Vapor Solidification of Saturated Air in Two-Dimensional Stagnation Flow

Ali Shokrgozar Abbasi\textsuperscript{1*}, Mohsen Ghayeni\textsuperscript{2}

1- Assistant Professor, Department of mechanical engineering, Payame Noor University, Iran. (Corresponding Author, Email: shokrgozar.ali@gmail.com)
Address: Payame Noor University, Moallem 71 street, Mashhad, Iran. Tel: 0985138683887 Mobile: 098915313811 Fax: 0985138451620
2- Graduate Student, Department of mechanical engineering, Payame Noor University, Iran.

**ABSTRACT**

Stagnation flow solidification of vapor from saturated air is investigated. Saturated air with strain rate $\alpha$ impinges on a flat plate and, because the plate temperature is below the freezing temperature of water, condensation occurs and an ice layer forms on the plate. The ice surface is modeled as an accelerated flat plate moving toward the impinging fluid. The unsteady Navier-Stokes equations were subjected to a similarity transformation to obtain a single ordinary differential equation for the velocity distribution. Two methods of solution were used for the energy equation: a finite-difference numerical technique and a numerical solution of a similarity equation; these two results were compared to establish accuracy.

Freezing time first increases as the far-field temperature decreases from above $0^\circ$C and then rapidly approaches zero as the far-field temperature approaches $0^\circ$C. Despite the expectation that in a physical experiment condensation would begin at the substrate, here the size of the cell next to the substrate controls the time at which condensation begins. It was found that the maximum time before freezing begins at about $5^\circ$C air temperature for size of both 0.1 and 0.2 mm. The ultimate frozen thickness for two saturated air temperatures are also presented.

**Keywords:** water vapor solidification, stagnation flow, unsteady flow, viscous fluid, incompressible flow, similarity solution.

**Nomenclature**

<table>
<thead>
<tr>
<th>English</th>
<th>Greek</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(t)$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>$\delta^*$</td>
</tr>
<tr>
<td>$a_0$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>$b_{1 \text{ to } 5}$</td>
<td>$\Theta$</td>
</tr>
<tr>
<td>$C$</td>
<td>$\psi$</td>
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<tr>
<td>$E_{\text{vap}}$</td>
<td>$\xi$</td>
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<tr>
<td>$H$</td>
<td>$\rho$</td>
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<tr>
<td>$H_{\text{ls}}$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>$H_{\text{vl}}$</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$k$</td>
<td>Viscous layer thickness</td>
</tr>
<tr>
<td>$K$</td>
<td>Displacement viscous layer thickness</td>
</tr>
<tr>
<td>$m$</td>
<td>Similarity variable</td>
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<tr>
<td>$\bar{m}$</td>
<td>Dimensionless temperature</td>
</tr>
<tr>
<td>$N$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$p$</td>
<td>Dimensionless x-axis</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Density</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Dimensionless time</td>
</tr>
<tr>
<td>$S, \dot{S}, \ddot{S}$</td>
<td>Humidity ratio $= m_{\text{vapor}}/m_{\text{air}}$</td>
</tr>
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**Subscripts**

<table>
<thead>
<tr>
<th>$\text{Subscripts}$</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$0$</td>
<td>At the nearest calls of mesh to the ice (or first row of cells in fluid region)</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Free stream (far-field)</td>
</tr>
<tr>
<td>$i, k$</td>
<td>Node (or cell) row and column numbers in $x$ and $z$ directions, respectively</td>
</tr>
<tr>
<td>$j$</td>
<td>1 to 3 numbers (air, vapor and water)</td>
</tr>
<tr>
<td>$l$</td>
<td>Liquid phase (water)</td>
</tr>
<tr>
<td>$ls$</td>
<td>Liquid to solid (phase change)</td>
</tr>
</tbody>
</table>
\( \xi, \xi \)  
Dimensionless thickness and velocity of ice evolution, respectively, in z-direction

\( t \)  
Time

\( T \)  
Temperature

\( T_{\text{air}} \)  
Temperature of air

\( T_{\text{sub,max}} \)  
The temperature of substrate for start of freezing

\( V \)  
Volume

\( u, w \)  
Velocity components near the plate in x, z directions

\( \bar{u}, \bar{w} \)  
Dimensionless velocity components near the plate in x, z directions

\( U, W \)  
Potential region velocity components in x, z directions

\( x, y, z \)  
Cartesian coordinates

\( \text{max} \)  
The maximum substrate temperature that the first row of cells (or nearest cells to ice) can freeze

\( N \)  
North

\( S \)  
South

\( ss \)  
Steady state

\( sub \)  
Substrate plate

\( total \)  
Total time of freezing the first cells row

\( vl \)  
Vapor to liquid (phase change)

\( v \)  
Vapor

**Superscripts**

-  
Dimensionless

\( n \)  
Timing step (old)

\( n + 1 \)  
Timing step (new)

1. **Introduction**

In the present study, freezing of water vapor from saturated air is investigated in two-dimensional Cartesian stagnation flow. Solidification is one of the most interesting phenomena in natural processes and industrial applications. This phenomenon comprises heat transfer accompanied by phase change. Glass, metal, plastic and oil, food and other corresponding industries require good insight into solidification behavior as the nature of solid growth. Similarly, studies of phase change in stagnant media helps to better understand convection effect upon interface behavior. Especially in the aerospace industry, freezing of the saturated air vapor and the conditions for its freezing in front of the aircraft is very important. This phenomenon may also occurs at the tip of the missile. Investigations in the field of heat transfer in phase change or solidification in stagnation flow with or without similarity solutions and related studies are as follows. The classic problem of ice formation in polar seas was solved using the analytical method by Stefan [1]. A one-dimensional heat flux for phase change issues was presented by Goodrich [2]. An experimental study for natural convection in the fluid border between liquid and solid phases was investigated by Sparrow and et al [3]. Numerical methods for solving problems of freezing the flow of natural convection between two isolated plates were presented by Lacroix [4]. Three-dimensional numerical study for free convection with phase change in a channel with rectangular a cross-section was considered by Yeoh et al [5]. An analysis about combining hydrodynamic behavior with behavior of the solid-liquid boundary layer of fluids, which is located between two freezing isolated plates, was investigated by Hadji and Schell [6]. A method for calculating time-dependent heat flux caused by natural convection during freezing fluid between two isolated plates was presented by Hanumanth [7]. Providing an integrated model for continuous phase change issues be investigated by Curtic et al [8]. The problem of freezing molten liquid drops on a rigid plate model was solved by Trapaça et al [9] and comparison between numerical modeling and laboratory results of deformation and solidification of a drop on a cold plate was conducted by Watanabe et al [10]. Evaluation of an existing model transformation and freezing adjusted to a drop impinging on the substrate plate was presented by Marchi et al [11]. The flow of heat transfer of viscous fluid produced by the axisymmetric stagnation flow on a flat plate that at its coordinates has damped oscillatory motion was investigated by Weidman and Mahalingam [12]. Investigation of flow and heat transfer in asymmetric three-dimensional viscous stagnation flow was presented by Shokrgozar and Rahimi [13]. Viscous stagnation flow and heat transfer in asymmetric three-dimensional system with suction and blowing was considered by Shokrgozar and Rahimi [14]. The stagnation point flow of Walters-B fluid induced by a Riga plate was investigated by Shafiq et al [15]. The problem of a steady MHD boundary-layer flow over a semi-infinite stretching surface with a power-law velocity was considered by Hammouch et al [16]. A hygro-thermo-mechanical multiphase model which describes the freezing behavior of partially saturated air-entrained concrete was investigated by Eriksson et al [17]. The unsteady stagnation-point boundary layer flow and heat transfer of a special third grade fluid past a permeable stretching/shrinking sheet was studied by Naganthran et al [18]. The impact of melting phenomenon in magnetohydrodynamic stagnation point flow of nanofluid towards a nonlinear stretching surface of variable thickness was considered by Farooq et al [19]. The simultaneous characteristics of thermal radiation and melting heat transfer effects in stagnation point flow of carbon nanotubes due to a stretching cylinder was investigated by Hayat et al [20]. The exact solution of the unsteady two-dimensional stagnation flow and heat transfer on a heated plate was presented by Shokrgozar and Rahimi [21]. The two-dimensional stagnation flow and heat transfer on an accelerated flat plate was investigated by Shokrgozar and Rahimi [22]. Axisymmetric stagnation flow and heat transfer on an accelerated flat plate was presented by Shokrgozar et al [23].
Stagnation flow solidification of an inviscid fluid that freezes at the common border was considered by Brattkus and Davis [24]. The problem of Stephen solidification for inviscid fluid in stagnation flow was solved by Rangel and Bian [25]. Freezing in sub-cooled liquid stagnation point (freezing point) was investigated in the two-dimensional Cartesian coordinate system by Lambert and Rangel [26]. Viscous fluid phase change in stagnation flow was considered by Joo-Sik [27]. The steady-state, viscous flow and heat transfer of Nano-fluid in the vicinity of an axisymmetric stagnation point of a stationary cylinder with constant wall heat flux was investigated by Mohammadiun et al [28]. The unsteady three-dimensional axisymmetric stagnation-point flow of a viscous compressible fluid on a flat plate was investigated by Rahimi and Mozayyeni [29]. The steady-state viscous flow and heat transfer in the vicinity of an unaxisymmetric stagnation-point of an infinite stationary cylinder was investigated by Rasool et al [30]. Solidification of incompressible fluid in two-dimensional stagnation flow was considered by Shokrgozar and Rahimi [31]. Freezing of incompressible fluid in unsteady axisymmetric stagnation flow was also investigated by Shokrgozar [32]; however, solidification of a vapor from a saturated gas has not been investigated what so ever. In this study, saturated air vapor freezing in stagnation flow is studied. The fluid flow is assumed incompressible, viscous, in laminar regime and long in y-direction. Due to fluid contact with a plate with a temperature lower than the temperature of water, the vapor becomes liquid and then, if the substrate temperature is below enough, ice will be produced. Note that some subcooling is usually needed for condensation of the vapor from the gas to occur. The same applies for the freezing of liquid water to ice. Here, if high heat transfer rate and adequate time to settle at the lowest node is not available, distilled water may turn into frost. This article assumes that there is adequate time available for the fluid to settle, so the frost formation assumption is disregarded; thus, frost formation requires a separate investigation. Since the velocities are low in this study, inertia and Magnus forces due to their rotation in response to shear are assumed negligible. The Magnus forces due to their rotation in response to shear are assumed negligible in this paper. The effects of mass diffusion, here are also neglected for simplicity.

2. Problem Formulation

In this study, the flow in y direction is so long that it is assumed two-dimensional. In Fig. 1, two-dimensional Cartesian coordinates with corresponding (u, w) velocities with respect to (x, z) are shown. Saturated air stagnation flow with a(t) strain in the z direction approaches to z = 0 perpendicular to the plate. The flow is assumed laminar with constant properties for air, vapor and water. If the plate temperature is sufficiently low, condensation occurs and the resulting liquid freezes. The ice surface is modeled as an accelerated flat plate moving toward an impinging fluid with variable velocity S(t) and acceleration S(t), respectively, by the distance S(t) in each time step. For more explanation over substitution of the imaginary flat plate in the solid-fluid interface, please refer to Shokrgozar and Rahimi [31].

Figure (1)

Due to temperature change in the air, densities of vapor and air are changed. These changes will be negligible, as the flow is assumed incompressible. However, the change in the volume conversion of vapor to water, which affects cell size in the mesh, has been taken into account. In this situation, there are particles of water and vapor together, so the fluid has a multiphase state. However, the mass of water particles is too small to destroy homogeneity. So the multiphase effects are neglected in equations. However, the same small mass of condensation is important in the energy equation. Note that the fine particles of water only at the lowest node will have the opportunity to settle as the lowest velocity and the highest water weight in this row. The high weight in the lowest node is due to the increase and accumulation of fine particles on each other. Otherwise, it may happen that water vapor in other nodes does have the opportunity to settle and frost is produced in this mode, which is outside the scope of this article and requires separate investigation. Mass and Navier–Stokes equations for Newtonian, laminar, incompressible, unsteady state with constant density and viscosity properties fluid give,

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]  
(1)

Momentum,

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]  
(2)

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right)
\]  
(3)
The Magnus forces due to their rotation in response to shear are assumed negligible. Note that the small mass of water is indeed a homogeneous mixture in saturated air and the continuous formula of partial derivatives is used with respect to time. In addition, the effects of mass diffusion are neglected for simplicity. Unsteady energy equation in the fluid region (dissipation and radiation heat transfer are neglected without internal source) gives:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\partial m_{ij}}{\partial t} \frac{H_{ij}}{\sum_{j=1}^{3} m_{ij} C_j}$$

(4)

Notice conductivity and heat capacity coefficient are constant ($k$ and $C$, respectively); also, $du/\partial t \approx c \partial T/\partial t$ is assumed where $p, \rho, \nu$ and $\alpha$ are the fluid pressure, density, kinematic viscosity, and thermal diffusivity.

Energy equation in solid phase (ice) is as follows:

$$\rho_s \frac{\partial S}{\partial t} = k_s \frac{\partial T}{\partial S} A_S - k_l \frac{\partial T}{\partial N} A_N$$

(5)

and the energy equation (boundary condition) at interface before reaching freezing temperature becomes:

$$\sum_{j=1}^{3} \rho_j C_j \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial z} \right) = k_s \frac{\partial T}{\partial S} A_S - k_l \frac{\partial T}{\partial N} A_N + \left( \frac{m^n_i - m^{n+1}_i}{\Delta t} \right) H_{ij}$$

(6)

where the left term in the equation is energy moving boundary and right terms are conductive heat transfers and heat source from left to right, respectively. Furthermore, the energy equation (boundary condition) at interface after reaching freezing temperature becomes:

$$\rho_l H_{il} \frac{\partial S}{\partial t} = k_s \frac{\partial T}{\partial S} - k_l \frac{\partial T}{\partial N}$$

(7)

Here, the left term is the source and the right terms are conductive heat transfers from up and down, respectively. Similarly, by using curve fitting, the humidity ratio from the psychometric chart can be defined as:

$$\omega_{n+1}^{n+1} = b_1 + b_2 T_{i,k}^{n+1} + b_3 \left( T_{i,k}^{n+1} \right)^2 + b_4 \left( T_{i,k}^{n+1} \right)^3 + b_5 \left( T_{i,k}^{n+1} \right)^4$$

(8)

The Eq. (8) can be used for a range of 0 to 70 degrees Celsius that after linearizing by Newton-Raphson method gives:

$$\omega_{n+1}^{n+1} = \omega_n^{n+1} + \left( b_2 + 2b_3 T_{i,k}^n + 3b_4 \left( T_{i,k}^n \right)^2 + 4b_5 \left( T_{i,k}^n \right)^3 \right) \left( T_{i,k}^{n+1} - T_{i,k}^n \right)$$

(9)

where subscripts $(i,k)$ are node numbers in $x$ and $z$ directions, respectively; Superscript $(n+1)$ is new time, $\omega_{n+1}^{n+1}$ is the humidity ratio for new temperature and $b (\text{C})^{9 \to -4}$ coefficients are:

$$b_1 = 3.744E - 03 \text{(C)}^0, \quad b_2 = 2.820E - 04 \text{(C)}^{-1}, \quad b_3 = 7.360E - 06 \text{(C)}^{-2}$$

$$b_4 = 2.200E - 07 \text{(C)}^{-3}, \quad b_5 = 3.270E - 09 \text{(C)}^{-4}$$

Note, the curve fitting method is used to find the coefficients, and Eq. (9) is obtained for linearizing the numerical solution of the energy equation.

3. Similarity Solution

3-1. Fluid Flow Solution

According to [16] and integrating equations (1)-(3) outside the boundary layer, the classical equations of potential flow solution is as follows:

$$U = a(r)x$$

(10)

Note, boundary layer is defined here as the edge of the points where their velocity is 99% of their corresponding potential velocity. By entering Eq.(10) in the continuity Eq.(1) and after integration, we have:

$$W = -a(r)z$$

(11)
where $\zeta = z - S(t)$ and $S(t)$ is the amount of plate displacement in the $z$-direction and is assumed to be positive when the plate moves toward the impinging flow. Hence, $S(t)$ and, then, $\zeta$ are functions of time. These recent equations (10)-(11) are the boundary condition equations in the viscous layer. A reduction of the Navier-Stokes equations is sought by the following coordinate separation in which the solution of the viscous problem inside the boundary layer is obtained by composing the inviscid and viscous parts of the velocity:

\[ u = a(t) \times f'(\eta) \quad (12) \]

\[ w = -\sqrt{\nu/a_0} \times a(t) \times f(\eta) \quad (13) \]

\[ \eta = \sqrt{\nu/a_0} \zeta \quad (14) \]

where the terms involving $f(\eta)$ in equations (12)-(13) denote the similarity form for unsteady stagnation-point flow and prime represents differentiation with respect to $\eta$. Moreover, $a_0$ is the reference potential flow strain rate at the start of time. By inserting the transformations (12)-(14) into equations (2)-(3) yields an ordinary differential equation in terms of $f(\eta)$:

\[ f'' + f' \left( \hat{S} + \hat{a} f' \right) + \left( -\hat{a} f' - \frac{1}{\hat{a} \hat{d}} \frac{d \hat{S}}{d \tau} \right) f' - \frac{1}{\hat{a} \hat{d} \hat{\zeta}} \frac{d \hat{\phi}}{d \tau} = 0 \quad (15) \]

where

\[ \frac{1}{\hat{a} \hat{d} \hat{\zeta}} \frac{d \hat{\phi}}{d \tau} = \frac{1}{\hat{a} \hat{d}} + \hat{a} \]

with Boundary conditions:

\[ \eta = 0: \quad f = 0, \quad f' = 0, \quad \eta \to \infty: \quad f' = 1 \quad (16) \]

Note, that in Eqs. (2) and (3), the $\frac{\partial}{\partial t}$ terms are not taken into account as the flow is assumed fully developed, however since the parameters $\hat{a}, \frac{d \hat{S}}{d \tau}$ and $\hat{S}$ are time-dependent Eq. (15) is also dependent on time. The ODE\(^1\) Eq. (15) was solved numerically using a shooting method trial and error based on the Runge-Kutta algorithm. For more explanations refer to Shokrgozar and Rahimi [16].

### 3.2. Heat Transfer Solution

Dimensionless temperature is defined as:

\[ \theta = \frac{T(\eta) - T_{\text{sub}}}{T_{\infty} - T_{\text{sub}}} \quad (17) \]

Making use of transformations (12)-(14), the energy equation Eq. (4) may be written as:

\[ \theta'' + Pr \theta' \left[ f + \hat{S} + \hat{S}_{\text{at}} \text{Evap} \right] = 0 \quad (18) \]

where,

\[ \text{Evap} = - \sum_{j=1}^{m} \frac{\dot{m}_{\text{air}} H_{\text{vj}}}{\sum_{j=1}^{m} \dot{m}_{\text{j}}} \left( b_2 + 2b_3 \theta + 3b_4 \theta^2 + 4b_5 \theta^3 \right) \]

Here, $\hat{S}_{\text{at}}$ dimensionless water vapor condensing velocity is defined as:

\[ \hat{S}_{\text{at}} = \hat{S} \times \frac{\rho_{\text{water}}}{\rho_{\text{vapor}}} \quad (19) \]

With boundary conditions as:

\[ \eta = 0: \quad \theta = 0, \quad \eta \to \infty: \quad \theta = 1 \quad (20) \]

---

\(^1\) Ordinary Differential Equation
where \( \theta \) is dimensionless temperature; the subscript \( \text{sub} \) and \( \infty \) refer to the conditions at the substrate and in the free stream, respectively, \( \text{Pr} = \frac{\nu}{\alpha} \) is Prandtl number and prime indicates differentiation with respect to \( \eta \). Again, the left term in Eq. (4), \( \frac{\partial \theta}{\partial \eta} \) is not taken into account, however since the \( u \) and v velocities are time-dependent Eq. (18) is also dependent on time.

4. Solution Approaches

The momentum equation Eq. (15) is solved numerically using a shooting method trial and error based on the fourth order Runge-Kutta algorithm. The velocity results are used in the energy equation Eq. (4) upon fluid region to convert this nonlinear equation into an ordinary one Eq. (18). The energy equation solution approach upon fluid region has been divided into two parts, exact solution, and numerical solution. In fact, the exact solution of the energy equation is the quasi-steady solution of the heat transfer equation and does not provide temperature profiles for all unsteady times; however, it is used to evaluate the numerical solution. In numerical solution, to solve the algebraic system of equations, TDMA\(^2\) within ADI\(^3\) method is used. The energy equation in the solid-fluid interface has been divided into two situations, solution before reaching the freezing point temperature Eq. (6), and after that Eq. (7). In the first situation, the energy equation terms quantities are dependent on relative humidity. However, the relationship between humidity ratio and temperature Eq. (8) is strongly nonlinear. The Newton-Raphson method for linearization is used in the case of relative humidity, which transforms the equation to linear one Eq. (9). In the second situation, the energy required for freezing the water is very smaller than that of the condensing latent enthalpy of vapor to water (about 1/7.5 times smaller). Thus, a big under relaxation factor is necessary for the latent enthalpy term in the energy equation to balance the energy between cells to prevent responses from divergence. While the method for solving the problem is implicit, trial and error is required with very small steps due to strong non-linearity. When the fluid temperature near the ice reaches the freezing point, the temperature of the solid-fluid interface remains constant, as a result, the temperature changes of fluid region are stopped and the energy equation in the solid-fluid interface is the same moving solidification boundary condition Eq. (7). The solution approaches of the solid-fluid interface equations are also numerical. All these equations have been solved at every step, simultaneously. Since the variation in the size of the mesh from 0.2 to 0.1mm result a negligible variation in the curve, \( T_{\text{sub,max}} \), it can be concluded that the mesh size of 0.2mm produces relatively accurate results.

5. Validation

The exact solution of momentum equations does not require validation. However, in this section, the numerical solution results of the energy equation are compared to the exact solution of this equation. In fact, the exact solution of the energy equation is the quasi-steady solution of the heat transfer equation, and does not provide temperature profiles for all unsteady times; however, it is used to evaluate the numerical solution for the last times of each step that the instantaneous temperature profile nears steady state. For better comparisons, parameter introduction in numerical and exact solutions are the same. Therefore, closing the temperature profiles together means validation of the numerical solution. The results of these comparisons are presented in the next section.

6. Presentation of Results

In Fig. 2, dimensionless velocity components \( \bar{u}, \bar{w} \) in the \( x, z \) directions are shown respectively; however, the velocities obtained in the solution of the energy equation are dimensional.

Figure (2)

Evolution of the fluid temperature profile to steady state is shown in Fig. 3 for far-field air temperature 20°C. For an infinity air temperature reaching the freezing point, the profile changes, and once the temperature arrives at the freezing point of water (zero degrees), the thermal profile becomes constant, and will remain unchanged until the ice layer is formed.

Figure (3)

As mentioned before, in order to validate the obtained heat transfer results, the exact solution of energy equation Eq. (18) is used. Nevertheless, the answers of the exact solution are useful only for the last times in each step nearing the steady state condition. In Fig. 4, the numerical and exact solutions of energy equations are compared together for far-field air temperature \( T_{\text{air}} = 20°C \). Here, there are good matches for these two diagrams. Note that the numerical solution data are at the end time of the steady state. Small differences between the values of numerical and exact solutions can be caused by linearization in the numerical solution in the humidity ratio term and accumulation of rounding errors.

\(^2\) Three Diagonal Matrix Algorithm
\(^3\) Alternating Direction Implicit
In Fig. 5, reduction of the vapor mass of saturated air versus reducing its temperature and increasing the water in the cell are represented for far-field temperature $T_{\text{air}} = 20^\circ\text{C}$. As expected, cell vapor mass in zero temperature approaches zero degree, thus, does not reach zero; however, from the point of $20^\circ\text{C}$, vapor begins to condensate, which is in fact a sign of saturation. Note, the amount of vapor in the air never exactly equals zero. In Fig. 6, humidity ratio with temperature changes is also shown.

One of the most important results in this paper is shown in Fig. 7 where the substrate temperatures needed to initiate freezing are provided for different far-field air temperatures. For example, for far-field air temperature $T_{\text{air}} = 5^\circ\text{C}$, maximum temperature of substrate should be lower than $T_{\text{sub},\text{max}} = -0.32^\circ\text{C}$ to start freezing, and for far-field air temperature $T_{\text{air}} = 20^\circ\text{C}$, substrate temperature should be lower than $T_{\text{sub},\text{max}} = -0.80^\circ\text{C}$. However, substrate temperature nears zero degree by decreasing the far-field temperature while decreasing water vapor mass in the air. The behavior in the Fig. 7 is generally plausible as suggested by the simple analysis that follows. Assume the viscous layer thickness is $\delta$ and $y$ is the distance from the substrate into the gas. The temperature $T$ profile in the viscous layer is simply represented as,

$$T - T_{\text{sub},\text{max}} = \frac{1}{f} y \delta$$

where $f$ is a variable factor. Since the amount of vapor in a saturated gas roughly doubles for each $15^\circ\text{C}$ temperature increase, a variable factor $f$ between $1/8$ – $1/3$ is plausible. The $y$ value of interest can be taken as a fraction, $1/N$, of the thermal viscous layer thickness; for a location in the middle of the cell next to the substrate, $N = 150$ where the computational cell size is $1/75$ the thermal viscous layer thickness. For freezing at the computational node nearest the substrate, Eq. (22) gives:

$$T_{\text{sub},\text{max}} = \left[15(N-1)\right]T - T_{\infty} / (15f)$$

For $0^\circ\text{C}$ subcooling of the vapor ($T = 0^\circ\text{C}$), Eq. (23) gives:

$$T_{\text{sub},\text{max}} = -T_{\infty} / (150f - 1)$$

If $f = \frac{1}{8}$ then $T_{\text{sub},\text{max}} \cong -T_{\infty}/18$, this result is in good agreement with the Fig. 7 for $T_{\infty}$ less than $15^\circ\text{C}$. For larger $T_{\infty}$, the effect of $1^\circ\text{C}$ subcooling of the vapor before condensation ($T = -1^\circ\text{C}$) and $N = 150$ in Eq. (23) results in:

$$T_{\text{sub},\text{max}} \cong \frac{-1 - T_{\infty}}{150f}$$

Thus, vapor subcooling would shift the result of Eq. (24) downward. Notice, since the variation in the size of the mesh from 0.2 to 0.1mm result a negligible variation in the $T_{\text{sub},\text{max}}$ (Fig. 7), it can be concluded that the mesh size of 0.2mm produces relatively accurate results.

The interesting and controversial point here is freezing time depicted in Fig. 8. According to this Figure, time increases linearly by moving from the far-field temperature $T_{\text{air}} = 50^\circ\text{C}$ to $T_{\text{air}} = 30^\circ\text{C}$, and then the time increment slope declines slowly. Thus, when the far-field temperature passes about $T_{\text{air}} = 5^\circ\text{C}$, time declines steeply toward zero. But why? Time condensation and freezing water vapor in the air is a function of two variables: far-field air temperature and water vapor content in the air. Of course, one cannot expect that these two phenomena are linear as the humidity ratio in the air is nonlinear. So far, for close to zero degrees Celsius far-field temperature, vapor and consequently the amount of water available to freeze is severely reduced. Therefore, the time required for condensation and freezing suddenly inclines toward zero. The behavior of the curve in Fig. 8 can be similarly explained. In the start-up transient, the thickness of the thermal viscous layer increases as the square root of elapsed time as:

$$\delta = K \sqrt{t}$$

Again, assume the thermal viscous layer thickness is $\delta$, which has the value $\delta_{ss}$ in the steady state. This is because a perpendicularly impinging flow results in a viscous layer of constant thickness on the substrate. Thus, the initial thermal disturbance is like that of a semi-infinite solid of initially uniform temperature whose surface temperature is suddenly changed, (Carslaw and Jaeger [33]). The Eq. (26) is also can be obtained by the scaling analysis in energy equation. For the
present problem, this transient ceases when the thermal viscous layer achieves its steady state value. The temperature distribution, here taken to be linear for simplicity, in Eq. (22), setting \( y = \frac{\delta_y}{N} \) as before and making use of the relationship of Eq. (26) results in:

\[
t = \left( \frac{\delta_y}{NK} \right)^2 \left( \frac{1}{f} \right)^2 \left( \frac{T_{sc} - T_{sub}}{T - T_{sub}} \right)^2 , \quad t < t_{ss}
\]

(27)

where \( (\frac{\delta_y}{NK}) \) is constant. Presuming that condensation begins at \( T = 0^\circ C \), no subcooling, and with substrate temperature \( T_{sub} = -1^\circ C \), Eq. (27) becomes:

\[
t = \left( \frac{\delta_y}{NK} \right)^2 \left( \frac{1}{f} \right)^2 \left( T_{sc} + 1 \right)^2 , \quad t < t_{ss}
\]

(28)

Equation (28) is a parabolic increase of \( t \) with increasing values of \( T_{sc} \), the same trend shown in the early going in Fig. 8, but with a constant value predicted after the thermal viscous layer is fully developed and with a constant value of \( \frac{1}{f} \). Remember that in the first region, \( T_{airsc} < 5^\circ C \) of Fig. 8, the factor \( (\frac{1}{f})^2 \) is about 64, however after this region it tends to 9. The effect of 0.5\(^\circ\)C subcooling (\( T = 0.5^\circ C \)) of the vapor before condensation and \( N = 150 \) in Eq. (27) results in:

\[
t = 4 \left( \frac{\delta_y}{NK} \right)^2 \left( \frac{1}{f} \right)^2 \left( T_{sc} + 1 \right)^2 , \quad t < t_{ss}
\]

(29)

The prediction of Eq. (29) is that 0.5\(^\circ\)C subcooling of the vapor before condensation increases the time lapse before condensation begins by a factor of 4. Since factor \( f \) is dependent on the amount of vapor in a saturated gas, after the first region, \( T_{airsc} > 5^\circ C \), \( (\frac{1}{f})^2 \) decreases faster than increasing of \( (\frac{T_{sc} - T_{sub}}{T - T_{sub}})^2 \), so the total time is reduced. Note that in a physical experiment, condensation should immediately begin at the substrate surface, however in numerical calculations, the size of the nearest cell to the substrate controls the time at which condensation begins because the low temperature imposed by the substrate reaches the center of that cell by diffusion. So computations are repeated with the cell size 0.01 mm and 0.02 mm in order to demonstrate this since the time to beginning of condensation should vary as the inverse square of cell size, according to Eq. (29). Therefore, the results in Fig. 8 show the trends of changes in the freezing start time versus far-field air temperature for two different mesh sizes. But then again, what will be happen if the temperature of the substrate (\( T_{sub} \)) is lower than that the temperature to start freezing (\( T_{sub,max} \))?

Figure (8)

In Fig. 9a, results of substrate temperature (\( T_{sub} \)) 10\(^\circ\)C lower than maximum temperature of the substrate for start of freezing (\( T_{sub,max} \)) for far-field temperature \( T_{airsc} = 5^\circ C \) are represented and Fig. 9b is the same for \( T_{airsc} = 20^\circ C \). As expected, the lower temperature substrate rises the thickness of the ice. Moreover, comparing between Figures 9a and 9b show that the same reduction of substrate temperature (10\(^\circ\)C) in the lower far-field temperature (\( T_{airsc} = 5^\circ C \)) exhibits a higher increase of ultimate ice thickness (about 50 mm) than the higher far-field temperature (\( T_{airsc} = 20^\circ C \)) that is about 12 mm.

Figure (9a), (9b)

7. Conclusions

In the present paper, saturated air water freezing in two-dimensional stagnation flow on a flat plate is investigated. Air with a relative humidity of 100\% vertically approaches toward the cooled flat plate. At first, water vapor condensation and then if the substrate is cold enough, solidification will occur. The first important result is the highest substrate temperature (\( T_{sub,max} \)) which the saturated vapor at that temperature becomes water and starts to freeze. These substrate temperatures have been provided for different far-field air temperatures (\( T_{airsc} \)). In addition, the time required for freezing the first row of cells has been presented. According to the obtained results, by increasing far-field air temperature (\( T_{airsc} \)) from 0\(^\circ\)C to about 5\(^\circ\)C, solidifying time increases, and then by more increasing it (\( T_{airsc} \)), solidifying time decreases. The temperature profiles, velocities in both directions and water quantity changes in the cells are also represented. Fluid temperature distribution, and most importantly, the ultimate frozen thickness for two different temperatures of far-field air and temperatures of substrate are also presented. The charts with a substrate temperature below the freezing start temperature indicate that a 10\(^\circ\)C drop in
substrate temperature causes a large increase in the ultimate thickness of the ice for far-field air at $T_{\text{air}} = 5^\circ\text{C}$, while the same degree drop slightly increases the ultimate thickness of the ice for the air at $T_{\text{air}} = 20^\circ\text{C}$.

6. References

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A brief technical biography of each author:

A. Dr. Ali Shokrgozar Abbasi
   Ali Shokrgozar Abbasi (1970), got his Ph.D. from Mechanical Engineering Department, Ferdowsi University of Mashhad (2010) in Iran. Main field of his study was solidification in stagnation-flow. During his Ph.D. he established a three-dimensional computer program to predict the flow, temperature and solidification of a fluid in stagnation-flow. In 2011 he joined as assistant professor to Payam Noor University of Mashhad, teaching more on CFD, advanced heat transfer, advanced numerical calculations and engineering mathematics. He works on techniques in computer programs of modeling process in heat and fluid flow with phase change. There are also analytical and experimental methods in his works. His main interests are solidification, phase change to liquid, heat and fluid flow. He has published about 10 ISI international Journal papers and a book, “Convective heat transfer”.

B. Mr. Mohsen Ghayeni,
   Mohsen Ghayeni (1990), got his bachelor's degree in Mechanical Engineering from Birjand University and is now a graduate student at the Mechanical Engineering Department at Payam Noor University in Mashhad. He is interested in freezing and melting, heat and fluid flow.

Figure and table captions:
Figure (1): Strain (Problem schematic diagram)
Figure (2): Dimensionless velocity component $\bar{u}, \bar{w}$ profiles in x and z directions, respectively
Figure (3): Temperature evolutions from unsteady to steady for $T_{air} = 20^\circ C$ ($T_{air}$ means $T_{air}$ at the nearest cells to ice)
Figure (4): Comparison of temperature profiles of numerical and exact solutions ($T_{\text{air}} = 20^\circ\text{C}$)
Figure (5): Variations in vapor and water masses in the cells for far-field saturated air $T_{\text{air}} = 20^\circ\text{C}$
Figure (6): Humidity ratio versus air temperature for far-field saturated air at $T_{\text{air}} = 20^\circ\text{C}$
Figure (7): Maximum temperature of substrate to start freezing versus far-field air temperature
Figure (8): Total time needed to initiate vapor freezing in the nearest cells to the substrate surface versus far-field air temperature for two different mesh sizes.
Figure (9a): Air and ice temperature profiles by reducing 10°C of start of freezing temperature
($T_{\text{air}} = 5^\circ\text{C}$, $T_{\text{sub}} = -10.32^\circ\text{C}$, $T_{\text{sub,max}} = -0.32^\circ\text{C}$)
Figure (9b): Air and ice temperature profiles by reducing 10°C of start of freezing temperature
($T_{\text{air}} = 20^\circ\text{C}$, $T_{\text{sub}} = -10.81^\circ\text{C}$, $T_{\text{sub,max}} = -0.81^\circ\text{C}$)

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\( T_{air} = 5°C, \ T_{sub} = -10.32°C, \ T_{sub,max} = -0.32°C \)
Figure (9b): Air and ice temperature profiles by reducing 10°C of start of freezing temperature
($T_{\text{air,\infty}} = 20^\circ\text{C}, \ T_{\text{sub}} = -10.81^\circ\text{C}, \ T_{\text{sub,\max}} = -0.81^\circ\text{C}$)