Coordination of traditional and online group-buying channels considering website promotion effort

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Abstract: The development of modern technology and e-commerce have given rise to the emergence of many new selling channels. Among one of them, group-buying attracts numerous new customers rapidly due to the characters of deep discounts and great convenience. Although deep discounts create sales growth for sellers, it also causes the loss in their profit margins. Meanwhile, the business model of group-buying websites is not thoroughly understood in literature. Based on a Stackelberg game framework, this paper studies the equilibrium between a group-buying website and a seller. The optimal pricing and channel decisions of the seller and the optimal group-buying pricing and promotion effort decisions for the website are investigated to offer guidance for their businesses. We find that the total profit of the whole system could be hurt when the agreement price or revenue sharing contracts are adopted by the firms. We propose a revenue-cost sharing contract that could coordinate the total profit. Finally, we show how the scale of the seller and the website and the unit cost influence the optimal decisions in the equilibrium.

Key words: group-buying, dual channels, channel management, revenue coordination, promotion effort

1. Introduction

Many companies have been selling their products or services to their consumers through group-buying websites (GBWs) since the early 21st century. On the GBWs, consumers are encouraged to buy together whereby products or services are offered with price discounts. This practice becomes popular in a variety of businesses, ranging from catering to entertainment services. With price discounts, GBWs can attract a large number of consumers in a relatively short period of time. According to the research report by Dholakia (2012) [1], GBWs can help the seller attract close to 80% new consumers even after running group-buying activities several times. Thus, for a seller, a GBW may help boost its brand awareness, increase its sales, and expand its markets.

A GBW serves as an additional sales channel for sellers, with promising potential benefits, but in practice not all sellers benefit from the GBW channel. Dholakia (2011a) [2] conducted a survey over 324 sellers who use GBWs (e.g. Groupon, LivingSocial, Open Table, Travelzoo
and Buywithme). It is shown that only 55.5% of the sellers made profits, 26.6% suffered loss and 17.9% broke even. Interestingly, in spite of the popularity of group-buying activities and behaviors, GBWs are not always making money, either. A survey released in China shows that, the number of GBWs reached almost 6000 in 2011. However, this number dropped to 123 by the middle of the year 2014, with an average reduction rate of 5.9 websites per day. The international giant GBWs are also experiencing a tough time. In December 2015, Groupon, the world’s largest GBW, announced its exit from the Nordic market after months’ business falling. It is reported that its market value has shrunk by 86% till 2015.

In view of the potential benefits brought by the GBWs, why do the sellers suffer from losses? Given the popularity of group-buying behaviors in consumer markets, why the GBWs fail to make money? Under what conditions, should a seller add the GBW channel, and if yes, by what contracts?

We study these research questions by the game theoretic approach. A seller provides products or services. It has a traditional offline channel, say retail stores, selling directly to end consumers. It can choose to add a GBW channel. By the GBW channel, the seller offers its products or services to GBW according to a certain contract, and GBW further sells them to end consumers at a discounted price (a price lower than the traditional channel). The GBW channel has two effects on the seller’s market demand. On one hand, the GBW can bring new consumers to the seller, possibly through sales efforts. On the other hand, because of the discounted price, the GBW cannibalizes the traditional channel by attracting consumers in the traditional channel to the GBW channel. We consider four types of contracts, an agreement price contract, a two-part tariff contract, a revenue sharing contract and a revenue-cost sharing contract. For each type of contract, we fully characterize the firms’ pricing and sales efforts decisions, and analyze the seller’s decision on the employment of the GBW channel.

The purpose of the study is two-folded. First, we try to generate insights into a seller’s strategic decision on whether or not to run the group-buying promotion (with costly efforts). Second, we propose a contracting mechanism between the seller and the GBW website firm such that the traditional channel and the group-buying channel are coordinated with the total profit reaches its maximum.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 introduces the model in the centralized scenario. Section 4 describes the model setting under agreement price contract. Section 5 study the conditions under which the seller and the website can be coordinated with two-part tariff contract, revenue sharing contract or a revenue-cost sharing contract. In Section 6, we extend our model to investigate the impact of product unit cost and seller’s offline consumer size. Concluding remarks are provided in Section 7. All the proofs are put into the appendix.

2. Literature review

Information technology has shown important and growing positive impact on business performance (Sahin and Topal 2018) [3]. Emerging technologies spawn myriad application that
have the potential to impact experienced market dramatically (Shugan 2004) [4] and group-buying, a dynamic consumer collective activity (Wang et al. 2013) [5], is one of such technologies. A large number of literature has explored the mechanism of group-buying and we category the literature most related to our paper into three dimensions: the profit mechanism of the sellers in group-buying, the profit model of the group-buying websites and the interplay between the websites and the sellers.

Group-buying takes advantage of group cohesion to benefit both consumers and participating sellers, as it can significantly increase sales volumes. Early research on group-buying deems it as an online auction activity, and focuses on the dynamic pricing mechanisms regarding the quantity and timing of it (e.g., Chen et al. 2002 [6], Anand and Aron 2003 [7], Chen et al. 2007 [8], Chen, Chen, Song et al. 2009 [9]). Such business model has been replaced by fixed-price group-buying business model nowadays.

Through two-year’s data collection from 2007 to 2009 in the context of urban China, Wang et al. (2013) [5] explore the value that group-buying creates for sellers, including the cost reduction and brand awareness building especially for middle-sized and less-known sellers. Dholakia (2011a [2], 2011b [10]) holds the same opinion that group-buying websites like Groupon can be an effective marketing tool for startup businesses by studying 150 small businesses that completed promotions on Groupon between June 2009 and August 2010. Heo (2016) [11] discusses how restaurants can utilize group-buying as a tool for revenue management. Wu and Zhu (2017) [12] investigate the quality decision for the seller to align with its group-buying strategy when consumers’ substitution effect is taken into consideration.

Group-buying, also known as voucher discounting, in which the sellers sell the vouchers through the group-buying websites and consumers will buy these vouchers and redeem them offline. Edelman et al. (2016) [13] investigate the profitability, pricing discrimination and advertising effect of discount vouchers. Gao and Chen (2015) [14] find that no show of voucher buyers may not be good for (large or start-up) sellers. Ni, Xu, Xu et al. (2015) [15] study the optimal pricing strategy of sellers by classifying customers into collectivist and individualistic customers according to actual market information. Taleizadeh et al. (2015) [16] investigate the joint multi discount pricing and ordering problem when demand for deteriorating product change with time.

Though the group-buying business model provides sellers with a new channel to sell their products, it is doubtful whether it will be profitable for the sellers in the long term due to the extremely deep discounts. With the deep discounts offered to consumers and payouts transferred to the websites (which can range from 20 to 50% of the revenue), the seller running the group-buying promotion can often be left with insufficient revenues to cover its costs (Dholakia 2011b) [10]. Hence, it is critical for us to explore the benefit when sellers adopting group-buying channel and to find how to guarantee the sellers’ profits.

For the extant research, the focus is primarily on the sellers’ strategies during the group-buying transaction, rather than the websites’ strategies. However, as the platform for
implementing group-buying, the corresponding websites should get more attention. The mechanism of group-buying website experienced the evolution from dynamic pricing model (e.g., Chen et al. 2002 [6], Anand and Aron 2003 [7], Chen et al. 2007 [8]) to fixed-pricing model (e.g., Zhao et al. 2016 [17], Gao and Chen 2015 [14], Ni, Xu, Xu et al. 2015 [15]). The dynamic model focuses on the quantity and pricing scheme, that is greater the quantity, lower the price. But the business model is too complex for the websites to operate, so it has gradually ceased operation (Kauffman and Wang 2002) [18]. In 2008, Groupon, originated in Chicago, introduces deal-of-the-day business model, which is the original form of fixed-pricing model (Ni, Xu, Xu et al. 2015) [15]. Groupon achieves great success, from the second quarter of 2009 to the first quarter of 2011, it has a revenue growth from $3.3 million to $644 million. The market expanded from 5 North American markets to 175 markets in 43 countries. The number of registered users increased from 152 thousand to 83 million and the number of sellers operating on its platform increased from 212 to 56781. In a short time, many sites began to clone Groupon’s business model. By the end of June 2011, China became one of the most competitive group-buying markets in the world (Zhao 2013) [19].

The worldwide rapid expansion of group-buying websites promotes the research for the profit mechanism of group-buying websites. Commission, sometimes called royalty fee that the sellers pay to the websites, is the major revenue source for the website (Zhao et al. 2016 [17], Edelman et al. 2016 [13], Gao and Chen 2015 [14]). Zhao et al. (2016) [17] find that commission charged by the website has a deep influence on sellers’ pricing strategies. When the commission is endogenous, it shows promotional effect for advertising sellers’ service quality. Edelman et al. (2016) [13] promote a theoretical decision framework for sellers with dual-channels, i.e. the online and offline channels, that is how to make the group-buying decision and set the optimal group-buying price when the deal quantity maximizing. Unlike other literature considering the cost of operating a group-buying website (Ni, Xu, Xu et al. 2015) [15], we ignore the cost of running group-buying business in our basic model. Otherwise, we take the effort cost of running group-buying business into consideration and assume that the website’s promotion effort level has a linear positive effect on the online demand. This kind of effort cost is quite common in dual-channel supply chains (Pu et al. 2017) [20].

After several years of rapid development, group-buying websites face a not so promising future. Take the development of group-buying websites in China for example, the number of the websites has dropped from nearly 6000 in 2011 to 123 in June 2014. Therefore, we should also attach great importance on the survival of group-buying websites.

Beyond the work on the separate strategies of the sellers and the websites, a branch of work has explored the interplay between the seller and the website, among which the cooperation between sellers and the websites are the most extensively studied. As mentioned above, commission is the traditional way that the sellers and the websites use to share total revenue.
Zhao et al. (2016) [17] consider a combination of the transaction-based commission and a fixed payment (a two-part tariff). They provide an effective way to balance the revenue between the seller and the website when there is a deep discount. Similarly, Tran and Desiraju (2017) [21] explore the channel coordination under asymmetric information of manufacturer and the retailer that can provide group-buying. They assume that besides the wholesale price, the retailer will charge a fixed fee for all the products. They find that when the retailer is more informed about the market size than about the level of consumer price sensitivity, the manufacturer will benefit more from group-buying. In our paper, we also suppose an agreement price combined with a fixed fee when exploring the coordination between the seller and the website in different market scenarios.

Besides of the transaction-based commission, more economic methodologies are used to research the interplay between the sellers and the websites. Based on the framework of Stackelberg game, Ni, Xu, Xu et al. (2015) [22] explore the optimal strategies of the seller and the website when they lead the market respectively. Similar to this, we also utilize the Stackelberg game model to simulate different marketing situations. The difference is we focus on the market structure, like the offline and online consumer size and product cost, nevertheless, they emphasize consumers’ behavior in group-buying, that is the sensibility for the cost to look for people to join group-buying together. Bhardwaj and Sajeesh (2017) [23] consider the bargaining power of sellers and the website, and find that when the two retailers competing with other, the website prefer to cooperate with one of them rather than both. Subramanian and Ram (2016) [24] use a traditional revenue sharing model to distribute the total group-buying revenue. They show that the cannibalization can be transformed into an advantage by displaying deal sales.

When sellers decide to sell products or services through the group-buying channel, they will not abandon their offline channel, which means they will sell their products/services through the dual channel. Cao et al. (2016) [25] focus on a retailer that sells through multiple distribution channels, and study how an “online-to-store” channel impact the retailer’s demand allocations and profitability. Chen, Liang, Yao et al. (2016) [26] investigates price and quality decisions in dual-channel supply chains, and they find the supply chain performance could be improved due to a new channel augmented. Zhang et al. (2017) [27] investigate the retailer’s decisions on channel structure and find that the optimal decision depends on customer acceptance rate for the online channel.

Our paper indicates that when deciding to sell through group-buying website, the seller will make its decision according to the website’s platform consumer size. And it’s the same for the website, that is to say they will decide whether to cooperate with each other according to the opposite side’s size. Our work importantly differs from its antecedents in that the prior literature, including the articles discussed above, has considered primarily the website’s and the seller’s respective optimal decision. In particular, the previous work on group-buying does not consider
what influence the website’s size for seller’s optimal marketing decision, which is a primary considering factor in our paper.

3. Model setting

3.1 The seller, the GBW, and the market

A seller provides its product or service to consumers, at a constant unit production cost $c$ ($c \geq 0$). Two selling channels are available for the seller. By the traditional offline channel, the seller directly sells to end consumers, say through its retail stores, at a regular price $p_o$. By the online group-buying channel, the seller promotes the product or service through an online group-buying website firm (i.e. a GBW) according to a certain contract; the GBW further sells the product or service to end consumers at a group-buying price $p_g$. The seller makes decisions on whether or not to sell through the GBW.

If the seller sells through the single traditional offline channel, we say it runs $O$ model. If the seller chooses to also sell through the GBW, we consider four types of contracts between the seller and the GBW: agreement price, two-part tariff, revenue sharing and revenue-cost sharing. Corresponding to the four types of contracts, we say the seller runs $O+GBW$ model, $O+GBW+T$ model, $O+GBW+R$ model and $O+GBW+RC$ model, respectively.

3.1.1 O model

Under the O model, the seller faces consumers on the spot market, with a potential size of $a$. The demand function is given by $q_o = a - p_o$. If the seller implements the O model, its profit is given by

$$\Pi_o = (p_o - c)(a - p_o),$$

leading to its optimal regular price $p_o^* = \frac{a + c}{2}$.

3.1.2 O+GBW model

Under the O+GBW model, the GBW will bring new consumers for the product or service, with a potential size $(b + \gamma)$, at a cost of $\frac{\gamma^2}{2}$. Here $b$ represents the number of the group-buying product hunters who can actively notice the product once it is advertised by, and available on, the GBW. In addition, the GBW firm can make sales efforts at cost $\frac{\gamma^2}{2}$ to attract $\gamma$ more new GBW buyers. For example, the GBW can analyze the purchasing histories and preferences of the consumers in its consumer-base to identify the targeted potential consumers for the product.

To eliminate the uninteresting cases, we assume, throughout the paper, that the parameter
\[ b \text{ is not too large. Mathematically, we assume } b < (a+c)/2. \text{ In fact, } b \text{ can be interpreted as the size of consumer-base for the product in the GBW channel. The parameter } b \text{ is usually related to the attractiveness of a GBW. If } b \text{ is too large, the GBW is too attractive and the demand from the traditional offline channel is negligible.}

Assume the seller’s regular price \( p_o^* \) is unchanged when adding the GBW channel. This assumption is consistent with practices. For example, Groupon states that the group-buying sellers’ advertised prices must be consistent with the price list of the local business who may be asked to show a written proof of the prices (Bhardwaj and Sajeesh 2017). Subramanian and Rao (2016) make similar assumption by arguing that the seller’s regular price will not be affected by a daily deal promotion that is of relatively short duration and offered infrequently.

Under the O+GBW model, the GBW cannibalizes the seller’s regular sales by transferring a fraction of spot-market consumers from the offline channel to the GBW online channel. The transfer rate is assumed to be \( 1 - \frac{p_g}{p_o} \), which relates to the ratio between the group-buying price and the regular price. Clearly, the lower the group-buying price \( p_g \), the more spot-market consumers transfer from the traditional offline channel to the GBW online channel. The market potential in the traditional offline channel becomes \( \frac{p_g}{p_o} a \) and that in the GBW channel is \( \left( 1 - \frac{p_g}{p_o} \right) a + b + \gamma \).

Under the O+GBW model, the seller and the GBW firm adopt a contract of an agreement price. It is the seller who sets the agreement price. With such a contract, the GBW pays an agreement price, \( p_s \) \( (c < p_s < p_g < p_o) \), to the seller for each deal sold online. The agreement price contract is similar to the wholesale price contract in the supply chain management literature. The O+GBW model is displayed in Fig.1.

If the seller implements the O+GBW model, its profit is given by

\[
\Pi_s = (p_s - c) \left[ \left( 1 - \frac{p_g}{p_o} \right) a + b + \gamma - p_g \right] + (p_o - c) \left( \frac{p_g}{p_o} a - p_o \right),
\]

where the first term is the profit obtained from selling to GBW, and the second term is the profit gained from selling to consumers through the traditional offline channel. The GBW’s profit is then given by

\[
\Pi_g = (p_g - p_s) \left[ \left( 1 - \frac{p_g}{p_o} \right) a + b + \gamma - p_g \right] - \frac{\gamma^2}{2}.
\]
From now on, we discuss a special case of \(a = 1 \) and \( c = 0 \), to simplify the analysis and obtain the key insights. In section 6, we extend our investigation to the impact of spot-market consumer size \( a \) and product (service) cost \( c \) on the equilibrium and optimal decisions for the seller and the GBW.

### 3.2 Benchmark: the centralized setting

Suppose the seller and the GBW firm are operated by a central planner. This centralized firm chooses which model to run, the O model or the O+GBW model. In this setting, the seller and the GBW do not need to sign any contract, because the two firms are viewed as a single firm to maximize the overall profits from both the offline and the GBW channels.

Under the O model, the centralized firm sells only through its traditional offline channel. Thus, the case is the same as described in section 3.1.1. The profit of the centralized firm (or the seller) can be written as \( \Pi_o = p_o (1 - p_o) \) (Recall that we now focus on the case with \( a = 1 \) and \( c = 0 \)). The optimal regular price \( p_o^* = \frac{1}{2} \), and the optimal profit is \( \Pi_o^* = \frac{1}{4} \).

Under the O+GBW model, the sales quantities in the GBW channel and in the traditional offline channel are, respectively,

\[
q^C_s = \begin{cases} 
1 - \frac{p^C_s}{p_o} + b + \gamma - p^C_s, & p^C_s < p^*_o \\
0, & p^C_s \geq p^*_o
\end{cases} \quad \text{and} \quad q^C_i = \begin{cases} 
\frac{p^C_i}{p_o} - p^*_o, & p^C_i < p^*_o \\
0, & p^C_i \geq p^*_o
\end{cases}
\]

where the superscript \( c \) denotes the centralized setting and \( p^*_o = \frac{1}{2} \) is the fixed offline regular price. Then, the maximization problem in the centralized firm can be described as:

\[
\begin{align*}
\max_{p^C_s, \gamma} \Pi^C &= p^C_s q^C_s + p^*_o q^C_i - \frac{\gamma^2}{2} \\
\text{s.t.} \quad &q^C_s \geq 0 \\
&q^C_i \geq 0
\end{align*}
\]

Solving the maximization problem, we obtain the following lemma.

**Lemma 1:** In the centralized setting, under the O+GBW model,

1. the optimal group-buying price and sales effort are \( p^C_s^* = \frac{2 + b}{5} \) and \( \gamma^C = \frac{2 + b}{5} \), respectively;

2. the optimal profit of the centralized firm is \( \Pi^C = \frac{3 + 8b + 12b^2}{20} \).

The consequent sales quantities through the offline and GBW channels are \( q^C_s = \frac{3 + 4b}{10} \) and \( q^C_i = \frac{1 + 3b}{5} \) respectively. The discount of GBW, defined as the ratio of the group-buying
price to the regular price, is \( \beta_{C^*} = \frac{p_{g^*}}{p_o} = \frac{2(2+b)}{5} \).

When having a greater GBW consumer base (larger \( b \)), the centralized firm adjusts its two instruments. First, when \( b \) increases, the centralized firm increases its group-buying price \( p_{g^*} \) to raise its unit margin. But this hurts its sales in the GBW channel (note that \( q_{g}^C \) decreases in \( p_{g}^C \)). Second, the centralized firm has to make more effort to attract more new consumers to compensate the shrink of sales caused by a higher \( p_{g}^C \). This explains that both \( p_{g}^C \) and \( \gamma_{C^*} \) increases in \( b \).

**Proposition 1:** Suppose the seller and the GBW are centralized to a single firm. The centralized firm adopts the O+GBW model if \( b \geq \frac{\sqrt{10} - 2}{6} \) and adopts the O model otherwise.

If the seller and the GBW are owned by a single firm, there is no double marginalization problem. The upside of the GBW channel for the centralized firm is that it brings new potential consumers. The downside is that it shifts the spot-market consumers in the offline channel to the GBW channel, which sacrifices the unit profit margin. When the GBW channel is attractive (indicated by large \( b \)), the centralized firm chooses the O+GBW model; otherwise, it simply runs the O model to sell solely on the spot market with a high margin.

4 **Agreement Price Contract**

In this section, we derive the seller’s and the GBW’s optimal prices under the agreement price contract, and based on this, we discuss the system performances.

The sequence of events is as follows (depicted in Fig.2.): (a) the seller decides whether to add the GBW channel or not; (b) if the GBW channel is added, then the seller announces its agreement price \( p_{s}^{A} \) to the GBW, and the GBW decides whether to accept it or not; (c) if the agreement price contract is agreed, the GBW decides the group-buying price \( p_{g}^{A} \) and the sales effort \( \gamma^{A} \); the demands and the profits are then realized. In the seller-leader case, the GBW’s decisions depends on the seller’s.

4.1 **Optimal decisions under the O model**

Under the O model, the seller’s optimal regular price and profit are the same as those in section 3.2. That is, \( p_{o}^{*} = \frac{1}{2} \) and \( \Pi_{o}^{*} = \frac{1}{4} \).

4.2 **Optimal decisions under the O+GBW model**

Under the O+GBW model, the sales quantities through the online GBW channel and the traditional offline channel are, respectively,
\[ q_s^A = \begin{cases} 1 - \frac{p_s^A}{p_o^*} - p_s^A + b + \gamma^A, & p_s^A < p_o^*, \\ 0, & p_s^A \geq p_o^* \end{cases} \quad \text{and} \quad q_s^A = \begin{cases} \frac{p_g^A}{p_o^*} - p_o^*, & p_g^A < p_o^*, \\ 0, & p_g^A \geq p_o^* \end{cases} \]

where the superscript \( A \) denotes the Agreement Price Contract scenario and \( p_o^* = \frac{1}{2} \) is the fixed offline regular price. We can formulate firms’ maximization problems in Agreement Price Contract scenario under O+GBW as follows:

\[
\begin{aligned}
\max_{p_s^A, p_g^A, q_s^A, q_g^A} & \Pi_s^A = p_s^A \cdot q_s^A + p_g^A \cdot q_g^A \\
\max_{p_s^A, p_g^A, q_s^A, q_g^A} & \Pi_g^A = (p_s^A - p_g^A) \cdot q_g^A - \frac{(\gamma^A)^2}{2} \\
\text{s.t.} & \quad q_s^A \geq 0 \\
& \quad q_g^A \geq 0.
\end{aligned}
\] (4)

This is a dynamic game. We use a backward induction to obtain firms’ optimal decisions on prices and sales efforts, summarized in the following lemma.

**Lemma 2:** Suppose the seller and the GBW implementing the O+GBW model under the Agreement Price Contract.

1. The optimal agreement price for the seller is \( p_s^{A*} = \frac{3b + 5}{18} \); The optimal group-buying price and effort for the GBW are \( p_g^{A*} = \frac{12b + 14}{45} \) and \( \gamma^{A*} = \frac{3b + 1}{30} \), respectively.

2. The seller’s optimal profit is \( \Pi_s^{A*} = \frac{9b^2 + 66b + 16}{180} \); The GBW’s optimal profit is \( \Pi_g^{A*} = \frac{(3b + 1)^2}{360} \); The total profit of two firms is \( \Pi^{A*} = \Pi_s^{A*} + \Pi_g^{A*} = \frac{33 + 138b + 27b^2}{360} \).

The seller’s sales quantity through the GBW channel and the traditional offline channel are \( q_s^{A*} = \frac{3b + 1}{10} \) and \( q_g^{A*} = \frac{48b + 11}{90} \), respectively. The discount on the GBW is \( \beta^{A*} = \frac{p_g^{A*}}{p_o^*} = \frac{2(12b + 14)}{45} \).

4.3 Whether or not to add the GBW channel

By comparing the seller’s profits under the O model and under the O+GBW model, we obtain the following proposition on the seller’s channel model choice.

**Proposition 2:** Under the Agreement Price Contract scenario, the seller adopts the O+GBW model if \( b \geq \frac{5\sqrt{6} - 11}{3} \) and adopts the O model otherwise.
The seller adds the GBW channel if and only if the GBW firm can attract sufficiently more new consumers (i.e., large $b$). This is the same as the centralized firm, shown in Proposition 1.

5 The pricing coordination of the seller and the group-buying website

5.1 The pricing coordination under the O+GBW+T model

In this part, the relationship between the seller and the GBW is governed by a two-part tariff (denoted by T) contract, denoted $(p_s^T, F)$, whose terms include an agreement price $p_s^T$ and a fixed fee $F$. Suppose that the website pays the seller a fixed payment and an agreement price for each transaction sold on the website if the seller decides to sell through the online channel.

5.1.1 Optimal decisions under the O+GBW+T model

The sequence of events is displayed in Fig. 3: (a) The seller decides whether to add the GBW channel or not; (b) If yes, the seller announces its agreement price $p_s^T$ and fixed payment $F$; (c) The GBW decides whether to accept the contract or not, and if yes, decides its group-buying price $p_g^T$ and the effort $\gamma^T$. The demands and the profits are then realized.

The group-buying and offline channel demands are

$$q_g^T = \begin{cases} 1 - \frac{p_s^T}{p_o} + b + \gamma^T - p_g^T, & p_g^T < p_o, \\ 0, & p_g^T \geq p_o \end{cases}$$

and

$$q_s^T = \begin{cases} \frac{p_s^T - a - p_o}{p_o}, & p_g^T < p_o, \\ 0, & p_g^T \geq p_o \end{cases}$$

We formulate the optimization problem as below

$$\max_{p_s^T, F} \Pi_s^T = p_s^T q_s^T + p_o q_o^T + F$$

subject to

$$q_s^T \geq 0$$

We can solve the optimal agreement price and
fixed payment regarding to seller’s profit, we get $p_s^* = \frac{3b+5}{18}$. We summarize all the results below in Lemma 3.

**Lemma 3:** When implementing $O+GBW+T$ model

(1) The optimal agreement price and fixed payment for the seller are $p_s^* = \frac{3b+5}{18}$ and

$$F^* = \frac{1+6b+9b^2}{360},$$

respectively; The optimal group-buying price and effort for the GBW are

$$p_g^* = \frac{14+12b}{45} \text{ and } \gamma^g = \frac{1+3b}{30},$$

respectively;

(2) The seller’s optimal profit is $\Pi_s^* = \frac{11+46b+9b^2}{120}$, the GBW’s optimal profit is $\Pi_g^* = 0$ and the optimal total profit of two firms is $\Pi^* = \frac{11+46b+9b^2}{120}$.

The seller’s sales quantity through the GBW channel and the traditional offline channel are

$$q_g^* = \frac{1+3b}{10} \text{ and } q_s^* = \frac{11+48b}{90},$$

respectively. The discount on GBW is $\beta^* = \frac{4(7+6b)}{45}$.

5.1.2 Whether or not to add the GBW channel

By comparing the seller’s profits under the O model and under the O+GBW+T model, we obtain the following proposition on the seller’s channel model choice.

**Proposition 3:** The seller adopts the O+GBW+T model if $b \geq \frac{10\sqrt{7} - 23}{9}$ and adopts the $O$ model otherwise.

**Proposition 4:** Under $O+GBW+T$ model, the fixed payment is increasing with the website’s scale, $b$.

5.2 The pricing coordination under the $O+GBW+R$ model

5.2.1 Optimal decisions under the $O+GBW+R$ model

In this part, the seller and the GBW use the revenue sharing contract (denoted by $R$), that is $(p_s^R, l)$. The decision sequence is showed in Fig. 4.

Under the revenue sharing contract, the sales quantities in the GBW channel and in the traditional offline channel are, respectively,

$$q_g^R = \begin{cases} 
1 - \frac{p_g^R}{p_o} + b + \gamma^R, & p_g^R < p_o \text{ and } q_s^R = \begin{cases} 
\frac{p_g^R}{p_o}, & p_g^R < p_o, \\
0, & p_g^R \geq p_o
\end{cases} \\
0, & p_g^R \geq p_o
\end{cases}$$

We can formulate firms’ maximization problems in seller-leader case under revenue sharing contract as follows:
\[
\begin{cases}
\max_{p_s^R} \Pi_s^R = p_s^R q_s^R + p_s^R q_s^R \\
\max_{p_s^R, q_s^R} \Pi_g^R = (p_s^R - p_s^R) q_s^R - \frac{(\gamma^R)^2}{2}
\end{cases}
\tag{10}
\]

\[s.t. \quad q_s^R \geq 0 \tag{11}\]
\[q_i^R \geq 0 \tag{12}\]

We summarize all results in the following lemma.

**Lemma 4**: Under the O+GBW+R model,

1. The optimal agreement price and revenue sharing proportion for the seller are 
   \[p_i^{g^*} = 0 \quad \text{and} \quad l^* = 1\], respectively; The optimal group-buying price and effort for the GBW are 
   \[p_s^{g^*} = \frac{2 + b}{6} \quad \text{and} \quad \gamma^R = 0\], respectively;

2. The optimal profit for the seller is 
   \[\Pi_s^{g^*} = \frac{b^2 + 4b + 1}{12}\] and the optimal profit for GBW
   \[\Pi_g^{g^*} = 0\]; The total profit of two firms is 
   \[\Pi^{g^*} = \frac{b^2 + 4b + 1}{12}\].

The seller’s sales quantity through the GBW channel and the traditional offline channel are 
\[q_s^{g^*} = \frac{b}{2} \quad \text{and} \quad q_i^{g^*} = \frac{2b + 1}{6} \], respectively. The discount on the GBW is 
\[\beta^{g^*} = \frac{p_s^{g^*}}{p_o} = \frac{2 + b}{3}\].

### 5.2.2 Whether or not to add the GBW channel

By comparing the seller’s profits under the O model and under the O+GBW+R model, we obtain the following proposition on the seller’s channel model choice.

**Proposition 5**: The seller adopts the O+GBW+R model if \[b \geq \sqrt{6} - 2\] and adopts the O model otherwise.

The seller adds the GBW channel if and only if the GBW firm can attract sufficiently more new consumers (i.e., large \(b\)). This is the same as the centralized firm, the O+GBW+T model and the O+GBW+R model, shown in Propositions 1,2 and 5. However, a further comparison on the two thresholds of \(b\) in Propositions 1 and 3 reveals the following Corollary.

**Corollary 1**: Comparing to the centralized setting, the seller is less likely to add the GBW channel under the Agreement Price Contract, the Two-part Tariff Contract and the Revenue Sharing contract.

When the seller and the GBW are decentralized firms, the GBW sets a group-buying price lower than the centralized setting (i.e. \(p_s^{g^*} < p_s^{c^*}\), \(p_i^{g^*} < p_i^{c^*}\), \(p_s^{g^*} < p_s^{c^*}\)). The GBW firm in essence competes with the seller by attracting the spot-market consumers to the GBW online channel. In addition, the GBW firm makes less sales effort than the centralized firm (i.e.,
\[ \gamma^{A_{\ast}} < \gamma^{C_{\ast}}, \ \gamma^{T_{\ast}} < \gamma^{C_{\ast}}, \ \gamma^{R_{\ast}} < \gamma^{C_{\ast}} \]. Therefore, unless the GBW can bring enough new potential buyers and boost the seller’s total sales, the seller runs the O model.

**Proposition 6:** Both the two-part tariff and the revenue sharing contract cannot coordinate the profit of the seller and GBW.

When under two-part tariff contract or revenue sharing contract, the whole profit of the seller and the GBW is less than the whole profit under the centralized setting. Thus, both the contracts cannot coordinate the profit of the seller and GBW.

5.3 The pricing coordination under O+GBW+RC

In this part, we will try to coordinate the whole profit with the revenue-cost sharing contract (denoted by RC). The decision sequence is showed in Fig. 5. With the RC contract, the seller and the GBW share their revenue with the proportion of \( l \) and \( 1 - l \), respectively; besides, the seller compensates the GBW \( e'(\gamma^2/2) \) for the sales effort cost incurred in effort level \( \gamma \). Without loss of generality, let \( e = e'/2 \). Thus, the seller chooses parameter pair \((l, e)\). The sequence of events is as follows.

Under the revenue-cost sharing contract, the sales quantities through the online GBW channel and the traditional offline channel are, respectively,

\[ q^{RC}_g = \begin{cases} \frac{p^{RC}_g}{p_o} - p^{RC}_o & p^{RC}_g < p_o, \\ 0 & p^{RC}_g \geq p_o \end{cases} \]
\[ q^{RC}_s = \begin{cases} p^{RC}_g - p_o & p^{RC}_g < p_o, \\ 0 & p^{RC}_g \geq p_o \end{cases} \]

We can formulate firms’ maximization problems in seller-leader case under the revenue-cost sharing contract as follows:

\[
\begin{align*}
\max_{p^{RC}_g, l, e} \Pi^{RC}_{s} &= (l, p^{RC}_g + p^{RC}_s) q^{RC}_g + p_o q^{RC}_s - e(\gamma^{RC})^2 \\
\max_{p^{RC}_g, \gamma^{RC}} \Pi^{RC}_{g} &= \left[(1-l) p^{RC}_g - p^{RC}_s\right] q^{RC}_g - \frac{(\gamma^{RC})^2}{2} + e(\gamma^{RC})^2 \\
\text{s.t.} \quad q^{RC}_g \geq 0 \\
\quad \quad q^{RC}_s \geq 0
\end{align*}
\]

All the results are summarized in lemma 5.

**Lemma 5:** When taking the revenue-cost sharing contract,

(1) The seller’s optimal agreement price, sales compensation and revenue sharing proportion are \( p^{RC*}_s = 0, \ e^{*} = \frac{1}{2} \) and \( l^{*} = 1 \), respectively.; The GBW’s optimal group-buying price and effort are \( p^{RC*}_g = \frac{2+b}{5} \) and \( \gamma^{RC*} = \frac{2+b}{5} \).
The seller’s optimal profit is $\Pi_s^{RC^*} = \frac{3 + 8b + 2b^2}{20}$; The GBW’s optimal profit is $\Pi_g^{RC^*} = 0$; The whole profit of the seller and GBW is $\Pi^{RC^*} = \frac{3 + 8b + 2b^2}{20}$.

The seller’s sales quantity through the GBW channel and the traditional offline channel are $q_s^{RC^*} = \frac{1 + 3b}{5}$ and $q_s^{RC^*} = \frac{3 + 4b}{10}$, respectively. The discount on GBW is $\beta^{RC^*} = \frac{P_s^{RC^*}}{P_o} = \frac{2(2 + b)}{5}$.

Note that the profit of the seller and GBW can be coordinated under the revenue-cost sharing contract. Thus, the condition for the seller to sell through group-buying channel under the revenue-cost sharing contract is same as in benchmark setting.

Proposition 7: The seller adopts the O+GBW+RC model if $b \geq \frac{\sqrt{10} - 2}{6}$ and adopts the O model otherwise.

Thus, we can get that the profit of the seller and GBW can be coordinated under the revenue-cost sharing contract. Then, comparing optimal results of the seller and GBW under these three coordination contracts, we get proposition 8.

Proposition 8: Under the O+GBW+R model, the seller obtains the least profit and the whole profit is also the least; Under the O+GBW+A model, the GBW obtains higher profit than under other contracts; Under the O+GBW+RC model, the seller and the GBW obtains the largest total profit.

We summarize all the results in Table 1.

By analyzing the results under different scenarios. We get following corollaries.

Corollary 2: As the website scale $b$ increases, the GBW will make more effort to attract new consumers.

Corollary 3: The seller will choose the group-buying strategy when the website scale $b$ is sufficiently large. As $b$ increases, both the seller and the GBW will obtain more profits.

When the seller uses group-buying, the profits of the seller and the GBW under all settings increase with the scale of the website, $b$. This explains the reality that the sellers prefer to cooperate with larger group-buying websites.

6 Model Extensions

In this section, we will investigate how the size of experienced consumers ($a$) and the unit cost ($c$) influence the seller and website’s decisions.

In centralized setting, the group-buying and offline demands are
the problem can be reformulated as

$$
\max_{p^C_g} \Pi^C_i = \left( p^C_g - c \right) q^C_g + \left( p_o - c \right) q^C_i - \frac{\gamma^C}{2} \left( p^C_g - p_o \right)
$$

s.t. \( q^C_g \geq 0 \)\n
(16)

$$
q^C_i \geq 0
$$

(17)

Under the Agreement Price Contract, the group-buying and offline demands are

$$
q^A_g = \left(1 - \frac{p^A_g}{p_o} \right) a + b + \gamma^A - p^A_g, \quad p^A_g < p_o \quad \text{and} \quad q^A_i = \left( \frac{p^A_g}{p_o} - a - p_o \right) p^A_g < p_o
$$

s.t. \( q^A_g \geq 0 \)\n
(18)

The profit of the seller and GBW can be reformulated as

$$
\max_{p^A_g} \Pi^A_s = \left( p^A_g - c \right) q^A_g + \left( p_o - c \right) q^A_i - \frac{\gamma^A}{2} q^A_g - \left( \frac{\gamma^A}{2} \right)^2
$$

s.t. \( q^A_g \geq 0 \)

(19)

$$
q^A_i \geq 0
$$

(20)

Under the O+GBW+RC model, the group-buying and offline demands are

$$
q^{RC}_g = \left(1 - \frac{p^{RC}_g}{p_o} \right) a + b + \gamma^{RC} - p^{RC}_g, \quad p^{RC}_g < p_o \quad \text{and} \quad q^{RC}_i = \left( \frac{p^{RC}_g}{p_o} - a - p_o \right) p^{RC}_g < p_o
$$

s.t. \( q^{RC}_g \geq 0 \)

(22)

$$
q^{RC}_i \geq 0
$$

(23)

The profit of the seller and GBW can be reformulated as

$$
\max_{p^{RC}_g} \Pi^{RC}_s = \left(1 - p^{RC}_g + p^{RC}_s - c \right) q^{RC}_g + \left( p_o - c \right) q^{RC}_i - e \left( \gamma^{RC} \right)^2
$$

s.t. \( q^{RC}_g \geq 0 \)

(24)

Solving the profit maximization problems of the GBW and the seller under different scenarios, we can get all the optimal decisions on group-buying prices, agreement prices, promotion effort, etc. All the optimal results are listed in Table 2 and more details are presented
in the Appendix B.

6.1 The impact of the service/product cost for the seller and the GBW

Former study shows that product or service unit cost plays a key role in the group-buying strategy. Edelman et al. (2016) [13] points out that when providing a service or product with lower marginal cost, the group-buying website will benefit more. The statistics of Tuan800 (one of the largest group-buying websites in China) in June 2015 show that, catering and entertainment (like cinema tickets and amusement) are the top two group-buying categories, which account for 61.89% and 16.78% of the total sales, respectively. However, their average deal prices are less than 80 and 50 RMB, respectively. When talking about their development orientation, Wang Xing, the CEO of the largest group-buying website, emphasizes that “Meituan will insist on providing service group-buying rather product group-buying, that is because the low marginal cost service group-buying will be the most attractive in price and quality” (Wang 2012) [28]. Therefore, in this section we go further to investigate the impact of unit cost on the optimal decisions and profits for the seller and the website.

Besides, for all the results in Table 2, the unit cost satisfies the condition that $0 \leq c \leq a$.

In Table 2, we list the optimal results of the seller and the website under different marketing scenarios. Based on these results, we will analyze the impact of product cost and experienced offline consumers for the optimal strategies of the seller and the website.

**Proposition 9:** As the unit cost $c$ increases, the followings hold:

1. The GBW will set higher group-buying price and the seller will set higher agreement price, respectively;
2. $\beta$ decreases, which means the group-buying discount is larger;
3. Group-buying and offline demands both decrease;
4. The website’s and the seller’s profits decrease.

As the unit cost increases, the seller will set a higher offline regular price $\left( \frac{\partial p_o}{\partial c} > 0 \right)$ and charge the website a higher agreement price $\left( \frac{\partial p_s}{\partial c} > 0 \right)$. The website will choose a higher group-buying price $\left( \frac{\partial p_g}{\partial c} > 0 \right)$ accordingly, but it’s increase rate is much smaller comparing to the offline regular price $\left( \frac{\partial p_o}{\partial c} > \frac{\partial p_s}{\partial c} \right)$. Therefore, $\beta$ decreases, which means the group-buying discount is larger. Although, increased offline regular price will transfer more experienced consumers to the group-buying website, the size of new consumers that the website attracts with the group-buying price decreases. Overall, the group-buying demand decreases.
\[
\left( \frac{\partial q_c^C}{\partial c} < 0, \frac{\partial q_A^C}{\partial c} < 0 \right). 
\]
The offline demand will apparently decrease due to a higher transfer rate from the offline channel to the group-buying channel \( \left( \frac{\partial q_c^C}{\partial c} < 0, \frac{\partial q_A^C}{\partial c} < 0 \right) \). The seller’s and the website’s profits suffer from the higher unit cost and the corresponding shrunken demands from both channels.

As the seller and the website both tend to offer products or services with lower marginal cost, it’s more likely for a seller with relatively low unit cost to choose group-buying strategy. This explains the phenomenon that catering and entertainment group-buying products account for an overwhelming majority of group-buying market. In 2010, Tuan800 investigated the average price per group-buying transaction of the top 10 Chinese group-buying websites. The results show that group-buying products with a price lower than 50 RMB are more attractive.

6.2 The impact of the experienced consumer size for the seller and the GBW

The group-buying website prefers to consociate with sellers with larger size of experienced consumers, better consumer evaluation and longer operating history in local area since these sellers generally invest more on their brand maintenance and product or service promotion. They will not damage their brand for petty profits (Tuan 800, 2015) [29]. This section investigates how the size of experienced consumers affect the group-buying market and the decisions of both sellers and websites. Analyzing the optimal results in Table 2, we obtain the following results.

**Proposition 10:** As the size of experienced consumers \( a \) increases

(1) The seller will set higher agreement price and the GBW will set higher group-buying price.

(2) \( \beta \) decreases, which means the group-buying discount is deeper.

(3) The GBW will enhance the promotion effort \( \gamma \).

(4) The group-buying demand and the offline demand increases.

(5) Both the seller and the website will obtain higher profit.

With larger size of experienced consumers, the seller will set a higher offline regular price

\[
\left( \frac{\partial p_s}{\partial a} > 0 \right). 
\]
Other than that, the seller will charge the website a higher agreement price

\[
\left( \frac{\partial q_c^C}{\partial a} > 0 \right), 
\]
which induces the website to enhance group-buying price

\[
\left( \frac{\partial p_s^A}{\partial a} > 0, \frac{\partial p_s^C}{\partial a} > 0, \frac{\partial p_s}{\partial a} > 0 \right). 
\]
The increase rate of offline regular price is higher than that of
group-buying price \( \left( \frac{\partial p_g}{\partial a} > \frac{\partial p^{A,B,C}_g}{\partial a} \right) \), thus \( \beta \) decreases \( \left( \frac{\partial \beta^C}{\partial a} < 0, \frac{\partial \beta^A}{\partial a} < 0, \frac{\partial \beta^B}{\partial a} < 0 \right) \), which means the group-buying discount is deeper. When taking O+GBW model, more experienced consumers will transfer to the GBW channel and the website will attract more new consumers, thus group-buying demand increases \( \left( \frac{\partial q^A_g}{\partial a} > 0 \right) \). Since both the seller and the website’s profits increases with the size of experienced consumers, the website will tend to work with a seller with larger size of experienced consumers.

7 Numerical study

The primary objectives of this numerical study are three-fold. First, we would like to verify the results we have proven in the previous section. We find that the seller will choose group-buying strategy when the website scale \( b \) is sufficiently large. And as \( b \) increases, both the seller and the GBW will obtain more profits, which verifies and complements Proposition 1, 2, 3, 5 and 7. Second, we aim to compare the advantage and profitability of different contracts, the \( O+GBW \) model, \( O+GBW+T \) model, \( O+GBW+R \) model and \( O+GBW+RC \) model, with the centralized setting. Third, we desire to verify the impact of group-buying service/product cost and experienced consumer size to different aspects, like the agreement price, group-buying price, group-buying discount \( \beta \), group-buying effort \( \gamma \) and the profit of seller and group-buying website.

7.1 The impact of group-buying website scale \( b \)

We first start by presenting the results of the profit of seller and website with \( b \) changing under different contracts. As shown in Fig. 6, when \( b \) is larger than a threshold value, the seller will obtain higher profit than operating only offline channel. This further verifies Proposition 1, 2, 3, 5, and 7. Moreover, we find that both the seller and the website will obtain higher profit with \( b \) increasing. Thus, we also further verify Corollary 3.

7.2 The profitability comparison between contracts

In this subsection, to further understand the profitability of different contracts for both the seller and the website, we desire to present the seller and the website’s profits with different website scale \( b \).

As shown in Fig. 7(a), the seller can always get the higher profit under the centralized setting than taking agreement price contract, two-part tariff contract and revenue sharing contract, no matter how \( b \) changes. This further verifies Corollary 1. In addition, comparing seller’s profit under agreement price contract, two-part tariff contract, revenue sharing contract and revenue-cost sharing contract, we can find that under the O+GBW+R model, the seller obtains the least profit.

In Fig. 7(b), it obviously shows that under the O+GBW+A model, the GBW obtains higher profit than under other contracts. In Fig. 7(c), comparing the total profit of the seller and
website under O+GBW+T and O+GBW+R, we prove that both the two-part tariff and the revenue sharing contract cannot coordinate the profit of the seller and GBW, further verifying Proposition 6. Besides, we can also get that under the O+GBW+A model, the GBW obtains higher profit than under other contracts and under the O+GBW+RC model, the seller and the GBW obtains the largest total profit. Thus, the Proposition 8 is also verified.

7.3 The impact of GB service/product cost and experienced consumer size

In this part, we will conduct more numerical studies to further reveal how the major parameters affect the performance of seller and website under different contracts.

In Fig. 8, with $b$ increasing, the seller will set higher fixed payment, which is exactly showed in Proposition 4. In Fig. 9, it is obviously showing that the GBW will offer more effort with its scale increasing. Thus, this exactly explains Corollary 2.

Since we will respectively analyze the impact of GB service/product cost $c$ and experienced consumer size $a$, thus, we assume the $a=1$ and $b=1$ when focusing on the impact of $c$ and assume $c=0$ and $b=1$ when investigating the impact of $a$.

In Fig. 10(a), the GB price and agreement price are always increasing with $c$. In Fig. 10(b), $\beta$ is decreasing with $c$, which means the GB discount is getting larger. In Fig. 10(c)(d)(e), it obviously shows that the GB demand, offline demand and the profit of the seller and GBW are always decreasing. Thus, we verify Proposition 9 in Section 6.

8 Conclusion

In this paper, we consider a seller who can choose to sell through a GBW, in addition to its traditional offline channel. GBW can make costly efforts to run group-buying, which will increase the number of new consumers. We study the seller’s decision on whether or not to implement group-buying, and how the contract between the seller and GBW impacts the seller’s profitability.

We find that when the GBW has an attractive consumer-base size, the seller will choose to use group-buying. When the seller and the GBW take two-part tariff or revenue sharing contracts, the total profit of the two channels are less than that of a centralized firm; that is, the two contracts could not coordinate the system. Based on this finding, we also propose a revenue-cost sharing contract that can coordinate the two channels.

We adopt the analytical modeling and the game theoretical approach in the paper. Although such analysis enables us to generate managerial insights, it usually lacks of data corroboration. In the future research, one can collect corresponding data to examine the exact relation between the group-buying promotions and a seller’s profitability. Besides, in the future, it is worth to explore some other easily implementable contracts that may better coordinate the revenue of the seller and the website.

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References


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**List of figures and tables**
Figure 1. O+GBW model for the seller
Figure 2. The sequence of events under the Agreement Price Contract
Figure 3. The sequence of events under the O+GBW+T model
Figure 4. The decision sequence of the seller and GBW under the O+GBW+R model
Figure 5. The sequence of events of the seller and GBW under the O+GBW+RC model
Figure 6. The impact of \( b \) on the seller and website’s profit
Figure 7. The profitability comparison under different contracts
Figure 8. The impact on \( F \) with \( b \) increasing
Figure 9. The impact on \( \gamma \) with \( b \) increasing
Figure 10. The impact of GB service/product cost \( c \) \((a=1 \text{ and } b=1)\)

Table 1. The seller and GBW’s optimal decisions under different scenarios
Table 2. The optimal results of the seller and GBW under different scenarios

![Fig.1. O+GBW model for the seller](image-url)
24

The seller decides whether to add the GBW channel or not

The seller decides its agreement price $p_s^A$

The GBW sets the group-buying price $p_g^A$ and the sales effort $\gamma^A$

---

**Fig. 2.** The sequence of events under the Agreement Price Contract

The seller decides whether to add the GBW channel or not

The seller sets $p_s^T$ and $F$

The GBW sets $p_g^T$ and $\gamma^T$

---

**Fig. 3.** The sequence of events under the O+GBW+T model

Seller decides if cooperate with GBW

If so, seller decides $p_s^R$ and the revenue sharing proportion $l$

GBW sets $p_g^R$ and $\gamma^R$ respectively

---

**Fig. 4.** The decision sequence of the seller and GBW under the O+GBW+R model

Seller decides if cooperate with GBW

If so, seller sets $p_s^{RC}$ and revenue sharing proportion $l$ and sales effort $e$

The GBW sets $p_g^{RC}$ and $\gamma^{RC}$, simultaneously

---

**Fig. 5.** The sequence of events of the seller and GBW under the O+GBW+RC model

(a)  (b)
Fig. 6 The impact of $b$ on the seller and website’s profit
Fig. 7 The profitability comparison under different contracts

Fig. 8 The impact on $F$ with $b$ increasing

Fig. 9 The impact on $\gamma$ with $b$ increasing
Table 1 The seller and GB’s optimal decisions under different scenarios

<table>
<thead>
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<td>$b \geq \frac{\sqrt{10} - 2}{6}$</td>
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Table 2 The optimal results of the seller and GBW under different scenarios

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<th>Conditions</th>
<th>Centralized setting</th>
<th>Agreement Price Contract</th>
<th>Revenue-Cost Sharing Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_s^*$</td>
<td>$(a + c)(2a + b) / 5a + c$</td>
<td>$(a + c)(12a^2b + 2ac^2 + 12a^2c + 12ac^2 + bc^2 + 14a^2 + 7abc) / (3a + c)^2(5a + c)$</td>
<td>$(a + c)(2a + b) / 5a + c$</td>
</tr>
<tr>
<td>$p_s^*$</td>
<td>--</td>
<td>$(a + c)(3ab + 6ac + bc + 5a^2 + c^2) / 2(3a + c)^2$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma^*$</td>
<td>$2a^2 - 3ac + ab - c^2 + bc / 5a + c$</td>
<td>$(a + c)(3ab - 4ac + bc + a^2 - c^2) / 2(3a + c)(5a + c)$</td>
<td>$2a^2 - 3ac + ab - c^2 + bc / 5a + c$</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>$2(2a + b) / 5a + c$</td>
<td>$28a^3 + 24a^2c + 24a^2b + 4ac^2 + 14abc + 2bc^2 / (3a + c)^2(5a + c)$</td>
<td>$2(2a + b) / 5a + c$</td>
</tr>
<tr>
<td>$q_g^*$</td>
<td>$b(3a + c) - 4ac + a^2 - c^2 / 2(5a + c)$</td>
<td>$a^2 - 4ac + 3ab - c^2 + bc / 5a + c$</td>
<td></td>
</tr>
<tr>
<td>$q_s^*$</td>
<td>$3a^2 - 6ac + 4ab - c^2 / 2(5a + c)$</td>
<td>$4ab(4a + c)(3a + c) + (a^2 - 4ac - c^2)(8ac + 11a^2 + c^2) / 2(5a + c)(3a + c)^2$</td>
<td>$3a^2 - 6ac + 4ab - c^2 / 2(5a + c)$</td>
</tr>
<tr>
<td>$\Pi_s^*$</td>
<td>--</td>
<td>$(a + c)(3ab + bc - 4ac + a^2 - c^2) / 8(5a + c)(3a + c)^2$</td>
<td>0</td>
</tr>
<tr>
<td>$\Pi_g^*$</td>
<td>--</td>
<td>$(a + c)(3ab + bc - 4ac + a^2 - c^2) / 8(5a + c)(3a + c)^2$</td>
<td>0</td>
</tr>
</tbody>
</table>