Optimal Pricing Strategies in Presence of Advance Booking Strategies for Complementary Supply Chains

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Abstract  
Due to high competition, companies tend to improve their market share by applying different selling mechanisms such as online to offline commerce as an efficient selling mechanism in which the companies sells their products via both online and real stores. This study deals with a selling problem for two complementary supply chains including a supplier and a shopping center where the commodities are sold by a virtual and a traditional shopping center who present items as complementary shopping centers. It is assumed that market demand depends on price and service level so that they can purchase the item via both shopping centers based on their priorities. Also to analyze the reactions of the chain’s partners, different games are deemed. The aim is to obtain the closed-form solutions of the decision variables of the networks’ members in order to maximize their profits where the prices of different selling periods at each echelon of the chains are the decision variables of the model. Then the closed-form solutions of the decision variables are derived and the solutions are examined by a numeric example. In following, several sensitivity analyses on key factors are performed to determine efficient ones on the variables and profits.

Keywords: Pricing; Selling; O2O Commerce; Advance Booking; Game Theory; Supply Chain.
1. Introduction and literature review

Recently, pricing policies have become a major concern for industries and corporations as an efficient tool for improving profitability. Because one of the factors influencing customer’s decisions is the price of the products. Then, companies strive to apply optimal pricing decisions to manage and control the market demand, in addition to raising their profits. Thus, competition between the companies and also between supply chains would be increased to satisfy customers’ demand and subsequently, enhancing their profits through attracting the market demand. Some authors have assumed pricing policies in their researches including Monahan (1984) [1] which he reviewed a proposed discount scheme proposed by a seller to buyers. The seller seeks to obtain the optimal price of his commodity, and in this research, the seller’s aim is to encourage customers to buy more of the desired product at reasonable prices, thereby increasing its profits and reducing the operational costs.

Whitin (1969) [2] examined pricing and inventory policies to minimize the cost of inventory systems. Benerjee (1986) [3] developed a model considering the optimal discount from seller's point of view. In this model, the seller buys his goods from another supplier, and the model is an extension of Monahan’s proposal. Ardalan (1991) [4] jointly studied optimal inventory and pricing policies for various combinations of sales periods and response times. Also, an algorithm is proposed to determine the combination of optimal ordering and pricing strategies.

Boyaci and Gallego (2002) [5] analyzed coordination issues in a two-tier supply chain including a wholesaler and one or more retailers. In this chain, customers’ demand is a function of the price of goods, and the operating costs include the cost of purchasing, commissioning, ordering and keeping inventory. Elmaghraby and Keskinocak (2003) [6] performed a review of dynamic pricing literature that has many uses in business. In their research, they focus on dynamic pricing on inventory issues. Agrawal et al. (2004) [7] presented a dynamic model of the allocation of inventory for a group of retailers to address the imbalance between inventory problems. They examined demand and net income information for inventory planning and balancing in a retail network, and addressed the issue with the aim of determining delivery time and inventory levels for retailers under a dynamic planning. Hua et al. (2010) [8] reviewed optimal pricing and delivery decisions in a centralized and decentralized supply chain, involving a producer and a retailer, with two channels (online sales and direct sales). Using a Stackelberg game, the impact of delivery time on the selling prices of producer and retailer has been evaluated and their results show that the delivery time has an efficient effect on the retail prices.
Chen et al. (2011) [9] examined a competitive decision-making model for two companies under a Stackelberg and a cooperative method and found that a non-cooperative approach leads to a higher income and lower cost for the retailer, and consequently, it brings lower profit for the producer. Chen and Hu (2012) [10] introduced a single-product pricing model with a common inventory under a finite planning horizon with a deterministic demand. In this model, the ordering and pricing decisions are determined simultaneously at the beginning of each decision period where demand depends on the price. Taleizadeh and Noori-daryan (2015a) [11] developed a non-linear programming model to examine the reactions of a three-echelon supply chain’s members in terms of pricing, manufacturing and ordering decisions.

Saha et al. (2017) [12], Yan et al. (2018) [13] and Jamali and Rasti-Barzoki (2018) [14] discussed different mathematical revenue management problems to model and study the behavior of supply chains’ partners under various making decision processes using Stackelberg and wide optimal methods. Noori-daryan et al. (2017) [15] considered a multinational supply chain including some capacity-constraint producers and a retailer in which the members make decision about their priorities such as price, order quantity, lead-time, and selection of supplier under different games. In addition, there are some investigations performing in revenue management area to optimize price decisions under various conditions such as Wee and Yu (1997) [16], Wee et al. (2003) [17], Sarkar et al. (2013) [18], Wei et al. (2013) [19], Shah et al. (2014) [20], Taleizadeh et al. (2015b) [21], Zhang et al. (2016) [22], Dan et al. (2016) [23], Niami Sadigh et al. (2016) [24], Jingxian et al. (2017) [25], Xu et al. (2017) [26], Noori-daryan et al. (2017) [27], Nazari et al. (2017) [28], Mokhlesian and Zegordi (2018) [29], Taleizadeh and Rasuli-Baghban (2018) [30], Gamasaee et al. (2018) [31].

Booking strategies as incentive mechanisms are applied by companies to manage their selling capacities. Under these types of policies, companies launch their items at a lower price than their real prices under capacity restriction to encourage customers and absorb the demand of market. Some researchers have been carried out to study pricing policies under booking strategies in different industries. Namely, Lott and Roberts (1991) [32], Gale and Holmes (1992) [33] and (1993) [34], Gale (1993) [35], Borenstein and Rose (1994) [36], Dana (1996 a, b, c) [37, 38, 39], (1998) [40] and (1999) [41] and Cho and Tang (2010) [42]. In 1992 and 1993, booking strategies are evaluated by Gale and Holmes and they found that the pre-sales strategies offered to passengers for pick flights are profitable for the airlines. Li (2001) [43] discussed a pricing issue for depreciable commodities or services such as tickets or hotel’s rooms. The results of this study showed that, based on the airlines’ expectations,
leisure travelers are more price sensitive than business travelers. Then, in 1998, Dana also investigated an advance booking policy in a competitive environment.

Chen and Schwartz (2006) [44] considered a reservation strategy for studying the behavior of customers and how much they bought from a hotel and an airline. Cho and Tang (2010) developed a Stackelberg game model to examine the interactions between a manufacturer and a retailer where each company sets their sales prices under reservation policies. They also found that the policy is always profitable for the producer, and the retailer benefits it only when there is a probability of shortage. Acciaro (2011) [45] offered a service-based model based on the carrier's reservation strategy, with the assumption that the service-level customers were different depending on the type of customer, cargo and route. Mei and Zhang (2013) [46] proposed a pricing issue with a reserved discount policy and presented a stochastic programming model. In this research, discount rates, prices, and optimal ordering were determined and numerical results express the more sensitivity and profitability of the profit regarding the sales price. Thus it is concluded that the reasonable prices lead to the greatest profit for the companies.

Ata and Dana Jr. (2015) [47] developed a mathematical model to examine a price differentiation strategy regarding different customers’ reservation times. Ling et al. (2015) [48] proposed an approach to enhance coordination between a hotel and an online travel agency to book hotel’s rooms. In this approach, customers are able to book their rooms through a hotel’s sales system or an online travel agency. Further, the hotel anticipates possible market demands with regard to the booked booking information, and the optimal availability approach for booking rooms, taking into account the maximum revenue from both methods (online and offline) is identified. Bilotkach et al. (2015) [49] examined the impact of reducing the flight ticket’s price on the revenue of two airlines, and found that reducing the standard rate of fares would increase the average consumers’ demand by an average of 2.7 percent. It should be noted that this fall in prices does not have much effect on the demand of customers for tourist trips. Guizzardi et al. (2017) [50] studied the impact of booking strategies, service quality, and service levels on customers’ purchasing decisions for a hotel. Zhao et al. (2017) [51] investigated the behaviors of firms in terms of capacity allocation and pricing under full-fare and discount market. Benitez-Aurioles (2018) [52] studied the relation between reservation and price in the tourist industry.

Game theory is a methodology for surveying the behavior of players under competition and collaborating settings in a supply chains. Several authors are employed this methodology to identify the decision variables which some of them are such as Dai et al. (2005) [53] has
been analyzed pricing strategies for several competing companies offering the same services to a group of customers to improve their revenue management. Companies have limited capacity and the market demand from each company depends on its sales price. They used game theory approaches to check systems when companies are faced with deterministic and probabilistic demand. They applied a Nash equilibrium point when demand for each company is a linear function of price. Liu et al. (2007) [54] considered a decentralized supply chain consisting of a retailer and a provider where demand depends on price and service level and there is a Stackelberg game between the partners. They concluded that decentralized decision-making results in lower performance and the supplier should improve its internal operations before pursuing a coordinating strategy with retailers.

Szmerekovsky and Zhang (2009) [55] reviewed pricing and two-level advertising decisions in single-producer-single-retailer supply chain. They assumed that customer’s demand was a function of a retail price and advertising cost paid by the manufacturer and retailer, and considered a Stackelberg game, in which the producer is a leader and retailer is a follower. They found that better performance would be achieved if the producer himself incurs the advertising costs and offered a lower wholesale price to the retailer. Grauberger and Kimms (2016) [56] presented a non-linear programming model to determine optimal pricing and reservation constraints under Nash games and it is found that the firms’ revenues in exclusive market are lower than that of in non-exclusive one and also Shah et al. (2014), Taleizadeh et al. (2015b) [57], and, Noori-daryan and Taleizadeh (2016) [58] considered game theory approaches to solve their presented models.

After reviewing the above researches, it is found that there is no investigation studying pricing decisions of two dual-channel supply chains under an advance booking strategy where demand is service-level and price sensitive and two complementary commodities are sold by the supply chains. In addition, upstream members offer an advance booking policy to downstream members and also to customers. All members determine their optimal pricing decisions under a non-cooperative game so that their profits are optimized. The rest of the paper is organized as follows. The problem is described in section 2. The introduced model is formulated in section 3. A solution procedure is proposed in section 4 and a real case and also the results are shown in section 5. Then, the study’s conclusion and findings are illustrated in section 6.

2. Problem statement
Here, a pricing problem for complementary items under an advance booking mechanism is studied where commodities are launched by the sellers at a lower price in advance and then, those are priced higher than before. The commodities are provided by two complementary supply networks in which a supplier sells own commodities through a virtual and a real shopping center, in each chain. Under this strategy, the networks aim to encourage them to order in advance in order to manage the market demand. The structure of the networks is presented in Figure 1.

The study’s purpose is to find the optimal decisions of the networks’ members, the suppliers and the retailer, about the selling prices of commodities within pre-selling and selling periods so that the members and the networks’ profit is optimized. In addition, the reactions of suppliers who supply the complementary products are surveyed applying a Nash game while a leader-follower game is assumed between the suppliers and its shopping center where a follower role is assigned to the retailer, real store, and the suppliers as leaders optimally determine their decision policies according to the reaction of the retailer.

The problem is modeled by utilizing following assumptions:

1. Price and service-level affect the customers’ demand.
2. Each network composes of a supplier and a shopping center.
3. Suppliers present complementary items to the customers.
4. Customers strive for purchasing both commodities as complement.
5. Suppliers launch their commodities through an online and a traditional shopping center.
6. A common shopping center is considered to sell both commodities to attract the customers of other complementary item.
7. Real shopping center’s service-level is more than that of online stores.
8. An advance booking strategy is applied to sell the commodities by online and offline shopping centers.

The on-hand problem is formulated by utilizing following notations indicated in Table 1.

### 3. Mathematical Model and Analysis

In this section, a pricing model for two complementary supply networks launching two complementary commodities under an advance booking policies. Each network involves a supplier and a retailer in which the suppliers sell its commodities through an online and a real shopping center. The formulation of the networks’ members’ model is presented as follows.
3.1. First Supplier’s Model

In this case, the first commodity is presented by the first supplier via its virtual shopping center and a real shopping center where it is priced in the virtual store lower than the real store, \( p_1^1 < p_R^1 \) and \( p_1^2 < p_R^2 \). Buyers decide about the purchasing based on the price and service-level, as their priorities, presented by the sellers. Therefore, the first supplier’s profit in virtual and real shopping centers is modeled as given.

3.1.1. First Supplier’s Model: Selling via an online shopping center

An advance booking strategy is employed by the supplier to persuade the buyers to order in advance. In fact, the supplier strives to manage the market demand by launching the item in two certain periods where it is cheaper within \([0, t_1]\) than the selling period \([t_1, T]\) indicated as \( p_1^1 < p_1^2 \). Additionally, whereas the buyers, who need more guidance, tend to purchase from real stores, supposed that the real shopping center’s service-level is more than that of online stores, \( s_1^1 < s_R^{11} \) and \( s_1^2 < s_R^{12} \).

According to the assumptions, buyers’ demand depends on the price and service-level and also they want to purchase both commodities. So, their demand from the first supplier in online store during time \([0, t_1]\) and \([t_1, T]\), respectively, are modeled as follows:

\[
d_1^1(p_1^1) = \eta_1 \alpha_1 - \beta_1 p_1^1 - \beta_2 p_1^2 + \beta_R p_R^1 + \gamma R \cdot s_1^1 - \gamma R \cdot s_R^{11} \tag{1}
\]
\[
d_1^2(p_1^2) = \eta_1 \alpha_1 - \beta_1 p_1^1 - \beta_2 p_1^2 + \beta_R p_R^1 + \gamma R \cdot s_1^2 - \gamma R \cdot s_R^{12} \tag{2}
\]

Then, the booking levels of the commodity within \([0, t_1]\) and \([t_1, T]\) are modeled differentially as follows:

\[
\frac{dq_1^1(t)}{dt} = d_1^1(p_1^1), \quad 0 \leq t \leq t_1
\]
\[
\frac{dq_1^2(t)}{dt} = d_1^2(p_1^2), \quad t_1 \leq t \leq T
\]

Since \( q_1^1(0) = 0 \) and \( q_1^2(T) = Z_1 \). Then, the first commodity’s booking levels at time \( t \) are as follows:

\[
q_1^1(t) = d_1^1(p_1^1) t = \left[ \eta_1 \alpha_1 - \beta_1 p_1^1 - \beta_2 p_1^2 + \beta_R p_R^1 + \gamma R \cdot s_1^1 - \gamma R \cdot s_R^{11} \right] t, \tag{5}
\]
\[
q_1^2(t) = d_1^2(p_1^2) (T - t) = \left[ \eta_1 \alpha_1 - \beta_1 p_1^1 - \beta_2 p_1^2 + \beta_R p_R^1 + \gamma R \cdot s_1^2 - \gamma R \cdot s_R^{12} \right] (T - t) \tag{6}
\]

Thereupon, the number of the item sold within \([0, t_1]\) can be demonstrated as follows:
\[ z_1^i = \int_0^t d_i^i(p_1^i)dt = d_i^i(p_1^i)t_1 = \left[ \eta_i\alpha_i - \beta_i\eta_i - \beta_i\eta_i + \beta_i p_2^i + \gamma_i s_1^i + \gamma_i s_1^i \right]t_1. \quad (7) \]

Hence, the number of the item sold within \([t_1, T]\) can be demonstrated as follows.

\[ z_2^i = \int_t^T d_i^i(p_1^i)dt = d_i^i(p_1^i)(T - t_1) = \left[ \eta_i\alpha_i - \beta_i\eta_i - \beta_i\eta_i + \beta_i p_2^i + \gamma_i s_1^i + \gamma_i s_1^i \right](T - t_1) \quad (8) \]

Therefore, the supplier’s profit earned by an online shopping center is as follows:

\[ \pi^0 = (p_1^i - k_1^i)z_1^i + (p_2^i - k_1^i)z_2^i \]

\[ = (p_1^i - k_1^i)\left[ \eta_i\alpha_i - \beta_i\eta_i - \beta_i\eta_i + \beta_i p_2^i + \gamma_i s_1^i - \gamma_i s_1^i \right]t_1 \]

\[ + (p_2^i - k_1^i)\left[ \eta_i\alpha_i - \beta_i\eta_i - \beta_i\eta_i + \beta_i p_2^i + \gamma_i s_1^i - \gamma_i s_1^i \right](T - t_1) \quad (9) \]

### 3.1.2. First Supplier’s Model: Selling via a real shopping center

The real common retailer orders the needed commodities based on the expected demand at its wholesale prices; then sells for a profit under a merchant model concluded between the retailer and the supplier. Note that the supplier presents its commodities to the real retailer under an advance booking policy where \( v_1^i < v_2^i \). So, the retailer’s demand from the first supplier during \([0, t_1]\) and \([t_1, T]\), respectively, are modeled as follows.

\[ d_i^{11}(p_{11}^i) = \eta_i(1 - \eta_i)\alpha_i - \beta_i\eta_i + \beta_i p_{21}^i + \beta_i p_1^i + \gamma_i s_1^i - \gamma_i s_1^i \quad (10) \]

\[ d_i^{12}(p_{12}^i) = \eta_i(1 - \eta_i)\alpha_i - \beta_i\eta_i + \beta_i p_{22}^i + \beta_i p_1^i + \gamma_i s_1^i - \gamma_i s_1^i \quad (11) \]

Then, the booking levels of the items within \([0, t_1]\) and \([t_1, T]\) are differentially formulated as follows:

\[ \frac{dq_1^{11}(t)}{dt} = d_i^{11}(p_{11}^i), \quad 0 \leq t \leq t_1 \quad (12) \]

\[ \frac{dq_1^{12}(t)}{dt} = d_i^{12}(p_{12}^i), \quad t_1 \leq t \leq T \quad (13) \]

Since \( q_1^{11}(0) = 0 \) and \( q_1^{12}(T) = Z_2^i \). Then, the first commodity’s booking levels at time \( t \) is as follows.

\[ q_1^{11}(t) = d_i^{11}(p_{11}^i)t = \left[ \eta_i(1 - \eta_i)\alpha_i - \beta_i\eta_i + \beta_i p_{21}^i + \beta_i p_1^i + \gamma_i s_1^i - \gamma_i s_1^i \right]t, \quad (14) \]

\[ q_1^{12}(t) = d_i^{12}(p_{12}^i)(T - t) = \left[ \eta_i(1 - \eta_i)\alpha_i - \beta_i\eta_i + \beta_i p_{22}^i + \beta_i p_1^i + \gamma_i s_1^i - \gamma_i s_1^i \right](T - t) \quad (15) \]

So, the number of the first item during \([0, t_1]\) is demonstrated as follows:

\[ z_1^{11} = \int_0^t d_i^{11}(p_{11}^i)dt = d_i^{11}(p_{11}^i)t_1 = \left[ \eta_i(1 - \eta_i)\alpha_i - \beta_i\eta_i + \beta_i p_{21}^i + \beta_i p_1^i + \gamma_i s_1^i - \gamma_i s_1^i \right]t_1, \quad (16) \]

Hence, the number of the first item during \([t_1, T]\) is demonstrated as follows.
\[ z_{i}^{R2} = \int_{t_i}^{T} d_i^{12}(p_{R}^{12}) dt = d_i^{12}(p_{R}^{12})(T - t) \]
\[ = \left[ \eta_{R} (1 - \eta_{i}) \alpha_{i} - \beta_{R} p_{R}^{12} - \beta_{i} p_{i}^{22} + \beta_{R}^{2} p_{R}^{11} + \gamma_{R} s_{R}^{12} - \gamma_{i} s_{i}^{22} \right] (T - t_{i}) \] (17)

Then, the first supplier’s profit earned by a real selling channel is as follows:
\[ \psi_{i}^{R} = (v_{i}^{1} - k_{i}^{1}) z_{i}^{R1} + (v_{i}^{2} - k_{i}^{2}) z_{i}^{R2} \]
\[ = (v_{i}^{1} - k_{i}^{1}) \left[ \eta_{R} (1 - \eta_{i}) \alpha_{i} - \beta_{R} p_{R}^{11} - \beta_{i} p_{i}^{21} + \beta_{R}^{2} p_{R}^{11} + \gamma_{R} s_{R}^{11} - \gamma_{i} s_{i}^{11} \right] t_{i} \]
\[ + (v_{i}^{2} - k_{i}^{2}) \left[ \eta_{R} (1 - \eta_{i}) \alpha_{i} - \beta_{R} p_{R}^{12} - \beta_{i} p_{i}^{22} + \beta_{R}^{2} p_{R}^{12} + \gamma_{R} s_{R}^{22} - \gamma_{i} s_{i}^{22} \right] (T - t_{i}) \] (18)

Eventually, the first supplier’s profit is as follows.
\[ \psi_{i} = \psi_{i}^{o} + \psi_{i}^{R} \]
\[ = (p_{i}^{1} - k_{i}^{1}) z_{i}^{1} + (p_{i}^{2} - k_{i}^{2}) z_{i}^{2} + (v_{i}^{1} - k_{i}^{1}) z_{i}^{R1} + (v_{i}^{2} - k_{i}^{2}) z_{i}^{R2} \] (19)

3.2. Second Supplier’s Model

Similarly, the second commodity is presented by the second supplier via its virtual shopping center and a real shopping center where it is priced in the virtual store lower than the real store, \( p_{2}^{1} < p_{R}^{21} \) and \( p_{2}^{2} < p_{R}^{22} \). Buyers decide about the purchasing based on the price and service-level, as their priorities, presented by the sellers. Therefore, the second supplier’s profit in virtual and real shopping centers is modeled as given.

3.2.1. Second Supplier’s Model: Selling via an online shopping center

An advance booking strategy is employed by the supplier to persuade the buyers to order in advance. Indeed, the supplier strives to manage the market demand by launching the item in two certain periods where it is cheaper within \([0, t_{i}]\) than the selling period \([t_{i}, T]\) indicated as \( p_{2}^{1} < p_{2}^{2} \). Additionally, whereas the buyers, who need more guidance, tend to purchase from real stores, supposed that the real shopping center’s service-level is more than that of online stores, \( s_{2}^{1} < s_{R}^{21} \) and \( s_{2}^{2} < s_{R}^{22} \).

According to the assumptions, buyers’ demand depends on the price and service-level and also they want to purchase both commodities. So, their demand from the second supplier in online store during time \([0, t_{i}]\) and \([t_{i}, T]\), respectively, are modeled as follows:
\[ d_{i}^{1} (p_{2}^{1}) = \eta_{2} \alpha_{2} - \beta_{2} p_{2}^{1} - \beta_{R} p_{R}^{21} + \beta_{R}^{2} p_{R}^{11} + \gamma_{2} s_{2}^{1} - \gamma_{R} s_{R}^{21} \] (20)
\[ d_{i}^{2} (p_{2}^{2}) = \eta_{2} \alpha_{2} - \beta_{2} p_{2}^{2} - \beta_{i} p_{i}^{1} + \beta_{R} p_{R}^{22} + \gamma_{2} s_{2}^{2} - \gamma_{R} s_{R}^{22} \] (21)

So, the booking levels of the items during periods \([0, t_{i}]\) and \([t_{i}, T]\) are modeled differentially as follows:
\[
\frac{dq_1^2(t)}{dt} = d_1^2(p_2^1), \quad 0 \leq t \leq t_1
\]  
(22)

\[
\frac{dq_2^2(t)}{dt} = d_2^2(p_2^1), \quad t_1 \leq t \leq T
\]  
(23)

Since \( q_1^2(0) = 0 \) and \( d_2^2(T') = Z_2 \). The second items’ booking levels at time \( t \) is as follows:

\[
q_1^2(t) = d_1^2(p_2^1)u = \left[ \eta_2 \alpha_2 - \beta_2 p_2^1 - \beta_1 p_1^1 + \beta_k \rho R_{21} + \gamma_2 \sigma_1^2 - \gamma_k \sigma_R^{21} \right] t,
\]  
(24)

\[
q_2^2(t) = d_2^2(p_2^1)(T - t) = \left[ \eta_2 \alpha_2 - \beta_2 p_2^2 - \beta_2 p_2^1 + \beta_k \rho R_{22} + \gamma_2 \sigma_1^2 - \gamma_k \sigma_R^{22} \right] (T - t)\]
(25)

So, the number of the commodities within \([0, t_1]\) is demonstrated as follows:

\[
z_1^2 = \int_0^t d_1^2(p_2^1)dt = d_1^2(p_2^1)u_1 = \left[ \eta_2 \alpha_2 - \beta_2 p_2^1 - \beta_1 p_1^1 + \beta_k \rho R_{21} + \gamma_2 \sigma_1^2 - \gamma_k \sigma_R^{21} \right] t_1,
\]
(26)

Therefore, the number of the items during period \([t_1, T]\) is indicated as follows.

\[
z_2^2 = \int_{t_1}^T d_2^2(p_2^1)dt = d_2^2(p_2^1)(T - t_1) = \left[ \eta_2 \alpha_2 - \beta_2 p_2^2 - \beta_2 p_2^1 + \beta_k \rho R_{22} + \gamma_2 \sigma_1^2 - \gamma_k \sigma_R^{22} \right] (T - t_1)
\]  
(27)

Furthermore, the second supplier’s profit earned by its online shopping center is as follows:

\[
\psi_2^\sigma = (p_2^1 - k_2^1)z_1^1 + (p_2^2 - k_2^2)z_2^2
\]
\[
= (p_2^1 - k_2^1) \left[ \eta_2 \alpha_2 - \beta_2 s_2^1 - \beta_1 s_1^1 + \beta_k \rho R_{21} + \gamma_2 \sigma_1^2 \right] t_1
\]
\[
+ (p_2^2 - k_2^2) \left[ \eta_2 \alpha_2 - \beta_2 p_2^2 - \beta_2 p_2^1 + \beta_k \rho R_{22} + \gamma_2 \sigma_2^2 \right] (T - t_1)
\]
(28)

3.2.2. Second Supplier’s Model: Selling via a real shopping center

In this case, like the previous one, the real store buys the second supplier’s item under an advance booking policy from the supplier at offered wholesale prices where \( v_{12}^1 < v_{12}^2 \); those are sold to buyers applying an advance booking policy in which \( p_{12}^{21} < p_{12}^{22} \). A merchant contract is considered between the retailer and supplier so that the retailer sells its ordered items for a profit based on own opinion about the item’s price.

So, the retailer’s demand from the second supplier during \([0, t_1]\) and \([t_1, T]\), respectively, are modeled as follow:

\[
d_{12}^{21}(p_{12}^{21}) = \eta_2 (1 - \eta_2) \alpha_2 - \beta_2 \rho R_{21} - \beta_2 p_2^1 + \beta_k \rho R_{11} - \gamma_2 \sigma_1^2
\]
(29)

\[
d_{12}^{22}(p_{12}^{22}) = \eta_2 (1 - \eta_2) \alpha_2 - \beta_2 \rho R_{22} - \beta_2 p_2^2 + \beta_k \rho R_{22} - \gamma_2 \sigma_2^2
\]  
(30)

Therefore, the booking levels of the items within \([0, t_1]\) and \([t_1, T]\) are differentially indicated as follows:

\[
\frac{dq_2^{21}(t)}{dt} = d_{12}^{21}(p_{12}^{21}), \quad 0 \leq t \leq t_1
\]  
(31)
\[
\frac{dq^{R2}_2(t)}{dt} = d^{22}_R(p^{22}_R), \quad t_1 \leq t \leq T
\]

Since \(q^{R1}_2(0) = 0\) and \(q^{R2}_2(T) = Z^2_R\), the items’ booking levels at time \(t\) is as follows:

\[
q^{R1}_2(t) = d^{21}_R(p^{21}_R) = \left[ \eta_R (1 - \eta_2) \alpha_2 - \beta_R p^{21}_R - \beta_R p^{21}_R + \beta_2 p^{11}_2 + \gamma_R s^{21}_R - \gamma s^1_2 \right] t,
\]

\[
q^{R2}_2(t) = d^{22}_R(p^{22}_R)(T - t_1) = \left[ \eta_R (1 - \eta_2) \alpha_2 - \beta_R p^{22}_R - \beta_R p^{12}_R + \beta_2 p^{12}_2 + \gamma_R s^{22}_R - \gamma s^2_2 \right](T - t)
\]

Then, the number of the items within \([0, t]\) is demonstrated as follows:

\[
z^{R1}_2 = \int_0^t d^{21}_R(p^{21}_R)dt = d^{21}_R(p^{21}_R)t_1 = \left[ \eta_R (1 - \eta_2) \alpha_2 - \beta_R p^{21}_R - \beta_R p^{11}_R + \beta_2 p^{11}_2 + \gamma_R s^{21}_R - \gamma s^1_2 \right] t_1,
\]

Hence, the number of the second items during \([t_1, T]\) is indicated as follows.

\[
z^{R2}_2 = \int_{t_1}^T d^{22}_R(p^{22}_R)dt = d^{22}_R(p^{22}_R)(T - t_1) = \left[ \eta_R (1 - \eta_2) \alpha_2 - \beta_R p^{22}_R - \beta_R p^{12}_R + \beta_2 p^{12}_2 + \gamma_R s^{22}_R - \gamma s^2_2 \right](T - t_1)
\]

Therefore, the second supplier’s profit earned by the real selling channel is as follows:

\[
\pi^R_2 = (p^{21}_R - \nu^1_2)z^{R1}_2 + (p^{22}_R - \nu^2_2)z^{R2}_2 = (p^{21}_R - \nu^1_2)\left[ \eta_R (1 - \eta_2) \alpha_2 - \beta_R p^{21}_R - \beta_R p^{11}_R + \beta_2 p^{11}_2 + \gamma_R s^{21}_R - \gamma s^1_2 \right] t_1 + (p^{22}_R - \nu^2_2)\left[ \eta_R (1 - \eta_2) \alpha_2 - \beta_R p^{22}_R - \beta_R p^{12}_R + \beta_2 p^{12}_2 + \gamma_R s^{22}_R - \gamma s^2_2 \right](T - t_1)
\]

Finally, the second supplier’s profit is as follows.

\[
\pi_2 = \pi^O_2 + \pi^R_2 = (p^1_2 - k^1_2)z^1_2 + (p^2_2 - k^2_2)z^2_2 + (\nu^1_2 - k^1_2)z^{R1}_2 + (\nu^2_2 - k^2_2)z^{R2}_2
\]

\[3.3. \text{Real Retailer’s Model}\]

Besides the online stores, the real shopping center launches the first and the second commodities simultaneously, as complementary items. It is supposed that the suppliers commonly sell the items through a real shopping center, along with online stores, to attract the buyers who need more guidance or prefer to touch the purchased commodities. Thus its profit composed of two components such as the profits of selling first and the second commodities. So, the booking level of its first item is formulated differentially as:

\[
\frac{dq^{11}_R(t)}{dt} = d^{11}_R(p^{11}_R), \quad 0 \leq t \leq t_1
\]

\[
\frac{dq^{12}_R(t)}{dt} = d^{12}_R(p^{12}_R), \quad t_1 \leq t \leq T
\]

Since \(q^{11}_R(0) = 0\) and \(q^{12}_R(T) = Z^1_R\). Hence, the first items’ booking levels at time \(t\) is as follows:
\[ q_R^{11}(t) = d^{11}_R(p_{R}^{11}) = \left[ \eta_R (1-\eta_R) \alpha_t - \beta_R p_{R}^{11} - \beta_2 p_{R}^{21} + \beta_1 p_{1}^{21} + \gamma_R s_{R}^{11} - \gamma_s^{11} \right] t, \quad (41) \]
\[ q_R^{12}(t) = d^{12}_R(p_{R}^{12})(T-t) = \left[ \eta_R (1-\eta_R) \alpha_t - \beta_R p_{R}^{12} - \beta_2 p_{R}^{22} + \beta_1 p_{1}^{22} + \gamma_R s_{R}^{12} - \gamma_s^{12} \right] (T-t) \quad (42) \]

Furthermore, the number of the first commodity within \([0,t_1]\) is indicated as follows:
\[ z_R^{11} = \int_{0}^{t_1} d^{11}_R(p_{R}^{11}) dt = \left[ \eta_R (1-\eta_R) \alpha_t - \beta_R p_{R}^{11} - \beta_2 p_{R}^{21} + \beta_1 p_{1}^{21} + \gamma_R s_{R}^{11} - \gamma_s^{11} \right] t_1, \quad (43) \]

Thus, the number of the first commodities during \([t_1,T]\) is indicated as follows.
\[ z_R^{12} = \int_{t_1}^{T} d^{12}_R(p_{R}^{12})(T-t) dt = \left[ \eta_R (1-\eta_R) \alpha_t - \beta_R p_{R}^{12} - \beta_2 p_{R}^{22} + \beta_1 p_{1}^{22} + \gamma_R s_{R}^{12} - \gamma_s^{12} \right] (T-t_1) \quad (44) \]

Then, the retailer’s profit earned by selling the first items is as follows:
\[ \psi_R^{1} = (p_{R}^{11}-v_1^1)z_R^{11} + (p_{R}^{12}-v_1^2)z_R^{12} \]
\[ = (p_{R}^{11}-v_1^1)\left[ \eta_R (1-\eta_R) \alpha_t - \beta_R p_{R}^{11} - \beta_2 p_{R}^{21} + \beta_1 p_{1}^{21} + \gamma_R s_{R}^{11} - \gamma_s^{11} \right] t_1 \]
\[ + (p_{R}^{12}-v_1^2)\left[ \eta_R (1-\eta_R) \alpha_t - \beta_R p_{R}^{12} - \beta_2 p_{R}^{22} + \beta_1 p_{1}^{22} + \gamma_R s_{R}^{12} - \gamma_s^{12} \right] (T-t_1) \quad (45) \]

In addition, the booking level of its second items is shown as:
\[ \frac{dq_R^{21}(t)}{dt} = d^{21}_R(p_{R}^{21}), \quad 0 \leq t \leq t_1 \quad (46) \]
\[ \frac{dq_R^{22}(t)}{dt} = d^{22}_R(p_{R}^{22}), \quad t_1 \leq t \leq T \quad (47) \]

Since \( q_R^{21}(0) = 0 \) and \( q_R^{22}(T) = Z_R^2 \), the second items’ booking levels at time \( t \) is as follows:
\[ q_R^{21}(t) = d^{21}_R(p_{R}^{21}) = \left[ \eta_R (1-\eta_R) \alpha_z - \beta_R p_{R}^{21} - \beta_2 p_{R}^{21} + \beta_1 p_{1}^{21} + \gamma_R s_{R}^{21} - \gamma_s^{21} \right] t, \quad (48) \]
\[ q_R^{22}(t) = d^{22}_R(p_{R}^{22})(T-t) = \left[ \eta_R (1-\eta_R) \alpha_z - \beta_R p_{R}^{22} - \beta_2 p_{R}^{22} + \beta_1 p_{1}^{22} + \gamma_R s_{R}^{22} - \gamma_s^{22} \right] (T-t) \quad (49) \]

So, the number of the second items within \([0,t_1]\) is demonstrated as follows:
\[ z_R^{21} = \int_{0}^{t_1} d^{21}_R(p_{R}^{21}) dt = d^{21}_R(p_{R}^{21}) t_1 \]
\[ = \left[ \eta_R (1-\eta_R) \alpha_z - \beta_R p_{R}^{21} - \beta_2 p_{R}^{21} + \beta_1 p_{1}^{21} + \gamma_R s_{R}^{21} - \gamma_s^{21} \right] t_1, \quad (50) \]

Then, the number of the second items within \([t_1,T]\) is shown as follows.
\[ z_R^{22} = \int_{t_1}^{T} d^{22}_R(p_{R}^{22}) dt = d^{22}_R(p_{R}^{22})(T-t_1) \]
\[ = \left[ \eta_R (1-\eta_R) \alpha_z - \beta_R p_{R}^{22} - \beta_2 p_{R}^{22} + \beta_1 p_{1}^{22} + \gamma_R s_{R}^{22} - \gamma_s^{22} \right] (T-t_1) \quad (51) \]

Therefore, the retailer’s profit earned by selling the second items is equal to:
Gradually, the total profit of retailer can be stated as follows.

\[
\psi_R = (p_R^{21} - v_1^1)z_R^{21} + (p_R^{22} - v_2^2)z_R^{22} \\
= (p_T^{21} - v_1^1)[\eta_R (1 - \eta_2)\beta_2 - \beta_R p_R^{11} - \beta_R p_R^{21} + \beta_T p_T^{11} + \gamma_T s_T^{11} - \gamma_T s_T^{11}]t_1 \\
+ (p_T^{22} - v_2^2)[\eta_R (1 - \eta_2)\beta_2 - \beta_R p_R^{11} - \beta_R p_R^{21} + \beta_T p_T^{11} + \gamma_T s_T^{11} - \gamma_T s_T^{11}](T - t_1) \\
\]

(52)

Gradually, the total profit of retailer can be stated as follows.

\[
\psi_R = (p_R^{11} - v_1^1)z_R^{11} + (p_R^{12} - v_1^1)z_R^{12} + (p_R^{21} - v_1^1)z_R^{21} + (p_R^{22} - v_2^2)z_R^{22} \\
\]

(53)

4. Solution Procedure

In this section, a solution procedure to find the closed-form solutions of the decisions variables is examined so that the reactions of the supply chains’ members are analyzed by considering game theory approaches such as Nash and Stackelberg game. In this case, it is assumed that there is a Nash game between the suppliers who supply the commodities independently while there is a Stackelberg game between the members of each chain where the real shopping center is a follower and the supplier is a leader of the market.

In the Stackelberg game, the optimal decision policies of the leaders depend on the follower’s optimal decisions so that the best reaction of the retailer is considered by the leaders. The commodities’ prices within \([0, t_1]\) and \([t_1, T]\) are the decision variables of the retailer, as the follower, while the price of the items within time \([0, t_1]\) and \([t_1, T]\) in online stores are determined by the suppliers, as the leaders. In addition, the optimality of the objective functions regarding the decision variables should be proven.

**Theorem 1.** The profit of the real retailer, \(\psi_R(p_R^{11}, p_R^{12}, p_R^{21}, p_R^{22})\), is concave.

**Proof.** If the conditions (B-12), (B-13), (B-14) and (B-15) are convinced (See appendix (B)), the concavity of the retailer’s profit function is proved. Thus, by taking the first order derivatives of the profit function, Eq. (53), regarding \(p_R^{11}, p_R^{12}, p_R^{21}, p_R^{22}\), the optimal values of \(p_R^{11}, p_R^{12}, p_R^{21}, p_R^{22}\) are obtained as follows:

\[
\frac{\partial \psi_R}{\partial p_R^{11}} = \left[\eta_R (1 - \eta_2)\alpha_1 - \beta_R p_R^{11} - \beta_T p_R^{21} + \beta_T p_T^{11} + \gamma_T s_T^{11} - \gamma_T s_T^{11}\right]t_1 - \left[(p_R^{11} - v_1^1) + (p_R^{21} - v_1^1)\right] \beta_R t_1 = 0 \\
\frac{\partial \psi_R}{\partial p_R^{12}} = \left[\eta_R (1 - \eta_2)\alpha_1 - \beta_R p_R^{11} - \beta_T p_R^{21} + \beta_T p_T^{11} + \gamma_T s_T^{11} - \gamma_T s_T^{11}\right](T - t_1) \\
- (p_R^{22} - v_2^2) \beta_R (T - t_1) - (p_R^{22} - v_2^2) \beta_R (T - t_1) = 0 \\
\frac{\partial \psi_R}{\partial p_R^{21}} = \left[\eta_R (1 - \eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11} + \beta_T p_T^{21} + \gamma_T s_T^{21} - \gamma_T s_T^{21}\right]t_1 - \left[(p_R^{21} - v_2^2) + (p_R^{21} - v_2^2)\right] \beta_R t_1 = 0 \\
\frac{\partial \psi_R}{\partial p_R^{22}} = \left[\eta_R (1 - \eta_2)\alpha_2 - \beta_R p_R^{21} - \beta_R p_R^{11} + \beta_T p_T^{21} + \gamma_T s_T^{21} - \gamma_T s_T^{21}\right](T - t_1) \\
- (p_R^{22} - v_2^2) \beta_R (T - t_1) - (p_R^{22} - v_2^2) \beta_R (T - t_1) = 0
\]
Then, the optimal decisions of the retailer regarding the suppliers’ policies after simplifying the above equations are as follows:

\[
p_{R1}^{11} = Y_1 + \frac{\beta_1 p_1^{11}}{2\beta_k}
\]

\[
p_{R1}^{11} = Y_2 + \frac{\beta_1 p_1^{11}}{2\beta_k}
\]

\[
p_{R1}^{21} = Y_3 + \frac{\beta_1 p_1^{21}}{2\beta_k}
\]

\[
p_{R1}^{21} = Y_4 + \frac{\beta_1 p_1^{21}}{2\beta_k}
\]

Where the equations utilized to simplify the relations are illustrated in Appendix (A).

Besides, based on the hypothesis, it is considered a Stackelberg game between the networks’ members where a real retailer decides about their optimal policies and the suppliers as the leaders characterize their variables regarding the optimal reactions of the retailer. A Nash game is assumed between the suppliers in order to obtain their decision variables, simultaneously.

**Theorem 2.** The first supplier’s profit, \(\psi_i(p_1^1, p_1^2)\), is concave.

**Proof.** The first supplier’s profit function’s concavity is proved (See Appendix (C)), the optimal values of \(p_{R1}^{11}, p_{R1}^{12}, p_{R1}^{21}, p_{R1}^{22}\) are replaced into Eq. (19) which is equal to:

\[
\psi_i = (p_i^1 - k_i)\left[\eta_i \alpha_i - \beta_i p_i^1 - \beta_2 p_i^2 + \beta_k p_{R1}^{11} + \gamma_1 s_i^1 - \gamma_k s_{R1}^{11}\right] t_1 +
\]

\[
(p_i^1 - k_i)\left[\eta_i \alpha_i - \beta_i p_i^1 - \beta_2 p_i^2 + \beta_k p_{R1}^{11} + \gamma_1 s_i^1 - \gamma_k s_{R1}^{11}\right] (T - t_1) +
\]

\[
(v_i^1 - k_i)\left[\eta_i (1 - \eta_i) \alpha_i - \beta_k p_{R1}^{11} - \beta_k p_{R1}^{12} + \beta_1 p_i^1 + \gamma_1 s_{R1}^{11} - \gamma_1 s_i^1\right] t_1 +
\]

\[
(v_i^1 - k_i)\left[\eta_i (1 - \eta_i) \alpha_i - \beta_k p_{R1}^{11} - \beta_k p_{R1}^{12} + \beta_1 p_i^1 + \gamma_1 s_{R1}^{11} - \gamma_1 s_i^1\right] (T - t_1)
\]

Then, the optimal values of \(p_i^1\) and \(p_i^2\) are obtained by taking the first order derivatives of the objective function, Eq. (62), regarding \(p_i^1\) and \(p_i^2\), are as follows:

\[
\frac{\partial \psi_i}{\partial p_i^1} = \left[\eta_i \alpha_i - \beta_i p_i^1 - \beta_2 p_i^2 + \beta_k p_{R1}^{11} + \gamma_1 s_i^1 - \gamma_k s_{R1}^{11}\right] t_1
\]

\[
- (p_i^1 - k_i) \beta_i t_1 + (v_i^1 - k_i) \beta_i = 0
\]
\[
\frac{\partial \psi_i}{\partial p_i^1} = [\eta_1 \alpha_1 - \beta_1 p_1^1 - \beta_2 p_2^1 + \beta_3 p_3^1 + \gamma_1 s_1^1 - \gamma_2 s_2^{12}] (T - t_i) \\
-(p_1^1 - k_1^1) \beta_i (T - t_i) + (v_1^1 - k_1^1) \beta_i (T - t_i) = 0
\] (64)

Hence, the first supplier’s closed-form solutions of the variables are as follow.

\[
p_1^{1*} = -\frac{Y_{17} + Y_{14}}{Y_{12} - \beta_1}
\] (65)

\[
p_2^{1*} = -\frac{-\beta Y_{13} + Y_{14}}{Y_{12} - \beta_1}
\] (66)

**Theorem 3.** The second supplier’s profit, \( \psi_2(p_1^2, p_2^2) \), is concave.

**Proof.** The second supplier’s profit function’s concavity is proved (See Appendix (D)), the optimal values of \( P_{R_1}^{11}, P_{R_2}^{12}, P_{R_1}^{21}, P_{R_2}^{22} \) are replaced into Eq. (38) which is equal to:

\[
\psi_2 = (p_1^1 - k_1^1) \left[ \eta_2 \alpha_2 - \beta_1 p_1^1 + \beta_4 p_4^1 + \beta_5 p_5^1 + \gamma_1 s_1^1 - \gamma_2 s_2^{12} \right] t_i + \\
(p_2^2 - k_2^2) \left[ \eta_2 \alpha_2 - \beta_2 p_2^2 - \beta_4 p_4^2 + \beta_5 p_5^2 + \gamma_2 s_2^1 - \gamma_3 s_3^{22} \right] (T - t_i) + \\
(P_{R_2}^{21} - v_1^1) \left[ \eta_2 (1 - \eta_2) \alpha_2 - \beta_2 p_2^1 + \beta_4 p_4^1 + \beta_5 p_5^1 + \gamma_2 s_2^1 - \gamma_3 s_3^{22} \right] t_i + \\
(P_{R_2}^{22} - v_2^2) \left[ \eta_2 (1 - \eta_2) \alpha_2 - \beta_2 p_2^2 + \beta_4 p_4^2 + \beta_5 p_5^2 + \gamma_2 s_2^1 - \gamma_3 s_3^{22} \right] (T - t_i)
\] (67)

The optimal values of \( p_1^1 \) and \( p_2^2 \) are derived by taking the first order derivatives of the objective function, Eq. (67), regarding \( p_1^1 \) and \( p_2^2 \), as given:

\[
\frac{\partial \psi_2}{\partial p_1^1} = [\eta_2 \alpha_2 - \beta_1 p_1^1 + \beta_4 p_4^1 + \beta_5 p_5^1 + \gamma_1 s_1^1 - \gamma_2 s_2^{12}] t_i - (p_1^1 - k_1^1) \beta_2 t_i + (v_1^1 - k_1^1) \beta_2 t_i = 0
\] (68)

\[
\frac{\partial \psi_2}{\partial p_2^2} = [\eta_2 \alpha_2 - \beta_2 p_2^2 - \beta_4 p_4^2 + \beta_5 p_5^2 + \gamma_2 s_2^1 - \gamma_3 s_3^{22}] (T - t_i) - (p_2^2 - k_2^2) \beta_2 (T - t_i) + (v_2^2 - k_2^2) \beta_2 (T - t_i) = 0
\] (69)

Thus, the second supplier’s closed-form solutions of the variables are as follow.

\[
p_1^{2*} = -\frac{\beta Y_{11} + Y_{14}}{Y_{12} - \beta_1}
\] (70)

\[
p_2^{2*} = -\frac{\beta Y_{13} + Y_{14}}{Y_{12} - \beta_1}
\] (71)

By replacing the closed-form solutions using Eq. (65), (66), (70) and (71) into the Eq. (58)–(61), the closed-form solutions of the variables are as follows:

\[
p_{R_1}^{11} = Y_1 + Y_7 \left( \frac{Y_{11} - Y_{14}}{Y_{12} - \beta_1} \right)
\] (72)
\[ p_{R}^{12} = Y_2 + Y_7 \left( \frac{Y_{13} - Y_{15}}{Y_{12} - \beta_1} \right) \] (73)

\[ p_{R}^{21} = \frac{Y_9}{2\beta_R} + \frac{\beta_2}{2\beta_R} \left( \frac{-\beta Y_{11} + Y_{14} Y_{12}}{\beta Y_{12} - \beta, \beta_2} \right) \] (74)

\[ p_{R}^{22} = \frac{Y_4}{2\beta_R} + \frac{\beta_2}{2\beta_R} \left( \frac{-\beta Y_{13} + Y_{15} Y_{12}}{\beta Y_{12} - \beta, \beta_2} \right) \] (75)

Note that the above used relations are illustrated in Appendix (A).

5. Numerical Example and Sensitivity Analysis

5.1. Real Case

As a real case, we set the introduced model for two tourism supply chains face customers who want to book an airline’s tickets and a hotel’s rooms for a travel. Supposed that the first chain launches the tickets and the second one provides the rooms to the buyers where their orders are sensitive to a price and a service-level. In the first chain, an airline and a travel agency are composed of the members while the second chain including a hotel and the same travel agency.

As it is common now, the airline and the hotel present their commodities through virtual stores and a common travel agency. Since the complementary items are presented by the airline and the hotel, companies prefer to sell their items through a common travel agency to increase their market shares. Thus the proposed model is examined numerically to show its applicability. The market demand depends on a service-level and a selling price. Then, the impacts of the key parameters on the optimal values of the decision variables and the profits of the supply networks are analyzed.

5.2. Numerical Example

Here, it is considered that \( k_1 = 10, k_2 = 12, \alpha_i = 40000, \eta_1 = 0.4, \beta_1 = 21, \beta_2 = 21.5, \beta_R = 20, \gamma_1 = 10, \gamma_R = 15, s_i^1 = 1, s_i^{11} = 2, s_i^2 = 1, s_i^{12} = 2, v_i = 20, v_i^2 = 25, \eta_R = 0.5, v_1 = 23, v_2 = 28, s_i^1 = 1, s_i^{11} = 2, s_i^2 = 1, s_i^{22} = 2, \gamma_2 = 19, \eta_2 = 0.4, k_1 = 12, k_2 = 17. \) The results are indicated in Table 2 and 3.

According to Table 2 and 3, when the airline and the hotel sell their commodities under an advance booking policy, their profits would be higher that of selling them at their real prices; thus they tend to apply an ADB selling strategy to improve their benefits with...
attracting the market demand because of proposed selling prices in different selling periods. In addition, the online stores’ launch the items at a lower price than that of the travel agency due to selling directly. Therefore, the firms succeed to improve their market shares using different selling channels in addition to considering an ADB policy.

5.3. Sensitivity Analysis

In this section, to analyze the members’ decisions and their profits with the parameters variations, the optimal values of the variables and the partners' profits with respect to some parameters changes are surveyed. The related results are shown in Table 4-7 and their diagrams are shown in Figure 2-5, respectively. After examining the results, the following managerial insights are obtained:

- Table 4 shows that increasing customers' price sensitivity of the first item (ticket) causes decreasing buyers' demand and also diminishing the first supplier's (airline) profit. On the other hand, demand for the second commodity (room) as a complementary one, is reduced; then, the profit of the second supplier (hotel) diminishes. Also the retailer's (travel agency) selling prices are reduced because he/she presents substitutable items; thus he/she should sells items at lower prices to attract customers and manage its market share. Nonetheless, his/her profit is decreased because of reducing its prices. The diagram of the firms’ profits regarding the price sensitivity of market demand of the first item changes is indicated in Figure 2.

- Based on Table 5, it is found that reducing customers' price sensitivity of the second item (room) leads to increasing second supplier's profit because of higher prices. In addition, if the price elasticity of customers' demand decreases, their demand slightly is influenced by price changes and in turn, the profit of the suppliers and the retailer would be increased and they price their commodities so that their profits are maximized. In other word, when the price sensitivity of customers for the room decreases, hotel tends to increase its booking prices while the airline goes to improve its price and subsequently, its profit. On the other hand, travel agency, as a competitive selling channel, increases selling prices with customers' sensitivity consideration. The diagram of the firms’ profits regarding the price sensitivity of market demand of the second item changes is indicated in Figure 3.

- The results of Table 6 state that the retailer launches the items at lower prices if the price elasticity of demand increases. So, the profit of the retailer (travel agency) decreases. Hence, customers do not want to purchase the commodities at higher
prices. Besides, the suppliers should be present items via virtual stores at lower prices to absorb the highly price dependent demands. The diagram of the firms’ profits regarding the customers’ price sensitivities of the ticket and the room changes is indicated in Figure 4.

- In addition, the effect of booking periods changes on the optimal values of decision variables and profits are indicated in Table 7. Based on the results, it is characterized that the suppliers and the retailer tend to employ an advance booking strategy to sell the commodities because of improving their profits. Also they benefit to sell the items during a longer pre-selling period to gain higher profit. The diagram of the firms’ profits regarding pre-selling period changes is indicated in Figure 5.

- According to the above analysis, it is concluded that price is a key variable which is suitably determined as the decision variable and has a significant impact on the customers' purchasing decisions so that increasing/decreasing the customers' sensitivity to the price changes causes decreasing/increasing the profit of the chain's members.

6. Conclusion

This paper develops a mathematical model for a selling problem under an advance booking policy where a market demand depends on service-level and price of commodities, launched by two complementary supply chains. Supposed that each chain comprises a supplier and a retailer in which the supplier presents own commodities through an online and a retailer, who presents both commodities to buyers, as a common member of both chains. The investigation's purpose is to find the closed-form solutions of the decision variables of the networks’ members in order to maximize their profits where the selling prices of items at each echelon of the chains in different selling periods are the decision variables of the proposed model.

Then, a real case is presented for the introduced model in which two tourism supply chains are considered which those are launched the complementary items such as thickets and rooms via its website and a travel agency. In this case, an airline and a hotel, as the suppliers of the chains, sell their commodities applying an advance booking selling strategy. A Nash game is assumed between the suppliers while there is a Stackelberg game between the members of each chain where the real shopping center is a follower and the supplier is a leader of the market.
In addition, there is a merchant agreement between the suppliers and a common travel agency to sell their items. Then, the model is numerically analyzed and the effects of some parameters changes on the decision variables and the members’ are evaluated. It is found that the members prefer to employ an advance booking policy to sell the commodities and manage the market demand. Based on the results, it is found that all the members of the supply chains benefit an advance booking strategy to launch their commodities. Also suppliers who sell their items via both channels earn higher profit than those of selling only through traditional one, due to essence of more customers, price and service oriented. For the future research, the following issues are recommended:

- Considering the proposed model under various booking strategies
- Considering the proposed model under a stochastic setting
- Considering the proposed model under competitive conditions
- Considering the proposed model in multi-echelon supply chains

References


Biographies

Mahsa Noori-daryan received her B.Sc. degree in physics from Science and Research Branch of Islamic Azad University, Tehran, Iran and also obtained her M.Sc. degree in industrial engineering from South Tehran Branch of Islamic Azad University, Tehran, Iran. She is a PhD candidate in Industrial Engineering at University of Tehran. Her research interests are in supply chain management, inventory control and operation research.

Ata Allah Taleizadeh is an associate professor in School of Industrial Engineering at University of Tehran in Iran. He received his Ph.D. degree in industrial engineering from Iran University of Science and Technology. Moreover he received his B.Sc. and M.Sc. degrees both in industrial engineering from Azad University of Qazvin and Iran University of Science and Technology, respectively. His research interest areas include inventory control and production planning, pricing and revenue optimization, Game Theory and uncertain programming. He has published several books and papers in reputable journals and he serves as the editor/editorial board member for a number of international journals.

Masoud Rabbani is a Professor at School of Industrial Engineering, at the University of Tehran, Iran. He received his Ph.D. in Industrial Engineering at Amirkabir University of Technology, Iran. His research interests include Manufacturing and Production Systems, Soft

**Appendix (A). Finding the closed-form solutions**

For finding the closed-form solutions of the decision variables, following parameters are utilized.

\[
Y_1 = \frac{\eta_R (1-\eta_i)\alpha_1 + \gamma_R s_R^{11} - \gamma_i s_i^1 + v_1^1 \beta_R}{2\beta_R}
\]  
(A-1)

\[
Y_2 = \frac{\eta_R (1-\eta_i)\alpha_1 + \gamma_R s_R^{12} - \gamma_i s_i^2 + v_1^2 \beta_R}{2\beta_R}
\]  
(A-2)

\[
Y_3 = \frac{\eta_R (1-\eta_i)\alpha_2 + \gamma_R s_R^{21} - \gamma_i s_i^1 + v_2^1 \beta_R}{2\beta_R}
\]  
(A-3)

\[
Y_4 = \frac{\eta_R (1-\eta_i)\alpha_2 + \gamma_R s_R^{22} - \gamma_i s_i^2 + v_2^2 \beta_R}{2\beta_R}
\]  
(A-4)

\[
Y_5 = \frac{(1-\eta_R)(1-\eta_i)\alpha_1 + (1-\eta_i)\alpha_2 + (v_1^1 + v_2^2)\beta_R}{2\beta_R}
\]  
(A-5)

\[
Y_6 = \frac{(1-\eta_R)(1-\eta_i)\alpha_1 + (1-\eta_i)\alpha_2 + (v_1^2 + v_2^2)\beta_R}{2\beta_R}
\]  
(A-6)

\[
Y_7 = \frac{\beta_1}{2\beta_R}
\]  
(A-7)

\[
Y_8 = 2\beta_2 \beta_R
\]  
(A-8)

\[
Y_9 = 2\beta_R Y_4
\]  
(A-9)

\[
Y_{10} = \frac{Y_8}{2\beta_R}
\]  
(A-10)
\[ Y_{11} = \eta_1 \alpha_1 + Y_1 \beta_R + \gamma_1 s^1_1 - \gamma_R s^1_R + (v^1_1 - 2k^1_1)(-\beta_1 + \beta_R Y) \quad \text{(A-11)} \]
\[ Y_{12} = 2(\beta_1 - \beta_R Y) \quad \text{(A-12)} \]
\[ Y_{13} = \eta_2 \alpha_2 + Y_1 \beta_R + \gamma_2 s^1_2 - \gamma_R s^1_R + (v^2_1 - 2k^2_1)(-\beta_1 + \beta_R Y) + (v^2_1 - k^2_1) \quad \text{(A-13)} \]
\[ Y_{14} = \eta_2 \alpha_2 + \gamma_2 s^2_2 - \gamma_R s^2_R + \left(\frac{\beta_2}{2}\right)(v^2_2 - 2k^2_2) \quad \text{(A-14)} \]
\[ Y_{15} = \eta_2 \alpha_2 + \beta_R Y_4 + \gamma_2 s^2_2 - \gamma_R s^2_R - \left(\frac{\beta_2}{2}\right)(v^2_2 - 2k^2_2) \quad \text{(A-15)} \]

**Appendix (B). Proofing the concavity of the real retailer's profit**

For proofing the real retailer’s profit’s concavity, the Hessian matrix firstly should be formed, as follows.

\[
H = \begin{bmatrix}
\frac{\partial^2 \psi_R}{\partial p_{R}^{11}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{12}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{21}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} \\
\frac{\partial^2 \psi_R}{\partial p_{R}^{21}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{12}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{21}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} \\
\frac{\partial^2 \psi_R}{\partial p_{R}^{21}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{12}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} \\
\frac{\partial^2 \psi_R}{\partial p_{R}^{22}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} & \frac{\partial^2 \psi_R}{\partial p_{R}^{22}} \\
\end{bmatrix}
\]

(B-1)

Where,

\[
\frac{\partial^3 \psi_R}{\partial p_{R}^{11}} = -2\beta_R t_1 \quad \text{(B-2)}
\]
\[
\frac{\partial^3 \psi_R}{\partial p_{R}^{12}} = \frac{\partial^3 \psi_R}{\partial p_{R}^{21}} = \frac{\partial^3 \psi_R}{\partial p_{R}^{22}} = 0 \quad \text{(B-3)}
\]
\[
\frac{\partial^3 \psi_R}{\partial p_{R}^{21}} = \frac{\partial^3 \psi_R}{\partial p_{R}^{11}} = -2\beta_2 f_1 \quad \text{(B-4)}
\]
\[
\frac{\partial^3 \psi_R}{\partial p_{R}^{12}} = \frac{\partial^3 \psi_R}{\partial p_{R}^{22}} = 0 \quad \text{(B-5)}
\]
\[
\frac{\partial^3 \psi_R}{\partial p_{R}^{22}} = -2\beta_R (T - t_1) \quad \text{(B-6)}
\]
\[
\frac{\partial^3 \psi_R}{\partial p_{R}^{22}} = \frac{\partial^3 \psi_R}{\partial p_{R}^{22}} = 0 \quad \text{(B-7)}
\]
\[
\frac{\partial^2 \psi_R}{\partial p_R^{12} \partial p_R^{22}} = \frac{\partial^2 \psi_R}{\partial p_R^{31} \partial p_R^{22}} = -2\beta_r(T - t_1) \quad \text{(B-8)}
\]

\[
\frac{\partial^2 \psi_R}{\partial p_R^{312}} = -2\beta_r t_1 \quad \text{(B-9)}
\]

\[
\frac{\partial^2 \psi_R}{\partial p_R^{31} \partial p_R^{22}} = \frac{\partial^2 \psi_R}{\partial p_R^{23} \partial p_R^{21}} = 0 \quad \text{(B-10)}
\]

\[
\frac{\partial^2 \psi_R}{\partial p_R^{223}} = -\beta_r (T - t_1) \quad \text{(B-11)}
\]

Then, it is found that \( \pi_R \) is concave if conditions (B-12)-(B15) are met.

\[
|H_1| = \left| \frac{\partial^2 \psi_R}{\partial p_R^{112}} \right| = -2\beta_r t_1 < 0 \quad \text{(B-12)}
\]

\[
|H_2| = \left| \begin{array}{cc}
-2\beta_r t_1 & 0 \\
0 & -2\beta_r (T - t_1) 
\end{array} \right| = 4\beta_r^2 t_1 (T - t_1) > 0 \quad \text{(B-13)}
\]

\[
|H_3| = \left| \begin{array}{ccc}
-2\beta_r t_1 & 0 & -2\beta_r t_1 \\
0 & -2\beta_r (T - t_1) & 0 \\
-2\beta_r t_1 & 0 & -2\beta_r (T - t_1) 
\end{array} \right| = 8(\beta_r^2 \beta_r - \beta_r^3) t_1^2 (T - t_1) < 0
\]

\[
|H_4| = \left| \begin{array}{cccc}
-2\beta_r t_1 & 0 & -2\beta_r t_1 & 0 \\
0 & -2\beta_r (T - t_1) & 0 & -2\beta_r (T - t_1) \\
-2\beta_r t_1 & 0 & -2\beta_r t_1 & 0 \\
0 & -2\beta_r (T - t_1) & 0 & -2\beta_r (T - t_1) 
\end{array} \right| = 16(\beta_r^4 + \beta_r^2 \beta_r^3) t_1^2 (T - t_1)^2 > 0 \quad \text{(B-15)}
\]

**Appendix (C). Proofing the concavity of the airline’s profit**

For proofing the first supplier’s profit concavity, the Hessian matrix firstly should be formed, as follows.

\[
H = \begin{bmatrix}
\frac{\partial^2 \psi_1}{\partial p_1^{12}} & \frac{\partial^2 \psi_1}{\partial p_1^{13}} \\
\frac{\partial^2 \psi_1}{\partial p_1^{23}} & \frac{\partial^2 \psi_1}{\partial p_1^{21}} \\
\frac{\partial^2 \psi_1}{\partial p_1^{22}} & \frac{\partial^2 \psi_1}{\partial p_1^{31}} \\
\frac{\partial^2 \psi_1}{\partial p_1^{21}} & \frac{\partial^2 \psi_1}{\partial p_1^{32}}
\end{bmatrix} \quad \text{(C-1)}
\]

Where,

\[
\frac{\partial^2 \psi_1}{\partial p_1^{12}} = -2\beta_1 t_1 \quad \text{(C-2)}
\]
\[
\frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_2^2} = \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^2} = 0 \quad \text{(C-3)}
\]

\[
\frac{\partial^2 \psi_A}{\partial p_1^2} = -2\beta_1 (T - t_1) < 0 \quad \text{(C-4)}
\]

Hence, it is found that \( \psi_1 \) is concave if the conditions (C-5) and (C-6) are met.

\[
|H|_1 = \frac{\partial^2 \psi_1}{\partial p_1^2} = -2\beta_1 t_1 < 0 \quad \text{(C-5)}
\]

\[
|H|_2 = \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^2} \cdot \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^2} - \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^2} \cdot \frac{\partial^2 \psi_1}{\partial p_1^2 \partial p_1^2} = 4\beta_1^2 t_1 (T - t_1) > 0 \quad \text{(C-6)}
\]

**Appendix (D). Proofing the concavity of the second supplier's profit**

For proofing the second supplier’s profit concavity, the Hessian matrix firstly should be formed, as follows.

\[
H = \begin{bmatrix}
\frac{\partial^2 \psi_2}{\partial p_1^2} & \frac{\partial^2 \psi_2}{\partial p_1^2} \\
\frac{\partial^2 \psi_2}{\partial p_2^2} & \frac{\partial^2 \psi_2}{\partial p_2^2} \\
\frac{\partial^2 \psi_2}{\partial p_2^2} & \frac{\partial^2 \psi_2}{\partial p_2^2} \\
\end{bmatrix} \quad \text{(D-1)}
\]

Where,

\[
\frac{\partial^2 \psi_2}{\partial p_2^2} = -2\beta_2 t_1 \quad \text{(D-2)}
\]

\[
\frac{\partial^2 \psi_2}{\partial p_1^2 \partial p_1^2} = \frac{\partial^2 \psi_2}{\partial p_1^2 \partial p_2^2} = 0 \quad \text{(D-3)}
\]

\[
\frac{\partial^2 \psi_2}{\partial p_2^2} = -2\beta_2 (T - t_1) < 0 \quad \text{(D-4)}
\]

Hence, it is found that \( \pi_2 \) is concave if conditions (D-5) and (D-6) are met.

\[
|H|_1 = \frac{\partial^2 \psi_2}{\partial p_2^2} = -2\beta_2 t_1 < 0 \quad \text{(D-5)}
\]

\[
|H|_2 = \frac{\partial^2 \psi_2}{\partial p_2^2} \cdot \frac{\partial^2 \psi_2}{\partial p_2^2} - \frac{\partial^2 \psi_2}{\partial p_2^2 \partial p_2^2} \cdot \frac{\partial^2 \psi_2}{\partial p_2^2 \partial p_2^2} = 4\beta_2^2 t_1 (T - t_1) > 0 \quad \text{(D-6)}
\]
Figure 1. Structure of supply chains

Figure 2. The first supplier's customers' price sensitivity changes versus the firms' profits
Figure 3. The second supplier's customers' price sensitivity changes versus the firms' profits

Figure 4. The real shopping center's customers' price sensitivity changes versus the firms' profits
**Figure 5.** The booking period changes versus the firms' profits

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1^1(t)$</td>
<td>The booking level of the first supplier within $[0, t_1]$;</td>
</tr>
<tr>
<td>$q_1^2(t)$</td>
<td>The booking level of the first supplier within $[t_1, T]$;</td>
</tr>
<tr>
<td>$q_2^1(t)$</td>
<td>The booking level of the second supplier within $[0, t_1]$;</td>
</tr>
<tr>
<td>$q_2^2(t)$</td>
<td>The booking level of the second supplier within $[t_1, T]$;</td>
</tr>
<tr>
<td>$q_{11}^1(t)$</td>
<td>The booking level of a first item in real store within $[0, t_1]$;</td>
</tr>
<tr>
<td>$q_{12}^1(t)$</td>
<td>The booking level of a first item in real store within $[t_1, T]$;</td>
</tr>
<tr>
<td>$q_{11}^2(t)$</td>
<td>The booking level of a second item in real store within $[0, t_1]$;</td>
</tr>
<tr>
<td>$q_{12}^2(t)$</td>
<td>The booking level of a second item in real store within $[t_1, T]$;</td>
</tr>
<tr>
<td>$d_1^1(p_1^1)$</td>
<td>The demand rate of customers from the first supplier at the selling price $p_1^1$ within $[0, t_1]$, per unit time;</td>
</tr>
<tr>
<td>$d_1^2(p_1^2)$</td>
<td>The demand rate of customers from the first supplier at the selling price $p_1^2$ within $[t_1, T]$, per unit time;</td>
</tr>
<tr>
<td>$d_2^1(p_2^1)$</td>
<td>The demand rate of customers from the second supplier at the selling price $p_2^1$</td>
</tr>
</tbody>
</table>
within \([0,t_1]\), per unit time;

\(d_2^2(p_2^2)\): The demand rate of customers from the second supplier at the selling price \(p_2^2\) within \([t_1,T]\), per unit time;

\(d_{R1}^{11}(p_{R1}^{11})\): The demand rate of customers from the real store for a first item at the selling price \(p_{R1}^{11}\) within \([0,t_1]\), per unit time;

\(d_{R1}^{12}(p_{R1}^{12})\): The demand rate of customers from the real store for a first item at the selling price \(p_{R1}^{12}\) within \([t_1,T]\), per unit time;

\(d_{R2}^{21}(p_{R2}^{21})\): The demand rate of customers from the real store for a second item at the selling price \(p_{R2}^{21}\) within \([0,t_1]\), per unit time;

\(d_{R2}^{22}(p_{R2}^{22})\): The demand rate of customers from the real store for a second item at the selling price \(p_{R2}^{22}\) within \([t_1,T]\), per unit time;

\(v_1^1\): The wholesale price of the first supplier for a first item within \([0,t_1]\), per item;

\(v_1^2\): The wholesale price of the first supplier for a first item within \([t_1,T]\), per item;

\(v_2^1\): The wholesale price of the second supplier for a second item within \([0,t_1]\), per item;

\(v_2^2\): The wholesale price of the second supplier for a second item within \([t_1,T]\), per item;

\(k_1^1\): The unit cost of the first supplier for a first item within \([0,t_1]\);

\(k_1^2\): The unit cost of the first supplier for a first item within \([t_1,T]\);

\(k_2^1\): The unit cost of the second supplier for a second item within \([0,t_1]\);

\(k_2^2\): The unit cost of the second supplier for a second item within \([t_1,T]\);

\(t_1\): The time of price-change;

\(T\): The length of pre-selling period;

\(\psi_1\): The total profit of the first supplier per unit time;

\(\psi_2\): The total profit of the second supplier per unit time;

\(\psi_R\): The total profit of the real store per unit time;

**Decision variables**

\(p_1^1\): The selling price of the first supplier within \([0,t_1]\), per item per unit time;

\(p_1^2\): The selling price of the first supplier within \([t_1,T]\), per item per unit time;
<table>
<thead>
<tr>
<th>Superscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The selling price of the second supplier within ([0,t_1]), per item per unit time;</td>
</tr>
<tr>
<td>2</td>
<td>The selling price of the second supplier within ([t_1,T]), per item per unit time;</td>
</tr>
<tr>
<td>11</td>
<td>The selling price of the real store for a first item within ([0,t_1]), per item per unit time;</td>
</tr>
<tr>
<td>12</td>
<td>The selling price of the real store for a first item within ([t_1,T]), per item per unit time;</td>
</tr>
<tr>
<td>21</td>
<td>The selling price of the real store for a second item within ([0,t_1]), per item per unit time;</td>
</tr>
<tr>
<td>22</td>
<td>The selling price of the real store for a second item within ([t_1,T]), per item per unit time;</td>
</tr>
</tbody>
</table>
Table 2. Results of example for \( \alpha = 0.5 \)

<table>
<thead>
<tr>
<th>Members</th>
<th>( p_1^1 )</th>
<th>( p_1^2 )</th>
<th>( \psi_1 )</th>
<th>( p_2^1 )</th>
<th>( p_2^2 )</th>
<th>( \psi_2 )</th>
<th>( p_R^1 )</th>
<th>( p_R^{12} )</th>
<th>( p_R^{21} )</th>
<th>( \psi_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Supplier</td>
<td>374.13</td>
<td>377</td>
<td><strong>2658662.86</strong></td>
<td>631.38</td>
<td>634.40</td>
<td><strong>8110090.40</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Second Supplier</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td><strong>634.40</strong></td>
<td><strong>8083506</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Real Shopping Center</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td><strong>548.42</strong></td>
<td><strong>730.27</strong></td>
<td><strong>100544.59</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Results of example for \( \alpha = 0 \)

<table>
<thead>
<tr>
<th>Partners</th>
<th>( p_1^1 )</th>
<th>( p_1^2 )</th>
<th>( \psi_1 )</th>
<th>( p_2^1 )</th>
<th>( p_2^2 )</th>
<th>( \psi_2 )</th>
<th>( p_R^1 )</th>
<th>( p_R^{12} )</th>
<th>( p_R^{21} )</th>
<th>( \psi_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Supplier</td>
<td>--</td>
<td>377</td>
<td><strong>2649773.95</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>Second Supplier</td>
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<td>-</td>
<td>-</td>
<td><strong>634.40</strong></td>
<td><strong>8083506</strong></td>
<td>-</td>
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<tr>
<td>Real Shopping Center</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td><strong>548.42</strong></td>
<td><strong>730.27</strong></td>
<td><strong>89447.64</strong></td>
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</tr>
</tbody>
</table>

Table 4. The effect of first supplier’s price sensitivity of demand changes on the optimal values of the variables and profits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( % \text{ changes} )</th>
<th>( p_1^1 )</th>
<th>( p_1^2 )</th>
<th>( \psi_1 )</th>
<th>( p_2^1 )</th>
<th>( p_2^2 )</th>
<th>( \psi_2 )</th>
<th>( p_R^1 )</th>
<th>( p_R^{12} )</th>
<th>( p_R^{21} )</th>
<th>( \psi_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>+0.75</td>
<td>224.07</td>
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<td>619.75</td>
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<td>560.23</td>
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<td>722.39</td>
</tr>
<tr>
<td></td>
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<td>261.32</td>
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<td>624.64</td>
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<td>556.29</td>
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<td>725.02</td>
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<td>307.59</td>
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<tr>
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<td>377</td>
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<td>631.38</td>
<td>634.40</td>
<td>8110090.40</td>
<td>544.42</td>
<td>548.42</td>
<td>726.14</td>
<td>730.27</td>
</tr>
<tr>
<td></td>
<td>-0.25</td>
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Table 5. The effect of second supplier’s price sensitivity of demand changes on the optimal values of the variables and profits

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<tr>
<th>Parameter</th>
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<th>( p_1^1 )</th>
<th>( p_1^2 )</th>
<th>( \psi_1 )</th>
<th>( p_2^1 )</th>
<th>( p_2^2 )</th>
<th>( \psi_2 )</th>
<th>( p_R^1 )</th>
<th>( p_R^{12} )</th>
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<th>( \psi_R )</th>
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<td>Infeasible</td>
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<td>537</td>
<td>539.39</td>
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34
Table 6. The effect of real shopping center’s price sensitivity of demand changes on the optimal values of the variables and profits

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<th>$\psi_2$</th>
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<th>$p_4$</th>
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Table 7. The effect of pre-selling period changes on the optimal values of profits

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