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Optimal pricing in the presence of advance booking strategies for complementary supply chains

M. Noori-daryan, A.A. Taleizadeh*, and M. Rabbani

School of Industrial Engineering, College of Engineering, University of Tehran, Iran.

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Abstract. With the high competition among companies, they tend to improve their market share by applying different selling mechanisms such as online to offline commerce as an efficient selling mechanism in which they sell their products via both online and real stores. This study deals with a selling problem for two complementary supply chains including a supplier and a shopping center where the commodities are sold by a virtual shopping center and a traditional one that present items as complementary shopping centers. It was assumed that market demand depended on price and service level so that the customers could purchase items via both shopping centers based on their priorities. Also, to analyze the reactions of the partners of the chain, different games were deemed. The aim was to obtain the closed-form solutions for the decision variables of the members of the network in order to maximize their profits. Prices of different selling periods at each echelon of the chains were decision variables of the model. The closed-form solutions for the decision variables were derived and the solutions were examined by a numerical example. Several sensitivity analyses of the key factors were performed to determine the efficient ones for the variables and profits.

1. Introduction and literature review

Recently, pricing policies have become a major concern for industries and corporates as an efficient tool for improving profitability, since one of the factors influencing decisions of the customers is the price of products. Companies strive to apply optimal pricing decisions to the management and control of the market demand in addition to raising their profits. Thus, competition among companies as well as supply chains increases to satisfy demands of the customers and subsequently, enhance profits through attracting the market demand.

*. *Corresponding author.*
E-mail addresses: m.nooridaryan@ut.ac.ir (M. Noori-daryan); taleizadeh@ut.ac.ir (A.A. Taleizadeh); mrabani@ut.ac.ir (M. Rabbani)

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Some authors have assumed pricing policies in their research. Monahan (1984) [1] reviewed a discount scheme proposed by a seller to buyers. The seller sought to obtain the optimal price of the commodity and their aim was to encourage customers to buy more amounts of the desired product at reasonable prices, thereby increasing profits and reducing operational costs.

Whitin (1969) [2] examined pricing and inventory policies to minimize the cost of inventory systems. Benerjee (1986) [3] developed a model considering the optimal discount from the point of view of the seller. In this model, which was an extension of Monahan's model, the seller bought their goods from another supplier. Ardalan (1991) [4] jointly studied optimal inventory and pricing policies for various combinations of sales periods and response times. Also, an algorithm was proposed to determine the combination of optimal ordering and pricing strategies.

Boyaci and Gallego (2002) [5] analyzed coordination issues in a two-tier supply chain including a wholesaler and one or more retailers. In this chain, the demand of the customers was a function of the price of goods and the operating costs included those of purchasing, commissioning, ordering, and keeping inventory. Elmaghraby and Keskinocak (2003) [6] performed a review of dynamic pricing literature as an influential field in business. They focused on dynamic pricing with inventory issues. Agrawal et al. (2004) [7] presented a dynamic model for the allocation of inventory to a group of retailers to address the imbalance among inventory problems. They examined demand and net income information for inventory planning and balancing in a retail network and addressed the issue with the aim of determining delivery time and inventory levels for retailers under a dynamic planning. Hua et al. (2010) [8] reviewed optimal pricing and delivery decisions in centralized and decentralized supply chains involving a producer and a retailer with two channels (online sales and direct sales). Using a Stackelberg game, the impact of delivery time on the selling prices of producer and retailer was evaluated and the results showed that delivery time had a considerable effect on the retail prices.

Chen et al. (2013) [9] examined a competitive decision-making model for two companies by a Stackelberg cooperative method and found that a non-cooperative approach would lead to a higher income and lower cost to the retailer and consequently, it brings about lower profit for the producer. Chen and Hu (2012) [10] introduced a single-product pricing model with a common inventory under a finite planning horizon with a deterministic demand. In this model, the ordering and pricing decisions were determined simultaneously at the beginning of each decision period where demand depended on the price. Taleizadeh and Noori-daryan (2015a) [11] developed a non-linear programming model to examine the reactions of the members of a three-echelon supply chain in terms of pricing, manufacturing, and ordering decisions.

Saha et al. (2017) [12], Yan et al. (2018) [13], and Jamali and Rasti-Barzoki (2018) [14] discussed different mathematical revenue management problems to model and study the behavior of the partners of a supply chain under various decision making processes using Stackelberg and wide optimal methods. Noori-daryan et al. (2017) [15] considered a multi-national supply chain including some capacity-constraint producers and a retailer in which the members made decisions about their priorities such as price, order quantity, lead-time, and selection of supplier under different games. In addition, some investigations have been carried out in revenue management area to optimize price decisions under various conditions, e.g. [16–31].

Booking strategies as incentive mechanisms are applied by companies to the management of selling capacities. Under these types of policies, companies launch their items at a lower price than their real prices under capacity restriction to encourage customers and absorb the demand of the market. Some researchers have studied pricing policies under booking strategies in different industries, e.g. [32–42]. In 1992 and 1993, Gale and Holmes evaluated booking strategies and found out that pre-sales strategies offered to passengers for pick flights were profitable for the airlines. Li (2001) [43] discussed a pricing issue for depreciable commodities or services such as tickets or rooms of a hotel. The results of this study showed that, as the airlines expected, leisure travelers were more price-sensitive than business travelers. Then, in 1998, Dana investigated an advance booking policy in a competitive environment.

Chen and Schwartz (2006) [44] considered a reservation strategy for studying the behavior of customers and how much they bought from a hotel and an airline. Cho and Tang (2010) developed a Stackelberg game model to examine the interactions between a manufacturer and a retailer where each company set its sales prices under reservation policies. They found that the policy was always profitable for the producer and the retailer benefited only when there was a probability of shortage. Acciaro (2011) [45] offered a service-based model based on reservation strategy of the carrier with the assumption that the service-level customers were different depending on the types of customer, cargo, and route. Mei and Zhang (2013) [46] proposed a pricing issue with a reserved discount policy and presented a stochastic programming model. In this research, discount rates, prices, and optimal ordering were determined and numerical results indicated higher sensitivity and profitability of the sales price. They concluded that reasonable prices would lead to the greatest profit for the companies.

Ata and Dana Jr. (2004) [47] developed a mathematical model to examine a price differentiation strategy regarding different reservation times of customers. Ling et al. (2015) [48] proposed an approach to enhancing coordination between a hotel and an online travel agency to book rooms of the hotel. In this approach, customers were able to book their rooms through the sales system of the hotel or an online travel agency. Furthermore, the hotel anticipated the possible market demands with regard to the booking information and the optimal availability approach to booking rooms, taking into account the maximum revenue through both methods (online and offline), was identified. Bilotkach et al. (2015) [49] examined the impact of reducing the price of flight ticket on the revenue of two airlines and found that reducing the standard rate of fares would increase the average demand of consumers by an average of 2.7 percent.

It should be noted that this fall in prices did not have considerable effect on the demand of customers for tourist trips. Guizzardi et al. (2017) [50] studied the impact of booking strategies, service quality, and service levels on purchasing decisions of customers for a hotel. Zhao et al. (2017) [51] investigated the behaviors of firms in terms of capacity allocation and pricing in full-fare and discount market. Benítez-Aurioles (2018) [52] studied the relation between reservation and price in the tourist industry.

Game theory is a methodology for surveying the behavior of players under competition and collaboration in a supply chain. Several authors have employed this methodology to identify the decision variables. For instance, Dai et al. (2005) [53] analyzed pricing strategies for several competing companies offering the same services to a group of customers to improve their revenue management. Companies had limited capacity and the market demand to each company depended on its sales price. They used game theory approaches to checking the systems when companies were faced with deterministic and probabilistic demand. They applied a Nash equilibrium point to the case in which the demand to each company was a linear function of price. Liu et al. (2007) [54] considered a decentralized supply chain consisting of a retailer and a provider in which demand depended on price and service level and there was a Stackelberg game among the partners. They concluded that decentralized decision-making resulted in lower performance and the supplier should improve its internal operations before pursuing a coordinating strategy with retailers.

Szemerekovsky and Zhang (2009) [55] reviewed pricing and two-level advertising decisions in a single-producer-single-retailer supply chain. They assumed that the demand of the customers was a function of retail price and advertising cost paid by the manufacturer and retailer, and considered a Stackelberg game in which the producer was the leader and the retailer was the follower. They found that better performance would be achieved if the producer incurred the advertising costs and offered a lower wholesale price to the retailer. Grauberger and Kimms (2016) [56] presented a non-linear programming model to determine optimal pricing and reservation constraints under Nash games and found that revenues of the firms in an exclusive market were lower than those in a non-exclusive one. Shah et al. (2014) [20], Taleizadeh et al. (2015b) [57], and, Noori-daryan and Taleizadeh (2016) [58] also considered game theory approaches to solving their presented models.

A look at the literature review given above indicates that there is no study investigating pricing decisions in two dual-channel supply chains under an advance booking strategy in which demand is service-level and price-sensitive and two complementary commodities are sold. In addition, upstream members offer an advance booking policy to downstream members as well as customers. All members determine their optimal pricing decisions under a non-cooperative game so that their profits are optimized. The rest of the paper is organized as follows. The problem is described in Section 2. Section 3 formulates the introduced model. In Section 4, a solution procedure is proposed and in Section 5, a real case and the results are given. Finally, Section 6 presents the conclusion and findings of the study.

2. Problem statement

Here, a pricing problem for complementary items under an advance booking mechanism is studied in which commodities are launched by the sellers at a lower price in advance and then, they are priced higher than before. The commodities are provided by two complementary supply networks in which the supplier sells them through a virtual and a real shopping center in each chain. Under this strategy, the networks aim to encourage the customers to order in advance in order to manage the market demand. The structure of the networks is presented in Figure 1.

The purpose of this study is to find the optimal decisions of the members of the networks, the suppliers, and the retailer on the selling prices of commodities within pre-selling and selling periods so that the profit of the members and the networks is optimized. In addition, the reactions of the suppliers who supply the complementary products are surveyed by applying a Nash game. A leader-follower game is assumed between the suppliers and its shopping center in which the role of the follower is assigned to the retailer and the real store, and the suppliers as the leaders optimally determine their decision policies according to the reaction of the retailer.

The problem is modeled by utilizing the following assumptions:

1. Price and service level affect the demand of the customers;
2. Each network is composed of a supplier and a shopping center;
3. Suppliers present complementary items to the customers;

Figure 1. Structure of supply chains.

4. Customers strive for purchasing both commodities as complementary ones;
5. Suppliers launch their commodities through an online and a traditional shopping center;
6. A common shopping center is considered to sell both commodities for attracting the customers of one item to the other complementary item;
7. Service level of the real shopping center is higher than that of the online stores;
8. An advance booking strategy is applied to selling the commodities by online and offline shopping centers.

The problem in hand is formulated by utilizing the following notation:

Parameters:

$q_1^1(t)$	Booking level of the first supplier within $[0, t_1]$
$q_1^2(t)$	Booking level of the first supplier within $[t_1, T]$
$q_2^1(t)$	Booking level of the second supplier within $[0, t_1]$
$q_2^2(t)$	Booking level of the second supplier within $[t_1, T]$
$q_R^{11}(t)$	Booking level of the first item in the real store within $[0, t_1]$
$q_R^{12}(t)$	Booking level of the first item in the real store within $[t_1, T]$
$q_R^{21}(t)$	Booking level of the second item in the real store within $[0, t_1]$
$q_R^{22}(t)$	Booking level of the second item in the real store within $[t_1, T]$
$d_1^1(p_1^1)$	Demand rate of customers from the first supplier at selling price p_1^1 within $[0, t_1]$ per unit time
$d_1^2(p_1^2)$	Demand rate of customers from the first supplier at selling price p_1^2 within $[t_1, T]$ per unit time
$d_2^1(p_2^1)$	Demand rate of customers from the second supplier at selling price p_2^1 within $[0, t_1]$ per unit time
$d_2^2(p_2^2)$	Demand rate of customers from the second supplier at selling price p_2^2 within $[t_1, T]$ per unit time
$d_R^{11}(p_R^{11})$	Demand rate of customers from the real store for the first item at selling price p_R^{11} within $[0, t_1]$ per unit time
$d_R^{12}(p_R^{12})$	Demand rate of customers from the real store for the first item at selling price p_R^{12} within $[t_1, T]$ per unit time
$d_R^{21}(p_R^{21})$	Demand rate of customers from the real store for the second item at selling price p_R^{21} within $[0, t_1]$ per unit time
$d_R^{22}(p_R^{22})$	Demand rate of customers from the real store for the second item at selling price p_R^{22} within $[t_1, T]$ per unit time
v_1^1	Wholesale price of the first supplier for the first item within $[0, t_1]$ per item
v_1^2	Wholesale price of the first supplier for the first item within $[t_1, T]$ per item
v_2^1	Wholesale price of the second supplier for the second item within $[0, t_1]$ per item
v_2^2	Wholesale price of the second supplier for the second item within $[t_1, T]$ per item
k_1^1	Unit cost of the first supplier for the first item within $[0, t_1]$
k_1^2	Unit cost of the first supplier for the first item within $[t_1, T]$
k_2^1	Unit cost of the second supplier for the second item within $[0, t_1]$
k_2^2	Unit cost of the second supplier for the second item within $[t_1, T]$
t_1	Time of price-change
T	Length of the pre-selling period
ψ_1	Total profit of the first supplier per unit time
ψ_2	Total profit of the second supplier per unit time
ψ_R	Total profit of the real store per unit time

Decision variables:

p_1^1	Selling price of the first supplier within $[0, t_1]$ per item per unit time
p_1^2	Selling price of the first supplier within $[t_1, T]$ per item per unit time
p_2^1	Selling price of the second supplier within $[0, t_1]$ per item per unit time
p_2^2	Selling price of the second supplier within $[t_1, T]$ per item per unit time
p_R^{11}	Selling price of the real store for the first item within $[0, t_1]$ per item per unit time
p_R^{12}	Selling price of the real store for the first item within $[t_1, T]$ per item per unit time
p_R^{21}	Selling price of the real store for the second item within $[0, t_1]$ per item per unit time

p_R^{22} Selling price of the real store for the second item within $[t_1, T]$ per item per unit time

3. Mathematical model and analysis

In this section, a pricing model is introduced for two complementary supply networks launching two complementary commodities under advance booking policy. Each network involves a supplier and a retailer in which the suppliers sell their commodities through an online and a real shopping center. The formulation of the model for the members of the networks is presented in the following.

3.1. Model for the first supplier

In this case, the first commodity is presented by the first supplier via its virtual shopping center and a real shopping center where it is priced in the virtual store lower than the real store, i.e., $p_1^1 < p_R^{11}$ and $p_1^2 < p_R^{12}$. Buyers decide about purchasing based on the price and service level presented by the sellers as their priorities. The profit of the first supplier in virtual and real shopping centers is modeled as given.

3.1.1. Model for the first supplier: Selling via an online shopping center

An advance booking strategy is employed by the supplier to persuade the buyers to order in advance. In fact, the supplier strives to manage market demand by launching the item in two certain periods. The item is cheaper within $[0, t_1]$ than in the selling period $[t_1, T]$, i.e., $p_1^1 < p_1^2$. Additionally, since the buyers who need more guidance tend to purchase from real stores, it is supposed that service level of the real shopping center is higher than that of online stores, i.e., $s_1^1 < s_R^{11}$ and $s_1^2 < s_R^{12}$.

According to the assumptions, the demand of the buyers depends on the price and service level and they want to purchase both commodities. Thus, their demands from the online store of the first supplier during $[0, t_1]$ and $[t_1, T]$ are respectively modeled as follows:

$$d_1^1(p_1^1)=\eta_1\alpha_1-\beta_1 p_1^1-\beta_2 p_2^1+\beta_R p_R^{11}+\gamma_1 s_1^1-\gamma_R s_R^{11}, \quad (1)$$

$$d_1^2(p_1^2)=\eta_1\alpha_1-\beta_1 p_1^2-\beta_2 p_2^2+\beta_R p_R^{12}+\gamma_1 s_1^2-\gamma_R s_R^{12}. \quad (2)$$

Then, the booking levels of the commodity within $[0, t_1]$ and $[t_1, T]$ are modeled differentially as follows:

$$\frac{dq_1^1(t)}{dt} = d_1^1(p_1^1), \qquad 0 \le t \le t_1, \quad (3)$$

$$\frac{dq_1^2(t)}{dt} = d_1^2(p_1^2), \qquad t_1 \le t \le T. \quad (4)$$

Since $q_1^1(0) = 0$ and $q_1^2(T) = Z_1$, booking levels of the first commodity at time t are as follows:

$$q_1^1(t) = d_1^1(p_1^1)t = \left[\eta_1\alpha_1 - \beta_1 p_1^1 - \beta_2 p_2^1 + \beta_R p_R^{A1} + \gamma_1 s_1^1 - \gamma_R s_R^{11}\right] t, \quad (5)$$

$$q_1^2(t) = d_1^2(p_1^2)(T-t) = \left[\eta_1\alpha_1 - \beta_1 p_1^2 - \beta_2 p_2^2 + \beta_R p_R^{12} + \gamma_1 s_1^2 - \gamma_R s_R^{12}\right] (T-t). \quad (6)$$

Thereupon, the number of items sold within $[0, t_1]$ can be calculated as follows:

$$z_1^1 = \int_0^{t_1} d_1^1(p_1^1)dt = d_1^1(p_1^1)t_1 = \left[\eta_1\alpha_1 - \beta_1 p_1^1 - \beta_2 p_2^1 + \beta_R p_R^{A1} + \gamma_1 s_1^1 - \gamma_R s_R^{11}\right] t_1, \quad (7)$$

and within $[t_1, T]$ can be identified as follows:

$$z_1^2 = \int_{t_1}^{T} d_1^2(p_1^2)dt = d_1^2(p_1^2)(T-t_1) = \left[\eta_1\alpha_1 - \beta_1 p_1^2 - \beta_2 p_2^2 + \beta_R p_R^{12} + \gamma_1 s_1^2 - \gamma_R s_R^{12}\right] (T-t_1). \quad (8)$$

Therefore, the profit earned by the supplier through an online shopping center is as follows:

$$\pi_1^O = (p_1^1 - k_1^1)z_1^1 + (p_1^2 - k_1^2)z_1^2 = (p_A^1 - k_A^1)\left[\eta_1\alpha_1 - \beta_1 p_1^1 - \beta_2 p_2^1 + \beta_R p_R^{11} + \gamma_1 s_1^1 - \gamma_R s_R^{11}\right] t_1 + (p_1^2 - k_1^2)\left[\eta_1\alpha_1 - \beta_1 P_1^2 - \beta_2 P_2^2 + \beta_R P_R^{12} + \gamma_1 S_1^2 - \gamma_R S_R^{12}\right] (T-t_1). \quad (9)$$

3.1.2. Model for the first supplier: Selling via a real shopping center

The real common retailer orders the needed commodities based on the expected demand at its wholesale prices. Then, it sells them for a profit under a merchant model achieved by an agreement between the retailer and the supplier. Note that the supplier presents its commodities to the real retailer under an advance booking policy by which $v_A^1 < v_A^2$. Therefore, the demands of the retailer from the first supplier during $[0, t_1]$ and $[t_1, T]$ are respectively modeled as follows:

$$d_R^{11}(p_R^{11}) = \eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{11} - \beta_R p_R^{21} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1, \quad (10)$$

$$d_R^{12}(p_R^{12}) = \eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{12} - \beta_R p_R^{22} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2. \quad (11)$$

The booking levels of the items within $[0, t_1]$ and $[t_1, T]$ are differentially formulated as follows:

$$\frac{dq_1^{R1}(t)}{dt} = d_R^{11}(p_R^{11}), \qquad 0 \leq t \leq t_1, \tag{12}$$

$$\frac{dq_1^{R2}(t)}{dt} = d_R^{12}(p_R^{12}), \qquad t_1 \leq t \leq T. \tag{13}$$

Since $q_1^{R1}(0) = 0$ and $q_1^{R2}(T) = Z_R^1$, booking levels of the first commodity at time t are as follows:

$$q_1^{R1}(t) = d_R^{11}\left(p_R^{11}\right) t = \left[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{11} - \beta_R p_R^{21} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1\right] t, \tag{14}$$

$$q_1^{R2}(t) = d_R^{12}\left(p_R^{12}\right)(T-t) = \left[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{12} - \beta_R p_R^{22} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2\right](T-t). \tag{15}$$

Thus, the quantity of the first item during $[0, t_1]$ is calculated as follows:

$$z_1^{R1} = \int_0^{t_1} d_R^{11}(p_R^{11})dt = d_R^{11}\left(p_R^{11}\right) t_1 = [\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{11} - \beta_R p_R^{21} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_A s_1^1] t_1, \tag{16}$$

and during $[t_1, T]$ is identified as follows:

$$z_1^{R2} = \int_{t_1}^{T} d_R^{12}(p_R^{12})dt = d_R^{12}(p_R^{12})(T-t) = [\eta_R(1-\eta_1)\alpha 1 - \beta_R p_R^{12} - \beta_R p_R^{22} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2](T - t_1). \tag{17}$$

Then, the profit earned by the first supplier through a real selling channel is as follows:

$$\begin{aligned}\psi_1^R = &(v_1^1 - k_1^1)z_1^{R1} + (v_1^2 - k_1^2)z_1^{R2} \\ = &(v_1^1 - k_1^1)\left[\eta_R(1-\eta_1)\alpha_1 - \beta_R P_R^{11} - \beta_R P_R^{21} + \beta_1 P_1^1 + \gamma_R S_R^{11} - \gamma_1 S_1^1\right] t_1 \\ &+ (v_1^2 - k_1^2)\left[\eta_R(1-\eta_1)\alpha_1 - \beta_R P_R^{12} - \beta_R P_R^{22} + \beta_1 P_1^2 + \gamma_R S_R^{A2} - \gamma_1 S_1^2\right](T - t_1).\end{aligned} \tag{18}$$

Eventually, profit of the first supplier is as follows:

$$\psi_1 = \psi_1^o + \psi_1^R = (p_1^1 - k_1^1)z_1^1 + (p_1^2 - k_1^2)z_1^2 + (v_1^1 - k_1^1)z_1^{R1} + (v_1^2 - k_1^2)z_1^{R2}. \tag{19}$$

3.2. Model for the second supplier

Similarly, the second commodity is presented by the second supplier via its virtual shopping center and a real shopping center and it is priced in the virtual store lower than in the real store, i.e., $p_2^1 < p_R^{21}$ and $p_2^2 < p_R^{22}$. Buyers decide about purchasing based on the price and service level offered by the sellers as their priorities. Therefore, the profit of the second supplier in virtual and real shopping centers is modeled as given.

3.2.1. Model for the second supplier: Selling via an online shopping center

An advance booking strategy is employed by the supplier to persuade the buyers to order in advance. Indeed, the supplier strives to manage the market demand by launching the item in two certain periods in which it is cheaper within $[0, t_1]$ than in the selling period $[t_1, T]$, i.e., $p_2^1 < p_2^2$. Additionally, since the buyers who need more guidance tend to purchase from real stores, it is supposed that service level of the real shopping center is higher than that of the online stores, i.e., $s_2^1 < s_R^{21}$ and $s_2^2 < s_R^{22}$.

According to the assumptions, the demand of the buyers depends on the price and service level and they want to purchase both commodities. Therefore, their demands from the online store of the second supplier during $[0, t_1]$ and $[t_1, T]$ are respectively modeled as follows:

$$d_2^1(p_2^1) = \eta_2\alpha_2 - \beta_2 P_2^1 - \beta_1 P_1^1 + \beta_R P_R^{21} + \gamma_2 S_2^1 - \gamma_R S_R^{21}, \tag{20}$$

$$d_2^2(p_2^2) = \eta_2\alpha_2 - \beta_2 p_2^2 - \beta_1 p_1^2 + \beta_R p_R^{22} + \gamma_2 s_2^2 - \gamma_R s_R^{22}. \tag{21}$$

Thus, the booking levels of the items during periods $[0, t_1]$ and $[t_1, T]$ are modeled differentially as follows:

$$\frac{dq_2^1(t)}{dt} = d_2^1(p_2^1), \qquad 0 \leq t \leq t_1, \tag{22}$$

$$\frac{dq_2^2(t)}{dt} = d_2^2(p_2^2), \qquad t_1 \leq t \leq T. \tag{23}$$

Since $q_2^1(0) = 0$ and $q_2^2(T) = Z_2$, booking levels of the second items at time t are as follows:

$$q_2^1(t) = d_2^1(p_2^1)t = \left[\eta_2\alpha_2 - \beta_2 p_2^1 - \beta_1 p_1^1 + \beta_R p_R^{21} + \gamma_2 s_2^1 - \gamma_R s_R^{21}\right] t, \tag{24}$$

$$q_2^2(t) = d_2^2(p_2^2)(T-t) = \left[\eta_2\alpha_2 - \beta_2 p_2^2 - \beta_1 p_1^2 + \beta_R p_R^{22} + \gamma_2 s_2^2 - \gamma_R s_R^{22}\right](T-t). \tag{25}$$

Hence, the number of commodities within $[0, t_1]$ is calculated as follows:

$$z_2^1 = \int_0^{t_1} d_2^1(p_2^1)dt = d_2^1(p_2^1)t_1 = \left[\eta_2\alpha_2 - \beta_2 p_2^1 - \beta_1 p_1^1 + \beta_R p_R^{21} + \gamma_2 s_2^1 - \gamma_R s_R^{21}\right] t_1, \tag{26}$$

and the number of items during period $[t_1, T]$ is identified as follows:

$$z_{H2}^2 = \int_{t_1}^{T} d_2^2(p_2^2)dt = d_2^2(p_2^2)(T-t) = \left[\eta_2\alpha_2 - \beta_2 p_2^2 - \beta_1 p_1^2 + \beta_R p_R^{22} + \gamma_2 s_2^2 - \gamma_R s_R^{22}\right](T - t_1). \tag{27}$$

Furthermore, the profit earned by the second supplier through its online shopping center is as follows:

$$\psi_2^o = (p_2^1 - k_2^1)z_2^1 + (p_2^2 - k_2^2)z_2^2 = (p_2^1 - k_2^1)\left[\eta_2\alpha_2 - \beta_2 s_2^1 - \beta_1 s_1^1 + \beta_R s_R^{21} + \gamma_2 s_2^1\right] t_1 + (p_2^2 - k_2^2)\left[\eta_2\alpha_2 - \beta_2 p_2^2 - \beta_1 p_1^2 + \beta_R p_R^{22} + \gamma_2 s_2^2\right](T - t_1). \tag{28}$$

3.2.2. Model for the second supplier: Selling via a real shopping center

In this case, similarly to the previous one, the real store buys the item under an advance booking policy from the second supplier at the offered wholesale prices where $v_2^1 < v_2^2$ and $p_R^{21} < p_R^{22}$. A merchant contract is considered between the retailer and the supplier so that the retailer sells its ordered items for a profit based on its own opinion about the price of the item.

Therefore, the demands of the retailer from the second supplier during $[0, t_1]$ and $[t_1, T]$ are respectively modeled as follows:

$$d_R^{21}(p_R^{21}) = \eta_R(1 - \eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1, \tag{29}$$

$$d_R^{22}(p_R^{22}) = \eta_R(1 - \eta_2)\alpha_2 - \beta_R p_R^{22} - \beta_R p_R^{12} + \beta_2 p_2^2 + \gamma_R s_R^{22} - \gamma_2 s_2^2. \tag{30}$$

Thus, the booking levels of the items within $[0, t_1]$ and $[t_1, T]$ are differentially indicated as follows:

$$\frac{dq_2^{R1}(t)}{dt} = d_R^{21}(p_R^{21}), \qquad 0 \le t \le t_1, \tag{31}$$

$$\frac{dq_2^{R2}(t)}{dt} = d_R^{22}(p_R^{22}), \qquad t_1 \le t \le T. \tag{32}$$

Since $q_2^{R1}(0) = 0$ and $q_2^{R2}(T) = Z_R^2$, booking levels of the items at time t are as follows:

$$q_2^{R1}(t) = d_R^{21}(p_R^{21})t = \left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{A1} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1\right] t, \tag{33}$$

$$q_2^{R2}(t) = d_R^{22}(p_R^{22})(T - t_1) = \left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{22} - \beta_R p_R^{12} + \beta_2 p_2^2 + \gamma_R s_R^{22} - \gamma_2 s_2^2\right](T - t). \tag{34}$$

Then, the number of items within $[0, t_1]$ is demonstrated as follows:

$$z_2^{R1} = \int_0^{t_1} d_R^{21}(p_R^{21})dt = d_R^{21}(p_R^{21})t_1 = \left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1\right] t_1. \tag{35}$$

Hence, the quantity of the second item during $[t_1, T]$ is calculated as follows:

$$z_2^{R2} = \int_{t_1}^{T} d_R^{22}(p_R^{22})dt = d_R^{22}(p_R^{22})(T - t_1) = \left[\eta_R(1 - \eta_2)\alpha_2 - \beta_T P_T^{H2} - \beta_T P_T^{A2} + \beta_H P_H^2 + \gamma_T S_T^{H2} - \gamma_H S_H^2\right](T - t_1). \tag{36}$$

Therefore, the profit earned by the second supplier through the real selling channel is as follows:

$$\pi_2^R = (p_R^{21} - v_2^1)z_2^{R1} + (p_R^{22} - v_2^2)z_2^{R2} = (p_T^{21} - v_2^1)\left[\eta_T(1 - \eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1\right] t_1 + (p_T^{22} - v_2^2)\left[\eta_T(1 - \eta_2)\alpha_2 - \beta_R p_R^{22} - \beta_R p_R^{12} + \beta_2 p_2^2 + \gamma_R s_R^{22} - \gamma_2 s_2^2\right](T - t_1). \tag{37}$$

Finally, the profit of the second supplier is as follows:

$$\pi_2 = \pi_2^O + \pi_2^R = (p_2^1 - k_2^1)z_2^1 + (p_2^2 - k_2^2)z_2^2 + (v_2^1 - k_2^1)z_2^{R1} + (v_2^2 - k_2^2)z_2^{R2}. \tag{38}$$

3.3. Model for the real retailer

Besides the online stores, the real shopping center launches the first and the second commodities, simultaneously, as complementary items. It is supposed that the suppliers commonly sell the items through a real shopping center, along with online stores, to attract the buyers who need more guidance or prefer to touch the purchased commodities. Thus, the profit comprises two components for selling the first and the second commodities. The booking level of the first item is formulated differentially as:

$$\frac{dq_R^{11}(t)}{dt} = d_R^{11}(p_R^{11}), \qquad 0 \le t \le t_1, \tag{39}$$

$$\frac{dq_R^{12}(t)}{dt} = d_R^{12}(p_R^{12}), \qquad t_1 \le t \le T. \tag{40}$$

Since $q_R^{11}(0) = 0$ and $q_R^{12}(T) = Z_R^1$, booking levels for the first item at time t are as follows:

$$q_R^{11}(t) = d_R^{11}(p_R^{11})t = \left[\eta_R(1 - \eta_1)\alpha_1 - \beta_R p_R^{11} - \beta_2 p_R^{21} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1\right] t, \tag{41}$$

$$q_R^{12}(t) = d_R^{12}(p_R^{12})(T - t) = \left[\eta_R(1 - \eta_1)\alpha_1 - \beta_R p_R^{12} - \beta_2 p_R^{22} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2\right](T - t). \tag{42}$$

Furthermore, the number of the first commodity within $[0, t_1]$ is calculated as follows:

$$z_R^{11} = \int_0^{t_1} d_R^{11}(p_R^{11})dt = d_R^{11}(p_R^{11})t_1 = [\eta_R(1-\eta_1)\alpha_1$$
$$-\beta_R p_R^{11} - \beta_2 p_R^{21} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1]\, t_1, \quad (43)$$

and within $[t_1, T]$ is identified as follows:

$$z_R^{12} = \int_{t_1}^{T} d_R^{12}(p_R^{12})(T-t)dt = [\eta_R(1-\eta_1)\alpha_1$$
$$-\beta_R p_R^{12} - \beta_2 p_R^{22} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2](T-t_1). \quad (44)$$

The profit earned by the retailer through selling the first item is as follows:

$$\psi_R^1 = (p_R^{11} - v_1^1)z_R^{11} + (p_R^{12} - v_1^2)z_R^{12}$$
$$= (p_R^{11} - v_1^1)\,[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{11} - \beta_2 p_R^{21}$$
$$+\beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1]\, t_1$$
$$+ (p_R^{12} - v_1^2)\,[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{12} - \beta_2 p_R^{22}$$
$$+\beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2]\,(T-t_1). \quad (45)$$

In addition, the booking level of the second item is:

$$\frac{dq_R^{21}(t)}{dt} = d_R^{21}(p_R^{21}), \qquad 0 \le t \le t_1, \quad (46)$$

$$\frac{dq_R^{22}(t)}{dt} = d_R^{22}(p_R^{22}), \qquad t_1 \le t \le T. \quad (47)$$

Since $q_R^{21}(0) = 0$ and $q_R^{22}(T) = Z_R^2$, booking levels of the second item at time t are as follows:

$$q_R^{21}(t) = d_R^{21}(p_R^{21})t = [\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{21}$$
$$-\beta_R p_R^{11} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1]\, t, \quad (48)$$

$$q_R^{22}(t) = d_R^{22}(p_R^{22})(T-t) = [\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{22}$$
$$-\beta_R p_R^{12} + \beta_2 p_2^2 + \gamma_R s_R^{22} - \gamma_2 s_2^2]\,(T-t). \quad (49)$$

Thus, the quantity of the second item within $[0, t_1]$ is calculated as follows:

$$z_R^{21} = \int_0^{t_1} d_R^{21}(p_R^{21})dt = d_R^{21}(p_R^{21})t_1 = [\eta_R(1-\eta_2)\alpha_2$$
$$-\beta_R p_R^{21} - \beta_R p_R^{11} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1]\, t_1, \quad (50)$$

and within $[t_1, T]$ is:

$$z_R^{22} = \int_{t_1}^{T} d_R^{22}(p_R^{22})dt = d_R^{22}(p_R^{22})(T-t_1)$$
$$= [\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{22} - \beta_R p_R^{12} + \beta_2 p_2^2$$
$$+\gamma_R s_R^{22} - \gamma_2 s_2^2]\,(T-t_1). \quad (51)$$

Therefore, the profit earned by the retailer through selling the second item is equal to:

$$\psi_R^2 = (p_R^{21} - v_2^1)z_R^{21} + (p_R^{22} - v_2^2)z_R^{22}$$
$$= (p_T^{21} - v_2^1)\,[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11}$$
$$+\beta_H p_H^1 + \gamma_T s_T^{H1} - \gamma_H s_H^1]\, t_1$$
$$+(p_T^{H2} - v_H^2)\,[\eta_T(1-\eta_H)\alpha_H - \beta_T p_T^{H2}$$
$$-\beta_T p_T^{A2} + \beta_H p_H^2 + \gamma_T s_T^{H2} - \gamma_H s_H^2]\,(T-t_1). \quad (52)$$

Gradually, the total profit of the retailer can be stated as follows:

$$\psi_R = (p_R^{11} - v_1^1)z_R^{11} + (p_R^{12} - v_1^2)z_R^{12} + (p_R^{21} - v_1^1)z_T^{21}$$
$$+ (p_R^{22} - v_2^2)z_R^{22}. \quad (53)$$

4. Solution Procedure

In this section, a solution procedure to find the closed-form solutions for the decision variables is examined in order to analyze the reactions of the members of supply chains by considering game theory approaches such as Nash and Stackelberg game. In this case, it is assumed that there is a Nash game between the suppliers who supply the commodities independently while there is a Stackelberg game between the members of each chain where the real shopping center is the follower and the supplier is the leader of the market.

In the Stackelberg game, the optimal decision policies of the leaders depend on the optimal decisions of the followers and the best reaction of the retailer is considered by the leaders. The prices of commodities within $[0, t_1]$ and $[t_1, T]$ are the decision variables of the retailer, as the follower, while the prices of the items within time $[0, t_1]$ and $[t_1, T]$ in online stores are determined by the suppliers, as the leaders. In addition, the optimality of the objective functions regarding the decision variables should be proven.

Theorem 1. The profit of the real retailer $\psi_R(p_R^{11}, p_R^{12}, p_R^{21}, p_R^{22})$ is concave.

Proof. If Eqs. (B.12) to (B.15) are met (see Appendix B), concavity of the profit function for the retailer is proven. Thus, by taking the first-order derivatives of the profit function in Eq. (53), the optimal values of p_R^{11}, p_R^{12}, p_R^{21}, p_R^{22} are obtained as follows:

$$\frac{\partial\psi_R}{\partial p_R^{11}} = \left[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{11} - \beta_2 p_R^{21} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1\right] t_1 - \left[(p_R^{11} - v_1^1) + (p_R^{21} - v_2^1)\right]\beta_R t_1 = 0, \tag{54}$$

$$\frac{\partial\psi_R}{\partial p_R^{12}} = \left[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{12} - \beta_2 p_R^{22} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2\right](T-t_1) - (P_R^{12} - v_1^2)\beta_R(T-t_1) - (P_R^{22} - v_2^2)\beta_R(T-t_1) = 0, \tag{55}$$

$$\frac{\partial\psi_R}{\partial p_R^{21}} = \left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1\right] t_1 - \left[(p_R^{21} - v_2^1) + (p_R^{11} - v_1^1)\right]\beta_R t_1 = 0, \tag{56}$$

$$\frac{\partial\psi_R}{\partial p_R^{22}} = \left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{22} - \beta_R p_R^{12} + \beta_2 p_2^2 + \gamma_R s_R^{22} - \gamma_2 s_2^2\right](T-t_1) - (p_R^{22} - v_2^2)\beta_R(T-t_1) - (p_R^{12} - v_1^2)\beta_R(T-t_1) = 0. \tag{57}$$

Then, the optimal decisions of the retailer with regard to the policies of the suppliers after simplifying the above equations are as follows:

$$p_R^{11*} = Y_1 + \frac{\beta_1 p_1^{1*}}{2\beta_R}, \tag{58}$$

$$p_R^{11*} = Y_2 + \frac{\beta_1 p_1^{2*}}{2\beta_R}, \tag{59}$$

$$p_R^{21*} = Y_3 + \frac{\beta_2 p_2^{1*}}{2\beta_R}, \tag{60}$$

$$p_R^{21*} = Y_4 + \frac{\beta_2 p_2^{2*}}{2\beta_R}, \tag{61}$$

for which the equations utilized to simplify the relations are illustrated in Appendix A. Besides, based on the hypothesis, a Stackelberg game is considered between members of the networks, where a real retailer decides about the optimal policies and the suppliers as the leaders characterize the variables regarding the optimal reactions of the retailer. A Nash game is assumed between suppliers in order to obtain their decision variables, simultaneously.

Theorem 2. The profit of the first supplier, $\psi_1(p_1^1, p_1^2)$, is concave.

Proof. Having the concavity of the profit function for the first supplier proven (see Appendix C), the optimal values of p_R^{11}, p_R^{12}, p_R^{21}, p_R^{22} are replaced into Eq. (19), which becomes:

$$\begin{aligned}\psi_1 =& (p_1^1 - k_1^1)\left[\eta_1\alpha_1 - \beta_1 p_1^1 - \beta_2 p_2^1 + \beta_R p_R^{A1*} + \gamma_1 s_1^1 - \gamma_R s_R^{11}\right] t_1 + (p_1^2 - k_1^2)\left[\eta_1\alpha_1 - \beta_1 p_1^2 - \beta_2 p_2^2 + \beta_R p_R^{12*} + \gamma_1 s_1^2 - \gamma_R s_R^{12}\right](T-t_1) \\ &+ (v_1^1 - k_1^1)\left[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{11*} - \beta_R p_R^{21*} + \beta_1 p_1^1 + \gamma_R s_R^{11} - \gamma_1 s_1^1\right] t_1 \\ &+ (v_1^2 - k_1^2)\left[\eta_R(1-\eta_1)\alpha_1 - \beta_R p_R^{12*} - \beta_R p_R^{22*} + \beta_1 p_1^2 + \gamma_R s_R^{12} - \gamma_1 s_1^2\right](T-t_1).\end{aligned} \tag{62}$$

Then, the optimal values of p_1^1 and p_1^2 are obtained by taking into account the first-order derivatives of the objective function, Eq. (62), with regard to p_1^1 and p_1^2 as follows:

$$\frac{\partial\psi_1}{\partial p_1^1} = \left[\eta_1\alpha_1 - \beta_1 p_1^1 - \beta_2 p_2^1 + \beta_R p_R^{11} + \gamma_1 s_1^1 - \gamma_R s_R^{11}\right] t_1 - (p_1^1 - k_1^1)\beta_1 t_1 + (v_1^1 - k_1^1)\beta_1 = 0, \tag{63}$$

$$\frac{\partial\psi_1}{\partial p_1^2} = \left[\eta_1\alpha_1 - \beta_1 p_1^2 - \beta_2 p_2^2 + \beta_R p_R^{12} + \gamma_1 s_1^2 - \gamma_R s_R^{12}\right](T-t_1) - (p_1^2 - k_1^2)\beta_1(T-t_1) + (v_1^2 - k_1^2)\beta_1(T-t_1) = 0. \tag{64}$$

Hence, the closed-form solutions for the variables of the first supplier are as follows:

$$p_1^{1*} = \frac{-Y_7 + Y_{11}}{Y_{12} - \beta_1}, \tag{65}$$

$$p_1^{2*} = \frac{-2\beta_R + Y_{13}}{Y_{12} - \beta_1}. \tag{66}$$

Theorem 3. The profit of the second supplier $\psi_2(p_2^1, p_2^2)$ is concave.

Proof. Having concavity of the profit function for the second supplier proven (see Appendix D), the optimal values of P_R^{11}, P_R^{12}, P_R^{21}, P_R^{22} are replaced into Eq. (38), which becomes:

$$\psi_2 =(p_2^1 - k_2^1)\left[\eta_2\alpha_2 - \beta_2 p_2^1 - \beta_1 p_1^1 + \beta_R p_R^{21*} +\gamma_2 s_2^1 - \gamma_R s_R^{21}\right] t_1 + (p_2^2 - k_2^2)\left[\eta_2\alpha_2 - \beta_2 p_2^2 -\beta_1 p_1^2 + \beta_R p_R^{22*} + \gamma_2 s_2^2 - \gamma_R s_R^{22}\right](T - t_1) + (P_R^{21} - v_2^1)\left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{21*} -\beta_R p_R^{11*} + \beta_2 p_2^1 + \gamma_R s_R^{21} - \gamma_2 s_2^1\right] t_1 + (P_R^{22} - v_2^2)\left[\eta_R(1-\eta_2)\alpha_2 - \beta_R p_R^{22*} -\beta_R p_R^{12*} + \beta_2 p_2^2 + \gamma_R s_R^{22} - \gamma_2 s_2^2\right](T - t_1). \quad (67)$$

The optimal values of p_2^1 and p_2^2 are derived by taking the first-order derivatives of the objective function in Eq. (67) with regard to p_2^1 and p_2^2 as:

$$\frac{\partial\psi_2}{\partial p_2^1} = \left[\eta_2\alpha_2 - \beta_2 p_2^1 - \beta_1 p_1^1 + \beta_R p_R^{21} + \gamma_2 s_2^1 - \gamma_R s_R^{21}\right] t_1 - (p_2^1 - k_2^1)\beta_2 t_1 + (v_2^1 - k_2^1)\beta_2 t_1 = 0, \quad (68)$$

$$\frac{\partial\psi_2}{\partial p_2^2} = \left[\eta_2\alpha_2 - \beta_2 p_2^2 - \beta_1 p_1^2 + \beta_R p_R^{22} + \gamma_2 s_2^2 -\gamma_R s_R^{22}\right](T - t_1) - (p_2^2 - k_2^2)\beta_2(T - t_1) + (v_2^2 - k_2^2)\beta_2(T - t_1) = 0. \quad (69)$$

The closed-form solutions for the variables of the second supplier are as follows:

$$p_2^{1*} = \frac{-\beta_1 Y_{11} + Y_{14}Y_{12}}{\beta_2 Y_{12} - \beta_1\beta_2}, \quad (70)$$

$$p_2^{2*} = \frac{-\beta_1 Y_{13} + Y_{15}Y_{12}}{\beta_2 Y_{12} - \beta_2\beta_1}. \quad (71)$$

By replacing the closed-form solutions using Eqs. (65), (66), (70), and (71) into Eqs. (58)–(61), the closed-form solutions for the variables are as follows:

$$p_R^{11*} = Y_1 + Y_7\left(\frac{Y_{11} - Y_{14}}{Y_{12} - \beta_1}\right), \quad (72)$$

$$p_R^{12*} = Y_2 + Y_7\left(\frac{Y_{13} - Y_{15}}{Y_{12} - \beta_1}\right), \quad (73)$$

$$p_R^{21*} = \frac{Y_9}{2\beta_R} + \frac{\beta_2}{2\beta_R}\left(\frac{-\beta_1 Y_{11} + Y_{14}Y_{12}}{\beta_2 Y_{12} - \beta_1\beta_2}\right), \quad (74)$$

$$p_R^{22*} = \frac{Y_4}{2\beta_R} + \frac{\beta_2}{2\beta_R}\left(\frac{-\beta_1 Y_{13} + Y_{15}Y_{12}}{\beta_2 Y_{12} - \beta_1\beta_2}\right). \quad (75)$$

Note that the above-used relations are illustrated in Appendix A.

5. Numerical example and sensitivity analysis

5.1. Real case

As a real case, we set the introduced model for two tourism supply chains with customers who wanted to book the tickets of an airline and the rooms of a hotel for a travel. It was supposed that the first chain launched the tickets and the second one provided the rooms for the buyers and their orders were sensitive to price and service level. In the first chain, an airline and a travel agency were the members and the second chain included a hotel and the same travel agency.

As it is common now, the airline and the hotel presented their commodities through virtual stores and a common travel agency. Since the complementary items were presented by the airline and the hotel, companies preferred to sell their items through a common travel agency to increase their market shares. Thus, this section numerically indicates applicability of the proposed model. Market demand depended on service level and selling price. The impacts of key parameters on the optimal values of decision variables and the profits of the supply networks were analyzed.

5.2. Numerical example

Suppose that $k_1^1 = 10$, $k_1^2 = 12$, $\alpha_1 = 40000$, $\eta_1 = 0.4$, $\beta_1 = 21$, $\beta_2 = 21.5$, $\beta_R = 20$, $\gamma_1 = 10$, $\gamma_R = 15$, $s_1^1 = 1$, $s_R^{11} = 2$, $s_1^2 = 1$, $s_R^{12} = 2$, $v_1^1 = 20$, $v_1^2 = 25$, $\eta_R = 0.5$, $v_2^1 = 23$, $v_2^2 = 28$, $s_2^1 = 1$, $s_R^{21} = 2$, $s_2^2 = 1$, $s_R^{22} = 2$, $\gamma_2 = 19$, $\eta_2 = 0.4$, $k_2^1 = 12$, and $k_2^2 = 17$. The results are indicated in Tables 1 and 2.

According to Tables 1 and 2, when the airline and the hotel sell their commodities under an advance booking policy, their profits would be higher than when they sell them at their real prices. Therefore, they tend to apply an ADB selling strategy to improve their benefits from attracting the market demand by the proposed selling prices in different selling periods. In addition, the online stores launch the items at a lower price than the travel agency due to direct selling. Therefore, the firms succeed to improve their market shares using different selling channels in addition to considering an ADB policy.

5.3. Sensitivity analysis

In this section, to analyze the decisions of the members and their profits with the variations of parameters, the optimal values of the variables and the profits of the partners with respect to the changes in some parameters are surveyed. The related results are shown in Tables 3–6 and the diagrams are shown in Figures 2–5. Examining the results leads to the following managerial insights:

- Table 3 shows that increase in sensitivity of customers to the price of the first item (ticket) results in decrease in the demand of buyers and diminishes the

Table 1. Results for the example with $\alpha = 0.5$.

Member	p_1^1	p_1^2	ψ_1	p_2^1	p_2^2	ψ_2	p_R^{11}	p_R^{12}	p_R^{21}	p_R^{22}	ψ_R
First supplier	374.13	377	**2658662.86**	—	—	—	—	—	—	—	—
Second supplier	—	—	—	631.38	634.40	**8110090.40**	—	—	—	—	—
Real shopping center	—	—	—	—	—	—	544.42	548.42	726.14	730.27	**100544.59**

Table 2. Results for the example with $\alpha = 0$.

Member	p_1^1	p_1^2	ψ_1	p_2^1	p_2^2	ψ_2	p_R^{11}	p_R^{12}	p_R^{21}	p_R^{22}	ψ_R
First supplier	—	377	**2649773.95**	—	—	—	—	—	—	—	—
Second supplier	—	—	—	—	634.40	**8083506**	—	—	—	—	—
Real shopping center	—	—	—	—	—	—	—	548.42	—	730.27	**89447.64**

Table 3. Effect of sensitivity of the prices of the first supplier to demand changes on the optimal values of the variables and profits.

Parameter	Change%	p_1^1	p_1^2	ψ_1	p_2^1	p_2^2	ψ_2	p_R^{11}	p_R^{12}	p_R^{21}	p_R^{22}	ψ_R
	+0.75	224.07	228.28	1545943.45	619.66	619.75	7762525.95	553.87	560.23	719.84	722.39	Infeasible
	+0.50	257.42	261.32	1792968.55	623.57	624.64	7877539.79	550.72	556.29	721.94	725.02	Infeasible
	+0.25	304.10	307.59	2139098.75	627.47	629.52	7993394.61	547.57	552.35	724.04	727.64	Infeasible
β_1	0	374.13	377	2658662.86	631.38	634.40	8110090.40	544.42	548.42	726.14	730.27	100544.59
	−0.25	490.84	492.65	3525094.79	635.29	639.29	8227627.17	541.27	544.48	728.24	732.89	830727.32
	−0.50	724.26	723.98	5258696.27	639.2	644.17	8346004.91	538.12	540.54	730.34	735.52	2192930.50
	−0.75	Infeasible	Infeasible	Infeasible	643.10	649.06	8465223.63	534.97	536.60	732.44	738.14	6080838.15

Table 4. Effect of sensitivity of the prices of the second supplier to demand changes on the optimal values of the variables and profits.

Parameter	Change%	p_1^1	p_1^2	ψ_1	p_2^1	p_2^2	ψ_2	p_R^{11}	p_R^{12}	p_R^{21}	p_R^{22}	ψ_R
	+0.75	Inf.*	Inf.	Inf.	372.62	376.91	4773323.89	537	539.39	737.21	743.81	1743897.04
	+0.50	364.71	365.52	2525873.27	430.12	434.13	5514177.43	539.47	542.4	733.56	739.3	1215973.09
	+0.25	369.42	371.25	2591890.34	510.62	514.24	6552152.54	541.94	545.41	729.85	734.78	668281.70
β_2	0	374.13	377	2658662.86	631.38	634.40	8110090.40	544.42	548.42	726.14	730.27	100544.59
	−0.25	378.84	382.72	2726190.83	Inf.	Inf.	Inf.	546.89	551.43	722.43	725.75	Inf.
	−0.50	383.55	388.45	2794474.24	Inf.	Inf.	Inf.	549.36	554.44	718.72	721.24	Inf.
	−0.75	388.26	394.19	2863513.11	Inf.	Inf.	Inf.	551.83	557.45	715.72	716.72	Inf.

*: Inf.: Infeasible

Table 5. Effect of sensitivity of the prices of the real shopping center to demand changes on the optimal values of the variables and profits.

Parameter	Change%	p_1^1	p_1^2	ψ_1	p_2^1	p_2^2	ψ_2	p_R^{11}	p_R^{12}	p_R^{21}	p_R^{22}	ψ_R
	+0.75	376.13	379.70	2689865.81	635.43	639.15	8222618.71	315.98	319.55	421.11	424.75	-185766.21
	+0.50	375.46	378.8	2679438.30	634.08	637.57	8185016.39	366.74	370.41	488.89	492.64	-122917.12
	+0.25	374.8	377.89	2669037.32	632.73	635.99	8147506.95	437.81	441.61	583.79	587.69	-33997.75
β_R	0	374.13	377	2658662.86	631.38	634.40	8110090.40	544.42	548.42	726.14	730.27	100544.59
	−0.25	373.46	376.08	2648314.92	630.03	632.82	8072766.73	722.09	726.42	963.39	967.89	326332.90
	−0.50	372.8	375.18	2637993.51	628.68	631.24	8035535.94	1077.44	1082.44	1437.89	1443.14	780236.13
	−0.75	372.13	374.27	2627698.62	627.33	629.66	7998398.04	2143.48	2150.48	2861.38	2868.88	2146599.04

profit of the first supplier (airline). Consequently, demand for the second commodity (room) as the complementary item is reduced and the profit of the second supplier (hotel) is also diminished. On the other hand, selling prices of the retailer (travel agency) are reduced, because they present substitutable items and should sell items at lower prices to attract customers and manage their market share. Nonetheless, their profit is decreased because of the reduction in prices. The diagrams of the profits

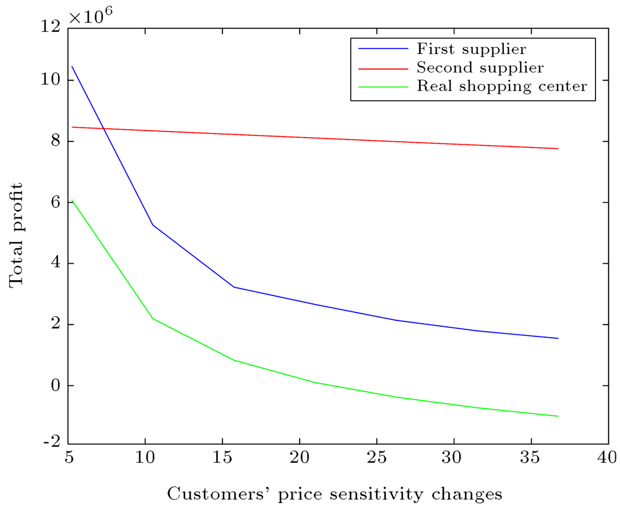

Figure 2. Sensitivity of the customers of the first supplier to changes in prices versus profits of the firms.

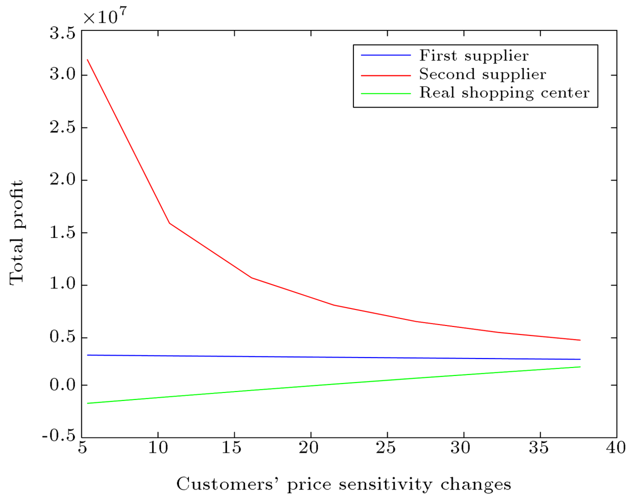

Figure 3. Sensitivity of the customers of the second supplier to changes in prices versus profits of the firms.

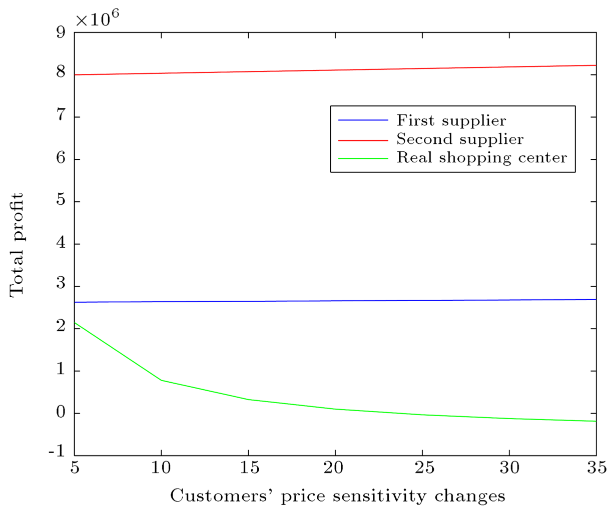

Figure 4. Sensitivity of the customers of the real shopping center to changes in prices versus profits of the firms.

Table 6. Effect of changes in the pre-selling period on the optimal values of profits.

Parameter	Value	ψ_1 Value	ψ_2 Value	ψ_R Value
α	0	2649773.95	8083506	89447.64
	0.10	2651551.73	8088822.88	91667.03
	0.20	2653329.51	8094139.76	93886.42
	0.30	2655107.3	8099456	96105.81
	0.40	2656885.08	8104773.52	98325.20
	0.50	2658662.86	8110090.40	100544.59
	0.60	2660440.64	8115407.28	102763.97
	0.70	2662218.42	8120724.16	104983.36
	0.80	2663996.20	8126041.04	107202.75
	0.90	2665774	8131357.92	109422.14

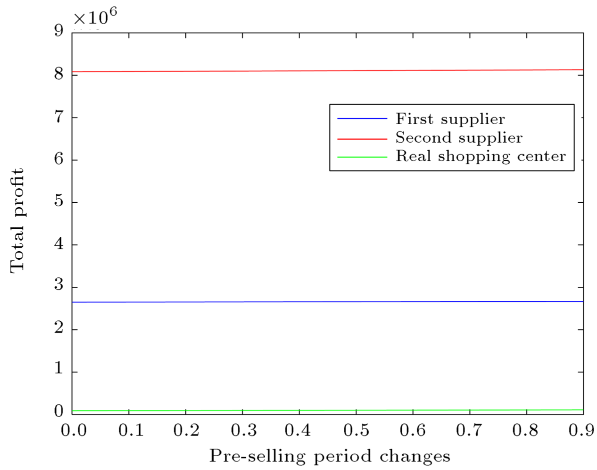

Figure 5. Changes in booking period versus profits of the firms.

of the firms with regard to sensitivity of market demand to the changes in prices for the first item are indicated in Figure 2;

- Based on Table 4, it is found that reducing sensitivity of customers to the price of the second item (room) leads to increase in the profit of the second supplier with higher prices. In addition, if price elasticity of the demand of customers decreases, their demand is slightly influenced by price changes and, in turn, the profit of the suppliers and the retailer will increase. Therefore, they price their commodities in a way to maximize their profits. In other word, when sensitivity of customers to the price of room decreases, the hotel tends to increase its booking prices and the airline as well increases its price and, subsequently, profit. In addition, the travel agency, as a competitive selling channel, increases its selling prices with consideration of the sensitivity of customers. The diagrams of the profits of the firms with regard to sensitivity of market demand to the changes in prices for the second item are presented in Figure 3;

- The results in Table 5 state that the retailer launches the items at lower prices if price elasticity of demand increases, as a result of which the profit of the retailer (travel agency) decreases. The customers do not want to purchase the commodities at higher prices. Hence, the suppliers should present items via virtual stores at lower prices to absorb the highly price-dependent demands. The diagrams of the profits of the firms with regard to sensitivity of customers to the changes in prices of the tickets and the rooms are given in Figure 4;
- The effects of changes in booking periods on the optimal values of decision variables and profits are illustrated in Table 6. From the results, it can be concluded that the suppliers and the retailer tend to employ an advance booking strategy to sell commodities for improving their profits. Also, they gain higher profit if they sell items during a longer pre-selling period. The diagrams of the profits of the firms with regard to the changes in the pre-selling period are provided in Figure 5;
- According to the above analysis, it is concluded that price is a key decision variable and it has a significant impact on purchasing decisions the customers. Increasing/decreasing sensitivity of the customers to the price changes leads to decrease/increase in the profit of the members of a chain.

6. Conclusion

A mathematical model was developed for a selling problem under an advance booking policy in which market demand depended on service level and price of commodities with two complementary supply chains. It was supposed that each chain comprised a supplier and a retailer; the supplier presented their own commodities both online and through a retailer who sold both commodities to buyers as a common member of both chains. The purpose was to find closed-form solutions for the decision variables of the members of the networks in order to maximize profit. Selling prices of items at each echelon of a chain in different selling periods were the decision variables of the proposed model.

A real case was presented for the introduced model in which two tourism supply chains were considered. They launched the complementary items of thickets and rooms via a website and a travel agency. In this case, an airline and a hotel, as the suppliers of the chains, sold their commodities by applying an advance booking selling strategy. A Nash game was assumed between the suppliers and a Stackelberg game between the members of each chain, where the real shopping center was the follower and the supplier was the leader of the market.

In addition, there was a merchant agreement between the suppliers and a common travel agency to sell their items. The model was numerically analyzed and the effects of changes in some parameters on the decision variables and the members were evaluated. It was found that the members preferred to employ an advance booking policy to sell the commodities and manage the market demand and the policy was beneficial to all the members of the supply chains. Also, suppliers who sold their items via both channels earned higher profit than those who performed only in the traditional way, because they attracted more customers and were price-and service-oriented. For the future research, the following issues are recommended:

- Considering the proposed model under various booking strategies;
- Considering the proposed model under a stochastic setting;
- Considering the proposed model under competitive conditions; and
- Considering the proposed model in multi-echelon supply chains.

References

1. Monahan, J.P. "A quantity discount pricing model to increase vendor profits", *Management Science*, **30**, pp. 720–726 (1984).

2. Whitin, T.T. "Inventory control and price theory", *Management Science*, **2**, pp. 61–68 (1969).

3. Banerjee, A. "On 'a quantity discount pricing model to increase vendor profits' ", *Management Science*, **32**, pp. 1513–1517 (1986).

4. Ardalan, A. "Combined optimal price and optimal inventory replenishment policies when a sale results in increase in demand", *Computers and Operations Research*, **18**(9), pp. 721–730 (1991).

5. Boyaci, T. and Gallego, G. "Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers", *International Journal of Production Economics*, **77**, pp. 95–111 (2002).

6. Elmaghraby, W. and Keskinocak, P. "Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions", *Management Science*, **49**, pp. 1287–1309 (2003).

7. Agrawal, V., Chao, X., and Seshadri, S. "Dynamic balancing of inventory in supply chains", *European Journal of Operational Research*, **159**, pp. 296–317 (2004).

8. Hua, G., Wang, S., and Cheng, T.E. "Price and lead time decisions in dual-channel supply chains", *European Journal of Operational Research*, **205**, pp. 113–126 (2010).

9. Chen, Y.C., Fang, S.C., and Wen, U-P. "Pricing policies for substitutable products in a supply chain with internet and traditional channels", *European Journal of Operational Research*, **224**(3), pp. 542–551 (2013).

10. Chen, X. and Hu, P. "Joint pricing and inventory management with deterministic demand and costly price adjustment", *Operations Research Letters*, **40**, pp. 385–389 (2012).

11. Taleizadeh, A.A. and Noori-daryan, M. "Pricing, manufacturing and inventory policies for raw material in a three-level supply chain", *International Journal of System Science*, **47**(4), pp. 919–931 (2015a).

12. Saha, S., Sarmah, S.P., and Modak, N.M. "Single versus dual-channel: A strategic analysis in perspective of retailer's profitability under three-level dual-channel supply chain", *Asia Pacific Management Review* (In Press) (2017). DOI: http://dx.doi.org/10.1016/j.apmrv.2017.03.001

13. Yan, B., Jin, Z., Liu, Y., and Yang, J. "Decision on risk-averse dual-channel supply chain under demand disruption", *Commun Nonlinear Sci Numer Simulat*, **55**, pp. 206–224 (2018).

14. Jamali, M-B. and Rasti-Barzoki, M. "A game theoretic approach for green and non-green product pricing in chain-to-chain competitive sustainable and regular dual-channel supply chains", *Journal of Cleaner Production*, **170**, 1029–1043 (2018).

15. Noori-daryan, M., Taleizadeh, A.A., and Jolai, F. "Analyzing pricing, promised delivery lead time, supplier-selection, and ordering decisions of a multinational supply chain under uncertain environment", *International Journal of Production Economic*, **209**, pp. 236–248 (March 2019). DOI: https://doi.org/10.1016/j.ijpe.2017.12.019

16. Wee, H.M. and Yu, J. "A deteriorating inventory model with a temporary price discount", *International Journal of Production Economics*, **53**(6), pp. 81–90 (1997).

17. Wee, H.M., Chung, S.L., and Yang, P.C. "Technical note a modified EOQ model with temporary sale price derived without derivatives", *The Engineering Economist*, **48**(2), pp. 190–195 (2003).

18. Sarker, B. and Wee, H.M. "An inventory model with variable demand component cost and selling price for deteriorating items", *Economic Modelling*, **30**, pp. 306–310 (2013).

19. Wei, Y., Hu, Q., and Xu, C. "Ordering, pricing and allocation in a service supply chain", *International Journal of Production Economics*, **144**(2), pp. 590–598 (2013).

20. Shah, N.H., Widyadana, G.A., and Wee, H.M. "Stackelberg game for two level supply chain with price markdown option", *International Journal of Computer Mathematics*, **91**(5), pp. 1054–1060 (2014).

21. Taleizadeh, A.A., Noori-daryan, M., and Cárdenas-Barrón, L.E. "Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items", *International Journal of Production Economics*, **159**, pp. 285–295 (2015b).

22. Zhang, X., Huang, S., and Wan, Z. "Optimal pricing and ordering in global supply chain management with constraints under random demand", *Applied Mathematical Modelling*, **40**(23–24), pp. 10105–10130 (2016).

23. Dan, B., He, Q., Zheng, K., and Liu, R. "Ordering and pricing model of retailers' preventive transshipment dominated by manufacturer with conditional return", *Computers & Industrial Engineering*, **100**, pp. 24–33 (2016).

24. Naimi Sadigh, A., Kamal Chaharsooghi, S., and Sheikhmohammady, M. "Game-theoretic analysis of coordinating pricing and marketing decisions in a multi-product multi-echelon supply chain", *Scientia Iranica, E*, **23**(3), 1459–1473 (2016).

25. Jingxian, C., Liang, L., and Shengnan, S. "Price and quality decisions in dual-channel supply chains", *European Journal of Operational Research*, **259**(3), 935–948 (2017).

26. Xu, X., Zhang, W., He, P., and Xu, X. "Production and pricing problems in make-to-order supply chain with cap-and-trade regulation", *Omega*, **66**, Part B, pp. 248–257 (2017).

27. Noori-daryan, M., Taleizadeh, A.A., and Govindan, K. "Joint replenishment and pricing decisions with different freight modes considerations for a supply chain under a composite incentive contract", *Journal of the Operational Research Society*, **69**(6), pp. 876–894 (2018). DOI: https://doi.org/10.1057/s41274-017-0270-z

28. Nazari, L., Seifbarghy, M., and Setak, M. "Modeling and analyzing pricing and inventory problem in a closed loop supply chain with return policy and multiple manufacturers and multiple sales channels using game theory", *Scientia Iranica, E*, **25**(5), pp. 2759–2774 (2017).

29. Mokhlesian, M. and Zegordi, S.H. "Pricing and advertising decisions in a dominant-retailer supply chain: A multi-follower bi-level programming approach", *Scientia Iranica, E*, **25**(4), pp. 2254–2266 (2018).

30. Taleizadeh, A.A. and Rasuli-Baghban, A. "Pricing and Lot sizing of a Decaying item under group dispatching with time-dependent demand and decay rates", *Scientia Iranica, E*, **25**(3), pp. 1656–1670 (2018).

31. Gamasaee, R. and Fazel Zarandi, M.H. "Incorporating demand, orders, lead time, and pricing decisions for

reducing bullwhip effect in supply chains", *Scientia Iranica, E*, **25**(3), pp. 1724–1749 (2018).

32. Lott, J.R. and Roberts, R.D. "A guide to the pitfalls of identifying price discrimination", *Economic Inquiry*, **29**, pp. 14–23 (1991).

33. Gale, I.L. and Holmes, T.J. "The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty", *International Journal of Industrial Organization*, **10**, pp. 413–437 (1992).

34. Gale, I.L. and Holmes, T.J. "Advance-purchase discounts and monopoly allocation of capacity", *American Economic Review*, **83**, pp. 135–146 (1993).

35. Gale, I.L. "Price dispersion in a market with advance-purchases", *Review of Industrial Organization*, **8**, pp. 451–464 (1993).

36. Borenstein, S. and Rose, N.L. "Competition and price dispersion in the US airline industry", *Journal of Political Economy*, **102**, pp. 653–683 (1994).

37. Dana, J.D., Jr., *Equilibrium Price Dispersion Under Uncertain Demand: The General Demand Case*, Kellogg School Working Paper, Northwestern University, Evanston, IL (1996a).

38. Dana, J.D., Jr., *Equilibrium Price Dispersion Under Uncertain Demand: The Role of Costly Capacity and Market Structure*, Kellogg School Working Paper, Northwestern University, Evanston, IL (1996b).

39. Dana, J.D., Jr., *Monopoly Price Dispersion Under Demand Uncertainty*, Kellogg School Working Paper, Northwestern University, Evanston, IL (1996c).

40. Dana, J.D., Jr. "Advance-purchase discounts and price discrimination in competitive markets", *Journal of Political Economy*, **106**(2), pp. 395–422 (1998).

41. Dana, J.D. Jr. "Using yield management to shift demand when the peak time is unknown. Advance-purchase discounts and price discrimination in competitive markets", *RAND Journal of Economic*, **30**(3), pp. 456–474 (1999).

42. Cho, S.H. and Tang, C.S. "Advance booking programs for managing supply, demand, and price risks", *Manufacturing and Service Operations Management*, **20**(2), pp. 1–44 (2010).

43. Li, M.Z.F. "Pricing non-storable perishable goods by using a purchase restriction with an application to airline fare pricing", *European Journal of Operational Research*, **134**, pp. 631–647 (2001).

44. Chen, C.C. and Schwartz, Z. "The importance of information asymmetry in customers' booking decisions", *Cornell Hotel and Restaurant Administration Quarterly*, **47**(3), pp. 272–285 (2006).

45. Acciaro, M. "Service differentiation in liner shipping: advance booking and express services", *International Journal of Shipping and Transport Logistics*, **3**(4), pp. 365–383 (2011).

46. Mei, W-X. and Zhang, X-H. "Study on advance booking discount policy with demand information updated", *6th International Conference on Information Management, Innovation Management and Industrial Engineering* (2013).

47. Ata, B. and Dana, J.D., Jr. "Price discrimination on booking time", *International Journal of Industrial Organization*, **43**, pp. 175–181 (2004).

48. Ling, L., Dong, Y., Guo, X., and Liang, L. "Availability management of hotel rooms under cooperation with online travel agencies", *International Journal of Hospitality Management*, **50**, pp. 145–152 (2015).

49. Bilotkach, V., Gaggero, A.A., and Piga, C.A. "Airline pricing under different market conditions: Evidence from European low-cost carriers", *Tourism Management*, **47**, pp. 152–163 (2015).

50. Guizzardi, A., Pons, F.M.E., and Ranieri, E. "Advance booking and hotel price variability online: Any opportunity for business customers?", *International Journal of Hospitality Management*, **64**, pp. 85–93 (2017).

51. Zhao, X., Atkins, D., and Zhang, W. "Revenue management under joint pricing and capacity allocation competition", *European Journal of Operational Research*, **257**(3), 957–970 (2017).

52. Benítez-Aurioles, B. "Why are flexible booking policies priced negatively?", *Tourism Management*, **67**, pp. 312–325 (2018).

53. Dai, Y., Chao, X., Fang, S.C., and Nuttle, H.L.W. "Pricing in revenue management for multiple firms competing for customers", *International Journal of Production Economics*, **98**, pp. 1–16 (2005).

54. Liu, L., Parlar, M., and Zhu, S.X. "Pricing and lead time decisions in decentralized supply chains", *Management Science*, **53**, pp. 713–725 (2007).

55. Szmerekovsky, J.G. and Zhang, J. "Pricing and two-tier advertising with one manufacturer and one retailer", *European Journal of Operational Research*, **192**, pp. 904–917 (2009).

56. Grauberger, W. and Kimms, A. "Airline revenue management games with simultaneous price and quantity competition", *Computers & Operations Research*, **75**, pp. 64–75 (2016).

57. Taleizadeh, A.A., Noori-daryan, M., and Tavakkoli-Moghaddam, R. "Pricing and ordering decisions in a supply chain with imperfect quality items and inspection under buyback of defective items", *International Journal of Production Research*, **53**(15), pp. 4553–4582 (2015b).

58. Noori-daryan, M. and Taleizadeh, A.A. "Coordinating pricing and ordering decisions in a multi-echelon pharmacological supply chain under different market power using game theory", *Journal of Industrial and Systems Engineering*, **9**(1), pp. 35–56 (2016).

Appendix A

Finding the closed-form solutions

For finding the closed-form solutions for the decision variables, the following parameters are utilized:

$$Y_1 = \frac{[\eta_R(1-\eta_1)\alpha_1) + \gamma_R s_R^{11} - \gamma_1 s_1^1 + v_1^1 \beta_R]}{2\beta_R}, \quad (A.1)$$

$$Y_2 = \frac{[\eta_R(1-\eta_1)\alpha_1) + \gamma_R s_R^{12} - \gamma_1 s_1^2 + v_1^2 \beta_R]}{2\beta_R}, \quad (A.2)$$

$$Y_3 = \frac{[\eta_R(1-\eta_2)\alpha_2) + \gamma_R s_R^{21} - \gamma_2 s_2^1 + v_2^1 \beta_R]}{2\beta_R}, \quad (A.3)$$

$$Y_4 = \frac{[\eta_R(1-\eta_2)\alpha_2) + \gamma_R s_R^{22} - \gamma_2 s_2^2 + v_2^2 \beta_R]}{2\beta_R}, \quad (A.4)$$

$$Y_5 = \frac{[1-\eta_R)[(1-\eta_1)\alpha_1 + (1-\eta_2)\alpha_2] + (v_1^1 + v_2^1)\beta_R]}{2\beta_R}, \quad (A.5)$$

$$Y_6 = \frac{(1-\eta_R)[(1-\eta_1)\alpha_1 + (1-\eta_2)\alpha_2] + (v_1^2 + v_2^2)\beta_R}{2\beta_R}, \quad (A.6)$$

$$Y_7 = \frac{\beta_1}{2\beta_R}, \quad (A.7)$$

$$Y_8 = 2\beta_2\beta_R, \quad (A.8)$$

$$Y_9 = 2\beta_R Y_4, \quad (A.9)$$

$$Y_{10} = \frac{Y_8}{2\beta_R}, \quad (A.10)$$

$$Y_{11} = \eta_1\alpha_1 + Y_1\beta_R + \gamma_1 s_1^1 - \gamma_R s_R^{11} + (v_1^1 - 2k_1^1)(-\beta_1 + \beta_R Y_7), \quad (A.11)$$

$$Y_{12} = 2(\beta_1 - \beta_R Y_6), \quad (A.12)$$

$$Y_{13} = \eta_1\alpha_1 + Y_2\beta_T + \gamma_1 s_1^2 - \gamma_R s_R^{12} + (v_1^2 - 2k_1^2)(-\beta_1 + \beta_R Y_7) + (v_1^2 - k_1^2), \quad (A.13)$$

$$Y_{14} = \eta_2\alpha_2 + \gamma_2 s_2^1 - \gamma_R s_R^{21} + \left(\frac{\beta_2}{2}\right)(v_2^1 - 2k_2^1), \quad (A.14)$$

$$Y_{15} = \eta_2\alpha_2 + \beta_R Y_4 + \gamma_2 s_2^2 - \gamma_R s_R^{22} - \left(\frac{\beta_2}{2}\right)(v_2^2 - 2k_2^2). \quad (A.15)$$

Appendix B

Proving concavity of the profit of the real retailer

To prove concavity of the profit of the real retailer, first, the Hessian matrix should be formed as follows:

$$H = \begin{bmatrix} \frac{\partial^2\psi_R}{\partial p_R^{11\,2}} & \frac{\partial^2\psi_R}{\partial p_R^{11}\partial p_R^{12}} & \frac{\partial^2\psi_R}{\partial p_R^{11}\partial p_R^{21}} & \frac{\partial^2\psi_R}{\partial p_R^{11}\partial p_R^{22}} \\ \frac{\partial^2\psi_R}{\partial p_R^{12}\partial p_R^{11}} & \frac{\partial^2\psi_R}{\partial p_R^{A22}} & \frac{\partial^2\psi_R}{\partial p_R^{12}\partial p_R^{21}} & \frac{\partial^2\psi_R}{\partial p_R^{12}\partial p_R^{22}} \\ \frac{\partial^2\psi_R}{\partial p_R^{21}\partial p_R^{11}} & \frac{\partial^2\psi_R}{\partial p_R^{21}\partial p_R^{12}} & \frac{\partial^2\psi_R}{\partial p_R^{21\,2}} & \frac{\partial^2\psi_R}{\partial p_R^{21}\partial p_R^{22}} \\ \frac{\partial^2\psi_R}{\partial p_R^{22}\partial p_R^{11}} & \frac{\partial^2\psi_R}{\partial p_R^{22}\partial p_R^{12}} & \frac{\partial^2\psi_R}{\partial p_R^{22}\partial p_R^{21}} & \frac{\partial^2\psi_R}{\partial p_R^{22\,2}} \end{bmatrix}, \quad (B.1)$$

where:

$$\frac{\partial^2\psi_R}{\partial p_R^{11\,2}} = -2\beta_R t_1, \quad (B.2)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{11}\partial p_R^{12}} = \frac{\partial^2\psi_R}{\partial p_R^{12}\partial p_R^{11}} = 0, \quad (B.3)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{11}\partial p_R^{21}} = \frac{\partial^2\psi_R}{\partial p_R^{21}\partial p_R^{11}} = -2\beta_2 t_1, \quad (B.4)$$

$$\frac{\partial^2\psi_R}{\partial p_T^{11}\partial p_T^{22}} = \frac{\partial^2\psi_R}{\partial p_R^{22}\partial p_R^{11}} = 0, \quad (B.5)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{12\,2}} = -2\beta_R(T - t_1), \quad (B.6)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{12}\partial p_R^{21}} = \frac{\partial^2\psi_R}{\partial p_R^{21}\partial p_R^{12}} = 0, \quad (B.7)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{12}\partial p_R^{22}} = \frac{\partial^2\psi_R}{\partial p_R^{22}\partial p_R^{12}} = -2\beta_2(T - t_1), \quad (B.8)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{21\,2}} = -2\beta_R t_1, \quad (B.9)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{21}\partial p_R^{22}} = \frac{\partial^2\psi_R}{\partial p_R^{22}\partial p_R^{21}} = 0, \quad (B.10)$$

$$\frac{\partial^2\psi_R}{\partial p_R^{22\,2}} = -\beta_R(T - t_1). \quad (B.11)$$

Then, it is found that π_R is concave if Eqs. (B.12)–(B.15) are met:

$$|H_1| = \left|\frac{\partial^2 \psi_R}{\partial p_R^{11^2}}\right| = -2\beta_R t_1 < 0, \tag{B.12}$$

$$|H_2| = \begin{vmatrix} -2\beta_R t_1 & 0 \\ 0 & -2\beta_R(T-t_1) \end{vmatrix}$$

$$= 4\beta_T^2 t_1(T-t_1) > 0, \tag{B.13}$$

$$|H_3| = \begin{vmatrix} -2\beta_R t_1 & 0 & -2\beta_2 t_1 \\ 0 & -2\beta_R(T-t_1) & 0 \\ -2\beta_2 t_1 & 0 & -2\beta_R t_1 \end{vmatrix}$$

$$= 8\left(\beta_2^2\beta_R - \beta_R^3\right) t_1^2(T-t_1) < 0, \tag{B.14}$$

$$|H_4| =$$

$$\begin{vmatrix} -2\beta_R t_1 & 0 & -2\beta_2 t_1 & 0 \\ 0 & -2\beta_R(T-t_1) & 0 & -2\beta_2(T-t_1) \\ -2\beta_2 t_1 & 0 & -2\beta_R t_1 & 0 \\ 0 & -2\beta_2(T-t_1) & 0 & -2\beta_R(T-t_1) \end{vmatrix}$$

$$= 16\left(\beta_R^4 + \beta_2^4\right) t_1^2(T-t_1)^2 > 0. \tag{B.15}$$

Appendix C

Proving concavity of the profit of the airline

To prove concavity of the profit of the first supplier, first, the Hessian matrix should be formed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \psi_1}{\partial p_1^{1^2}} & \frac{\partial^2 \psi_1}{\partial p_1^1 \partial p_1^2} \\ \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^1} & \frac{\partial^2 \psi_1}{\partial p_1^{2^2}} \end{bmatrix}, \tag{C.1}$$

where:

$$\frac{\partial^2 \psi_A}{\partial p_1^{1^2}} = -2\beta_1 t_1, \tag{C.2}$$

$$\frac{\partial^2 \psi_1}{\partial p_1^1 \partial p_1^2} = \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^1} = 0, \tag{C.3}$$

$$\frac{\partial^2 \psi_A}{\partial p_1^{2^2}} = -2\beta_1(T-t_1) < 0. \tag{C.4}$$

Hence, it is found that ψ_1 is concave if conditions (C.5) and (C.6) are met:

$$|H|_1 = \frac{\partial^2 \psi_1}{\partial p_1^{1^2}} = -2\beta_1 t_1 < 0, \tag{C.5}$$

$$|H|_2 = \frac{\partial^2 \psi_1}{\partial p_1^{1^2}} \cdot \frac{\partial^2 \psi_1}{\partial p_1^{2^2}} - \frac{\partial^2 \psi_1}{\partial p_1^1 \partial p_1^2} \cdot \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^1}$$

$$= 4\beta_1^2 t_1(T-t_1) > 0. \tag{C.6}$$

Appendix D

Proving concavity of the profit of the second supplier

To prove concavity of the profit of the second supplier, first, the Hessian matrix should be formed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \psi_2}{\partial p_2^{1^2}} & \frac{\partial^2 \psi_2}{\partial p_2^1 \partial p_2^2} \\ \frac{\partial^2 \psi_2}{\partial p_2^2 \partial p_2^1} & \frac{\partial^2 \psi_2}{\partial p_2^{2^2}} \end{bmatrix}, \tag{D.1}$$

where:

$$\frac{\partial^2 \psi_2}{\partial p_2^{1^2}} = -2\beta_2 t_1, \tag{D.2}$$

$$\frac{\partial^2 \psi_2}{\partial p_2^1 \partial p_2^2} = \frac{\partial^2 \psi_2}{\partial p_2^2 \partial p_2^1} = 0, \tag{D.3}$$

$$\frac{\partial^2 \psi_2}{\partial p_2^{2^2}} = -2\beta_2(T-t_1) < 0. \tag{D.4}$$

Hence, it is found that π_2 is concave if Conditions (D.5) and (D.6) are met:

$$|H|_1 = \frac{\partial^2 \psi_2}{\partial p_2^{1^2}} = -2\beta_2 t_1 < 0, \tag{D.5}$$

$$|H|_2 = \frac{\partial^2 \psi_2}{\partial p_2^{1^2}} \cdot \frac{\partial^2 \psi_2}{\partial p_2^{2^2}} - \frac{\partial^2 \psi_2}{\partial p_2^1 \partial p_2^2} \cdot \frac{\partial^2 \psi_2}{\partial p_2^2 \partial p_2^1}$$

$$= 4\beta_2^2 t_1(T-t_1) > 0. \tag{D.6}$$

Biographies

Mahsa Noori-daryan received her BSc degree in Physics from Science and Research Branch of Islamic Azad University, Tehran, Iran, and her MSc degree in Industrial Engineering from South Tehran Branch of Islamic Azad University, Tehran, Iran. She is a PhD candidate in Industrial Engineering at University of Tehran. Her research interests are in supply chain management, inventory control, and operation research.

Ata Allah Taleizadeh is an Associate Professor in the School of Industrial Engineering at University of Tehran in Iran. He received his PhD degree in Industrial Engineering from Iran University of Science and Technology. Moreover, he received his BSc and MSc degrees both in Industrial Engineering from Azad University of Qazvin and Iran University of Science and Technology, respectively. His areas of research interest include inventory control and production planning, pricing and revenue optimization, game theory, and uncertain programming. He has published several books and papers in reputable journals and serves as the editor/editorial board member for a number of international journals.

Masoud Rabbani is a Professor in the School of Industrial Engineering at University of Tehran, Iran. He received his PhD in Industrial Engineering from Amirkabir University of Technology, Iran. His research interests include manufacturing and production systems, soft computing techniques, Multi-Criteria Decision Making (MCDM), transportation, and scheduling. He has published papers in journals such as the International Journal of Advanced Manufacturing Technology, Applied Mathematics and Computation, International Journal of Production Research, Soft Computing, Advances in Engineering Software, Applied Mathematical Modeling, Engineering Optimization, International Journal of Manufacturing Technology and Management, Advanced Engineering Informatics, European Journal of Operational Research, Expert Systems with Applications, International Journal of Computer Mathematics, Applied Soft Computing, Computers and Operations Research, Journal of Manufacturing Systems, and Journal of Applied Sciences among others.