A bi-objective hierarchical hub location model with facility failure

Mohsen Babashahi[a], Kamran Shahanaghi[a], Mohammad Reza Gholamian*[a], Arash Yavari[a]
[a] School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

Abstract- In the most past hub location problem (HLP) studies, failure probability is limited to a constant value; while in reality, it depends on various factors including natural disasters such as floods and earthquakes, commodity flow, institutional elements such as strikes, and etc. This paper looks into the problem of hub network design through a new model. The problem is to maximize the network reliability and to minimize the total routing cost simultaneously. In order to provide a more realistic model, the hub's failure probabilities were considered as a function of the hub's incoming traffic. Reserve hub elements are also taken into account in the model with the aim of increasing network reliability. To solve the model, a non-dominated sorting genetic algorithm (NSGA-II) is presented. The parameters of the algorithm are tuned using Taguchi method. The proposed solution is then carried out in a case study where the results confirm the acceptable performance of the model and also sensitivity analysis is performed in order to describe the effects of critical parameters into objective functions.

Keywords: Hierarchical Hub Location, Backup Facility, Reliability, Hub Failure, NSGA-II.

1. INTRODUCTION

The decisions about locating hubs and the allocation of demand nodes to hubs, are strategic decisions which require heavy investments because the system should be used for many coming years with these conditions [1]. The overall performance of the network extremely depends on proper performance of networks elements (hubs and arcs). Happening failure in one elements of the network (hubs or arcs), may lead to the network breakdown or poor service levels. So; the priority is to design networks with higher reliability. Reliability is defined as the probability that a facility (hub and arcs) or a route performs its operations (e.g. transmitting flows over a time period without failure, delay or congestion) over a time period without failure, delay or congestion [2].

Kim and O'Kelly [2] used the concept of reliability in HLP for the first time and presented a reliable p-HLP which focused on maximizing the network performance in terms of reliability by locating hubs for delivering flows among city nodes. In their study, the relationship between network performance and hub facility location by using telecommunication networks in United States was explored. Davari et al. [3] designed a single-allocation hub location, so that the reliability of the network is maximized. Fazel et al. [4] presented a new method for obtaining the reliability of the paths in a hub-and-spoke network and then designed a single-allocation hub location with maximizing the weighted reliability of the network as its first objective and maximizing minimum reliability in the network as its second objective. Karimi and Bashiri [5] provided a model that among considering the minimum cost of the network establishment, ensures also that the reliability of paths that pass through one hub in not less than a specified threshold value. However, their study considered the reliability of the arcs only for determining the reliability of paths. Recently Hamidi et al. [6] developed a new type of reliability naming "prevention reliability" which makes the network reliable by adding fake items to the network.

One of the famous variants of HLP are hierarchical hub networks which are used in many types of the networks. Yaman [7] is one of the first researchers in this field. She considered the problem of designing a three level hub network with the following structure; the central hubs are connected in a complete network on the top level, and the remaining hubs and the demand centers are connected to the central hubs through star networks in the second and third levels, respectively. Hence, a hierarchical network structure was formed. As some recent works in this area, we can mentioned to Hamzaoui and Ben-Ayed [8] which developed an effective time-tabling program in a hierarchical HLP of parcel distribution network. A star/star two-level HLP model for telecommunication networks was developed by Yaman and Elloumi [9] which minimizes the longest path at the first level and routing cost at the

---

* Corresponding Author. Tel: +98 21 7322 5067, Mobile: +98 9126150947, Email Address: Gholamian@iust.ac.ir

second level. Meanwhile, two algorithms are developed for solving the models. Alumur et al. [10] introduced a hierarchical multimodal HLP for a Turkish cargo delivery network. They supposed two transportation modes; a highway segment and an airline segment at the levels of hierarchy structure. Davari and Fazel [11] extend the Yaman's model [7] with fuzzy variables. The model was developed based on credibility measures. Also Fazel et al. [12] developed two meta-heuristics for hierarchical HLP and compare the results. An application of capacitated hierarchical HLP model for public transportation between urban and rural areas has been developed by Zhong et al. [13]. The model has been solved using a hybrid genetic-tabu search optimization algorithm. Also, in healthcare systems, Smith et al. [14] developed a hierarchical HLP model for the network of laboratories which are responsible for blood sample collection for HIV/AIDS. They developed two mixed integer programming models in a way that in the first model total number of laboratories was minimized and in the second model total travel time was minimized. The models were run in real case study at South Africa and the results were compared with the current situation. Sedehzadeh et al. [15] used both fuzzy modelling and meta-heuristics solution approach in his work. The model seeks to minimize cost and fuel consumption simultaneously by considering various modes of transportation in a tree hierarchical HLP. It is assumed that the costs, capacities and speed of vehicles are uncertain and so a fuzzy programming with triangular numbers was developed for the model. Finally, the model was solved with two multi-objective meta-heuristics (i.e. NSGAII and MOICA).

Another extension was proposed by Korani and Sahraeian [16]. They developed a hierarchical hub covering problem where the nodes are assigned to hubs based on a cover radiuses. Also, Li et al. [17] developed a hierarchical HLP for passenger hub layout by considering both hub coverage and hub number of urban agglomeration. They extend their work into multi-period model and solved the model using a variation of Adaptive clonal selection algorithm [18]. Also, recently Dükkanci and Kara [19] introduced a hierarchical multimodal hub covering problem whereas the upper level is configured by ring structure instead of star structure which is used in middle and lower levels. So, the routing and scheduling constraints are added to the model. The model was solved by sub-gradient based heuristics and implemented in a case study of cargo deliver application in Turkey.

On the other hand, Ryerson and Kim [20] stablished a hub hierarchy using machine learning techniques. Specifically, they developed a triple hub clustering indices and then used k-means clustering method on data from U.S. airline network. Karimi et al. [21] proposed a capacitated hierarchical HLP model which is in fact an extension of previous capacitated models into hierarchical structure. Considering refreshment operations in different levels of a hierarchical HLP is another concept which is introduced for perishable items by Esmizadeh and Bashiri [22]. Da Costa Fontes and Goncalves [23] presented a hierarchical HLP model with multi-allocation of sub-hubs. They developed their work on the case of liner shipping network with deep sea/short sea operations. In this work, using sub-hub linkages creates alternative paths which allows multiple allocations. Mahmutogullari and Kara [24] introduced a bi-level model for competitive hub location in duopoly market in which each player decides the location of it hubs; specifically using the medianoid model at the first level and centroid model at the second level to reach the maximum amount of market share. Kim et al. [25] developed a two-layered hierarchical network to determine the hurricane evacuation routes to safe areas. They developed the model considering four cases of corridor line generation (with/without contraflow) and then the models have been solved using a rule-based multi-agent simulation method. The model has been applied at Haeundae Beach, Busan, Korea, to prevent Tsunami disasters. Torkestani et al. [26] used a dynamic system approach in a hierarchical HLP considering different probabilities of disruption at each level and each time period of the model. The model has been solved using Monte Carlo simulation approach.

As shown, in the previous works, the failure is either assumed constant or is modeled by a Bernoulli random variable, i.e., facility i fails with probability p_i. In reality however, the failure depends on various factors including natural disasters and practical disruptions.

In our model, we consider a hierarchical three-level hub network. In the top level, the central hubs are connected through a complete network. In the second level, the remaining hubs are connected to central hubs through star networks. Finally, in the third level, the demand centers are connected to the hubs through star networks. In this model, the probability of a hub facility failure is considered dependent on both the commodity flow and natural effects. The dependency to the flow is assumed to be linear; therefore, the probability of hub failure is a continuous function that changes among different levels. In order to increase the reliability of the total network, for each central hub, back-up components are provided. These back-up components can replace the faulty elements within each hub to prevent the failure.

Based on above descriptions, the remainder of the paper is structured as follows. In Section 2, the proposed model is introduced as a bi-objective mathematical programming model. Section 3 is devoted to provide numerical examples and analytical results and finally in section 4 conclusions and future studies will be presented.

2. THE PROPOSED MODEL

In this section, we consider the model introduced by Yaman [7] as our reference model. In what follows, we describe the model in more detail.

Assumptions and conditions:

The model possesses the following assumptions:

- The network is considered in three levels.
- The number of hubs and central hubs are assumed to be p and p_0, respectively.
- A single assignment scheme is allowed for the network.
- The central hubs network is defined with full connection.
- The demand centers and the hubs are connected to the central hubs through star networks.
- The demand centers are connected to the hubs through star networks.
- Hub failures are assumed to be independent.
- The hub failure depends on external elements, and on its entering flow.
- The demands at the demand centers are not completely fulfilled.

The major decisions

The major decision in the model is to determine:

- The location of the hubs and the central hubs.
- The traffic in the network links.
- The number of backup components at each hub.

Definitions and notations

Parameters:

- I: The set of nodes.
- $H \subseteq I$: The set of available locations for the hubs.
- $C \subseteq I$: The set of available locations for the central hubs.
- d_{ij}: The routing cost of unit traffic from node $i \in I$ to node $j \in I$. ($d_{ij} = d_{ji}$; $d_{ii} = 0$)
- α_H: The discount factor for the routing cost from the hubs to the central hubs.
- α_C: The discount factor for the routing cost among the central hubs.
- p: The number of hubs which need to be located and opened.
- p_0: The number of central hubs which need to be located and opened.
- λ_j: The probability of failure for hub *j* due to external effects.
- M: A very large number.
- f_k^B: The fixed cost of one backup component *B* in hub *k*.
- Ca_j: The capacity of the j^{th} hub

Intermediate decision variables:

- t_{im}: The traffic flowing from node $i \in I$ to node $m \in I$.
- O_j: The inward (outward) flow of hub *j*.
- P_j: The probability of flow-enforced failure for hub *j*.
- RR_j: The probability that hub *j* does not fail.
- FR_j: The failure probability for hub *j*.
- TR_j: The total failure probability for hub *j*, taking into account the backup components.

Final decision variables:

- g_{jl}^{i}: The traffic related to node $i \in I$ as the origin or destination nodes which flows between hub $j \in H$ and the central hub $l \in C$.
- f_{kl}^{i}: The traffic of node $i \in I$ as an origin node, which flows between the central hub $k \in C$ and the central hub $l \in C \backslash \{k\}$.
- z_{ijl}: is equal to 1 if node $i \in I$ is assigned to hub $j \in H$ and hub j is assigned to the central hub $l \in C$; otherwise it is equal to 0.
- x_k^B: The number of backup components for hub *k*.

Now that the parameters and notations are introduced, the developed model is brought as follows:

$$Min \sum_{i\in I}\sum_{m\in I}(\mathrm{t}_{im}+\mathrm{t}_{mi})\sum_{j\in H}d_{ij}\sum_{l\in C}Z_{ijl}+\sum_{i\in I}\sum_{j\in H}\sum_{l\in C\backslash\{\mathrm{j}\}}\alpha_H d_{jl}g_{jl}^{i}+\sum_{i\in I}\sum_{j\in H}\sum_{l\in C\backslash\{\mathrm{j}\}}\alpha_C d_{jl}f_{jl}^{i}+\sum_{k\in C}f_k^B x_k^B \tag{1}$$

$$Max\ \min_j\ (\mathrm{TR}_j) \tag{2}$$

$$\sum_{j\in H}\sum_{l\in C}z_{ijl}=1 \quad \forall i\in I \tag{3}$$

$$z_{ijl}\le z_{jjl} \quad \forall i\in I, j\in H\backslash\{i\}, l\in C \tag{4}$$

$$\sum_{m\in H}z_{jml}\le z_{lll} \quad \forall j\in H, l\in C\backslash\{j\} \tag{5}$$

$$\sum_{j\in H}\sum_{l\in C}z_{jjl}=p \tag{6}$$

$$\sum_{l\in C}z_{lll}=p_0 \tag{7}$$

$$g_{jl}^{i}\ge\sum_{m\in I\backslash\{\mathrm{j}\}}(\mathrm{t}_{im}+\mathrm{t}_{mi})(\mathrm{z}_{ijl}-\mathrm{z}_{mjl}) \quad \forall i\in I, j\in H, l\in C\backslash\{j\} \tag{8}$$

$$\sum_{k\in C\backslash\{l\}}f_{lk}^{i}-\sum_{k\in C\backslash\{l\}}f_{kl}^{i}=\sum_{m\in I}t_{im}\sum_{j\in H}(\mathrm{z}_{ijl}-\mathrm{z}_{mjl}) \quad \forall i\in I, l\in C \tag{9}$$

$$O_j\ge\sum_{i\in I}\sum_{l\in C}g_{jl}^{i}-M(1-\sum_{l\in C\backslash\{j\}}z_{jjl}) \quad \forall j\in H \tag{10}$$

$$O_j\ge\sum_{i\in I}\sum_{k\in C}f_{jk}^{i}-M(1-z_{jjj}) \quad \forall j\in C \tag{11}$$

$$P_j=1-\min\{1,\max\{0,\frac{\mathrm{O}_j-Ca_j}{Ca_j}\}\} \quad \forall j\in H \tag{12}$$

$$RR_j=(1-P_j)(1-\lambda_j) \quad \forall j\in H \tag{13}$$

$$FR_j=(1-RR_j) \quad \forall j\in H \tag{14}$$

$$TR_j = (1 - FR_j^{x_B+1}) \quad \forall j \in H \tag{15}$$

$$x_B \in \{0,1,2,3\} \tag{16}$$

$$z_{ljl} = 0 \quad \forall j \in H, l \in C \setminus \{j\} \tag{17}$$

$$g_{jl}^{i} \geq 0 \quad \forall i \in I, j \in H, l \in C \tag{18}$$

$$f_{kl}^{i} \geq 0 \quad \forall i \in I, \mathrm{k} \in \mathrm{C}, \mathrm{l} \in \mathrm{C} \setminus \{\mathrm{j}\} \tag{19}$$

$$t_{im} \geq 0 \quad \forall i \in I, m \in I \tag{20}$$

$$O_j, P_j, RR_j, FR_j, TR_j \geq 0 \quad \forall j \in H \tag{21}$$

$$z_{ijl} \in \{0,1\} \quad \forall \mathrm{i} \in \mathrm{I}, \mathrm{j} \in \mathrm{H}, \mathrm{l} \in \mathrm{C} \tag{22}$$

In this model, the first objective function (1) illustrates the backup establishment cost plus the costs of all the possible routing traffic between the nodes, including the traffic between demand nodes and their hub, between the hubs and their central hubs, and among central hubs. In Contrast, the second objective function (2) maximizes the lowest level of the network reliability.

Constraint (3) guarantees that each demand node is assigned to a hub or a central hub. Constraint (4) states that if node i is assigned to hub j and central hub l, then node j should be a hub that is assigned to central hub l. According to constraint (5), if node j is assigned to central hub l, then hub l must be a central hub. Constraints (6) and (7) reflect the fixed number of hubs and central hubs to be opened, i.e., p and p_0, respectively. The values of g^{i}_{jl} in terms of the assignment variables are determined by constraints (8) and (18). Constraint (9) balances the flow between nodes in the hierarchy. Constraints which are already introduced are approximately the same as the ones used in the basic model. In the following, the new additional constraints are introduced.

Constraints (10) and (11) demonstrate the incoming flow in the j'th and l'th hub, respectively. Constraint (12) gives the probability of failure for the l'th hub, in terms of its incoming flow. If the incoming flow to this hub exceeds the maximum tolerable level, this probability increases to 1, and the hub will fail. This constraint can be converted into set of linear constraints by adding new variables as shown in equations (23) and (24):

$$v = \max(0, \frac{o_j - Ca_j}{Ca_j}) \Rightarrow v \geq \frac{o_j - Ca_j}{Ca_j} \,\&\, v \geq 0 \tag{23}$$

$$u = \min(1, v) \Rightarrow u \leq v \,\&\, u \leq 1 \tag{24}$$

Constraint (13) gives the functioning probability of the l'th hub. The constraint (14) provides the total failure probability for the l'th hub. Constraint (15) determines the functioning probability of l'th hub considering the backup elements. Constraints (16)-(22) represent integrality and non-negativity of variables. Although, it should be noted that redundant constraints (19) are considered to further strengthen the LP relaxation.

The proposed model in this paper is a nonlinear model. In order to reduce the nonlinearity of the model, a heuristic with an acceptable approximation is proposed. For this reason, first the range [0,1] is partitioned logarithmically into partitions indexed by n=1,2,…10. Each of these values contains four cases based on the range of x_j^B.

Consider the following new parameters.

a_{nt}: the approximate number of values that fall in partition *n*, with a power of *t*.

y_{jn}: is equal to 1 if the approximate FR_j falls in the *n*'th partition, otherwise it is equal to 0.

x_{jt}:is equal to 1 if *t* backup facilities are deployed in the *j*'th hub, otherwise it is equal to 0.

Now, the following constraints can be added to the model,

$$FR_j \leq \sum_n a_{n1} y_{jn} \quad \forall j \in H \tag{25}$$

$$TR_j \le 1 - \sum_t a_{nt} x_{jt}^B + M(1 - y_{jn}) \quad \forall j \in H, n \tag{26}$$

$$\sum_t x_{kt}^B = 1 \quad \forall k \in C \tag{27}$$

Constraint (25) demonstrate the approximate value for FR_j. The value of y_{jn} equals to 1 only for one value of n, otherwise it is equal to 0; therefore, a_{n1} corresponding to each FR_j is obtained through this constraint. Constraint (26) computes the approximate value for TR_j.Constraint (27) ensures that a certain number of backup elements are assigned to each central hub. Also, the objective function (1) can be rewritten as follows in equation (28):

$$Min \sum_{i\in I}\sum_{m\in I}(\mathrm{t}_{im}+\mathrm{t}_{mi})\sum_{j\in H} d_{ij}\sum_{l\in C} z_{ijl} + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C\setminus\{j\}} \alpha_H d_{jl} g_{jl}^i + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C\setminus\{j\}} \alpha_C d_{jl} f_{jl}^i + \sum_{k\in C}(t-1) f_k^B x_{kt}^B \tag{28}$$

The bi-objective model has been solved using ε-constraint method [27]. To do so, the first objective function is restricted to the bounds (L) which are defined on the range from the worst (TC^D) to the best (TC^U) values of this objective function considering predefined step size (r) as shown in equation (29):

$$L_n = TC^D + \frac{n}{r-1}(TC^U - TC^D) \qquad n = 0, 1, 2, \ldots, r-1 \tag{29}$$

Then the model is converted into single objective by considering the first objective function as a constraint as shown in equation (30):

$$\sum_{i\in I}\sum_{m\in I}(t_{im}+t_{mi})\sum_{j\in H} d_{ij}\sum_{l\in C} z_{ijl} + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C\setminus\{j\}} \alpha_H d_{jl} g_{jl}^i + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C\setminus\{j\}} \alpha_C d_{jl} f_{jl}^i + \sum_{k\in C}(t-1) f_k^B x_{kt}^B \le L_n \tag{30}$$

Meanwhile, the second objective function can be simplified by considering:

$$Max: TR_m = \min_j TR_j \tag{31}$$

and adding new set of constraints to the model, as shown in equation (32):

$$TR_j \ge TR_m \tag{32}$$

Solving this single objective model at each $n = 0, 1, 2, \ldots, r-1$ will generate the plot of Pareto frontier as shown in the next section.

3. NUMERICAL RESULTS

Now that the model is mathematically described, we present the solutions for our model. For small scale problems, we solve the problem via the LINGO - CPLEX solver. As for the problems in larger scales, we use the NSGAII where Taguchi method was used to obtain the optimal parameters of the model.

In order to show the efficiency of our proposed solution, we solve the problem in low dimensions using a CPLEX solver, and then compare the results with those obtained by NAGAII. Finally, the model is verified by conducting sensitivity analyses for the critical parameters of the model.

The computational experiments are carried out on a PC with Windows 7, version 2012, and Core-I 5, 2.5 GHz CPU with a 4 Giga bytes of RAM.

The case study:

The case study considered in this paper is based on a real-life postal network with 20 nodes, whose data is reported by Ernest and Krishnamurthy [28]. Since, there is no complete database available in the literature, we use random data generator for the missing parameters. The failure probabilities due to natural effects are considered independently and identically distributed over [0.2 0.3], according to a uniform distribution. Moreover, the cost for each substitute component is assumed to be random and uniformly distributed over [15000, 20000].

In Table 1, the outputs for $N = 20$, $p_1 = 5$ and along with different values of α_C and α_H have been shown. Note that in larger dimension problems, global optimization with OR applications such as LINGO is impossible.

Meanwhile, some heuristics such as Benders' decomposition have been examined. But unfortunately no results were achieved in large scale problems and we are forced to tackle these problems with meta-heuristics such as NSGAII.

Table 1- The hubs and central hubs assignment for N=20,P_1=5

As a sample, Fig. 1, depicts the hub and central hub location assignments for (α_C, α_H) = (1,1), i.e., the first line of the Table 1.

Figure 1- the hub and central hub location assignments for N=20,P_1=5

As observed in Fig. 1 the nodes 10, 14, 20, 7, and 11 are chosen as hubs whilst node 10 is chosen as the central hub. Notice that the X and Y in the Fig. 1, are distances of locations from each other. Fig. 2 further shows the Pareto solution set for the studied problem. The represented result is obtained by ε-constraint method. The bounds of the first objective function is considered in the range [6.495 6.527] × 10^7 with step size 20,000 as brought in Table 2. The spacing metric suggested by Schott [29] is also calculated in this Table.

Figure 2- The Pareto solution for N=20,P_1=5

Table 2-The results of running ε-constraint method

In Table 2, the first and second columns represent the optimal Pareto solutions obtained from solving the model using ε-constraint method. The third column represents the relative distance measure between consecutive Pareto optimal solutions (d_i) as shown in equation (33).

$$d_i = \underset{k \in N_i}{Min}\{Z_i - Z_k\} \tag{33}$$

Where N_i is the set of Pareto solutions in neighborhood of the point i and Z_i is the optimal objective vector at i.
The last row represents spacing metric calculated based on Euclidean distance of d_i's from the mean as follow equation (34):

$$S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d_i - \bar{d})^2} \tag{34}$$

Where n is number of Pareto solutions and $\bar{d}$ represents the mean of distance measures.
In Fig. 2, the reliability of the total network changes in an acceptable range. In Table 3, the number of backup (reserve) elements for the hubs are brought. Also in order to reach the maximum reliability, the hubs should utilize the reserve elements.

Table 3-Number of reserve elements for each hub

<u>Selecting the parameters for solving the problem:</u>

Heuristic and Meta-Heuristic methods usually need to appropriately set some parameters in their searching process. These parameters should be set in a way that the algorithm reaches the optimal solution in the shortest possible time. The complexity of difficult problems highly depends on the size of the problem. In this paper, the size of the problem mainly depends on a few parameters including the number of demand nodes, the number of hubs, and the number of central hubs. We design the experiments in three size levels: small, moderate, and large. The values for the parameters are randomly chosen according to the ranges given in the Table 4. At each level, 10-30 problems are considered for parameter selection and four main parameters involved in the solution are analyzed and their optimal levels are obtained. In order to reduce the number of experiments, Taguchi's method is carried out. Table 5 summarizes the network parameters and the NSGAII factors for each pre-defined level.

Table 4- Network parameters for the three complexity (size) levels.

Table 5- The factors involved in the solution at each complexity level.

Finally, each problem is named according to its network parameters. For example, the problem in which the network consists of 15 demand nodes, 7 hubs and 4 central hubs is entitled by N15_H7_C5_P1, where P denotes the number of the problem in its relevant level.

Problem Evaluations:

As mentioned before, in order to evaluate the efficiency of the proposed algorithm, sample problems have been solved using NSGAII and Lingo 10. Table 6, summarizes the results for the first objective. Note that the reported results for the NSGAII method illustrate the minimum values over 5 repetitions. The results indicate the proximity of NSGAII results into Lingo outputs with a good and acceptable approximation. Although, whereas the size of the problem increases, the OR applications fail to solve the problem.

Table 6- A comparison of the results from NSGAII and Lingo

According to the results summarized in Table 6, the performance of the proposed algorithm can be evaluated very well. Furthermore, in order to demonstrate the efficiency of the approximate linear model, the small size problems have been solved as brought in Table 7. As shown, in small size problems, the linear model gives results that are quite close to those of nonlinear model.

Table 7- A comparison of the results from the linear model and those of the nonlinear model

Sensitivity Analysis:

In this section, the effect of critical parameters on the final results are analyzed. Specifically, we study the effect of the discount factor for the routing cost between the hubs and the central hubs (α_C), the discount factor for the routing cost between the central hubs (α_H), the number of central hubs (p_0), the constant cost of one backup unit at the k'th hub (f_k^B), and the probability of failure due to environmental effects (λ).
Fig. 3 depicts how the objective function changes with α_H, and α_C ,respectively. As shown, the cost function increases as the discount factors increase. Comparing the trends of the curves in this Figure however indicate that the cost function is more sensitive to α_H.

Figure 3- Sensitivity analysis of the cost function, with respect to the discount factors

Fig. 4 shows how the cost function changes with the number of central hubs and f_k^B. As illustrated, the sum of all routing costs and costs of backup elements decreases as the number of central hubs increases. This ensures the positive effect of applying central hubs in decreasing the total cost. As the number of central hubs increases in the network, the discount factors in routing between the hubs would be more profitable and hence the transportation costs are decreased in the network and whilst the cost of back up units have risen, the total cost is also decreasing.

Figure 4- Sensitivity analysis of the cost function, with respect to the number of central hubs and extra tools.

Finally, Fig. 5 shows the changes of the cost function with respect to the probability of failure due to environmental effects (λ). As illustrated, the network reliability decreases as the hub failure probability increases due to environmental effects, which is reasonable because the total failure probability of hub is directly proportional to λ.

Figure 5- Sensitivity analysis with respect to λ.

4. CONCLUSION

In the context of hub allocation, a hierarchical hub location problem was introduced in this work as a mixed integer programming model. It can be seen that the p-hub median problem and the star p-hub median problem are two special cases of the studied problem. Specifically, if the number of central hubs is equal to the number of hubs, then the problem turns into the p-hub median problem as a special case; whereas if the number of central hubs to be opened is equal to 1, then the problem turns into a star p-hub median problem where the candidate set is singleton. By solving the developed model with the same previous instance in the literature and with the number of central hubs varying from 1 to p, the effect of using central hubs on total cost and total reliability was investigated.

The main model maximizes the network reliability and minimizes the total routing cost simultaneously. In order to provide a more realistic model, the hub failure probabilities are considered as a function of the hub's incoming traffic while in previous studies these probabilities were set to be fixed. Reserve hub elements are also taken into account in the model, to potentially increase the reliability of the network. The proposed model is then carried out in case study with different sample size problems and the result of which was verified numerically. Meanwhile, the sensitivity analysis is performed on critical parameters and in general, it observed that the effects of extra tools were important in increasing the total reliability, and the total cost is reduced by increasing the number of central hubs.

As recommendations for future studies, some conceptual developments can be considered. For example, the reliability can be defined on arcs instead of nodes and the model can be again developed and solved based on new conditions. In addition, other probability distribution functions can be used as failure probability instead of uniform distribution function in this study. In our model the hub functioning probability follows the concept of geometric distribution function. This concept can be changed into more familiar distribution functions such as exponential, Weibull and so on. On the other hand, some structural developments can be offered. For example a budget constraint can be defined on backup establishment costs. The model is developed in deterministic conditions while the model with robust conditions is closer to real-world problems. Moreover, the model can be extended into a game theory-based model by considering the realistic conflicts of players.

REFERENCES

1. Sim T.K.T. (2007). "The hub covering flow problem and the stochastic p-hub center problem", PhD Thesis, University of Iowa.
2. Kim H., and O'Kelly M.E. "Reliable p-Hub Location Problems in Telecommunication Networks", *Geographical Analysis*, **41**(3), pp. 283-306, 2009. doi: 10.1111/j.1538-4632.2009.00755.x.
3. Davari S., Fazel M.H., and Turksen I. "The fuzzy reliable hub location problem", *Fuzzy Information Processing Society (NAFIPS), 2010 Annual Meeting of the North American Fuzzy Information Processing Society: IEEE*, pp. 1-6, (2010). doi: 10.1109/NAFIPS.2010.5548271.
4. Fazel M.H., Davari S., and Haddad Sisakht S.A. "The Q-coverage multiple allocation hub covering problem with mandatory dispersion", *Scientia Iranica*, **19**(3), pp. 902-911, (2012). doi: 10.1016/j.scient.2012.03.007.
5. Karimi H., and Bashiri M. "Hub covering location problems with different coverage types", *Scientia Iranica*, **18**(6), pp. 1571-1578, (2011). doi: 10.1016/j.scient.2011.09.018.
6. Hamidi, M., Gholamian, M.R., and Shahanaghi, K. "Developing prevention reliability in hub location models", *Journal of Risk and Reliability*, **228**(4), pp. 337-346, (2014). doi: 10.1177/1748006X13519247.
7. Yaman H. "The hierarchical hub median problem with single assignment", *Transportation Research Part B: Methodological*, **43**(6), pp. 643-58, (2009). doi: 10.1016/j.trb.2009.01.005.
8. Hamzaoui, S., and Ben-Ayed, O. "Parcel distribution timetabling problem", *Operations Management Research*, **4**(3-4), pp.138-149, (2011). doi: 10.1007/s12063-011-0056-4.
9. Yaman, H, and Elloumi, S. "Star p-hub center problem and star p-hub median problem with bounded path lengths", *Computers and Operations Research*, **39**(11), pp. 2725-2732, (2012). doi: 10.1016/j.cor.2012.02.005.
10. Alumur, S.A., Yaman, H., and Kara, B.Y. "Hierarchical multimodal hub location problem with time-definite deliveries", *Transportation Research Part E: Logistics and Transportation Review*, **48**(6), pp. 1107-1120, (2012). doi: 10.1016/j.tre.2012.04.001.
11. Davari, S., and Fazel, M.H. "The single-allocation hierarchical hub-median problem with fuzzy flows", *Advances in Intelligent Systems and Computing*, **195**, pp. 165-181. Springer, (2013). doi: 10.1007/978-3-642-33941-7_17.
12. Fazel, M.H., Davari, S., and Haddad Sisakht, S.A. "An empirical comparison of simulated annealing and iterated local search for the hierarchical single allocation hub median location problem", *Scientia Iranica*, **22**(3), pp. 1203-1217, (2015).

13. Zhong, W., Juan, Z., Zong, F., et al. "Hierarchical hub location model and hybrid algorithm for integration of urban and rural public transport", *International Journal of Distributed Sensor Networks*, **14**(4), (2018). doi: 10.1177/1550147718773263.
14. Smith, H., Cakebread, D., Battarra, M., et al. "Location of a hierarchy of HIV/AIDS test laboratories in an inbound hub network: case study in South Africa", *Journal of the Operational Research Society*, **68**(9), pp. 1068-1081, (2017). doi:10.1057/s41274-017-0240-5.
15. Sedehzadeh, S., Tavakkoli-Moghaddam, R., Baboli, A., et al. "Optimization of a multi-modal tree hub location network with transportation energy consumption: A fuzzy approach", *Journal of Intelligent and Fuzzy Systems*, **30**(1), pp. 43-60, (2016). doi: 10.3233/IFS-151709.
16. Korani, E., and Sahraeian, R. "The hierarchical hub covering problem with an innovative allocation procedure covering radiuses", *Scientia Iranica*, **20**(6), pp. 2138-2160, (2013).
17. Li, T.-T., Song, R., He, S.-W., et al. "Optimization model of comprehensive passenger hub in urban agglomeration based on hierarchical layout", *China Journal of Highway and Transport*, **29**(2), pp. 116-122, (2016).
18. Li, T.-T., Song, R., He, S.-W., et al. "Multiperiod Hierarchical Location Problem of Transit Hub in Urban Agglomeration Area", *Mathematical Problems in Engineering*, online, (2017). doi: 10.1155/2017/7189060.
19. Dükkanci, O., and Kara, B.Y. "Routing and scheduling decisions in the hierarchical hub location problem", *Computers and Operations Research*, **85**, pp. 45-57, (2017). doi: 10.1016/j.cor.2017.03.013.
20. Ryerson, M.S., and Kim, H. "Integrating airline operational practices into passenger airline hub definition", *Journal of Transport Geography*, **31**, pp. 84-93, (2013). doi: 10.1016/j.jtrangeo.2013.05.013.
21. Karimi, M., Eydi, A.R., and Korani, E. "Modeling of the capacitated single allocation hub location problem with a hierarchical approach", *International Journal of Engineering, Transactions A: Basics*, **27**(4), pp. 573-586, (2014).
22. Esmizadeh, Y., and Bashiri, M. "Applying hierarchical hub location problem on perishable good distribution systems", *Joint International Symposium on The Social Impacts of Developments in Information, Manufacturing and Service Systems*, Istanbul, Turkey, pp. 260-269, (2014).
23. Da Costa Fontes, F.F., and Goncalves, G. "Routing problem with pendular and cyclic service in a hierarchical structure of hub and spoke with multiple allocation of sub-hubs", *International Conference on Industrial Engineering and Systems Management*, Seville, Spain, pp. 561-567, (2015). doi: 10.1109/IESM.2015.7380214.
24. Mahmutogullari, A.I., and Kara, B.Y. "Hub location under competition", *European Journal of Operational Research*, **250**(1), pp. 214-225, (2016). doi: 10.1016/j.ejor.2015.09.008.
25. Kim, J., Lee, S., and Lee, S. "An evacuation route choice model based on multi-agent simulation in order to prepare Tsunami disasters", *Transportmetrica B: Transport Dynamics*, **5**(4), pp. 385-401, (2017). doi:10.1080/21680566.2016.1147002.
26. Torkestani, S., Seyedhosseini, S.M., Makui, A., et al. "The reliable design of a hierarchical multi-modes transportation hub location problems (HMMTHLP) under dynamic network disruption (DND)", *Computers & Industrial Engineering*, **122**, pp. 39-86, (2018). doi: 10.1016/j.cie.2018.05.027
27. Cohon, J.L. Multi-objective Programming and Planning, Chapter 6.2, 2nd edition, *Dover Publications*, 2013.
28. Ernest A.T., and Krishnamoorthy M. "Efficient algorithms for the uncapacitated single allocation-hub median problem", *Location science*, **4**(3), pp. 139-54, (1996). doi: 10.1016/S0966-8349(96)00011-3.
29. Schott, J.R. (1995) "Fault Tolerant Design Using Single and Multicriteria Genetic Algorithm Optimization". Master's Thesis, MIT University.

FIGURE CAPTIONS:

Figure 1- the hub and central hub location assignments for N=20,P_1=5
Figure 2- The Pareto solution for N=20,P_1=5
Figure 3- Sensitivity analysis of the cost function, with respect to the discount factors
Figure 4- Sensitivity analysis of the cost function, with respect to the number of central hubs and extra tools.
Figure 5- Sensitivity analysis with respect to λ.

TABLE CAPTIONS:

Table 1- The hubs and central hubs assignment for N=20,P_1=5
Table 2-The results of running ε-constraint method
Table 3-Number of reserve elements for each hub
Table 4- Network parameters for the three complexity (size) levels.
Table 5- The factors involved in the solution at each complexity level.
Table 6- A comparison of the results from NSGAII and Lingo
Table 7- A comparison of the results from the linear model and those of the nonlinear model

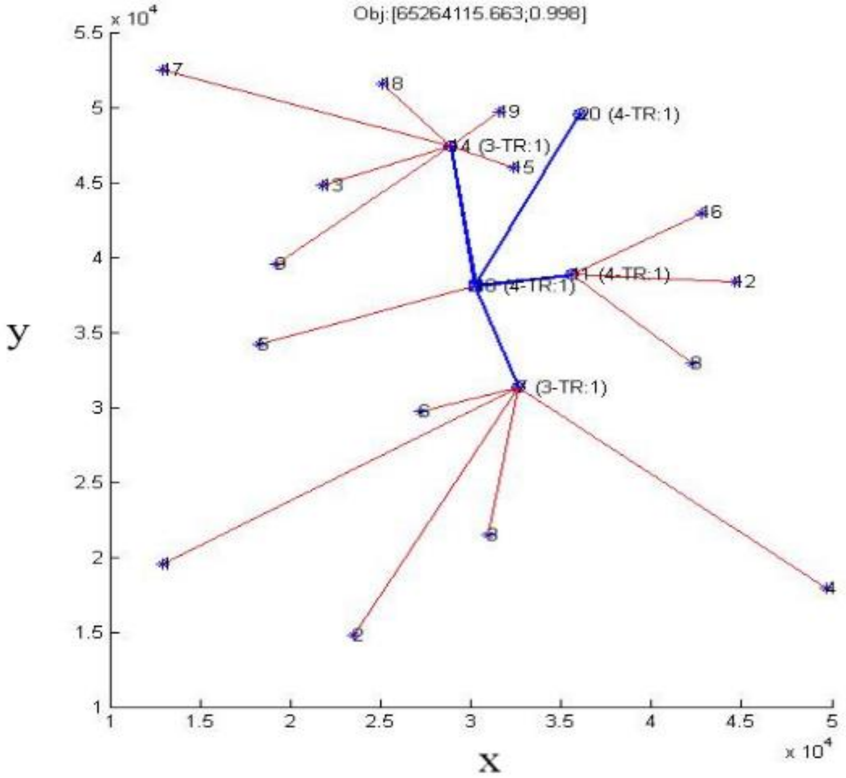

Figure 1- the hub and central hub location assignments for N=20,P_1=5

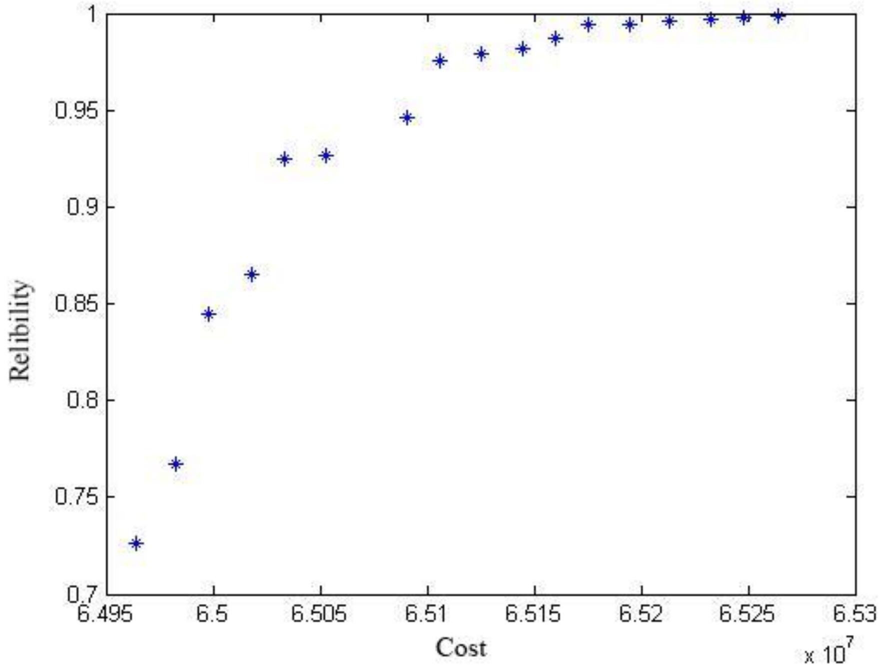

Figure 2- The Pareto solution for N=20,P_1=5

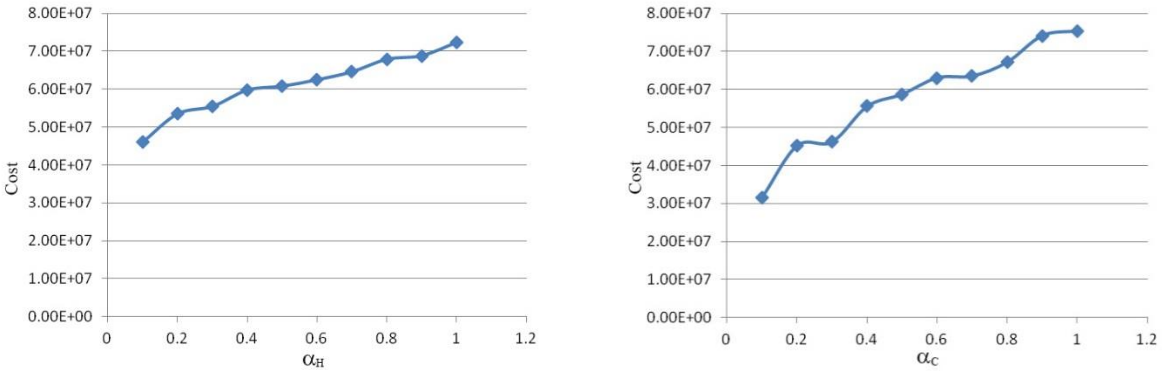

Figure 3- Sensitivity analysis of the cost function, with respect to the discount factors

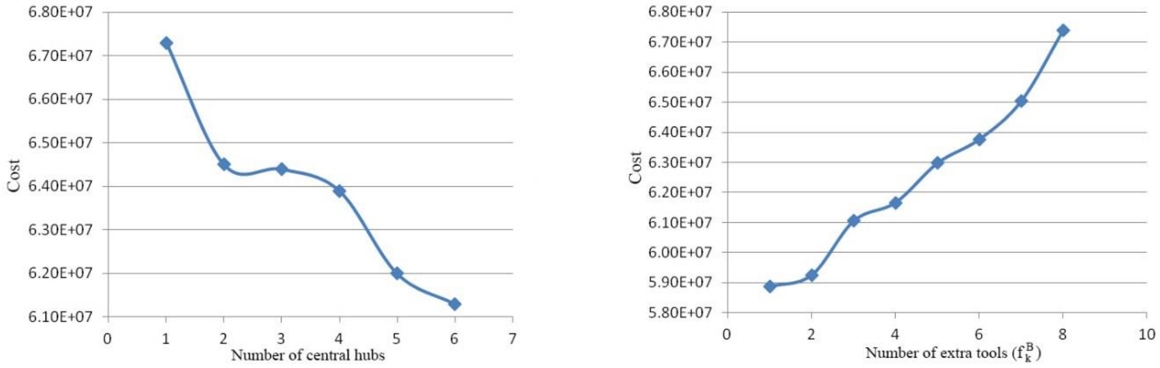

Figure 4- Sensitivity analysis of the cost function, with respect to the number of central hubs and extra tools.

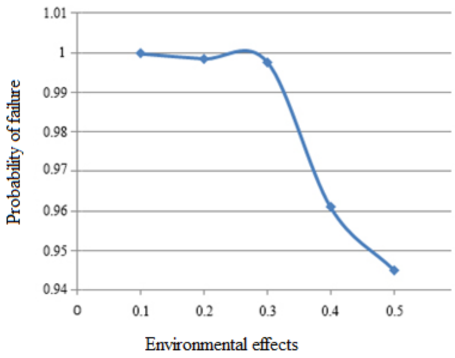

Figure 5- Sensitivity analysis with respect to λ.

Table 1- The hubs and central hubs assignment for N=20,P_1=5

N=20 , P1=5 The Best Case						
(α_C, α_H)	***P2***	***Central Hubs (GA)***	***Hubs (GA)***	***Objective 1***	***Objective 2***	***CPU Time (GA)***
(1,1)	1	10	10,14,20,7,11	6.52e+7	0.9985	15019
	3	10,15,14	13,11,15,14,10	6.21e+7	.0.9985	5201
	4	10,15,14,11	11,15,14,10,13	6.01e+7	.9985	4926
(0.9,0.9)	2	10,6	14,7,11,6,10	6.39e+7	0.9985	4914
	3	15,10,6	10,6,15,14,18	6.01e+7	0.9985	5506
	4	15,10,11,6	6,14,11,15,10	5.98e+7	0.9985	4929
(0.8,0.8)	2	15,10	14,6,10,15,11	6.26e+7	0.9985	4941
	3	19,14,15	15,11,14,10,19	5.72e+7	0.9985	4980
	4	14,19,15,10	6,10,15,19,14	5.65e+7	0.9985	4661
(0.9,0.8)	2	19,15	14,11,19,20,15	6.27e+7	0.9985	5245
	3	19,15,14	20,19,15,11,14	6.12e+7	0.9985	4964
	4	15,19,10,14	19,7,15,14,10	6.015e+7	0.9985	5582

Table 2-The results of running ε-constraint method

Cost (10^7)	Reliability	d%
6.495	0.726	4.1
6.497	0.767	4.1
6.499	0.847	1.6
6.501	0.863	1.6
6.503	0.922	0.3
6.505	0.925	0.3
6.507	0.947	2.2
6.509	0.976	0.4
6.511	0.98	0.2
6.513	0.982	0.2
6.515	0.989	0.7
6.517	0.996	0
6.519	0.996	0
6.521	0.997	0.1
6.523	0.998	0.1
6.525	0.999	0.1
6.527	1	0.1
Spacing Metric =		1.31244

Table 3-Number of reserve elements for each hub

No. of Reserved Elements	Number of Hub
4	10
3	14
4	20
3	7
4	11

Table 4- Network parameters for the three complexity (size) levels.

	Small	Moderate	Large
Number of demand nodes	5-7	7-15	15-25
The number of hubs	2-3	4-7	8-10
The number of central hubs	1-2	3-4	5-7

Table 5- The factors involved in the solution at each complexity level.

	Level 1	Level 2	Level 3
Population	30	45	60
Crossover −Mutation Rate Rate	0.3-0.7	0.5-0.5	0.3-0.7
Iteration	500	2000	4000
Improvement rate	0.2	0.5	−

Table 6- A comparison of the results from NSGAII and Lingo

The Problem (size)	***The First Objective Function***			***The Elapsed Time (sec)***	
	NSGAII	***Lingo***	***GAP***	***GA***	***Lingo***
N10_H3_C1_P1	14903593	14902354	8.314e-5	4517	20
N10_H5_C1_P2	14639385	14617236	0.0015	4446	129
N10_H5_C2_P3	15286284	15262153	0.0016	4870	10542
N15_H3_C1_P4	36919222	36400128	0.0143	4926	65423
N15_H5_C2_P5	36329565	-	-	4953	-
N15_H7_C3_P6	34878527	-	-	5019	-
N15_H7_C4_P7	34667458	-	-	5169	-
N20_H3_C1_P8	64634229	-	-	5260	-
N20_H5_C2_P9	63698681	-	-	5290	-
N20_H7_C2_P10	62250930	-	-	5384	-
N20_H7_C3_P11	67587991	-	-	5688	-
N20_H7_C4_P12	59149467	-	-	5800	-

Table 7- A comparison of the results from the linear model and those of the nonlinear model

The Problem (size)	***The Objective Function***			***The Elapsed Time (sec)***	
	Nonlinear	***Linear***	***GAP***	***Nonlinear***	***Linear***
N5_H2_C1_P1	1.92e+6	1.99e+6	-0.0352	4086	
N5_H3_C1_P2	1.66e+6	1.85e+6	-0.1027	4683	
N7-H2-C1_P3	8.39e+6	8.58e+6	-0.0221	3936	
N7_H3_C1_P4	8.16e+6	8.41e+6	-0.0297	4561	
N10_H3_C1_P5	1.49e+7	1.68e+7	-0.113	20	
N10_H5_C1_P6	1.46e+7	1.7e+7	-0.141	129	
N10_H5_C2_P7	1.52e+7	1.74e+7	-0.126	10542	

TECHNICAL BIOGRAPHY

Mohsen Babashahi *is an M.S. Graduate student in School of Industrial Engineering at the Iran University of Science and Technology (IUST), Tehran, Iran. His research interests include Network and Reliability, Facility Location, Simulation and System Dynamics.*

Kamran Shahanaghi *is an Associate Professor of Industrial Engineering at Iran University of Science and Technology, Tehran, Iran. His research interests include Maintenance and Reliability, Multiple Criteria Decision-Making, Uncertain Programming and Operations Research. He has published papers in journals such as Computers & Industrial Engineering, Expert Systems with Applications, Applied Mathematical Modelling, Engineering Failure Analysis, Journal of Manufacturing Systems, Reliability Engineering and System Safety and IEEE Transactions on Reliability among others.*

Mohammad Reza Gholamian *is an Associate Professor in School of Industrial Engineering at the Iran University of Science and Technology (IUST), Tehran, Iran. He received his M.S. degree in Industrial Engineering from Isfahan University of Technology (IUT), Isfahan in 1998 and obtained Ph.D. in Industrial Engineering from Amirkabir University of Technology (AUT), Tehran in 2005. Presently he is faculty member of Systems Engineering Group in School of Industrial Engineering and is actively engaged in conducting Academic, Research and Development Programs in the field of Industrial Engineering. He has contributed more than 172 research papers to many national and international journals and conferences. Besides this, he has published 5 books by reputed publishers. His research interests are in the areas of Inventory Models, Supply Chain Network Design and Multi-Criteria Decision Making.*

Arash Yavari *is an M.S. Graduate student in School of Industrial Engineering at the Iran University of Science and Technology (IUST), Tehran, Iran. His research interests include Facility Design, Supply Chain Management and Optimization Methods.*