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Analysis of the second-grade fluid flow in a porous channel by Cattaneo-Christov and generalized Fick's theories

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Abstract. This study investigates the combined heat mass transport features in steady Magneto-Hydro-Dynamic (MHD) viscoelastic fluid flow through stretching walls of channel. The channel walls were considered porous. The analysis of heat transport was carried out with the help of Cattaneo-Christov heat diffusion formula and generalized Fick's theory was developed for the study of mass transport. The system of partial differential expressions was changed into an ordinary differential set by introducing suitable variables. The homotopic scheme was employed for solving the resultant equations and then, validity of the results was verified by various graphs. Moreover, an extensive analysis was performed on the influence of involved constraints on liquid velocity, concentration, and temperature profiles. It was observed that the normal component of velocity decreased by increasing Reynolds number or the viscoelastic constraint. Both temperature and concentration profiles were enhanced by increasing combined parameter and Reynolds number. The presence of thermal relaxation number and concentration relaxation number decreased temperature and concentration profiles, respectively.

1. Introduction

The phenomenon of heat transport occurs by thermal energy movement from an object to another one due to temperature difference. Such object may be solid, liquid, gas, or solid within a gas or liquid. The interest in heat transfer phenomenon is substantially increasing due to its countless industrial and technological applications. Instances of such applications are in fuel cells, energy production, and cooling process in various atomic devices. Fourier [1] developed one of the most effective fluid models for heat transport analysis. The unique feature of this model is its capability of heat transfer analysis in macroscopic systems. Furthermore, Fourier's mathematical modeling obtains the parabolic form of energy expression. Cattaneo [2] presented the generalization of this theory by adding relaxation time phenomenon, which transformed the expression of energy into hyperbolic type. This formula enabled the transport of heat by means of thermal waves with confined speed. Christov [3] extended Cattaneo's theory [2] by changing the time derivative with Oldroyd-B upper convected derivative. Straughan [4] utilized this law to elaborate the aspects of thermal convection in the flow of horizontal-layer viscous liquids. Han

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et al. [5] observed that the presence of thermal relaxation constraints suppressed thermal thickness of the boundary layer. Khan et al. [6] presented a numerical solution by using shooting technique to examine heat transport characteristics based on Cattaneo-Christov expressions. Later on, Meraj et al. [7] employed this heat diffusion model to address the heat transport effects on the flow of Jeffrey fluid with variable conductivity.

In several medical and engineering processes, the combination of stretched Magneto-Hydro-Dynamic (MHD) flows and electrically conducting materials has various applications, e.g., in metal working, nuclear reactors, plasma, thermal insulators, modern metallurgy, oil exploration, extraction of geothermal energy, and MHD generators. MHD flows in arteries in several physiological processes have significantly attracted the interest of researchers and scientists. An applied magnetic field is used to control the fluid flows, mixing of samples, heat transfer rate and biological transportation. Many researchers have contributed to the extensive analysis of the flow of various fluids in the presence of applied magnetic field [8–18].

Porous media have manifold applications to different engineering fields including petroleum technology, solar collectors, porous insulation, drying processes, geothermal energy, geophysics, oil recovery, packed beds, etc. Fluid flows in porous media mainly depend on the differential expression of the macroscopic motion of liquid. Darcy [19] in an experimental study observed that applied pressure gradient and flow velocity were linearly proportional in case of unidirectional flow under uniform medium. Darcy's theory has a very effective role in different applications of biomedical engineering, e.g., biological tissues [20]. Attia [21] computed asymptotic solutions for viscous liquid flow through an insulated disk by utilizing Darcy's model. In another attempt, Attia et al. [22] presented flow characteristics for time-dependent non-Newtonian liquid over rotating insulated disk with porous medium. Siddiq et al. [23] examined the Darcy-Forchheimer porous theory for convectively heated nanofluid flow. Some novel numerical computations regarding the flow of alumina nanoparticles through a permeable porous enclosure were presented by Sheikholeslami et al. [24].

In the past few decades, great attention has been devoted to the analysis and mathematical modeling of flow between stretching boundaries because of its scientific applications, e.g., in extrusion process in plastic and metal industries, artificial fibers, metal spinning, glass blowing, drawing plastic films, and metal industries. Many authors have studied this phenomenon in various fluids models with different physical properties. Ashraf and Batool [25] developed the algorithm of shooting method to find a numerical solution for buoyancy driven-flow of micropolar fluid in a disk. Turkyilmazoglu [26] computed the numerical expressions of flow of viscous fluid by radially stretchable rotating disk employing spectral numerical integration scheme. Hayat et al. [27] employed the Homotopy Analysis Method (HAM) to discuss steady-state flow of Newtonian fluid generated by stretchable rotating disk.

The flow problem associated with pulsating motion of the walls of channel has been another hot research topic in recent years. The magnetohydrodynamic channel flow has various theoretical and practical applications, e.g., in space vehicle reentry, accelerators, astrophysical flows, and solar power technology. An extensive analytical study of a viscoelastic fluid flow caused by stretching walls of the channel has been presented in [28]. Misra et al. [29] captured the novel features of a viscous fluid model by considering MHD flow in a channel with pulsating walls by using finite difference method. Misra et al. [30] carried out numerical investigations into electrically conducing flow of non-Newtonian fluid through a stretching wall channel. They considered the blood as non-Newtonian fluid in their investigation. In another contribution, Misra et al. [31] explored the effects of induced magnetic field on flow of the second grade between channel walls. They also utilized the heat transfer phenomenon by using well Fourier law of heat conduction. Raftari and Vajravelu [32] showed that the series solution obtained by HAM had an excellent agreement with the results achieved by Misra et al. [31]. Abbasi et al. [33] presented slip flow of electrically conducting Maxwell fluid in porous channel with stretchable walls.

This study is aimed at combined mass transport analysis of electrically conducting viscoelastic fluid in a porous channel with stretching walls by using Cattaneo-Christov heat diffusion. The results of this study may be of interest to fluid dynamicity researchers and physiologists in their investigations into blood flow in arteries under chemical reaction. The solution to the modeled problem is obtained via the HAM. The physical properties of various flow parameters are explored with various graphs in this study.

2. Flow analysis

We consider the steady-state situation of laminar, incompressible, second-grade fluid flow confined by planes $y = \pm a$. It is assumed that the fluid particles flow solely by the motion of stretching walls. A magnetic force of strength B_0 is imposed perpendicular to the walls of the channel (see Figure 1). It is noteworthy that the dominance of the induced magnetic force can be neglected, because we assume low magnetic Reynolds numbers. Let T_w represent the constant temperature of the walls. The equations of

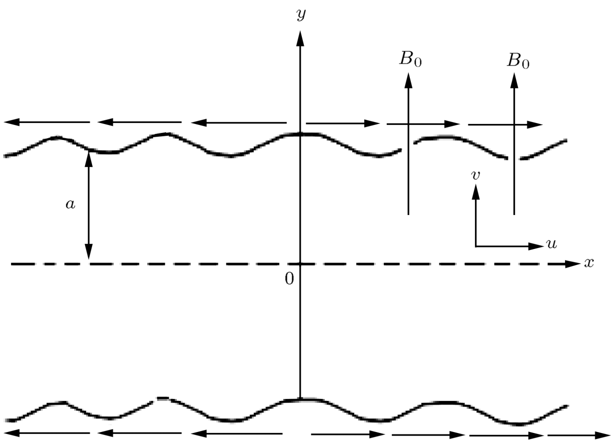

Figure 1. Geometry of problem.

flow dynamics in the channel can be developed as [24]:

$$\frac{\partial u}{\partial \bar{x}}+\frac{\partial v}{\partial \bar{y}}=0, \tag{1}$$

$$u\frac{\partial u}{\partial \bar{x}}+v\frac{\partial v}{\partial \bar{y}}=\nu\frac{\partial^2 u}{\partial \bar{y}^2}+k_0\left[u\frac{\partial^3 u}{\partial \bar{x}\partial \bar{y}^2}+v^2\frac{\partial^3 u}{\partial \bar{y}^3}+\frac{\partial u}{\partial \bar{x}}\frac{\partial^2 u}{\partial \bar{y}^2}-\frac{\partial u}{\partial \bar{y}}\frac{\partial^2 u}{\partial \bar{x}\partial \bar{y}^2}\right]-\frac{\sigma B_0^2}{\rho}u-\frac{\nu\vartheta}{\rho k^*}u. \tag{2}$$

where u and ν reflect velocity components along the coordinate axes $\bar{x}$ and $\bar{y}$, respectively. Moreover, ν is the kinematic viscosity, σ the electric intensity, B_0 strength of the applied magnetic field, ρ fluid density, k_0 the elasticity parameter, ϑ the porosity parameter, and k^* the permeability parameter. The imposed boundary conditions are [24]:

$$u=b\bar{x},\ v=0,\ \text{at}\ \bar{y}=a,$$

$$\frac{\partial u}{\partial \bar{y}}=0,\ \ v=0,\ \text{at}\ \bar{y}=0. \tag{3}$$

Let us introduce dimensionless quantities [21]:

$$u=bxf'(\eta),\quad v=-abf(\eta),\quad \eta=\frac{y}{a}. \tag{4}$$

Using the above dimensionless variables, Eqs. (2) and (3) can be transformed to:

$$f'''+K\left[2f'f'''-(f'')^2-ff''''\right]-\mathrm{Re}\left[(f')^2-ff''\right]-Mf'=0, \tag{5}$$

$$f(0)=0,\quad f(1)=0,\quad f'(1)=1,\quad f''(0)=0, \tag{6}$$

where $K=\alpha_1 b/\mu$ is the viscoelastic parameter, $M=\sqrt{\frac{\sigma}{\mu}}B_0 a+\frac{\nu\vartheta}{bk^*}$ the combined magnetic and porosity parameter, and $\mathrm{Re}=a^2b/\nu$ the Reynolds number.

2.1. Heat and mass transfer analysis

The generalized expressions of Fourier's and Fick's theories are [5,6]:

$$\mathbf{q}+\lambda_E\left[\frac{\partial \mathbf{q}}{\partial t}+(\nabla\cdot\mathbf{V})\,\mathbf{q}+\mathbf{V}\cdot\nabla\mathbf{q}-\mathbf{q}\cdot\nabla\mathbf{V}\right]=-k\nabla T, \tag{7}$$

$$\mathbf{J}+\lambda_C\left[\frac{\partial \mathbf{J}}{\partial t}+(\nabla\cdot\mathbf{V})\,\mathbf{J}+\mathbf{V}\cdot\nabla\mathbf{J}-\mathbf{J}\cdot\nabla\mathbf{V}\right]=-D_B\nabla C, \tag{8}$$

where $\mathbf{q}$ and $\mathbf{J}$ denote the heat and mass fluxes, respectively; D_B stands for molecular diffusivity of the species; and λ_E and λ_C are the thermal and concentration relaxation times, respectively. It is noteworthy that for $\lambda_E=\lambda_C=0$ in Eqs. (7) and (8), the expressions for Classical Fourier's and Fick's laws have been retained. Thus, the energy equation and the presence of thermal radiation and concentration equation are defined as:

$$u\frac{\partial T}{\partial \bar{x}}+v\frac{\partial T}{\partial \bar{y}}+\lambda_E\Omega_E=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{9}$$

$$u\frac{\partial C}{\partial \bar{x}}+v\frac{\partial C}{\partial \bar{y}}+\lambda_C\Omega_C=D_B\frac{\partial^2 C}{\partial \bar{y}^2}, \tag{10}$$

where α represents the liquid thermal diffusivity and Ω_E and Ω_C can be expresses as:

$$\Omega_E=\left[u\frac{\partial u}{\partial \bar{x}}\frac{\partial T}{\partial \bar{x}}+v\frac{\partial v}{\partial \bar{y}}\frac{\partial T}{\partial \bar{y}}+2uv\frac{\partial^2 T}{\partial \bar{x}\partial \bar{y}}+u\frac{\partial v}{\partial \bar{x}}\frac{\partial T}{\partial \bar{y}}+v\frac{\partial u}{\partial \bar{y}}\frac{\partial T}{\partial \bar{x}}+u^2\frac{\partial^2 T}{\partial \bar{x}^2}+v^2\frac{\partial^2 T}{\partial \bar{y}^2}\right], \tag{11}$$

$$\Omega_C=\left[u\frac{\partial u}{\partial \bar{x}}\frac{\partial C}{\partial \bar{x}}+v\frac{\partial v}{\partial \bar{y}}\frac{\partial C}{\partial \bar{y}}+2uv\frac{\partial^2 C}{\partial \bar{x}\partial \bar{y}}+u\frac{\partial v}{\partial \bar{x}}\frac{\partial C}{\partial \bar{y}}+v\frac{\partial u}{\partial \bar{y}}\frac{\partial C}{\partial \bar{x}}+u^2\frac{\partial^2 C}{\partial \bar{x}^2}+v^2\frac{\partial^2 C}{\partial \bar{y}^2}\right]. \tag{12}$$

We impose the following boundary conditions for governing energy and concentration equations:

$$T=T_w,\quad C=C_w\quad \text{at}\quad \bar{y}=a,$$

$$\frac{\partial T}{\partial \bar{y}}=0,\quad \frac{\partial C}{\partial \bar{y}}=0,\quad \text{at}\quad \bar{y}=0. \tag{13}$$

The governing equations are made dimensionless by introducing the following dimensionless variable:

$$\theta=\frac{T}{T_w},\quad \varphi=\frac{C}{C_w}, \tag{14}$$

$$\frac{1}{\mathrm{Pr}}\theta''+\mathrm{Re}f\theta'-\mathrm{Re}\delta_T\left(ff'\theta'+f^2\theta''\right)=0, \tag{15}$$

$$\frac{1}{Sc}\varphi'' + \mathrm{Re}f\varphi' - \mathrm{Re}\delta_C\left(ff'\varphi' + f^2\varphi''\right) = 0, \quad (16)$$

where $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number, $\delta_T = \lambda_E b$ thermal relaxation constraint, $\mathrm{Sc} = \nu/D_B$ the Schmidt number, and $\delta_C = \lambda_C b$ the concentration relaxation constraint. The dimensionless boundary conditions are:

$$\theta'(0) = 0, \quad \theta(1) = 1, \ \varphi'(0) = 0, \varphi(1) = 1. \quad (17)$$

3. Homotopy Analysis Method (HAM)

Mathematical modeling of many engineering problems involves the differential equations of highly nonlinear nature. It remains a challenge for engineers to compute such equations by either the analytical solution or the numerical one. The famous analytical HAM is a powerful technique to derive the series solution to differential expressions without the restrictions of large or small constraints. This useful method offers great freedom to adopt and control the region of convergence. The main benefit of this method over various numerical techniques is that it is round-off-error-free thanks to its discretization process. The computations of this method do not make large demands on time and computer memory. Since the time the approach was proposed by Liao [34], various researchers have adopted it to solve the governing differential equations [35–41]. In the present section, we introduce one of the most powerful techniques to solve Eqs. (5), (15) and (16) by using homotopy analysis scheme for discussing the analytical solution for all the values of given parameters. In the beginning, we suggest the following initial approximations for the given flow problem:

$$f_0(\eta)=\frac{1}{2}\left(\eta^3-\eta^2\right), \quad \theta_0(\eta)=\frac{1}{2}\eta^2, \quad \varphi_0(\eta)=\frac{1}{2}\eta^2. \quad (18)$$

The differential operators are:

$$\pounds_f = \frac{\partial^3}{\partial\eta^3}, \ \pounds_\theta = \frac{\partial^2}{\partial\eta^2}, \ \pounds_\varphi = \frac{\partial^2}{\partial\eta^2}, \quad (19)$$

satisfying:

$$\pounds_f\left[A_1 + A_2\eta + A_3\eta^2 + A_4\eta^3\right] = 0, \quad (20)$$

$$\pounds_\theta\left[A_5\eta + A_6\eta^2\right] = 0, \quad (21)$$

$$\pounds_\varphi\left[A_7\eta + A_8\eta^2\right] = 0, \quad (22)$$

where A_i $(i = 1, 2, ..., 8)$ denotes constants.

4. Convergence of solution

The series solution obtained in the previous section is highly dependent on auxiliary constraints h_f, h_θ, and h_φ. The appropriate values of these parameters are necessary for convergence of the HAM solution. Graphs of these auxiliary parameters are plotted at the 16th order of approximation for the selection of the suitable range for these parameters. The admissible values for $-0.8 \leq h_f \leq -0.2$, $-1.5 \leq \hbar_\theta < -0.3$, and $-1.5 \leq h_\varphi < -0.3$ are given in Figure 2. The role of convergence in the given region is quite vital for guaranteeing the attainment of the proper solution. It is commonly known that when the values of the involved parameters are not properly selected, comparison of HAM with other exact or numerical methods becomes quite difficult and time consuming. Hence, suitable selection of the parameters also guarantees achieving the solution for various sundry parameters [42]. In order to validate the achieved solution, the presented results are compared with the already available numerical values reported by Hayat et al. [28] (see Table 1). It will be observed that our results are in excellent agreement with the reported results.

Table 1. Comparison of the presented results with those of Hayat et al. [28] for $\beta = 6.0$.

K	Re	Hayat et al. [28]	Presented results
0.0	10	7.7958	7.79583
0.4		15.4094	15.40952
0.6		22.1710	22.17100
0.6	0.0	17.7101	17.71010
	5.0	18.2930	18.29329
	15.0	19.4301	19.43020

5. Results and discussion

After achieving the explicit analytical solution discussed in the previous section, the aim of this section is to provide a deeper view of the presented results with various flow parameters. A graphical analysis for the distinct governing parameters of velocity, temperature, and concentration profiles is presented in Figures 3–17. The viscoelastic parameter K on normal velocity component $f(\eta)$ is graphically shown in Figure 3. The values of K range from 0 to 5.5. It is observed that the magnitude of velocity increases with larger values of K. Figure 4 indicates that back mass flow rate decreases when M increases. The combined parameter, which is a combination of both Hartmann number and porosity parameter, effectively reduces the magnitude of the velocity of fluids particles. Figure 5 depicts the influence of Reynolds number (Re) on the velocity

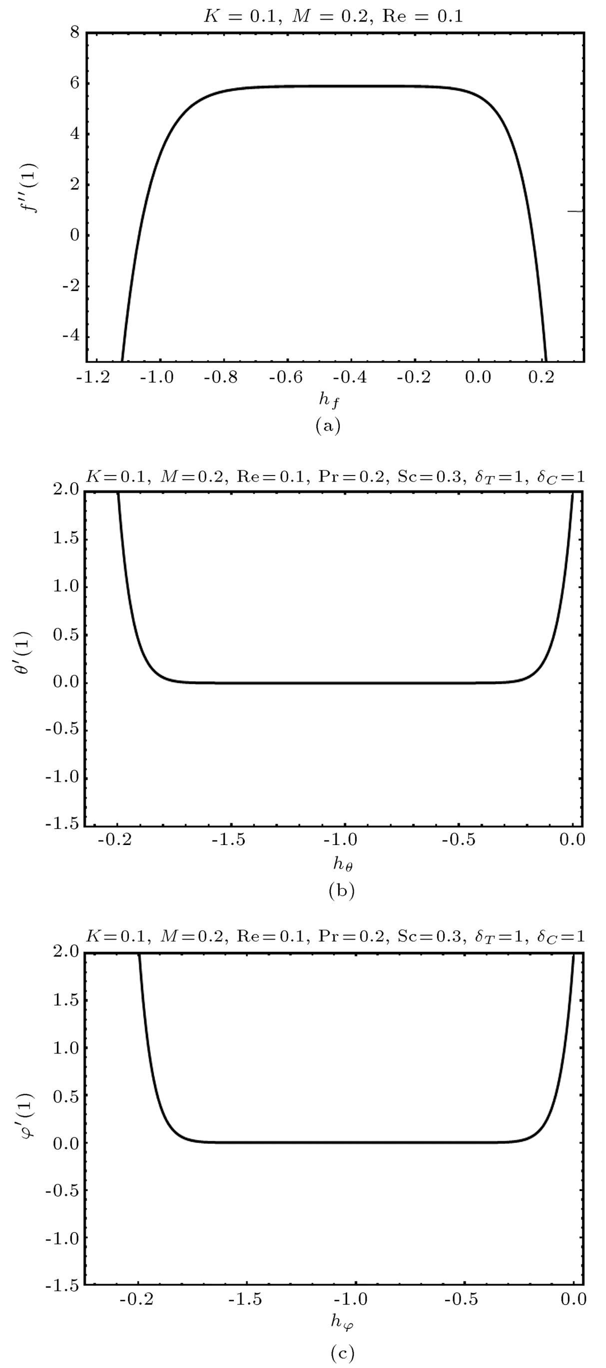

Figure 2. h-curves for (a) velocity, (b) temperature, and (c) concentration profile.

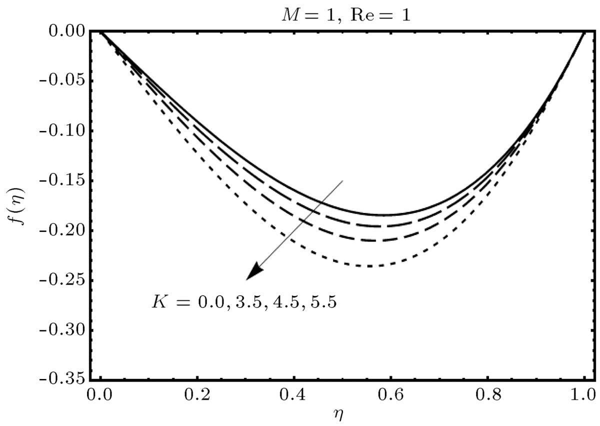

Figure 3. Graph of K on f.

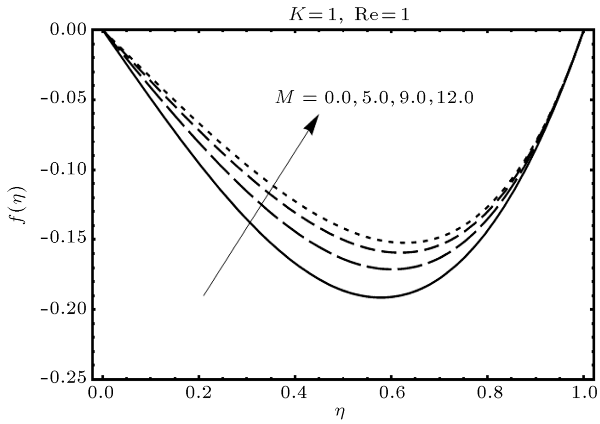

Figure 4. Graph of M on f.

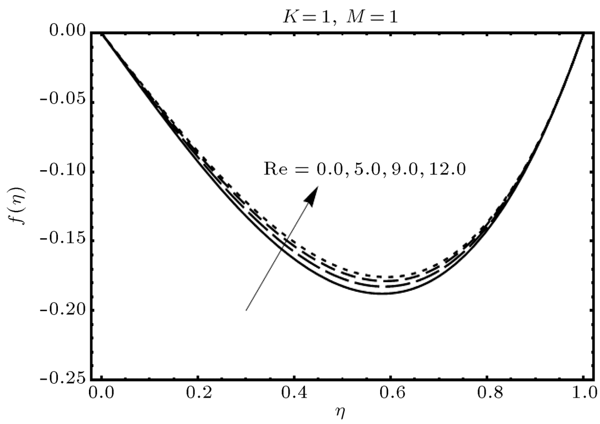

Figure 5. Graph of Re on f.

component $f(\eta)$. The magnitude of velocity $f(\eta)$ decreases in the whole region by increasing Reynolds number (Re). Figure 6 demonstrates that when M and Re are constant and K is varying, the back flow velocity in the center line decreases, but the region of back flow increases. Here, an observation is of significance within the region of the channel. While the flow smoothly increases near the central line, it is reduced near the walls of the channel.

The variation of velocity f' against η for distinct values of M is depicted in Figure 7. It is evident from the figure that the horizontal component of velocity $f'(\eta)$ has increasing behavior near the walls, but a decreasing trend near the central line of the channel. By increasing the magnetic field, back flow velocity in the central line decreases, but the region of back flow increases. The influence of Re ranging from 0 to 12 on $f'(\eta)$ is sketched in Figure 8. The movement of stretching walls results in a back flow near the central

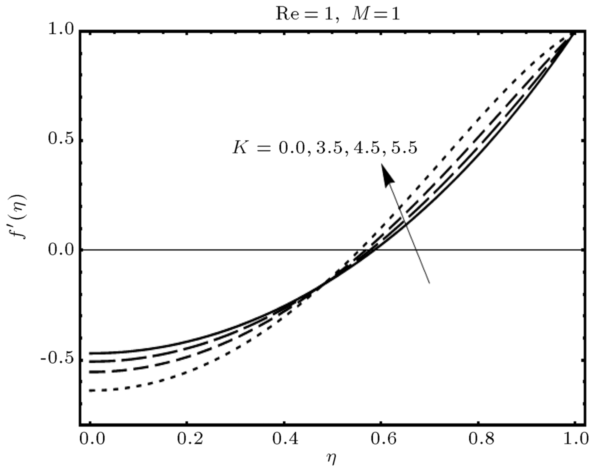

Figure 6. Graph of K on f'.

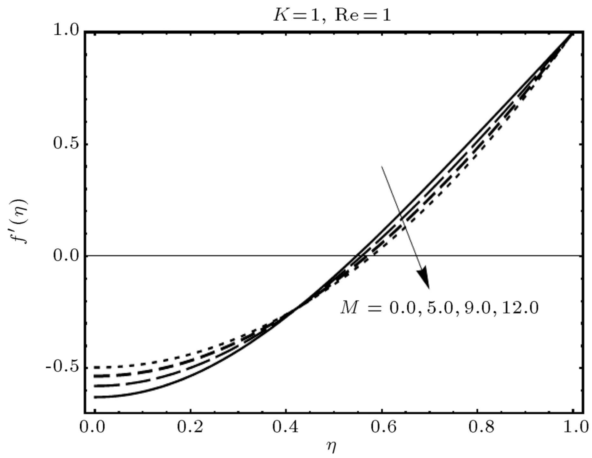

Figure 7. Graph of M on f'.

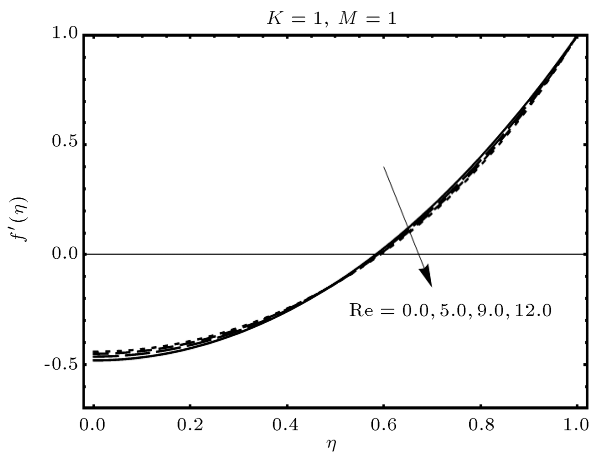

Figure 8. Graph of Re on f'.

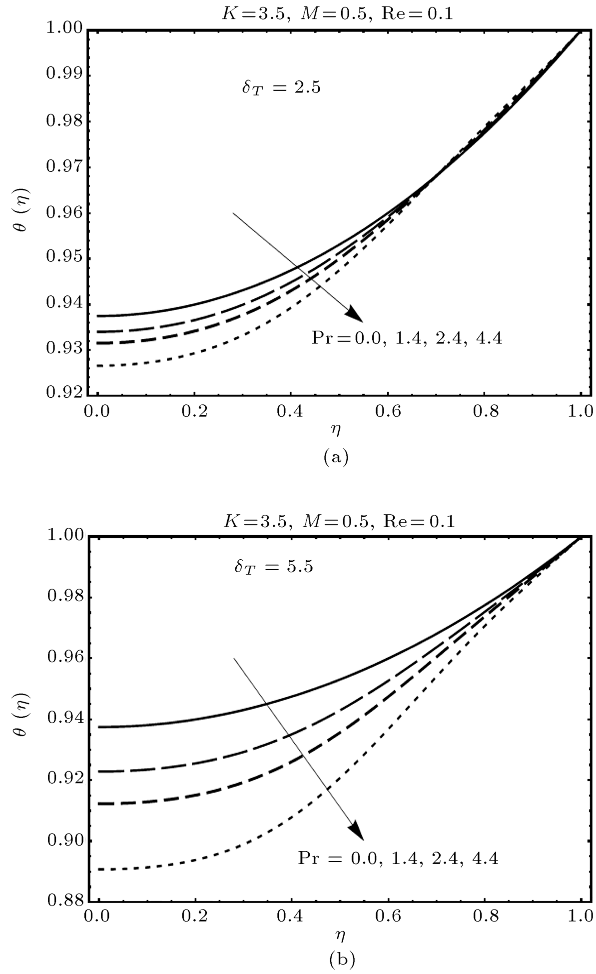

Figure 9. Graph of Pr on θ for (a) $\delta_T = 2.5$ and (b) $\delta_T = 5.5$.

line of the channel. It is observed that velocity near the walls of the channel decreases down to $\eta = 0.5$. However, an opposite behavior is observed far away from the center of the channel.

The influence of different values of Prandtl number (Pr) on θ is demonstrated in Figure 9. In Figure 9(a), variation of Prandtl number, Pr = 0.0, 1.4, 2.4, 4.4, is presented taking $\delta_T = 2.5$. The temperature profile shows decreasing behavior at low thermal diffusivity. Figure 9(b) also gives the variation of Pr for $\delta_T = 5.5$. A similar trend is observed again. However, the rate of heat transfer is smaller in this case. Thus, proper selection of δ_T can be more useful for increasing or decreasing the fluid temperature. Figure 10 indicates superiority of the combined parameter M over θ. The flow field develops a Lorentz force, which acts like a frictional force and hence, increases temperature of the fluid. In Figure 11, temperature of the fluid decreases in the whole domain by increasing the viscoelastic parameter K. In Figure 12, the influence of Reynolds number (Re) on $\theta(\eta)$ is depicted. A rise in temperature is observed with the dominant values of Re. Moreover, a rise in the value of Re increases the boundary layer thickness. Figure 13 illustrates the influence of relaxation time constant δ_T on temperature profile θ. It is observed that the temperature profiles with the related thermal boundary layer thickness are reduced with increase in δ_T in the whole domain.

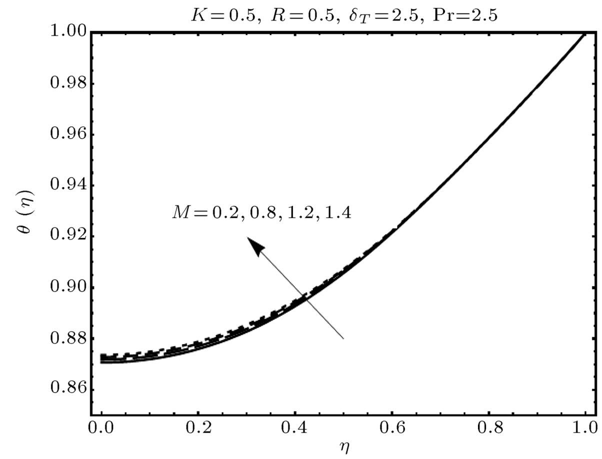

Figure 10. Graph of M on θ.

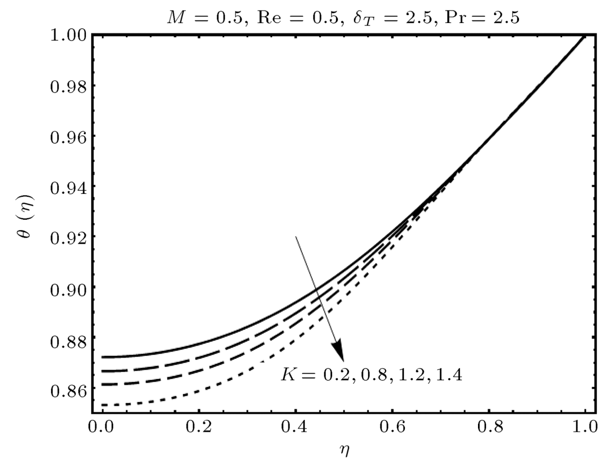

Figure 11. Graph of K on θ.

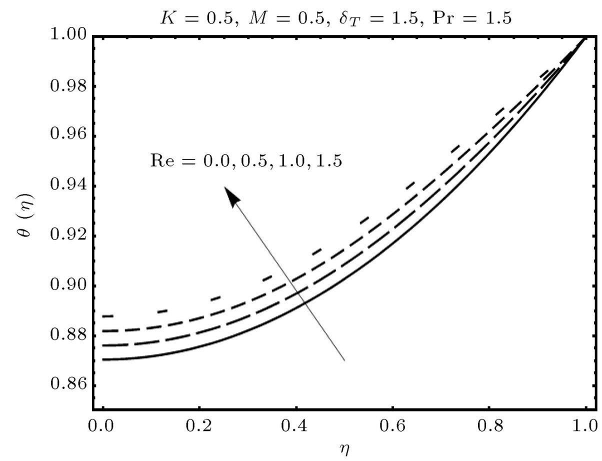

Figure 12. Graph of Re on θ.

Figure 14 shows the behavior of concentration profile φ with the viscoelastic parameter. The concentration field decreases by increasing viscoelastic parameter. Contrarily, it is found to increase with increase in M (Figure 15). Figure 16 elucidates the outcomes for Re with concentration distribution. It is noted

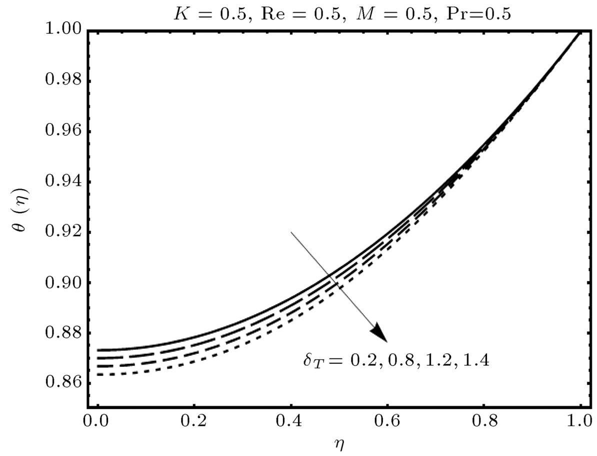

Figure 13. Graph of δ_T on θ.

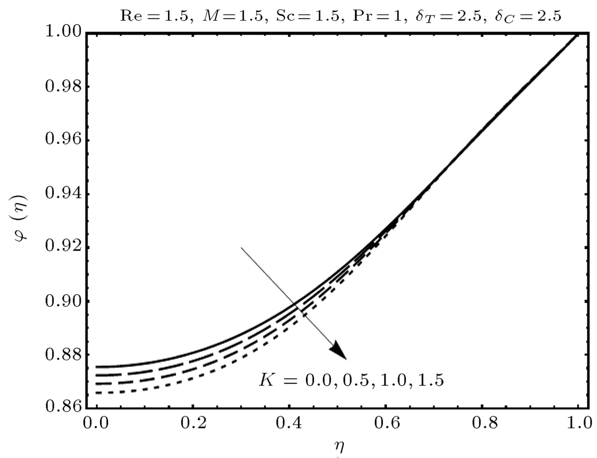

Figure 14. Graph of K on φ.

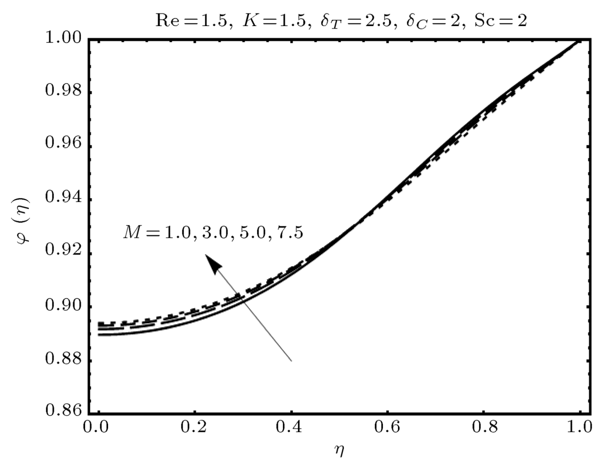

Figure 15. Graph of M on φ.

that concentration profile is enhanced by increasing Re. Finally, Figure 17 presents the effects of relaxation time constant δ_C on concentration profile. It is seen that concentration profile has a decreasing behavior by increasing δ_C.

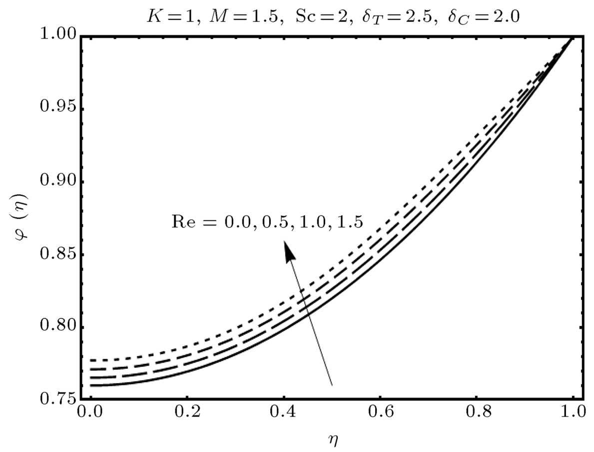

Figure 16. Graph of Re on φ.

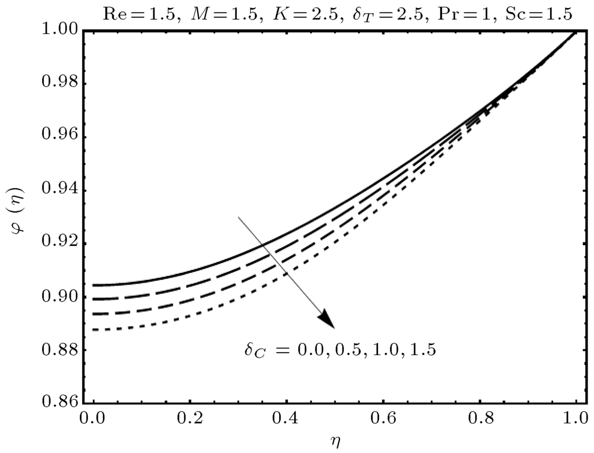

Figure 17. Graph of δ_C on φ.

6. Concluding remarks

Magneto-Hydro-Dynamic (MHD) steady-state flow of second-grade fluid in a channel with stretching walls in the presence of porous medium was considered. The partial differential system governing the flow was first transformed into a fourth-order nonlinear differential equation and then, solved by employing Homotopy Analysis Method (HAM). This method can be useful in other analytical methods that use the series solution. The conclusions drawn in the present study are the following:

- A decrease in the normal component of velocity was observed by increasing combined parameter and Reynolds number, while it had an opposite trend by increasing viscoelastic parameter;
- By increasing combined parameter and Reynolds number, the fluid temperature in whole domain increased and demotion in profile was observed by increasing viscoelastic parameter, Prandtl number, and thermal relaxation number;
- Larger values of thermal relaxation number reduced temperature profile.

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