMS software can generate optimal solutions, the relative high performance of PSO-MR and GA-MR is reported in terms of the time spent into the modeling and solving by GAMS.

### 2.5. Comparing PSO-MR with GA-MR

Given that the modeling and problem solving by GAMS optimization was only appropriate for very small problems and cannot be justified for larger problems due to the required long times, the meta-heuristic algorithms can be used to generate near optimal solutions in short time. Therefore, in this paper, two improved meta-heuristic algorithms, known as PSO-MR and GA-MR, are presented to solve this problem. In order to evaluate performance of PSO-MR and GA-MR in large-sized problems, a number of 50 test problems in different scales are designed and their results are shown in Table 4. The designed problems are also solved separately with the presented PSO-MR, and the results are registered in terms of quality and time spent of the solutions obtained and compared with what is obtained by the presented GA-MR.
For problems with ID 001 to 020, observing the results recorded in Table 4 and Figure 9, it can be observed that 85% of the solutions obtained by PSO-MR is equal to best solutions resulting from the problem solving by GA-MR, and merely in 15% of the cases, results are better than PSO-MR. The solutions obtained from the proposed PSO-MR and GA-MR algorithms are on average 1449.45 and 1447.6, respectively. Thus, it can be concluded that two improved algorithms can generate the solutions with nearly identical quality for very small-scale problems.

On the other hand, as shown in Table 4 and Figure 10, the required time for achieving the solutions provided by PSO-MR and GA-MR algorithms is on average 25.88s and 17.51s, respectively. It can be argued that in very small-scale problems, the GA-MR has better performance than PSO-MR in terms of time performance.

For small-scale problems, by observing the results recorded in Table 5 and Figure 11, it can be denoted that solutions obtained from the PSO-MR are on average 1.4% worse than the best solutions obtained from solving problems by the GA-MR presented in this paper, while GA-MR has obtained the results through the saving time by 58.72% into PSO-MR. The solutions obtained from the proposed PSO-MR and GA-MR algorithms are on average 2131.6 and 2102.1, respectively. Therefore, it can be stated that the quality of generation solutions by these two algorithms is approximately the same, and the major difference is in the time of achieving the solutions.

As shown in Table 5 and Figure 12, the required time for achieving the solutions provided by PSO-MR and GA-MR algorithms is on average 389.77s and 160.86s, respectively. It can be argued that in small-sized problems, the GA-MR is better than PSO-MR in terms of time performance.

For the medium problems (ID 031 to 040) by observing the results recorded in Table 6 and Figures 13 and 14, PSO-MR is obtained on average quality of 5766.4 in time expending 2618.5s, while GA-MR is obtained on average 3.14% better results in time saving 965.7s into PSO-MR. Thus, it can be indicated that the quality of solutions generated by the two algorithms is almost the same, and the main difference is in the time of achieving the solutions. The PSO-MR algorithm requires a longer time than GA-MR algorithm for achieving the solutions in the medium sized problems.
For large-sized problems (ID 041-050) by observing the results recorded in Table 7 and Figures 15 and 16, it can be stated that the solutions obtained from the proposed PSO-MR and GA-MR are on average 10401.2 and 9899.4, respectively. Also, time spent for solving the problems is in average 7702.2 and 5048.9 using PSO-MR and GA-MR, respectively. Considering the quality of solutions, it can be denoted that GA-MR algorithm has achieved the best solutions in all the cases, and the PSO-MR algorithm has generated near solutions. Thus, it can be said that in large sized problems, the quality of solutions generated by GA-MR is better than PSO-MR. Also, the PSO-MR algorithm requires longer time for achieving the appropriate solution on the problems than GA-MR algorithm.

6. Conclusions

Regarding some technologies for performing several parts of the project is a significant issue, which were not properly examined in the resource constrained project scheduling problem (RCPSP) in the literature. Best selection of technology is critical due to changes in the main network of a project from the perspective of the number of activities required for the implementation, precedence relationships, resources allocations, and consequently, the time and cost of project completion. Hence, in this paper, RCPSP with multiple routes for flexible project parts, namely RCPSP-MR, was proposed with the aim of minimizing the cost of project completion. The cost of project completion, in this paper, was divided into three sections: 1) fixed cost of the availability of limited resources during the implementation of the project; 2) cost of using limited resources with different implementation levels for each project activity; and 3) cost of using non-renewable resources for implementing any project activity. In this paper, a new mathematical formulation of RCPSP-MR is firstly presented to find optimal solutions of the problem. Also, in order to solve large-sized problems, two improved meta-heuristic algorithms, named PSO-MR and GA-MR, were presented. Firstly, in the proposed genetic algorithm (GA), due to the importance of producing initial solutions, each chromosome in problem space was divided into seven parts, in which two parts were considered the sequencing of total of project activities (such as fixed and flexible activities); one part related to route was selected for flexible parts, and finally, four parts were regarded to assigned reassures and level of quality for implementation activities. Solutions were reinforced with crossover and mutations operations frequently and at each iteration, a specification of high quality solution was saved. Secondly, considering that the solution space presented, for the problem in this paper, was discrete and the proposed particle swarm optimization (PSO) algorithm was used in continuous space; it is necessary to use an appropriate method to transform the production solutions into continuous space. The method used in this paper was as follows: For each of the number of project activities an equal interval between 0 to 1 was considered. Selection of numbers inside each interval, at each iteration, indicated the selection of the corresponding activity. Also, in this proposed algorithm, in order to
produce quality solutions, using mutation was presented at GA algorithm, initial solution was presented. According to velocity vector in each iteration, obtained solutions with the proposed algorithm moved to the optimal solution. In order to assess the performance of the mathematical model and the proposed algorithms, number of 50 test problems (in very small, small, medium, and large sized) were designed and solved using the mathematical model and the proposed algorithms and comparisons were provided. The computational results showed that the proposed GA-MR generated high-quality solutions compared to the PSO-MR in a timely fashion. In very small problems, the results obtained from PSO-MR, GA-MR and mathematical model were approximately the same. Therefore, it can be concluded that proposed methods had decent performance in the perspective quality solution. In small and medium problems, it can be shown that PSO-MR and GA-MR were able to demonstrate relatively appropriate performance from the perspective of the quality of solution. But, GA-MR can obtain the results in less time than PSO-MR. Finally, in large sized problems, the quality of solutions generated by GA-MR was %100 better than PSO-MR. Also, the PSO-MR algorithm required longer time for achieving the appropriate solution on the large sized problems than GA-MR algorithm. The RCPSP-MR was interesting, and research potential was high. In this paper, a MINLP model and two meta-heuristic algorithms (i.e., PSO-MR and GA-MR) were proposed to solve the problem with the objective of minimizing the total cost of project end. In future researches, it is suggested to incorporate other well-known meta-heuristic algorithms, (e.g., SA, TS, and ACO) with other objective regarded to economic such as maximization of net present value or resource-based such as resource investment problem or regarded to cost, such as minimization ordering costs during the project implementation. Also, the problem should be extended in a connection with real world problems; therefore, another recommendation can be the use of concepts including preemptive scheduling, setup times, time lags, release dates and deadlines.

Acknowledgements

The authors are thankful to the editor and anonymous reviewers for constructive comments and recommendations that lead to improving this paper remarkably.

References


**Figures’ captions**

**Figure 1.** Project activity network

**Figure 2.** Crossover operation on a solution

**Figure 3.** Types of mutation operations

**Figure 4.** Flowchart of the proposed GA-MR

**Figure 5.** Flowchart of proposed PSO-MR

**Figure 6.** Network of activity in numerical example

**Figure 7.** Sequencing of activities

**Figure 8.** Results obtained from the problem solving by the mathematical model and PSO-MR and GA-MR

**Figure 9.** Results obtained from solving very small problems by PSO-MR and GA-MR in terms of TC

**Figure 10.** Results obtained from solving very small problems by PSO-MR and GA-MR in terms of time

**Figure 11.** Results obtained from solving small problems by PSO-MR and GA-MR in terms of TC

**Figure 12.** Results obtained from solving small problems by PSO-MR and GA-MR in terms of time

**Figure 13.** Results obtained from solving medium problems by PSO-MR and GA-MR in terms of TC

**Figure 14.** Results obtained from solving the medium problems by PSO-MR and GA-MR in terms of time

**Figure 15.** Results obtained from solving large problems by PSO-MR and GA-MR in terms of TC

**Figure 16.** Results obtained from solving large problems by PSO-MR and GA-MR in terms of time

**Tables’ captions**
Table 1. Total information of the first numerical example

Table 2. General characteristics of designed problems

Table 3. Results obtained from problem solving by the mathematical model and PSO-MR and GA-MR

Table 4. Computational results (very small problems)

Table 5. Computational results (small sized problems)

Table 6. Computational results (medium sized problems)

Table 7. Computational results (large sized problems)
Figures:

Figure 1.

Figure 2.
**Current solutions**

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| C.Flex | 1 | 3 | 2 | 4 |
| C.FR   | 2 | 1 | 1 | 2 |
| C.FL   | 3 | 3 | 1 | 1 |

**Current solutions**

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Figure 3.
Obtain necessary data and parameters
\( t = 1 \)

Generate initial solutions from fixed activities

Generate feasible solutions from fixed activities (C.Main)

Assign resource to fixed activities (C.R)

Determine quality level of implementation fixed activities by resource (C.L)

Assign a route out of alternative routes for each flexible activity (C.Route)

Generate initial solutions from flexible activities

Generate feasible solutions from flexible activities (C.Flex)

Assign resource to flexible activities (C.FR)

Determine quality level of implementation flexible activities by resource (C.FL)

Calculate the total cost of the project and value of each solution

Generate new solutions using crossover operation

Calculate the total cost of the project and value of each solution

Generate new solutions using mutation operation

Calculate the total cost of the project and value of each solution

Is the stopping conditions satisfied?

Delete low value solutions

Figure 4.
Obtain necessary data and parameters $t=1$

1. Generate initial solutions from fixed activities
2. Generate feasible solutions from fixed activities (C.Main)
3. Assign resource to fixed activities (C.R)
4. Determine quality level of implementation fixed activities by resource (C.L)

Start

5. Generate initial solutions from flexible activities
6. Assign a route out of alternative routes for each flexible activity (C.Route)
7. Generate feasible solutions from flexible activities (C.Flex)
8. Assign resource to flexible activities (C.FR)
9. Determine quality level of implementation flexible activities by resource (C.FL)

Calculate the objective function

10. Generate initial velocity vectors
11. Is the stopping conditions satisfied?
12. Generate new solutions using mutation operation
13. Calculate the objective function
14. Generate initial position vectors
15. Convert generated solutions to numbers in continuous space
16. Generate initial position vectors
17. Convert generated solutions to numbers in discrete space
18. Calculate the objective function
19. Determine values Pbest and Gbest
20. Yes

Finish

21. Update velocity vector
22. Update position vector
23. Convert generated solutions to numbers in discrete space
24. Determine values Pbest and Gbest
25. Yes

No

Determine quality level of implementation flexible activities by resource (C.FL)

Generate feasible solutions from flexible activities (C.Flex)

Assign resource to flexible activities (C.FR)

Generate initial solutions from flexible activities

Figure 5.
Figure 6
Figure 7.

Figure 8.
Figure 9.
Figure 10.

![Small Problems Graph](image)

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Figure 11.

![Small Problems Graph](image)

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Figure 12.
Figure 13.

Figure 14.
Figure 15.

Figure 16.
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**Route selection 1 for flexible activity 1**

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**Route selection 2 for flexible activity 1**

| $d$ | 15  | 8   | 9   | 12  | 12  | 14  | 13  | 15  |
|-----|-----|-----|-----|-----|-----|-----|-----|
| $C$ | 9   | 7   | 6   | 8   | 7   | 7   | 6   | 7   |
| $nr$| 1   | 2   | 1   | 2   | 1   | 2   | 1   | 2   |
| $\gamma$| 3 | 3   | 2   | 5   | 0   | 3   | 2   | 5   |

**Route selection 3 for flexible activity 1**

| $d$ | 15  | 8   | 9   | 12  | 14  | 10  | 13  | 15  |
|-----|-----|-----|-----|-----|-----|-----|-----|
| $C$ | 9   | 7   | 6   | 8   | 7   | 7   | 6   | 7   |
| $nr$| 1   | 2   | 1   | 2   | 1   | 2   | 1   | 2   |
| $\gamma$| 3 | 3   | 2   | 5   | 0   | 3   | 2   | 5   |

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**Problems’ specifications**

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Biographies:

Alireza Birjandi received his BSc and MSc in Industrial Engineering from Tafresh University, Tafresh, Iran, in 2010 and 2013, respectively. He is PhD student in Industrial Engineering at Islamic Azad University, South Tehran Branch, Tehran, Iran. His research interest is scheduling, project management and control. He has published several papers in international journals and conference proceedings.

Seyed Meysam Mousavi is an Associate Professor at Department of Industrial Engineering, Faculty of Engineering, Shahed University in Tehran, Iran. He received a PhD degree from the School of Industrial Engineering at University of Tehran, Iran, and is currently a member of Iran’s National Elite Foundation. He is now the Head of Industrial Engineering Department at Shahed University and a member of the Iranian Operational Research Association. His main research interests include: cross-docking systems planning, logistics planning and scheduling, quantitative methods in project management, multiple criteria decision making under uncertainty, and applied soft computing. He has published many papers and book chapters in reputable journals and international conference proceedings.

Behnam Vahdani is an Assistant Professor at Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University in Iran, and is a member of Iran’s National Elite Foundation. His current research interests include: Supply chain network design, facility location and design, multi-criteria decision making, uncertain programming, meta-heuristics algorithms and operations research applications. He has published numerous papers and book chapters in the aforementioned areas.